



STRONG-NA7 Workshop & HFHF Theory Retreat  
28. Sep. – 4. Oct. 2023, Taormina, Italy

# UNDERSTANDING HARMONIC FLOW OF PROTONS, LIGHT- & HYPERNUCLEI

---

Tom Reichert

Institut für Theoretische Physik, Goethe-Universität Frankfurt

In collaboration with: Oleh Savchuk, Apiwit Kittiratpattana, Nihal Buyukcizmeci, Alexander Botvina, Jan Steinheimer, Marcus Bleicher, et al.



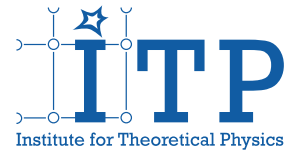
FIAS Frankfurt Institute  
for Advanced Studies



Stiftung  
Polytechnische  
Gesellschaft  
Frankfurt am Main

HFHF

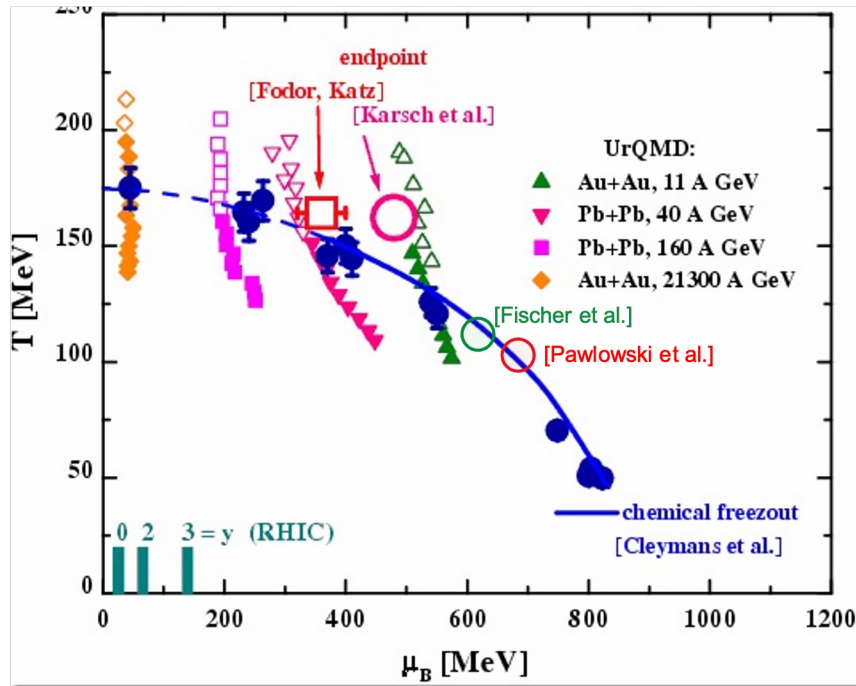
Helmholtz Forschungsakademie Hessen für FAIR



Institute for Theoretical Physics

# Overarching goal and outline

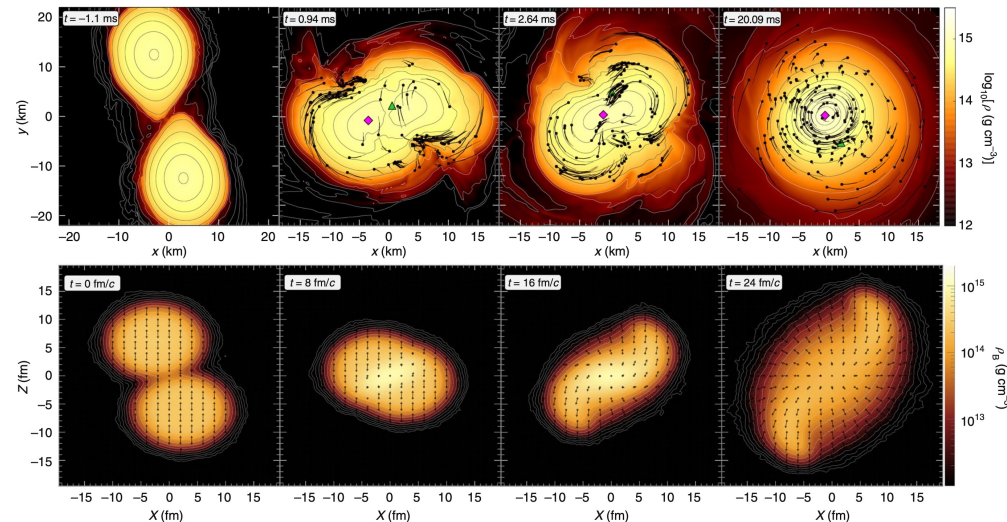
- Understanding the nature of the QCD phase diagram
- Bridge the gap from heavy-ion collisions to neutron stars



L. Bravina et al. JPG 1999  
I. Arsene et al. PRC 2007

→ Talk: Christian Fischer

## Neutron star simulation



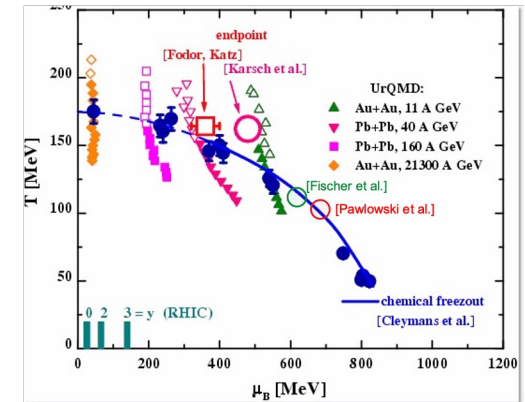
## Heavy-ion simulation (UrQMD)

HADES. Nature Phys. 15 (2019) 10, 1040-1045

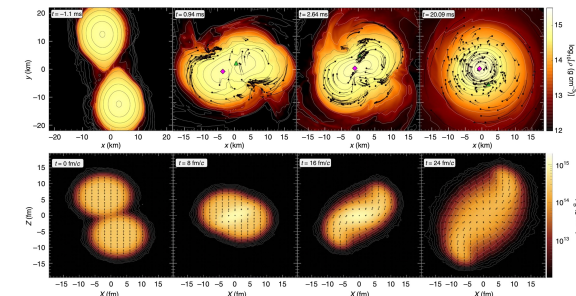
# Overarching goal and outline

- Understanding the nature of the QCD phase diagram
- Bridge the gap from heavy-ion collisions to neutron stars
- Heavy-ions are collided at GSI, RHIC, LHC
- Compare to numerical simulations solving e.g. Boltzmann-Equation

$$\begin{aligned} & \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f \\ &= - \int \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 p'_1}{(2\pi)^3} \frac{d^3 p'_2}{(2\pi)^3} \sigma v_{12} [f f_2 (1 - f_{1'}) (1 - f_{2'}) \\ & \quad - f_{1'} f_{2'} (1 - f) (1 - f_2)] (2\pi)^3 \delta^3(p + p_2 - p'_1 - p'_2) \end{aligned}$$

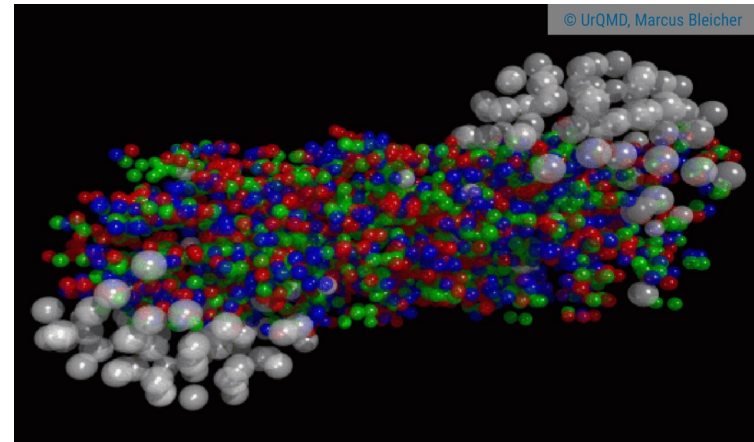


L. Bravina et al. JPG 1999  
I. Arsene et al. PRC 2007



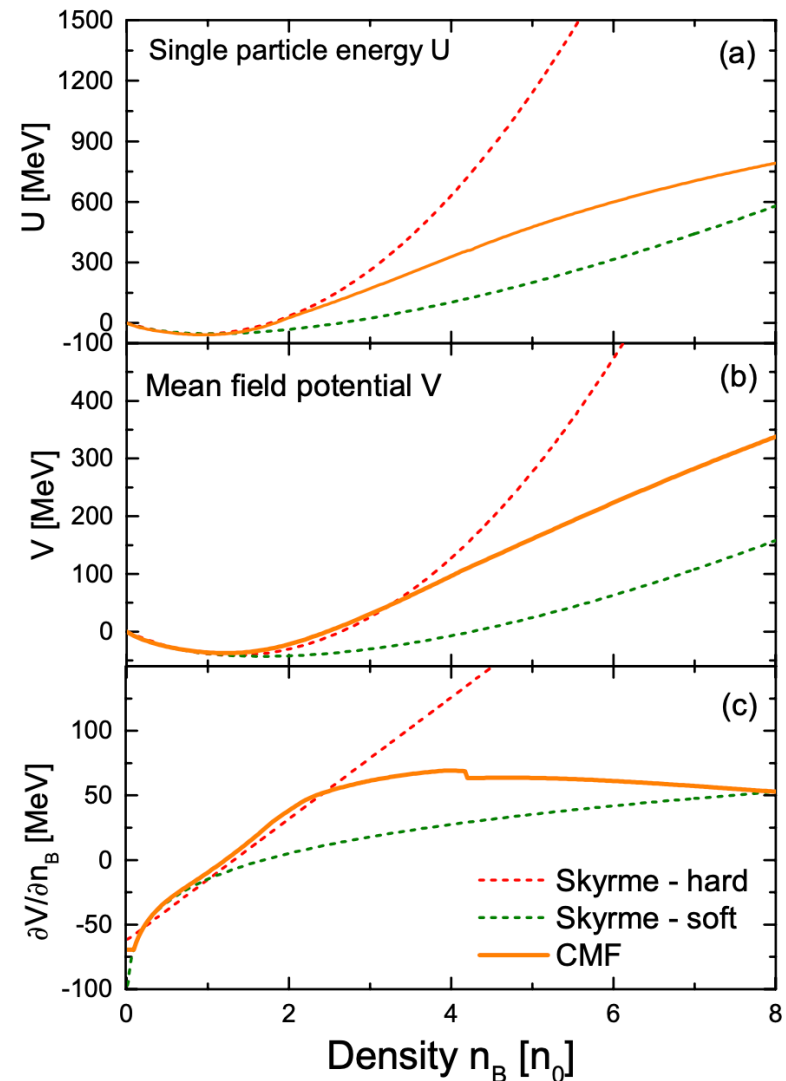
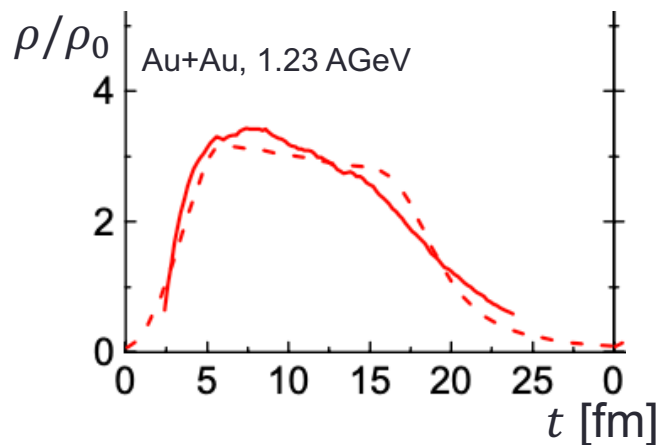
# Ultra-relativistic Quantum Molecular Dynamics

- Hadron/String transport approach
- Based on propagation of hadrons
- Rescattering among hadrons fully included
- String excitation and decay (LUND model, PYTHIA)
- Solution for the time dependent n-body distribution of hadrons
- Collision term includes more than 100 hadrons up to 4 GeV in mass
- Soft/Hard or CMF EoS can be switched on



# UrQMD with Chiral Mean Field EoS

- $m_{b\pm}^* = \sqrt{[(g_{\sigma b}^{(1)}\sigma + g_{\zeta b}^{(1)}\zeta)^2 + (m_0 + n_s m_s)^2]} \pm g_{\sigma b}^{(2)}\sigma$ ,
- $V_{CMF} = E_{\text{field}}/A = E_{CMF}/A - E_{FFG}/A$ ,
- CMF EoS hard up to  $3\rho_0$  then soft
- Easy implementation of phase transition

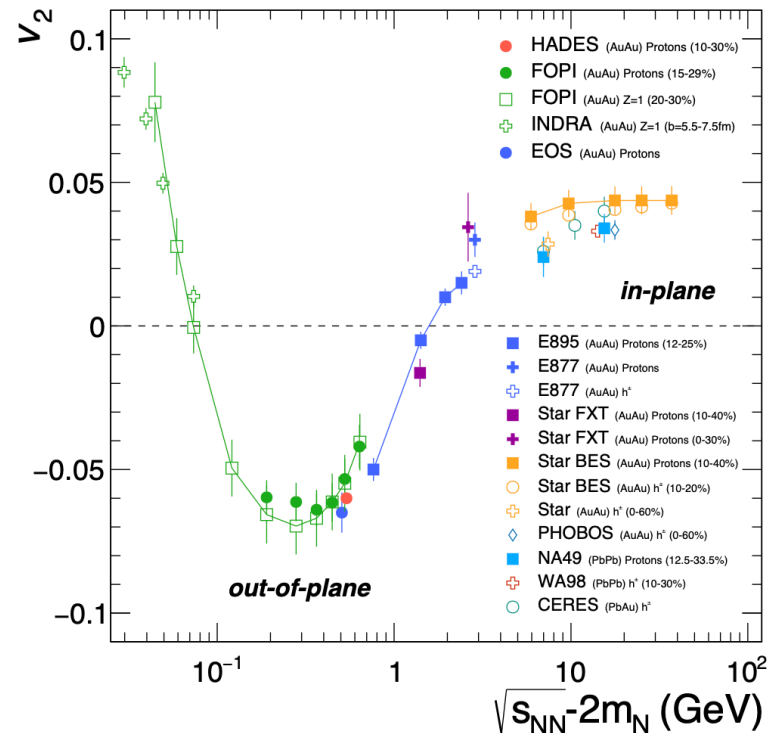
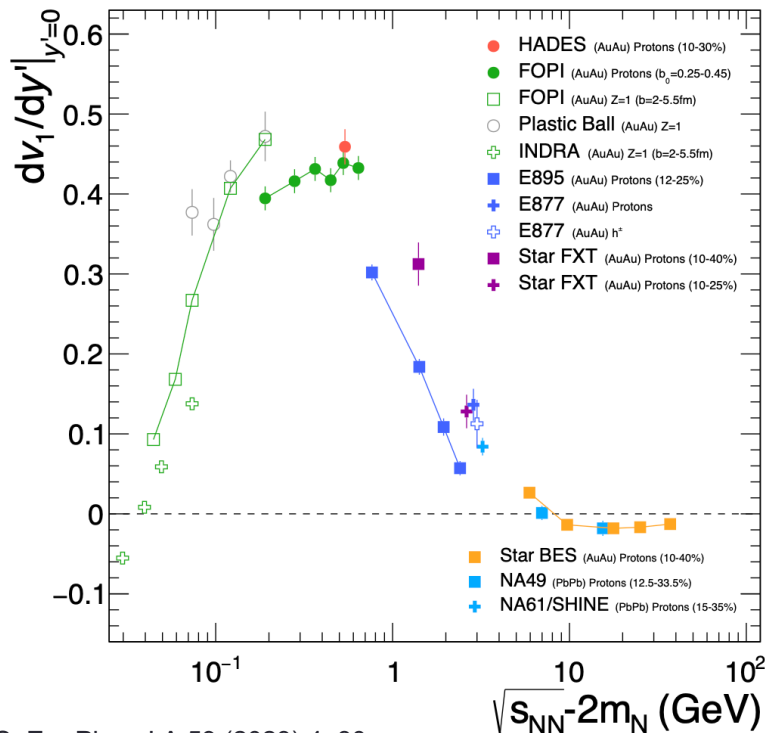
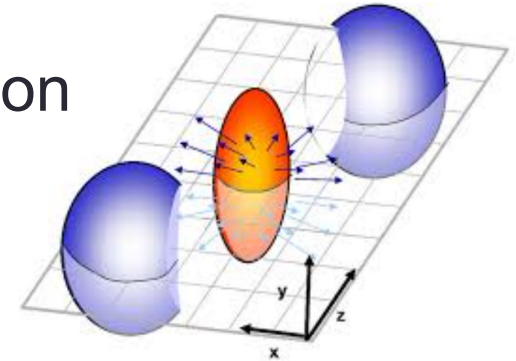


# Harmonic flow

- Fourier series of azimuthal angle distribution

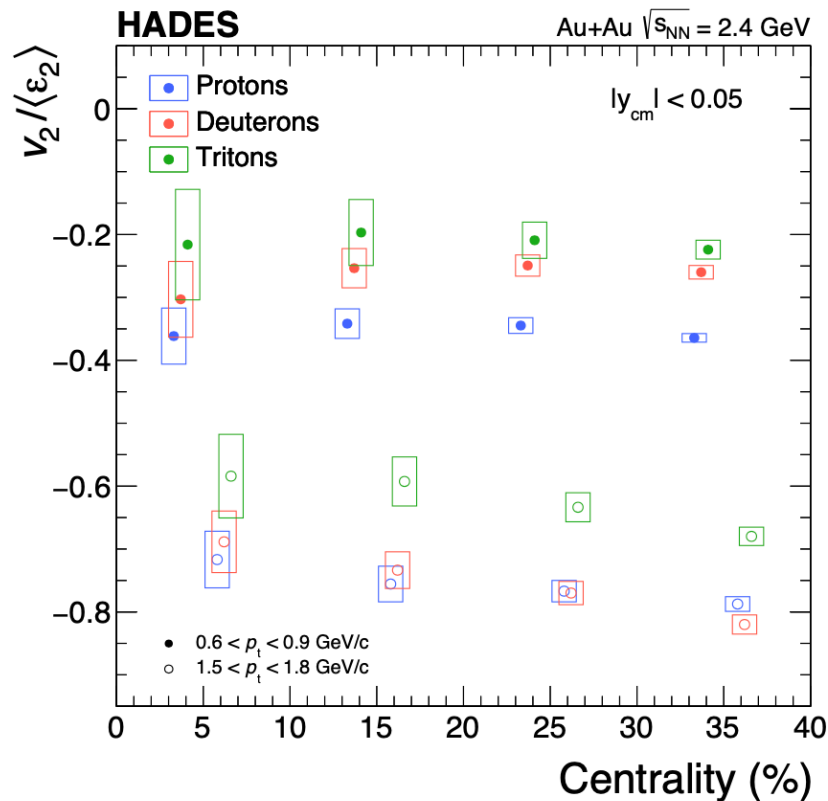
$$\frac{dN}{d\varphi} = 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\varphi - \Psi_{RP}))$$

- $v_1$ : Directed flow,  $v_2$ : Elliptic flow

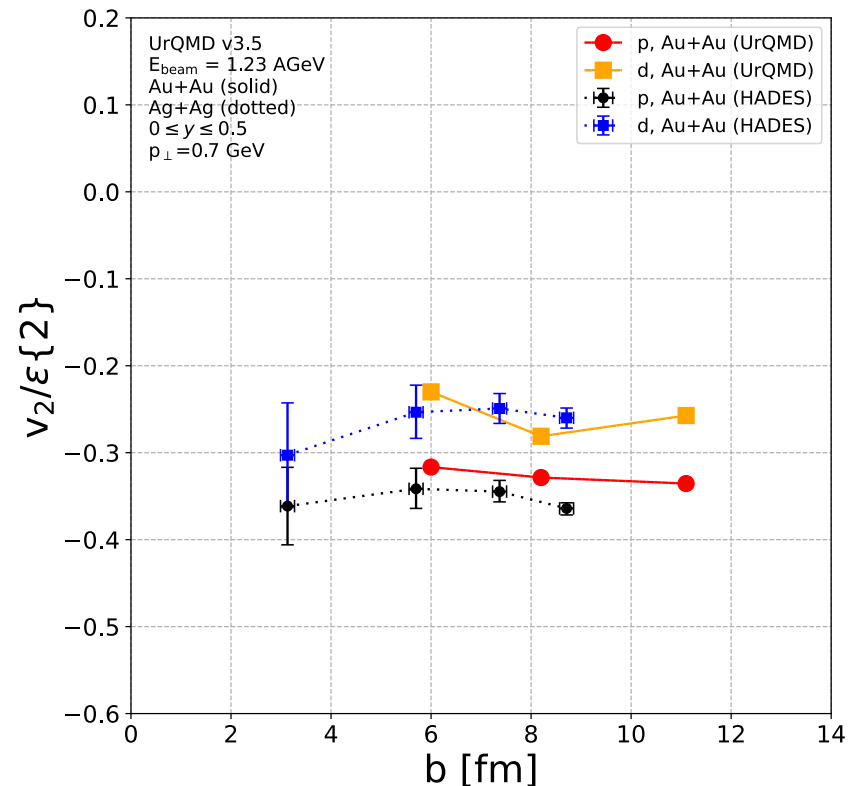


# Elliptic flow scaling with eccentricity

- LHC & RHIC: initial  $\varepsilon_2 \rightarrow -\nabla P \rightarrow$  final  $v_2$
- GSI: Negative scaling observed by HADES



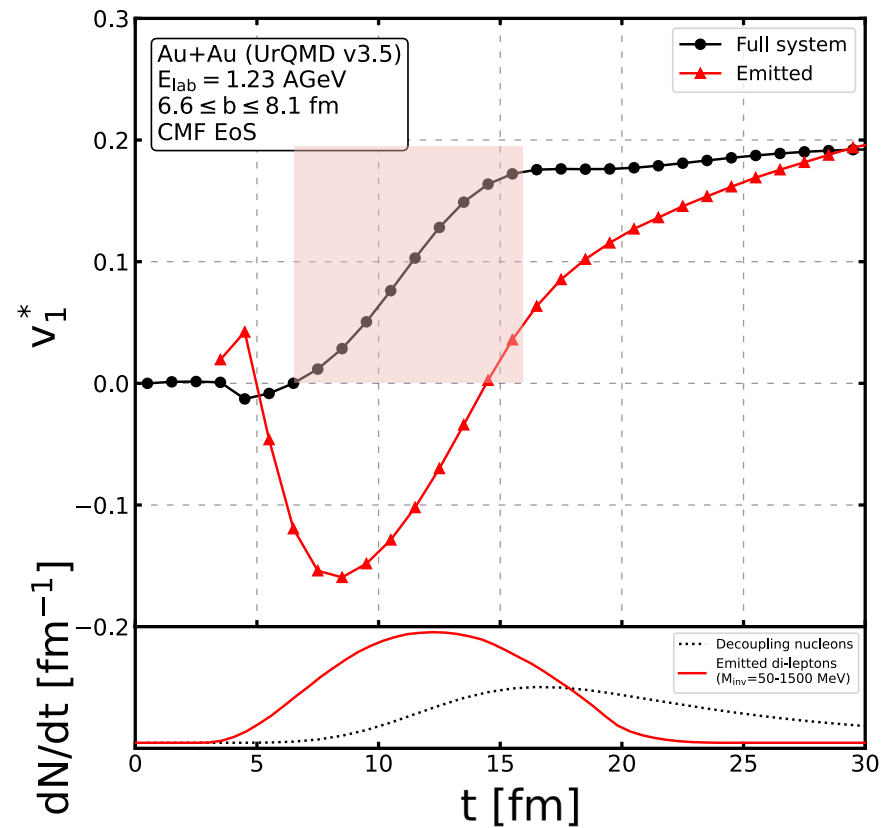
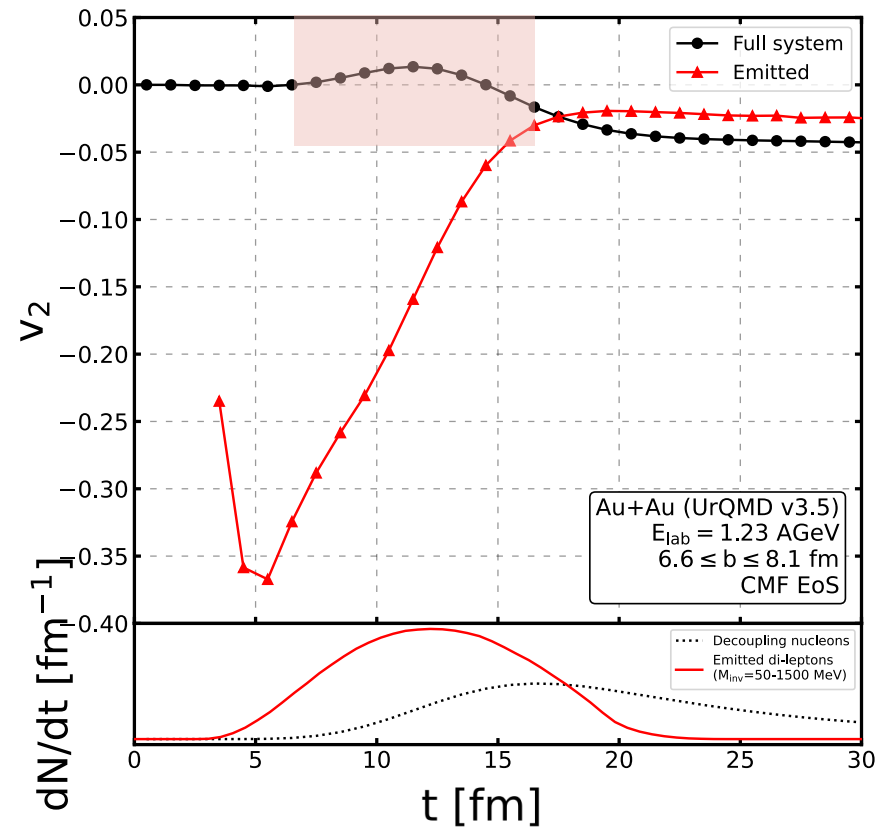
HADES. Eur.Phys.J.A 59 (2023) 4, 80



T. Reichert et al. J.Phys.G 49 (2022) 5, 055108



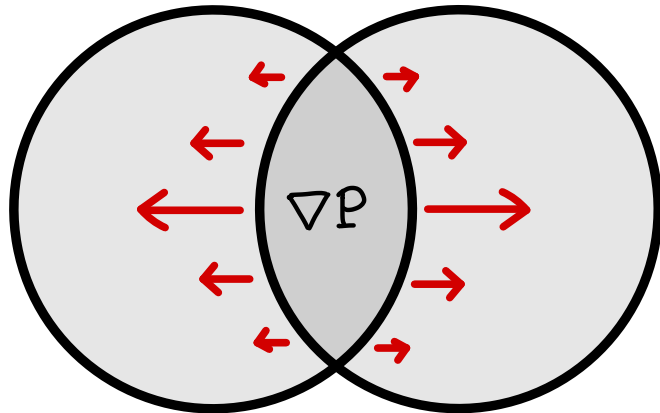
# Time development of the flow



- Flow is directly sensitive to the EoS
- Tight connection between  $v_1$  and  $v_2$
- Is the positive  $v_2$  measurable?



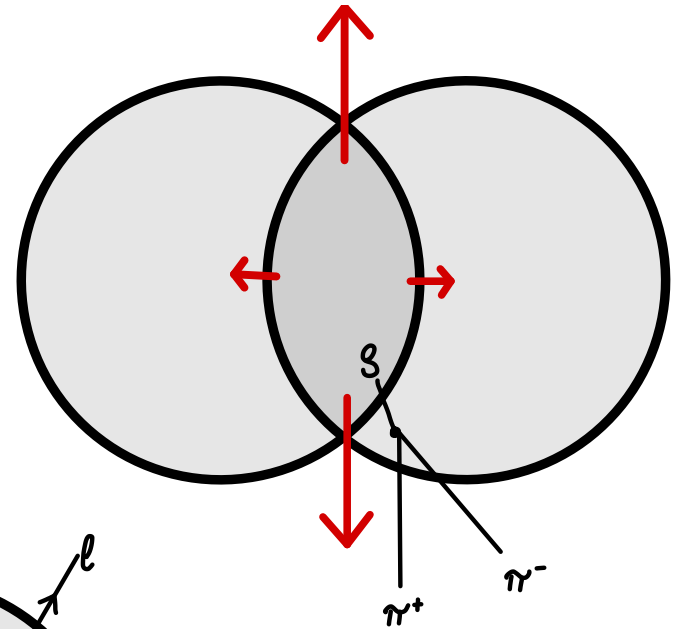
# Time evolution



Bulk dynamics

$t = 7-15$  fm

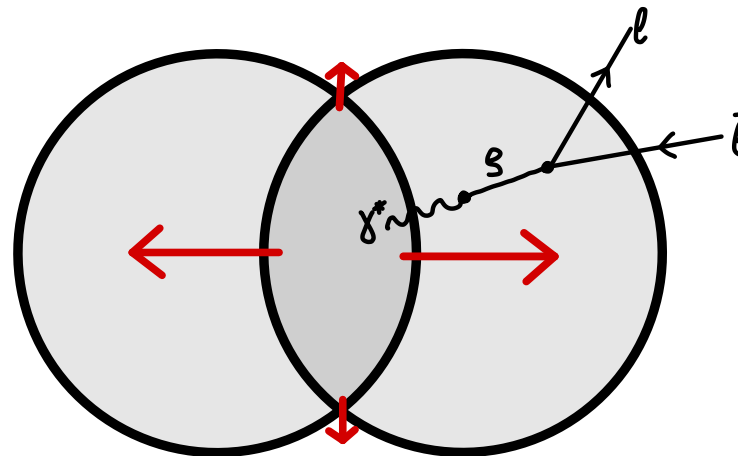
Pos.  $v_2$  creates pos.  $v_1$



Hadron decoupling

$t = 20-30$  fm

Shadowing  $\rightarrow$  neg.  $v_2$



Dilepton emission

$t = 12$  fm

Observation of in-plane expansion

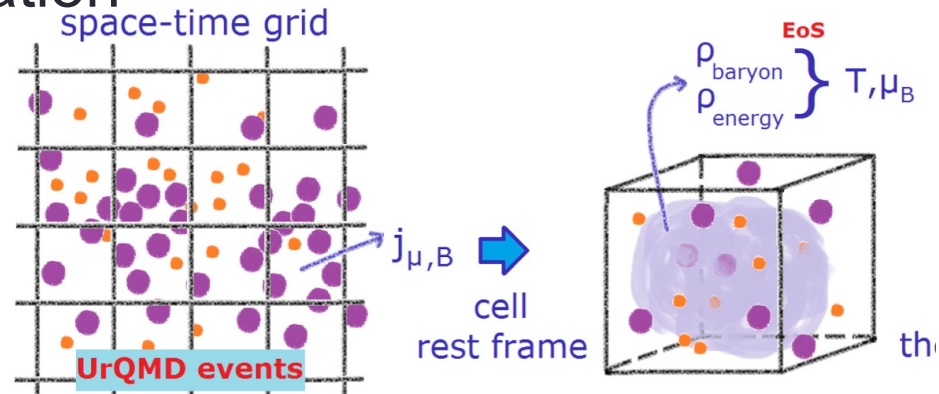
# Dileptons

$$\bullet \frac{dN_{\ell^+\ell^-}}{d^4x d^4q} = -\frac{\alpha^2}{3\pi^3} \frac{q^2 + 2m_\ell^2}{(k^2)^2} \sqrt{1 - \frac{4m_\ell^2}{k^2}} \eta_{\mu\nu} \text{Im} \Pi_{\text{ret}}^{\mu\nu}(M, \vec{q}) n_B(u \cdot q)$$

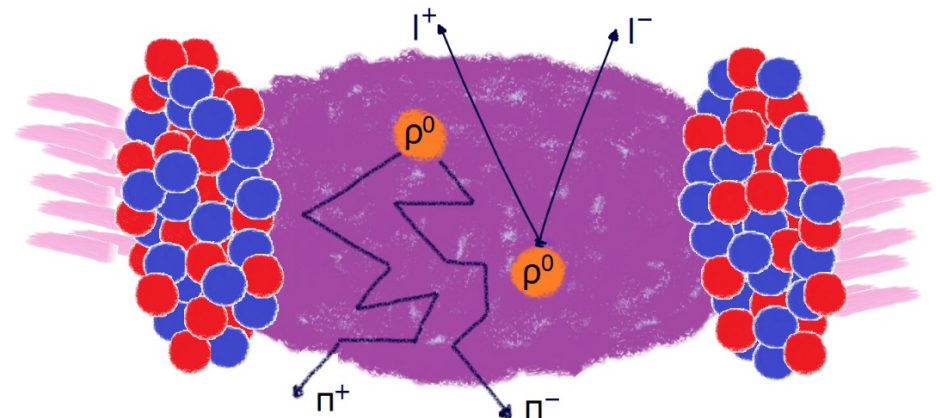
C. Gale et al. Nucl. Phys. B357 (1991) 65

- **Spectral** and **thermal** information

- UrQMD + coarse-graining
- Evaluate  $\langle T^{\mu\nu} \rangle$  and  $\langle j_B^\mu \rangle$  in each cell and obtain  $T, \mu_B$
- Calculate dileptons using Rapp spectral functions
- Shining method (collisional broadening included)

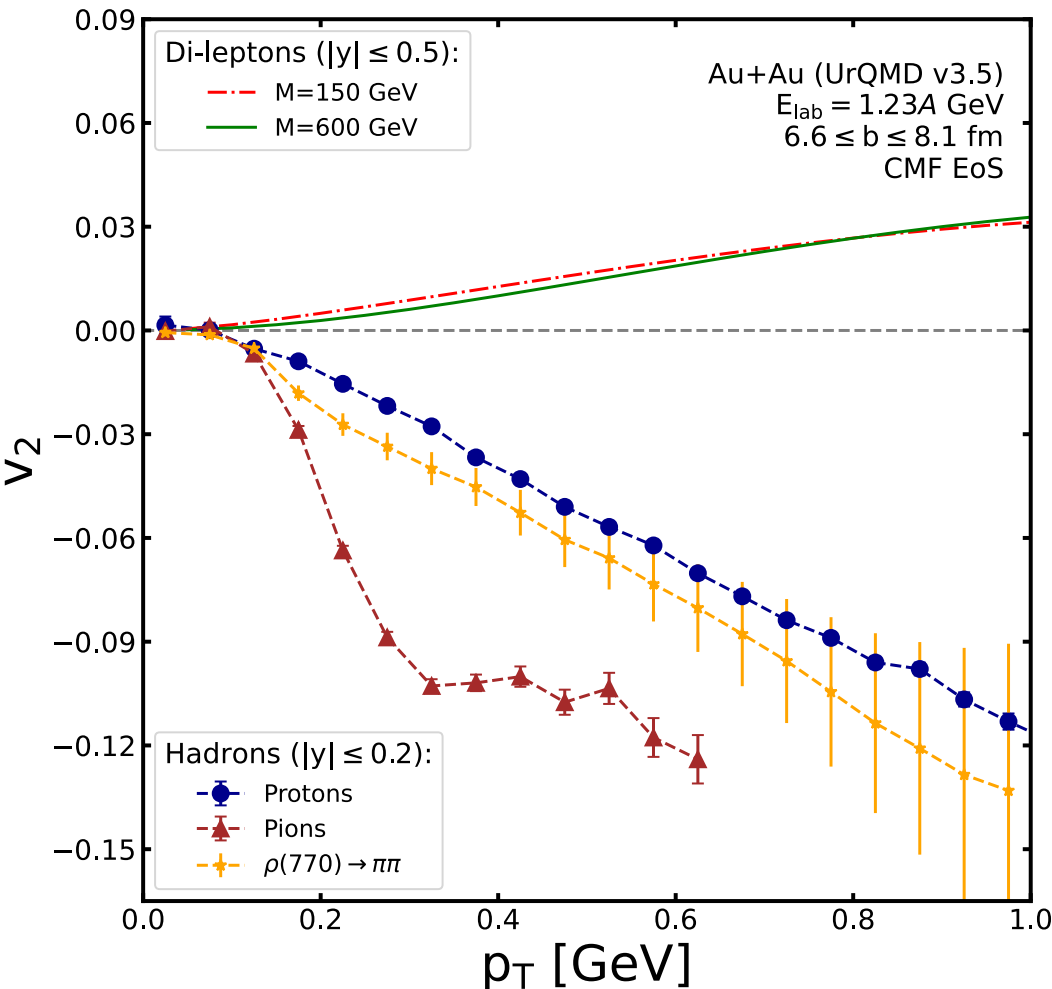


S. Endres et al. Phys.Rev.C 91 (2015) 5, 054911



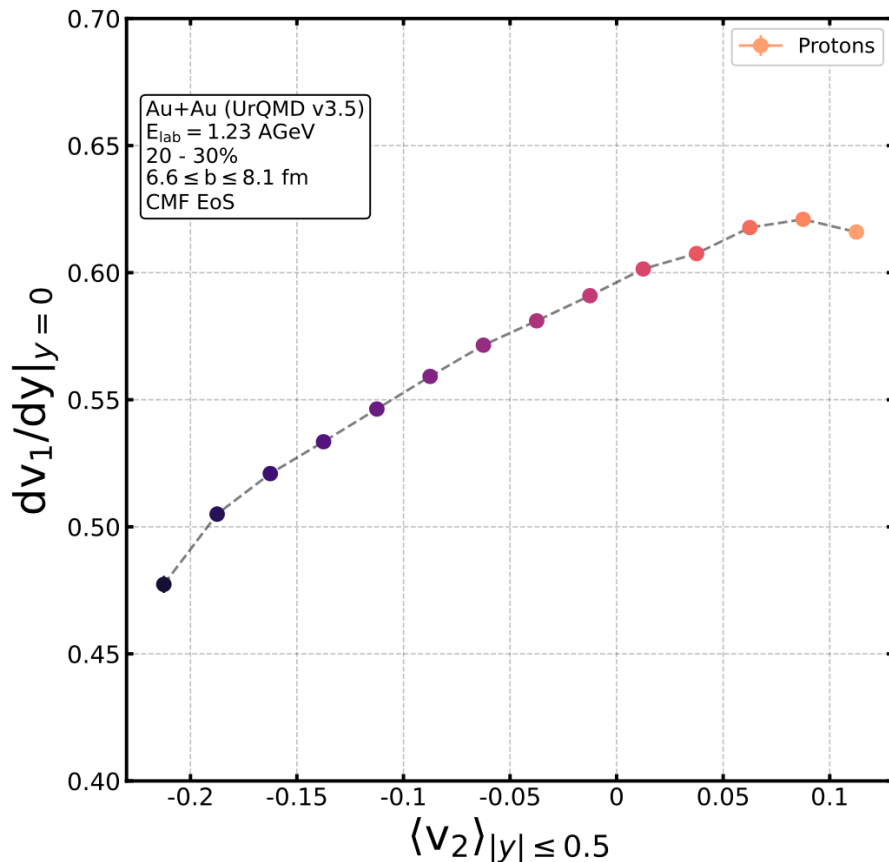
- Talk: Hendrik van Hees
- Talk: Renan Hirayama

# Measuring the initial flow and early EoS with Di-Leptons



- Hadrons show negative  $v_2$
- Simulation in line with HADES data
- Dileptons have positive  $v_2$
- Dileptons show hydro-mass scaling
- Direct measurement of EoS
- Direct correlation of  $v_2$  and  $v_1$

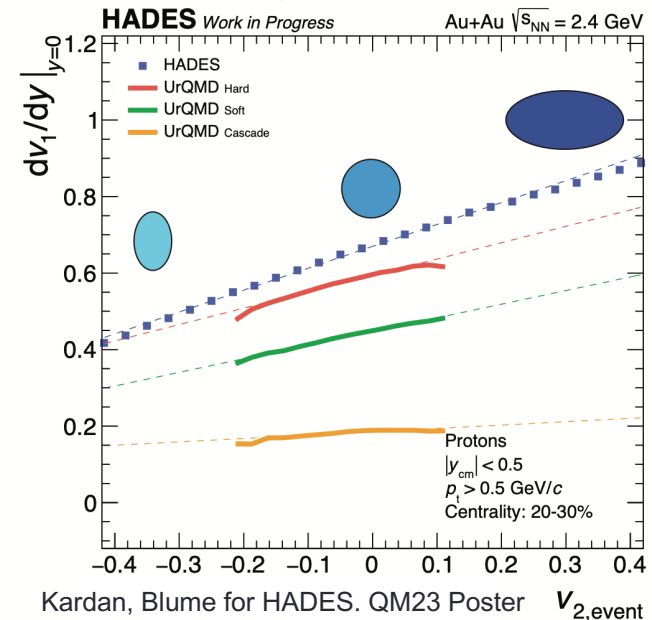
# Measuring the initial flow and early EoS with Correlations



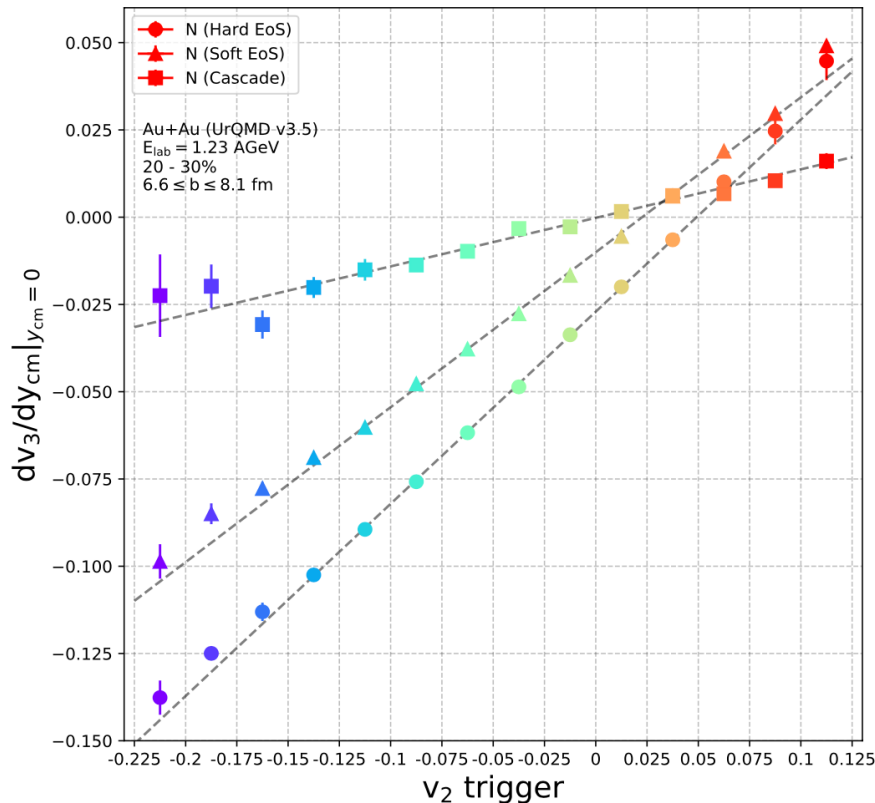
Reichert et al. Phys.Lett.B 841 (2023) 137947

- HADES (prel!) data prefers hard EoS

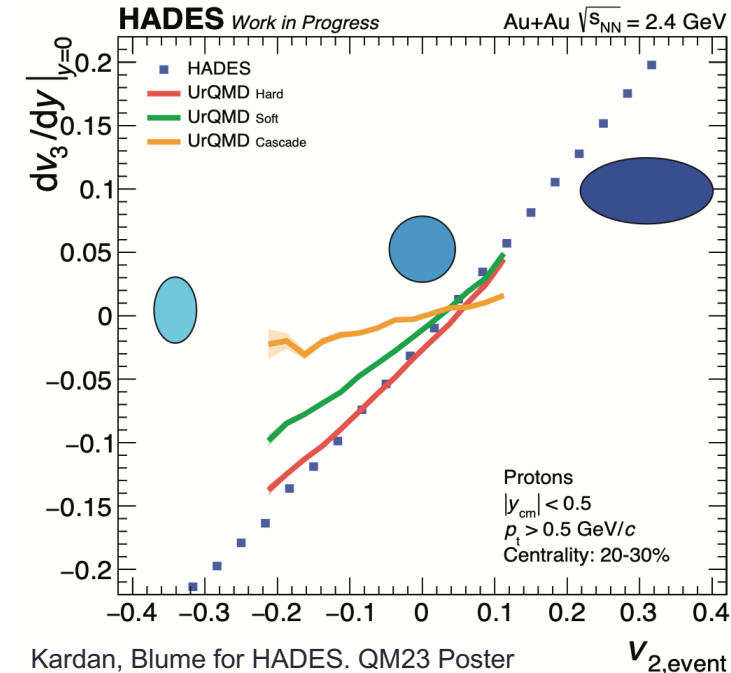
- Select events based on integrated final  $v_2$
- Measure  $dv_1/dy$  as function of  $v_2$  trigger
- Strong correlation observed
- Explained by pressure gradient and shadowing



# Measuring the initial flow and early EoS with Correlations



- Select events based on integrated final  $v_2$
- Measure  $dv_3/dy$  as function of  $v_2$  trigger
- Strong sensitivity to stiffness of EoS



Kardan, Blume for HADES. QM23 Poster

 $v_{2,\text{event}}$ 

Reichert et al. Phys.Lett.B 841 (2023) 137947

- HADES (prel!) data prefers hard EoS

# Light cluster and hypernuclei

- Light clusters

→ Deuteron (pn), Triton (pnn),  $^3\text{He}$  (ppn),  $^4\text{He}$  (ppnn)

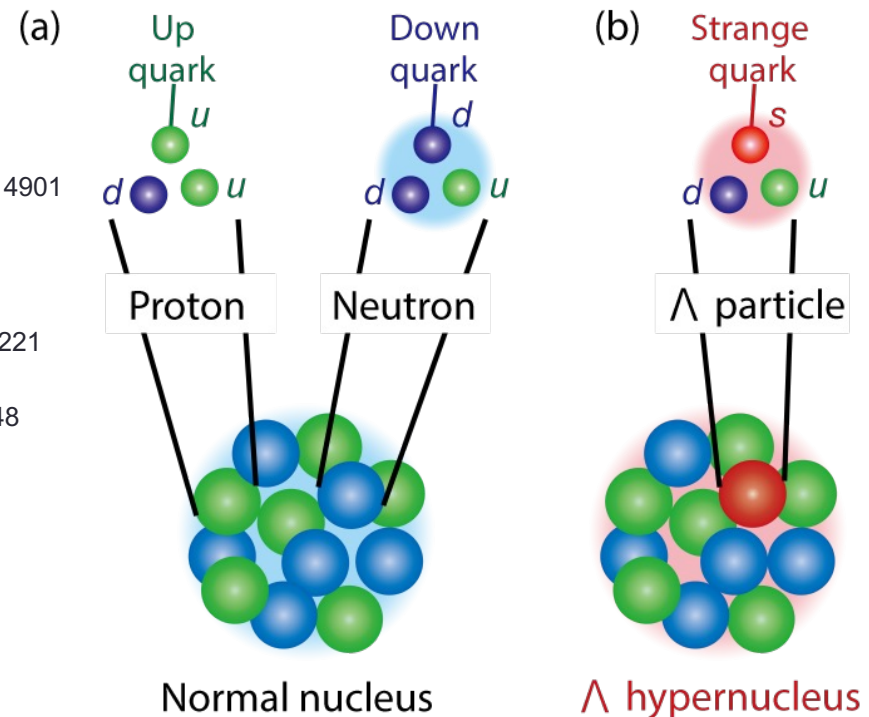
- Hypernuclei

→ Hypertriton (pn $\Lambda$ ),  $^4_{\Lambda}\text{H}$  (pnn $\Lambda$ )

- Production?

- **Coalescence** S. Sombun et al. Phys.Rev.C 99 (2019) 1, 014901
- **Potential** J. Aichelin, et al. Phys.Rev.C 101 (2020) 4, 044905
- **Fragmentation** Bondorf et al. Phys.Rept. 257 (1995) 133-221
- **Thermal** P. Braun-Munzinger, et al. Phys.Lett.B 344 (1995) 43-48
- **Wigner** Mattiello et al. Phys.Rev.C 55 (1997) 1443-1454
- **Kinetic** Oliinychenko et al. Phys.Rev.C 99 (2019) 4, 044907

- Talk: Apiwit Kittiratpattana
- Talk: Gabriele Coci
- Talk: Tim Neidig



# Coalescence

- Clusters are weakly bound compared to momentum transfer (temperature)
- Clusters are formed after kinetic freeze-out
- Coalescence: Cluster is formed if correct constituents occupy certain phase space volume

$$\frac{dN}{d^3k} = g \int dp_1^3 dp_2^3 dx_1^3 dx_2^3 f_A(p_1, x_1) f_B(p_2, x_2) \rho_{AB}(\Delta x, \Delta p) \delta(k - (p_1 + p_2))$$

- Need realistic phase space distribution functions of nucleons  
→ Use microscopic transport model keeping all n-body correlations



# Box-Coalescence

1. Boost into local rest frame of each possible nucleon+nucleon pair with the correct isospin combination at kinetic freeze-out. If relative distance  $\Delta x < \Delta x_{max}$  and relative momentum  $\Delta p < \Delta p_{max}$  the two-nucleon system is marked a deuteron candidate.
2. Boost into local rest frame of deuteron+nucleon and check again if  $\Delta x < \Delta x_{max}$  and  $\Delta p < \Delta p_{max}$ . A triton or  ${}^3\text{He}$  is then formed with a probability of 1/12 at the position  $r_{NNN} = (r_1 + r_2 + r_3)/3$  and with momentum  $p_{NNN} = p_1 + p_2 + p_3$

# Statistical Multifragmentation

- Break up of thermal nuclear system
- Microcanonical ensembles
- Break up is modeled according to statistical weight of entropy of decay channel

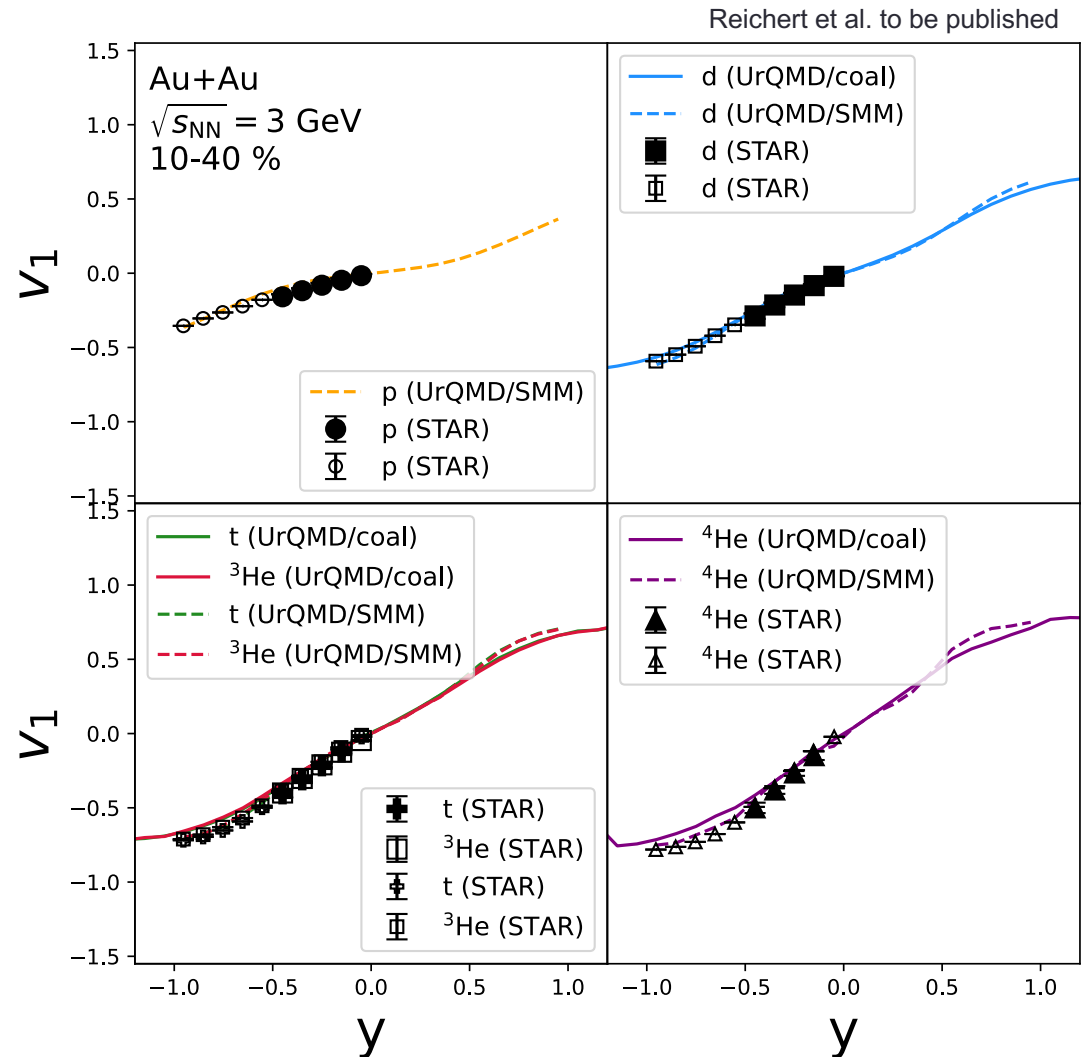
$$W_f \sim \exp[S_f(A_0, Z_0, E^*, V)]$$

- Deexcitation via Fermi break up

Bondorf et al. Phys.Rept. 257 (1995) 133-221  
Steinheimer et al. Phys.Lett.B 714 (2012) 85-91  
Botvina. Phys. Rev.C76 (2007) 024909

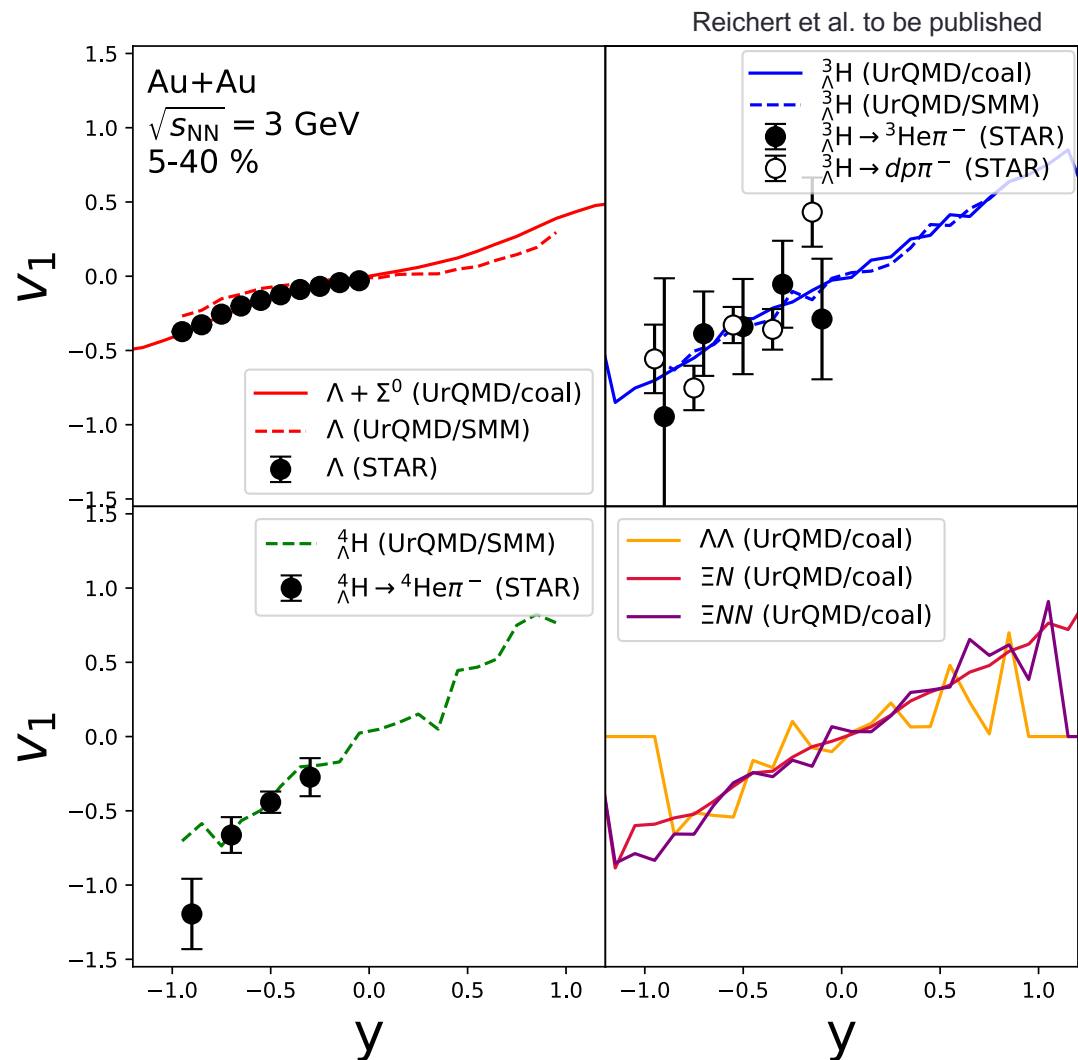
# Directed flow of light nuclei

- Data is described very good in large rapidity acceptance
- Coalescence and statistical multi-fragmentation yield similar results
- Prominent bounce-off visible in  $v_1$
- $v_1$  of the clusters follows the  $v_1$  of the nucleons



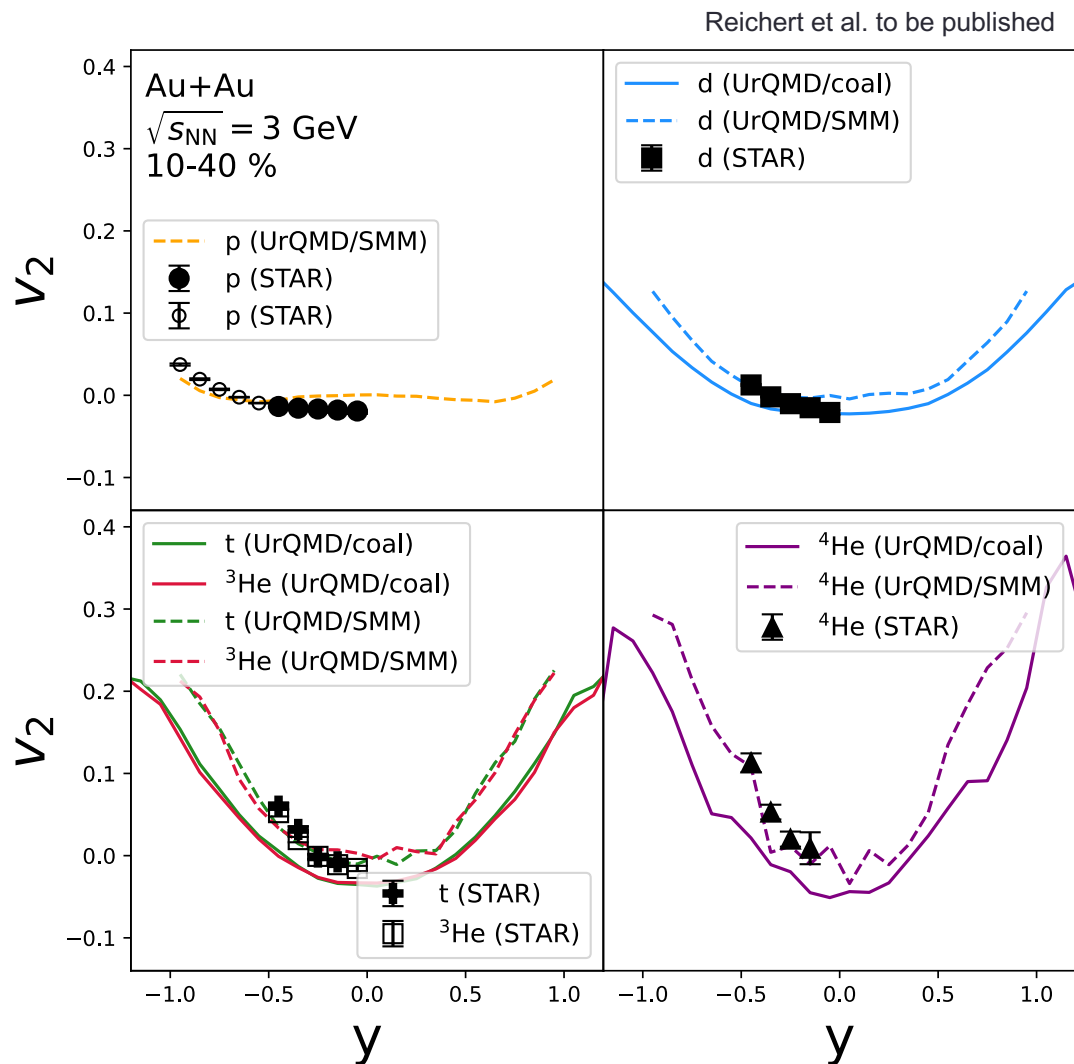
# Directed flow of hypernuclei

- Coalescence and multifragmentation describe  $v_1$  of  $\Lambda\Lambda$ ,  ${}^3_{\Lambda}\text{H}$ ,  ${}^4_{\Lambda}\text{H}$  good
- $v_1$  of hypernuclei follows  $v_1$  of the nucleons and Lambda hyperons
- Predict  $v_1$  of exotic objects,  $|\Lambda\Lambda\rangle$ ,  $|\Xi N\rangle$ ,  $|\Xi NN\rangle$
- Allows to constrain the YN interaction more precisely



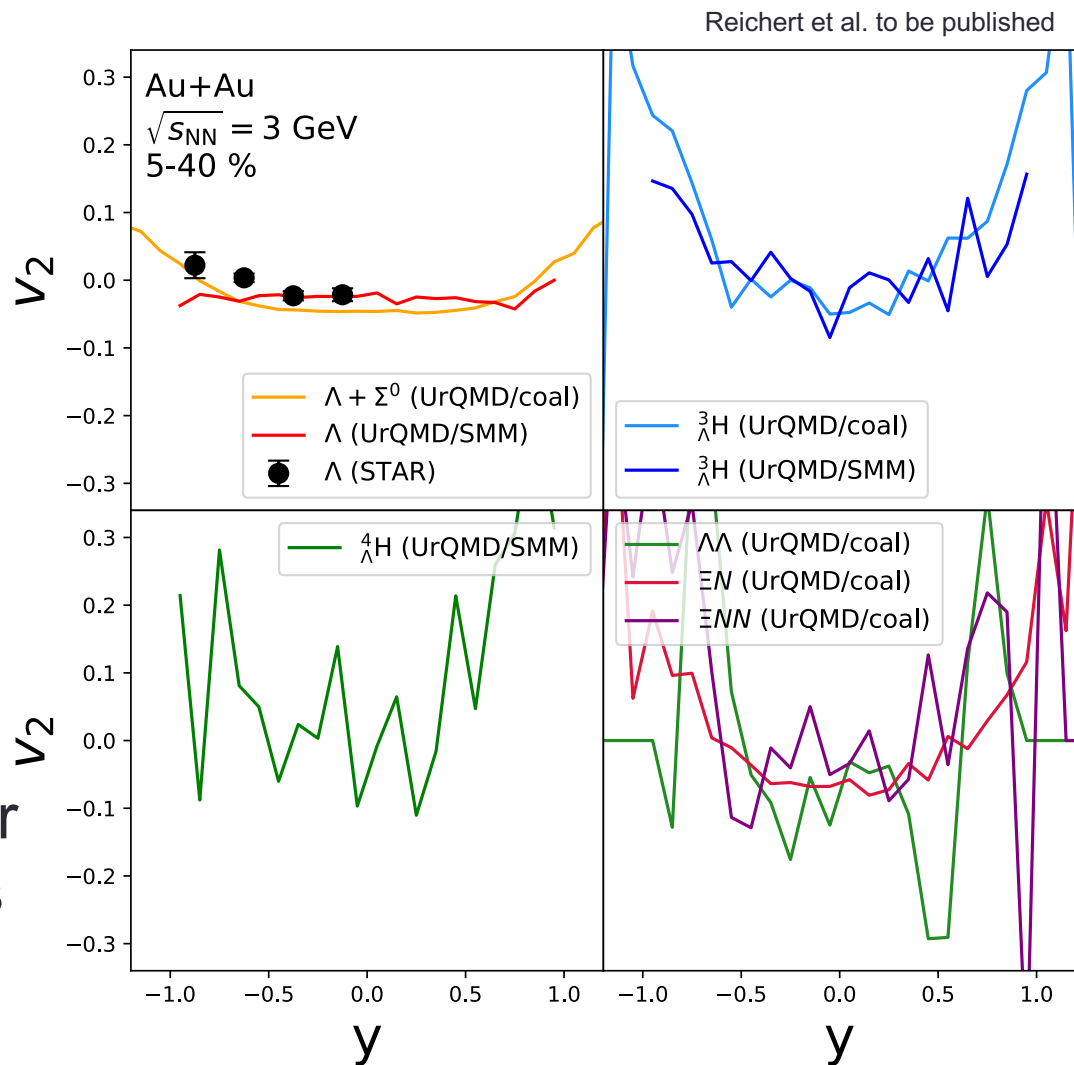
# Elliptic flow of light nuclei

- Data is very well described in large rapidity acceptance
- Coalescence yields a slightly smaller  $v_2$  than multifragmentation
- $v_2$  of light clusters follow  $v_2$  of the nucleons



# Elliptic flow of hypernuclei

- Data is described very well
- Coalescence yields a smaller  $v_2$  than multifragmentation
- Hints at intricate time evolution of  $v_2$
- Constrain the YN interaction
- Benchmark potentials for neutron star calculations





# Summary

- Pressure gradient turns positive  $v_2$  into positive  $v_1$ , therefore creating bounce-off
- Final  $v_2$  is negative due to immense shadowing, momentum transfer to (semi-) spectators
- Measurement of initial  $v_2$  possible with di-leptons
- Correlations of  $v_2$ -  $v_1$ ,  $v_2$ -  $v_3$  are sensitive to EoS
- Coalescence and Multifragmentation describe and predict flow of light- and hypernuclei
- $v_1$  &  $v_2$  pose opportunity to study YN interaction



FIAS Frankfurt Institute  
for Advanced Studies



Stiftung  
Polytechnische  
Gesellschaft  
Frankfurt am Main

HFHF

Helmholtz Forschungsakademie Hessen für FAIR

