Leveling up Quantum Trajectories for $Q\bar{Q}$

Quarkonia as Open Quantum Systems

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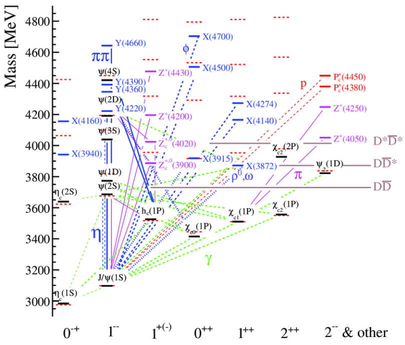
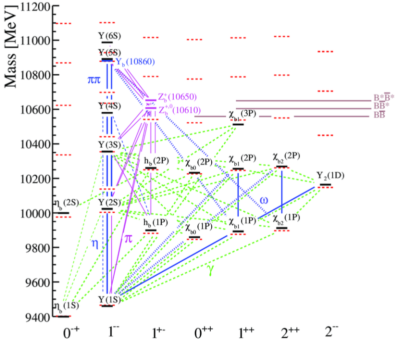
Quarkonia as an OQS

Can we describe how quarkonia propagates through a medium from *first principles*?

Quarkonia

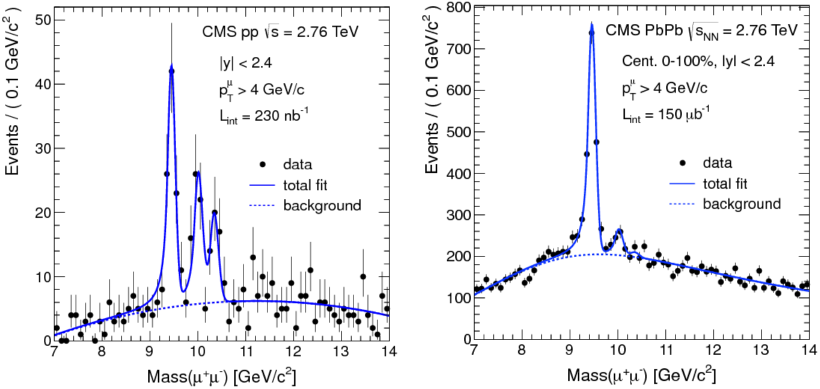
They are **bound states of a heavy quark-antiquark pair ($Q\bar{Q}$) of the same kind** (Olsen et al., 2017) which are stable with respect to strong decay into open charm/bottom (Sarkar et al., 2010).





Observations

Experimental evidence (Chatrchyan et al., 2012) of nuclear effects in the creation and propagation of quarkonia.

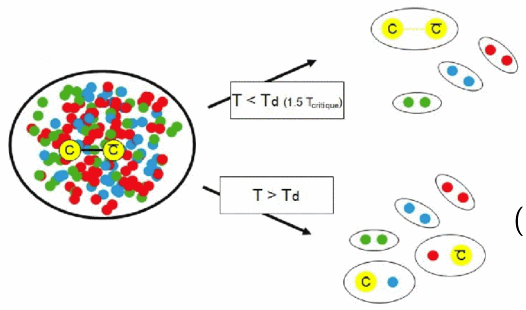


Why quarkonia as a probe

1. Hard scale: quarkonia mass $m_{Q\bar{Q}}, m_Q \gg \Lambda_{QCD}$. *Easy* to be described by EFT.
2. Small radius: harder to dissociate from color screening than light quark matter.

$$\Delta E_{J/\psi} = 2M_D - M_{J/\psi} \approx 0.6 \text{ GeV} \gg \Lambda_{QCD} \approx 0.2 \text{ GeV}. \quad (1)$$

3. Well-known probe. Experimentally, clean signal through dilepton decays.

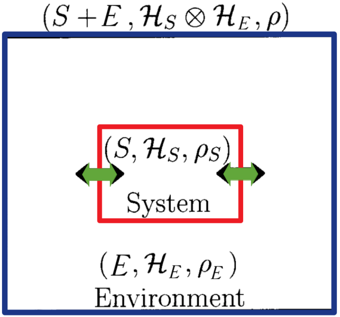


(Porteboeuf, 2011)

Open Quantum Systems 101

We divide the full quantum system (T) into well-differentiated parts: the subsystem (S) and the environment (E) (Breuer and Petruccione, 2002).

The full quantum dynamics of the subsystem is kept whereas the environment is traced out.



Main character (**density matrix**, ρ) and **observables** $\langle\mathcal{O}\rangle$:

$$\rho = p_i \sum_i |\psi_i\rangle\langle\psi_i| \longrightarrow \langle\mathcal{O}\rangle = Tr\{\rho\mathcal{O}\}. \qquad (2)$$

Hamiltonian: $H_T = H_S \otimes \mathbb{I}_E + \mathbb{I}_S \otimes H_E + H_I$, where $H_I = V_S \otimes V_E$.

Evolution: Liouville - von Neumann equation.

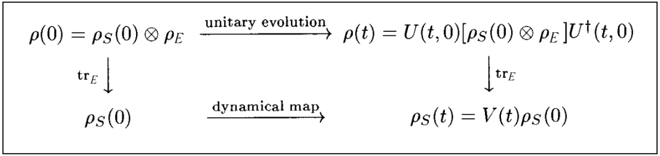
$$\frac{d}{dt}\rho_T(t) = -i[H_T, \rho_T(t)] \Longrightarrow \rho_T(t) = -i\int_0^{\infty} dt'[H_T, \rho_T] \quad (3)$$

1. Go to the interaction picture.
2. Iterate the integral equation into the differential one.

$$\frac{d\rho_{I,T}(t)}{dt} = -\int_0^t dt'[H_I(t), [H_I(t'), \rho_{I,T}(t')]] \quad (4)$$

3. Divide DoF into subsystem + environment.
4. Trace out the environmental DoF $\longrightarrow$ loss of unitarity.

$$Tr_E[\rho_T] = \rho_S \quad (5)$$



Further assumptions

$$\boxed{\frac{d\rho_S(t)}{dt} = -\int_0^t dt' \, Tr_E\left[[H_I(t),[H_I(t'),\rho_{I,T}(t')]]\right]} \tag{6}$$

1. Born approximation $\longrightarrow$ weakly interacting system.

$$\rho_T(t) \approx \rho_S(t) \otimes \rho_E(t) \approx \rho_S(t) \otimes \rho_E(0) \tag{7}$$

2. Markov approximation $\longrightarrow$ no memory in the system.

$$(0,t) \longrightarrow (-\infty,0), \quad \rho_T(t') \approx \rho_T(t) \tag{8}$$

3. Born-Oppenheimer approximation $\longrightarrow$ the light degrees of freedom of the plasma accommodate very fast to changes produced by quarkonia ($\sim$ atomic physics).

Timescales

These approximations also refer to the characteristic timescales τ_i of the different parts of the system, namely:

$$\tau_S = 1/E, \quad \tau_E \sim 1/T, \quad \tau_R \sim M/T^2. \tag{9}$$

Here E is the binding energy of the state, T is the temperature and M the particle mass.

We look for the regime where:

$$\tau_E \ll \tau_R \longrightarrow \text{Born and Markov approximations,} \tag{10}$$

$$\tau_E \ll \tau_S \longrightarrow \text{Born-Oppenheimer approximation.} \tag{11}$$

These considerations will help out with the algebraic manipulations to reach the desired and consistent OQS shape of the equation of evolution.

Open Quantum Systems for Quarkonia

The explicit form of the full hamiltonian (using NRQCD) would be:

$$H_T = \frac{1}{2M}\left(p_Q^2 + p_{\bar{Q}}^2\right) \otimes \mathbb{I}_E + \mathbb{I}_S \otimes H_{q+A} + \int d^3x[\delta(\mathbf{x}-\mathbf{x}_Q)t_Q^a - \delta(\mathbf{x}-\mathbf{x}_{\bar{Q}})t_{\bar{Q}}^{a*}] \otimes gA_o^a(\mathbf{x}) \quad (12)$$

We know that:

$$Tr_E\left[T[A_0^a(t_1,\mathbf{x}_1)A_0^b(t_2,\mathbf{x}_2)]\rho_E\right] = -i\delta^{ab}\Delta(t_1-t_2,\mathbf{x}_1-\mathbf{x}_2) \quad (13)$$

As we will be using pNRQCD, we can profit from the fact that propagators of the chromoelectric field can be linked with a real and a imaginary potentials like (Blaizot and Escobedo, 2017):

$$V(\mathbf{r}) = -\Delta^R(\omega=0,\mathbf{r}), \qquad W(\mathbf{r}) = -\Delta^<(\omega=0,\mathbf{r}) \quad (14)$$

Lindblad form

As a result, after some rearranging, we get the Lindblad equation:

$$\frac{d\rho_S(t)}{dt} = -i[H_S(t), \rho_S(t)] + \sum_k \left(L_k \rho_S L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_S(t)\} \right), \quad (15)$$

$L_k \sim D_{env.}(t, \mathbf{x}) \cdot (V_S^k(t) + \frac{i}{4T}\frac{dV_S^k(t)}{dt})$ is called the Lindblad operator (Akamatsu, 2022).

1. $k > 1$, if more than one kind of operator (decay channel).
2. $D_{env.}(t, \mathbf{x}) \sim \Delta(t, \mathbf{x})$, from tracing out the environmental DoF.

Conceptually, Lindblad operators are going to produce **jumps** between states (modifying the internal quantum numbers of the system). Thus, they are also called **jump operators**.

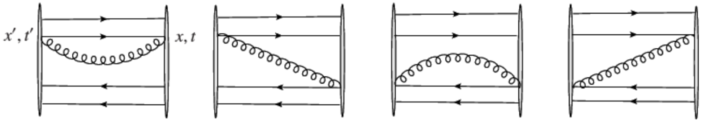
Quantum trajectories: an algorithm to solve Lindblad's.

We redefine the subsystem hamiltonian by adding the 1-loop contributions, H_{1-loop} (Akamatsu, 2022; Blaizot and Escobedo, 2018; Yao and Mehen, 2019).
It becomes a **non-hermitian hamiltonian.**

$$H_{eff} = H_S + H_{1-loop} = H_S - \frac{i}{2}\sum_k \gamma_k L_k^\dagger L_k \tag{16}$$

$$\boxed{\frac{d\rho_S(t)}{dt} = -i[H_{eff}(t), \rho_S(t)]} + \sum_k L_k \rho_S L_k^\dagger, \tag{17}$$

The state is evolved in Schrödinger-like way (norm decreases).
When the norm goes below a certain value, a projection (jump) is performed according to certain selection rules.



Description of the algorithm

1. Non-hermitian hamiltonian evolution step is performed. Its non-unitarity makes the norm of the state decrease.

$$\langle \psi(t_1) | \psi(t_1) \rangle > \langle \psi(t_2) | \psi(t_2) \rangle \, , \text{ where } t_1 < t_2 \tag{18}$$

2. A random number decides if the jump is performed. The state will normally evolve until the norm goes below this value.

$$\text{When} \quad \langle \psi(t) | \psi(t) \rangle < \text{Random Number} \longrightarrow \text{jump.} \tag{19}$$

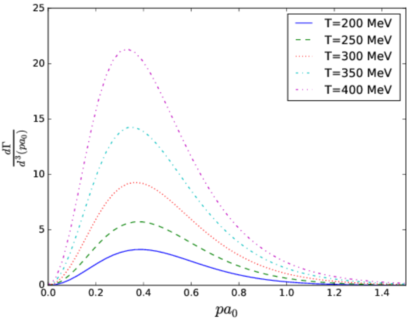
3. Randomly select the jump according to certain selection rules. We project using the corresponding jump operator:

$$|\psi_{new}\rangle = \hat{L}^x(\vec{q}) \, |\psi_{old}\rangle \tag{20}$$

4. Renormalize and back again.

Jumps

Selection rules are enacted via the partial decay rates $\Gamma(p)$ (Blaizot and Escobedo, 2018). These explicitly depend on the shape of the Lindblad operators.



Decay rates are defined as:

$$\Gamma_k(p) = L_k(p)L_k^\dagger(p). \quad (21)$$

QTRAJ $(1.0 + \epsilon)$

QTRAJ 1.0 (Ba Omar et al., 2022): C-based code which simulates through the quantum trajectories algorithm and shows the relative population of colour and wave states for quarkonia.

The current potential available compatible with the Lindblad formalism is the Munich potential. This approach is adequate for a regime where $rT \ll 1$ and is performed with a finite number of Lindblad operators.

Goal of $+\epsilon$ **:** New potentials $\longrightarrow$ Infinite number of Lindblad operators $\longrightarrow$ reach regime where $rT \approx 1$.

How?:

1. Adding definitions of new potentials to QTRAJ.
2. Modifying the selection rules $\longleftrightarrow$ Defining new Lindblad operators.

Current efforts

New potential, less restrictive, to try to perform up to $rT \approx 1$. We will use the general expression:

$$\Delta(\omega = 0, \mathbf{r}) = -\Delta^R(\omega = 0, \mathbf{r}) + i\Delta^<(\omega = 0, \mathbf{r}), \tag{22}$$

which was found when tracing out the environment to get the Lindblad operators. These correspond to real and imaginary potentials, in our case: .

$$Re\{H_I(r)\} = -C_F\alpha_s(1/a_0)\frac{e^{-m_D r}}{r}, \tag{23}$$

$$Im\{H_I(r)\} = \frac{g^2 T}{2\pi}\int_0^\infty dx \frac{x}{(x^2+1)^2}\left[1 - \frac{sin(xrm_D)}{xrm_D}\right] \tag{24}$$

where m_D is the Debye mass:

$$m_D = \sqrt{\frac{2N_c + N_f}{6}} gT \tag{25}$$

New Lindblad operators:

Lindblad operators are in this framework:

$$\hat{L}^x(\vec{q}) = K_x\sqrt{\Delta(\vec{q})}cs(\frac{\vec{q}\cdot\hat{\vec{r}}}{2}), \tag{26}$$

where cs stands for $\sin\left(\vec{q}\cdot\hat{\vec{r}}/2\right)$ if $x \in \{s \to o, o \to s, o \to o\ (1)\}$ and $\cos\left(\frac{\vec{q}\cdot\hat{\vec{r}}}{2}\right)$ is $x \in \{o \to o\ (2)\}$. Using:

$$e^{-i\vec{k}\vec{r}} = \sum_{\ell=0}^{\infty}(-i)^{\ell} j_{\ell}(kr) Y_{\ell m}(\vec{k}_u) Y^{*}_{\ell,m}(\vec{r}_u), \tag{27}$$

we get:

$$\hat{L}^x(\vec{q}) = K_x\sqrt{\Delta(\vec{q})}\sum_{t}^{\infty}\sum_{m=-\ell}^{\ell} j_{\ell}(qr) Y_{\ell}^{m}(\Omega_r) = \sum_{t}^{\infty}\hat{L}^x_{\alpha}(\vec{q}), \tag{28}$$

where for the case of the cosine $\alpha = 2t$ and for the sine $\alpha = 2t+1$.

New rules

The change affects how selection rules are implemented:

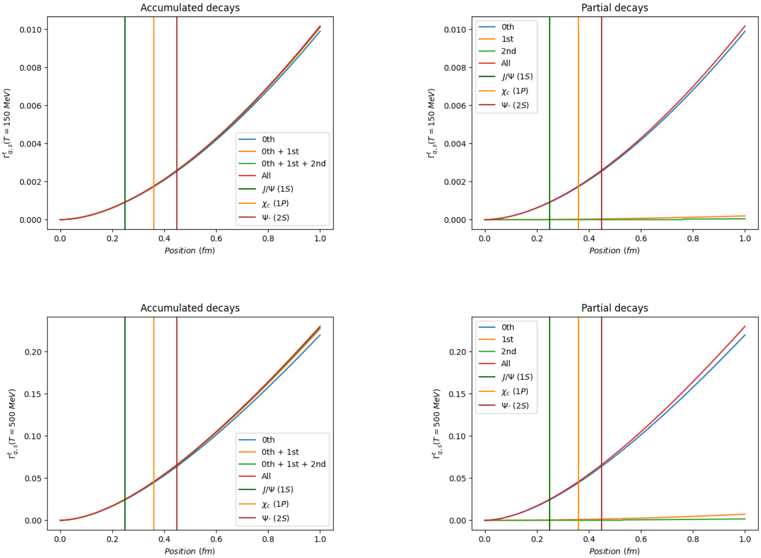
1. We choose, according to the current state, the kind of transition that quarkonia will undergo to apply its proper Lindblad operator:

$$\hat{L}^{s\longrightarrow o}(\vec{q}), \quad \hat{L}^{o\longrightarrow s}(\vec{q}), \quad ,\hat{L}^{o\longrightarrow o(1)}(\vec{q}), \quad \hat{L}^{o\longrightarrow o(2)}(\vec{q}). \quad (29)$$

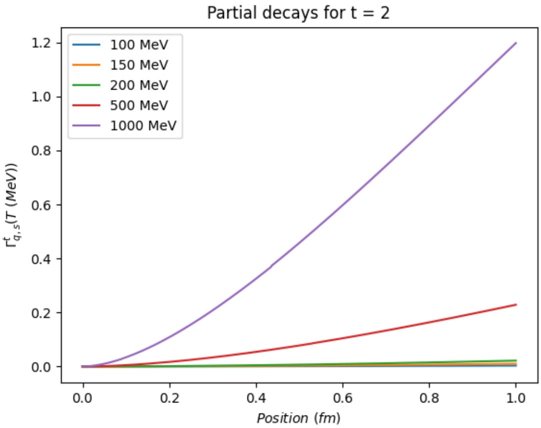
2. We choose, once the jump channel is known, the value of t of $\hat{L}_t^x(\vec{q})$, which we understand it as the virtual angular momentum of the one gluon exchange.
3. We choose q from its momentum distribution.
4. We apply the Lindblad operator so:

$$\hat{L}_t^x(\vec{q}) \left|\psi_{old}\right\rangle = \left|\psi_{new}\right\rangle . \quad (30)$$

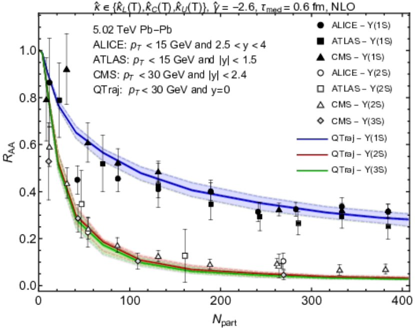
Behaviour of the jump operators

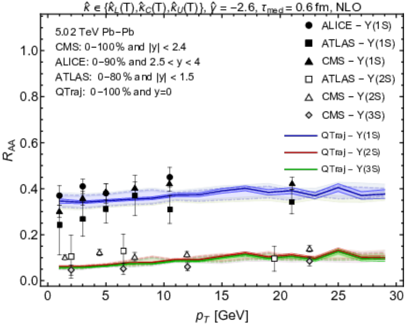


Tendency of the jumps



Plots that can be retrieved.





These results are from Strickland's original code (Brambilla et al., 2022).

Conclusions

1. The inclusion of less restrictive potentials allows the expansion the regime of validity of the simulations.
2. This means two things: either temperature does not have to be as high as before for applying this formalism or the small dipole approximation implicit in the Boltzmann equation is no longer applied. The latter case is of our greater interest.
3. The new shape of the Lindblad operators depend on the momentum exchanged with the medium particles. In the region of interest, $\Delta J = 1$ dominates.

Thank you!

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Approximations: Born approximation

It is a weak coupling between the subsystem and the environment, $H_I \ll 1$.

$$\rho_T(t) = \rho_S(t) \otimes \rho_E(t) + \rho_{corr}(t) \approx \rho_S(t) \otimes \rho_E(t), \tag{31}$$

where ρ_{corr} is the correlation component between the environment and the subsystem.

$$\frac{d\rho_{T,I}(t)}{dt} \approx -\int_0^t d\tau [H_I(t), [H_I(\tau), \rho_{S,I}(\tau) \otimes \rho_{E,I}(0)]] \tag{32}$$

Approximations: Markov approximation

Taking into account only the current step in order to obtain the next one $\rho_{S,I}(\tau) \longrightarrow \rho_{S,I}(t)$. We will perform the change of variable $\tau \longrightarrow \tau' = t - \tau$ so:

- $\tau = 0 \longrightarrow \tau' = t - \tau = t$
- $\tau = t \longrightarrow \tau' = t - \tau = 0$
- Since the correlation time of the environment is much less than the average relaxation time of the system we can take $t \longrightarrow \infty$.

If we also trace over the environment, we get:

$$\frac{d\rho_{S,I}(t)}{dt} \approx -\int_0^\infty d\tau \; tr_E\{[H_I(t), [H_I(t-\tau), \rho_{S,I}(t) \otimes \rho_{E,I}(0)]]\}. \tag{33}$$

Redfield equation.

Approximations: Born-Oppenheimer approximation

The environmental degrees of freedom move much faster than the quarkonium so effectively they instantly change to any changes that the quarkonium may induce.

$$V_S(t-s) \approx V_S(t) - s\frac{dV_S(t)}{dt} + \cdots = V_S(t) - is[H_S, V_S(t)] + \ldots \tag{34}$$

Gradient expansion for Brownian motion.

1. Projecting $\rho_S(t)$ into spherical harmonics.
2. Also, split into the singlet-octet colour basis.

$$\rho_S(t) = diag(\rho_S^{sing,s}, \rho_S^{oct,s}, \rho_S^{sing,p}, \rho_S^{oct,p}) \tag{35}$$

Great computational advantage: 3D $\longrightarrow$ 1D $\cdot Y_m^\ell(\theta, \phi)$.

Quark-gluon plasma

It is a deconfined phase on the QCD phase diagram [12].