Retreat 2022

13 September 2022

Chiral spin symmetry and the QCD phase diagram

Owe Philipsen

Based on: Glozman, O.P., Pisarski, arXiv:2204.05083
Lowdon, O.P., arXiv:2207.14718

Chiral spin symmetry

Trafo: Generators:

Dirac: $\psi \rightarrow \psi' = \exp\left(i\frac{\varepsilon^n \Sigma^n}{2}\right)\psi$ $\Sigma^n = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$ $k = 1, 2, 3, 4$

Weyl: $\begin{pmatrix} R \\ L \end{pmatrix} \rightarrow \begin{pmatrix} R' \\ L' \end{pmatrix} = \exp\left(i\frac{\varepsilon^n \sigma^n}{2}\right)\begin{pmatrix} R \\ L \end{pmatrix}$ $[\Sigma^a, \Sigma^b] = 2\mathrm{i}\epsilon^{abc}\Sigma^c$ $su(2)$

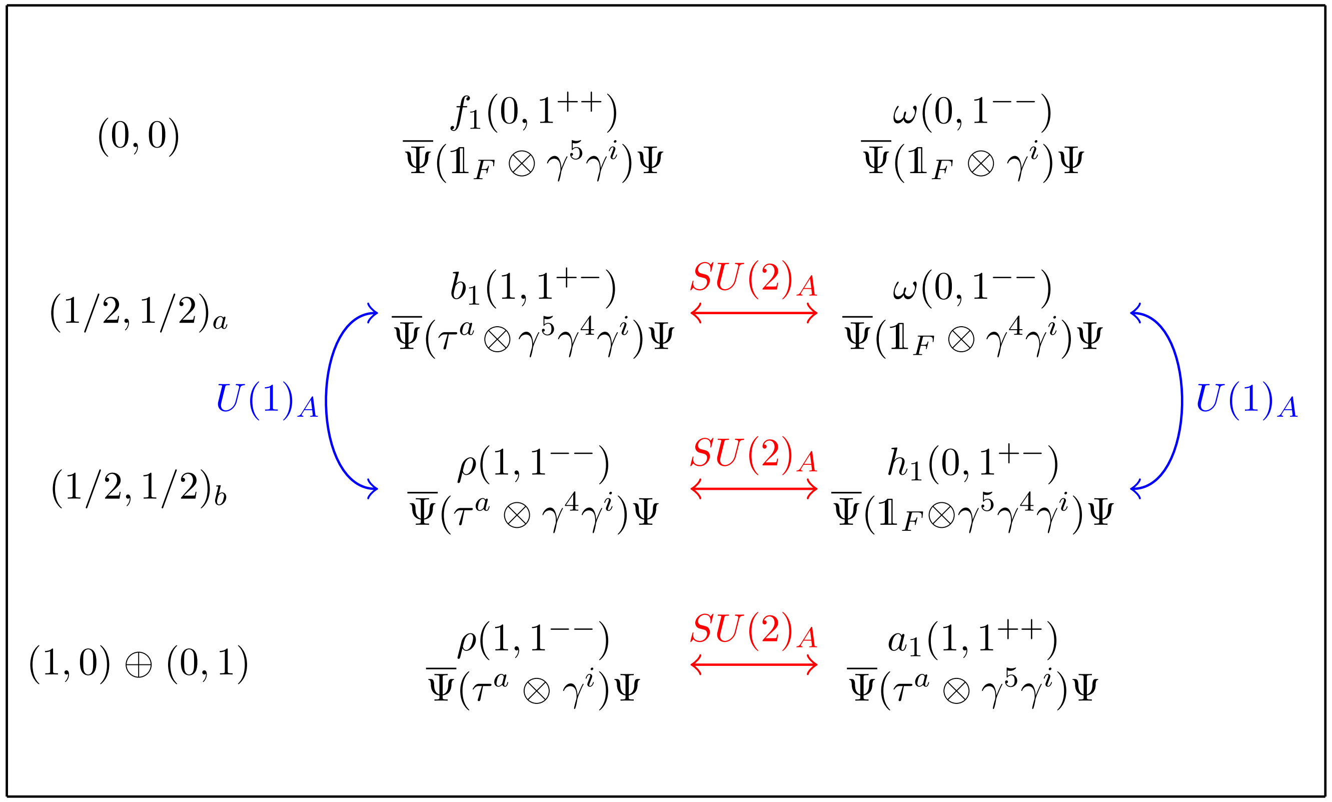
Obviously: $SU(2)_{CS} \supset U(1)_A$

Not so obvious $SU(2)_{CS} \otimes SU(2)_F :$ $\{(\vec{\tau} \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \vec{\Sigma}_k), (\vec{\tau} \otimes \vec{\Sigma}_k)\}$ 15 generators

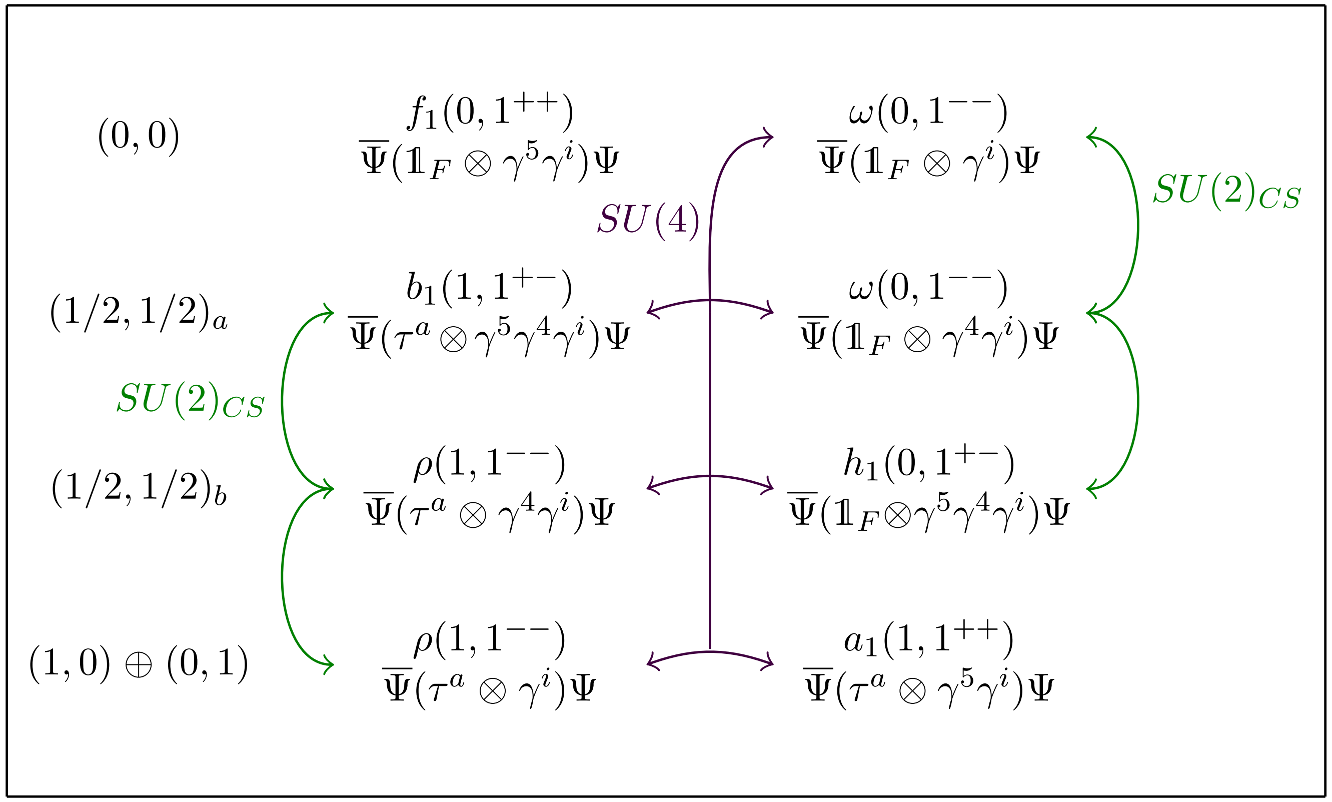
↓

$$SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A$$

Relations in multiplets



$$\begin{tikzcd}
(0,0) & \substack{f_1(0,1^{++})\\ \overline{\Psi}(\mathbb{1}_F\otimes\gamma^5\gamma^i)\Psi} & \substack{\omega(0,1^{--})\\ \overline{\Psi}(\mathbb{1}_F\otimes\gamma^i)\Psi} \\
(1/2,1/2)_a & \substack{b_1(1,1^{+-})\\ \overline{\Psi}(\tau^a\otimes\gamma^5\gamma^4\gamma^i)\Psi} \arrow[r, leftrightarrow, "SU(2)_A"] \arrow[d, leftrightarrow, bend right, "U(1)_A"'] & \substack{\omega(0,1^{--})\\ \overline{\Psi}(\mathbb{1}_F\otimes\gamma^4\gamma^i)\Psi} \arrow[d, leftrightarrow, bend left, "U(1)_A"] \\
(1/2,1/2)_b & \substack{\rho(1,1^{--})\\ \overline{\Psi}(\tau^a\otimes\gamma^4\gamma^i)\Psi} \arrow[r, leftrightarrow, "SU(2)_A"] & \substack{h_1(0,1^{+-})\\ \overline{\Psi}(\mathbb{1}_F\otimes\gamma^5\gamma^4\gamma^i)\Psi} \\
(1,0)\oplus(0,1) & \substack{\rho(1,1^{--})\\ \overline{\Psi}(\tau^a\otimes\gamma^i)\Psi} \arrow[r, leftrightarrow, "SU(2)_A"] & \substack{a_1(1,1^{++})\\ \overline{\Psi}(\tau^a\otimes\gamma^5\gamma^i)\Psi}
\end{tikzcd}$$



$$\begin{tikzcd}
(0,0) & \substack{f_1(0,1^{++})\\ \overline{\Psi}(\mathbb{1}_F\otimes\gamma^5\gamma^i)\Psi} & \substack{\omega(0,1^{--})\\ \overline{\Psi}(\mathbb{1}_F\otimes\gamma^i)\Psi} \arrow[d, leftrightarrow, bend left, "SU(2)_{CS}"] \\
(1/2,1/2)_a & \substack{b_1(1,1^{+-})\\ \overline{\Psi}(\tau^a\otimes\gamma^5\gamma^4\gamma^i)\Psi} \arrow[r, leftrightarrow] \arrow[ur, "SU(4)"] \arrow[d, leftrightarrow, bend right, "SU(2)_{CS}"'] & \substack{\omega(0,1^{--})\\ \overline{\Psi}(\mathbb{1}_F\otimes\gamma^4\gamma^i)\Psi} \arrow[d, leftrightarrow, bend left] \\
(1/2,1/2)_b & \substack{\rho(1,1^{--})\\ \overline{\Psi}(\tau^a\otimes\gamma^4\gamma^i)\Psi} \arrow[r, leftrightarrow] \arrow[d, leftrightarrow, bend right] & \substack{h_1(0,1^{+-})\\ \overline{\Psi}(\mathbb{1}_F\otimes\gamma^5\gamma^4\gamma^i)\Psi} \\
(1,0)\oplus(0,1) & \substack{\rho(1,1^{--})\\ \overline{\Psi}(\tau^a\otimes\gamma^i)\Psi} \arrow[r, leftrightarrow] & \substack{a_1(1,1^{++})\\ \overline{\Psi}(\tau^a\otimes\gamma^5\gamma^i)\Psi}
\end{tikzcd}$$

chiral symmetry

CS symmetry

Rohrhofer et al., Phys. Lett. B802 (2020)

Emergent CS symmetry: where does it come from?

QCD quark action, chiral limit: $\bar{\psi}\gamma^{\mu}D_{\mu}\psi = \bar{\psi}\gamma^{0}D_{0}\psi + \bar{\psi}\gamma^{i}D_{i}\psi$

$[\Sigma^a, \gamma_0] = 0, [\Sigma^a, \gamma_i] \neq 0,$

CS invariant breaks CS

The classical QCD action in the chiral limit is not CS symmetric!

The free quark action in the chiral limit is not CS symmetric!

Quark gluon interactions:

colour-electric

$\bar{\psi}\gamma_0 T^a \psi \; A_0^a$

CS invariant

colour-magnetic

$\bar{\psi}\gamma_i T^a \psi \; A_i^a$

breaks CS

Necessary condition for approximate CS symmetry:

Quantum effective action Γ_k dominated by colour-electric interactions!

Spatial and temporal correlators at finite T

Chiral symmetry restoration at finite T

$$C_\Gamma(\tau, \boldsymbol{x}) = \langle O_\Gamma(\tau, \boldsymbol{x})\, O_\Gamma(0, \mathbf{0}) \rangle$$

$$C_\Gamma(\tau, \boldsymbol{p}) = \int_0^\infty \frac{d\omega}{2\pi}\, K(\tau, \omega) \rho_\Gamma(\omega, \boldsymbol{p})\ ,$$

$$K(\tau, \omega) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}\ .$$

$$C^s_\Gamma(z) = \sum_{x,y,\tau} C_\Gamma(\tau, \boldsymbol{x})$$

$$C^\tau_\Gamma(\tau) = \sum_{x,y,z} C_\Gamma(\tau, \boldsymbol{x})$$

Spectral function contains all information about degrees of freedom

Inversion from discrete data ill-posed problem

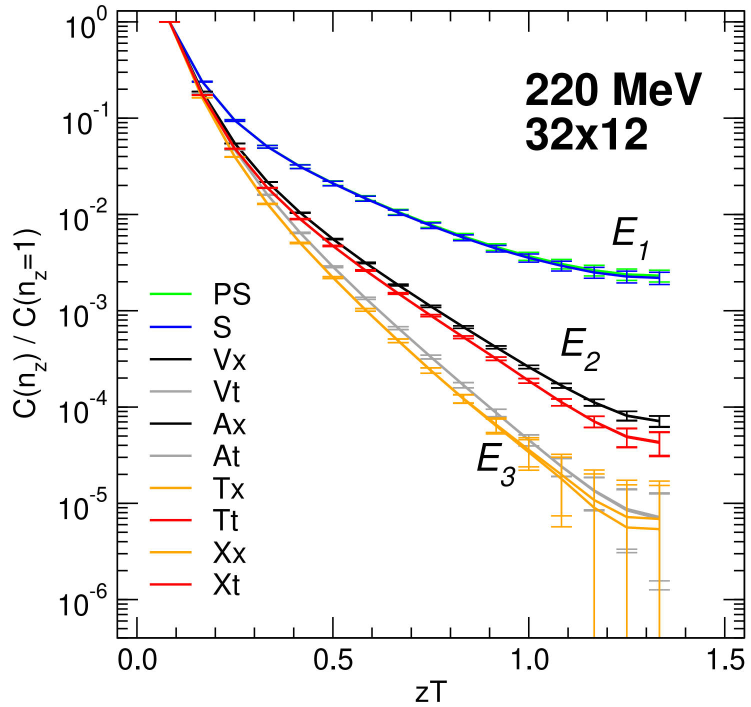
Finite T has preferred reference frame: colour-electric and colour magnetic distinguishable!

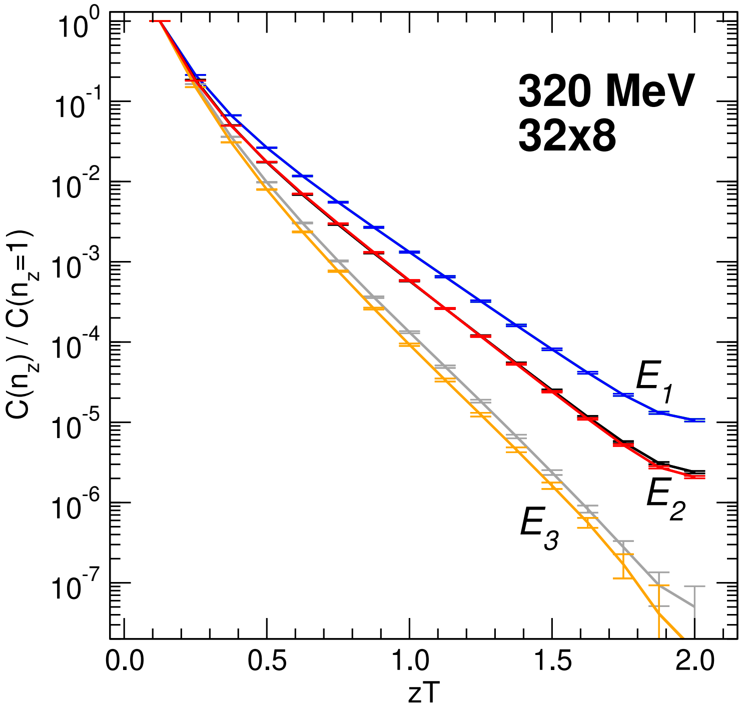
Symmetry in spatial and temporal correlators sufficient for symmetry of spectral function

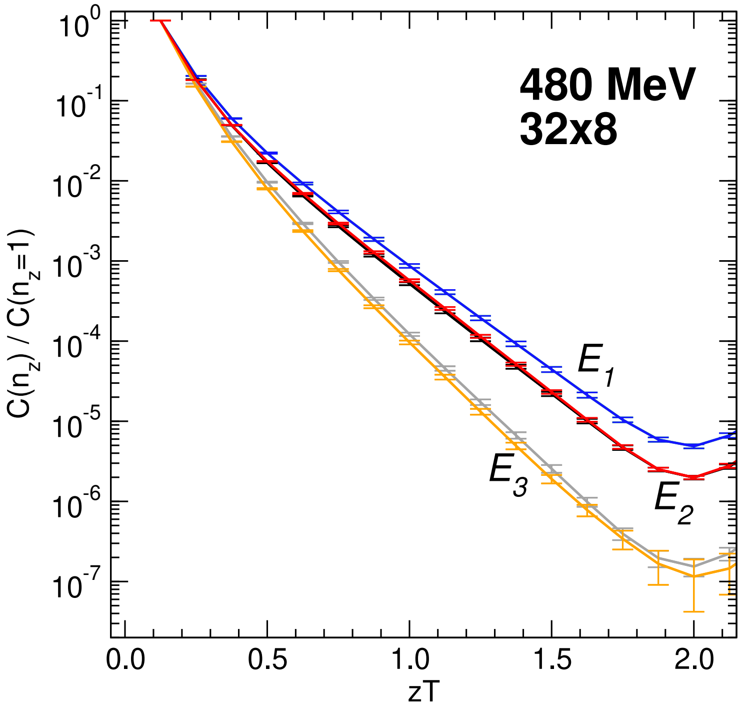
Spatial correlators at finite T

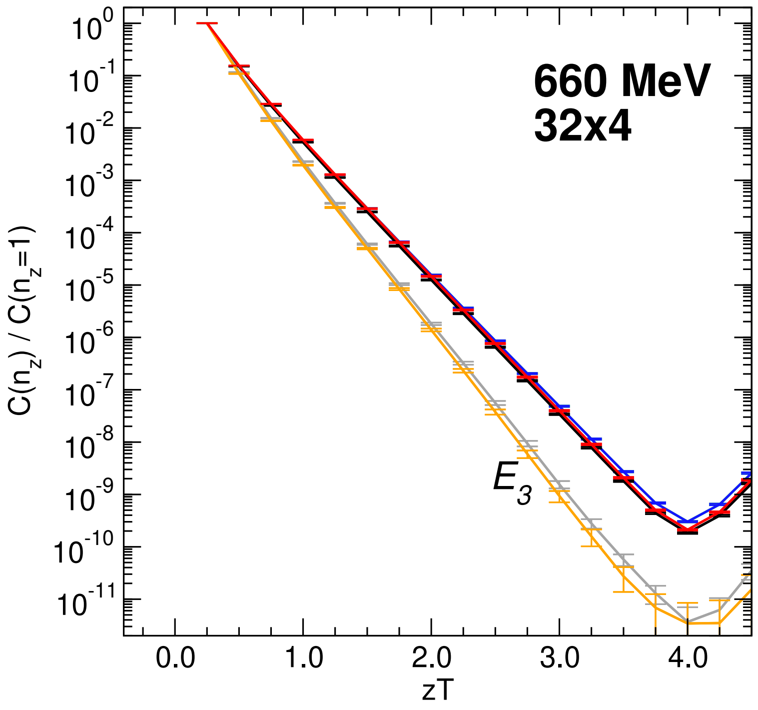
Multiplet structure

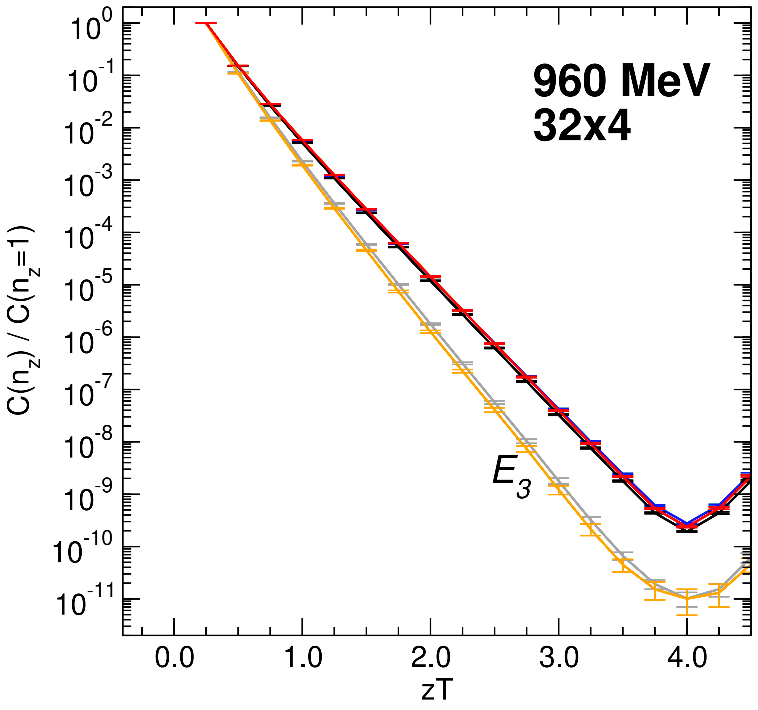
$$\begin{array}{llll} E_1 : & PS \leftrightarrow S \,, & & U(1)_A \\ E_2 : & V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x \,, & & SU(4) \\ E_3 : & V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t \,. & & SU(2)_L \times SU(2)_R \times U(1)_A \end{array}$$











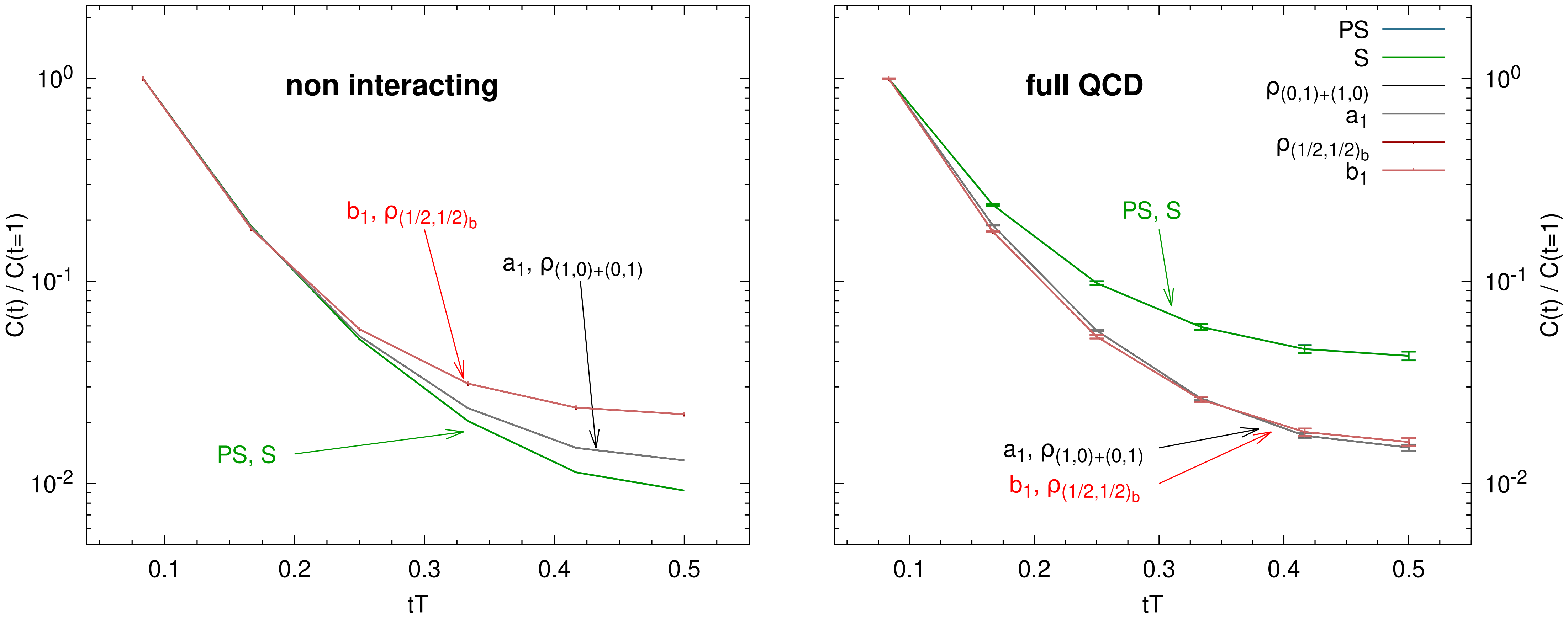
JLQCD domain wall fermions

Rohrhofer et al., Phys. Rev. D100 (2019)

Temporal correlators at finite T

JLQCD domain wall fermion configurations

Rohrhofer et al., Phys. Lett. B802 (2020)



$$48^3 \times 12 \qquad T = 220\text{MeV}\ (1.2T_c) \qquad (a = 0.075\ \text{fm})$$

Three temperature regimes of QCD

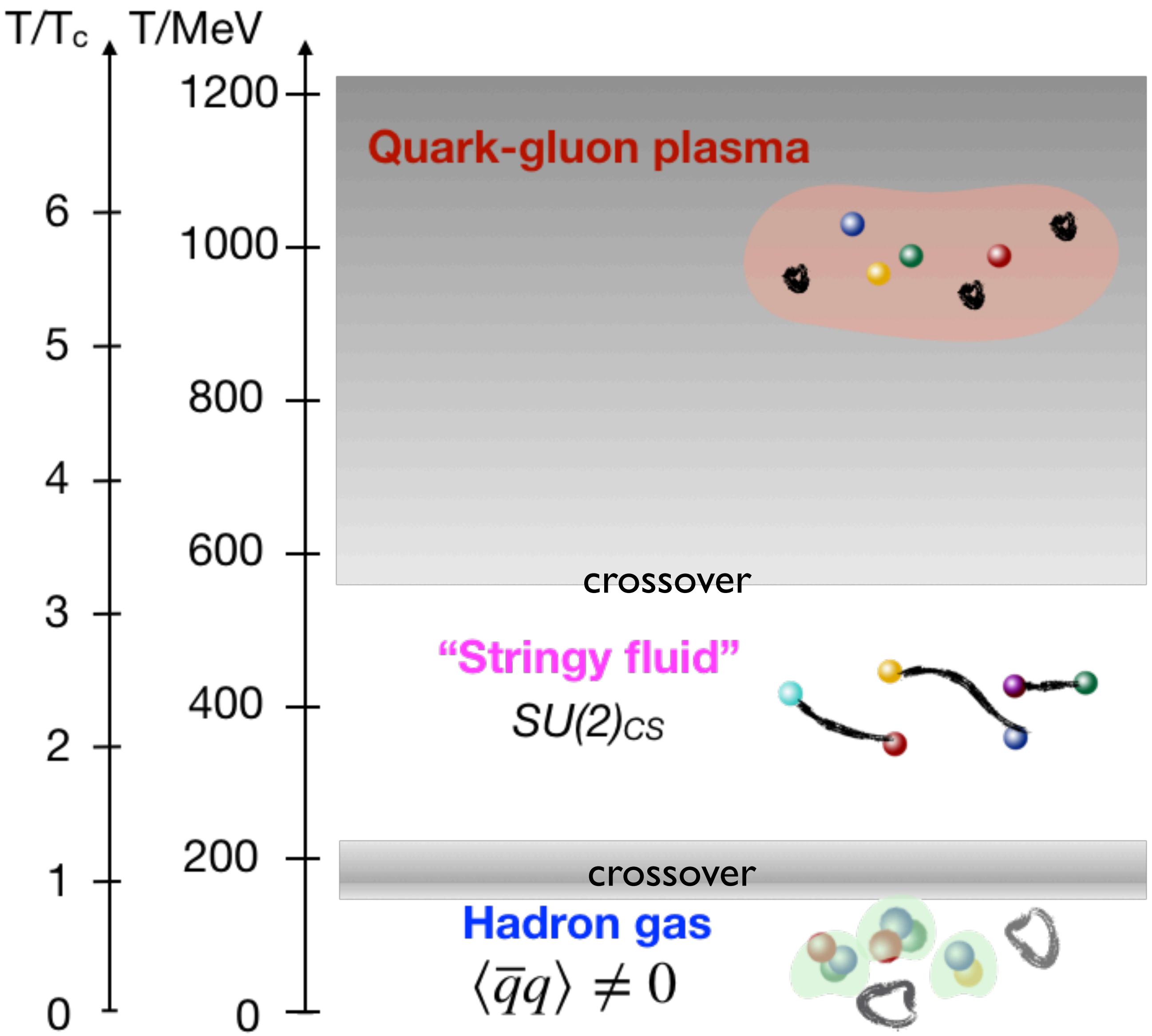
Symmetries (verified):

Chiral symmetry (approximate)

Chiral spin symmetry (approximate)

Chiral symmetry broken

Degrees of freedom (to be verified):



Rohrhofer et al., Phys. Rev. D100 (2019)

How to classify effective degrees of freedom?

No universal, or generally accepted, definition of "confinement"

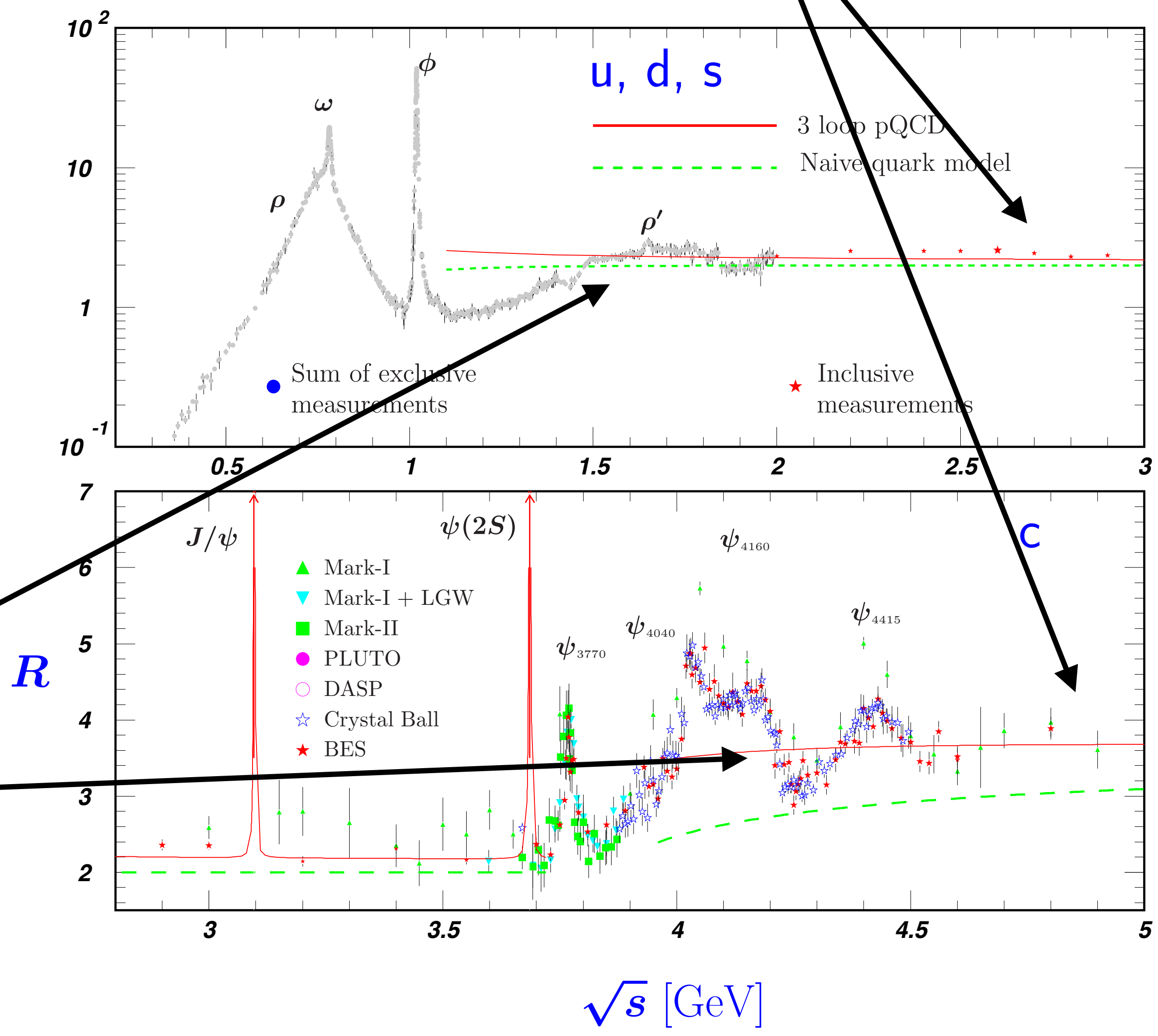
Vacuum QCD: Quark Hadron Duality [e.g. M. Shifman, hep-ph/0009131]

Experimental observables are always hadronic. Quark hadron duality holds, when these follow perturbative predictions for partonic (sub-)processes

Example:

$$R \equiv \frac{\sigma(e^+e^- \to \text{Hadrons}\ )}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

duality violations



Check well-studied observables: screening masses

$$C^s_\Gamma(z) = \sum_{x,y,\tau} C_\Gamma(\tau, \boldsymbol{x}) \quad \xrightarrow{z\rightarrow\infty} \text{const.}\ e^{-m_{scr} z}$$

Directly related to the partition function and equation of state

by transfer matrices: $T = e^{-aH}, T_z = e^{-aH_z}$

$$\begin{aligned} e^{pV/T} &= Z = \mathrm{Tr}(e^{-aHN_\tau}) \\ &= \mathrm{Tr}(e^{-aH_z N_z}) = \sum_{n_z} e^{-E_{n_z} N_z} \end{aligned}$$

Screening masses: eigenvalues of H_z

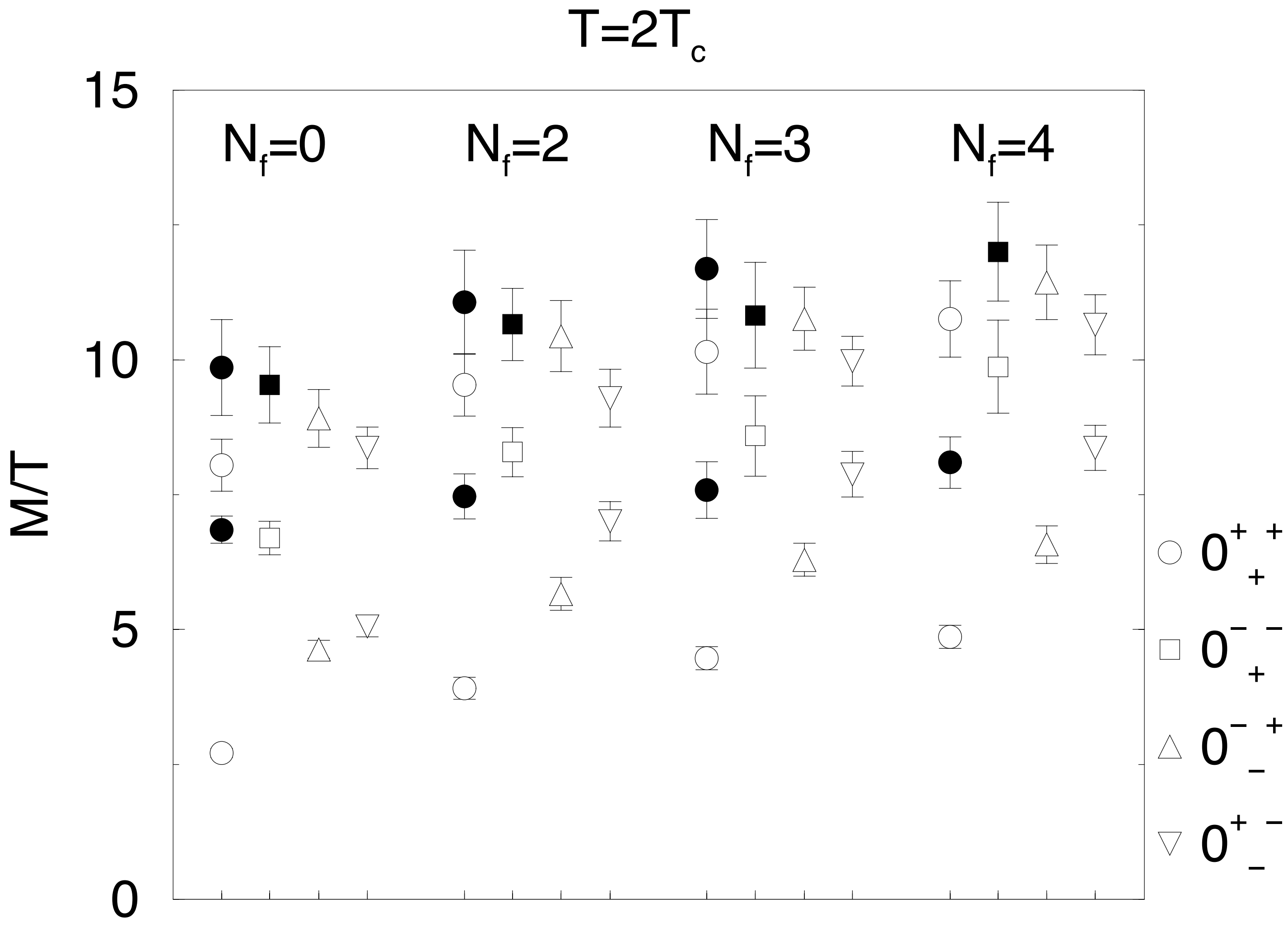
For T=0 equivalent to eigenvalues of H, for $T \neq 0$ “finite size effect”

Colour-electric vs. colour magnetic fields

Scales at finite T: $\text{Matsubara} \sim \pi T,$ $\text{hard modes, fermions}$ QCD

$\text{Debye/electric} \sim gT,$ A_0 EQCD

$\text{magnetic} \sim g^2T,$ A_i MQCD



[Hart, Laine, O.P., NPB 00]

Screening masses in EQCD

- ● ops. constructed only from A_i
- ○ ops. containing also A_0

Colour-electric fields dynamically dominant, perturbative ordering reversed!

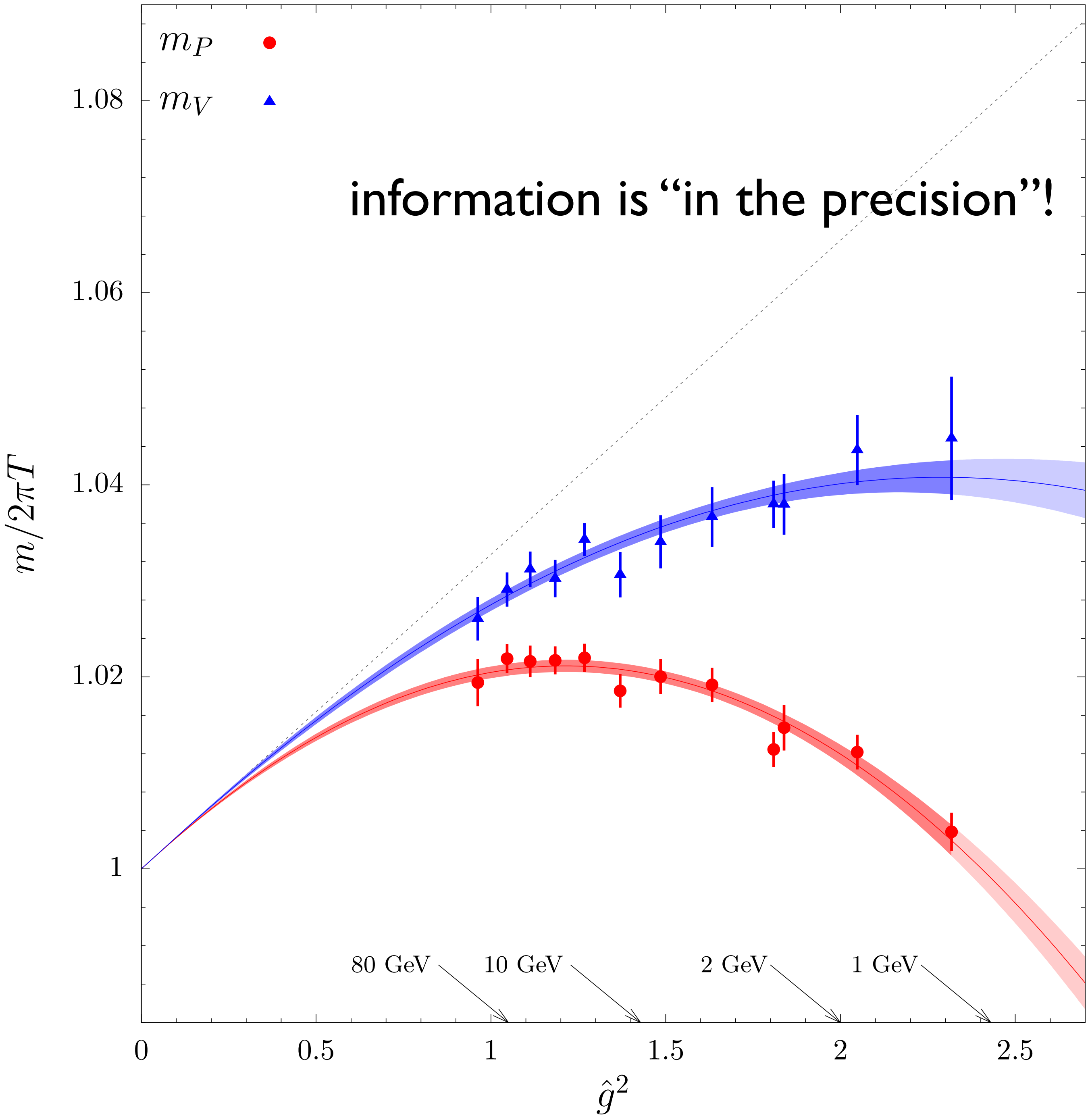
No quark hadron duality; expected for soft scales of EQCD at low T

Meson screening masses at high temperatures

[Dalla Brida et al., JHEP 22]

Nf=3, T=1 GeV -160 GeV

Highly non-trivial technically:
shifted b.c. + step-scaling techniques
(Alpha-Collaboration)



information is “in the precision”!

$$\frac{m_{PS}}{2\pi T} = 1 + p_2\,\hat{g}^2(T) + p_3\,\hat{g}^3(T) + p_4\,\hat{g}^4(T)$$
$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4\,\hat{g}^4(T)$$

$$p_2 = 0.032739961$$

[Laine,Vepsäläinen., JHEP 04]

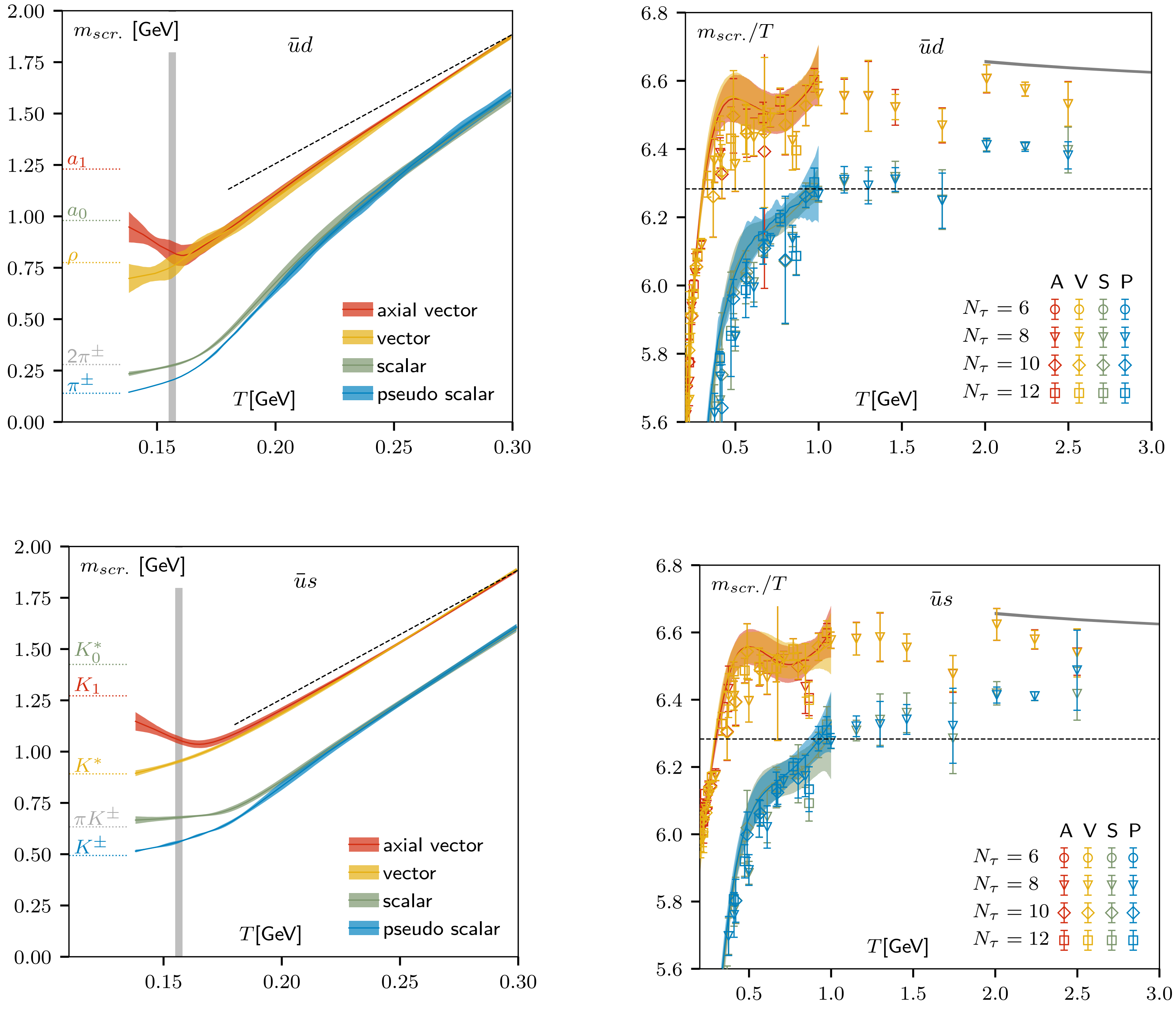
p_3, p_4, s_4 fitted, excellent $\chi^2_{\rm dof}$

$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2}\ln\frac{2\pi T}{\Lambda_{\overline{\rm MS}}} + \frac{4}{9\pi^2}\ln\left(2\ln\frac{2\pi T}{\Lambda_{\overline{\rm MS}}}\right)$$

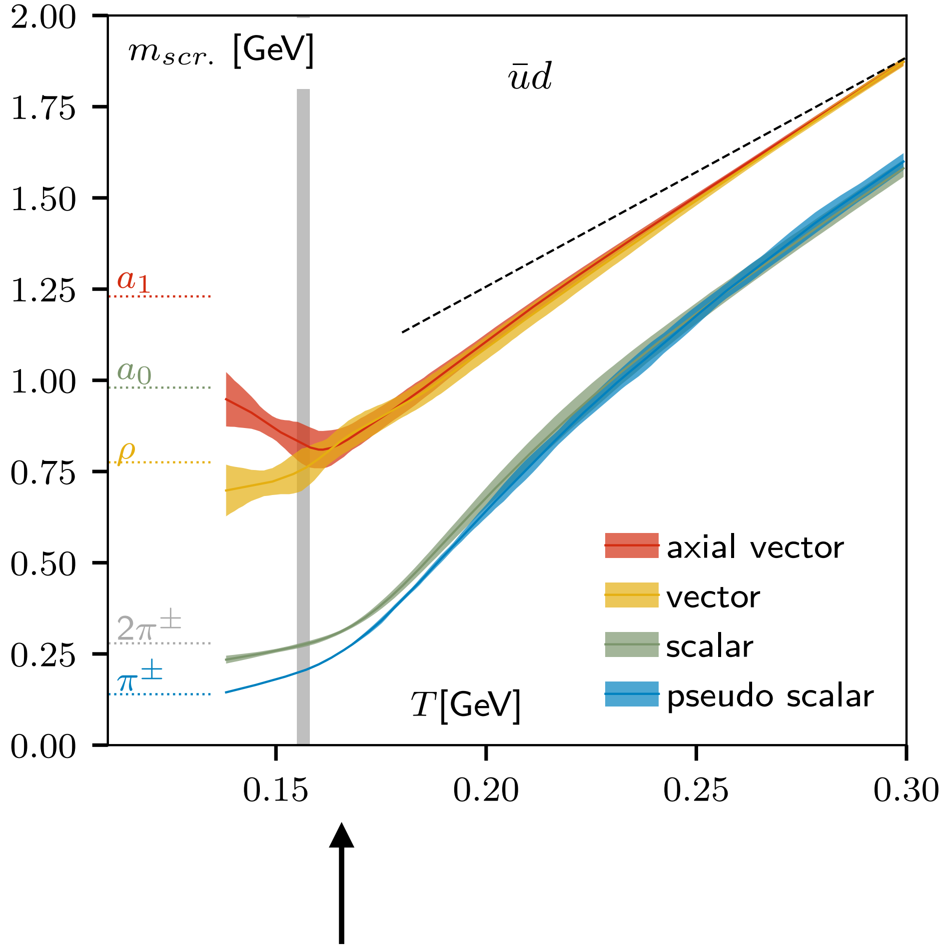
Quark hadron duality holds

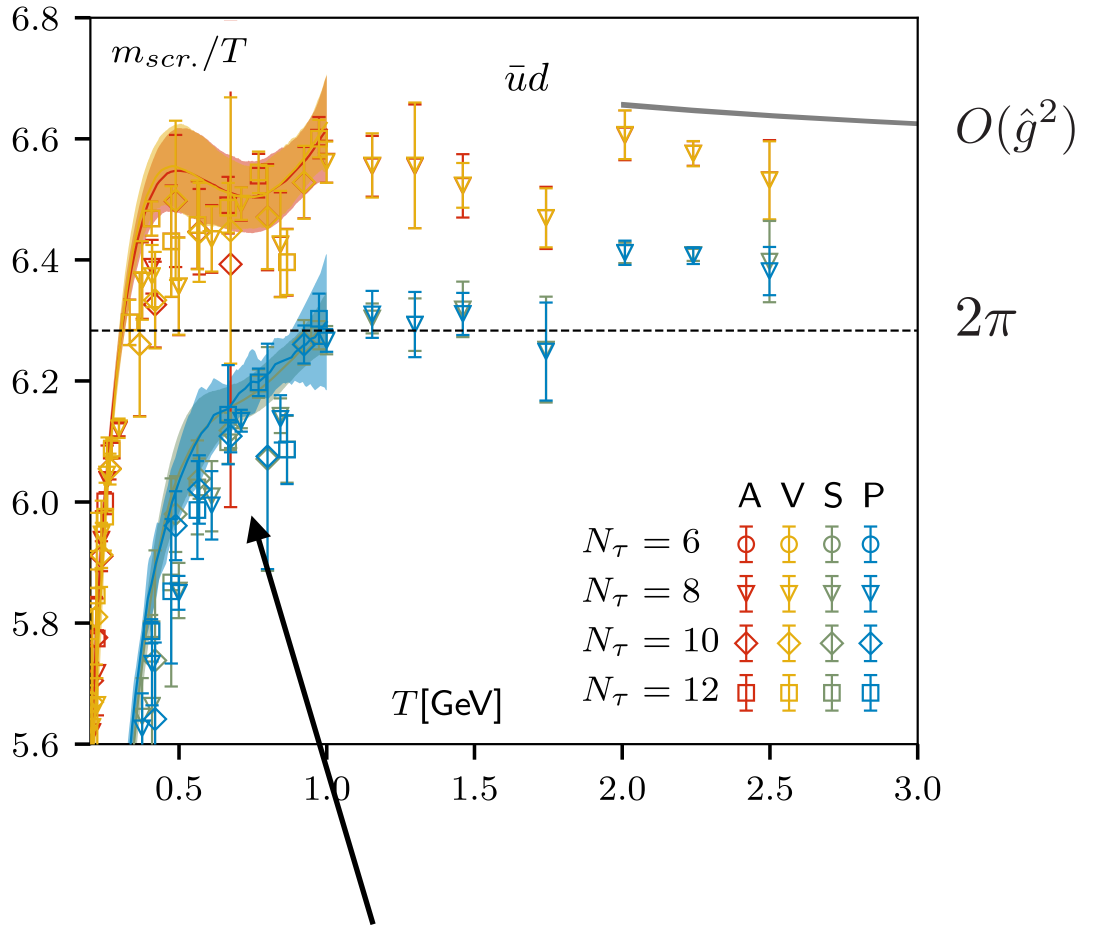
Meson screening masses at intermediate temperatures

HotQCD, Phys. Rev. D100 (2019) staggered fermions, physical point, continuum extrapolated



….and the same pattern also for $\bar{s}s$





Chiral symmetry restoration

drastic change: "vertical" - "horizontal"

Heavy chiral partners "come down" in all flavour combinations

Remember resumed pert. theory:

$$\frac{m_{PS}}{2\pi T} = 1 + p_2\, \hat{g}^2(T) + p_3\, \hat{g}^3(T) + p_4\, \hat{g}^4(T)\ ,$$
$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4\, \hat{g}^4(T)\ ,$$

➡ pressure increases

Cannot describe the "bend"

No quark hadron duality for T<0.5 GeV in 12 lightest meson channels! CS symmetry!

Finite density

- Finite density: $\mu\bar{\psi}\gamma_0\psi$ is CS invariant; regime must continue to finite density

- Upper "boundary" of CS band: screening mass at "bend" (one possible def.)

$$r_V^{-1} \equiv m_V(\mu_B = 0, T_{\rm s}) = C_0 T_{\rm s}$$

➡

$$\begin{aligned} &T < T_{\rm s} \quad \text{unscreened} \\ &T > T_{\rm s} \quad \text{screened} \end{aligned}$$

- For small μ_B

$$\frac{m_V(\mu_B)}{T} = C_0 + C_2\left(\frac{\mu_B}{T}\right)^2 + \dots$$

$$C_2 > 0$$

➡

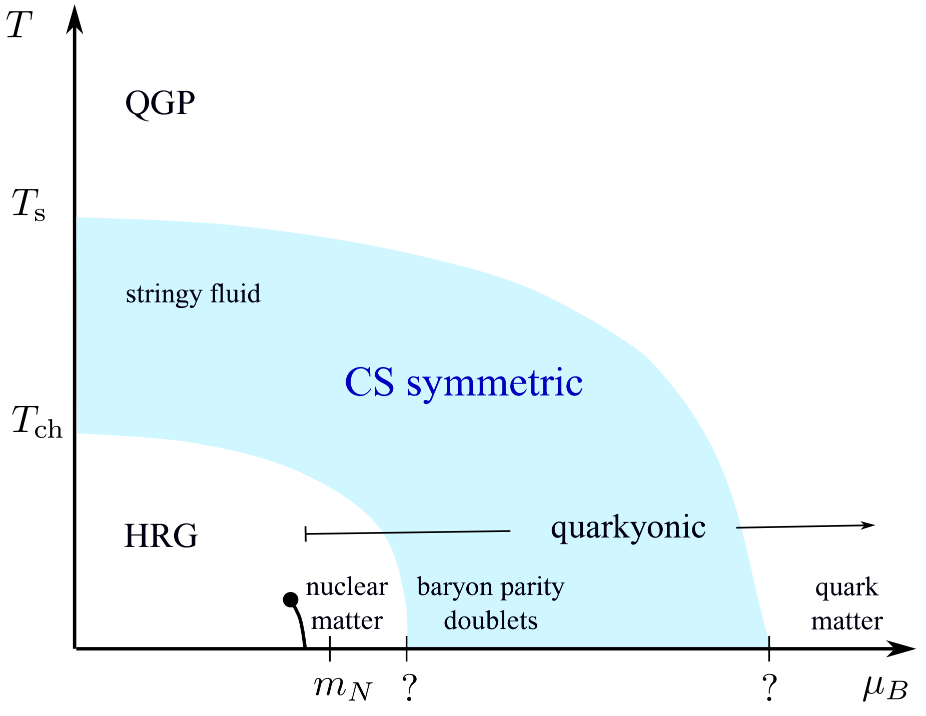
$$\frac{dT_{\rm s}}{d\mu_B} = -\frac{2C_2}{C_0}\frac{\mu_B}{T} - \frac{2C_2^2}{C_0^2}\left(\frac{\mu_B}{T}\right)^3 + \dots$$

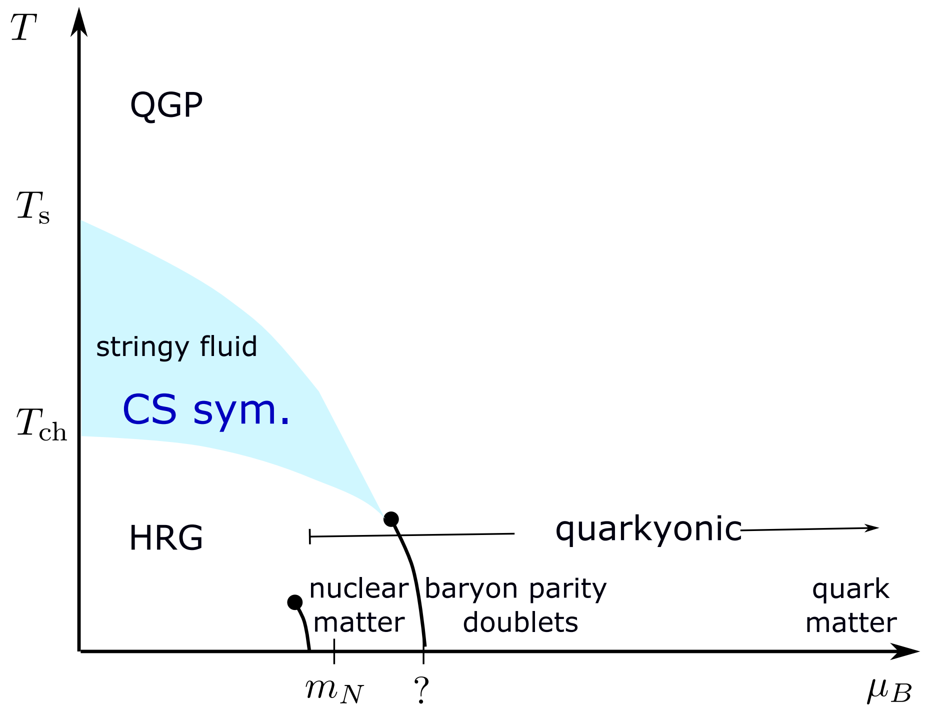
- Lower "boundary" of CS band: (this is a lower bound only)

$$\frac{T_{\rm pc}(\mu_B)}{T_{\rm pc}(0)} = 1 - 0.016(5)\left(\frac{\mu_B}{T_{\rm pc}(0)}\right)^2 + \dots \qquad \approx \frac{T_{\rm ch}(\mu_B)}{T_{\rm ch}(0)}$$

Separate order parameters for $SU(2)_A, U(1)_A, SU(4)$?

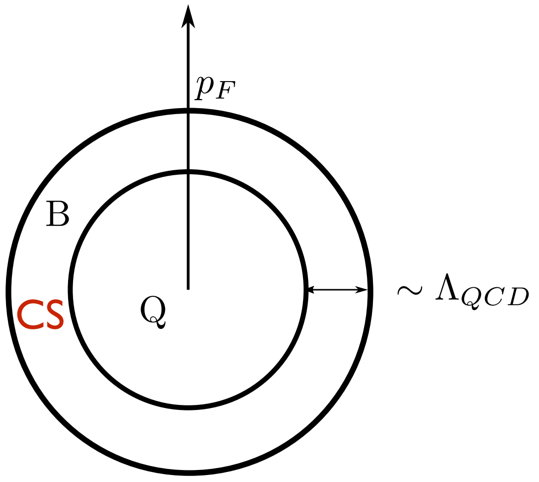
Possibilities for the QCD phase diagram





etc …

- Cold and dense candidate: baryon parity doublet models CS symmetric [Glozman, Catillo PRD 18]
- Quarkyonic matter [McLerran, Pisarski, NPA 07; O.P., Scheunert JHEP 19]
Contains regime with chirally symmetric baryon matter
Fully consistent with transient intermediate CS regime!
- Can be realized wit or without non-analytic chiral phase transition!



Effective degrees of freedom...? ➡ Spectral functions

Based on micro-causality of scalar, local quantum fields at finite T:

[Bros, Buchholz., NPB 94, Ann. Inst. Poincare Phys. Theor. 96]

$$\rho_{\mathrm{PS}}(p_0,\vec{p}) = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\, \epsilon(p_0)\, \delta\big(p_0^2 - (\vec{p}-\vec{u})^2 - s\big)\, \underset{\uparrow}{\widetilde{D}_\beta(\vec{u},s)}$$

Exact, goes to Källen-Lehmann representation for $T \to 0$

thermal spectral density

For stable massive particle with gap to continuum states (QCD pions!):

Ansatz $$\widetilde{D}_\beta(\vec{u},s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s-m^2) + \widetilde{D}_{c,\beta}(\vec{u},s)$$

Analytic structure inherited from vacuum, in absence of phase transition

➡ low energy behaviour influenced (at low T dominated) by vacuum particle states

Why this is plausible

V,A correlators in the chiral limit using PCAC, $\epsilon = T^2/(6f_\pi^2)$

[Dey, Eletsky, Ioffe PLB 90]

$$C_V(p,T) = (1-\epsilon)C_V(p,0) + \epsilon C_A(p,0)$$
$$C_A(p,T) = (1-\epsilon)C_A(p,0) + \epsilon C_V(p,0)$$

General spectral decomposition of spatial correlators

$$C_\Gamma(\tau,\mathbf{0},T) = \sum_{m,n} |\langle m|O_\Gamma(0,\mathbf{0})|n\rangle|^2 \, e^{-\tau(E_n-E_0)}e^{-(T-\tau)(E_m-E_0)}$$

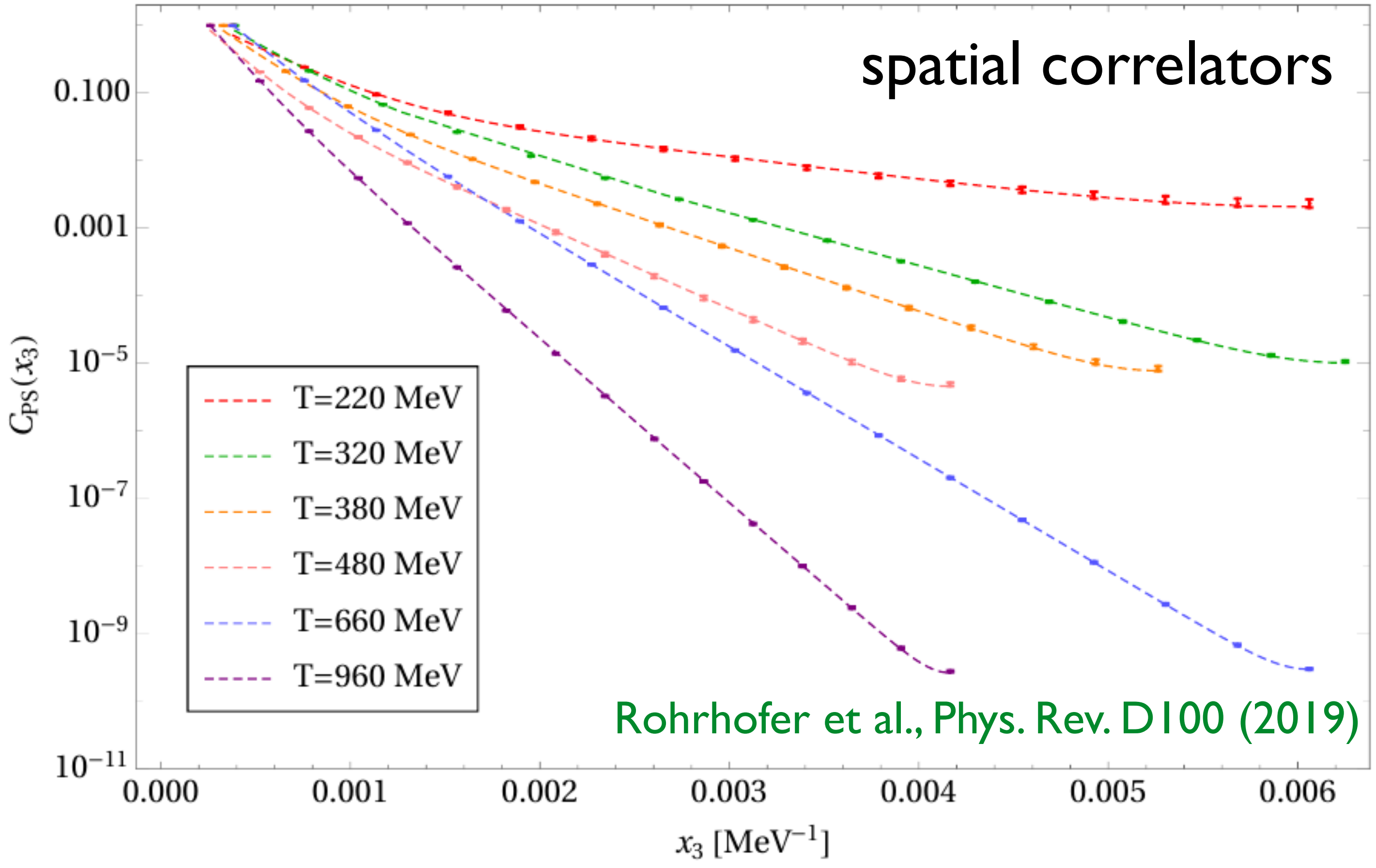
Analytic structure of vacuum still dominant for low temperatures

The pion spectral function

[Lowdon, O.P., arXiv:2207.14718]

2-state fits π, π^*

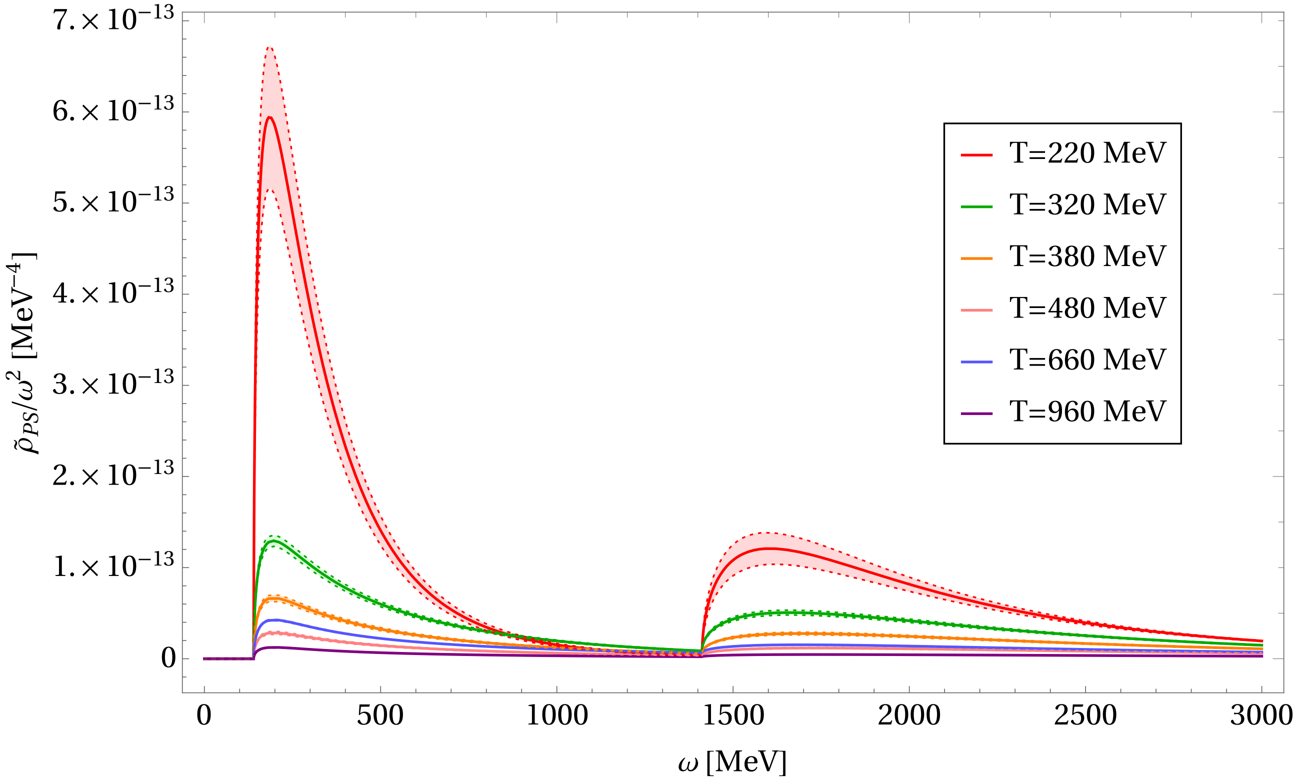
$D_{m,\beta}$

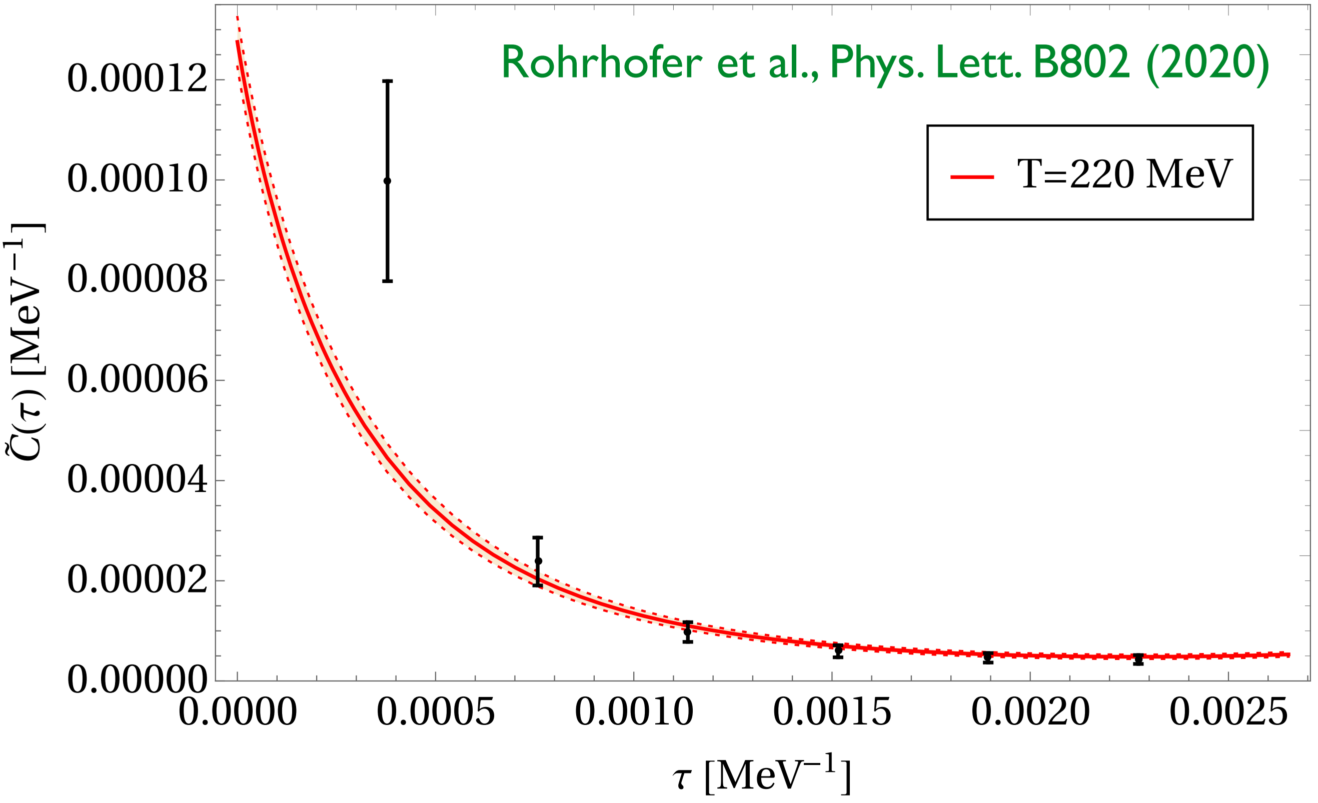


Rohrhofer et al., Phys. Rev. D100 (2019)

spectral functions

predict temporal correlators, compare with data





Rohrhofer et al., Phys. Lett. B802 (2020)

Conclusions

- QCD has an emergent approximate Chiral Spin symmetry in an intermediate temperature and density range
- Screening masses entirely non-perturbative in that window
- New spectral representation based on old locality principles: spectral functions from spatial lattice correlators
- Effective degrees of freedom in CS-regime consistent with hadron-like states
- CS-regime extends as a band into QCD phase diagram; natural connection to quarkyonic matter, investigate imag. chem. pot.