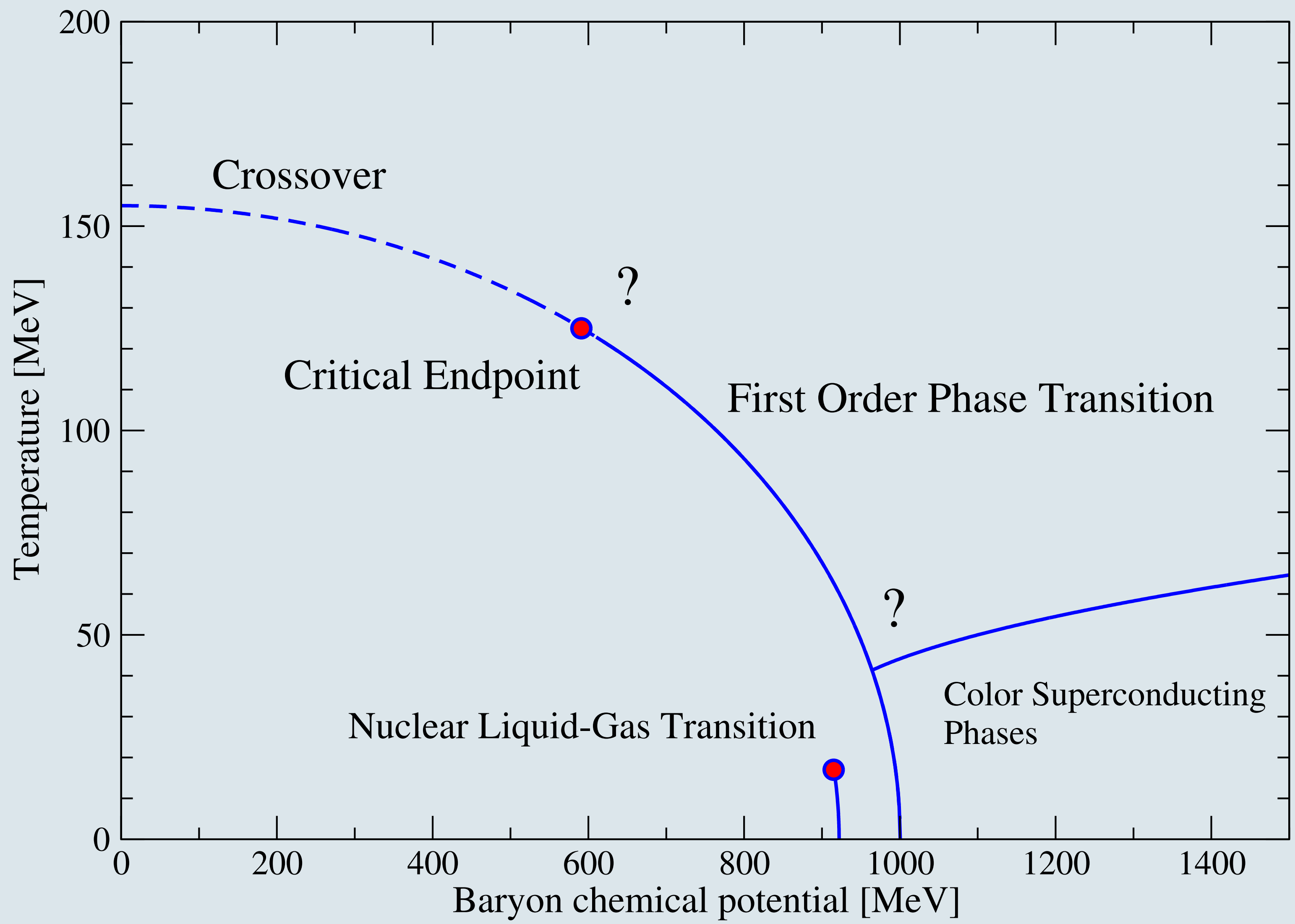
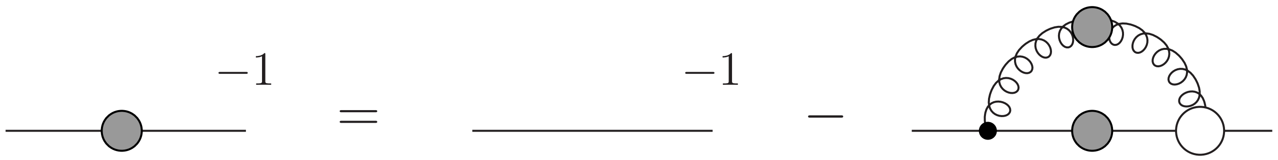


QCD-phase diagram with functional methods

Review: CF, PPNP 105 (2019) [1810.12938]

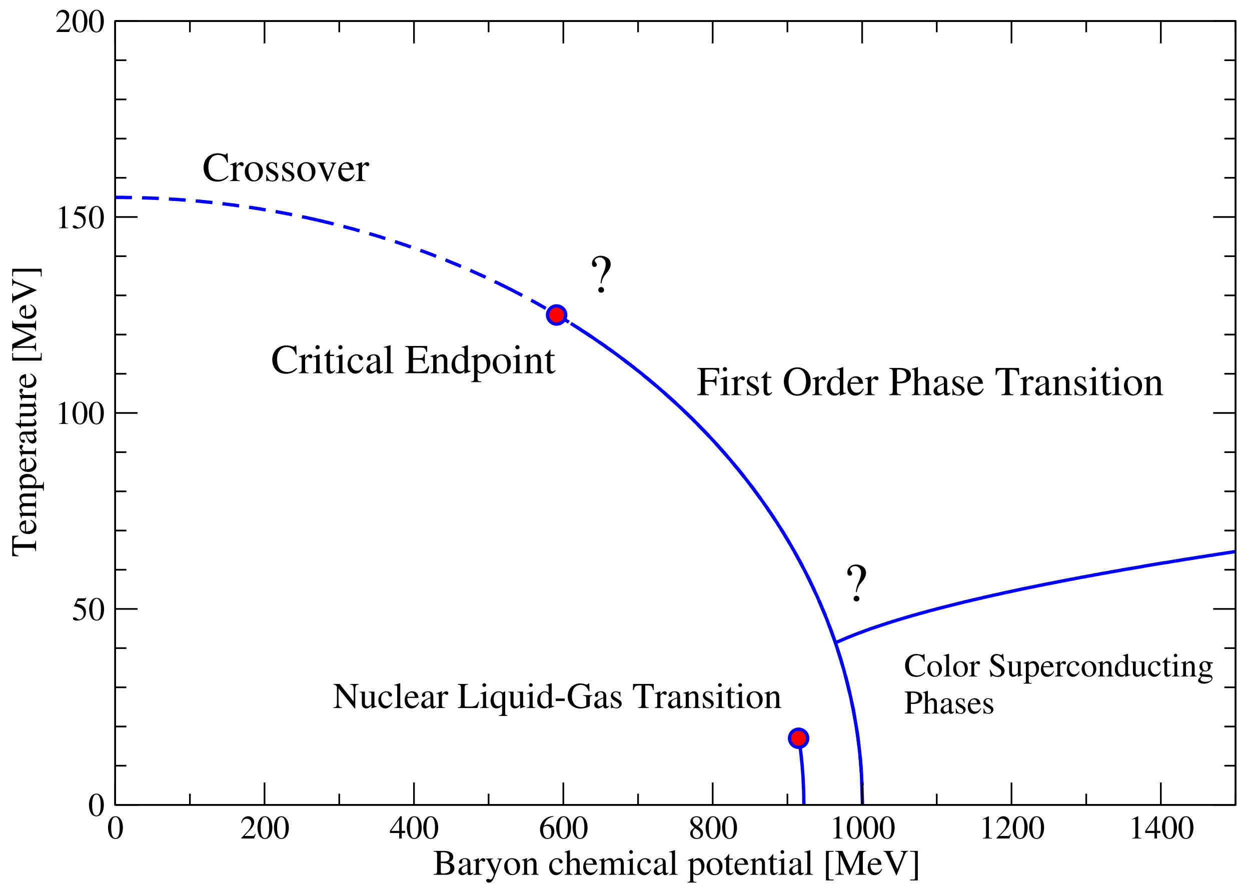
Overview

1. Introduction: dynamical mass generation

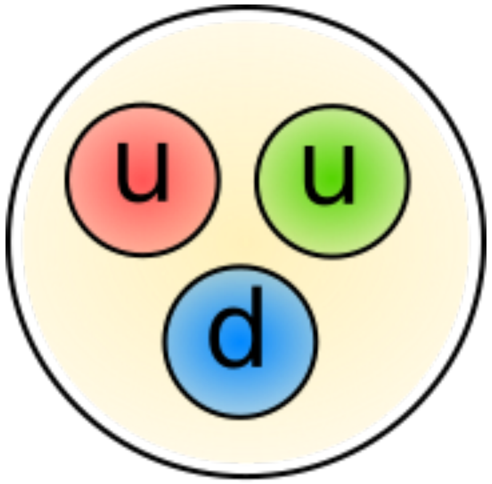


2. Large T, small μ: the quest for the critical end point

3. Small T, large μ: the quest for the equation of state



Dynamical mass generation I

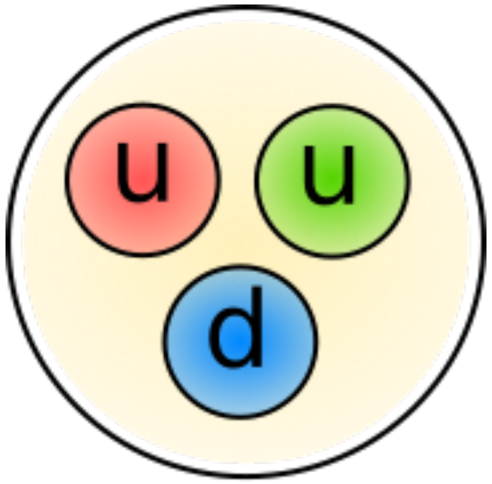


m_{proton} = 938 MeV

Dynamical quark masses via weak force

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000

Dynamical mass generation I

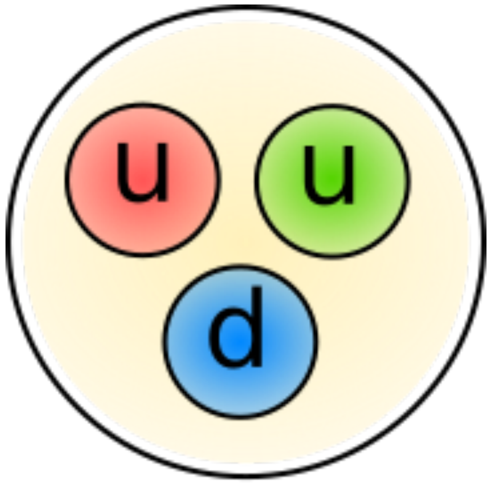


$m_{proton} = 938$ MeV

Dynamical quark masses via weak force and strong force:

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000
M_{strong} [MeV]	350	350	350	350	350	350

Dynamical mass generation I



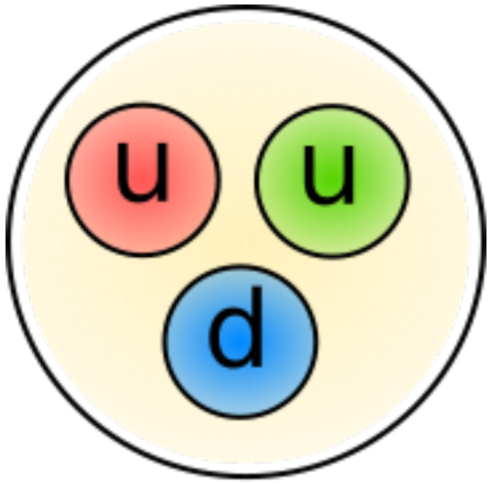
$m_{proton} = 938$ MeV

Yoichiro Nambu,
Nobel prize 2008

Dynamical quark masses via weak force and strong force:

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000
M_{strong} [MeV]	350	350	350	350	350	350

Dynamical mass generation I

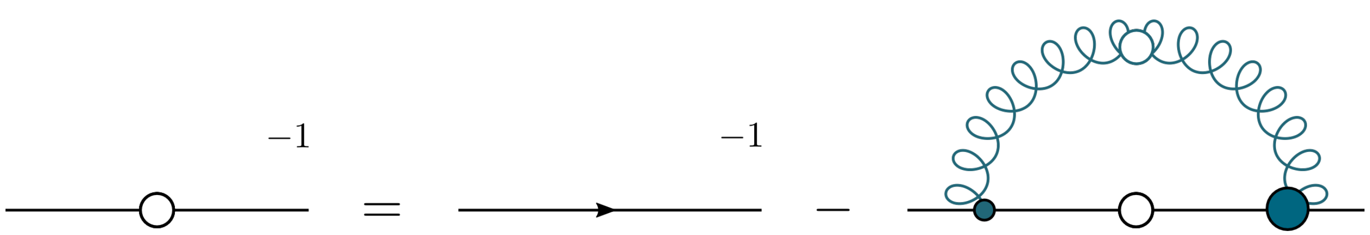


m_{proton} = 938 MeV

Yoichiro Nambu,
Nobel prize 2008

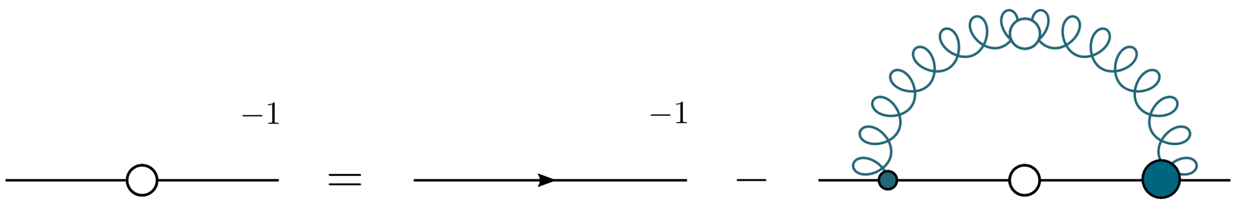
Dynamical quark masses via weak force and strong force:

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000
M_{strong} [MeV]	350	350	350	350	350	350

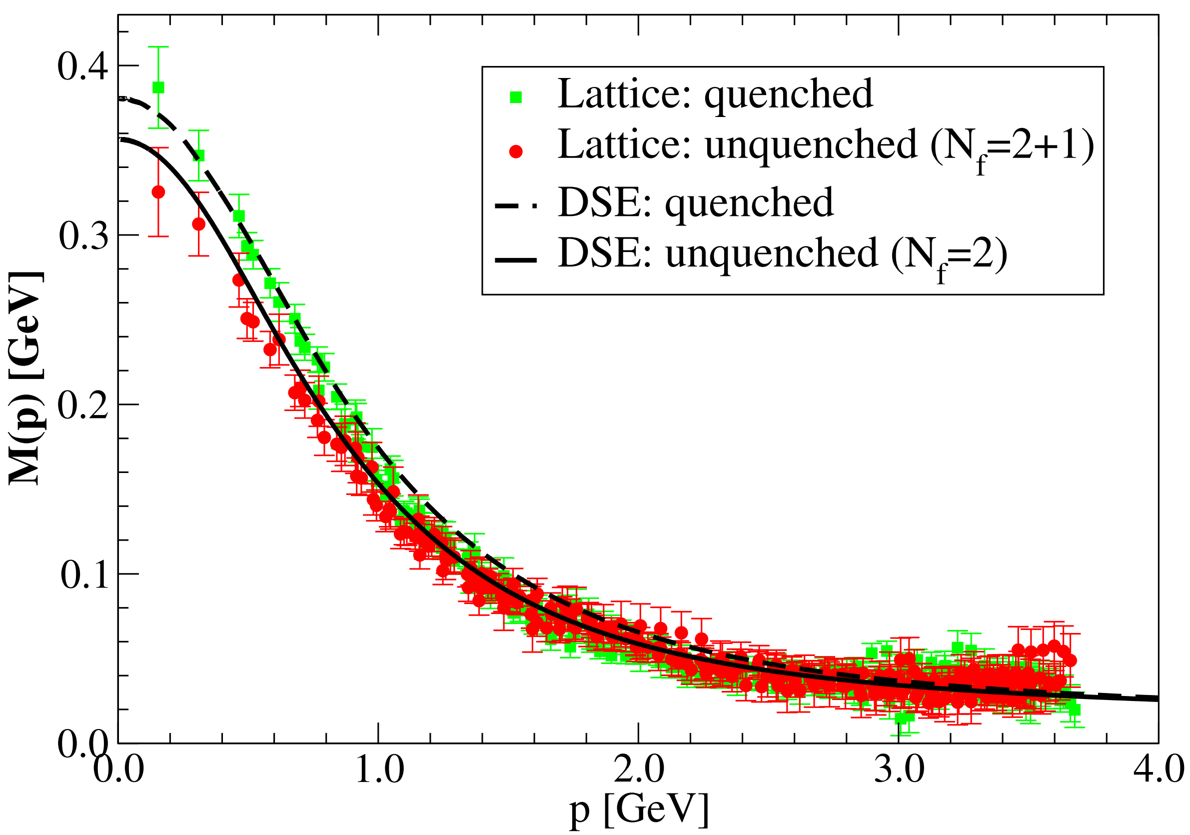


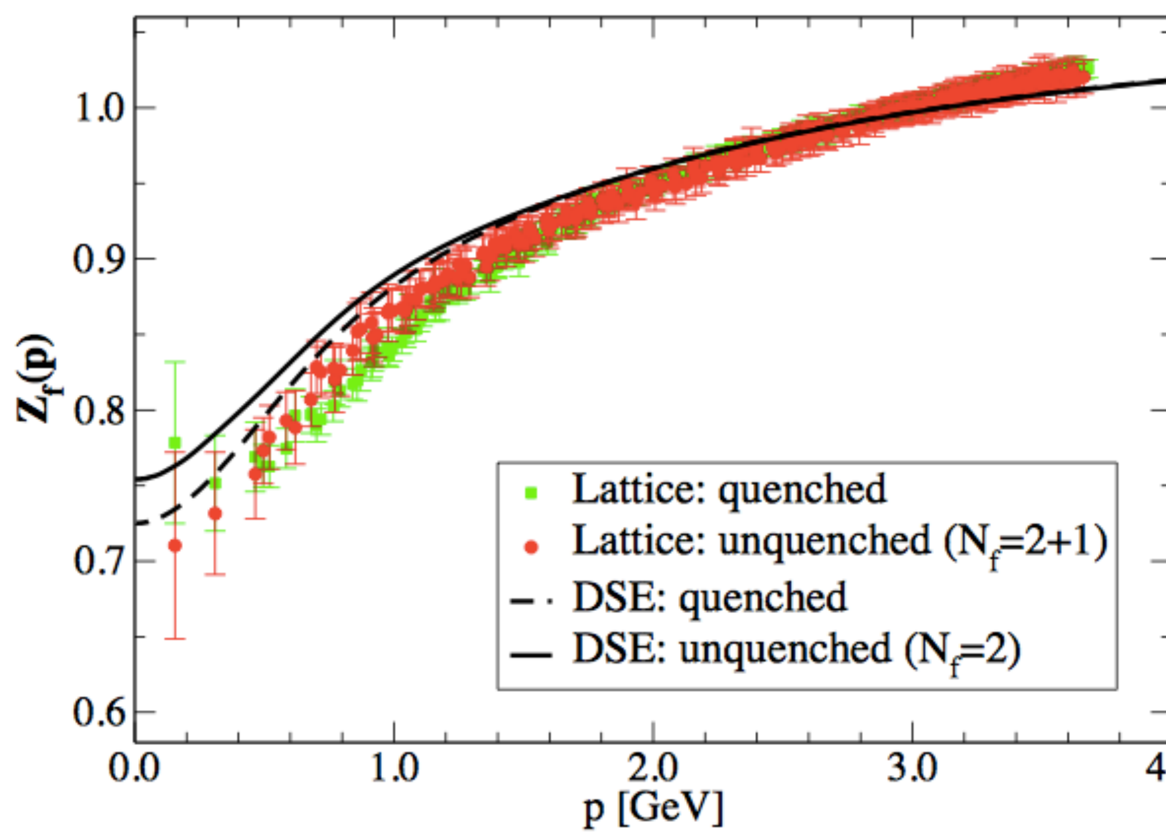
DSE

$$S^{-1}(p) = \frac{(i\not{p} + M(p^2))}{Z_f(p^2)}$$



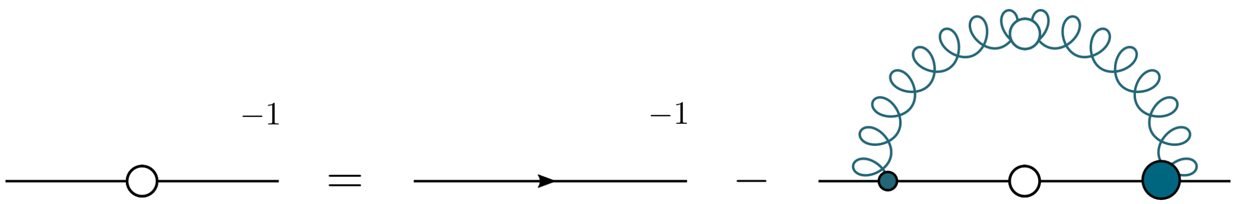
DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507



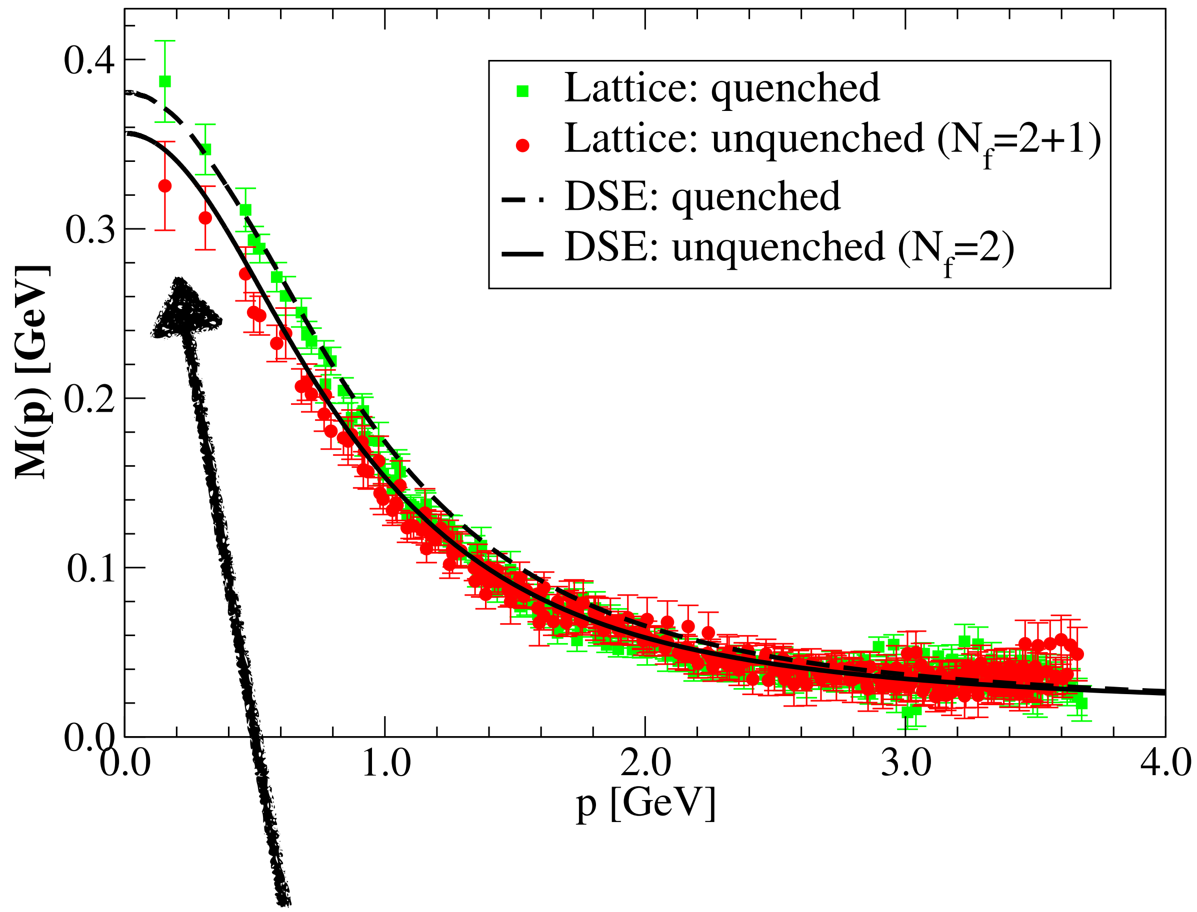


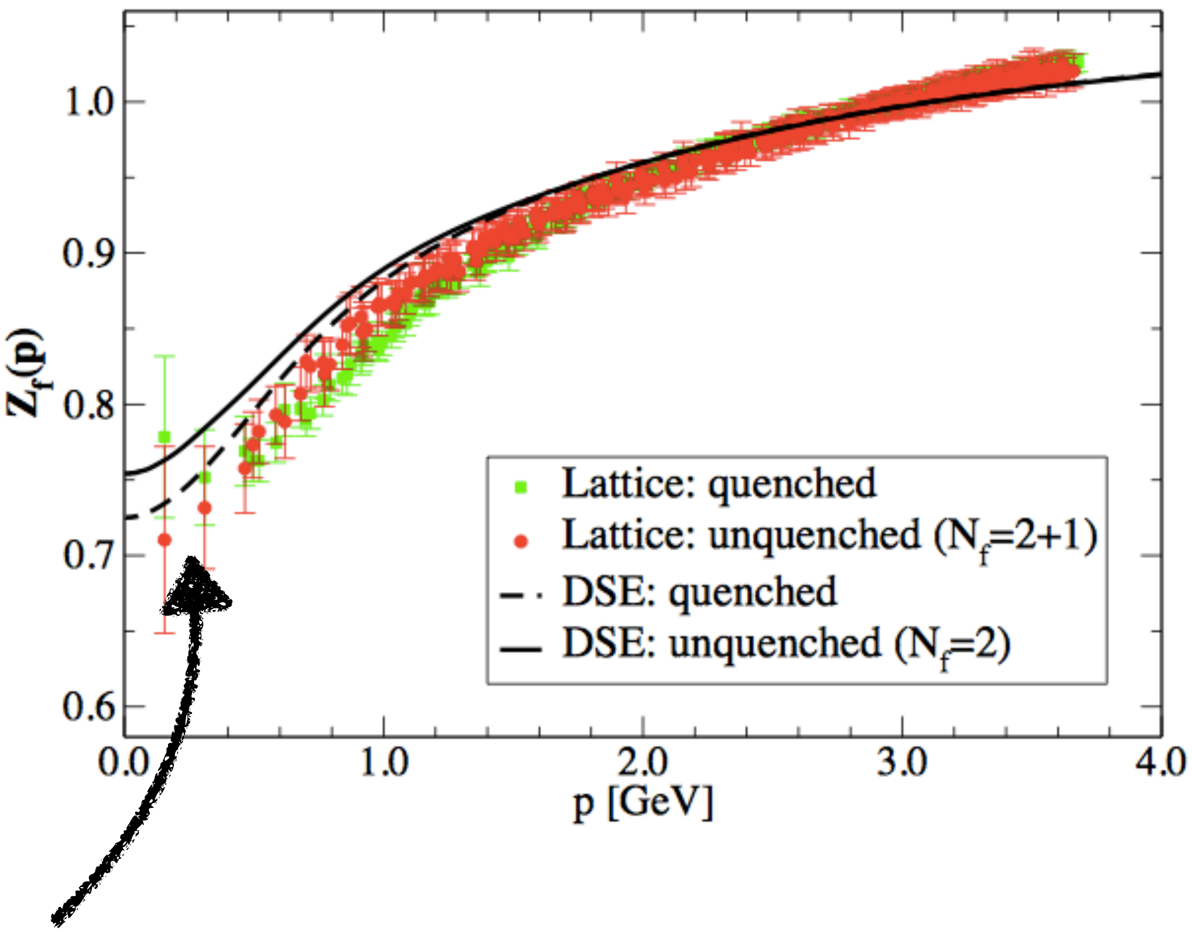
Dynamical mass generation II

$$S^{-1}(p) = \frac{(i\not{p} + M(p^2))}{Z_f(p^2)}$$



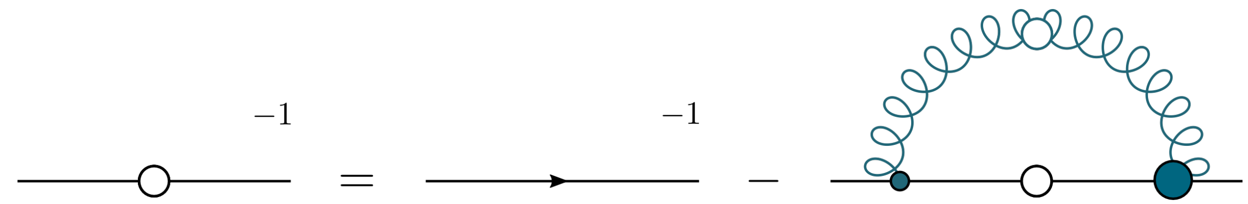
DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





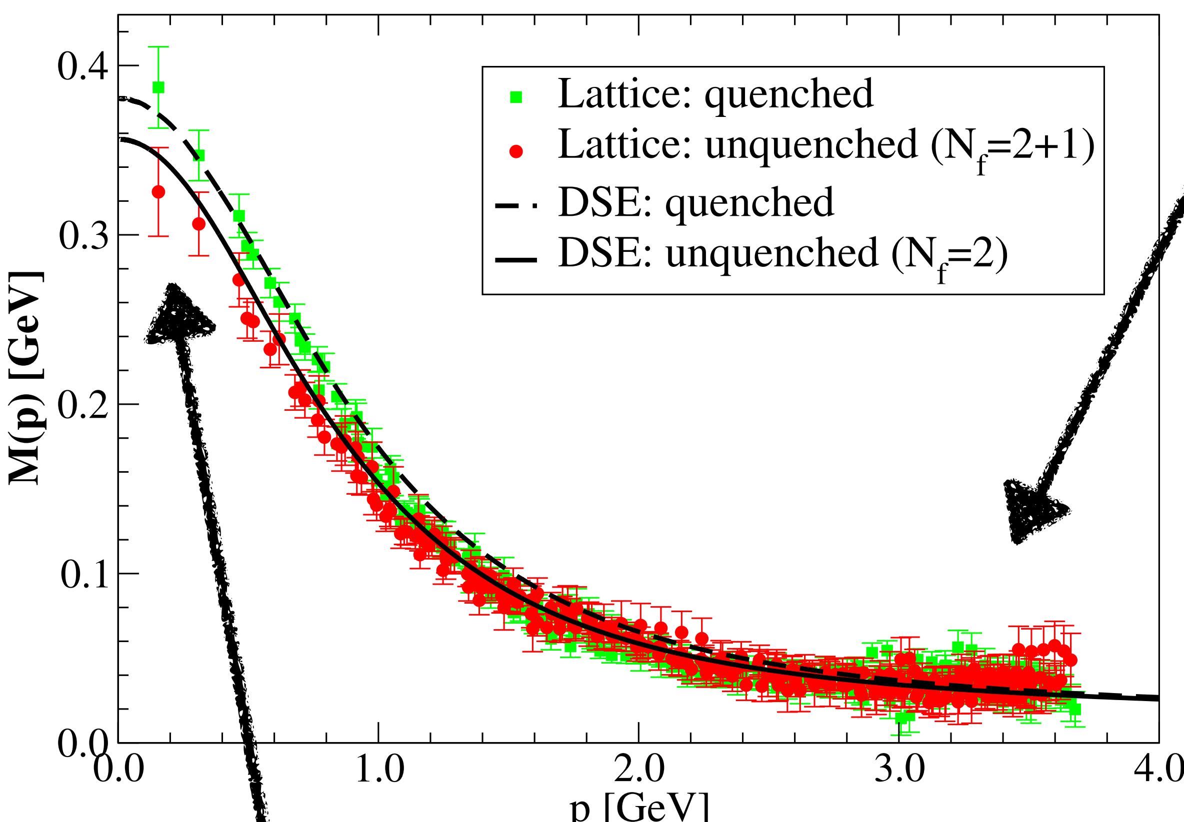
‘constituent quark’: large mass - very composite

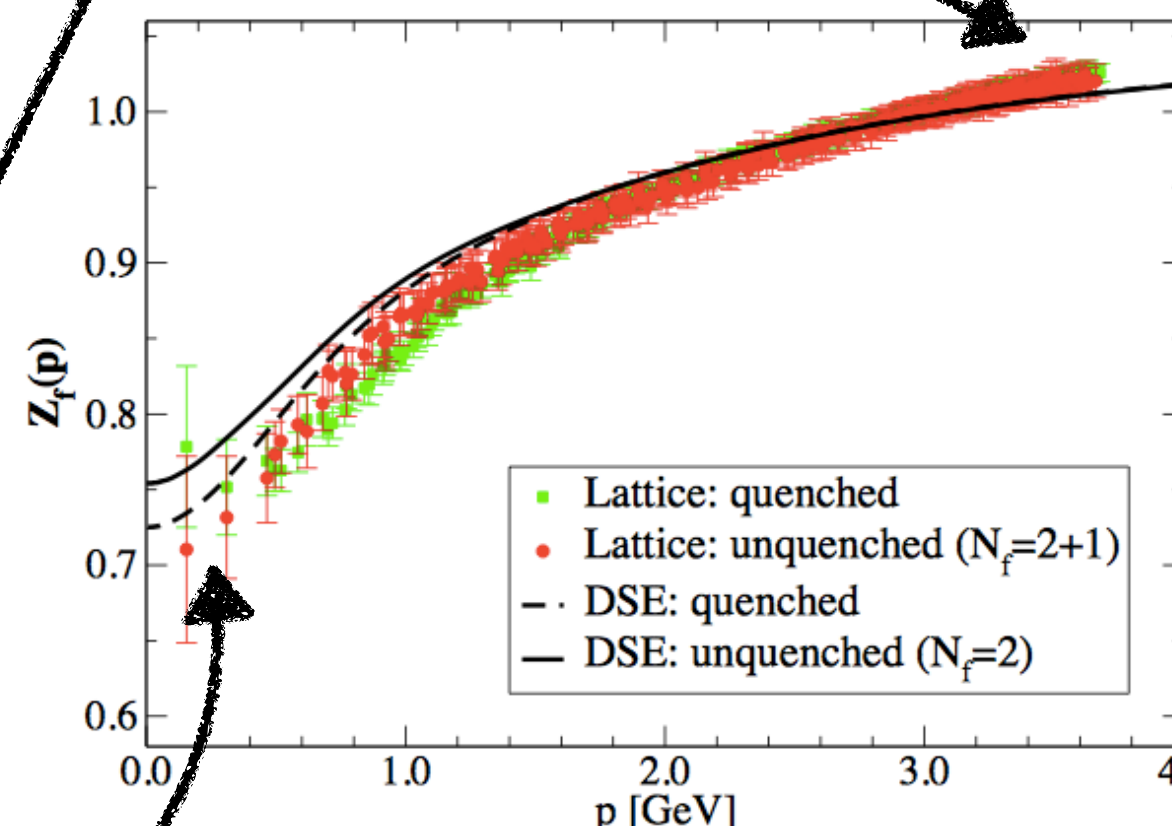
$$S^{-1}(p) = \frac{(i\not{p} + M(p^2))}{Z_f(p^2)}$$



DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

‘current quark’: small mass; non-composite



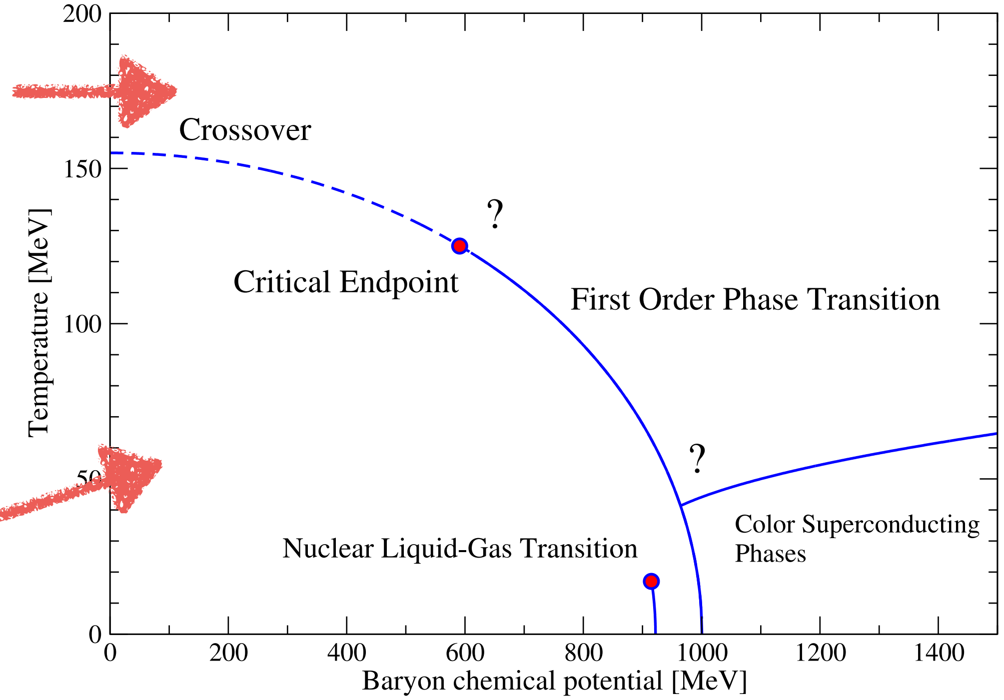


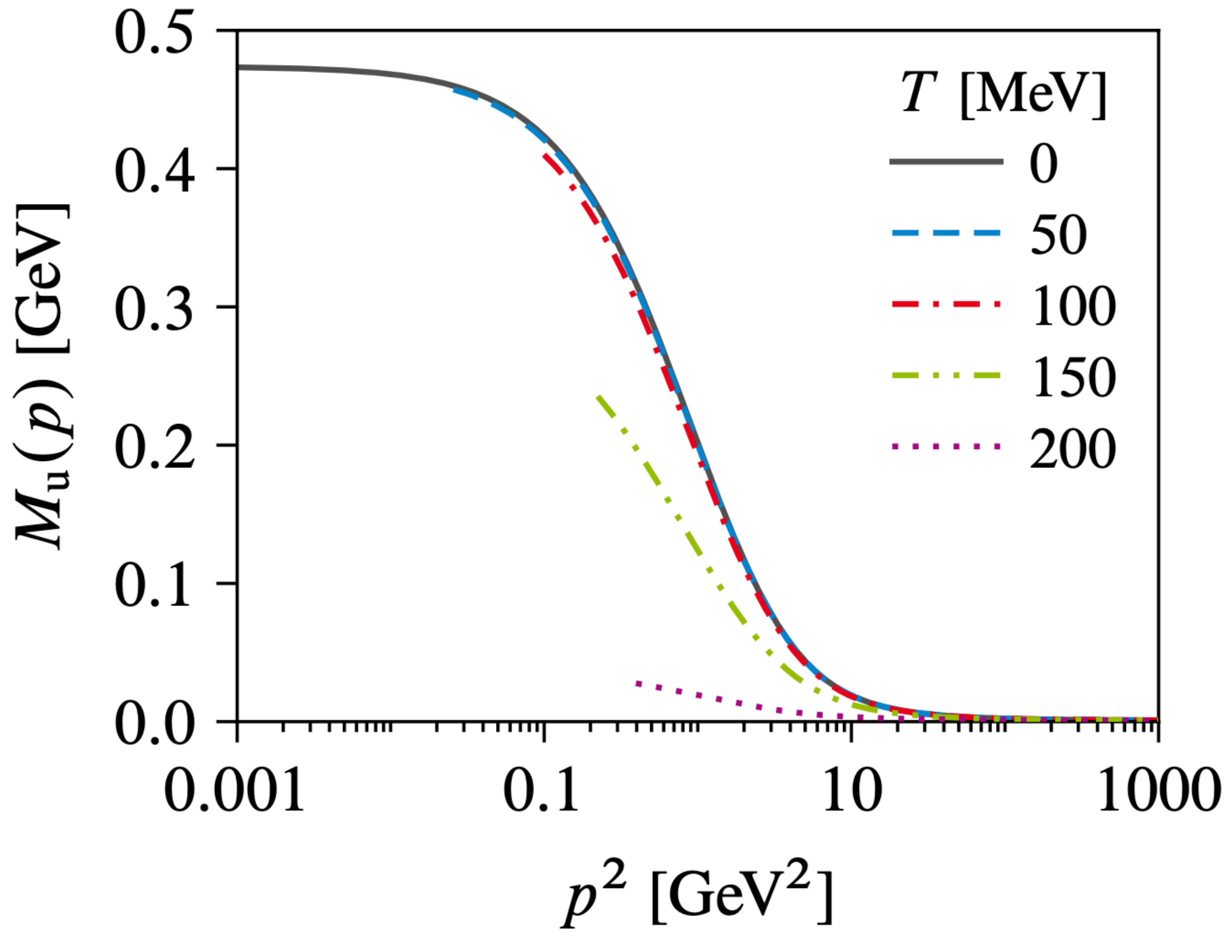
‘constituent quark’: large mass - very composite

QCD phase transitions: 3 quark flavors

Quarks (almost) massless

Quarks massive

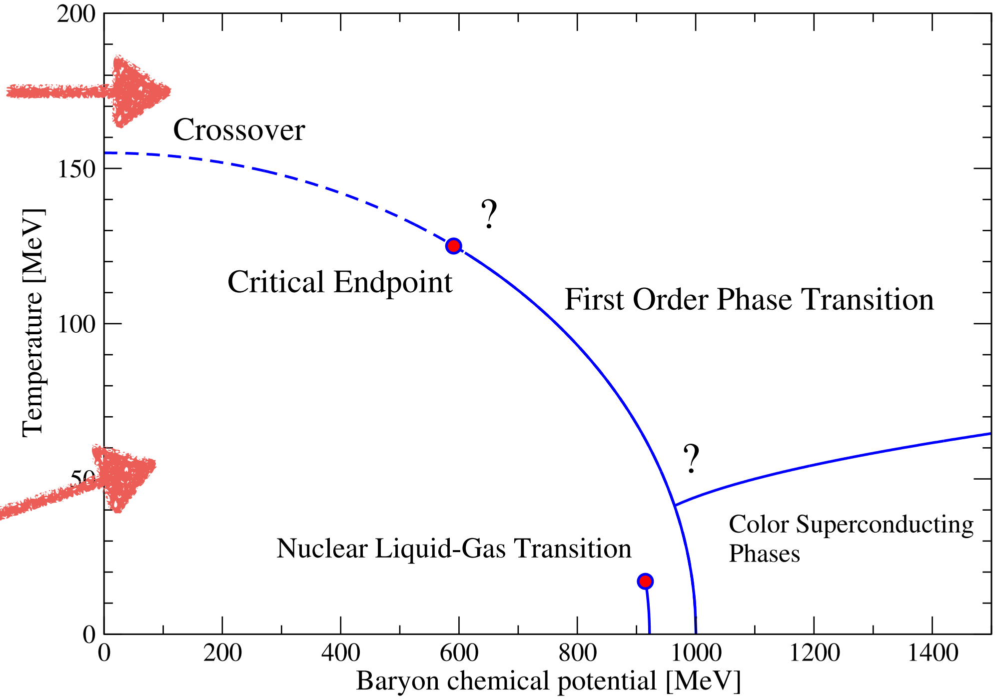


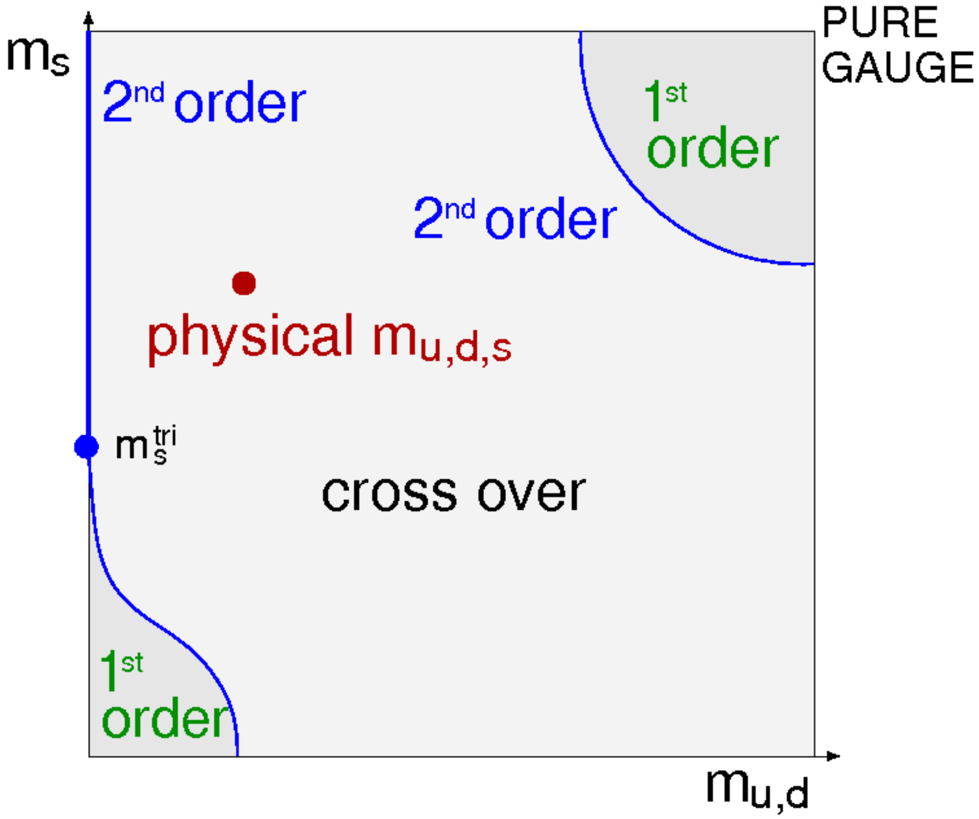


QCD phase transitions: 3 quark flavors

Quarks (almost) massless

Quarks massive

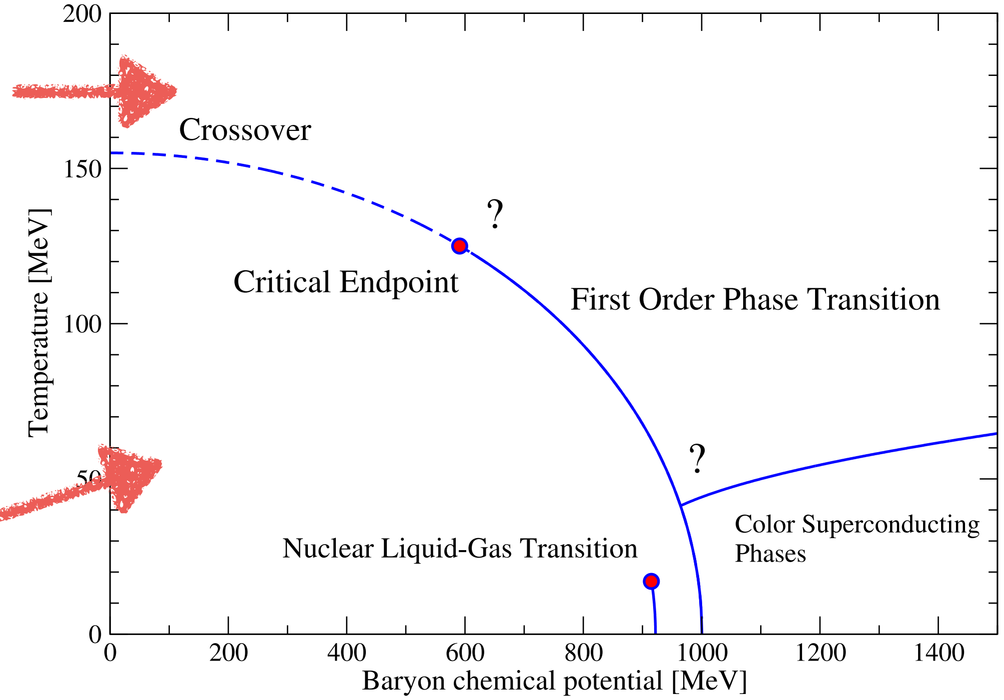


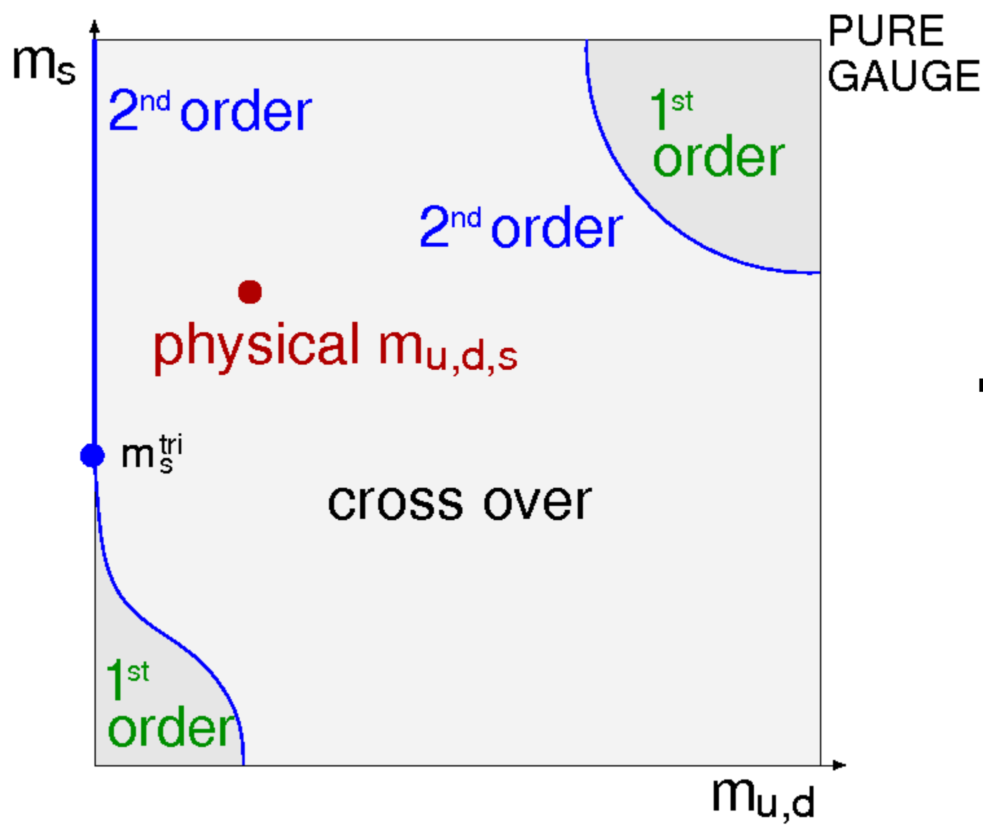


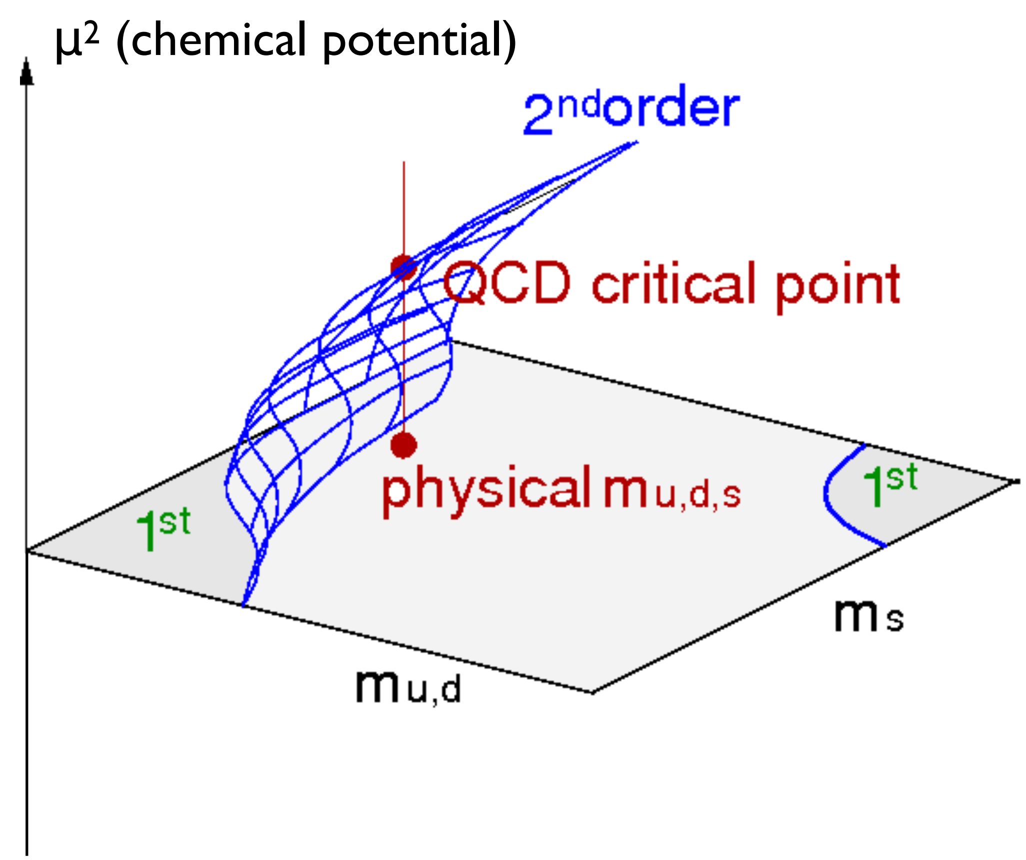
QCD phase transitions: 3 quark flavors

Quarks (almost) massless

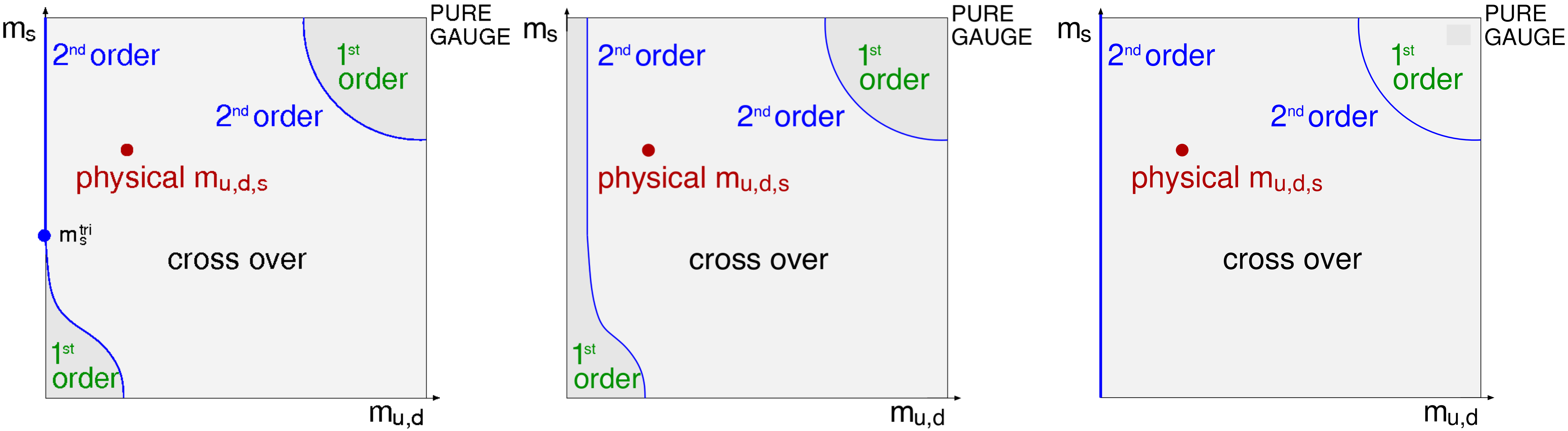
Quarks massive







QCD phase transitions



$U_A(1)$ broken at T_c $U_A(1)$ restored at T_c

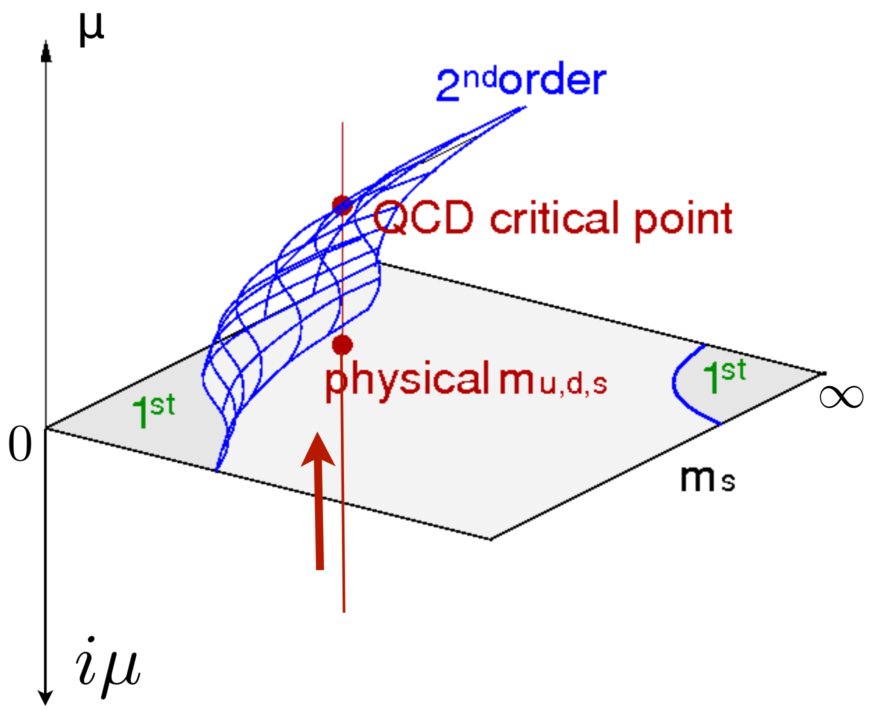
Is there chiral 1^{st} order at all?

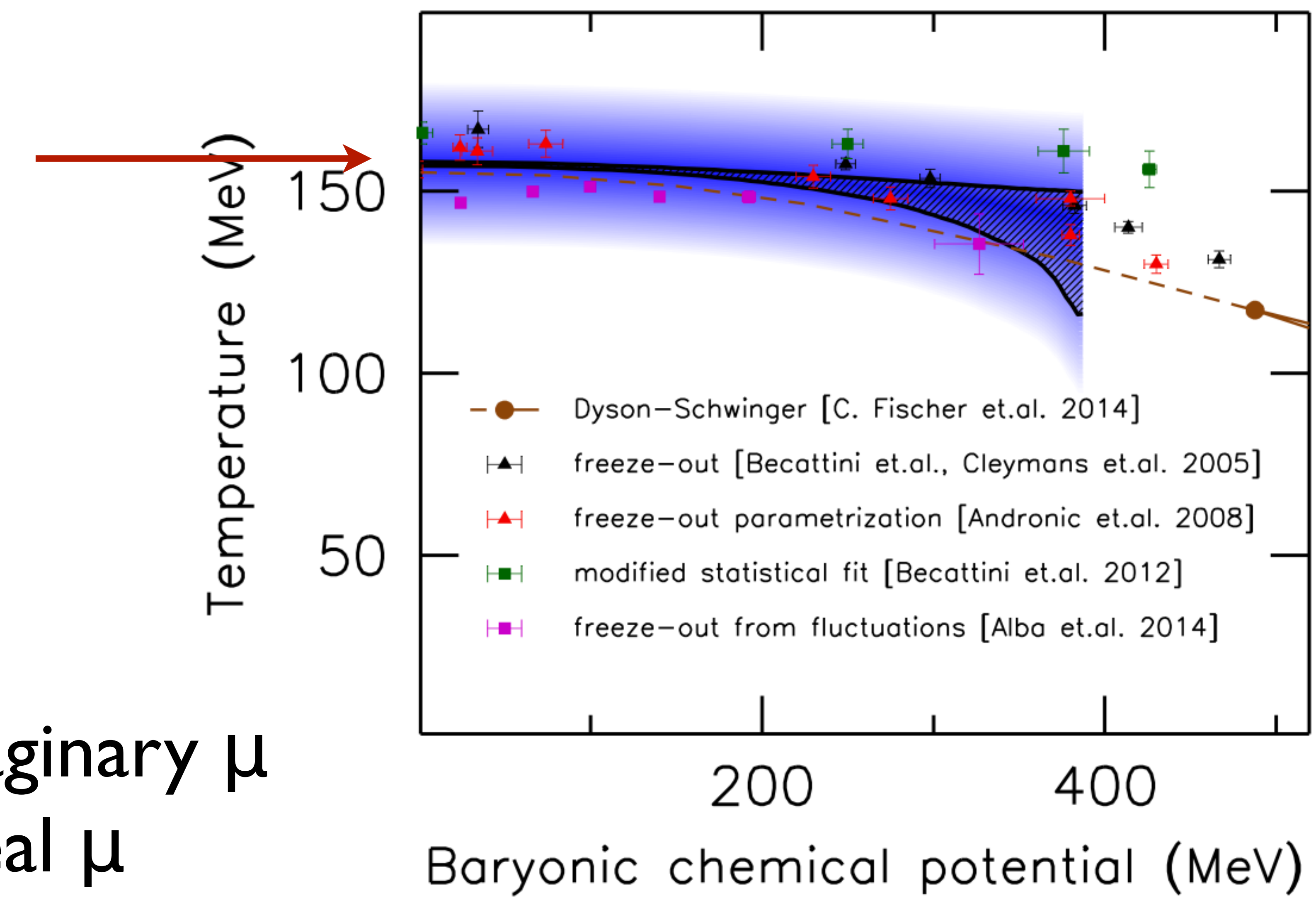
Pisarski and Wilczek, PRD 29 (1984), 338-341
Resch, Rennecke and Schaefer, PRD 99 (2019)

and many more…

Cuteri, Philipsen and Sciarra, JHEP 11 (2021), 141
Dini, et al, PRD 105 (2022) no.3, 034510
Fejos, PRD 105 (2022) no.7, L071506

Chiral transition line from analytic continuation





Bellwied, Borsanyi, Fodor, Günther, Katz, Ratti and Szabo, PLB 751 (2015) 559

Lattice method:

- Det. crossover at imaginary μ and extrapolate to real μ
- Control systematics

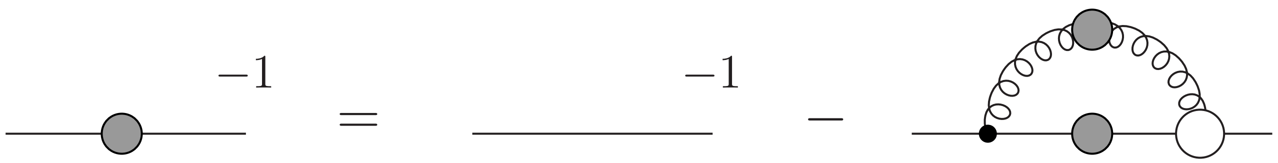
HOT-QCD: similar results

Main result:

- No transition for $\mu_B/T < 2\text{-}3$

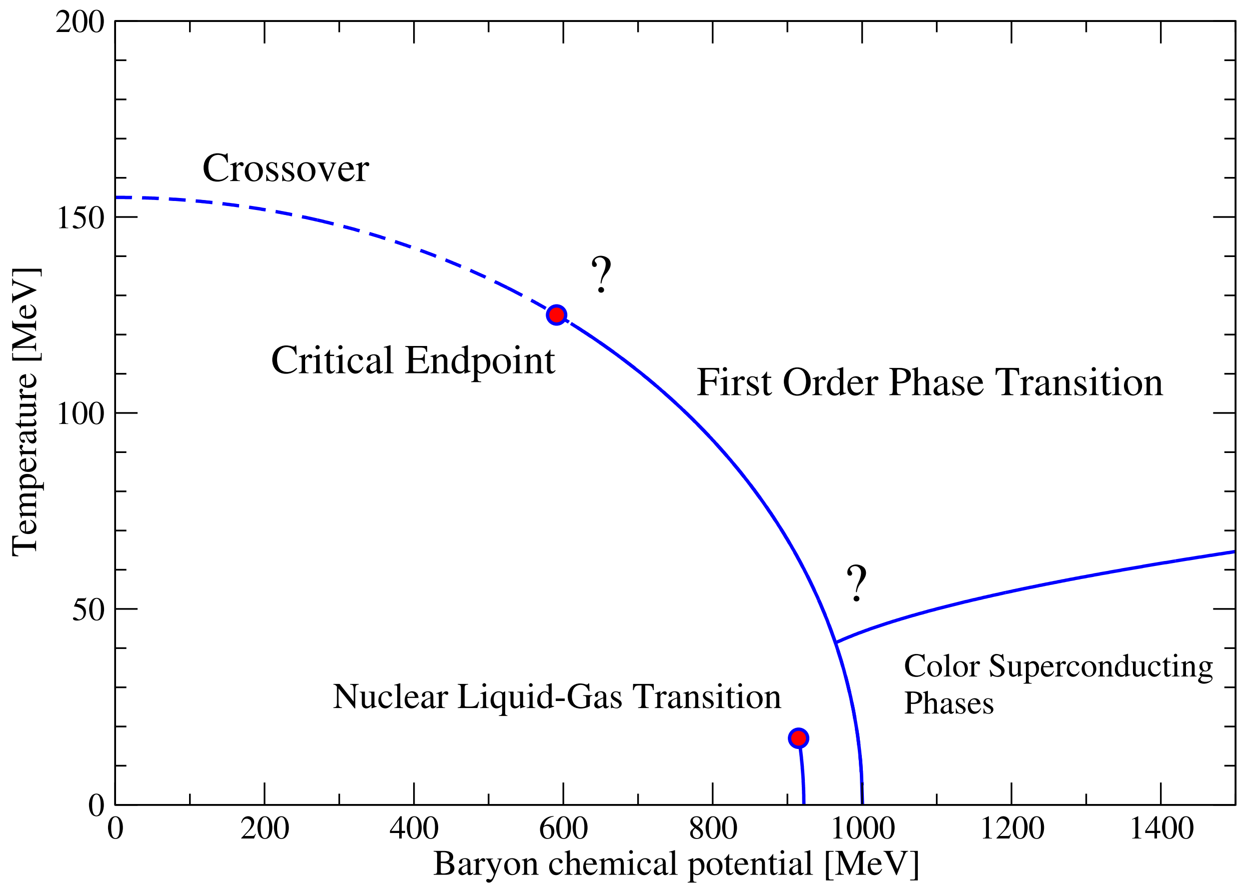
Overview

1.Introduction: dynamical mass generation

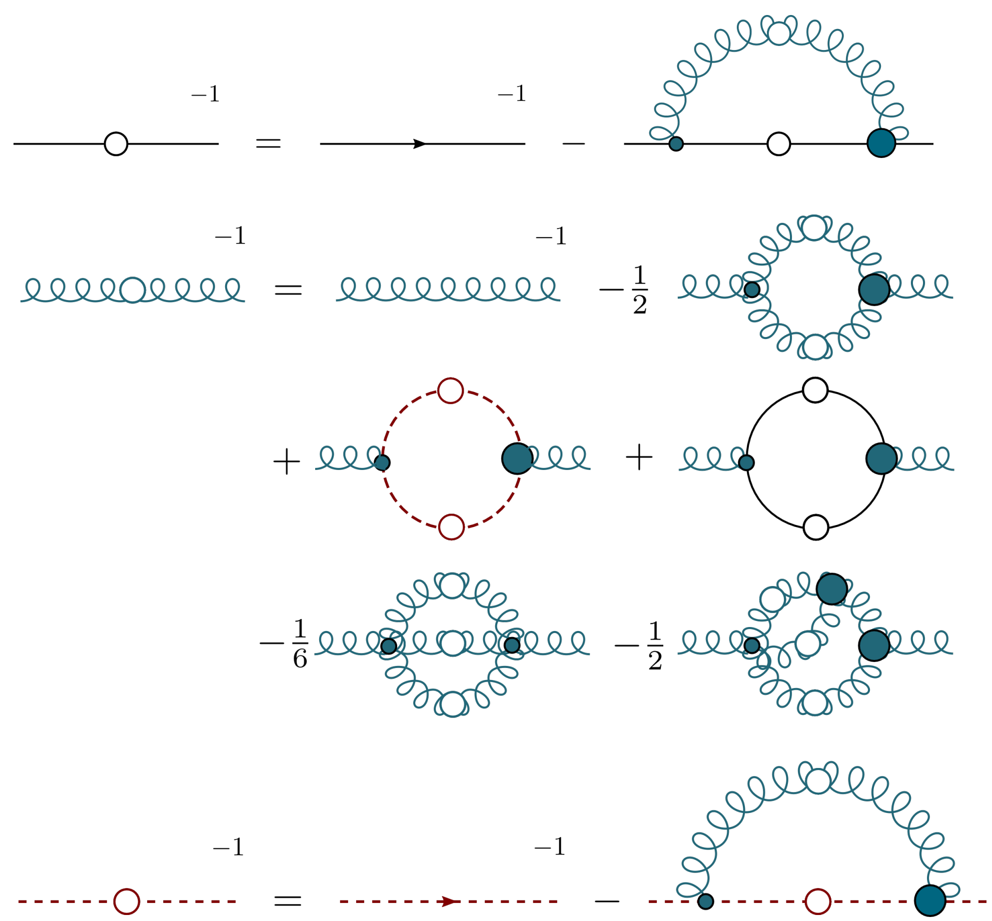


2.Large T, small μ: the quest for the critical end point

3.Small T, large μ: the quest for the equation of state

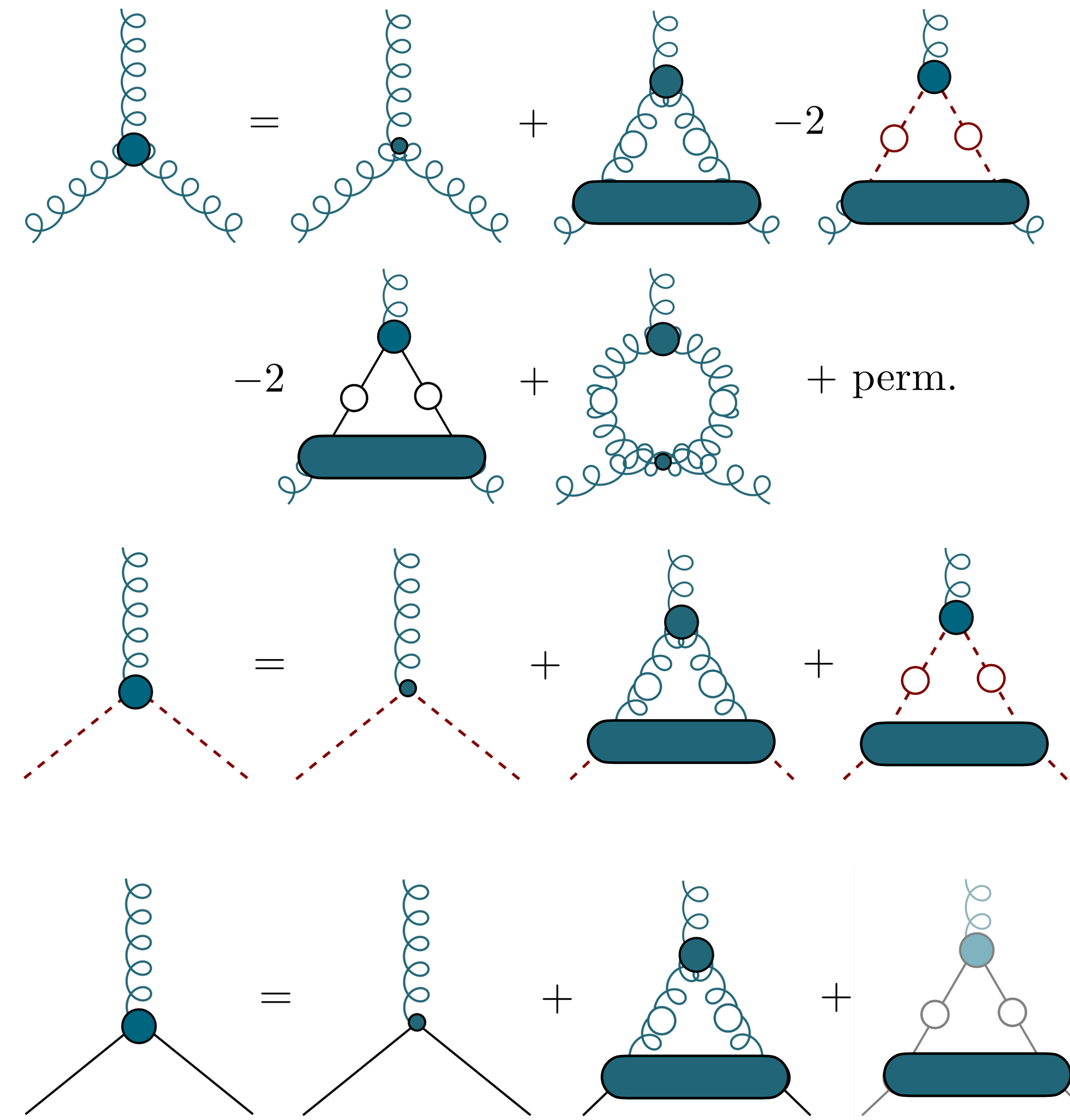


propagators



for different BRL approaches see work of
Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister, Huber, Maas, Mitter, Papavassiliou, Pawlowski, Roberts, Smekal, Strodthoff, Vujinovic, Watson, Williams…

vertices

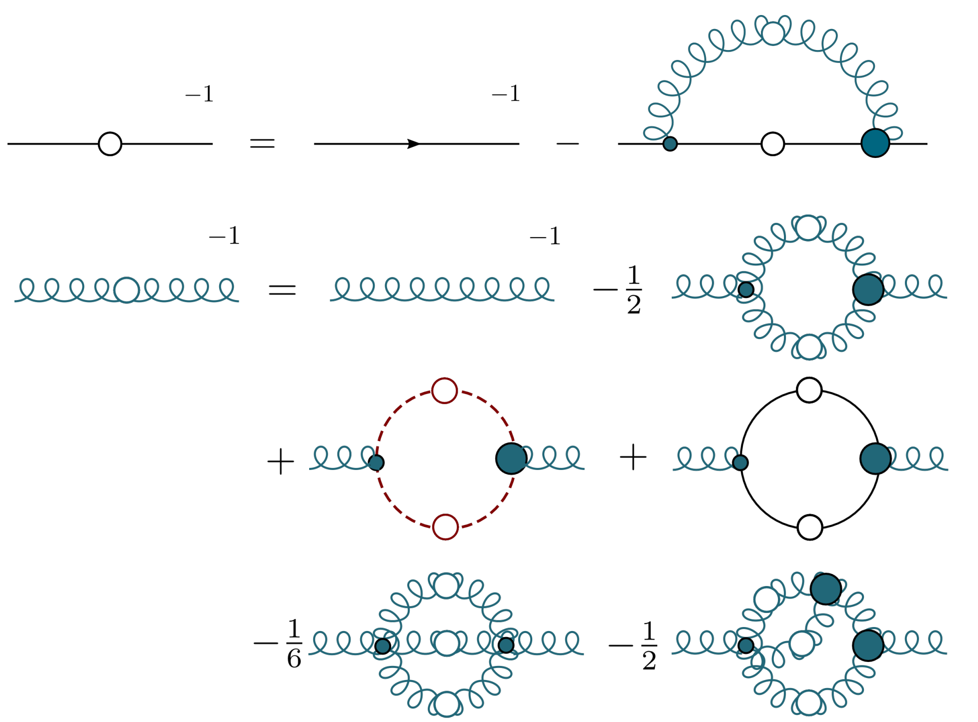


Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

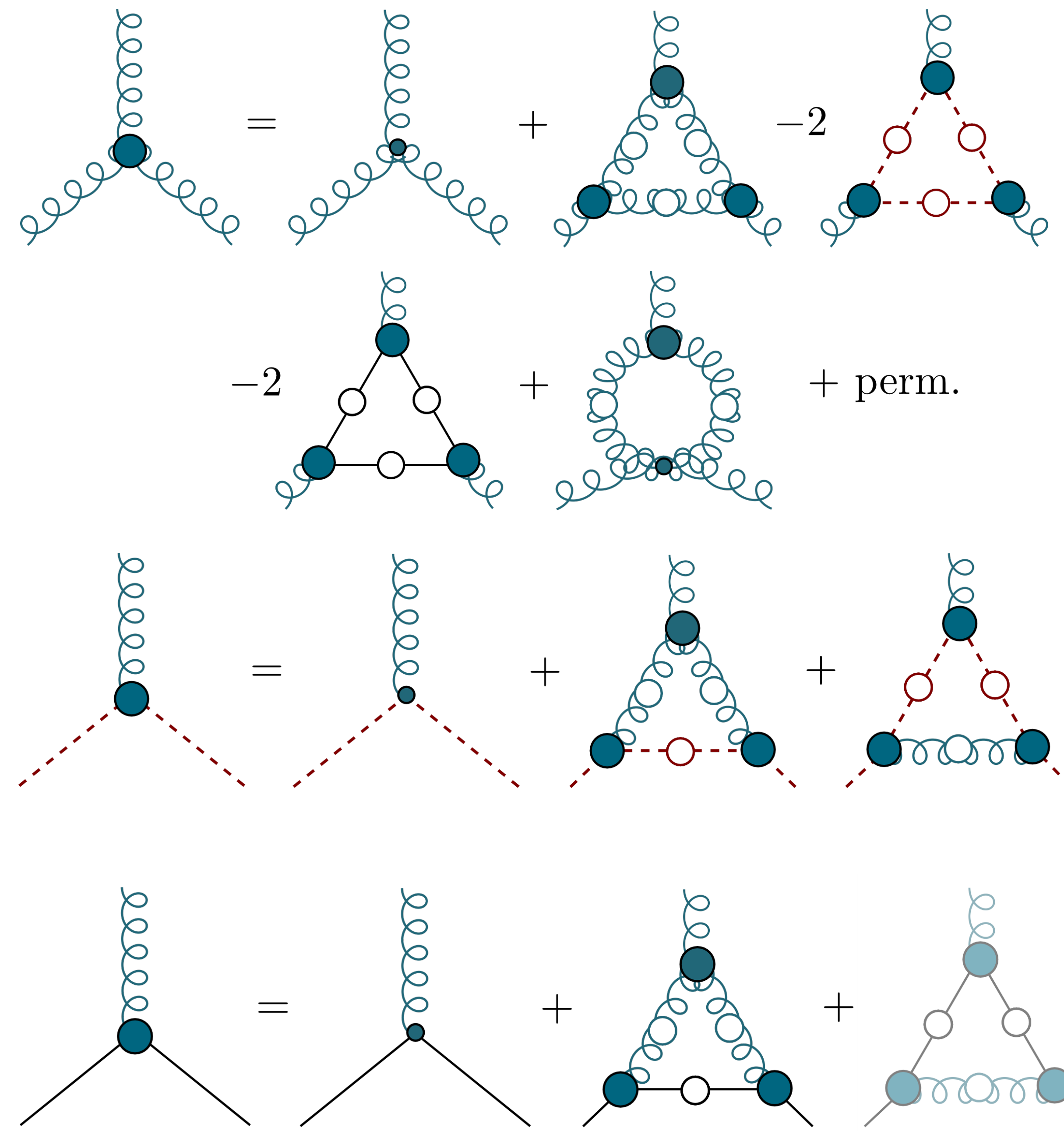
QCD with functional methods (T=0, μ=0)

propagators



for different BRL approaches see work of
Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister, Huber, Maas, Mitter, Papavassiliou, Pawlowski, Roberts, Smekal, Strodthoff, Vujinovic, Watson, Williams…

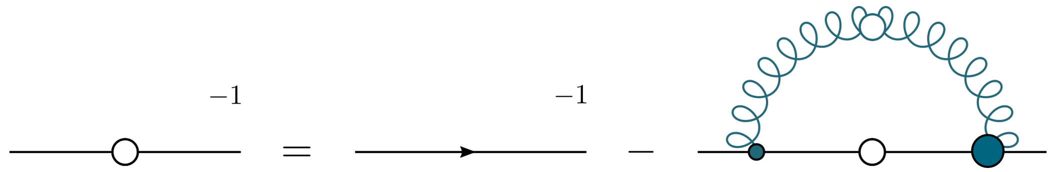
vertices

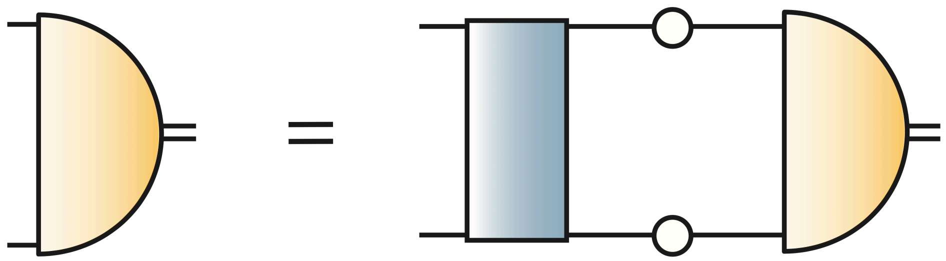


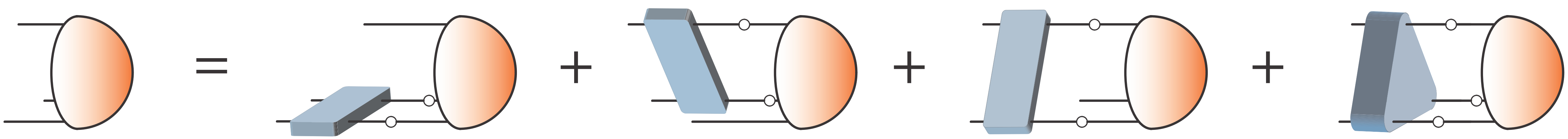
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

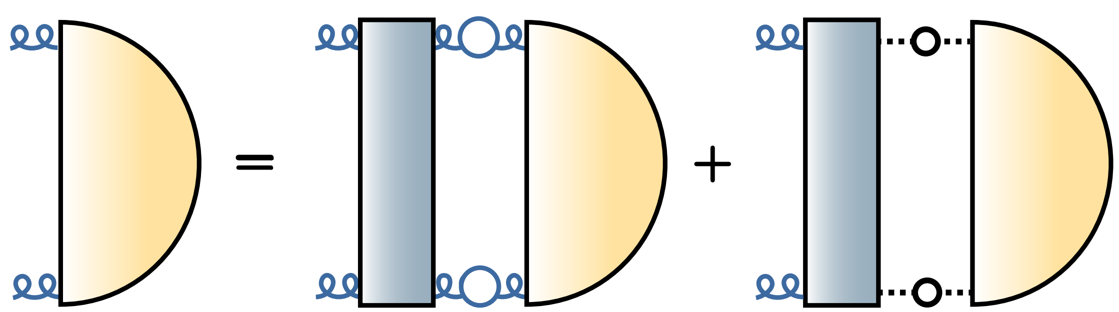
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

BSEs:





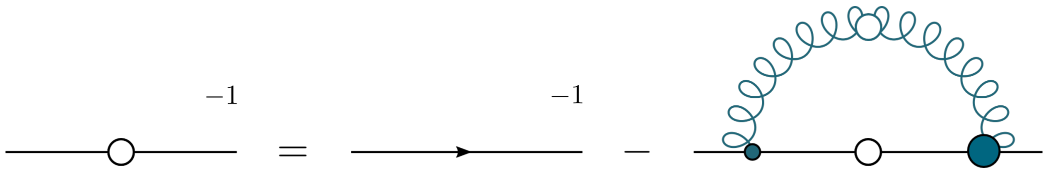




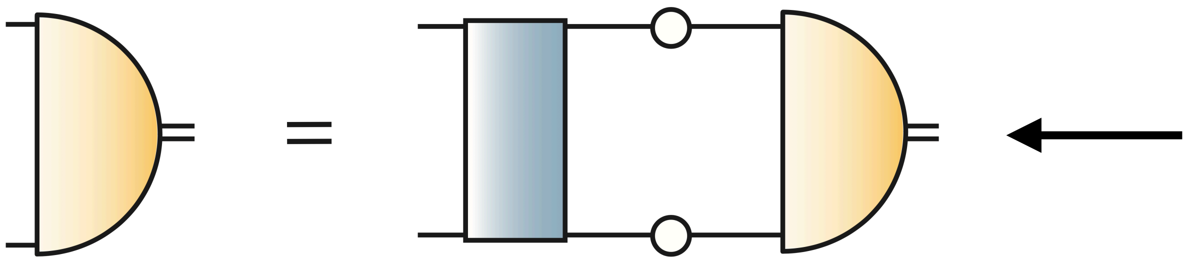
Eigenvalue equations: masses and wave functions

Bound states and Bethe-Salpeter equations

BSEs:

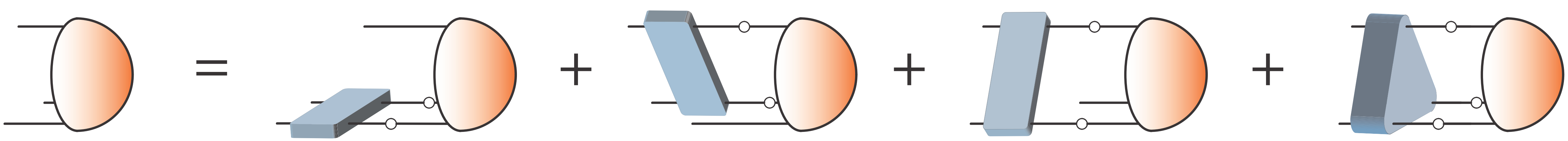


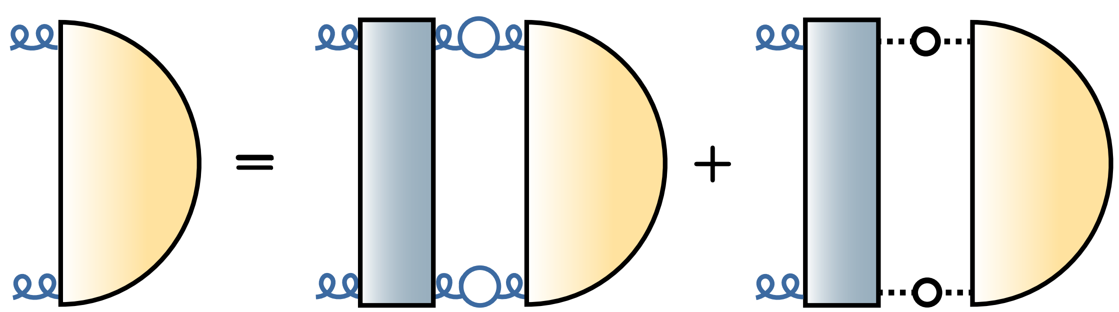
$\lambda(P^2)=1$



$P^2 = -m_\pi^2$

Bethe-Salpeter Amplitude

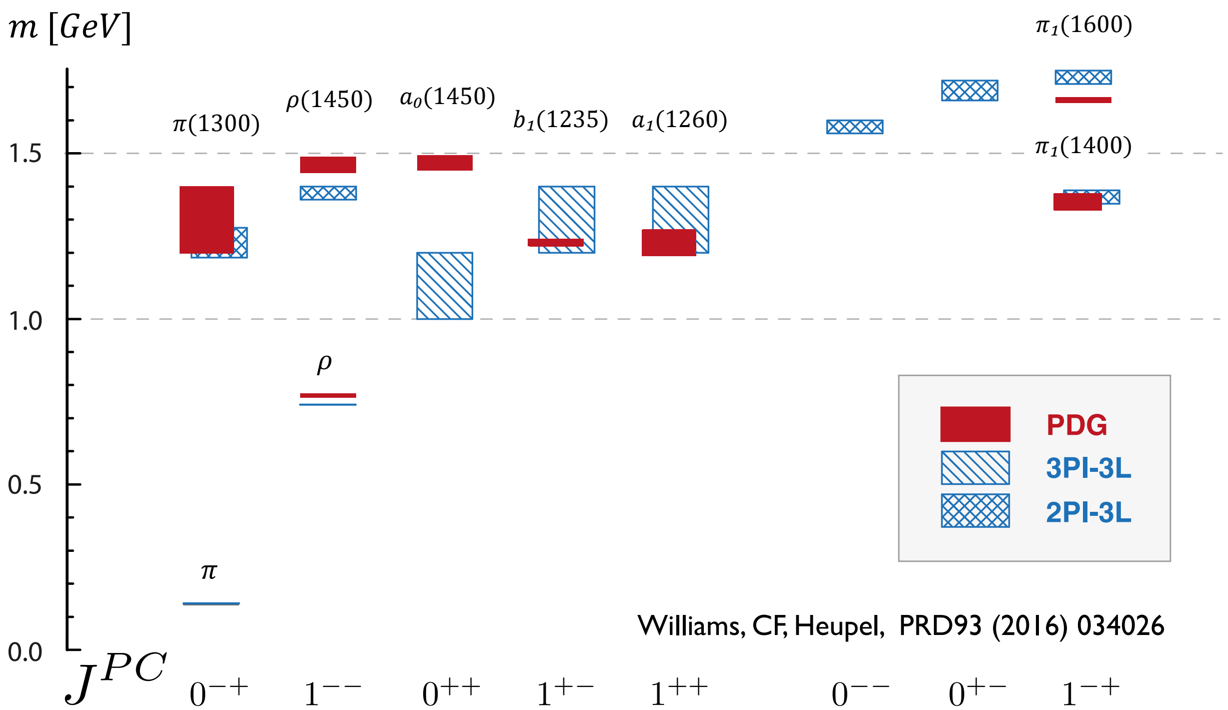




Eigenvalue equations: masses and wave functions

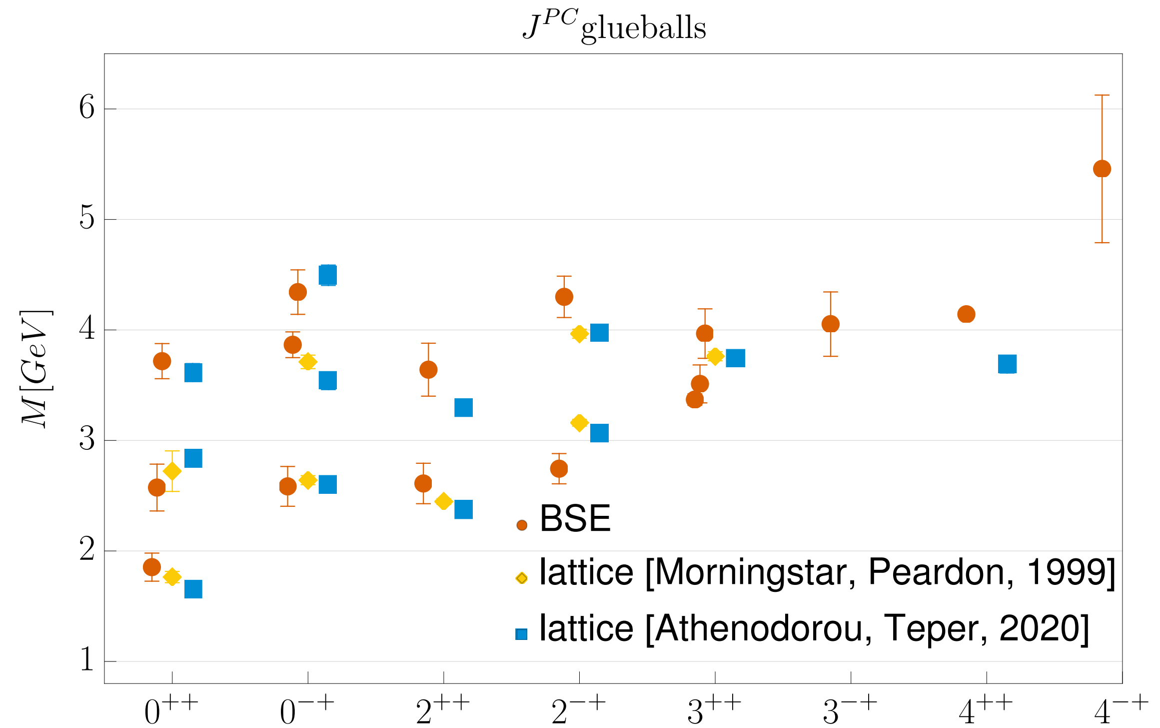
Hadron spectra: mesons, baryons, glueballs

Mesons:



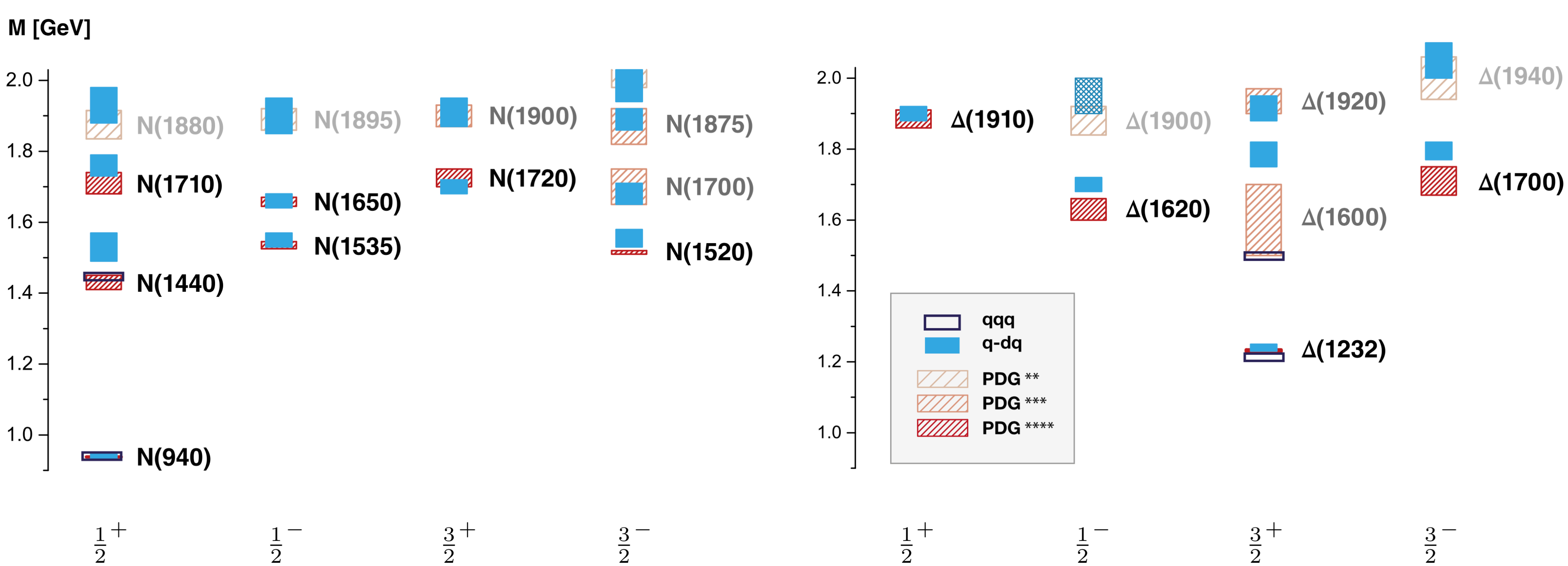
Williams, CF, Heupel, PRD93 (2016) 034026

Glueballs:



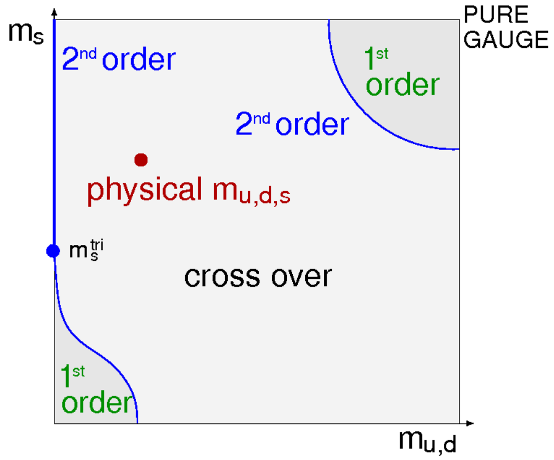
CF, Huber, Sanchis-Alepuz, EPJC 80 (2020)
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021)

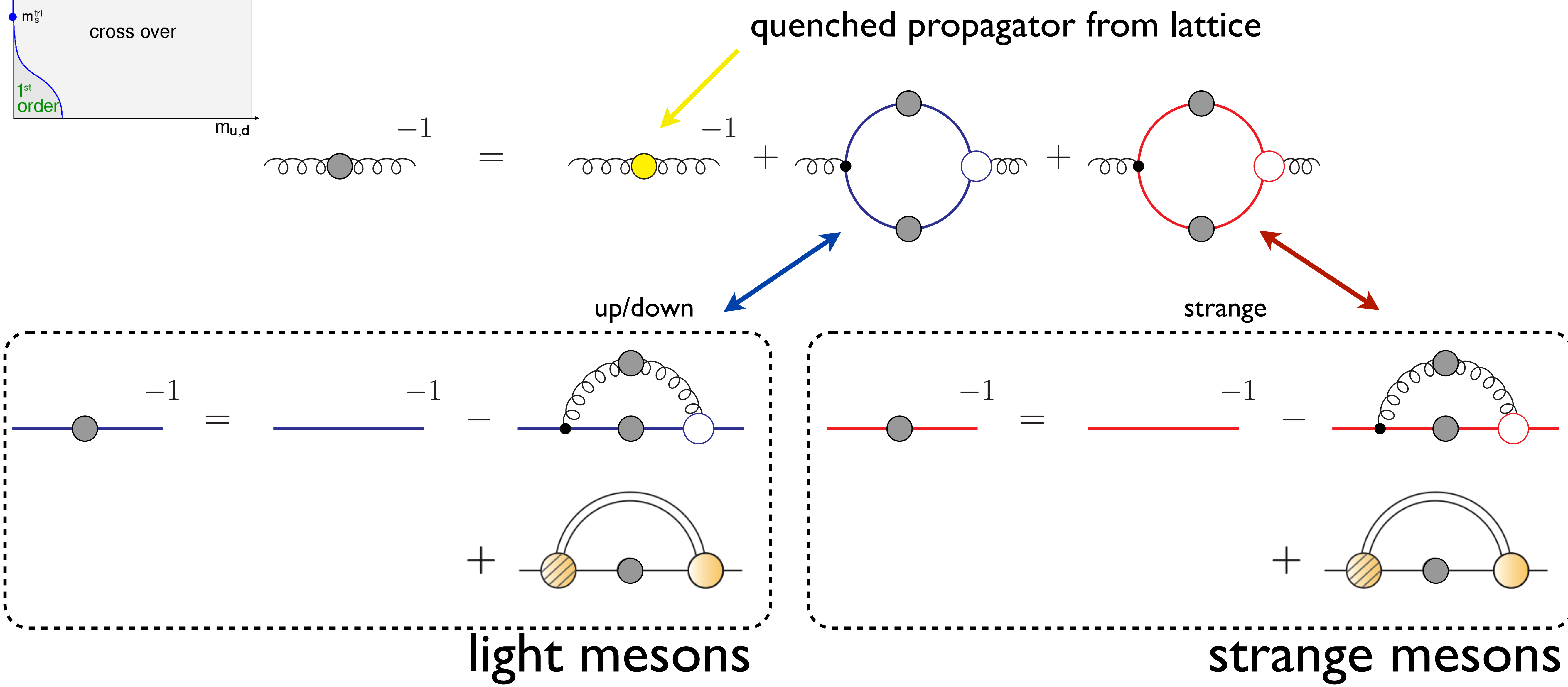
Baryons:



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

N_f=2+1-QCD with DSEs and meson backcoupling

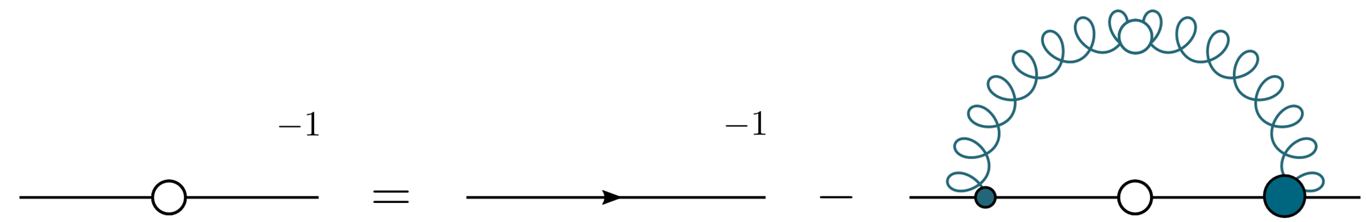




Chiral order parameter:

$$\langle \bar{\Psi}\Psi \rangle = Z_2 N_c Tr_D \frac{1}{T} \sum_{\omega} \int \frac{d^3p}{(2\pi)^3} S(\vec{p},\omega)$$

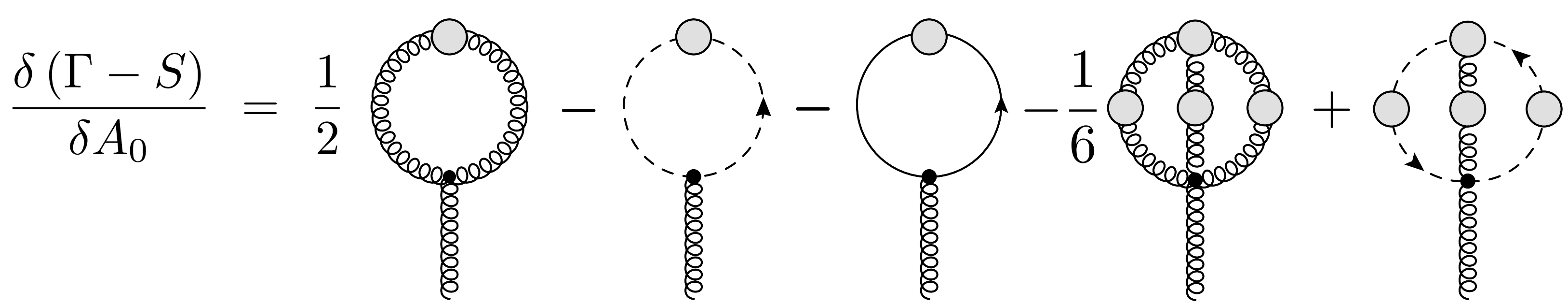
spatially homogenuous



Deconfinement:

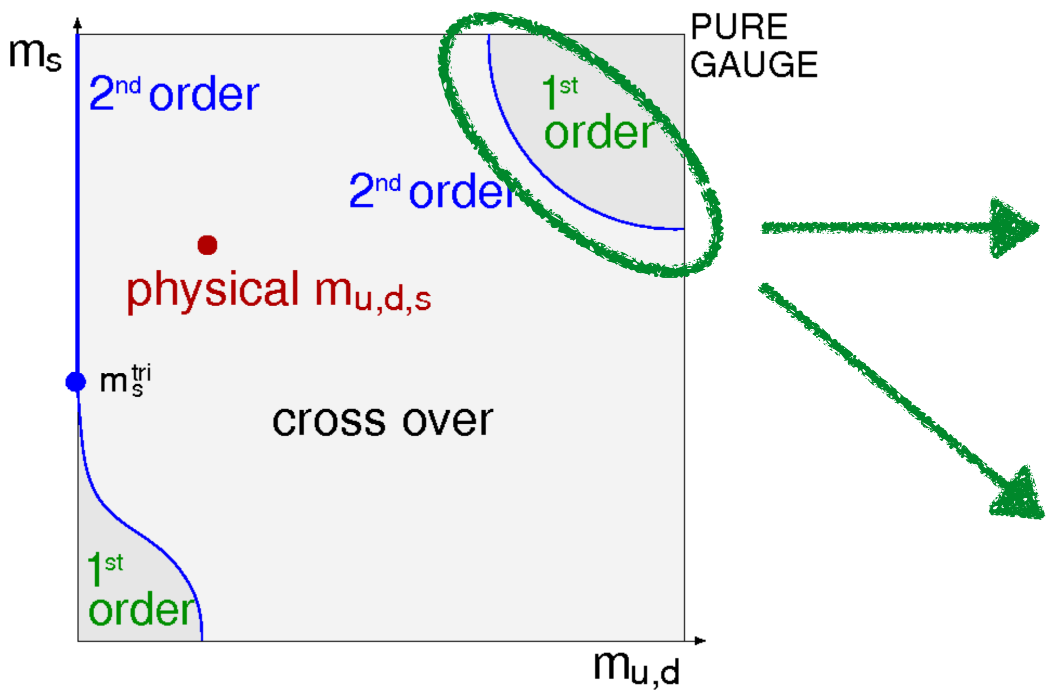
- Polyakov loop potential

$$L = \frac{1}{N_c} Tr\, e^{ig\beta A_0}$$

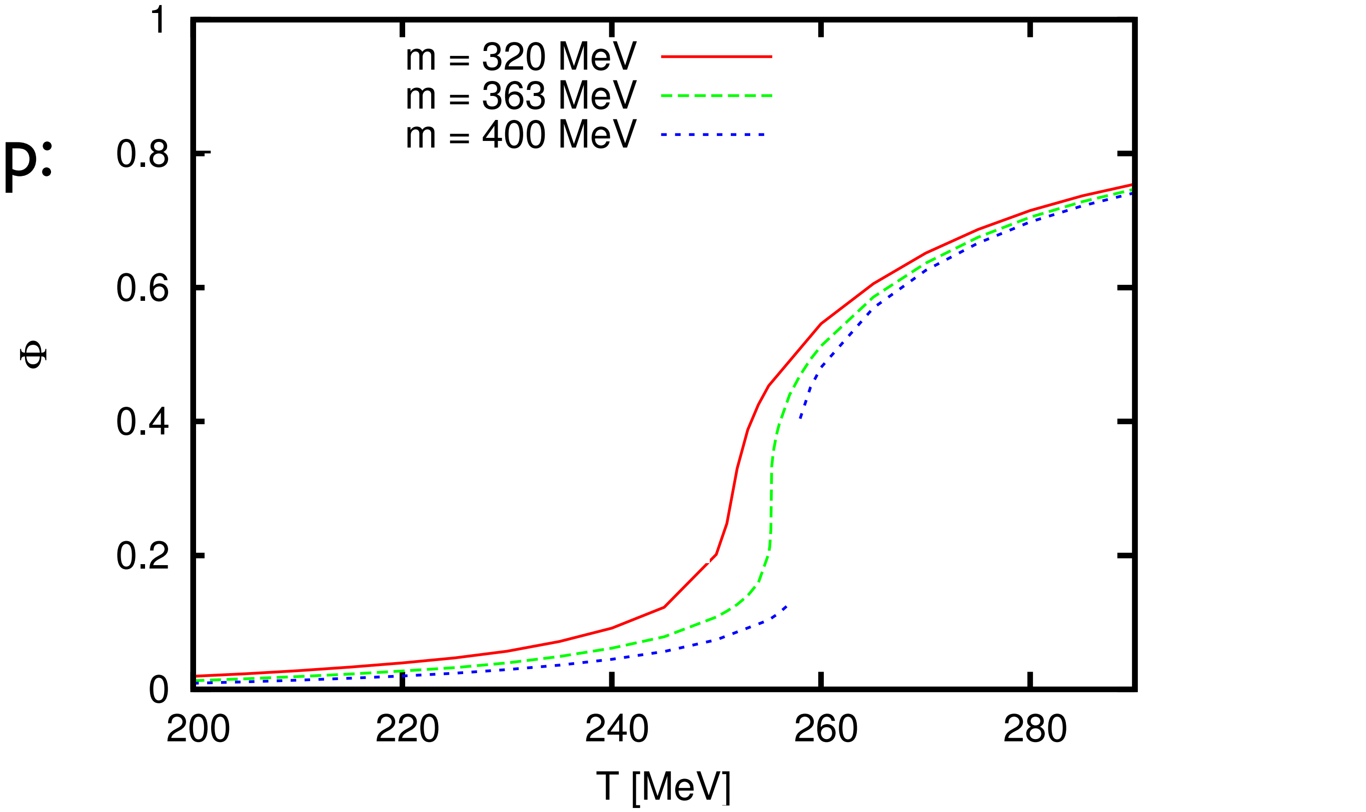


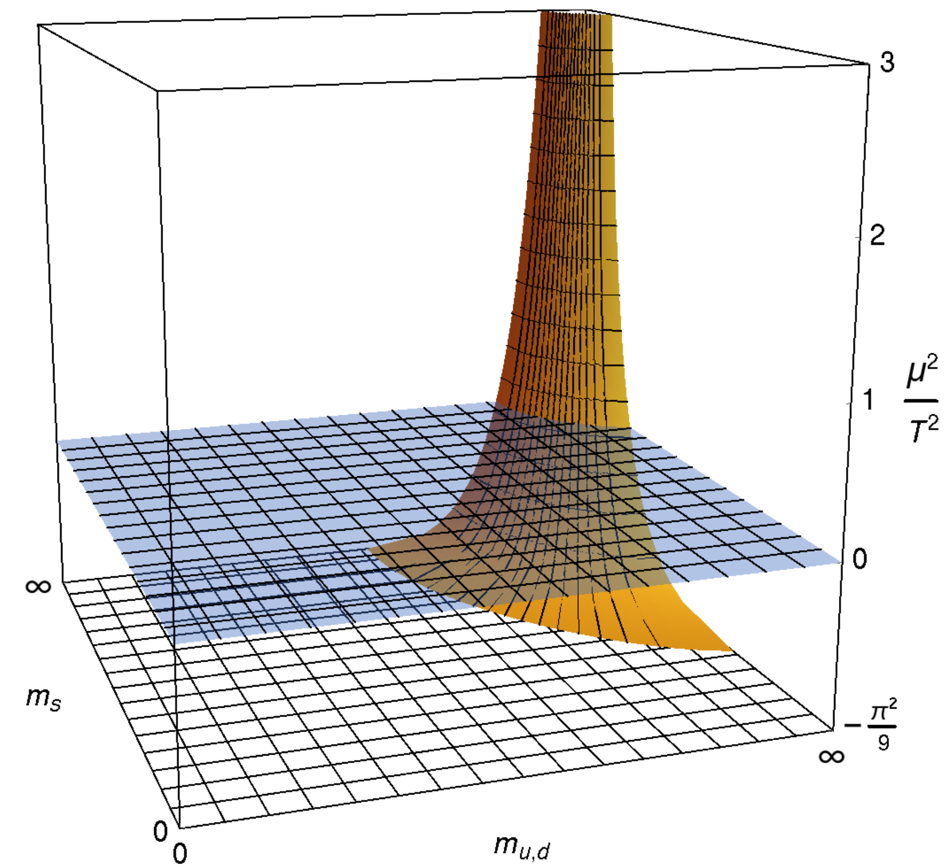
Braun, Gies, Pawlowski, PLB 684, 262 (2010)
Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)
Fister, Pawlowski, PRD 88 045010 (2013)
CF, Fister, Luecker, Pawlowski, PLB 732 (2013)

Critical line/surface for heavy quarks



Polyakov Loop:



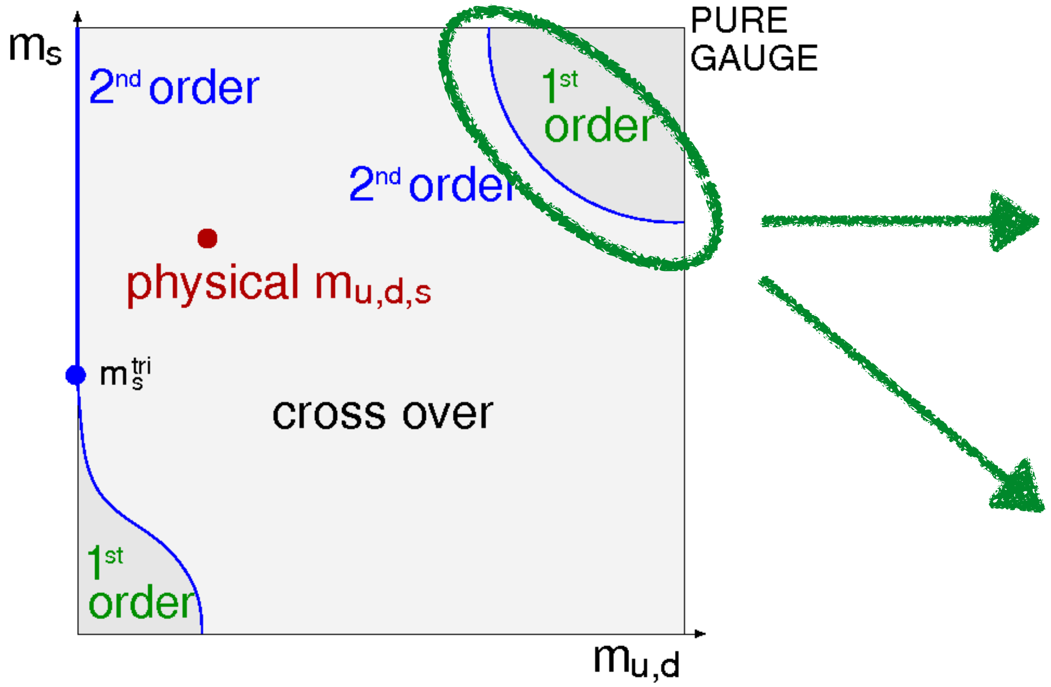


- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen

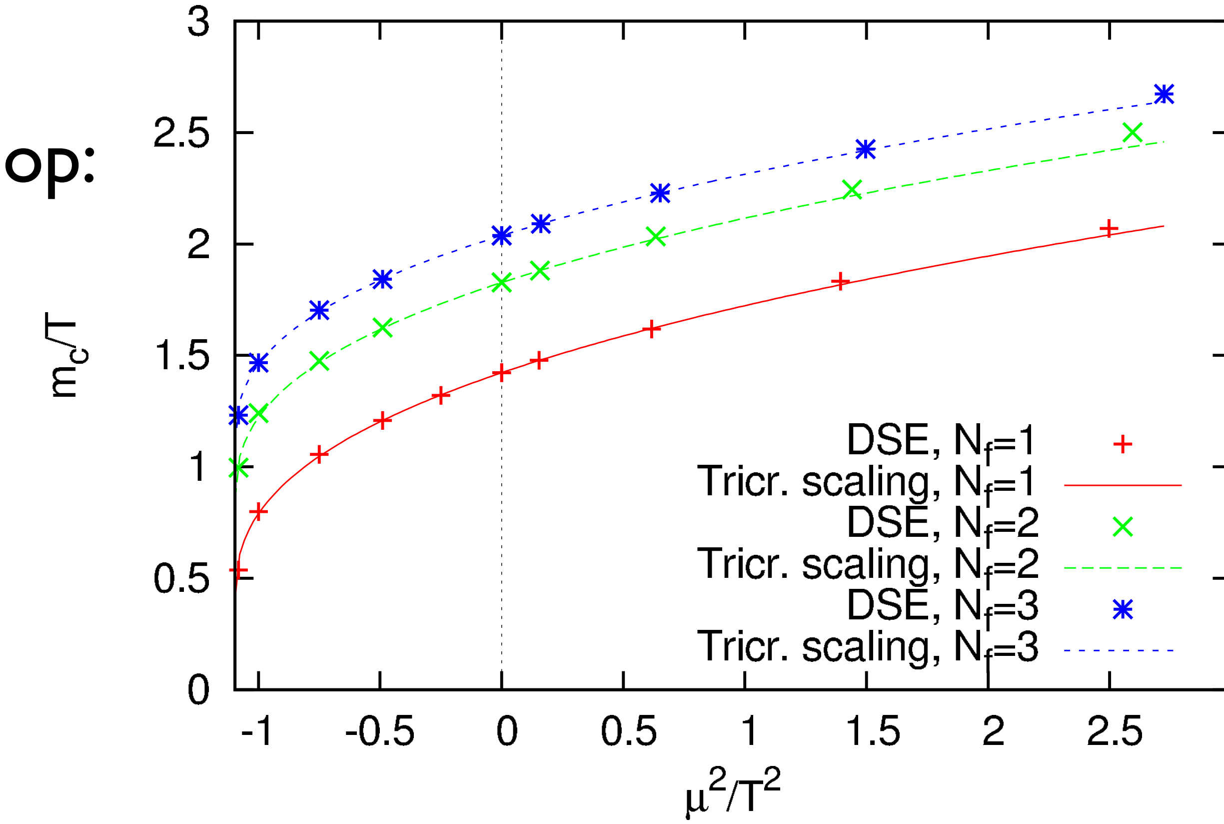
Lattice:
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

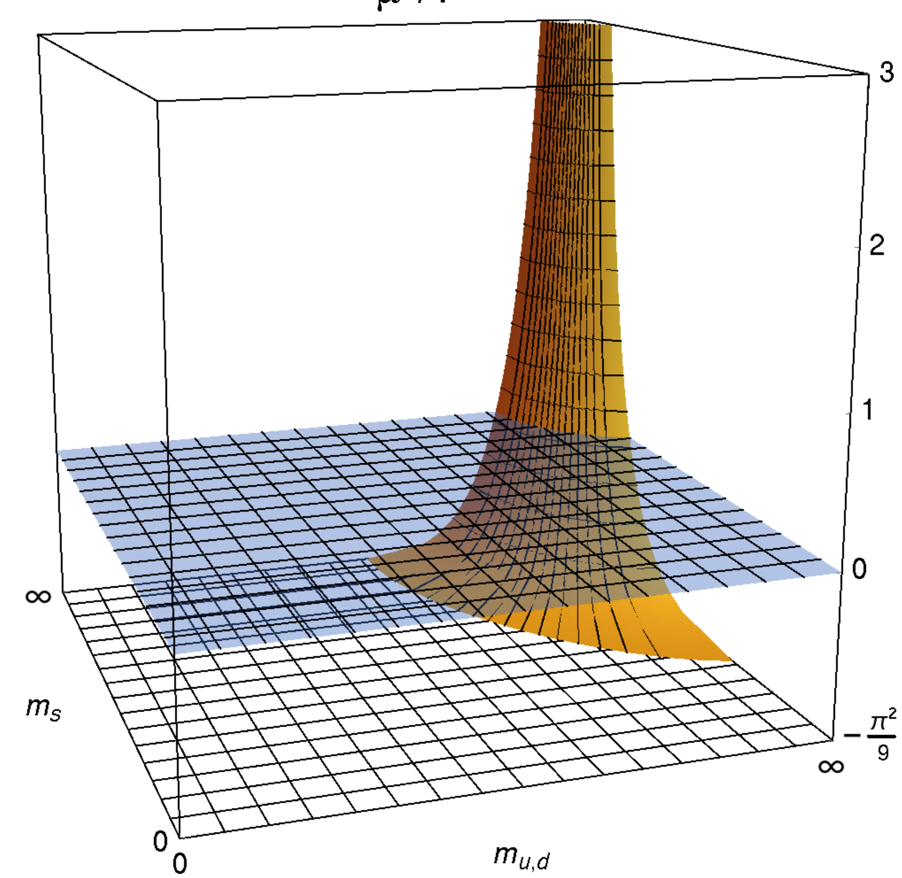
CF, Luecker, Pawlowski, PRD 91 (2015) 1

Critical line/surface for heavy quarks



Polyakov Loop:

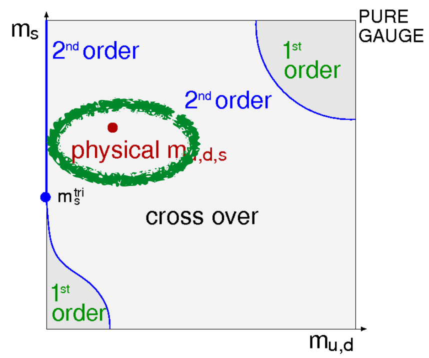


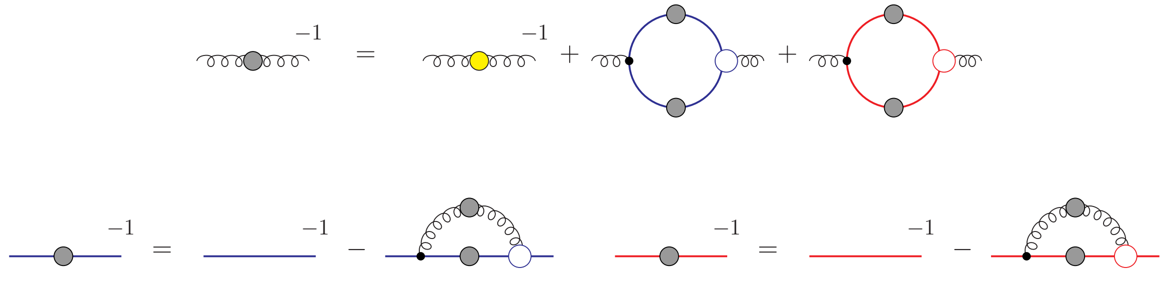


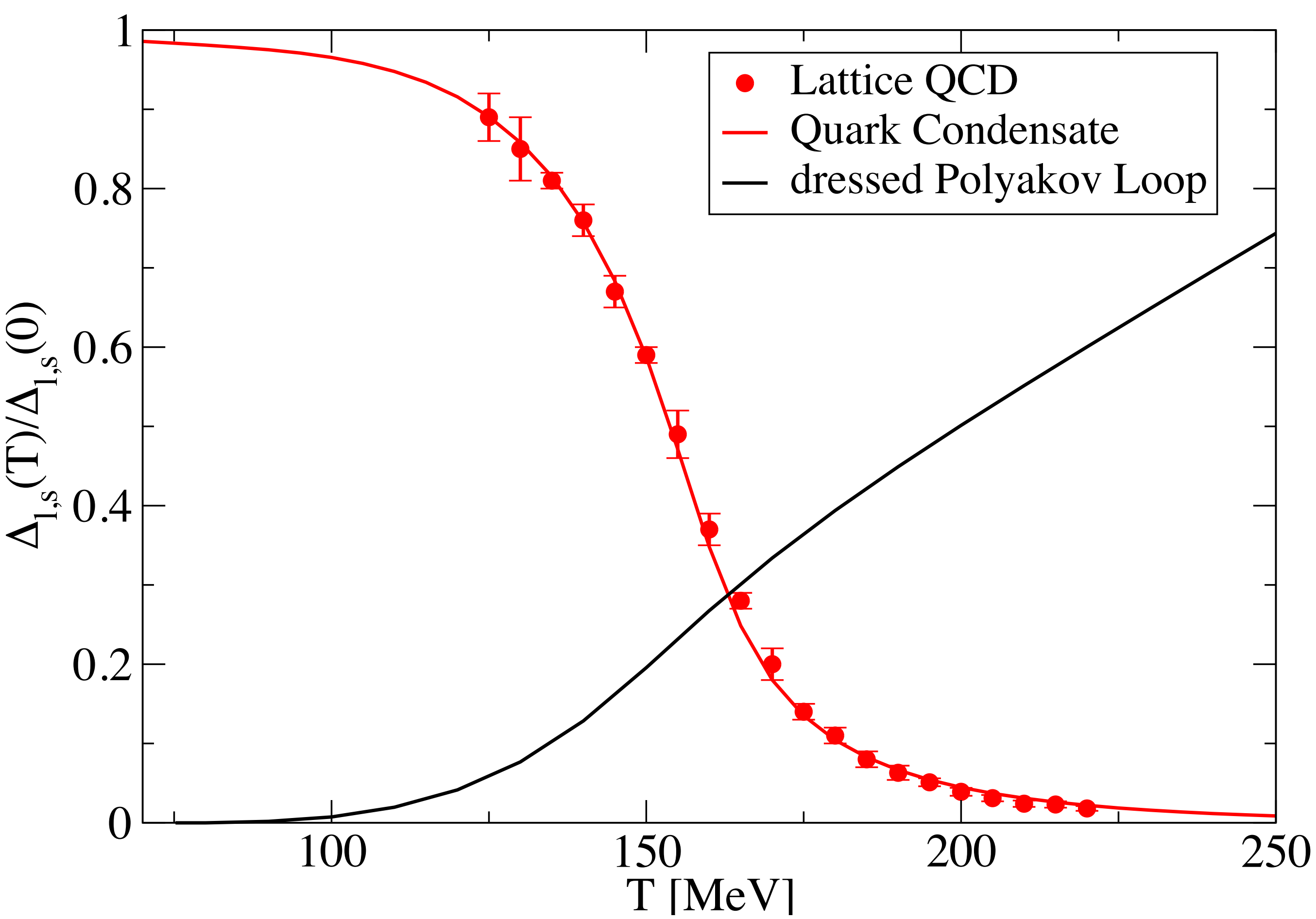
- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen

Lattice:
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

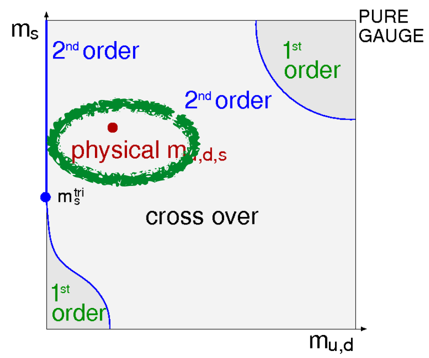
CF, Luecker, Pawlowski, PRD 91 (2015) 1

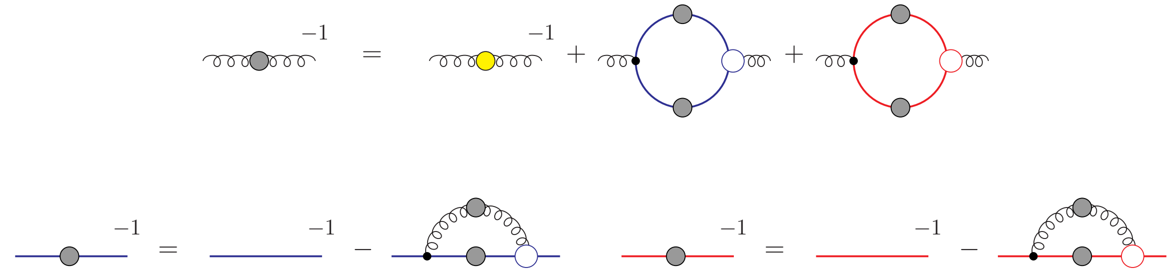


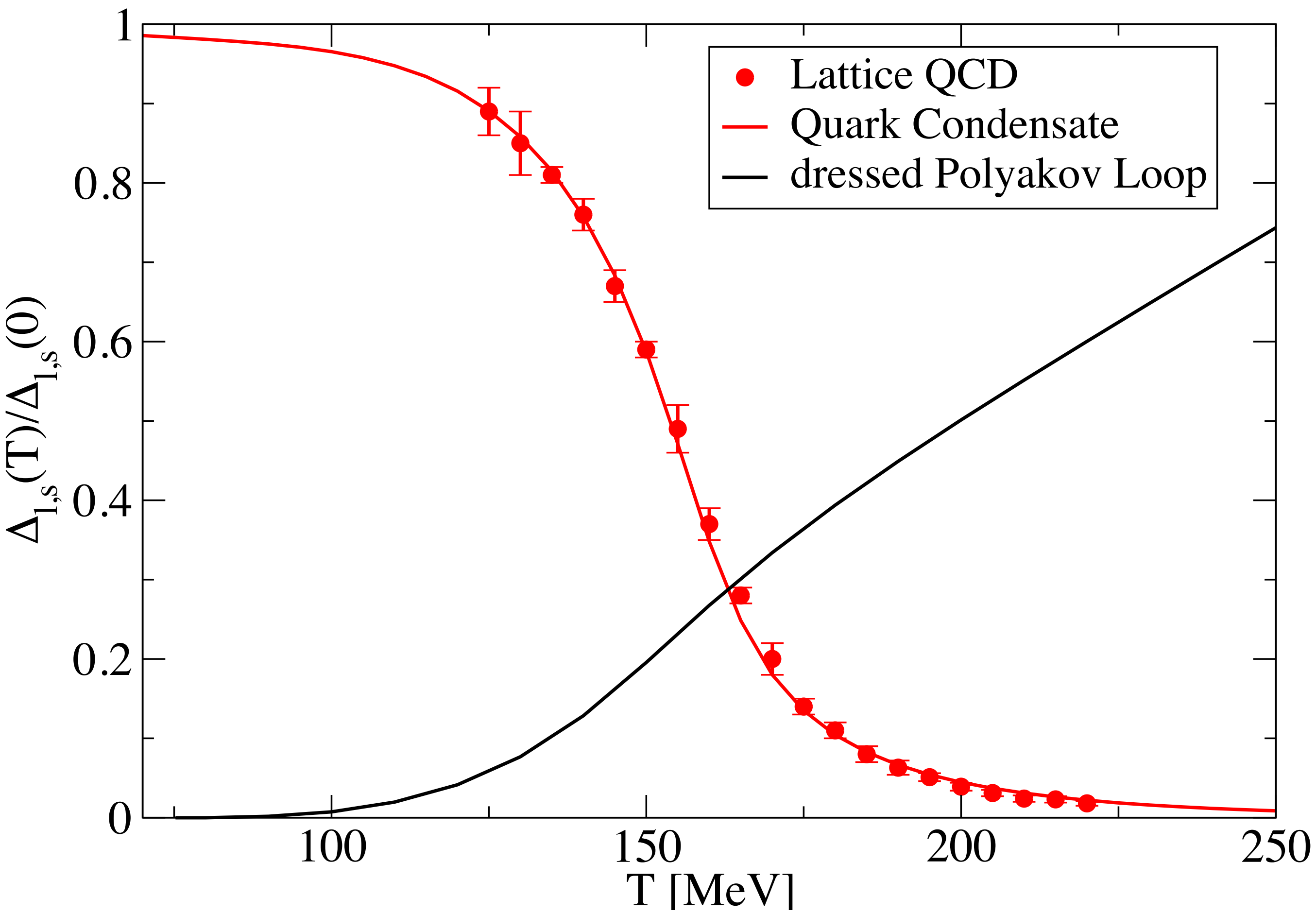


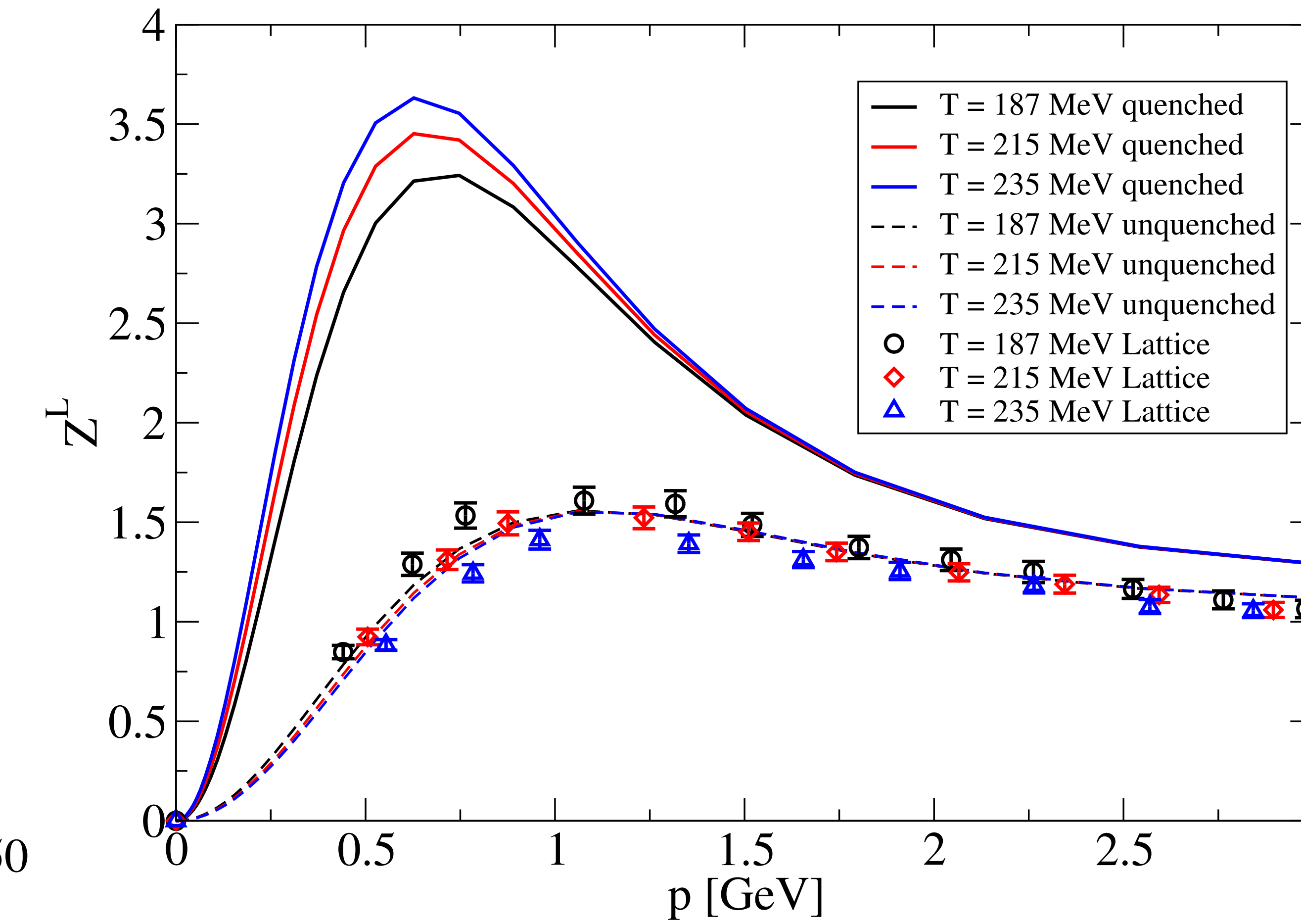


Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022







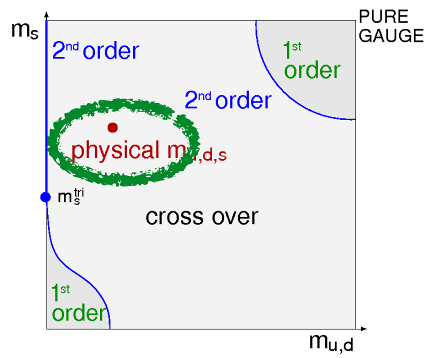


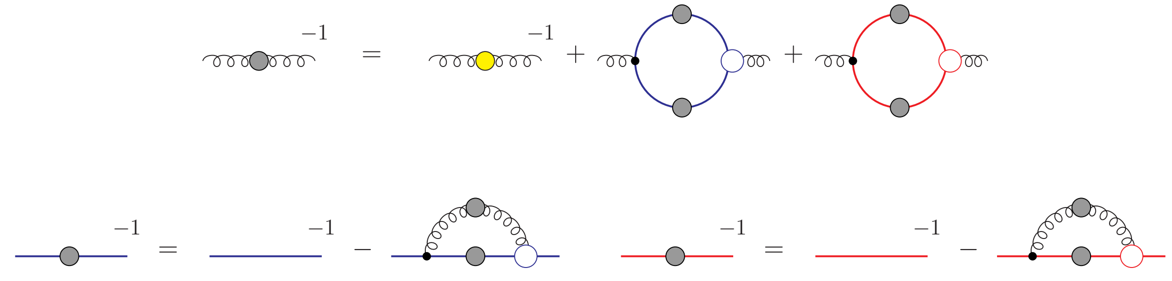
Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

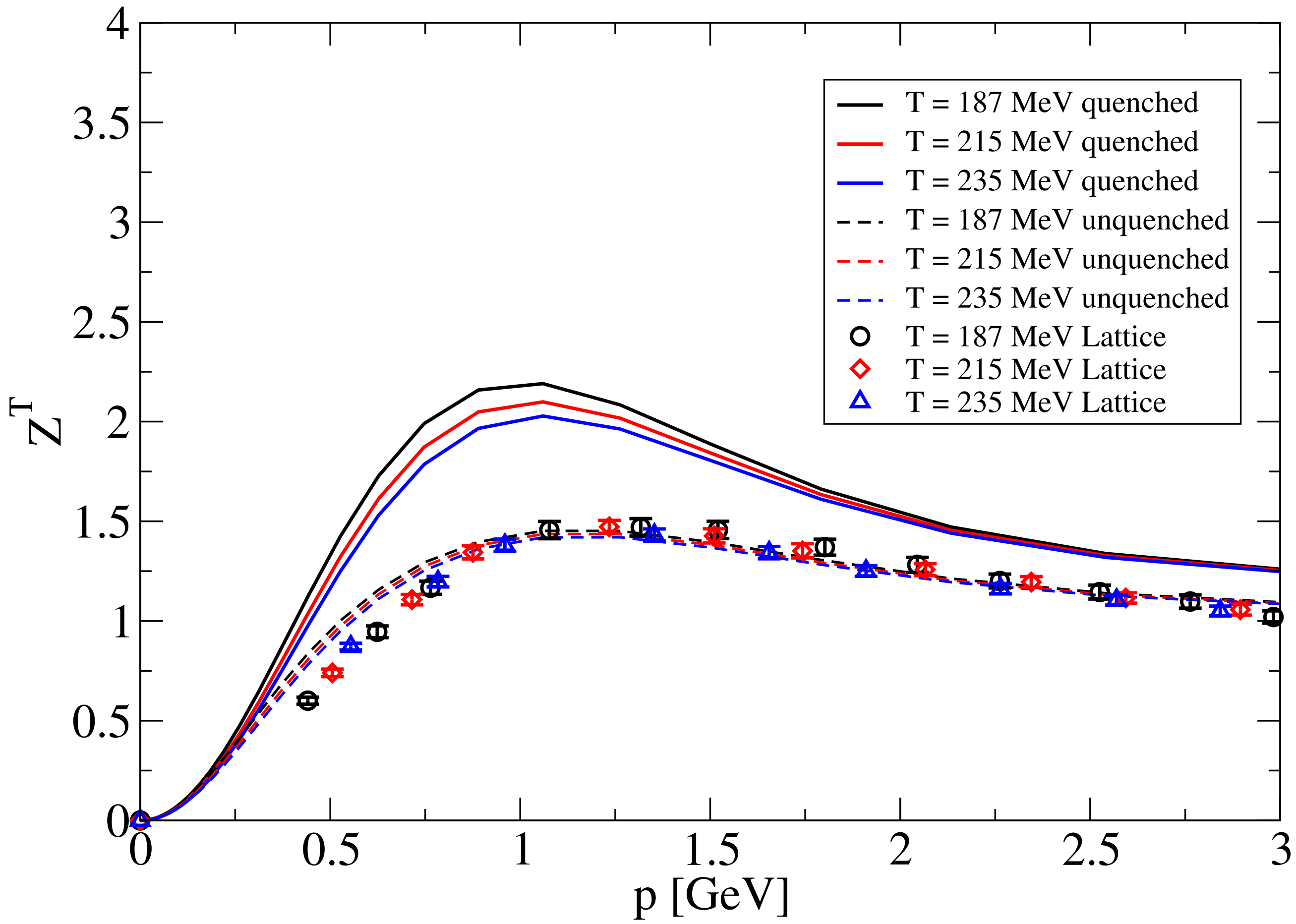
Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

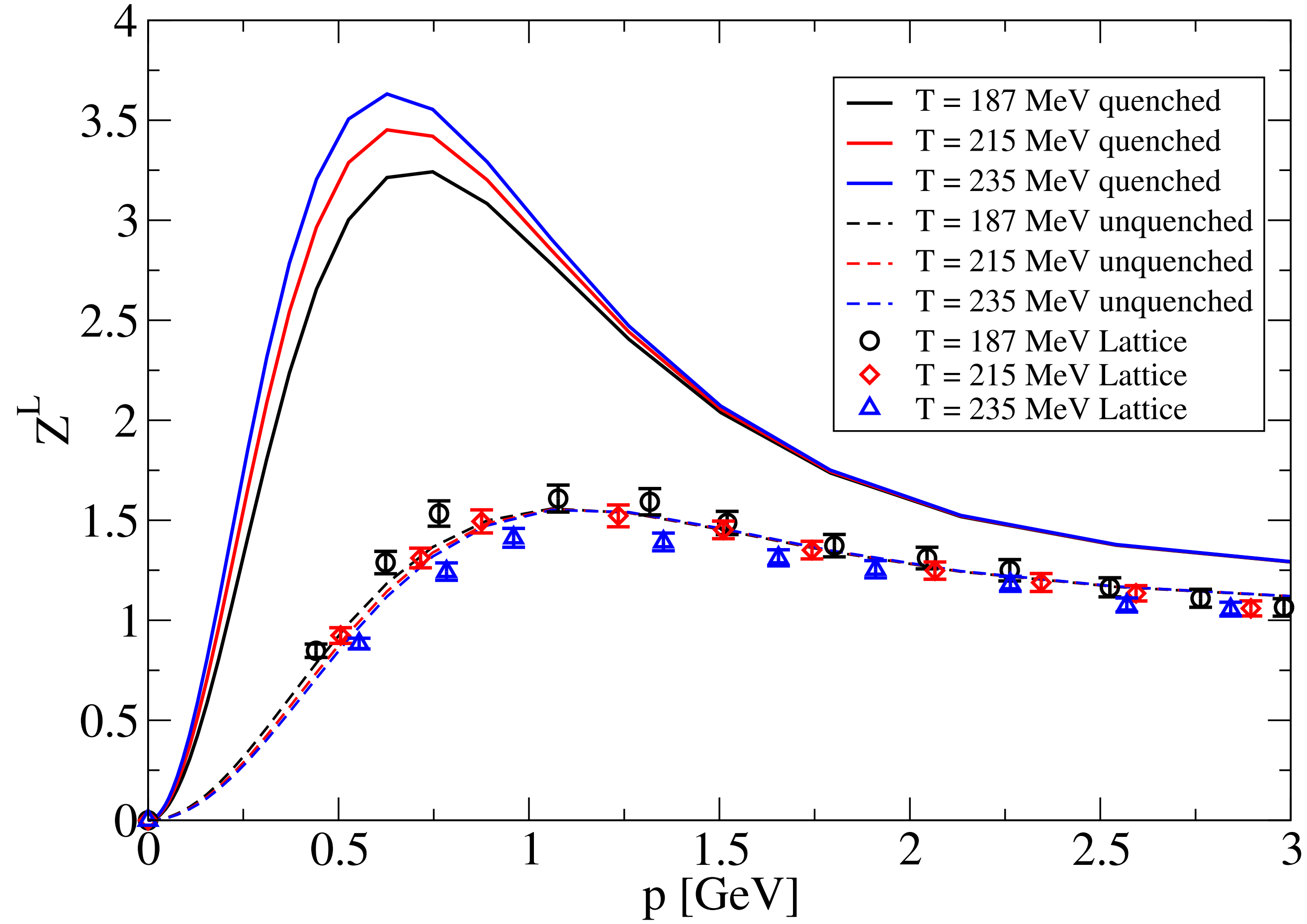
- quantitative agreement: DSE prediction verified by lattice

N_f=2+1, μ=0, physical point





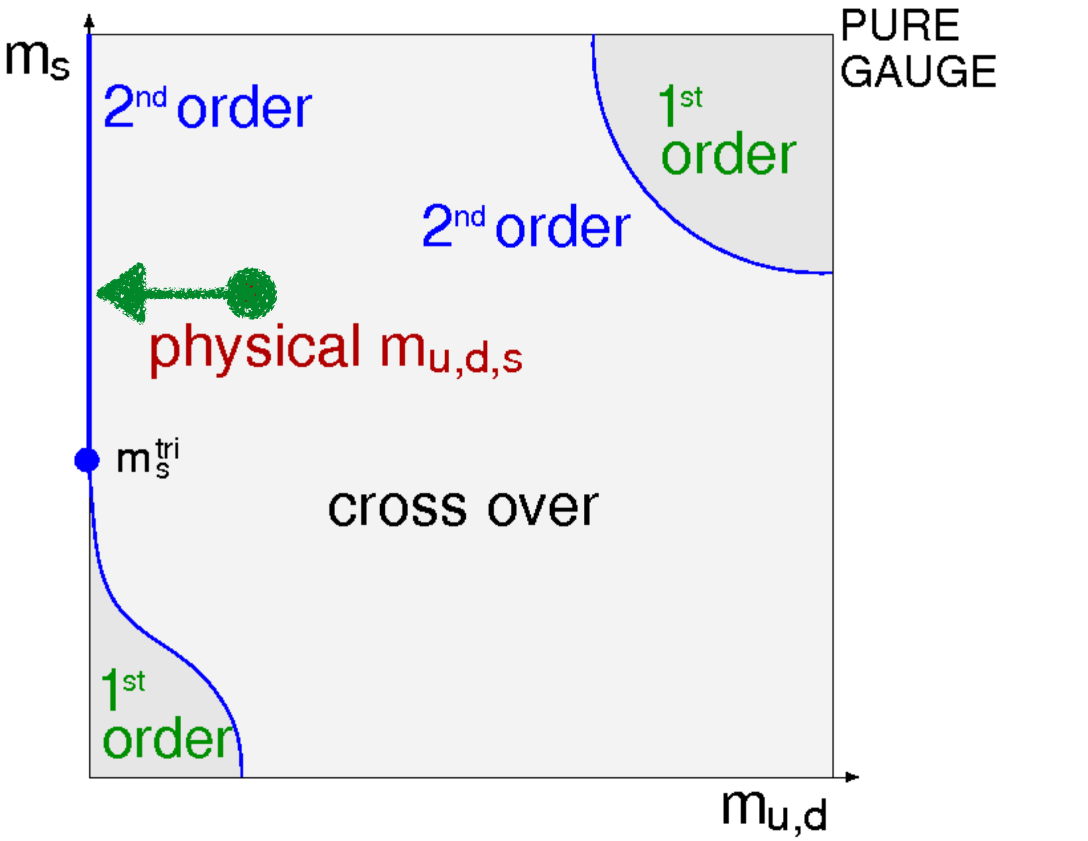


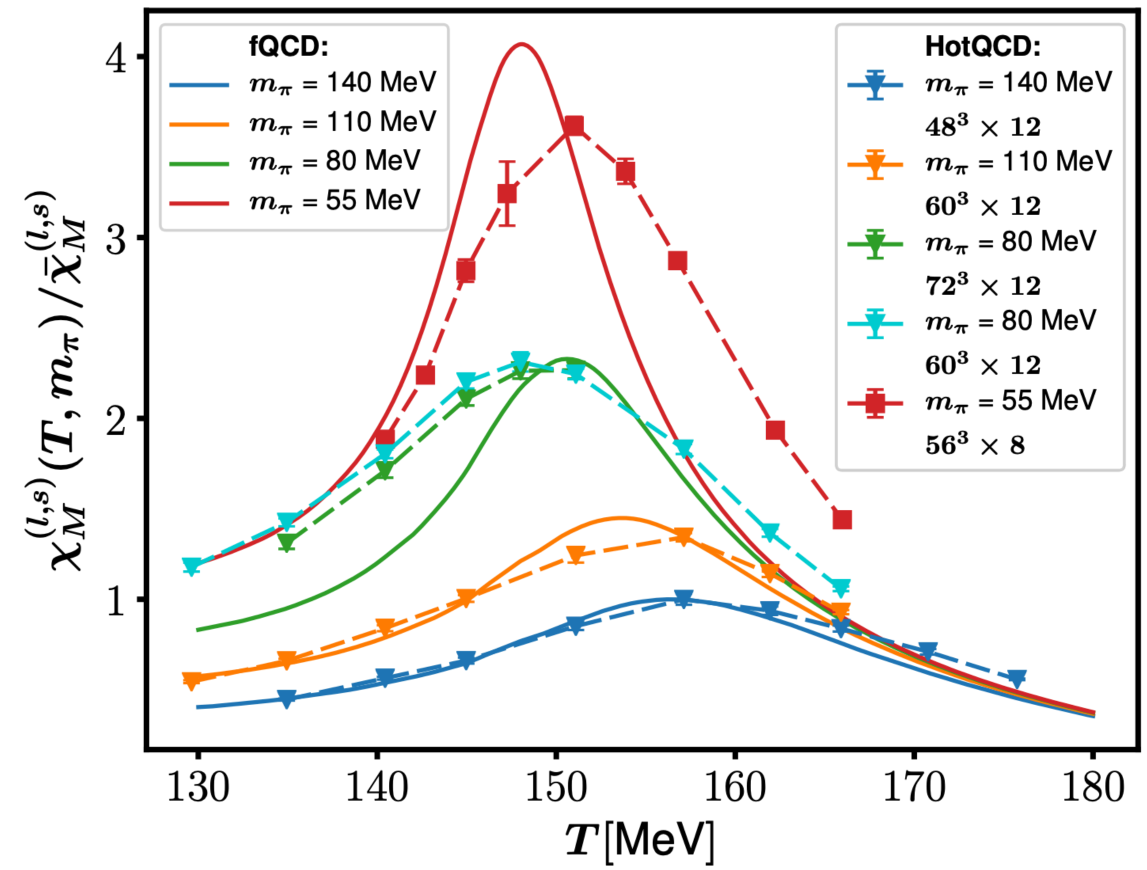


Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

- quantitative agreement: DSE prediction verified by lattice

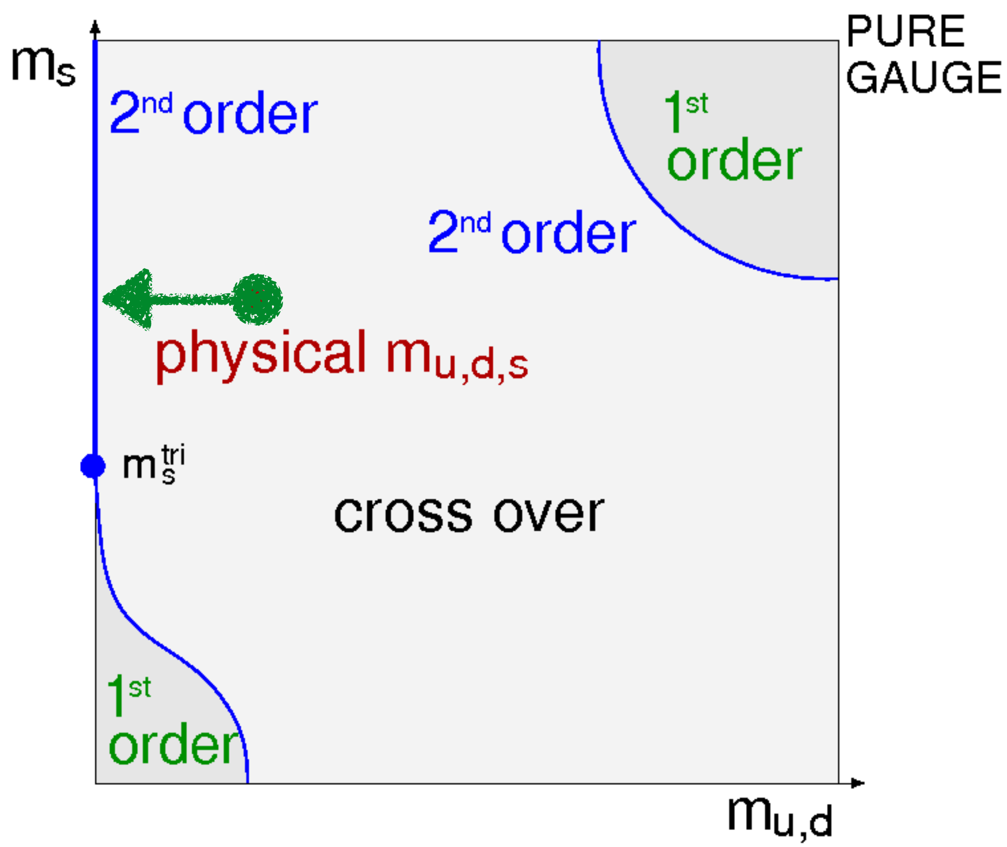
Towards the chiral limit…

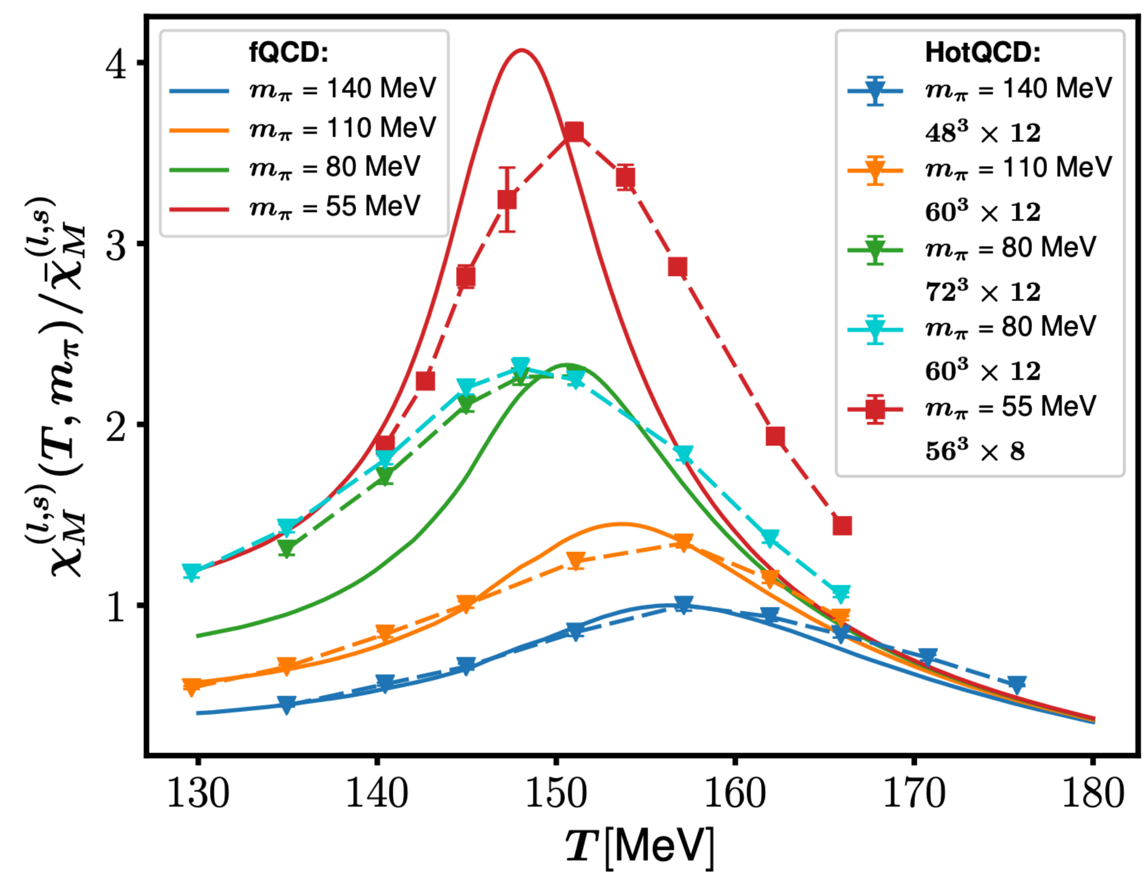


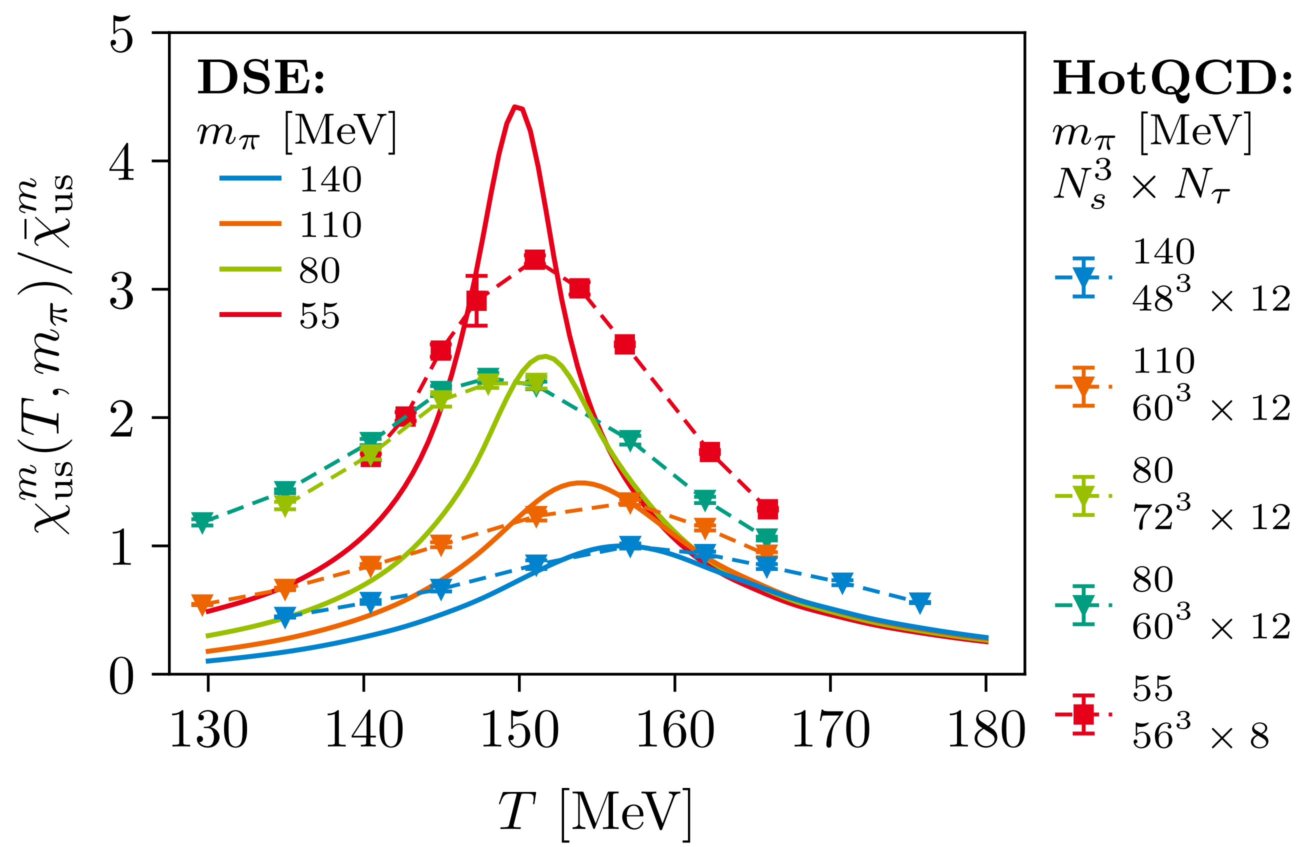


see talk of Julian Bernhardt

HotQCD: Ding et al. PRL 123, 062002 (2019)
FRG: Braun et al, PRD 102 (2020) 5, 056010
DSE: Bernhardt and CF, arXiv:2309.06737



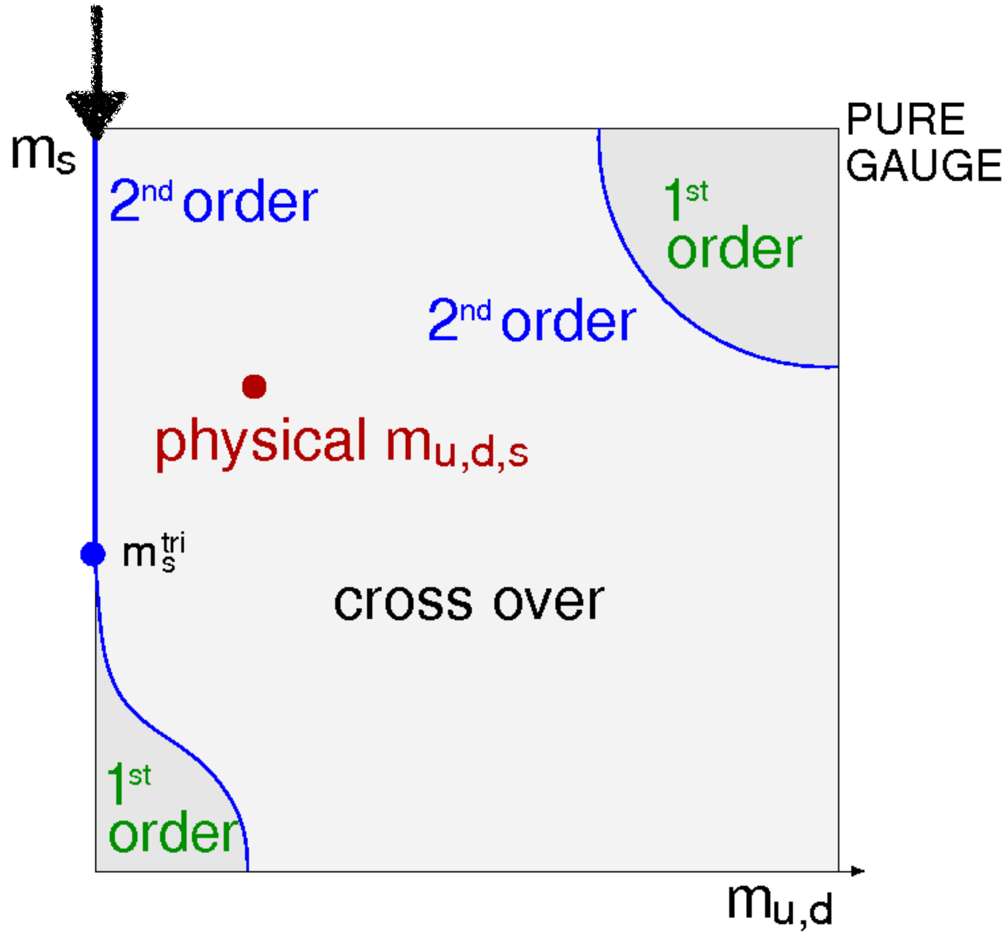


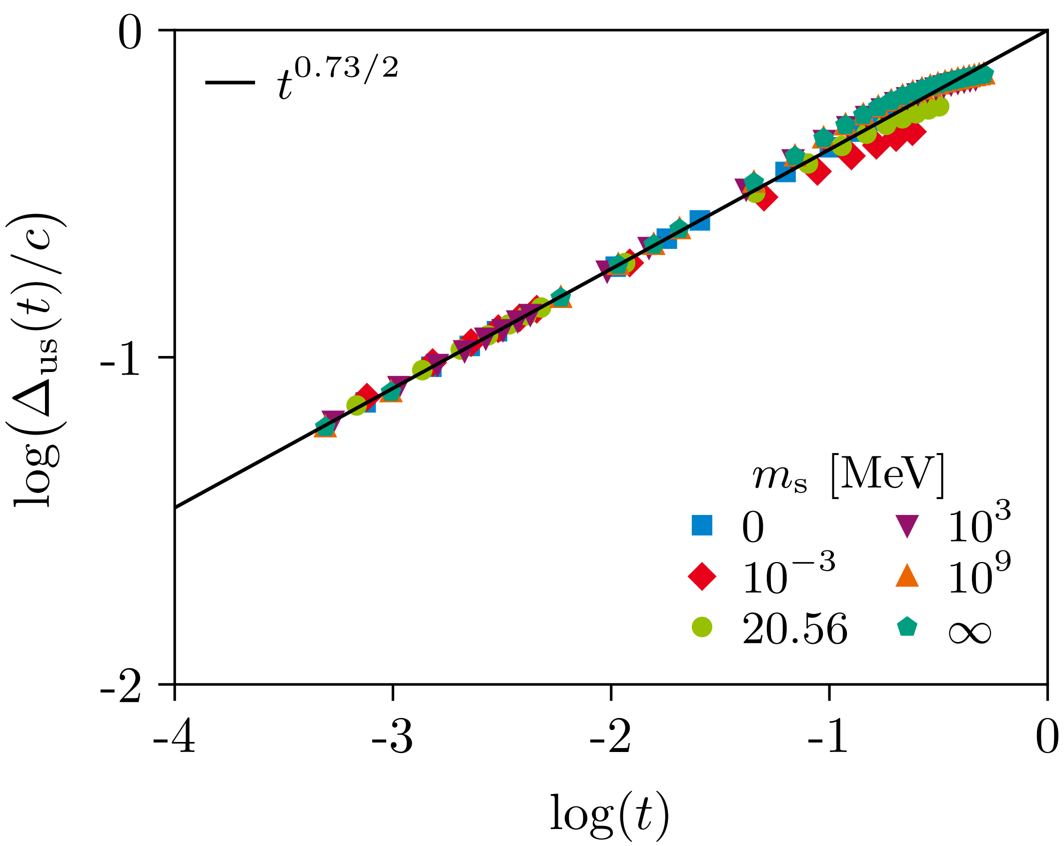


see talk of Julian Bernhardt

HotQCD: Ding et al. PRL 123, 062002 (2019)
FRG: Braun et al, PRD 102 (2020) 5, 056010
DSE: Bernhardt and CF, arXiv:2309.06737

At the chiral limit…





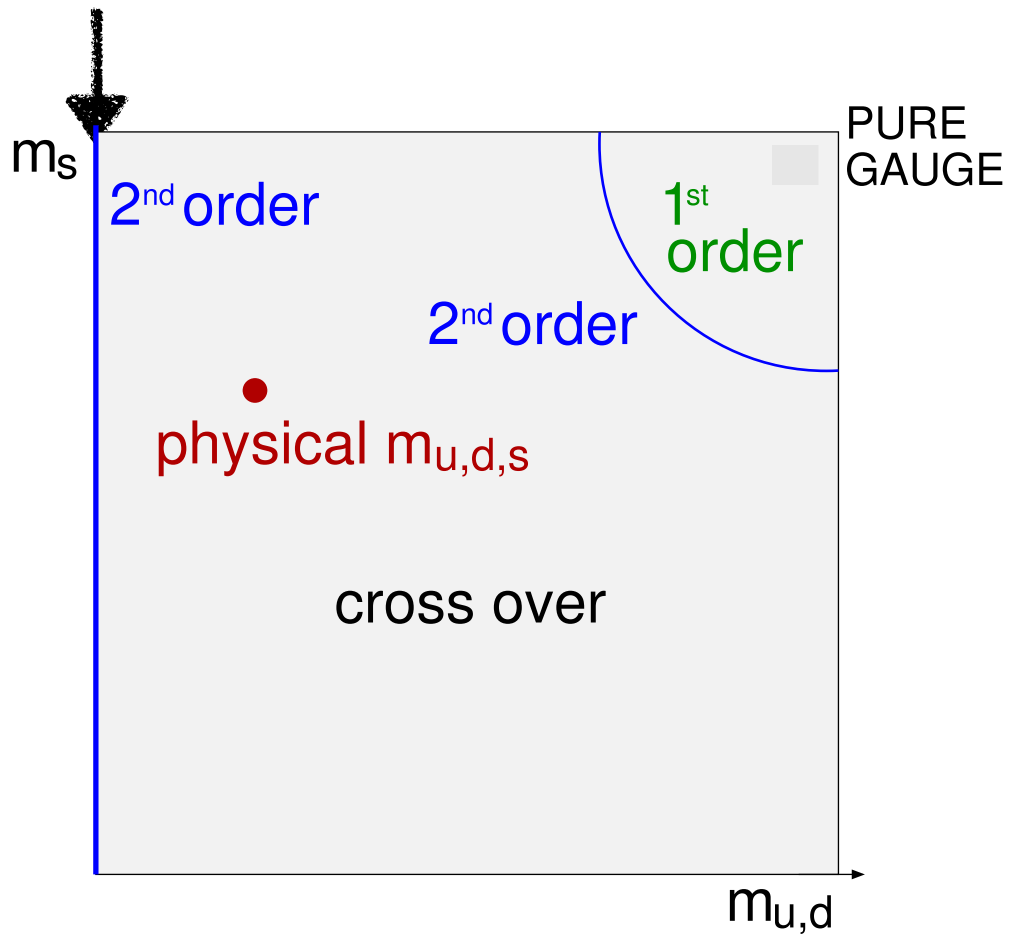
see talk of Julian Bernhardt

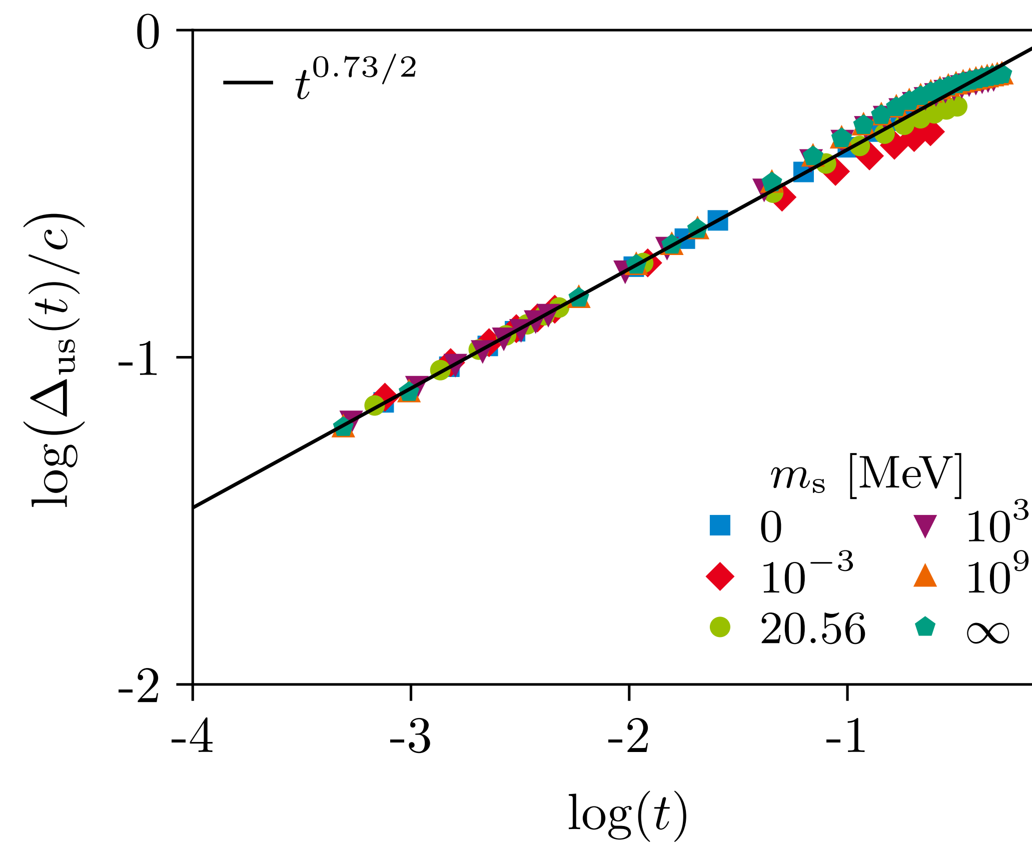
reproduce CF and Mueller, PRD 84 (2011) 054013

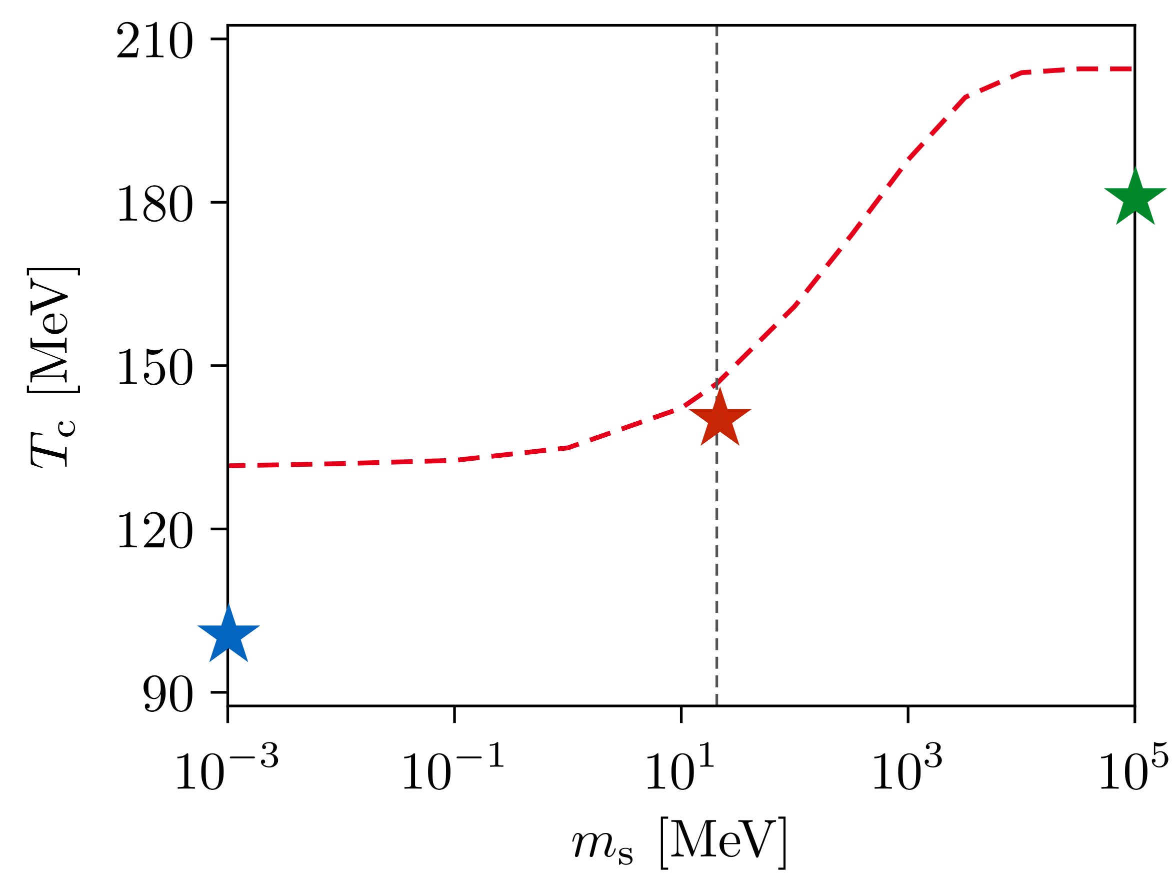
DSE: Bernhardt and CF, arXiv:2309.06737

Lattice: Dini, et al, PRD 105 (2022) no.3, 034510
Ding et al. PRL 123, 062002 (2019)
Bornyakov et al. PRD 82, 014504 (2010)

At the chiral limit...







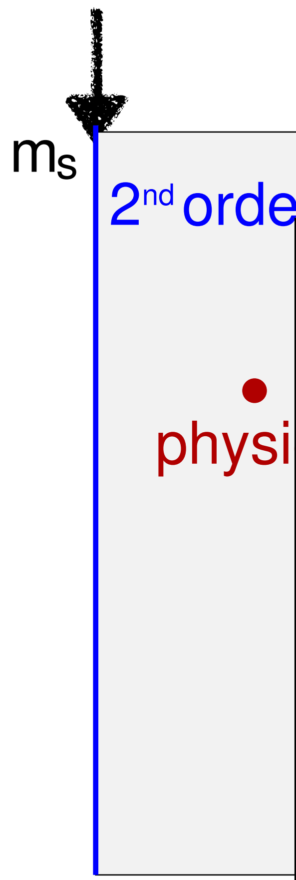
see talk of Julian Bernhardt

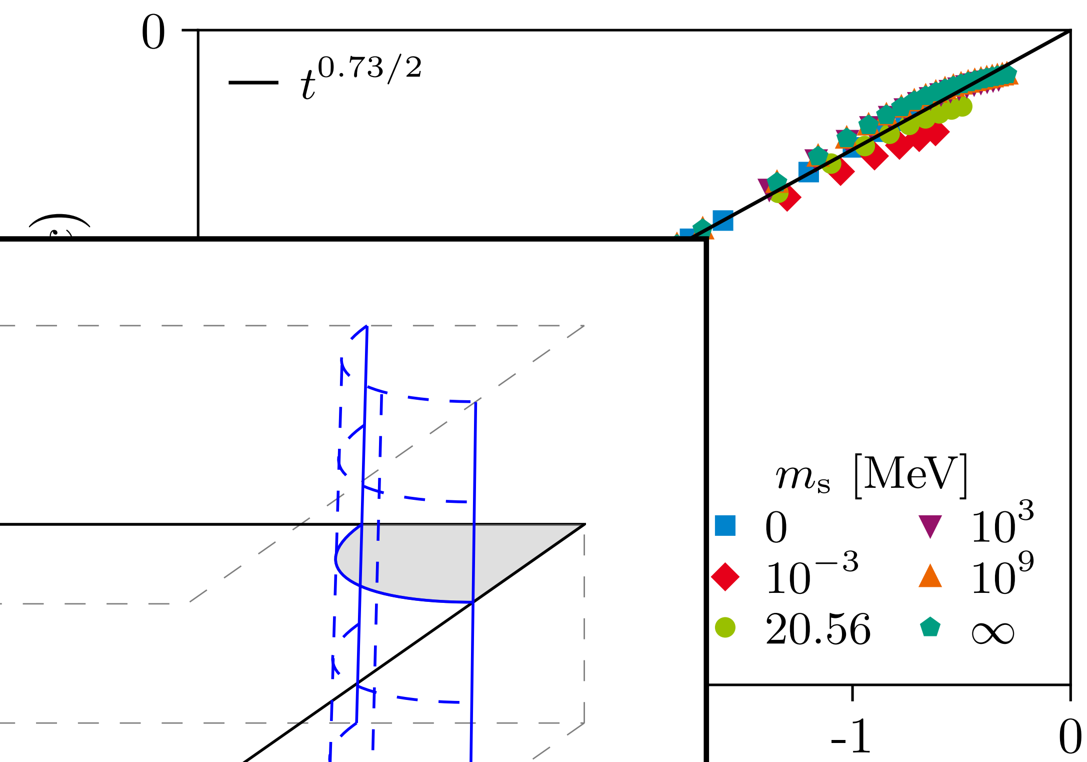
reproduce CF and Mueller, PRD 84 (2011) 054013

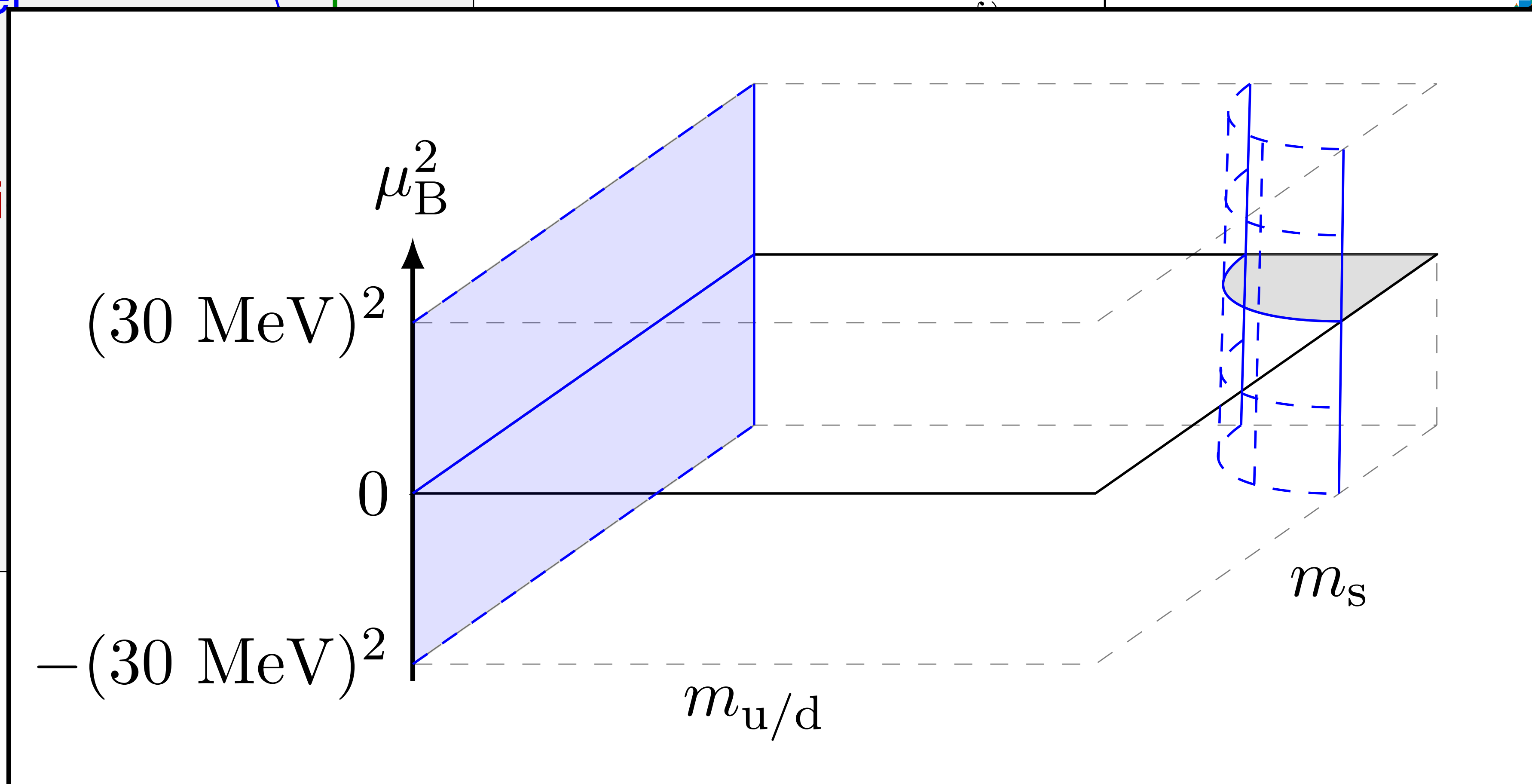
DSE: Bernhardt and CF, arXiv:2309.06737

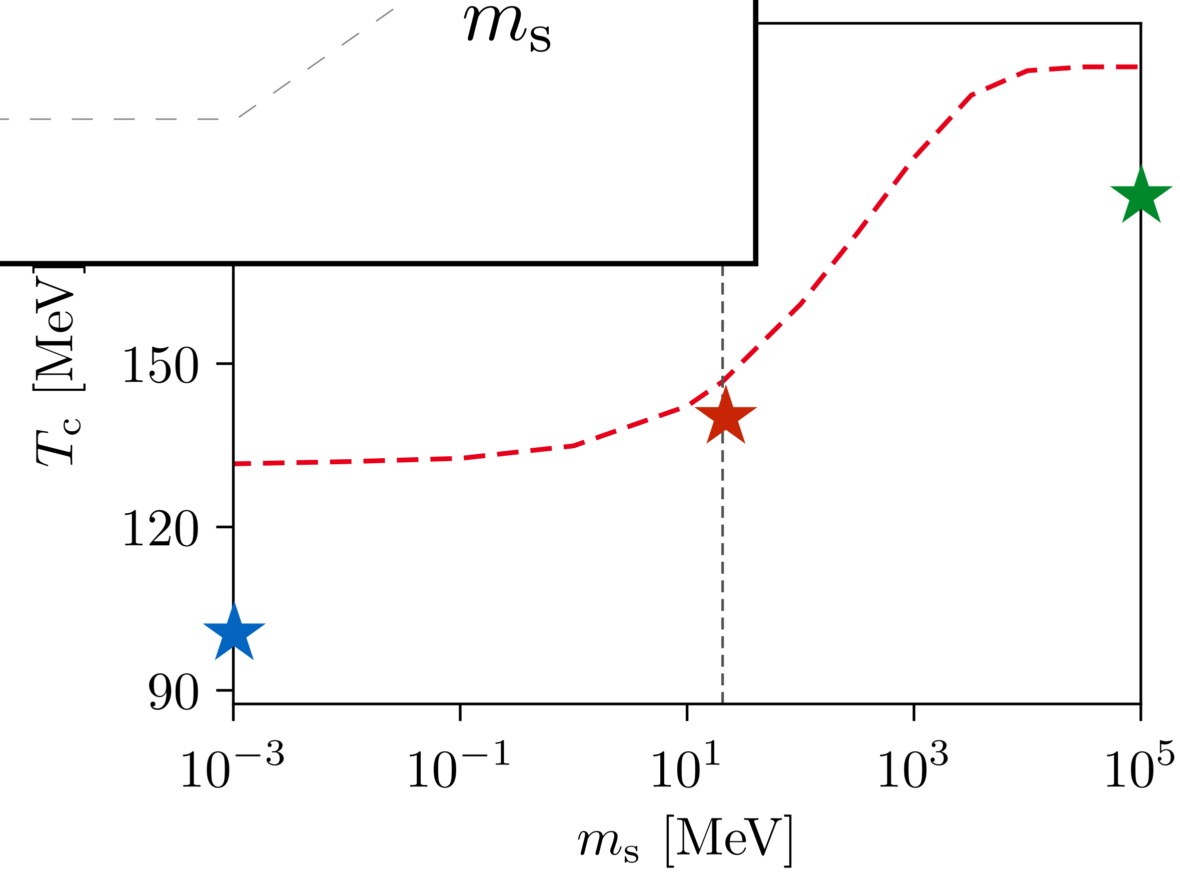
Lattice: Dini, et al, PRD 105 (2022) no.3, 034510
Ding et al. PRL 123, 062002 (2019)
Bornyakov et al. PRD 82, 014504 (2010)

At the chiral limit…







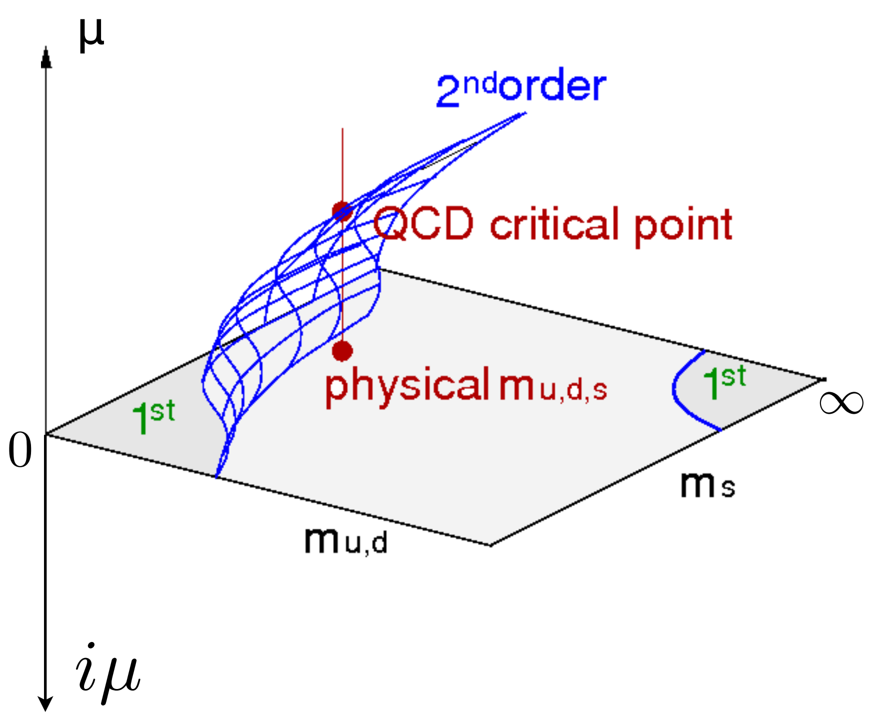


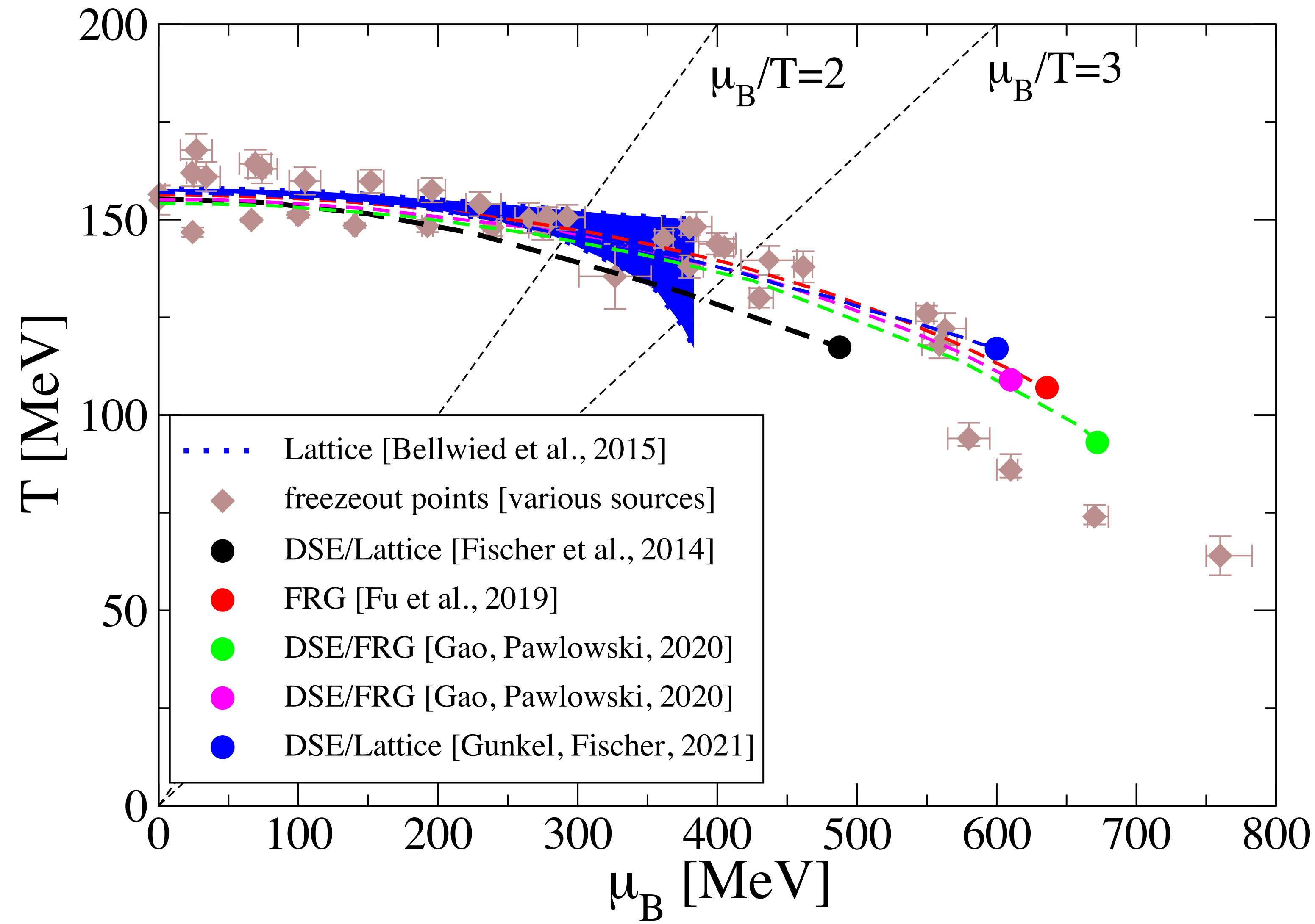
see talk of Julian Bernhardt

reproduce CF and Mueller, PRD 84 (2011) 054013

DSE: Bernhardt and CF, arXiv:2309.06737

Lattice: Dini, et al, PRD 105 (2022) no.3, 034510
Ding et al. PRL 123, 062002 (2019)
Bornyakov et al. PRD 82, 014504 (2010)

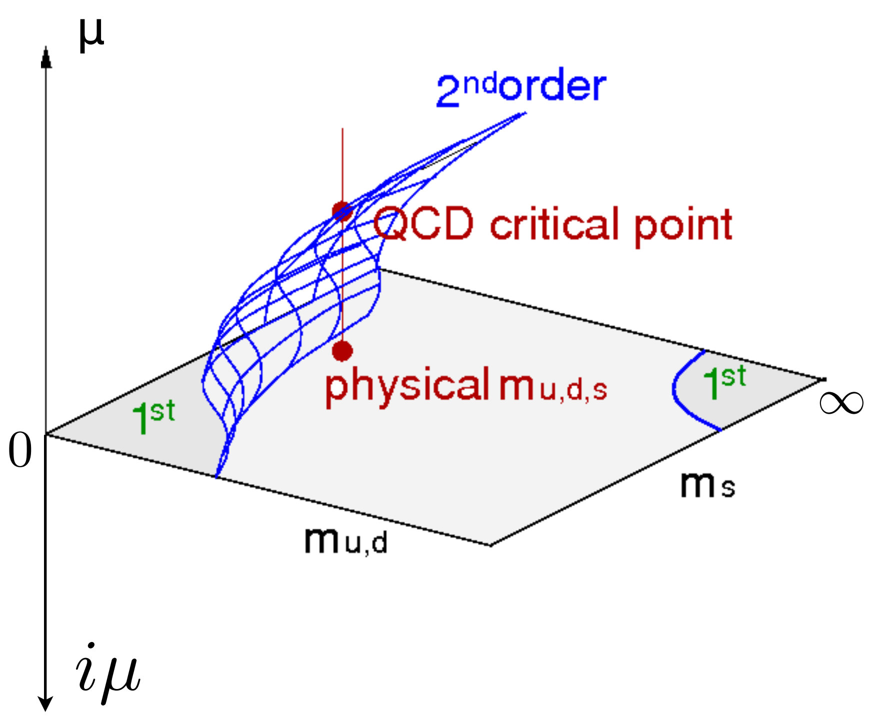


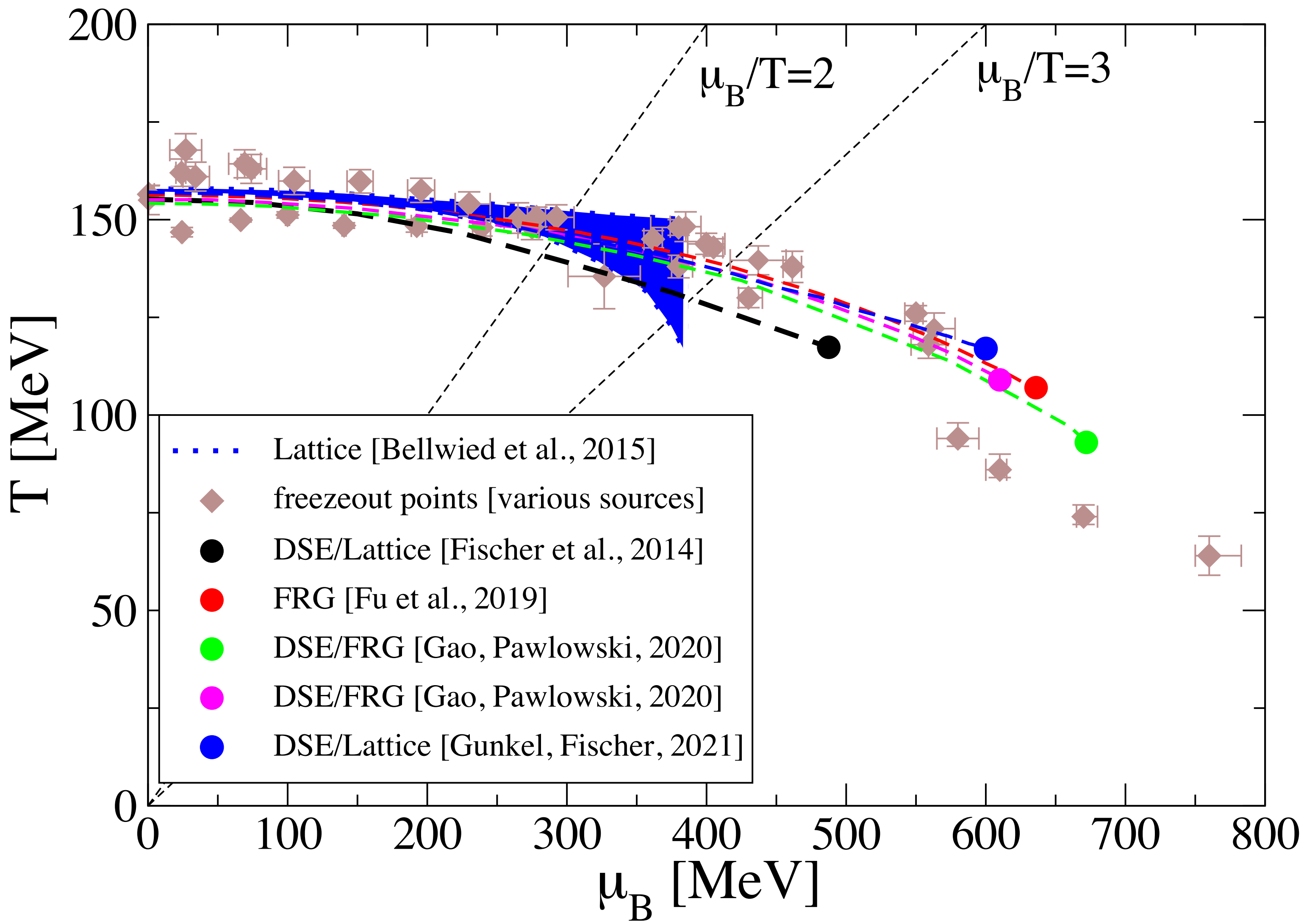


- how stable is this result ??
  - crosscheck with FRG ✔

see talk of Theo Motta

Location of CEP



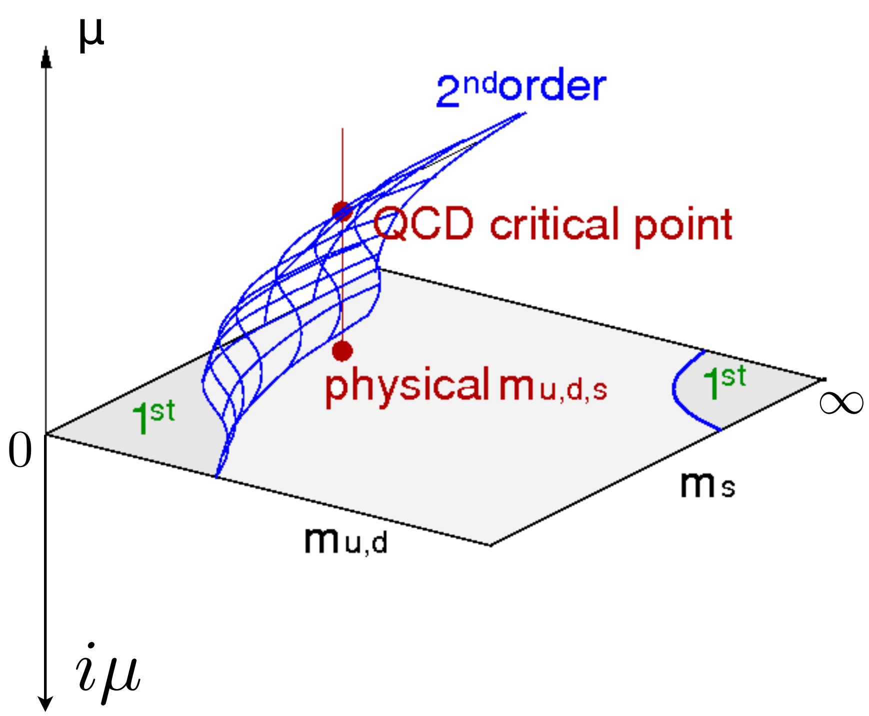


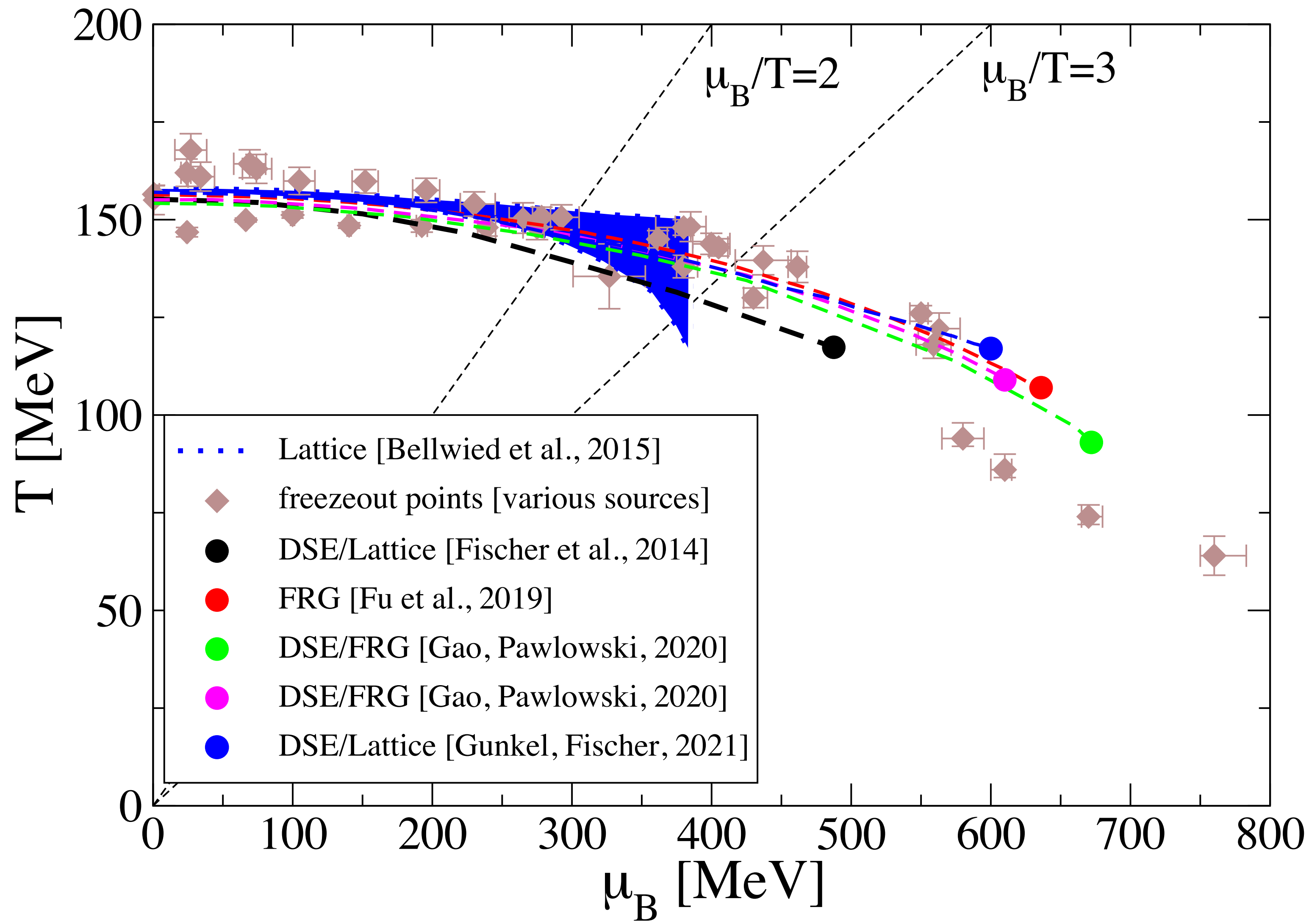
- how stable is this result ??
  - crosscheck with FRG ✔
  - N_f=2+1+1 ✔

CF, Luecker, Welzbacher, PRD 90 (2014) 034022

see talk of Theo Motta

Location of CEP





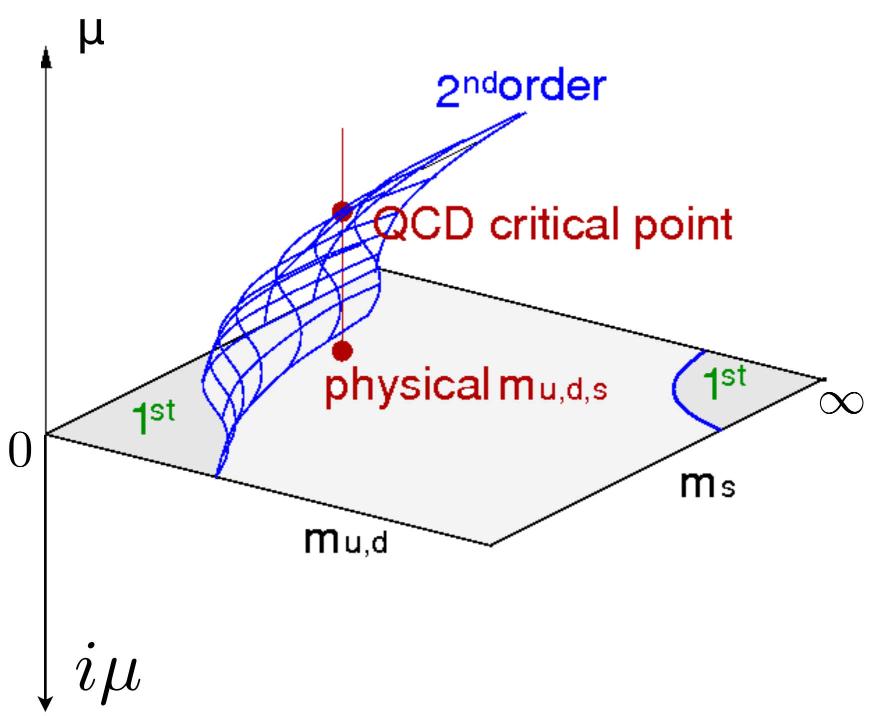
- how stable is this result ??
  - crosscheck with FRG ✔
  - $N_f=2+1+1$ ✔
  - baryon and meson effects ✔

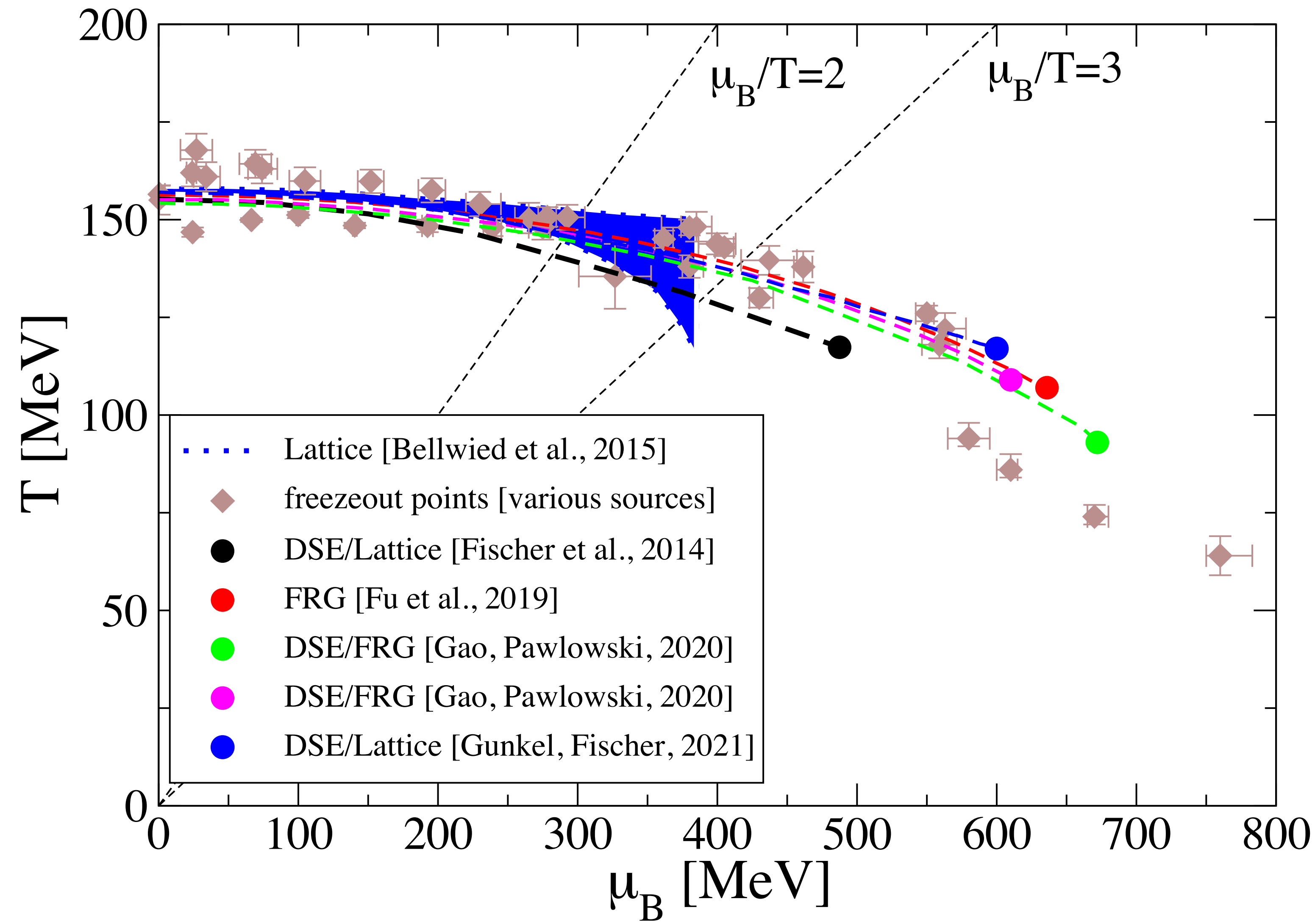
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Eichmann, CF, Welzbacher, PRD93 (2016)

see talk of Theo Motta

Location of CEP





- how stable is this result ??
  - crosscheck with FRG ✔
  - N_f=2+1+1 ✔
  - baryon and meson effects ✔
- inhomogeneous phases

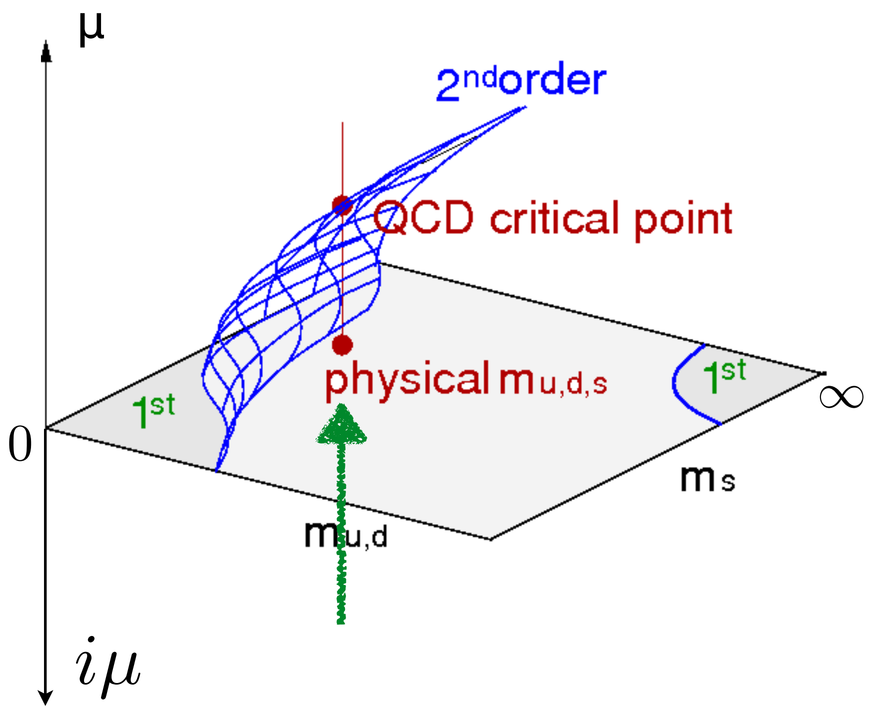
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

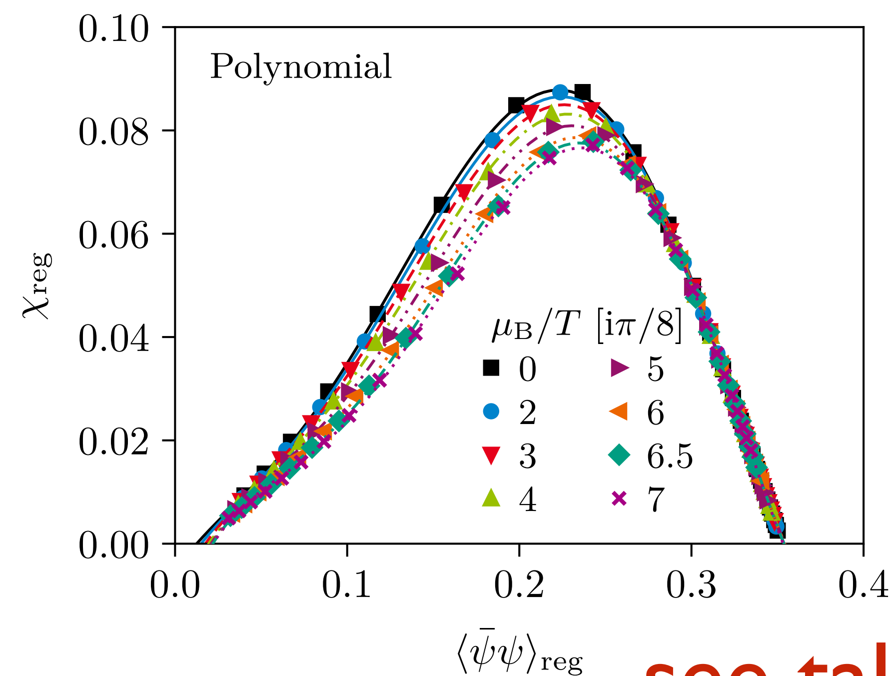
Eichmann, CF, Welzbacher, PRD93 (2016)

see talk of Theo Motta

Buballa and Carignano, PPNP 81 (2015) 39

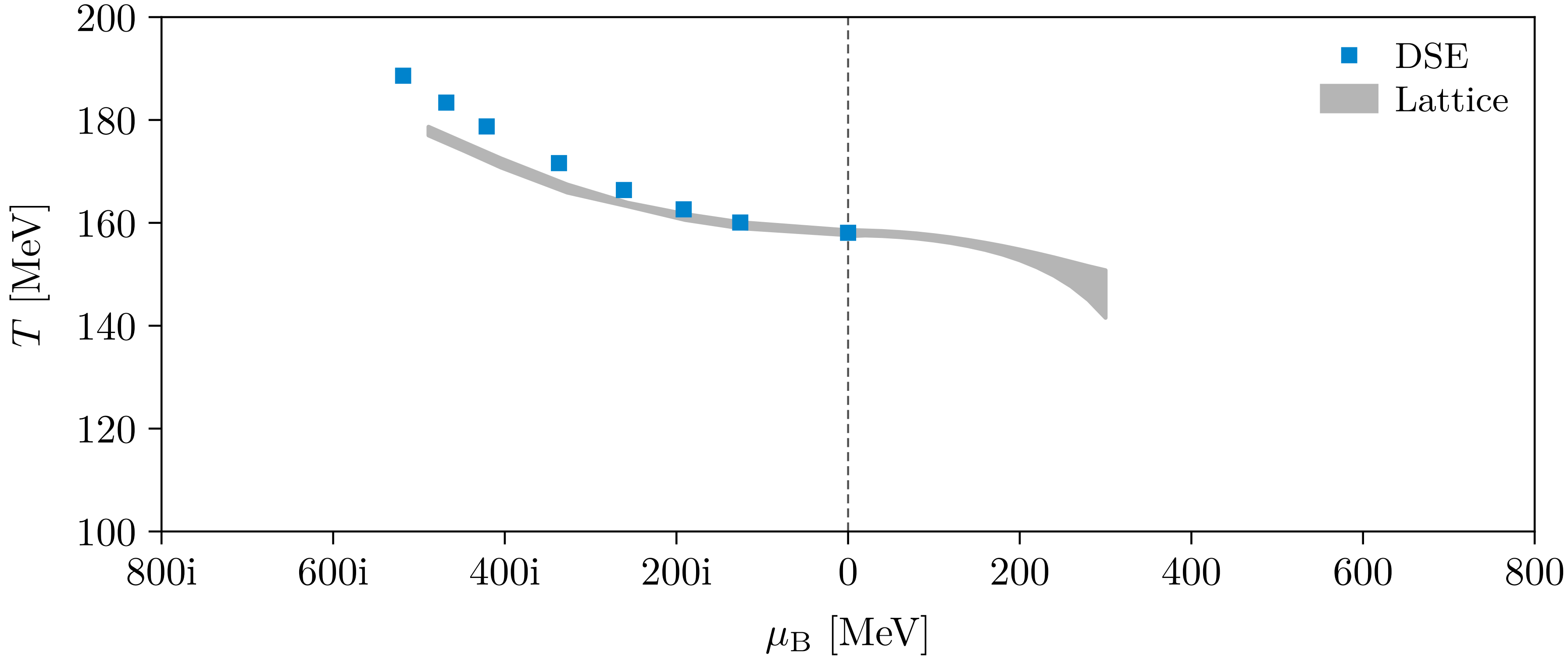
Extrapolation from imaginary chemical potential





$$\chi(T) = \frac{\partial\langle\bar{\psi}\psi\rangle(T)}{\partial m_u}$$

see talk of Julian Bernhardt

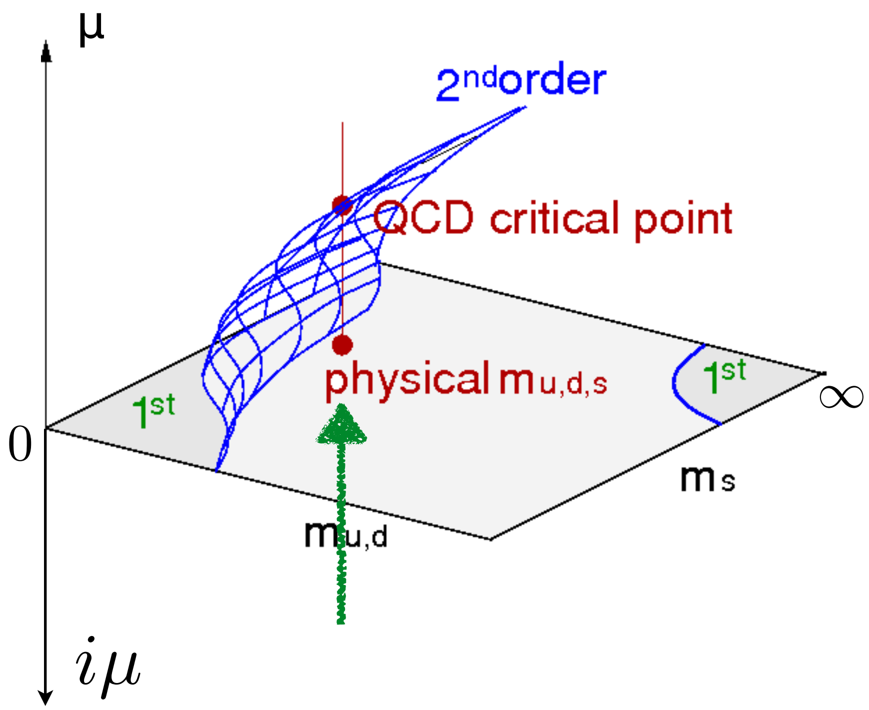


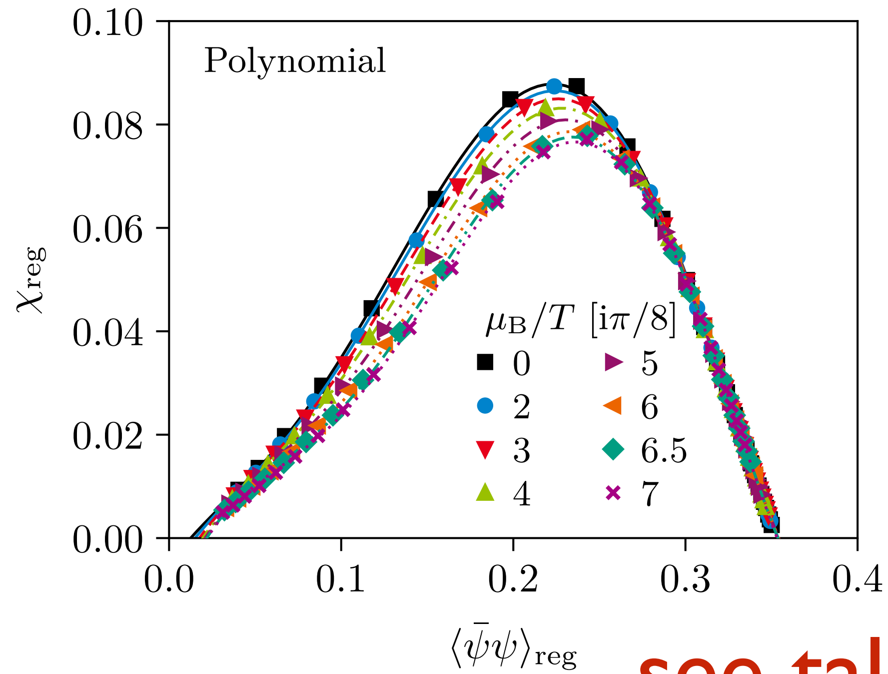
Lattice: Borsanyi et al. PRL 125 052001 (2020)
DSE: Bernhardt, CF, arXiv: 2305.01434

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2\left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_c}\right)^4$$

$$\kappa_2^{\mathrm{poly}} = 0.0196\,, \quad \kappa_4^{\mathrm{poly}} = 0.00015\,,$$

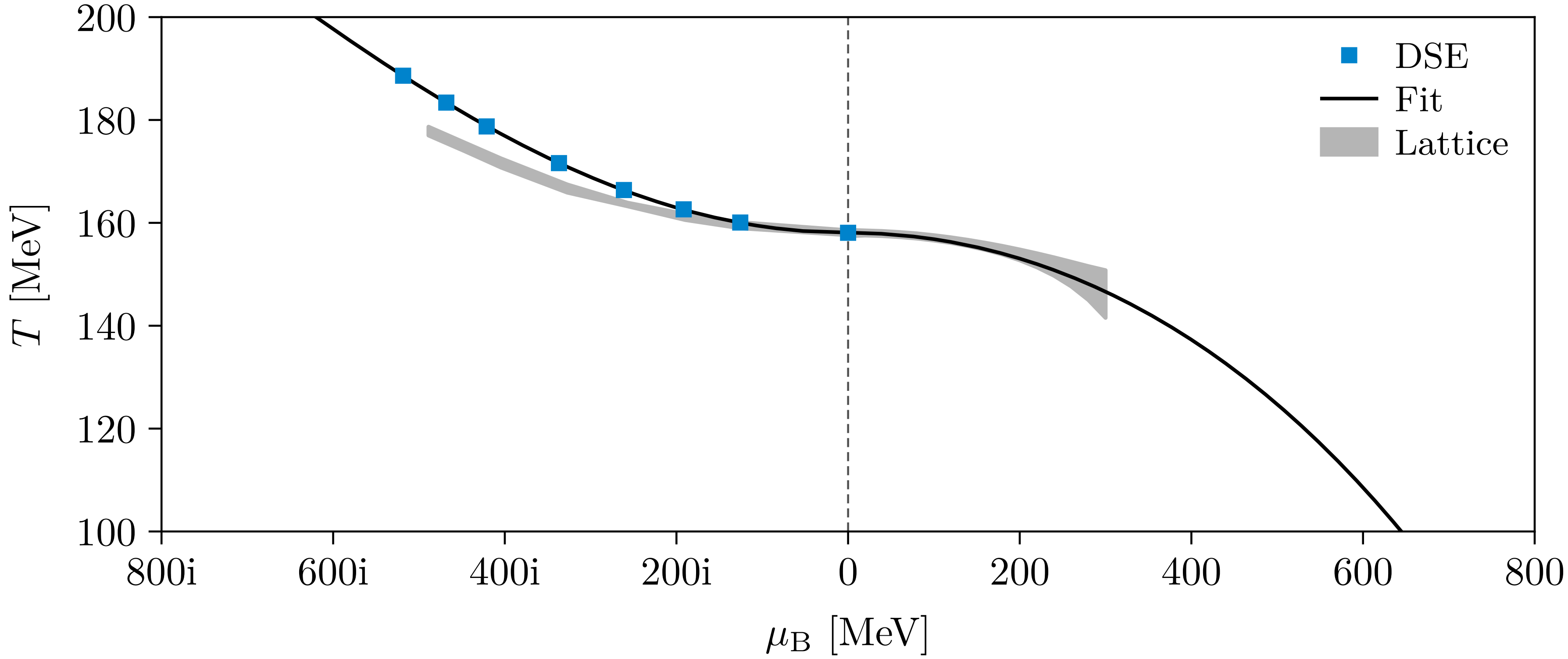
Extrapolation from imaginary chemical potential





$$\chi(T) = \frac{\partial\langle\bar{\psi}\psi\rangle(T)}{\partial m_{\mathrm{u}}}$$

see talk of Julian Bernhardt

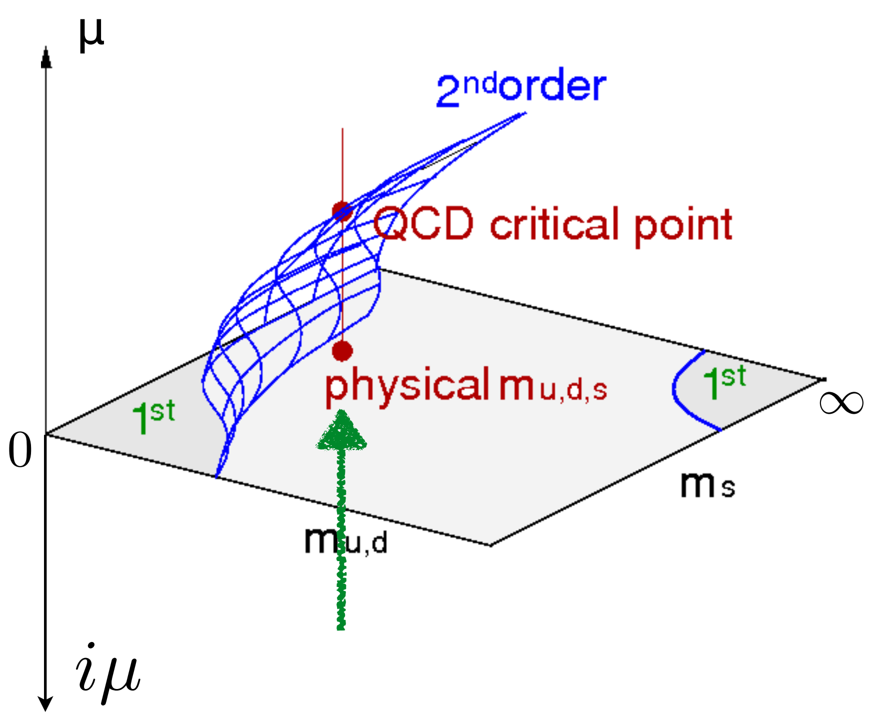


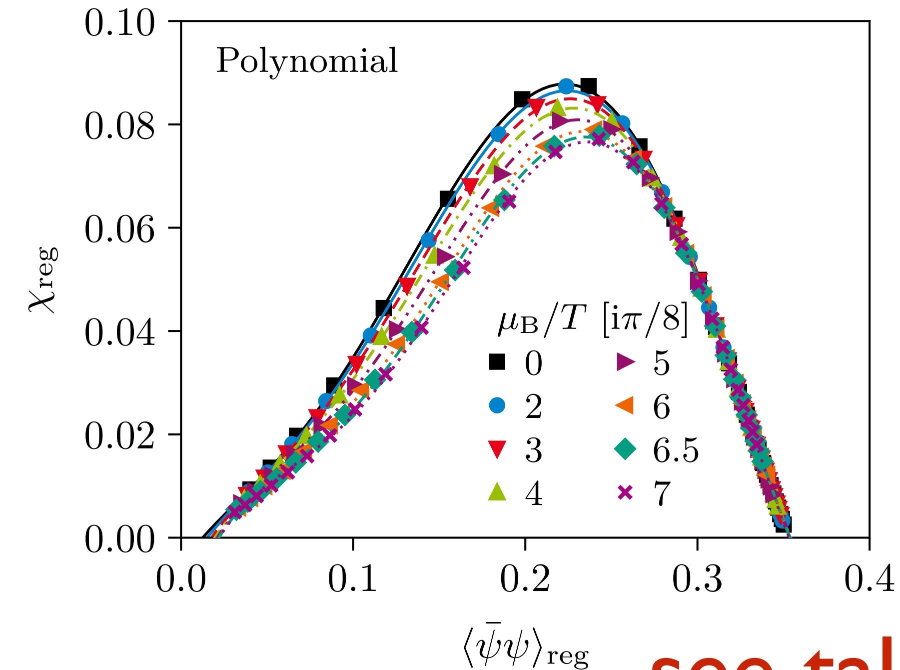
Lattice: Borsanyi et al. PRL 125 052001 (2020)
DSE: Bernhardt, CF, arXiv: 2305.01434

$$\frac{T_{\mathrm{c}}(\mu_{\mathrm{B}})}{T_{\mathrm{c}}} = 1 - \kappa_2\left(\frac{\mu_{\mathrm{B}}}{T_{\mathrm{c}}}\right)^2 - \kappa_4\left(\frac{\mu_{\mathrm{B}}}{T_{\mathrm{c}}}\right)^4$$

$$\kappa_2^{\mathrm{poly}} = 0.0196\,, \quad \kappa_4^{\mathrm{poly}} = 0.00015\,,$$

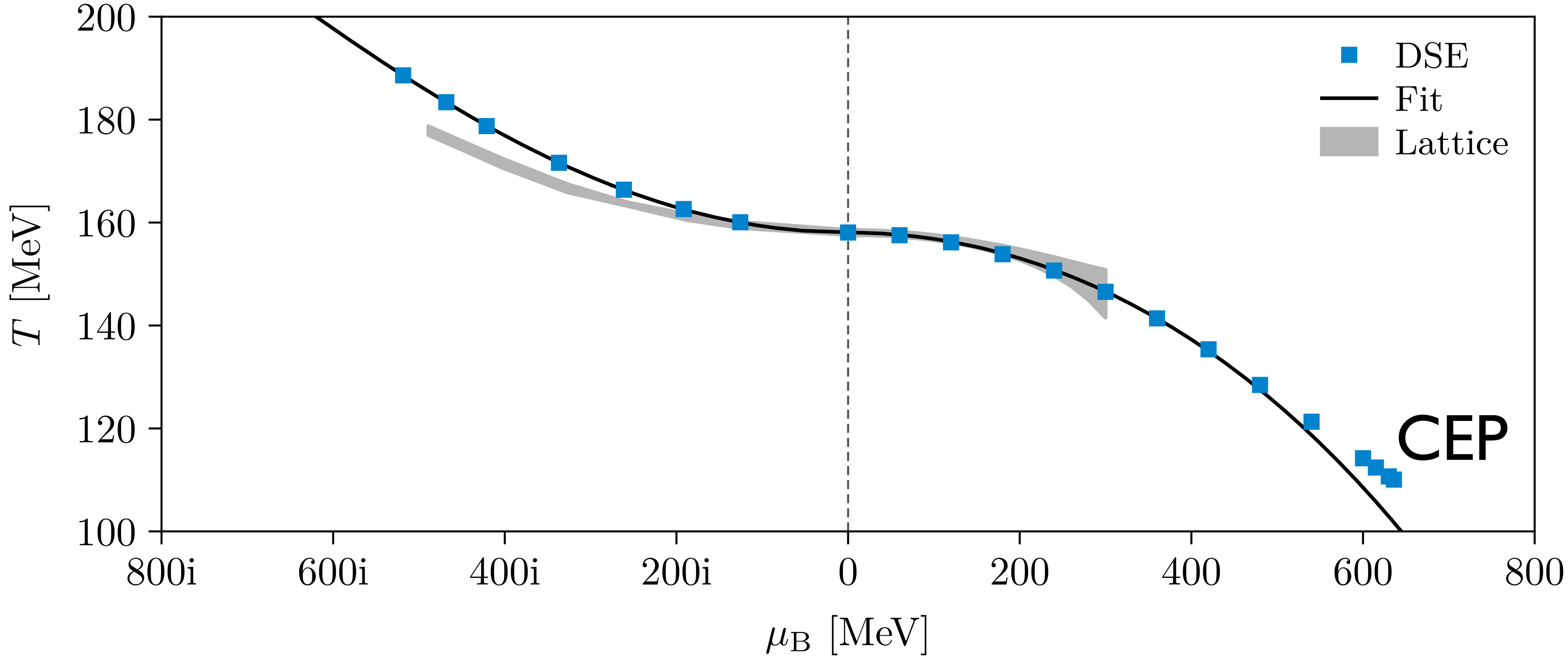
Extrapolation from imaginary chemical potential





$$\chi(T) = \frac{\partial\langle\bar{\psi}\psi\rangle(T)}{\partial m_{\mathrm{u}}}$$

see talk of Julian Bernhardt



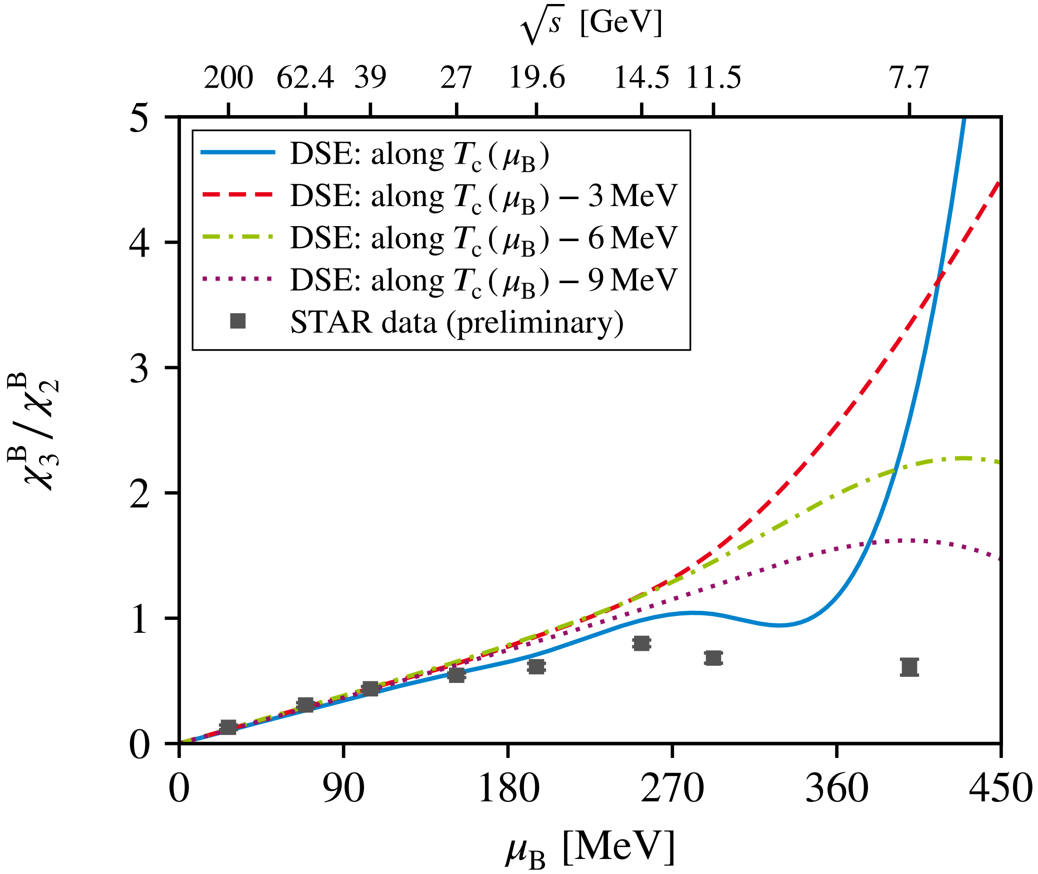
Lattice: Borsanyi et al. PRL 125 052001 (2020)
DSE: Bernhardt, CF, arXiv: 2305.01434

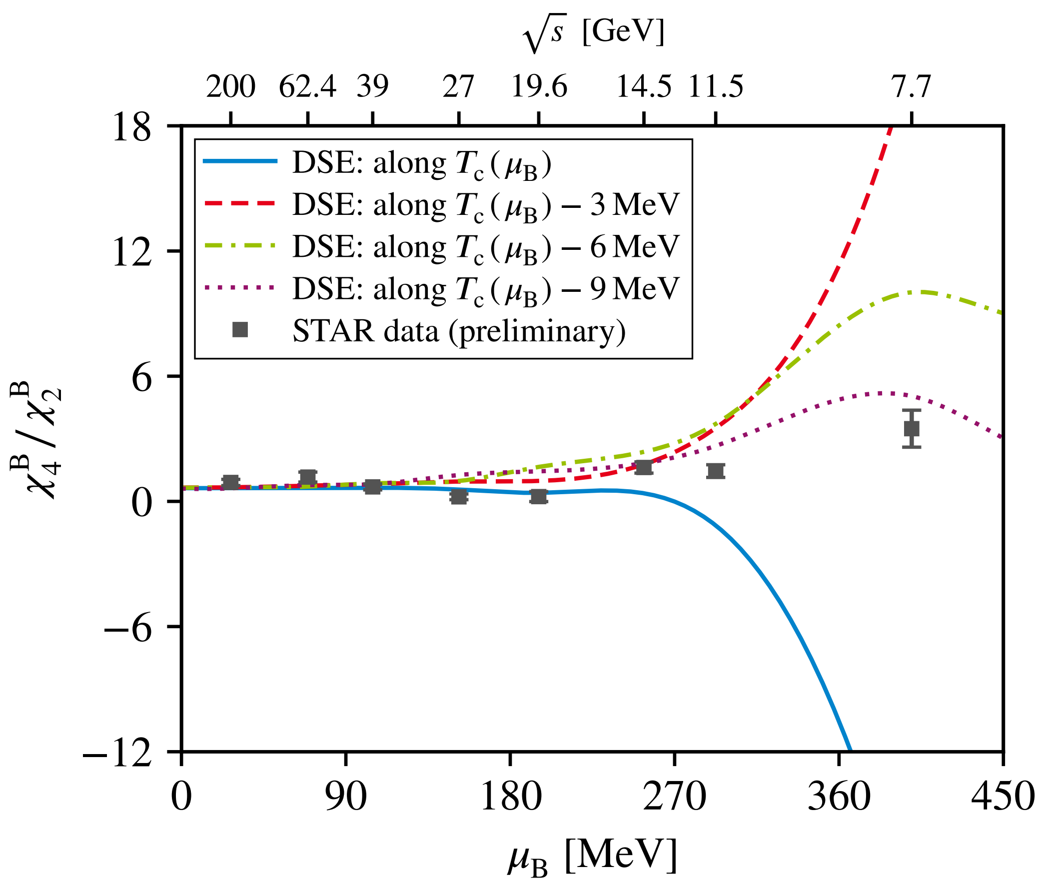
$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2\left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_c}\right)^4$$

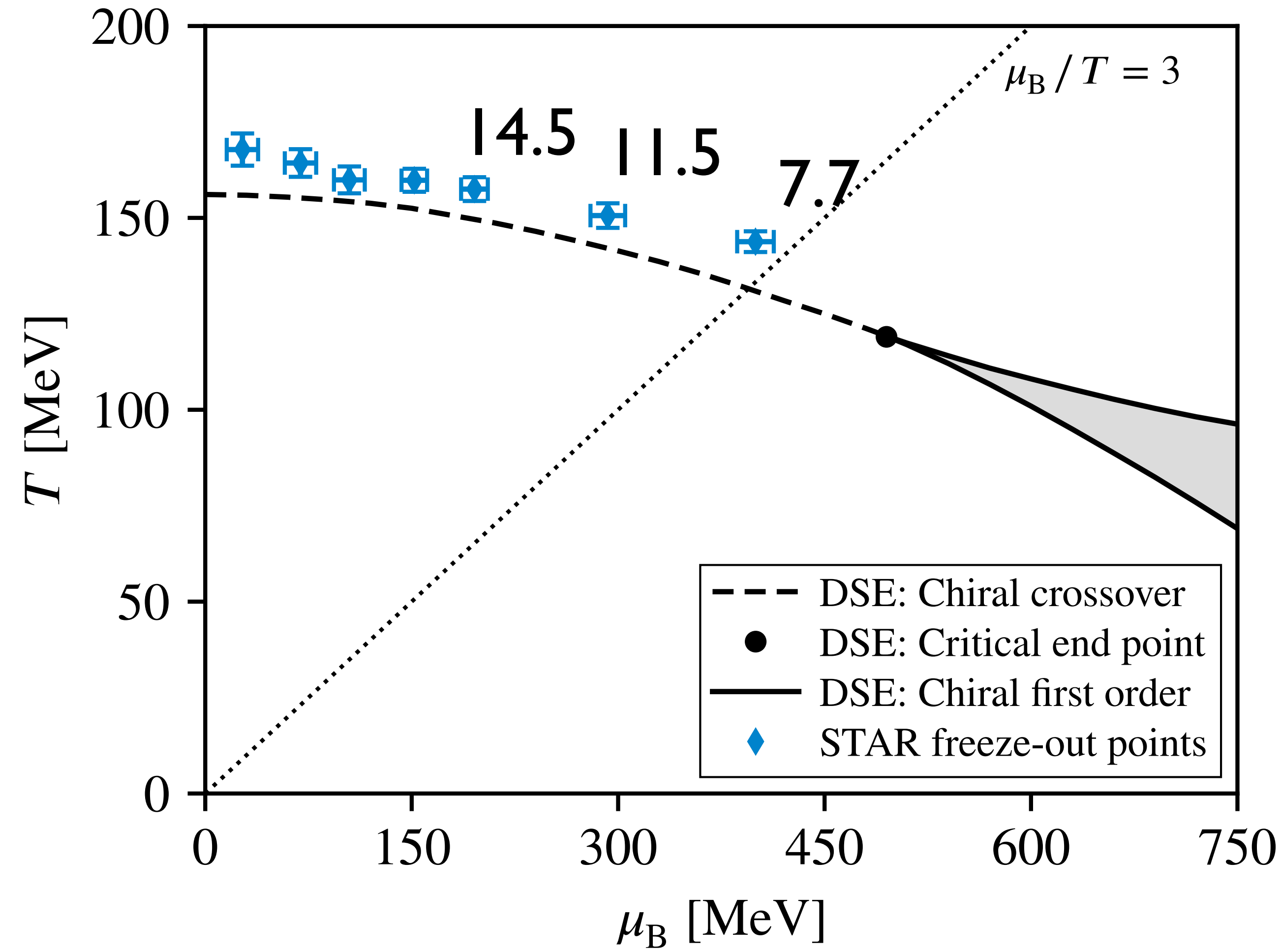
$$\kappa_2^{\mathrm{poly}} = 0.0196\,, \quad \kappa_4^{\mathrm{poly}} = 0.00015\,,$$

- Extrapolation works very well!

Contact with experiment: skewness and curtosis







$\sqrt{s} \geq 14.5$: good agreement

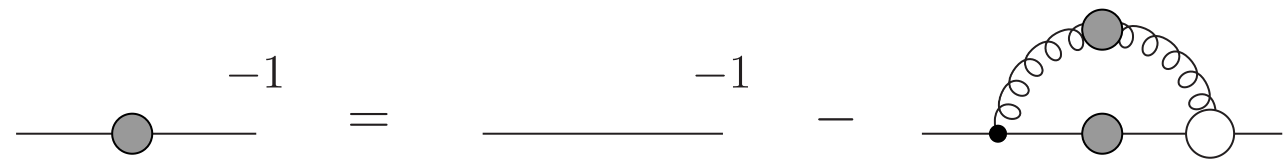
$\sqrt{s} = 11.5$: trend ok !

$\sqrt{s} \leq 7.7$: freezeout line $\neq$ transition line ?!

Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

Overview

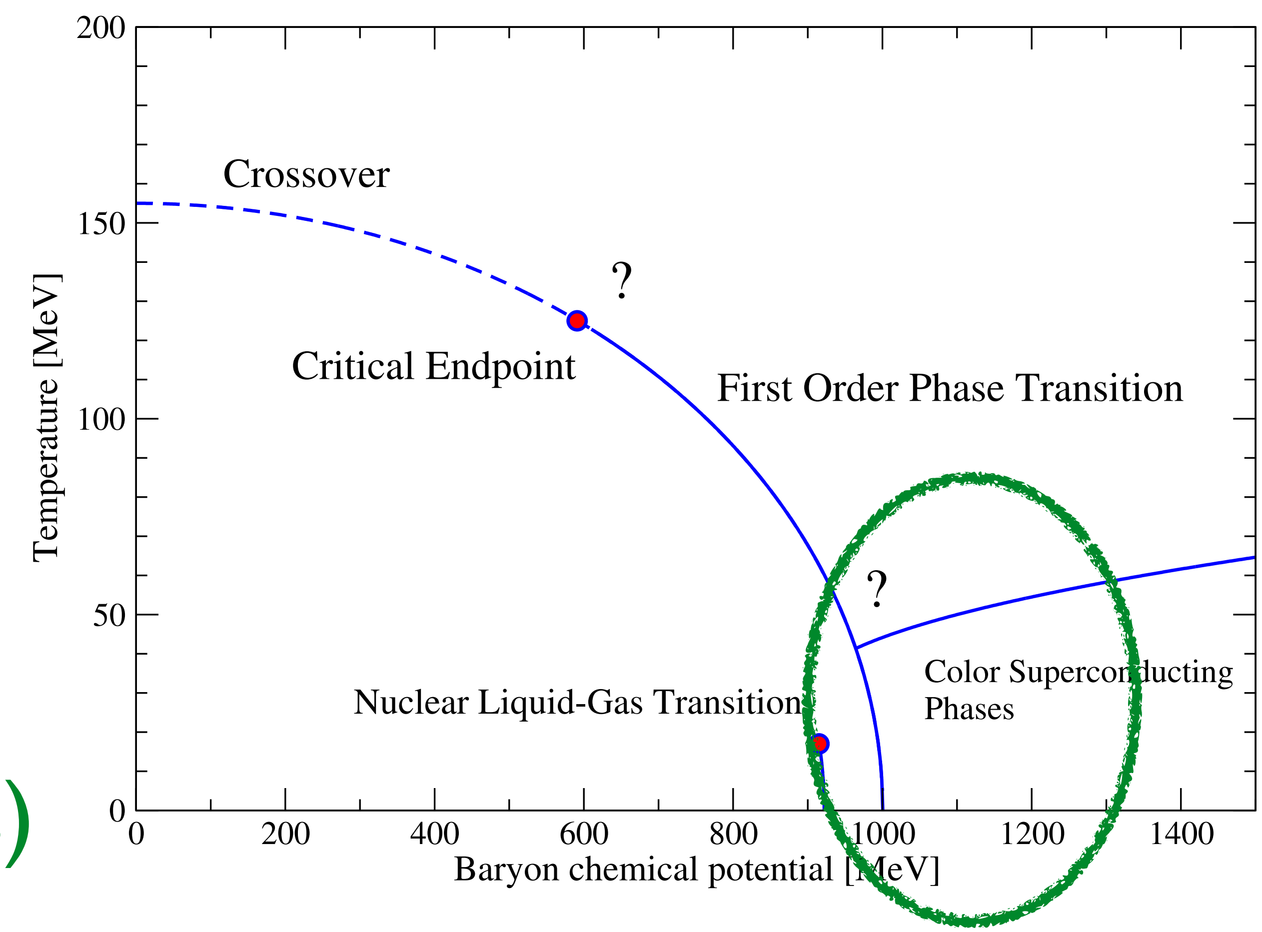
1. Introduction: dynamical mass generation



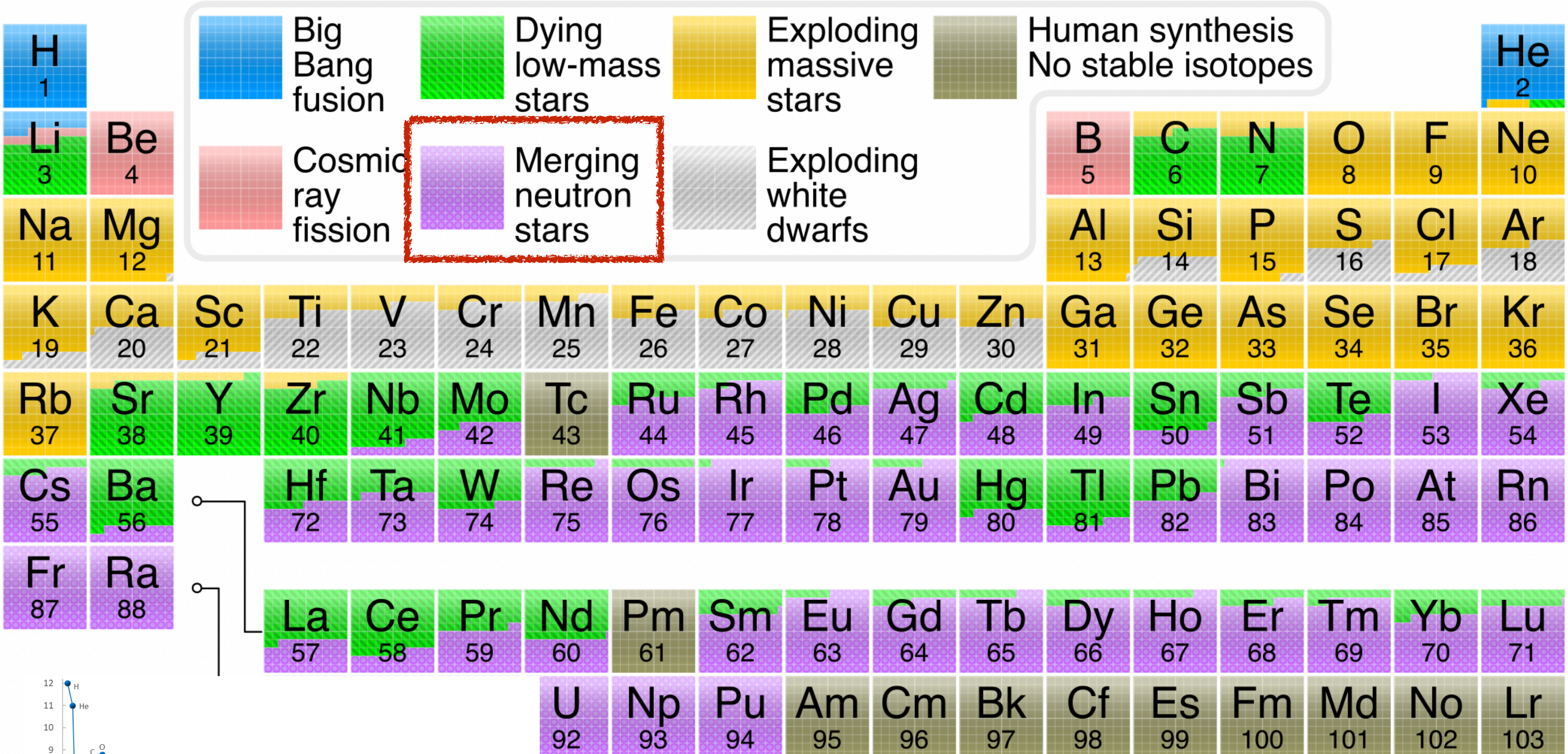
2. Large T, small μ: the quest for the critical end point

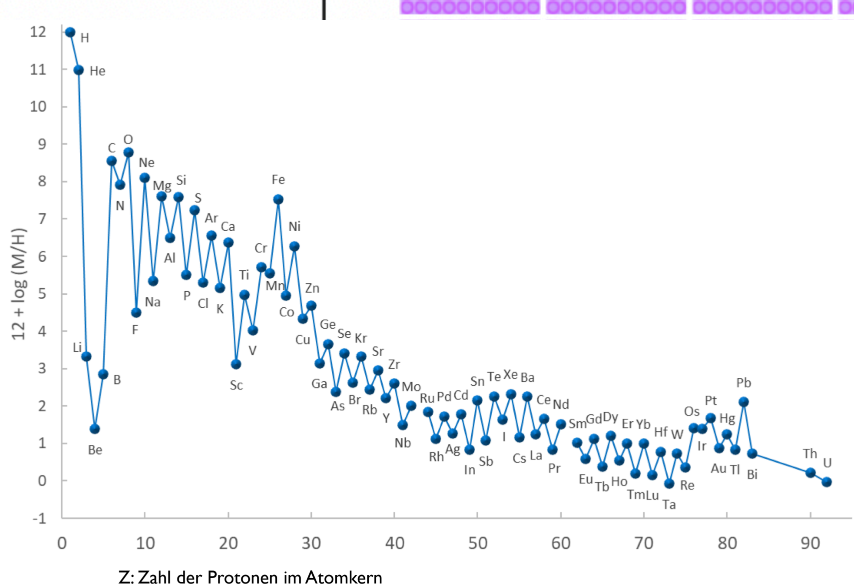
3. Small T, large μ: the quest for the equation of state

Neutron star (mergers)



Nucleo synthesis via r-process

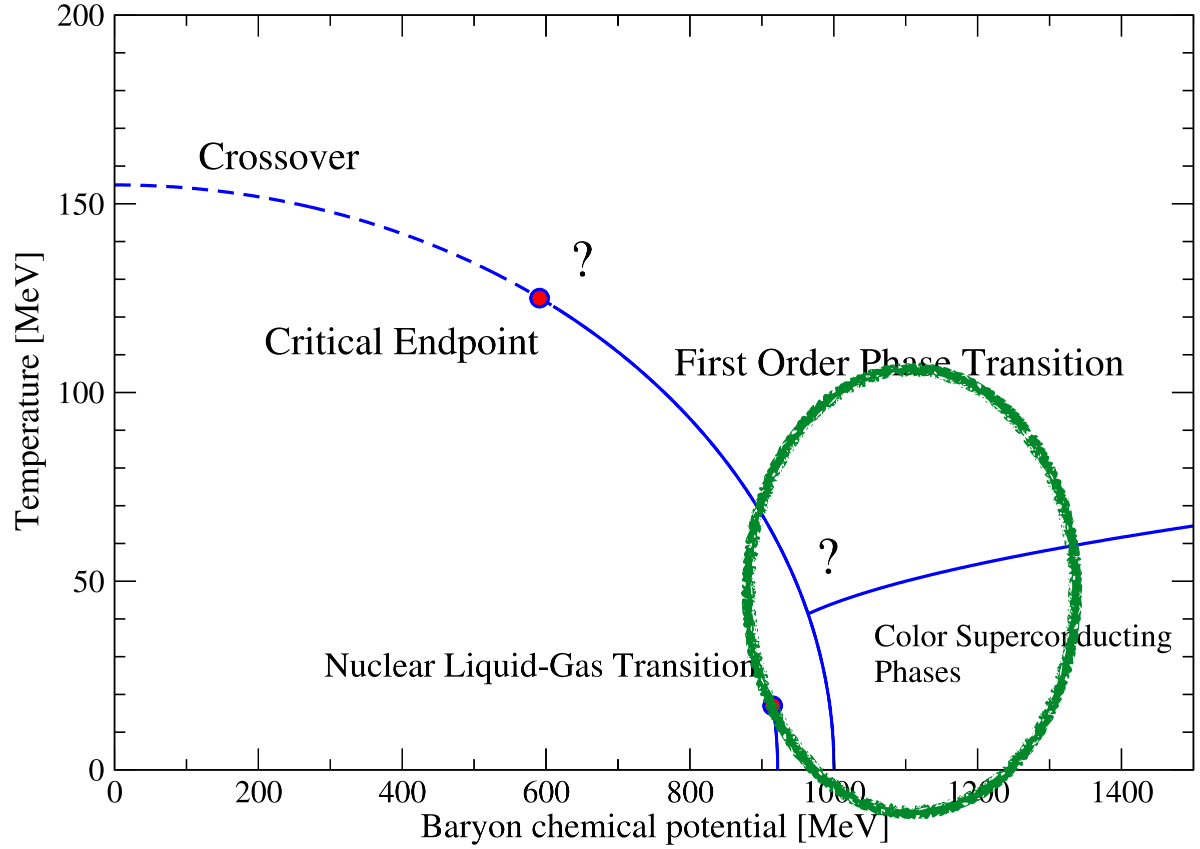




Wikimedia, CC-BY-SA-3.0, nach Jennifer Johnson, Ohio State University

Equation of state from QCD

Gefördert durch DFG Deutsche Forschungsgemeinschaft

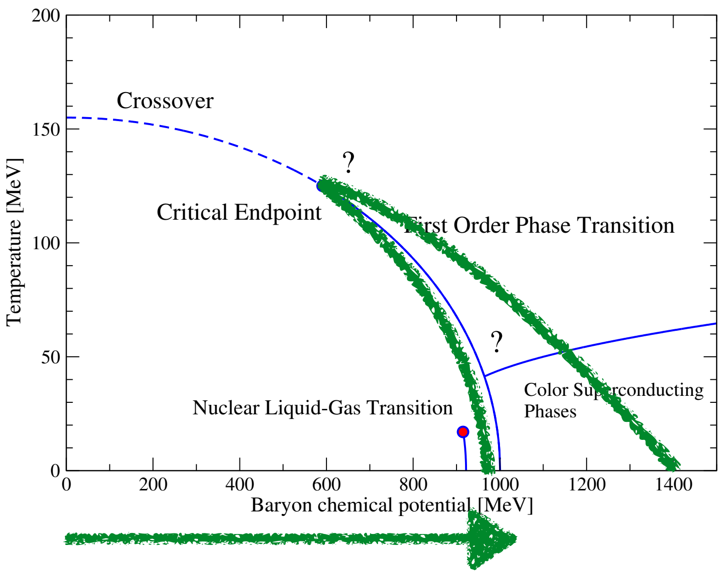


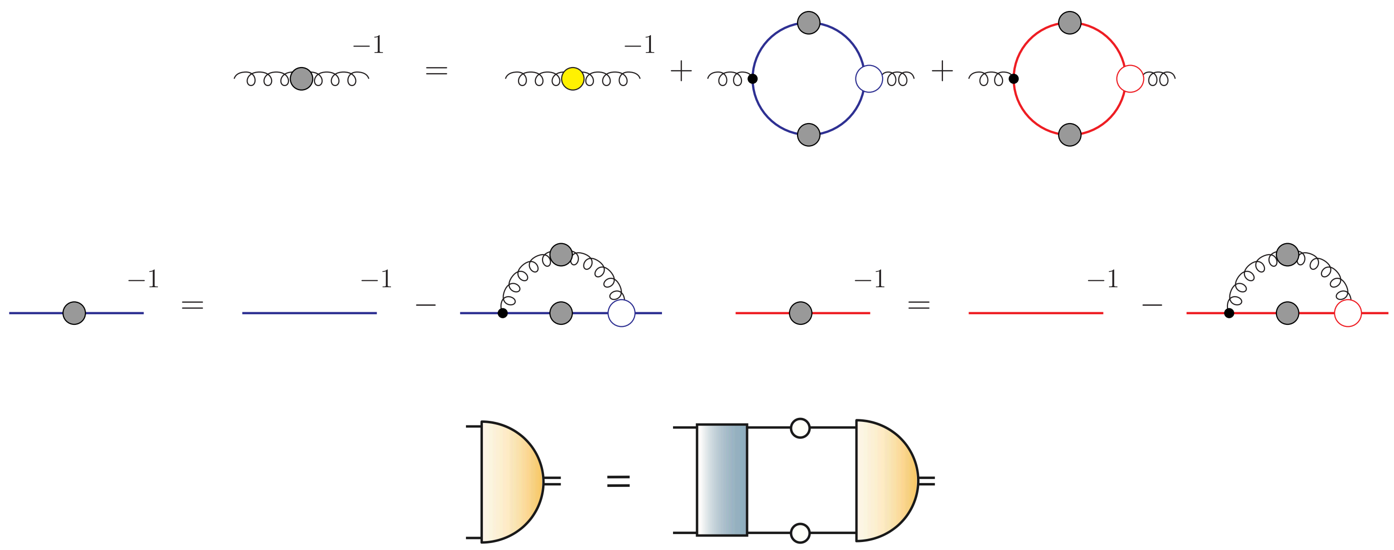
EoS from microscopic QCD (functional approach):

- chirally broken phase
  - quarks, mesons — ✔ our work
  - baryons — work in progress (DFG-ind.)
- superconducting phase(s) — ✔ Buballa et al.
  Müller, Buballa, Wambach, arXiv:1603.02865
- inhomogenuous broken (‘cristaline’) phase(s) — work in progress (CRC, A03)
  Motta, Bernhardt, Buballa, CF, arXiv:2306.09749

see talk of Theo Motta

Meson properties at finite chemical potential

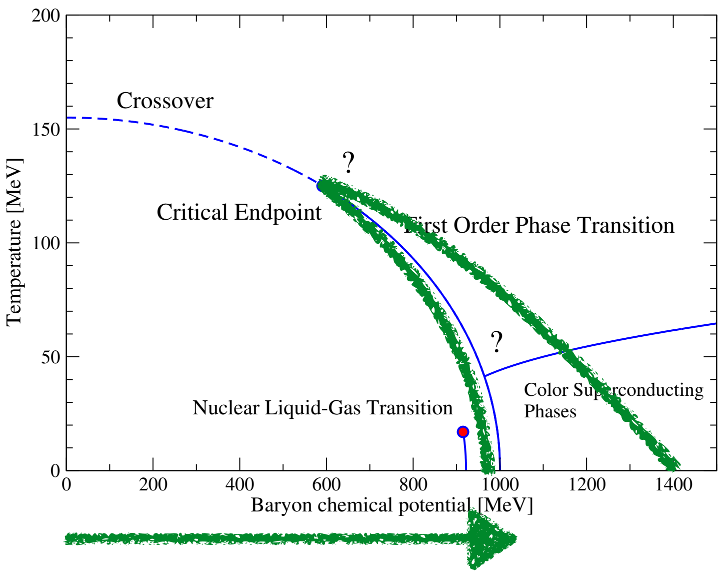


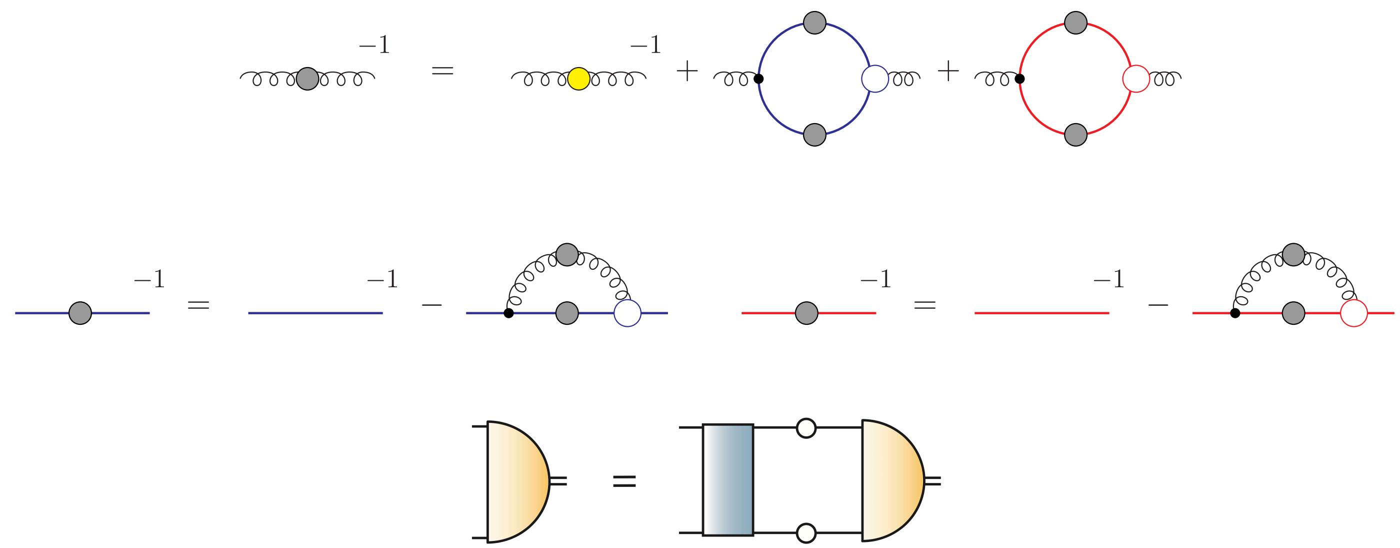


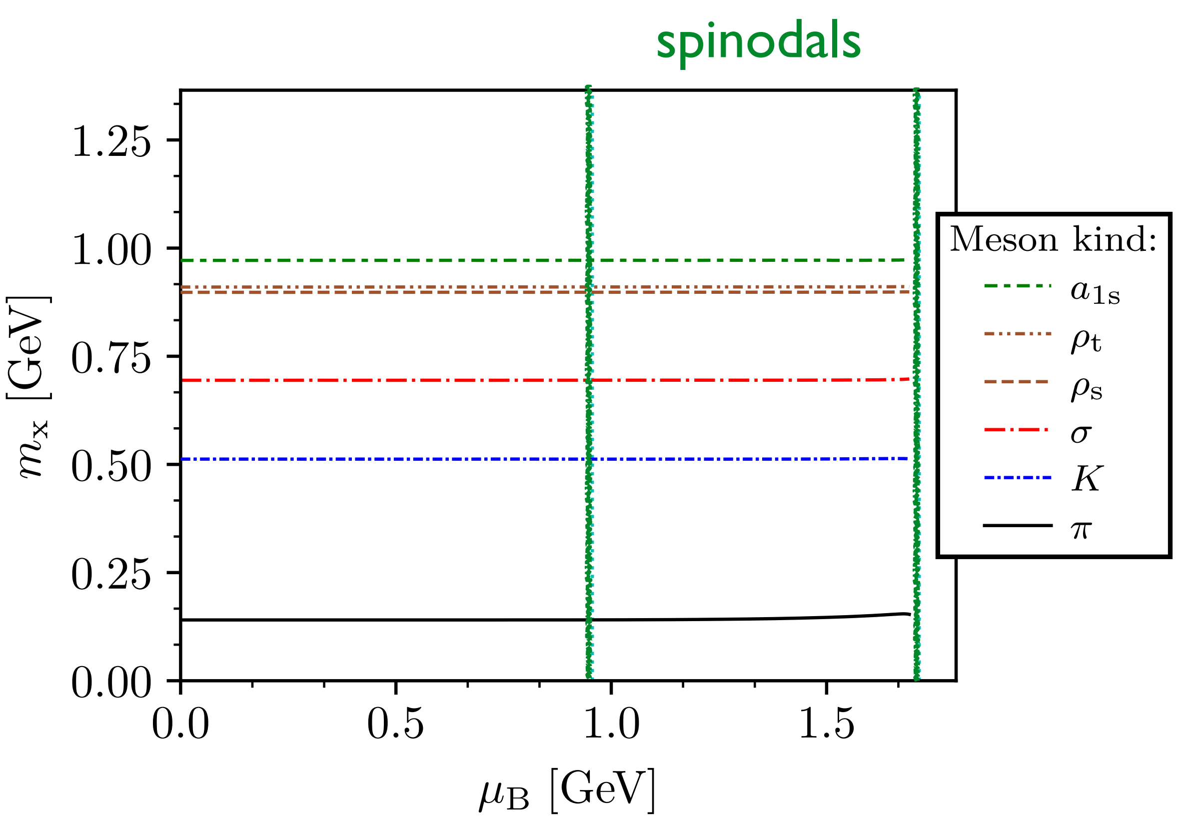
- Quarks/meson wave functions do change !

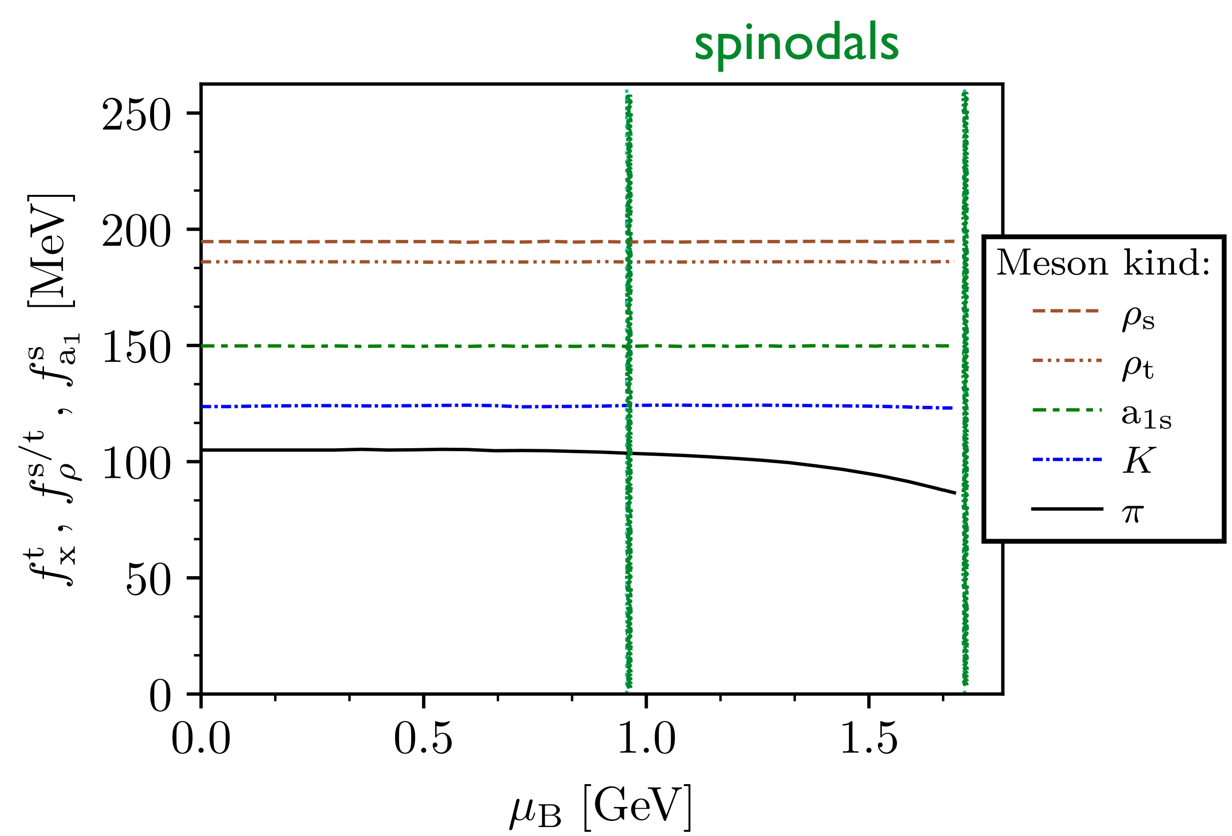
Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147

Meson properties at finite chemical potential









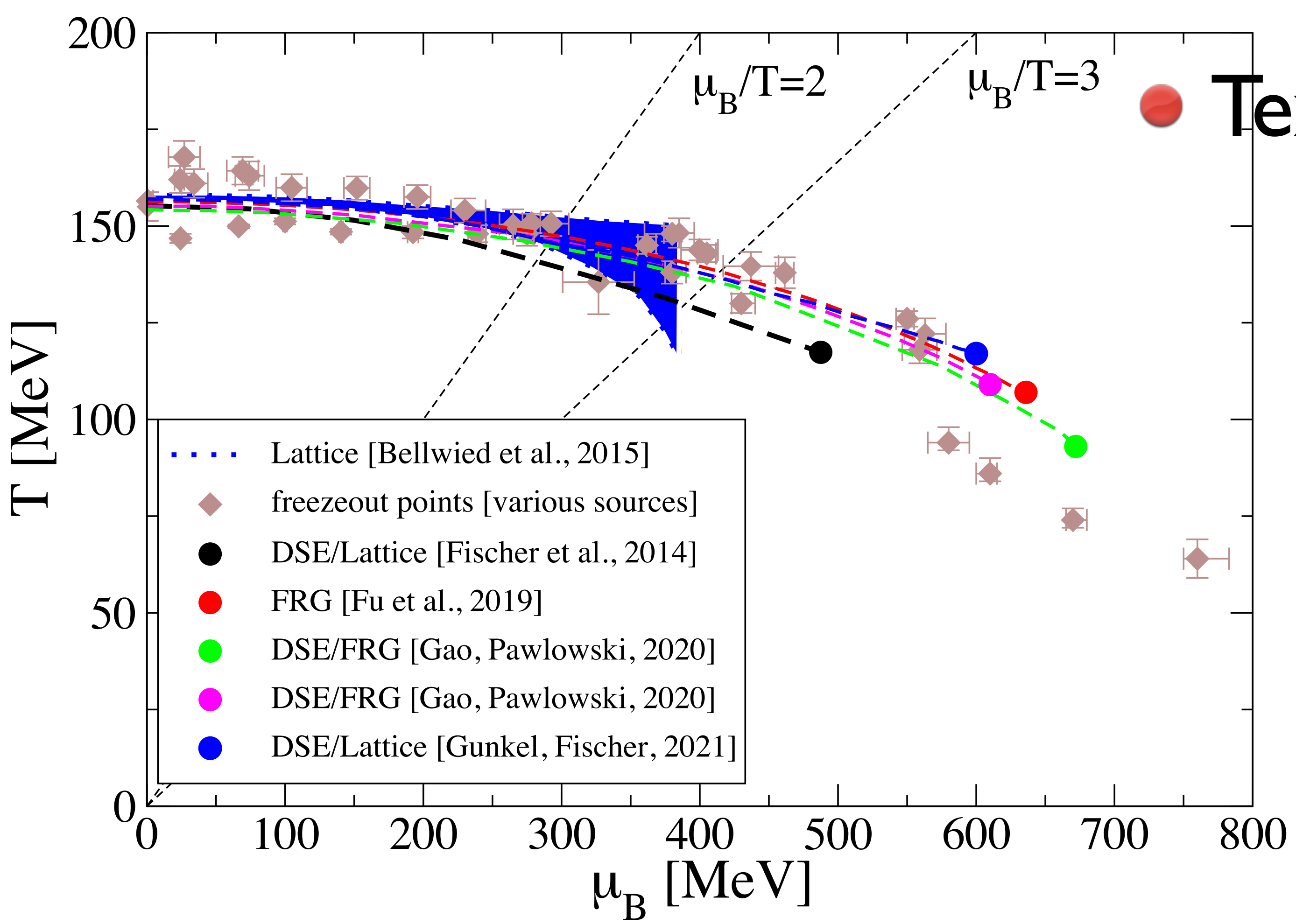
- Quarks/meson wave functions do change !
- But: Silver blaze satisfied

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147
T. D. Cohen, PRL 91 , 222001 (2003)

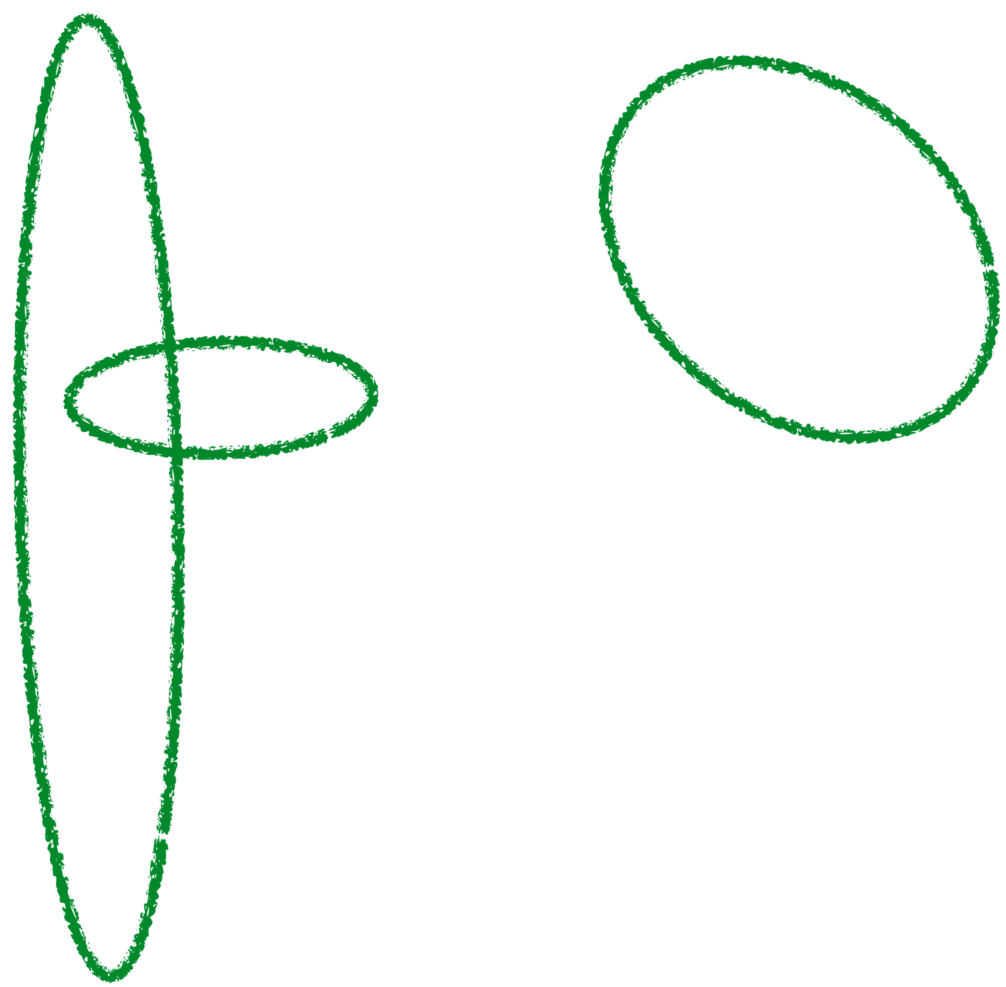
Main goals:

- **one** framework for all areas of hadron physics:
  mesons, baryons, 'exotic states', form factors,
  hadronic contributions to precision observables (g-2)
- **same** framework for QCD phase diagram

Main results:



- Text

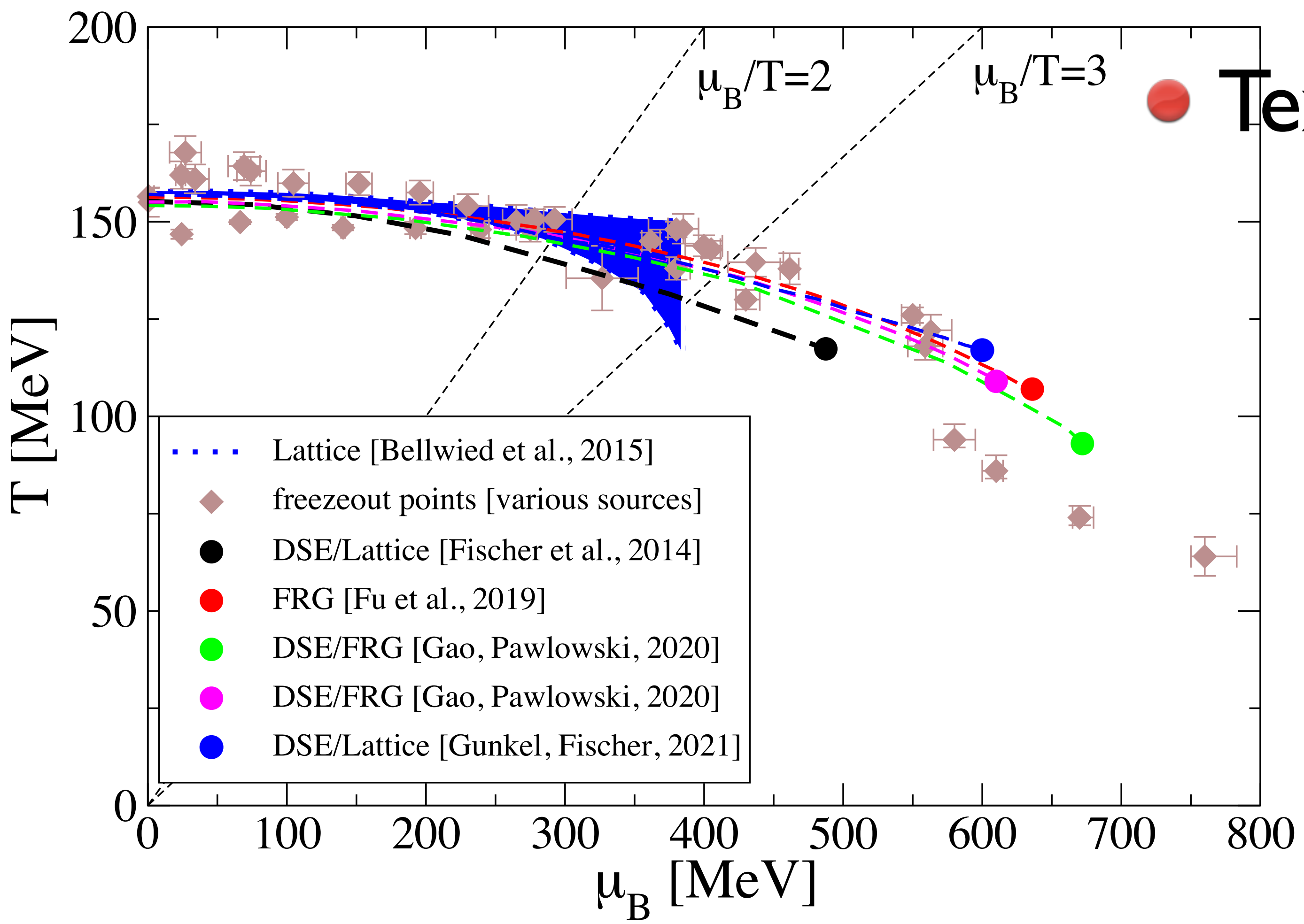


Summary: QCD with functional methods

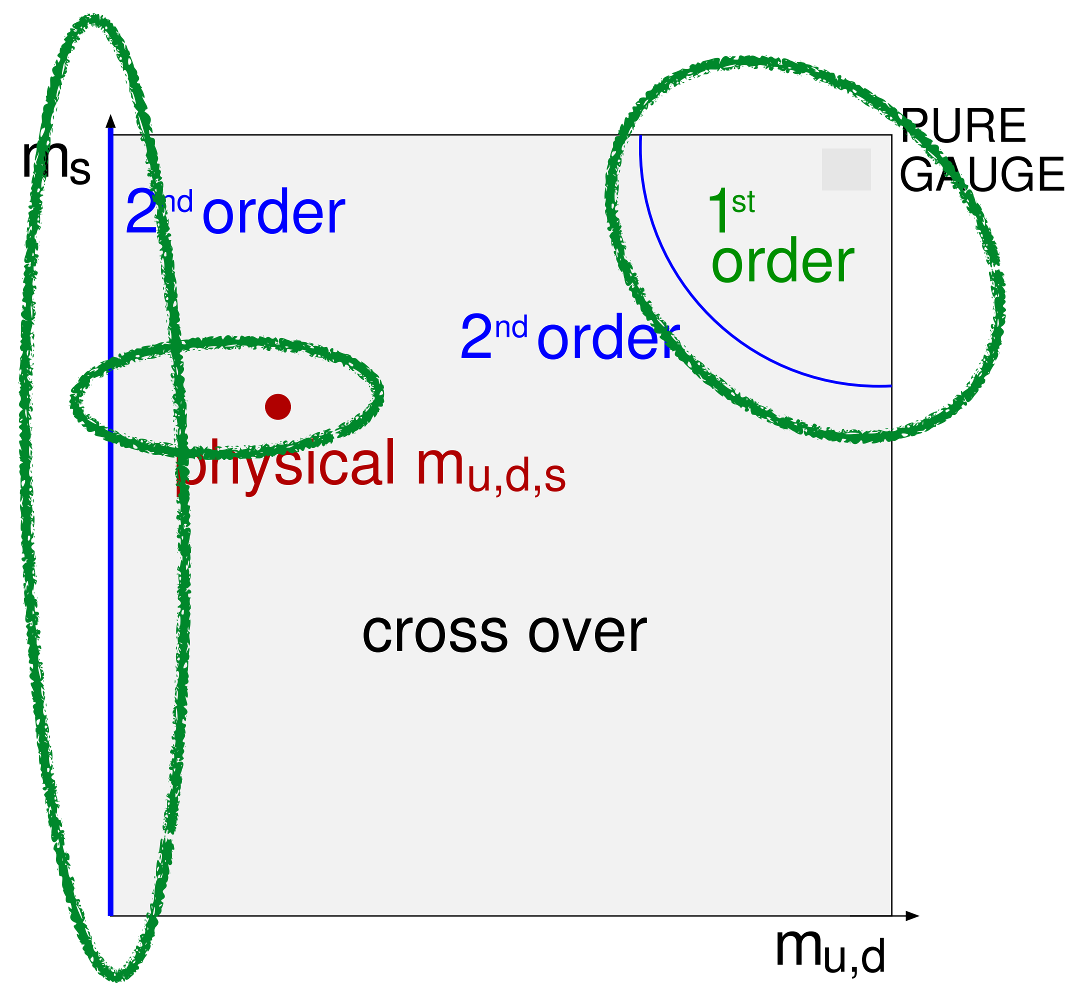
Main goals:

- **one** framework for all areas of hadron physics:
  mesons, baryons, 'exotic states', form factors,
  hadronic contributions to precision observables (g-2)
- **same** framework for QCD phase diagram

Main results:



- Text

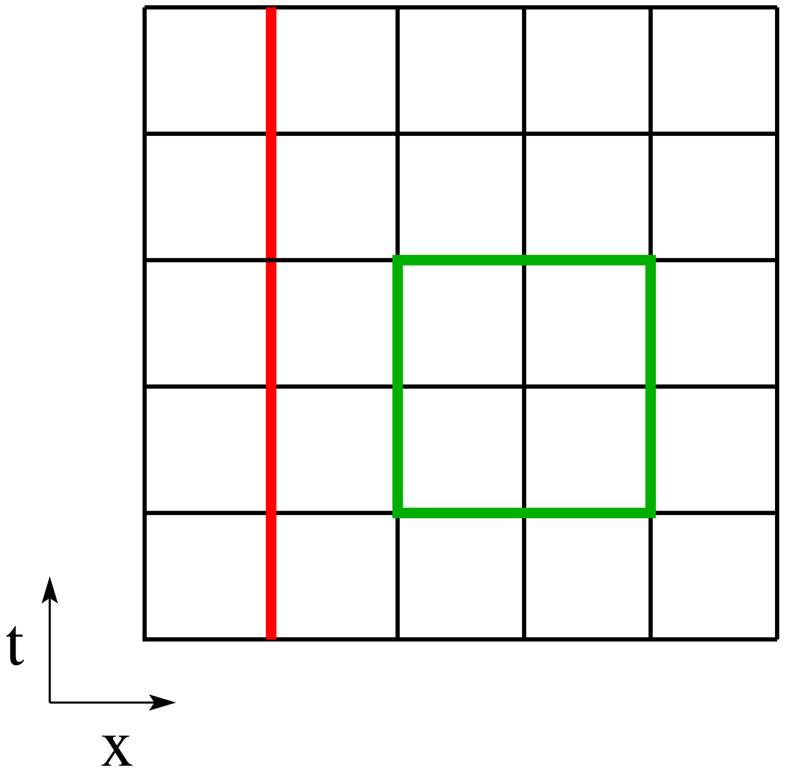


Backup

Polyakov-Loop and center symmetry

Wilson-Loop: $$U(C) = \hat{P} \exp \left[ig \oint_C dx^\mu A_\mu(x) \right]$$

Polyakov-Loop: $$\Phi = \hat{P} \exp \left[ig \int_0^{1/T} d\tau A_4(\tau, \vec{x}) \right]$$



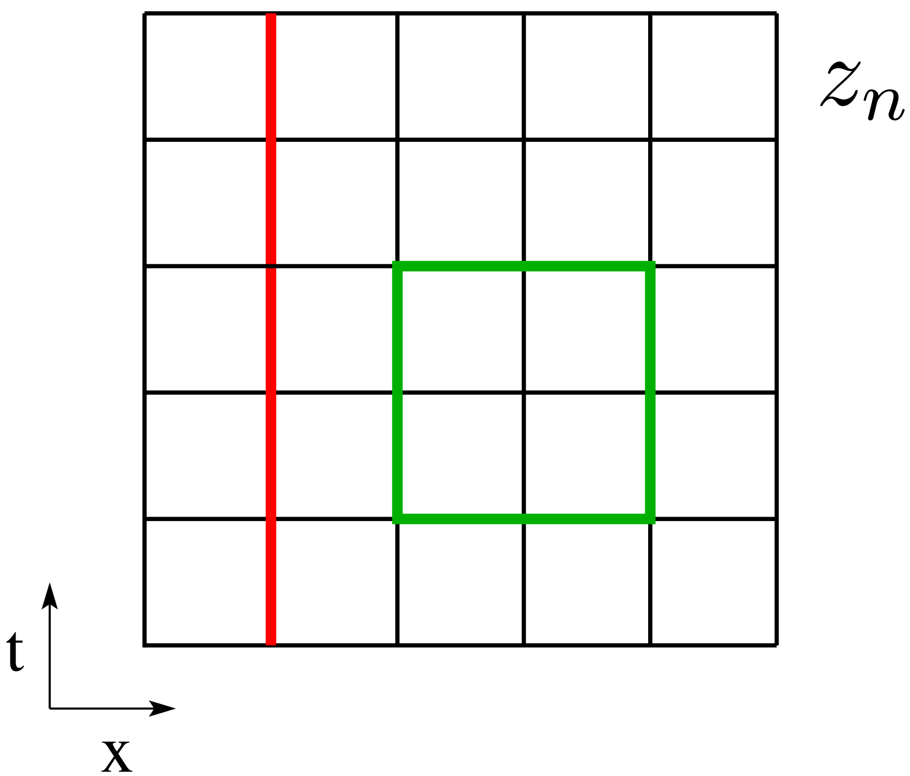
Center of gauge group SU(N_c):

$$z_n = \exp[2\pi i\, n/N_c] \mathbb{1}, \quad n = 0..N_c - 1$$

Polyakov-Loop and center symmetry

Wilson-Loop: $$U(C) = \hat{P} \exp\left[ig \oint_C dx^\mu A_\mu(x) \right]$$

Polyakov-Loop: $$\Phi = \hat{P} \exp\left[ig \int_0^{1/T} d\tau A_4(\tau, \vec{x}) \right]$$



Center of gauge group SU(N_c):

$$z_n = \exp[2\pi i\, n/N_c]\mathbb{1}, \quad n = 0..N_c - 1$$

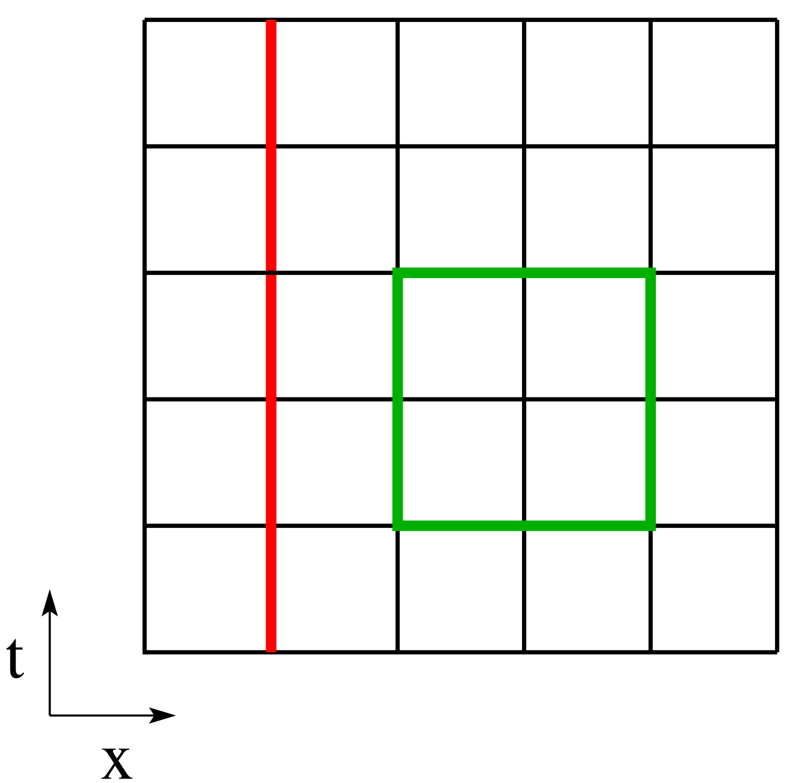
Center transformation:

$$S_{QCD} \to S_{QCD}$$
$$\Phi \to z_n \Phi$$

Polyakov-Loop and center symmetry

Wilson-Loop: $$U(C) = \hat{P} \exp\left[ig \oint_C dx^\mu A_\mu(x) \right]$$

Polyakov-Loop: $$\Phi = \hat{P} \exp\left[ig \int_0^{1/T} d\tau A_4(\tau, \vec{x}) \right]$$



z_n

Center of gauge group SU(N_c):

$$z_n = \exp[2\pi i \, n/N_c] \mathbb{1}, \quad n = 0..N_c - 1$$

Center transformation:

$$S_{QCD} \to S_{QCD}$$
$$\Phi \to z_n \Phi$$

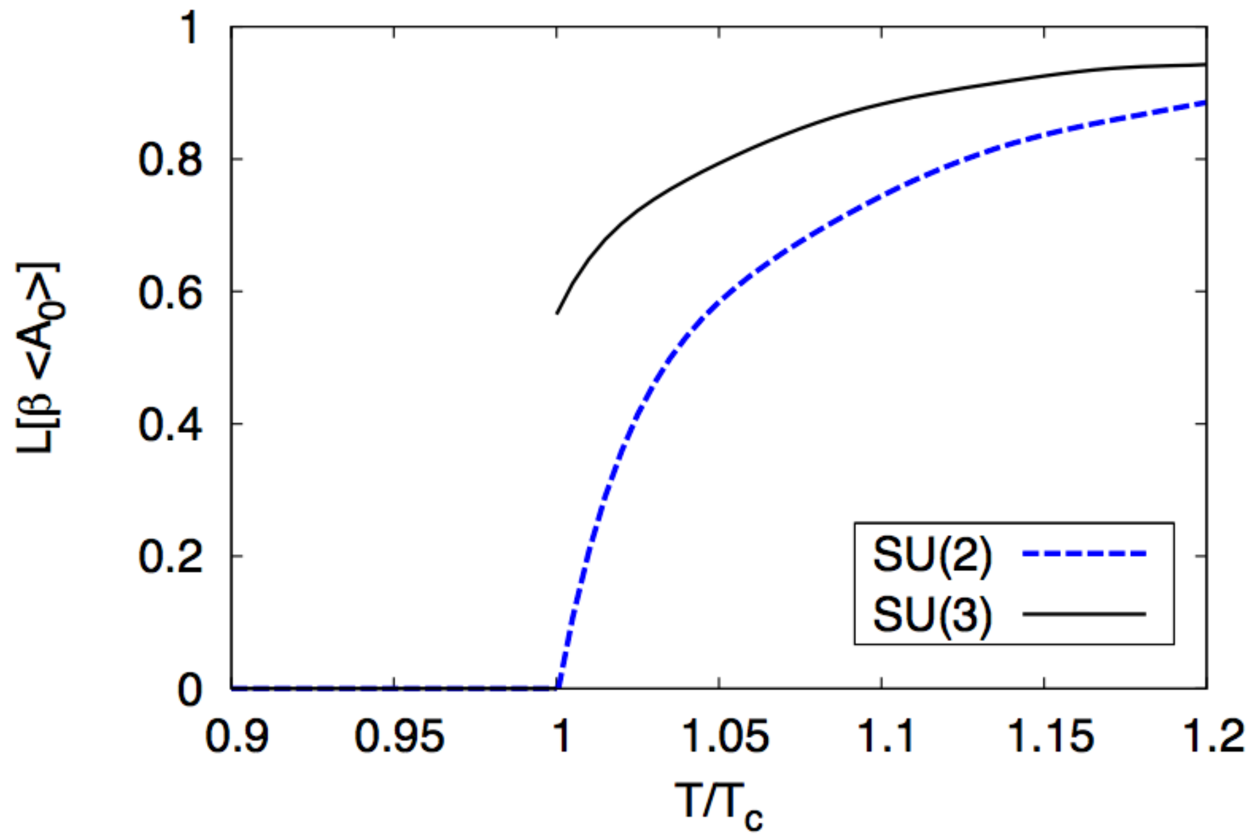
$$\langle Tr\, \Phi \rangle = \begin{cases} 0 & \text{unbroken } z_n \text{ symmetry} \\ \text{non-zero} & \text{broken } z_n \text{ symmetry} \end{cases}$$

Energy of an isolated quark

$$\langle Tr\,\Phi\rangle = \begin{cases} 0 & \text{unbroken } z_n \text{ symmetry} \\ \text{non-zero} & \text{broken } z_n \text{ symmetry} \end{cases}$$

$$\langle Tr\,\Phi\rangle \sim e^{-F_q/T} \qquad F_q = \begin{cases} \infty & \text{unbroken } z_n \text{ symmetry} \\ \text{finite} & \text{broken } z_n \text{ symmetry} \end{cases}$$

F_q: free energy of heavy quark



Braun, Gies, Pawlowski, PLB684 (2010)

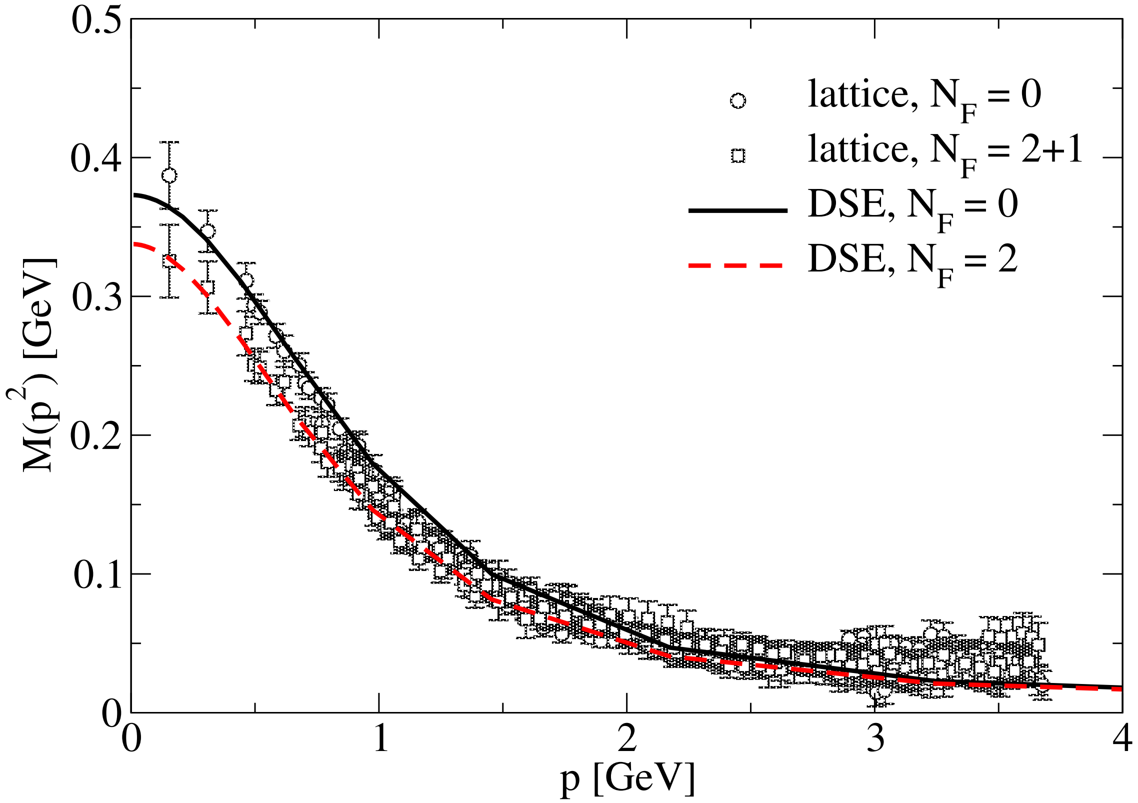
Order parameter!

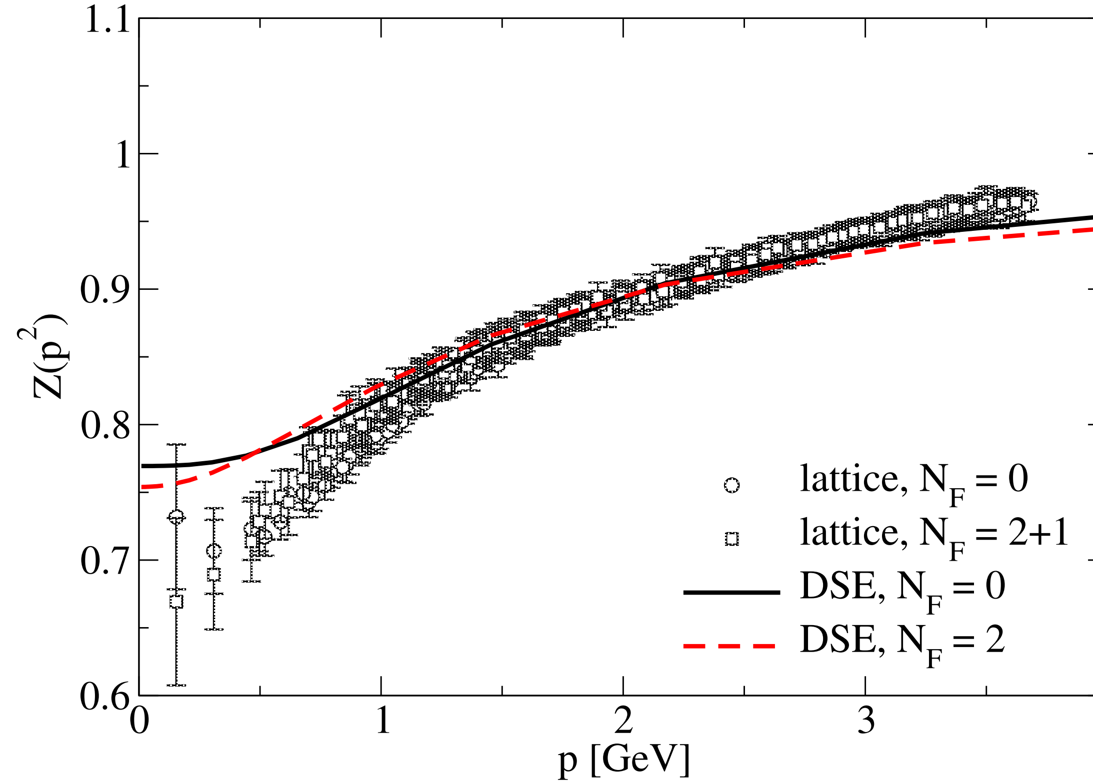
- SU(2): second order
- SU(3): first order

Selected results for Green's functions

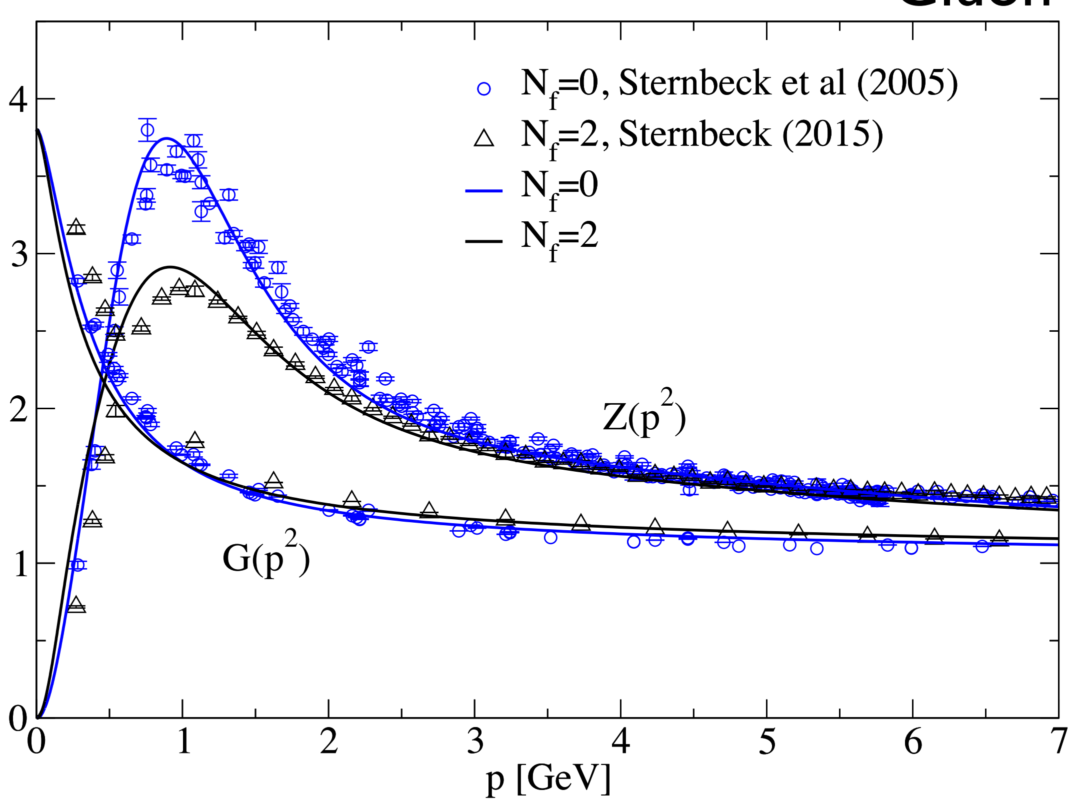
Quark

Williams, CF, Heupel, PRD 93 (2016) 034026

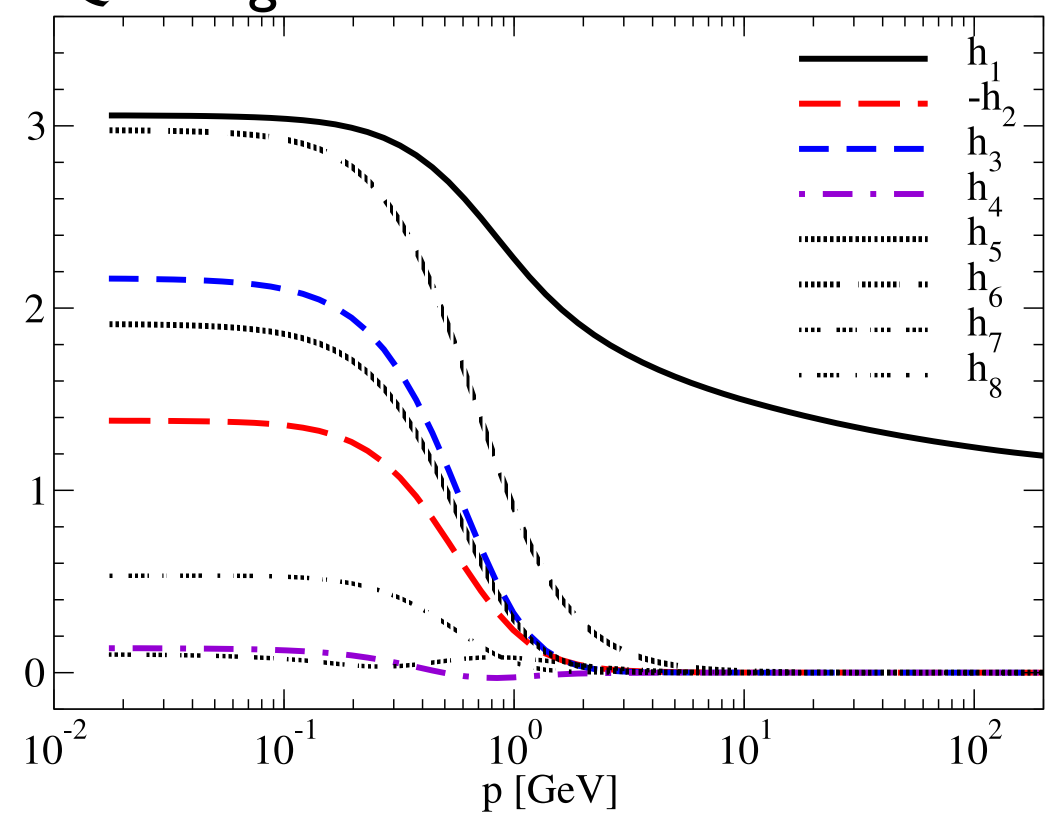




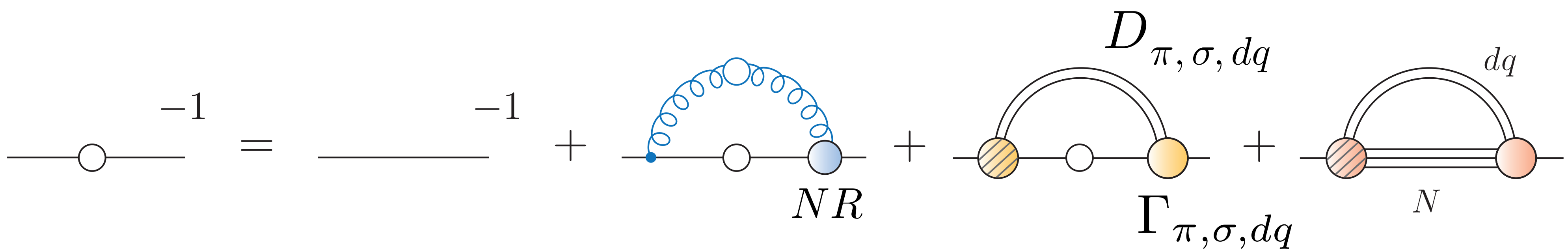
Gluon



Quark-gluon-vertex

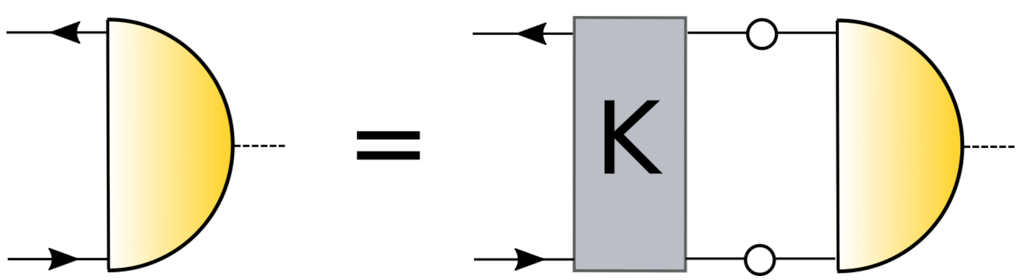


Meson effects at finite T and μ



$$D_\pi(p) = \frac{1}{p_4^2 + u^2(\vec{p}^2 + m_\pi(T,\mu)^2)} \qquad u = \frac{f_s}{f_t}$$

Son, Stephanov, PRD 66 (2002) 7

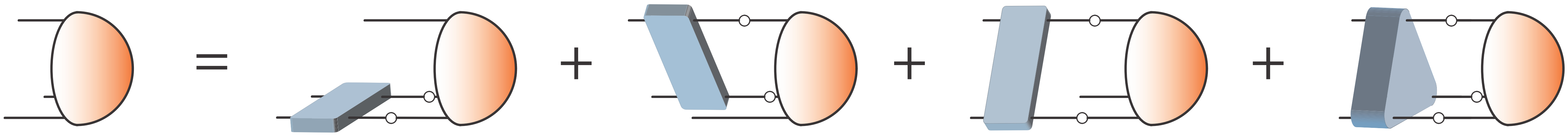


$$\Gamma_\pi(P,q) = \gamma_5\, E(P,q,T,\mu) + \ldots$$

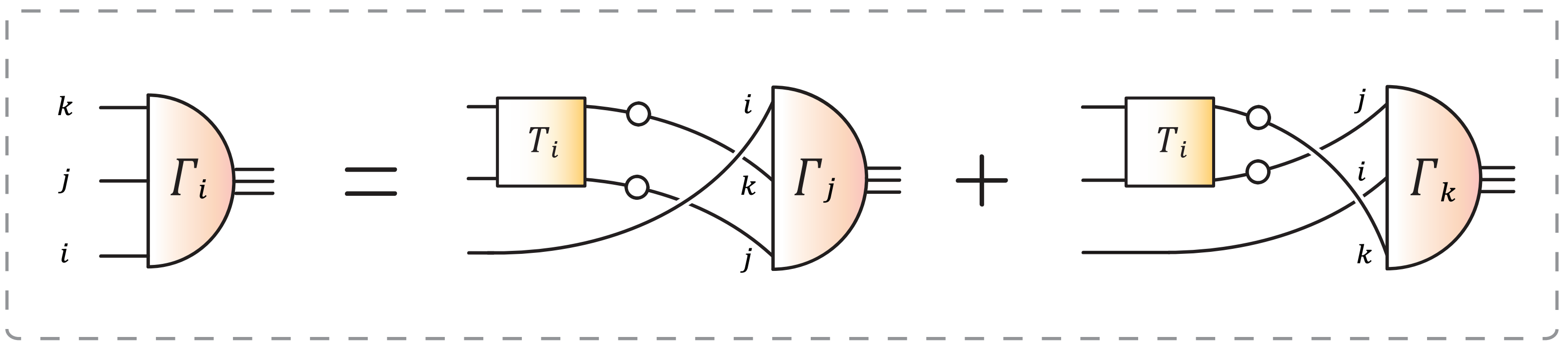
chiral limit: $\Gamma_\pi = \gamma_5 \dfrac{B}{f_t}$

Vacuum: Baryons from BSEs

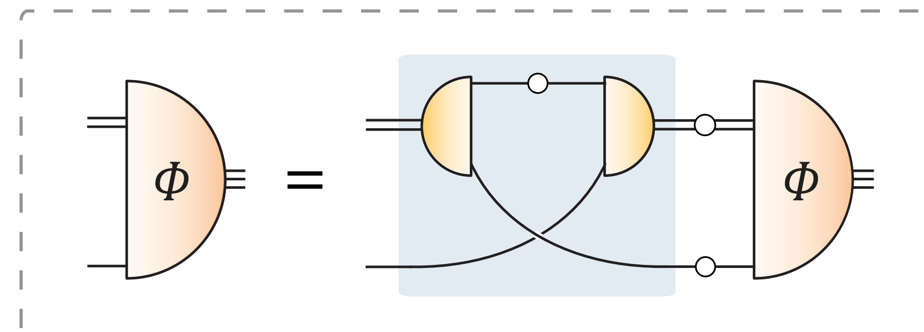
BSE for baryons (derived from equation of motion for G)



Faddeev equation (no three-body forces)

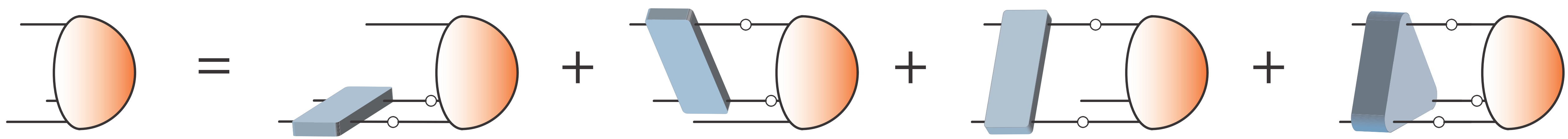


Diquark-quark

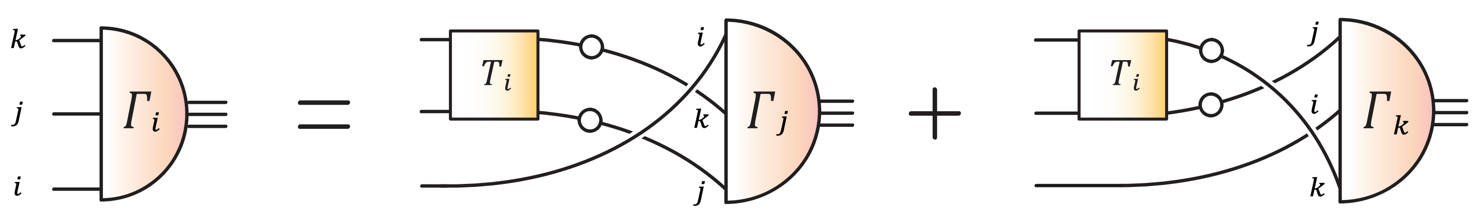


Vacuum: Baryons from BSEs

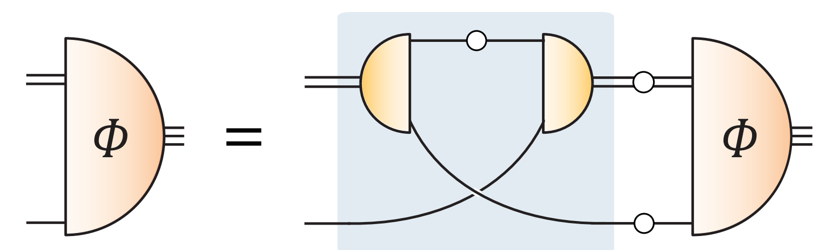
BSE for baryons (derived from equation of motion for G)

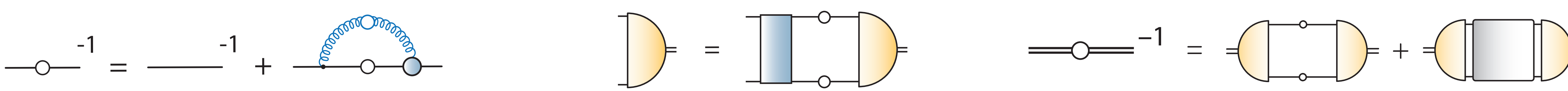


Faddeev equation (no three-body forces)



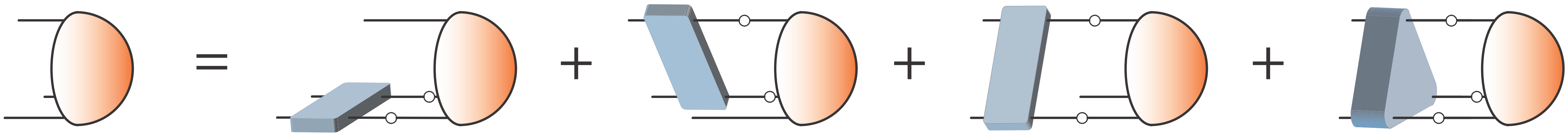
Diquark-quark



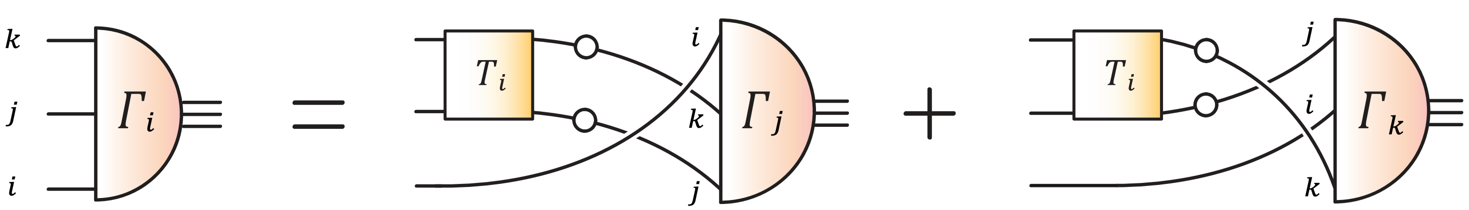


Vacuum: Baryons from BSEs

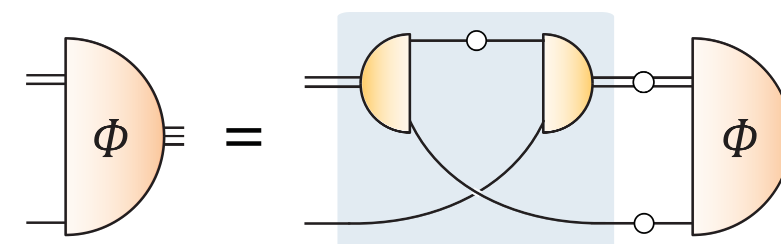
BSE for baryons (derived from equation of motion for G)

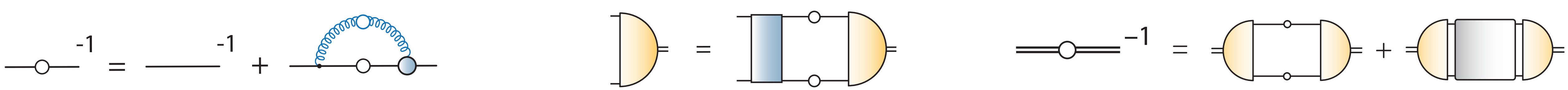


Faddeev equation (no three-body forces)

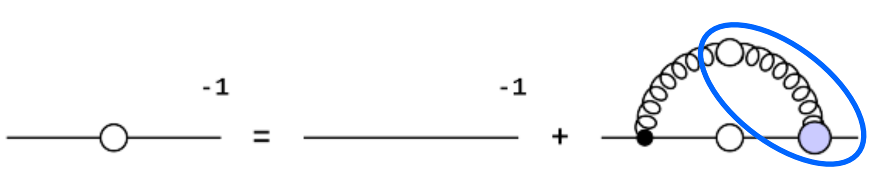


Diquark-quark





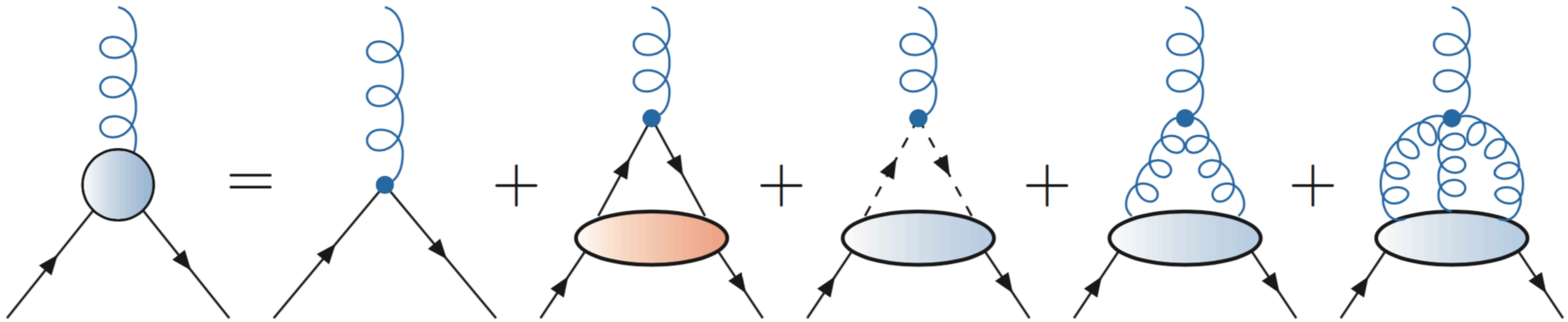
- Input: Non-perturbative quark, quark-gluon interaction (RL)

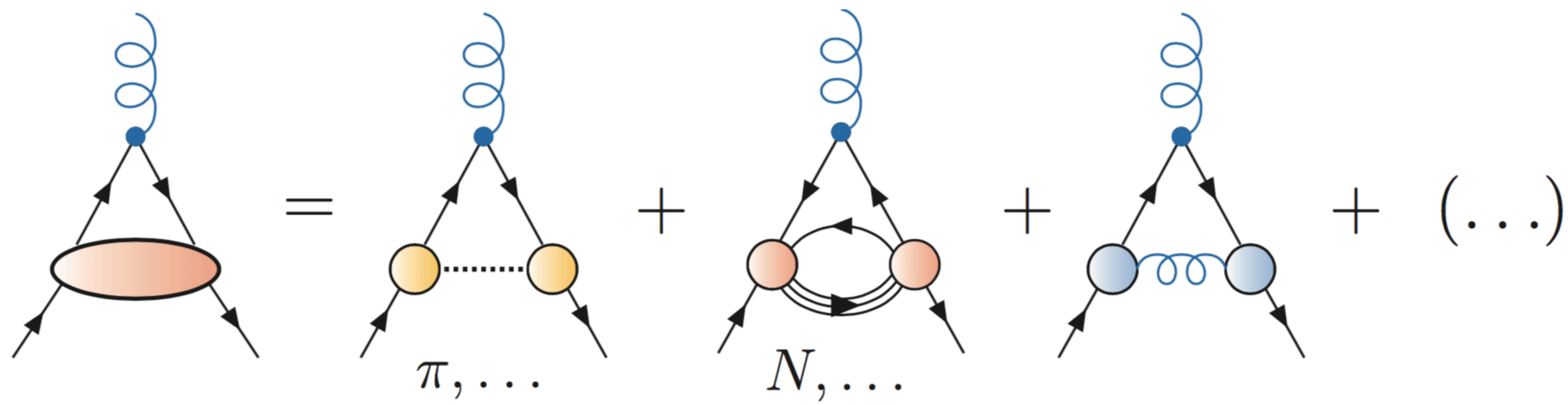


$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

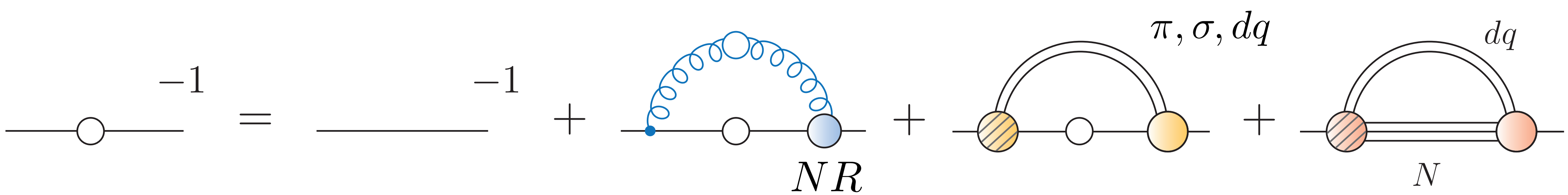
Hadron effects in quark-gluon interaction

quark-gluon vertex:





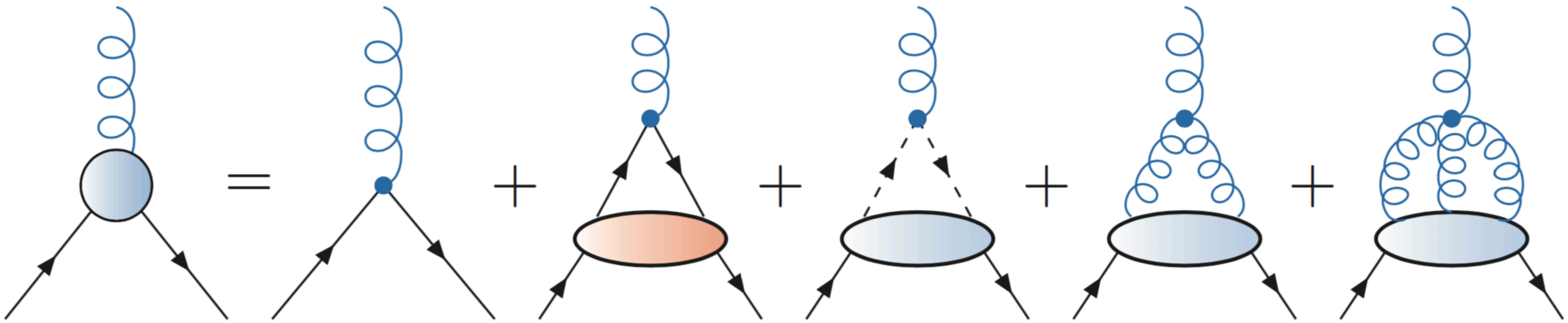
quark:

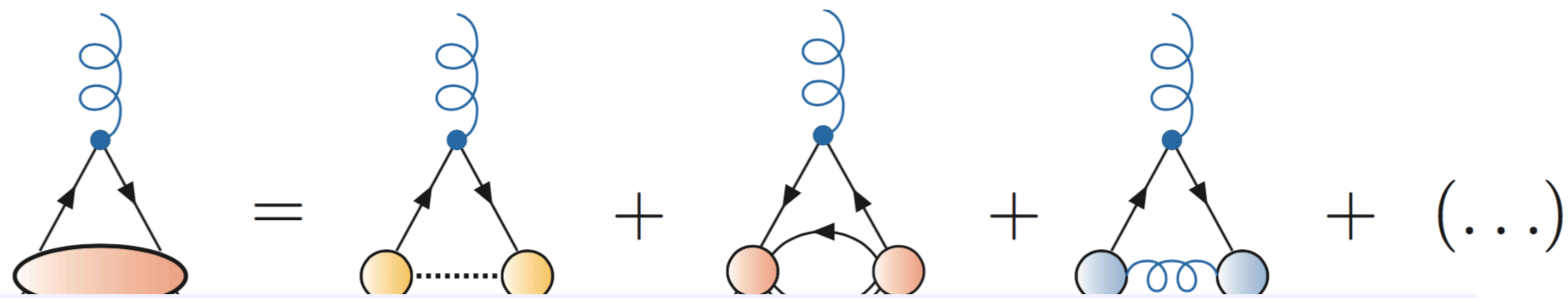


Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

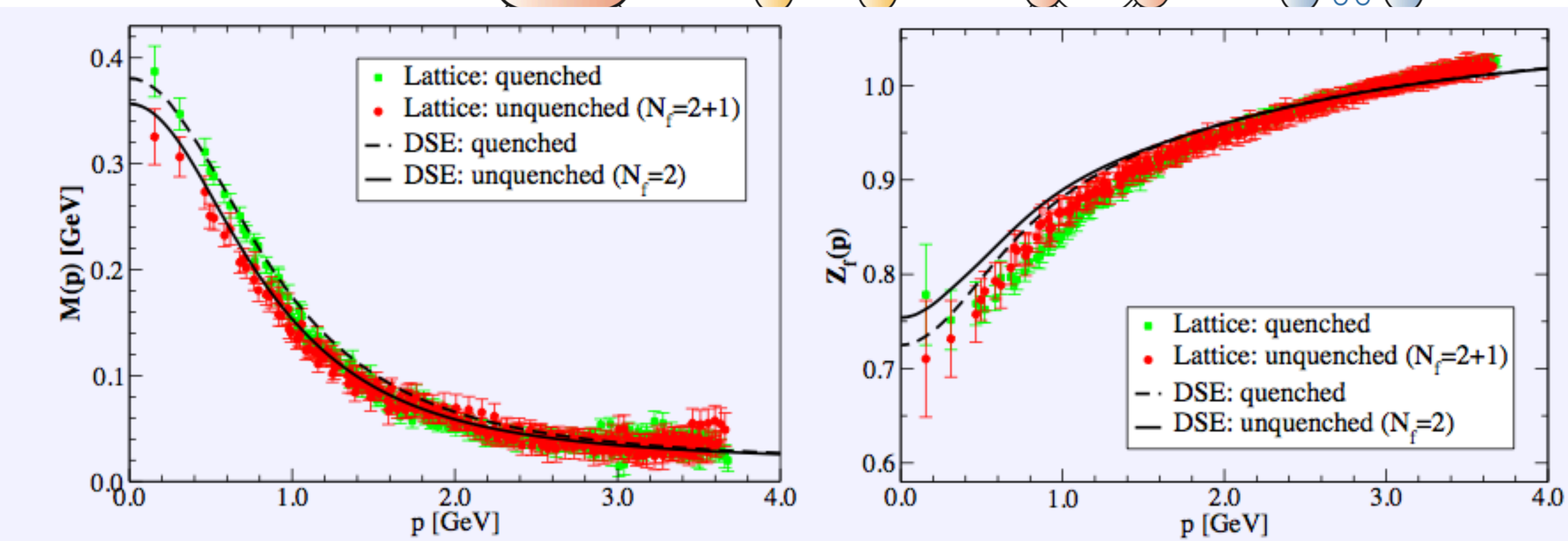
Hadron effects in quark-gluon interaction

quark-gluon vertex:





quar



CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

2]