MOAT REGIMES & THEIR SIGNATURES IN HEAVY-ION COLLISIONS

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- HFHF THEORY RETREAT -

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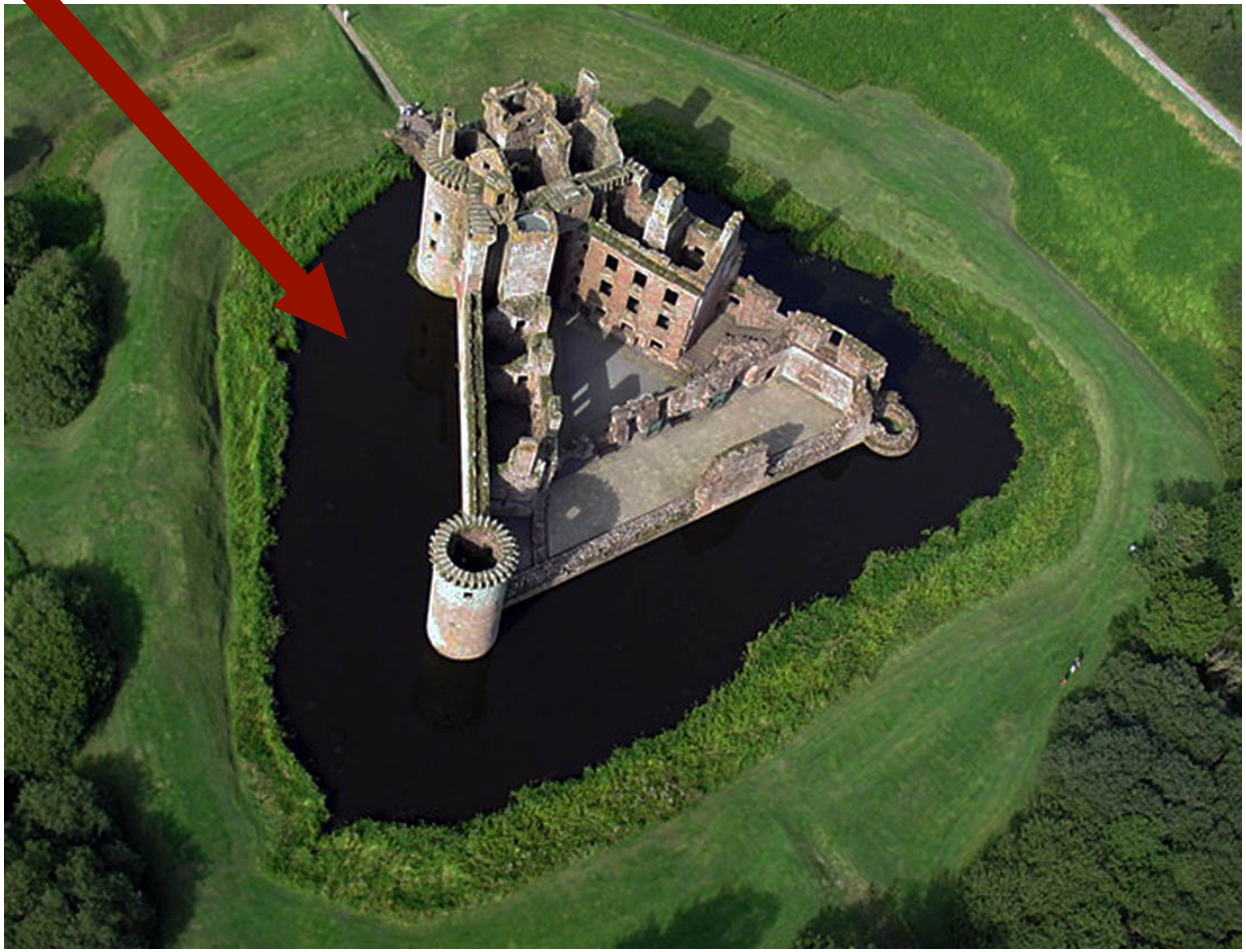
A MOAT

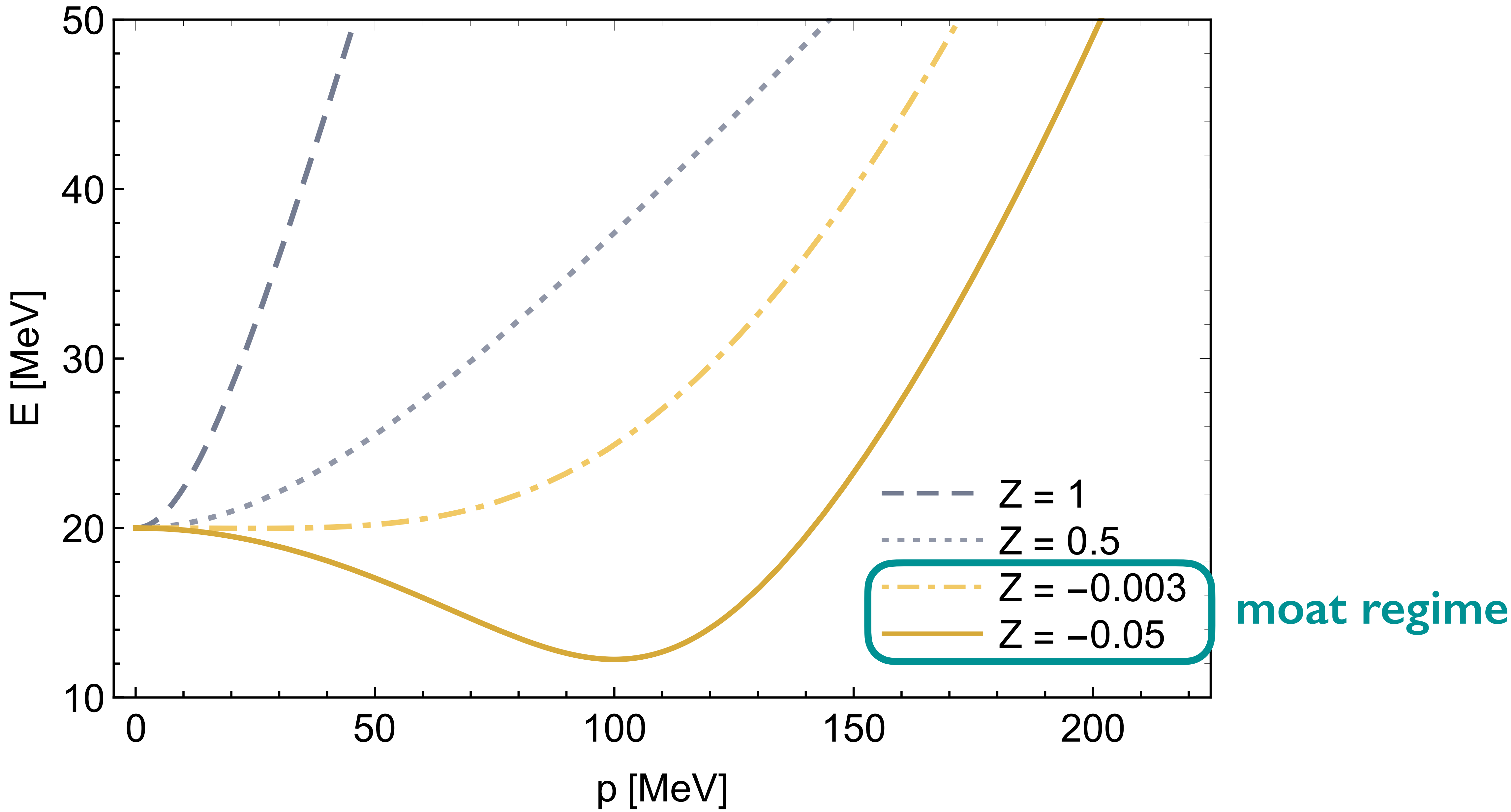


[Caerlaverock Castle, Scotland (source: Wikipedia)]

A MOAT

energy dispersion of particle ϕ:

$$E_\phi(\mathbf{p}^2) = \sqrt{Z\,\mathbf{p}^2 + W\left(\mathbf{p}^2\right)^2 + m^2}$$

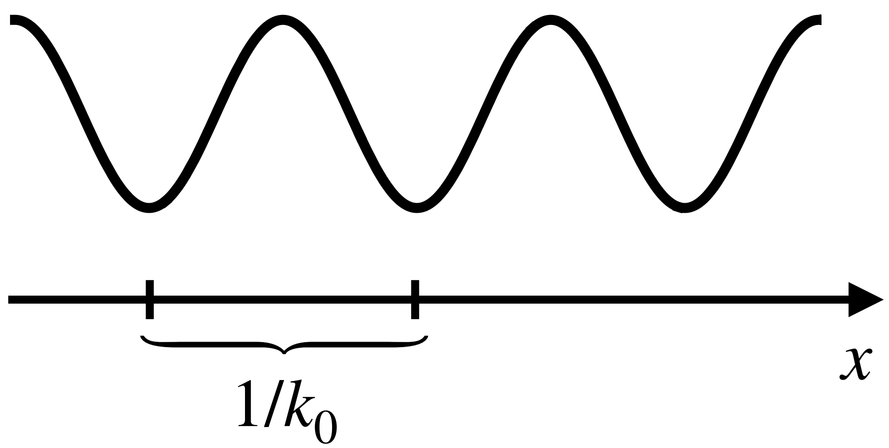


→ particles are favored to have a nonzero momentum
"gain energy by going faster"

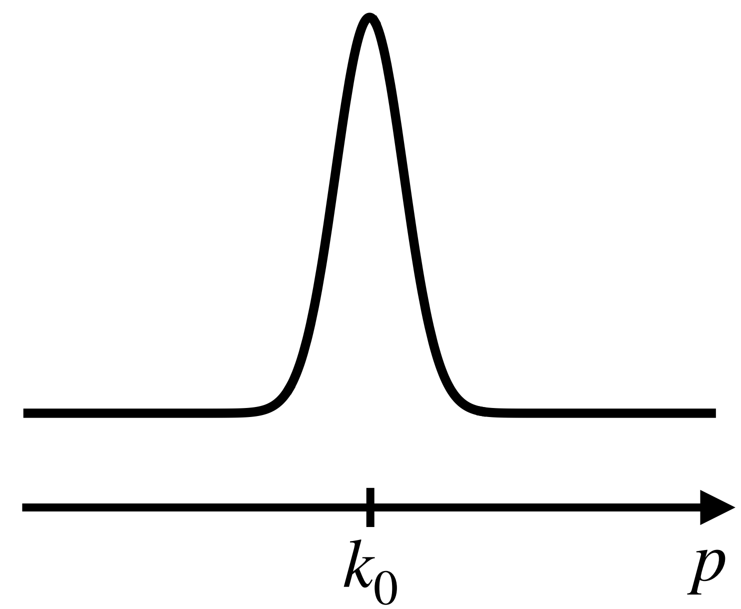
WHERE DOES THE MOAT COME FROM?

heuristic picture:

spatial oscillation
$\cos(2\pi k_0 x)$



momentum space peak
$\delta(p - k_0)$



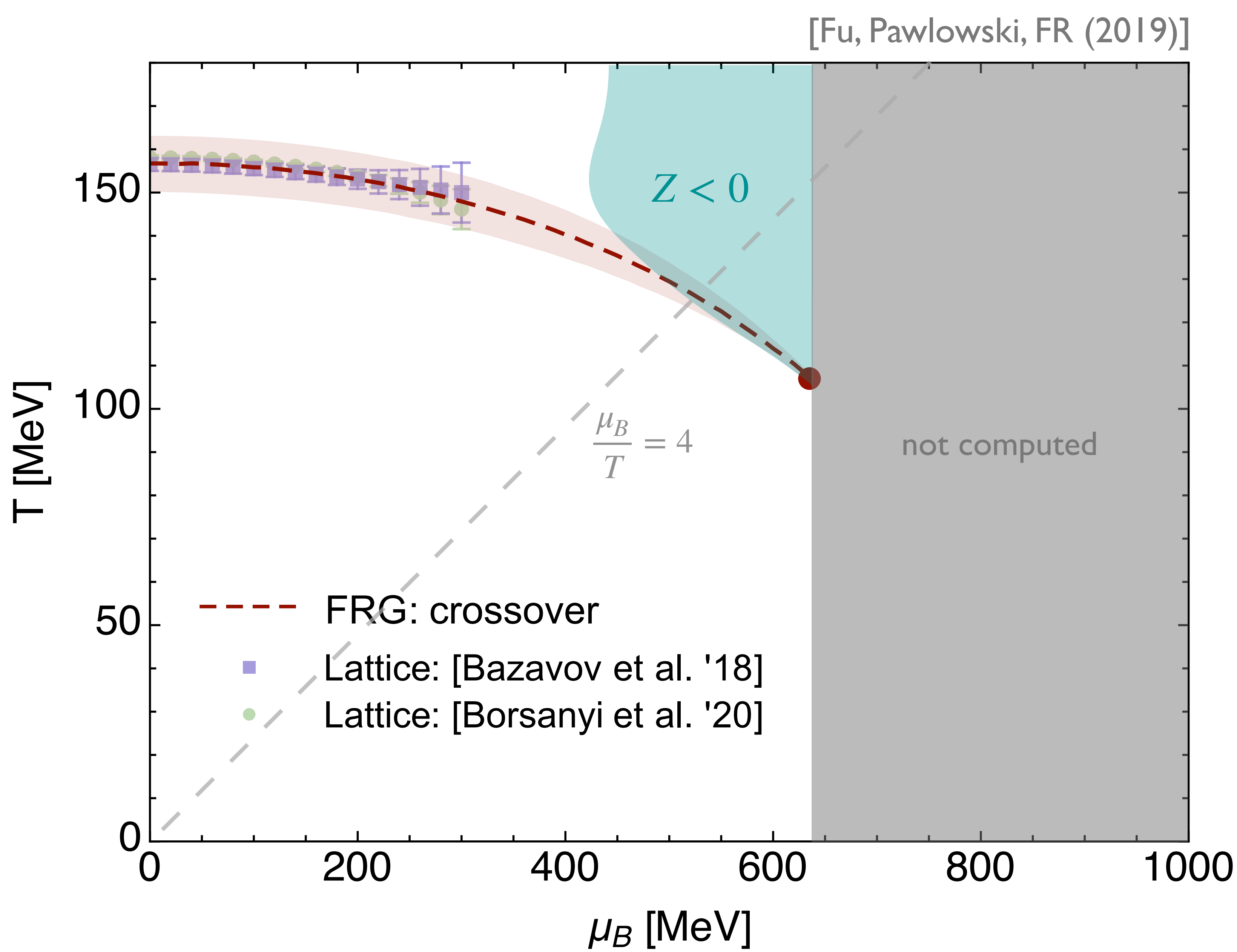
- particles subject to a spatial modulation are favored to have momentum k_0

$\longrightarrow$ moat energy dispersion
(minimal energy at k_0) $k_0^2 = -Z/(2W)$

- typical for inhomogeneous/crystalline phases or a quantum pion liquid (QπL)

WHERE CAN MOAT REGIMES APPEAR?

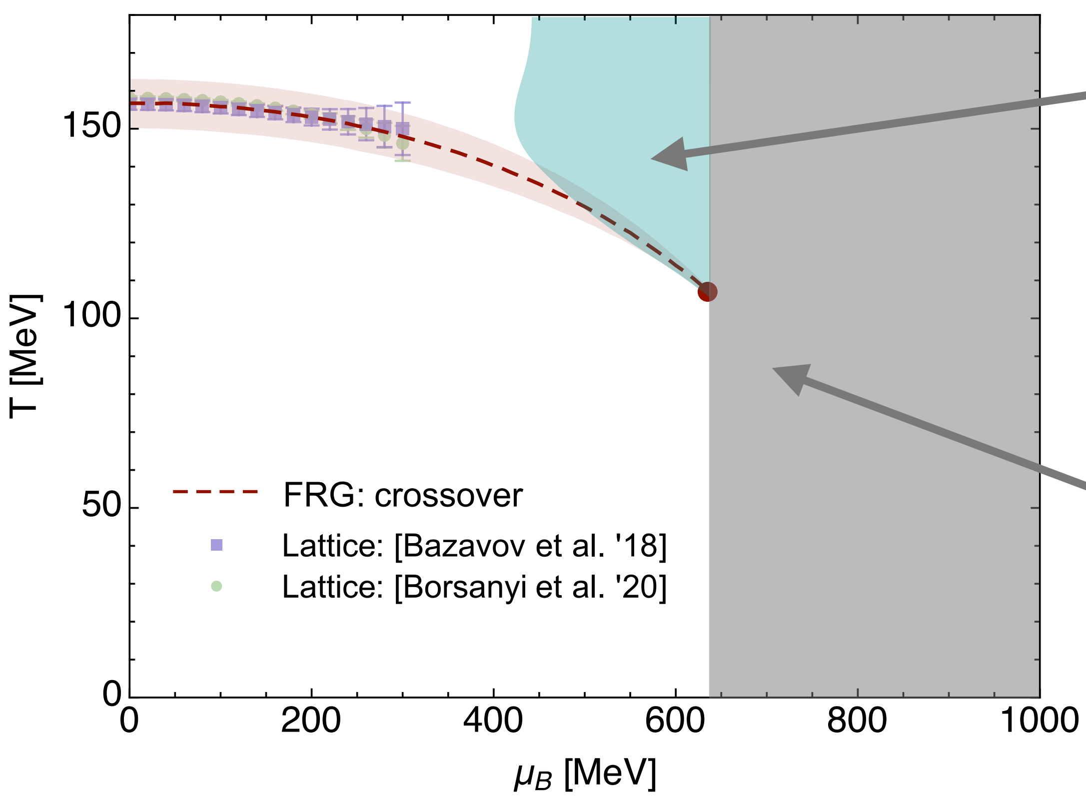
Expected at large μ. Also QCD phase diagram?!

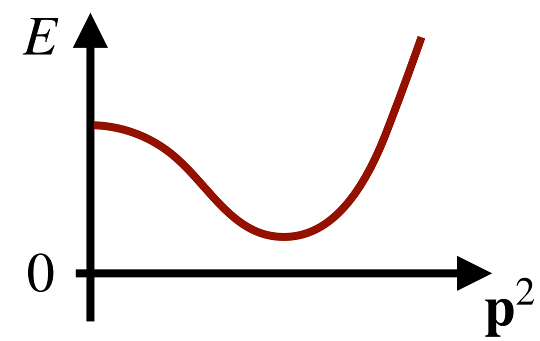


→ indication for extended region with $Z < 0$ in QCD: **moat regime**

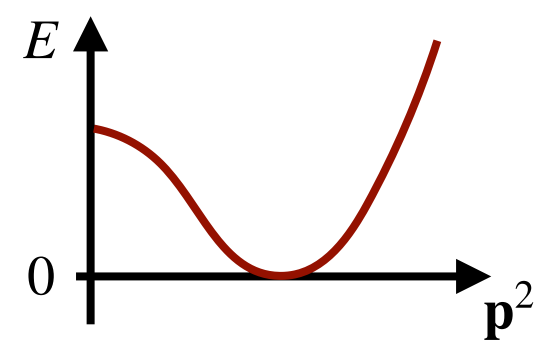
IMPLICATIONS OF THE MOAT

The energy gap might close at lower T and larger μ_B :





$E > 0$ for all $\mathbf{p}^2$



$E = 0$ at $\mathbf{p}^2 > 0$:

→ instability towards formation of an **inhomogeneous condensate**

INHOMOGENEOUS PHASE

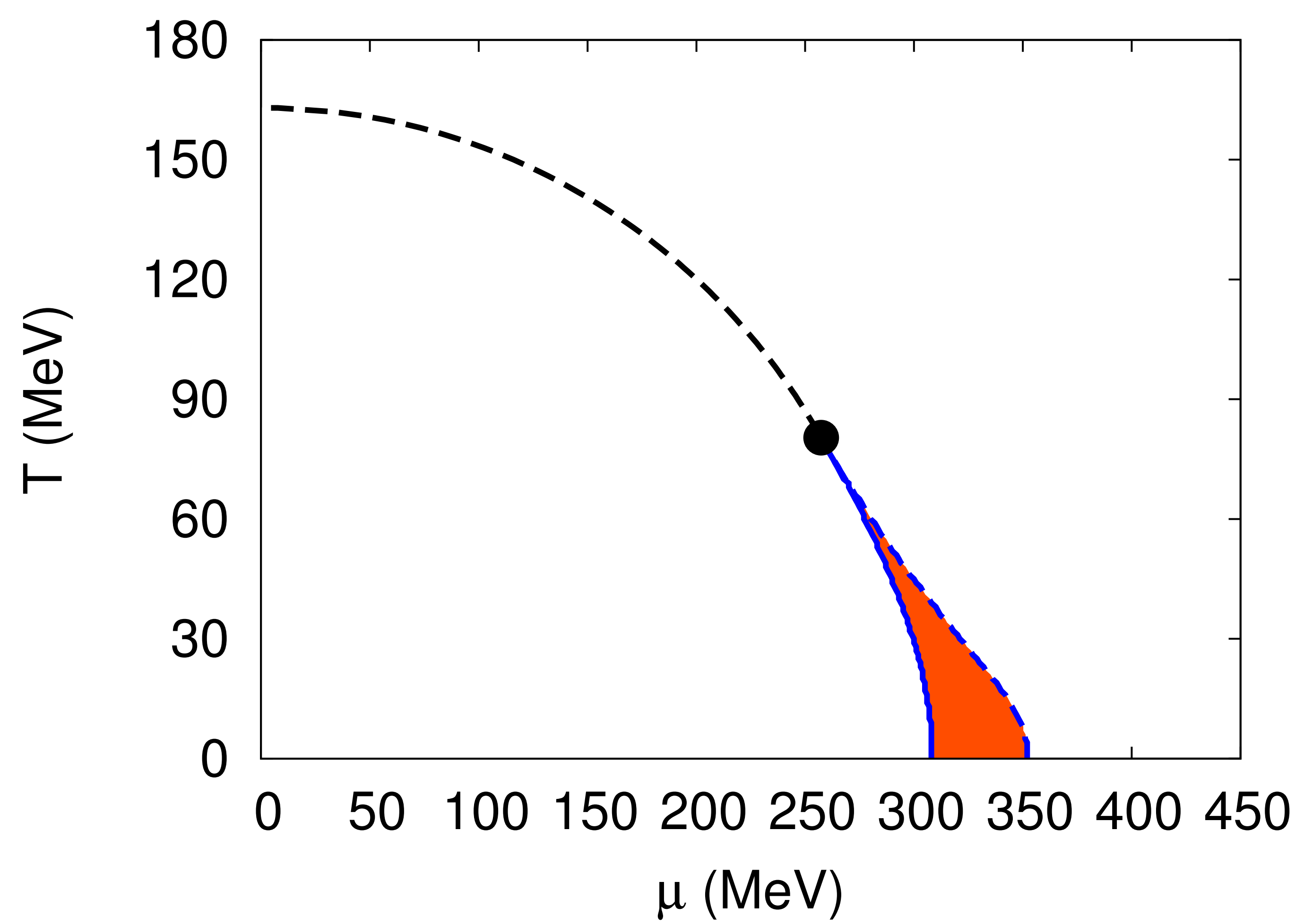
emerges if energy gap closes

- $E_\phi(k_0^2) = 0$: particles with momentum k_0 condense
- basic example: $O(N)$ chiral spiral

field condensate/VEV

$$\phi = \begin{pmatrix} \sigma \\ \pi_{N-1} \\ \pi_{N-2} \\ \vdots \\ \pi_1 \end{pmatrix}, \quad \phi_0 = \Delta \begin{pmatrix} \cos(k_0 z) \\ \sin(k_0 z) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

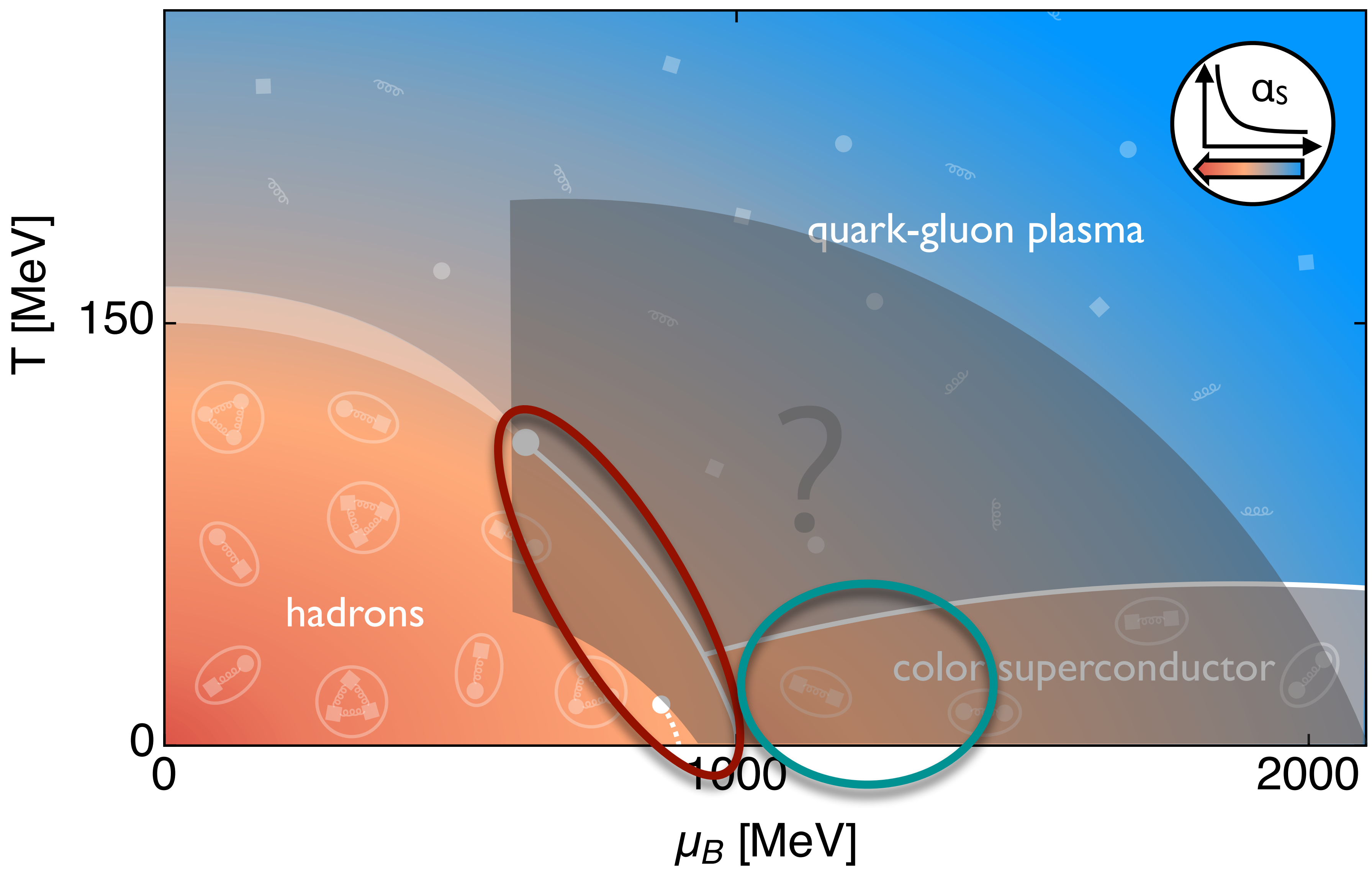
mean-field phase diagram



[Carignano, Buballa, Schaefer '14]

IMPLICATIONS OF THE MOAT

option 1: moat is a precursor for an inhomogeneous phase



possibilities: inhomogeneous chiral condensate or crystalline CSC

INHOM. PHASES & FLUCTUATIONS

Inhomogeneous phases are mostly studied in mean-field.

But associated spontaneous symmetry breaking gives rise to massless modes.

Their fluctuations must be relevant!

Two types of symmetry breaking for inhomogeneous phases:

- continuous spatial symmetries (rotations, translations) broken down to discrete ones
- global flavor symmetries are broken (e.g. $O(N) \to O(N-2)$ for chiral spiral)

SPATIAL SYMMETRY BREAKING

It has been argued that 1d modulations are favored against higher-dimensional ones

[Abuki, Ishibashi, Suzuki '12]
[Carignano, Buballa '12]

Goldstone bosons from spatial symmetry breaking (e.g. phonons) lead to Landau-Peierls instability of 1d inhomogeneous condensates (e.g. chiral spiral)

- Goldstone fluctuations lead to **logarithmic IR divergences**

  $\longrightarrow$ 1d condensate is destroyed; the system is disordered

- algebraically instead of exponentially decaying correlations still possible

  $\longrightarrow$ quasi-long-range order (e.g. liquid crystals)

[Landau, Lifshitz, Stat. Phys. I, §137]
[Lee, Nakano, Tsue, Tatsumi, Friman '15]

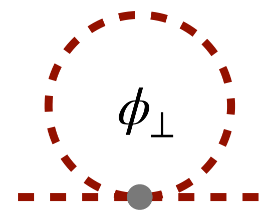
Option 2: moat is a precursor for a liquid-crystal-like phase

FLAVOR SYMMETRY BREAKING

even "worse" for fluctuations of Goldstones from broken flavor symmetry

- basic example: fluctuations around $O(N)$ chiral spiral

$$\phi = \Delta \begin{pmatrix} \cos(k_0\, z) \\ \sin(k_0\, z) \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \delta\phi_{\parallel} \\ \delta\phi_{\perp} \end{pmatrix} \longrightarrow$$



$$\sim \frac{T}{W}\, k_0^{d-3} \int_{|\mathbf{p}|\sim k_0} \frac{d\,|\mathbf{p}|}{(\,|\mathbf{p}| - k_0)^2}$$

- transverse fluctuations lead to linear IR divergences at finite T in any dimension

$\longrightarrow$ $\delta\phi_{\perp}$ disorders the system: **no inhomogeneous phase for $N > 2$**
not even quasi-long-range order (rigorous for $O(N)$ chiral spiral at $N \to \infty$)

instead, there is a **quantum pion liquid**

- disordered phase with a moat spectrum ($E > 0$ for all $\mathbf{p}^2$)
- spatial modulations: $\langle \phi(x)\phi(0)\rangle \sim e^{-m_r\, x} \cos(m_i\, x)$ for large x

[Pisarski, Tsvelik, Valgushev '20]
[Pisarski '21]

Option 3: moat signals a quantum pion liquid

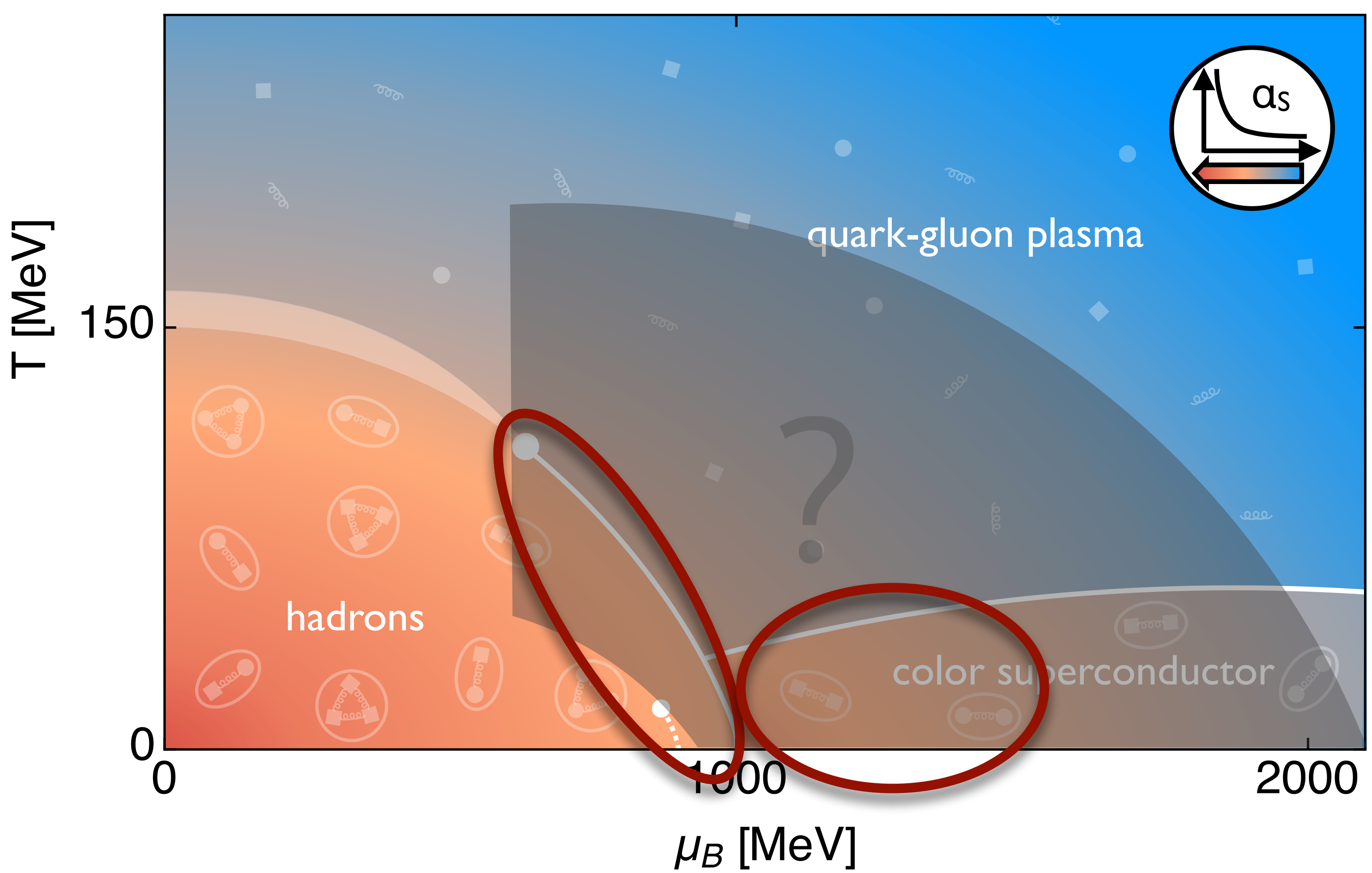
IMPLICATIONS OF THE MOAT

the moat regime could be an indication that dense QCD has:

option 1:
inhomogeneous phase
- only if there are no Goldstone bosons

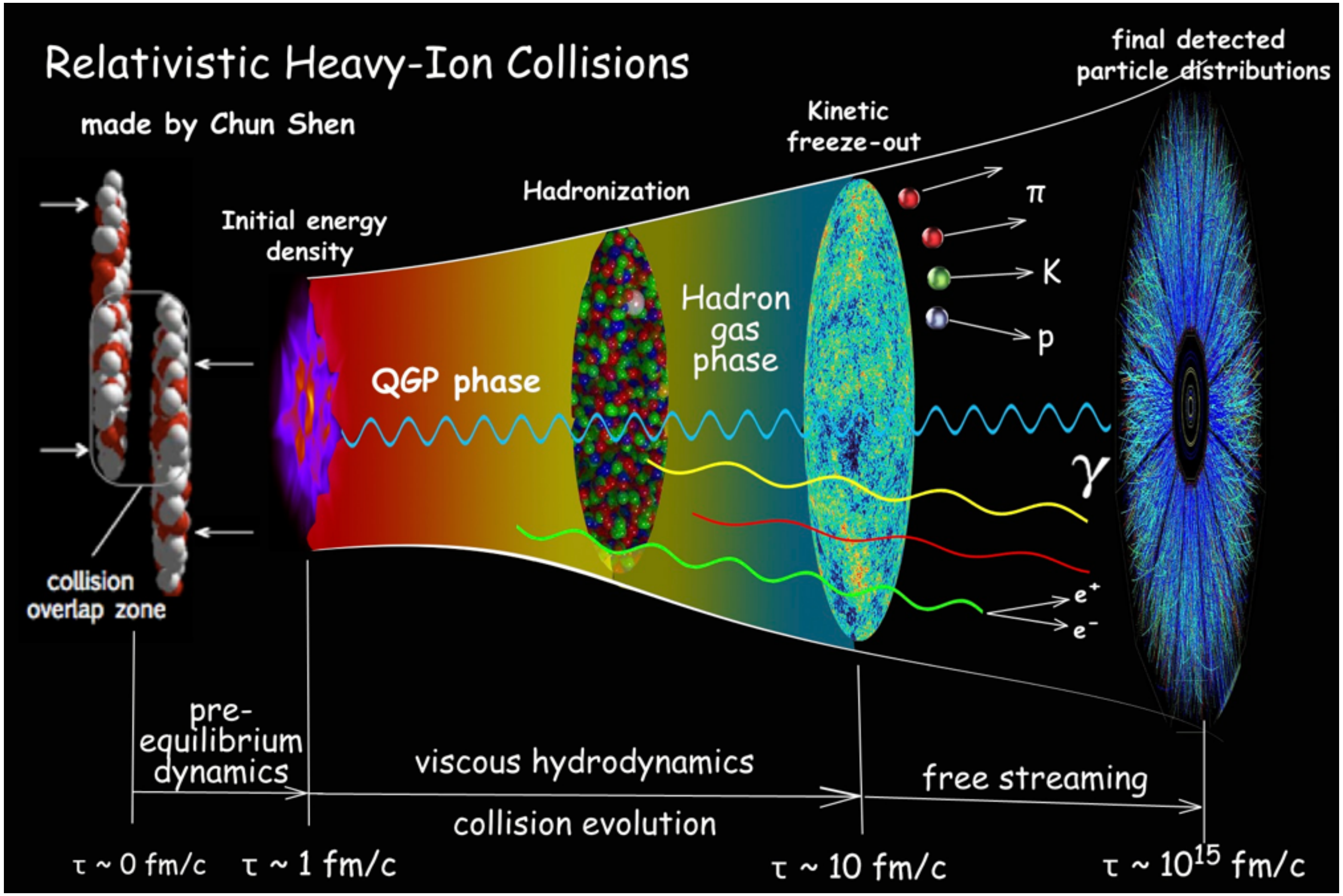
option 2:
liquid-crystal-like
- only if there are only Goldstones from spatial symmetry breaking from 1d condensates

option 3:
quantum pion liquid
- only if there are Goldstones flavor symmetry breaking



this will occur in the regions where inhomogeneous phases are expected

PROBING THE PHASE DIAGRAM



→ imprints of the phase structure at freeze-out?

SEARCH FOR MOAT REGIMES [Pisarski, FR '21]

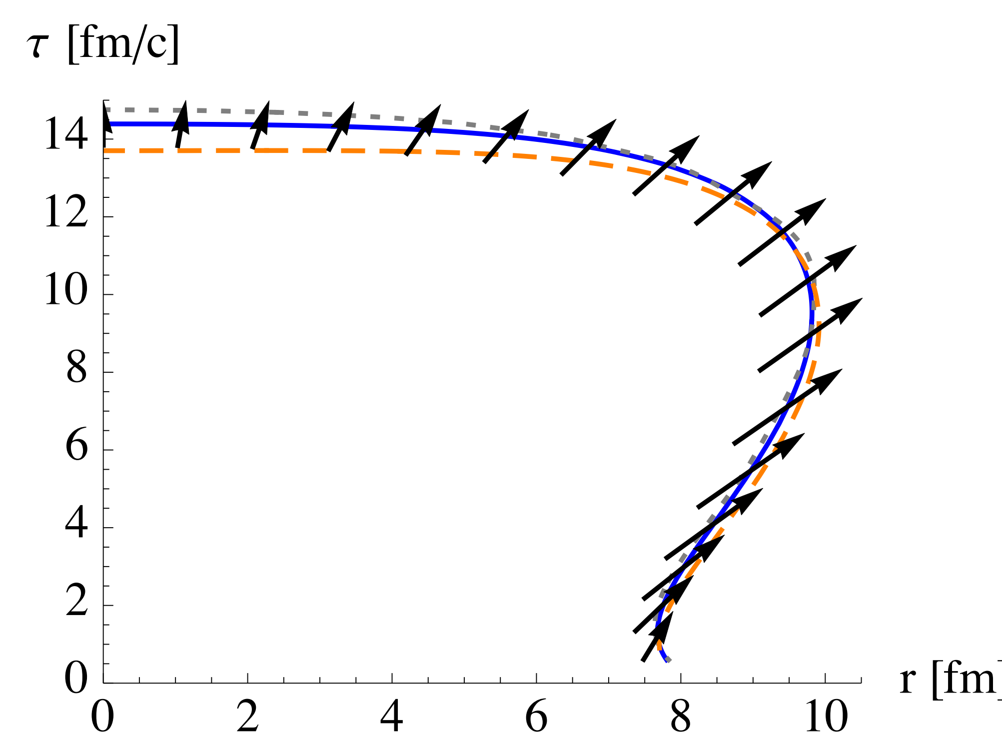
Characteristic feature: minimal energy at nonzero momentum

$\Rightarrow$ **enhanced particle production at nonzero momentum**

$\rightarrow$ look for signatures in the **momentum dependence** of particle numbers and correlations

- consider heavy-ion collision
- particles at freeze-out "mapped" onto detector
- freeze out at certain temperature T_f

$\longrightarrow$ defines 3d hypersurface: freeze-out surface Σ



[Floerchinger, Wiedemann '13]

How does the moat regime affect particles on Σ?

PARTICLE PRODUCTION

How does a moat regime affect particle production?

- study particle numbers and correlations, e.g.,

single-particle spectrum

$$(2\pi)^3 E_{\mathbf{p}} \frac{dN_1}{dp^3} = E_{\mathbf{p}} \langle a^\dagger_{\mathbf{p}} a_{\mathbf{p}} \rangle$$

two-particle spectrum: particle number correlation

$$(2\pi)^6 E_{\mathbf{p}} E_{\mathbf{q}} \frac{dN_2}{dp^3 dq^3} = E_{\mathbf{p}} E_{\mathbf{q}} \langle a^\dagger_{\mathbf{p}} a^\dagger_{\mathbf{q}} a_{\mathbf{p}} a_{\mathbf{q}} \rangle$$

- possible sources for correlations: thermodynamic fluctuations, critical fluctuations, interference (HBT) and all sorts of interactions

⟶ develop unified framework to study all this (work in progress)

- if particle number is not conserved, it can only be defined 'asymptotically' (quasi-particles). Then, e.g., for a real scalar field:

on-shell: $\bar{p}_0 = E_{\mathbf{p}}$

$$\sqrt{2\,\bar{p}_0}\, a_{\mathbf{p}} = i \int d^3x\, e^{i\bar{p}x} \left(\partial_{x_0} - i\bar{p}_0\right) \phi(x)$$

- cf. LSZ reduction, but here x_0 could be any time where a quasi-particle picture applies (not necessarily $x_0 = \pm\,\infty$)

⟶ $\bar{p}_0 = E_{\mathbf{p}}$ might differ from free dispersion

GENERALIZED COOPER-FRYE FORMULA

- QFT expression of (mixed) single-particle spectrum (relevant for HBT; work in progress)

$$\sqrt{\bar{p}_0\,\bar{q}_0}\,\langle a^\dagger_{\mathbf{p}} a_{\mathbf{q}}\rangle = \frac{1}{2(2\pi)^3}\int d^3X\, e^{i\bar{P}X}\int\frac{dQ_0}{2\pi}\left[\frac{1}{4}\partial^2_{X_0} - \frac{i}{2}P_0\partial_{X_0} + \left(Q_0+\bar{Q}_0\right)^2 - \frac{1}{4}\bar{P}_0^2\right]\left[F(X,Q) - \frac{1}{2}\rho(X,Q)\right]$$

relative momentum $P = p - q$ average momentum $Q = (p+q)/2$

- Wigner-transformed two-point functions

spectral function: $$\rho(X,Q) = \int d^4Y\, e^{iQY}\left\langle\left[\phi\left(X+\frac{1}{2}Y\right),\phi\left(X-\frac{1}{2}Y\right)\right]\right\rangle$$

statistical function: $$F(X,Q) = \int d^4Y\, e^{iQY}\left\langle\left\{\phi\left(X+\frac{1}{2}Y\right),\phi\left(X-\frac{1}{2}Y\right)\right\}\right\rangle$$

- **assume** local thermal equilibrium + only relative position matters: $F(Q) \approx \left[\frac{1}{2} + f(Q)\right]\rho(Q)$ e.g. $f = n_B$

- particles on (freeze-out) surface Σ move with fluid velocity u^μ, described by current N_μ with $N = u^\mu N_\mu$; set $p = q$

$\longrightarrow$ generalized Cooper-Frye formula:

$$\breve{\bar{Q}}_0\frac{dN_1}{dQ^3} = \frac{1}{2(2\pi^3)}\int d\Sigma^\mu\int\frac{dQ_0}{2\pi}\left(Q_\mu+\bar{Q}_\mu\right)\left(\breve{Q}_0+\breve{\bar{Q}}_0\right)f\left(\breve{Q}_0\right)\rho\left(\breve{Q}\right)$$

boosted momenta:
$\breve{Q}_0 = u^\mu Q_\mu$
$\breve{\mathbf{Q}}^2 = \left(u^\mu u^\nu - g^{\mu\nu}\right)Q_\mu Q_\nu$

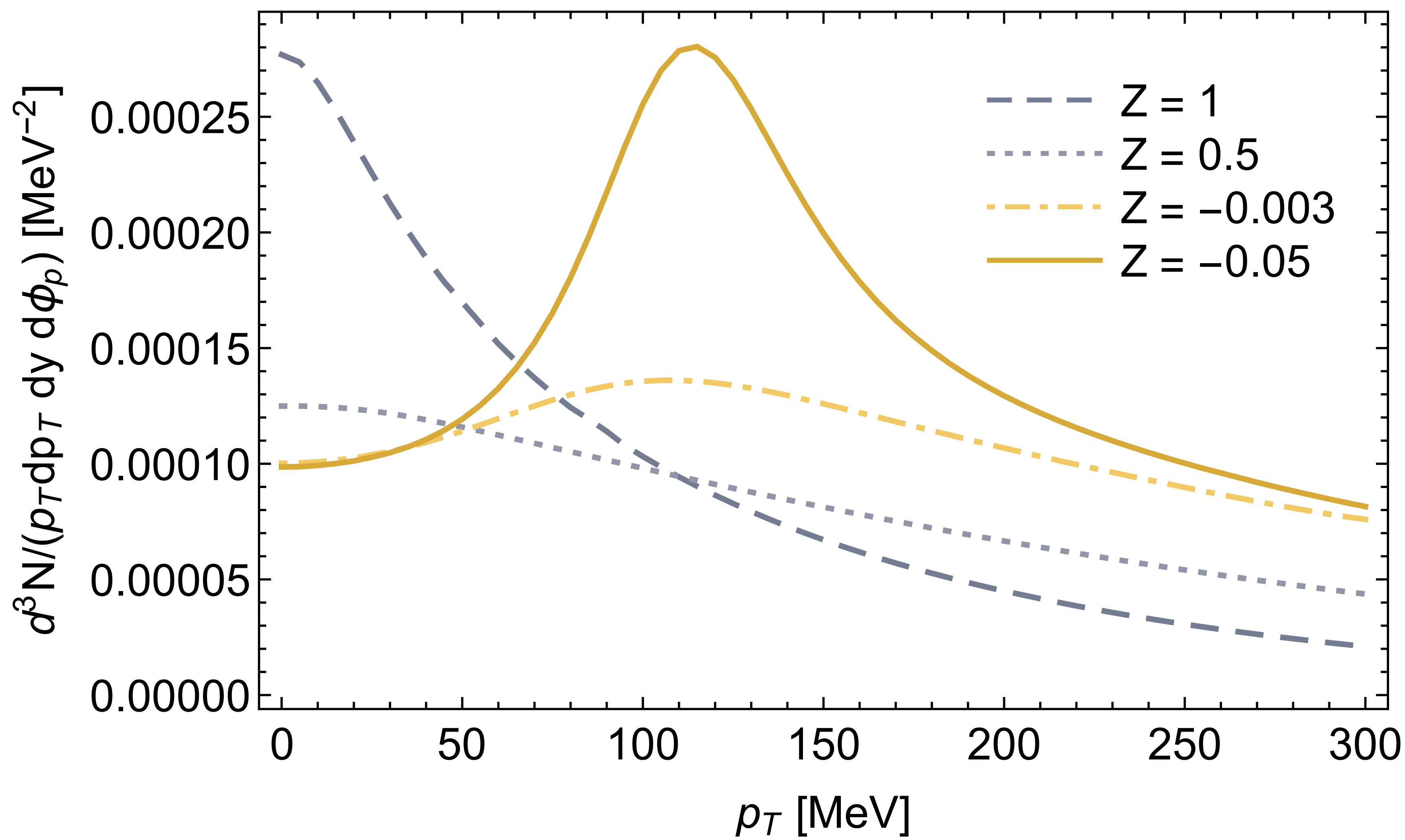
PARTICLE SPECTRUM IN A MOAT REGIME

transverse momentum spectrum

[Pisarski, FR (2021)]

- use simple models for illustration (quasi-particle in moat regime, boost-inv. and transverse isotropic freeze-out at fixed proper time, blast wave fluid velocity)
- compare normal phase (gray, $W = 0$) to moat phase (yellow, $W = 2.5\,\mathrm{GeV}^{-2}$)

$$\frac{dN_1}{p_T\, dp_T\, dy\, d\phi_p}$$



enhanced particle production at nonzero momentum!

maximum related to the wavenumber of the spatial modulation

PARTICLE NUMBER CORRELATIONS

- correlations sensitive to in-medium modifications
- consider only thermal fluctuations for now (no HBT, interaction effects etc.)
- moat regime is disordered: single particle distributions can capture relevant features

→ **correlations on Σ from generalized Cooper-Frye formula**

[Pisarski, FR (2021)]
[Floerchinger et al. (2022)]

n-particle correlation:

$$\left\langle \prod_{i=1}^{n} \breve{Q}_i^0 \frac{d^3N_1}{dQ_i^3} \right\rangle = \left[\prod_{i=1}^{n} \frac{1}{2(2\pi)^3} \int d(\Sigma_i)_\mu \int \frac{dQ_i^0}{2\pi} \left(Q_i^\mu + \bar{Q}_i^\mu\right)\left(\breve{Q}_i^0 + \breve{\bar{Q}}_i^0\right) \right] \left\langle \prod_{i=1}^{n} f(\breve{Q}_i^0)\, \rho(\breve{Q}_i) \right\rangle$$

thermodynamic average

- consider small thermodynamic fluctuations in T, μ_B, u, with $\kappa_i^\mu(x) = \big(T(x), \mu_B(x), u^\mu(x)\big)_i$

$$\left\langle f\rho\, f\rho \right\rangle_c = \frac{\partial(f\rho)}{\partial \kappa_i^\mu} \frac{\partial(f\rho)}{\partial \kappa_j^\nu} \bigg|_{\bar{\kappa}} \left\langle \delta\kappa_i^\mu \delta\kappa_j^\nu \right\rangle + \mathcal{O}\left(\delta\kappa^3\right)$$

connected correlator

fluctuations of T, μ_B, u

- fluctuations, e.g., of thermodynamic quantities generate particle correlations

THERMODYNAMIC CORRELATIONS

- correlations $\langle \ldots \rangle$ from thermodynamic average
- weight configurations with the change in entropy due to fluctuations, Δs^μ [Landau, Lifshitz (vol. 5)]

$\longrightarrow$ generating functional of (connected) thermodynamic correlations

$$W[J] = \ln \int \mathscr{D}\kappa(x)\, \exp \int d\Sigma_\mu \left[\Delta s^\mu(x) + J(x)_{i\nu}\, \hat{v}^\mu\, \kappa_i^\nu(x)\right]$$

normal to Σ

- connected n-point correlations $\langle \kappa^n \rangle_c$ from $\left.\dfrac{\delta^n W[J]}{\delta J^n}\right|_{J=0}$
- change of entropy in an ideal fluid $(T^{\mu\nu} = \epsilon\, u^\mu u^\nu + p\Delta^{\mu\nu})$ with Gaussian fluctuations:

$$\hat{v}_\mu \Delta s^\mu = -\frac{1}{2}\kappa_{i\mu}(x)\, \mathscr{F}_{ij}^{\mu\nu}(x)\, \kappa_{j\nu}(x) \qquad \mathscr{F}_{ij}^{\mu\nu} = \frac{1}{T}\begin{pmatrix} \hat{u}\,\dfrac{\partial s}{\partial T} & \hat{u}\,\dfrac{\partial s}{\partial \mu_B} & s\hat{v}^\nu \\ \hat{u}\,\dfrac{\partial s}{\partial \mu_B} & \hat{u}\,\dfrac{\partial n_B}{\partial \mu_B} & n_B\hat{v}^\nu \\ s\hat{v}^\mu & n_B\hat{v}^\mu & -\hat{u}\,(Ts + \mu_B n_B)g^{\mu\nu} \end{pmatrix}_{ij}$$

local fluctuations!

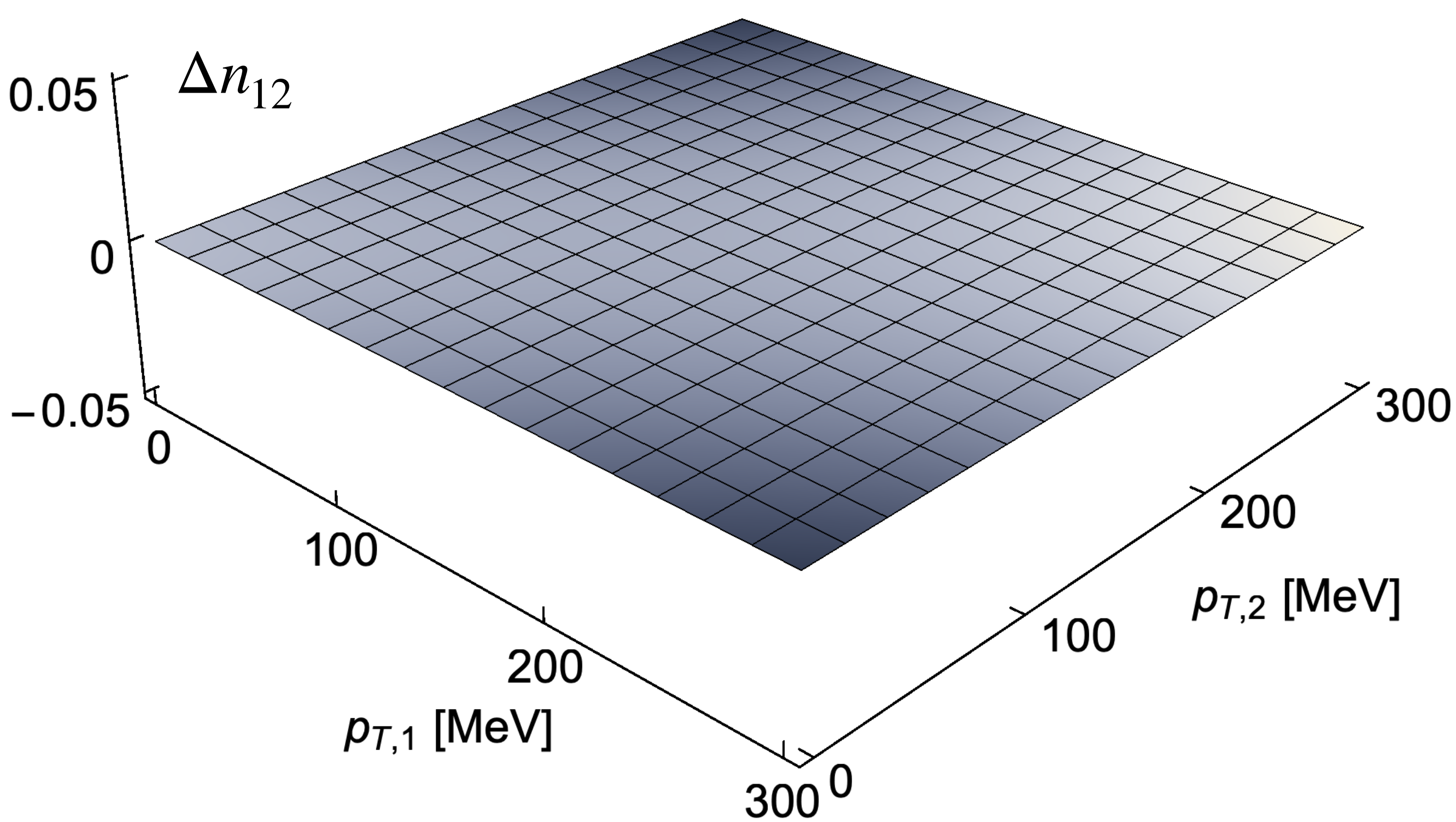
fluctuation matrix $(\hat{u} = \hat{v}^\mu\, u_\mu)$

TRANSVERSE MOMENTUM CORRELATIONS

[Pisarski, FR (2021)]

- normalized two-particle correlation $\Delta n_{12} = \dfrac{\left\langle \dfrac{dN_1}{dp_1^3} \dfrac{dN_1}{dp_2^3} \right\rangle_c}{\left\langle \dfrac{dN_1}{dp_1^3} \right\rangle \left\langle \dfrac{dN_1}{dp_2^3} \right\rangle}$

normal phase



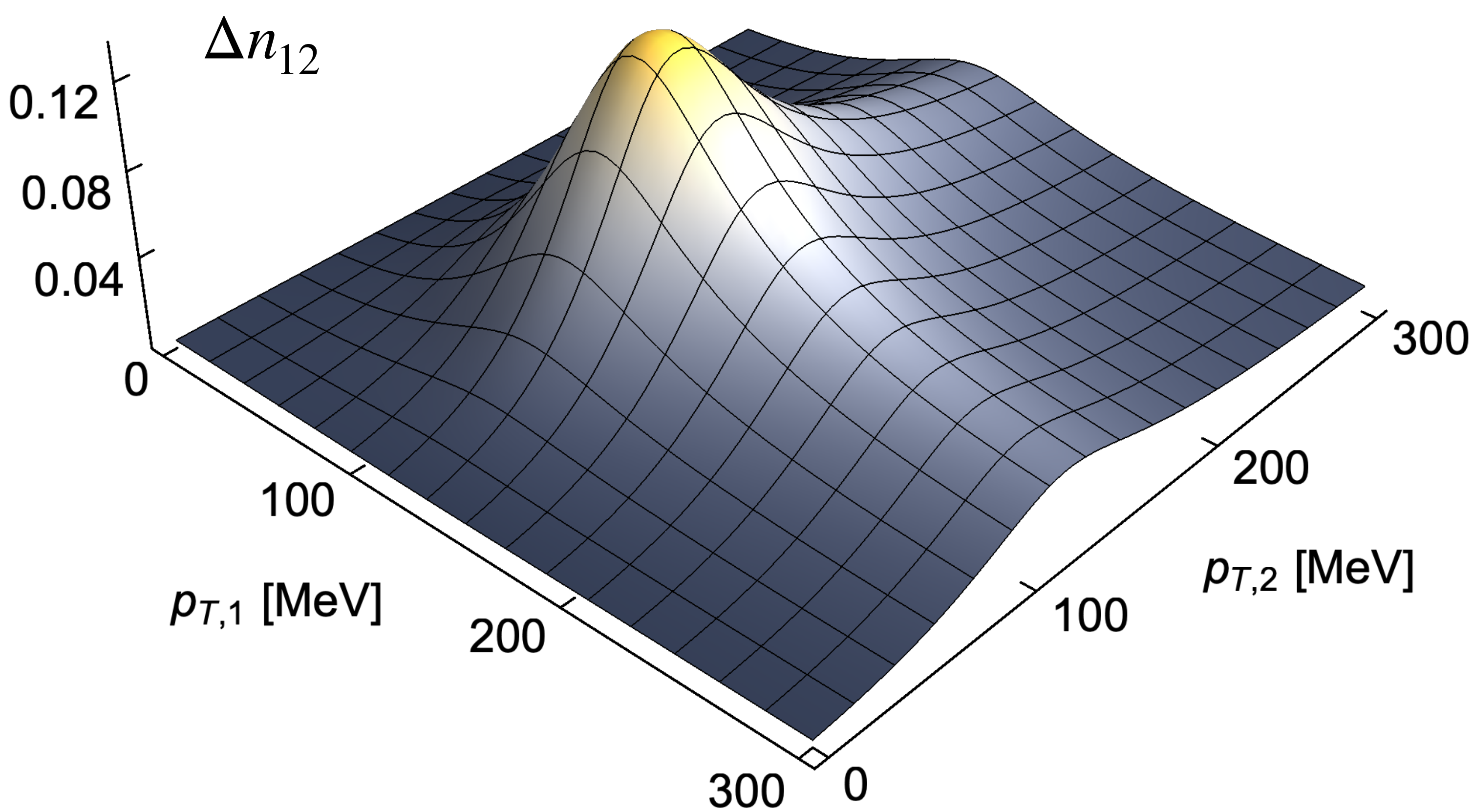
(relatively) flat two-particle p_T correlation in the normal phase

TRANSVERSE MOMENTUM CORRELATIONS

[Pisarski, FR (2021)]

- normalized two-particle correlation $\Delta n_{12} = \dfrac{\left\langle \dfrac{dN_1}{dp_1^3} \dfrac{dN_1}{dp_2^3} \right\rangle_c}{\left\langle \dfrac{dN_1}{dp_1^3} \right\rangle \left\langle \dfrac{dN_1}{dp_2^3} \right\rangle}$

moat regime



pronounced peak and ridges at nonzero p_T related to wavenumber of spatial modulation!

huge enhancement:

$$\frac{\Delta n_{12}(p_{\text{peak}})\big|_{\text{moat}}}{\Delta n_{12}(p_{\text{peak}})\big|_{\text{normal}}} \approx 10^2$$

CONCLUSION

I think this is an opportunity for FAIR!

- measure differential particle spectra
- good resolution at low momentum required

Questions to address here

- other sources of correlations?
- evolving through a moat regime with your favorite transport code?
- what's for dinner?

BACKUP

PARTICLE SPECTRUM IN A MOAT PHASE

use simple models to show general structure

Particle in a moat regime:

- **low-energy model** of free bosons in a moat regime $(Z < 0, W > 0)$:

$$\mathscr{L}_0 = \frac{1}{2}\left(\partial_0\phi\right)^2 + \frac{Z}{2}\left(\partial_i\phi\right)^2 + \frac{W}{2}\left(\partial_i^2\phi\right)^2 + \frac{m_{\text{eff}}^2}{2}\phi^2$$

- gives simple in-medium spectral function

$$\rho_\phi(p_0, \mathbf{p}^2) = 2\pi\,\mathrm{sign}(\mathrm{p}_0)\,\delta\big[\mathrm{p}_0^2 - \mathrm{E}_\phi^2(\mathbf{p}^2)\big] \quad \text{with} \quad E_\phi(\mathbf{p}^2) = \sqrt{Z\,\mathbf{p}^2 + W\big(\mathbf{p}^2\big)^2 + m_{\text{eff}}^2}$$

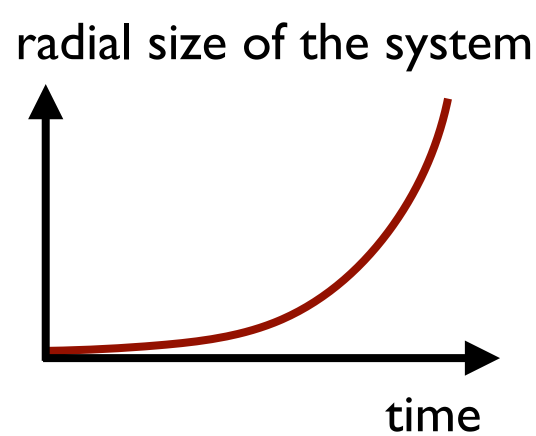
- boost symmetry broken! (but spatial rotation symmetry still intact)

Fluid velocity and freeze-out surface from hydro evolution

- boost invariant, transverse-isotropic freeze-out at fixed temperature T_f and fixed proper time τ_f $\left(=\sqrt{t^2 - z^2}\right)$

- blast wave approximation for the fluid velocity:

$$u^r = \bar{u}\,\frac{r}{\bar{R}}\,\theta(\bar{R} - r)$$



[Schnedermann, Sollfrank, Heinz (1993)]
[Teaney (2003)]

PARTICLE SPECTRUM IN A MOAT PHASE

use simple models to show general structure

model parameters:

- pick a beam energy of $\sqrt{s} = 5\,\mathrm{GeV}$ and read off thermodynamic and blast wave parameters:

$$T_f = 115\,\mathrm{MeV}$$
$$\mu_{B,f} = 536\,\mathrm{MeV}$$

[Andronic, Braun-Munzinger, Redlich, Stachel (2018)]

$$\bar{u} = 0.3$$
$$\bar{R} = 8\,\mathrm{fm}$$
$$\tau_f = 5\,\mathrm{fm/c}$$

[Zhang, Ma, Chen, Zhong (2016)]

- thermodynamics (used later) from a hadron resonance gas [Braun-Munzinger, Redlich, Stachel (2003)]

- moat parameters: purely illustrative

$$\text{if } Z < 0: \quad W = 2.5\,\mathrm{GeV}^{-2}$$