JETS IN STRONGLY INTERACTING MATTER

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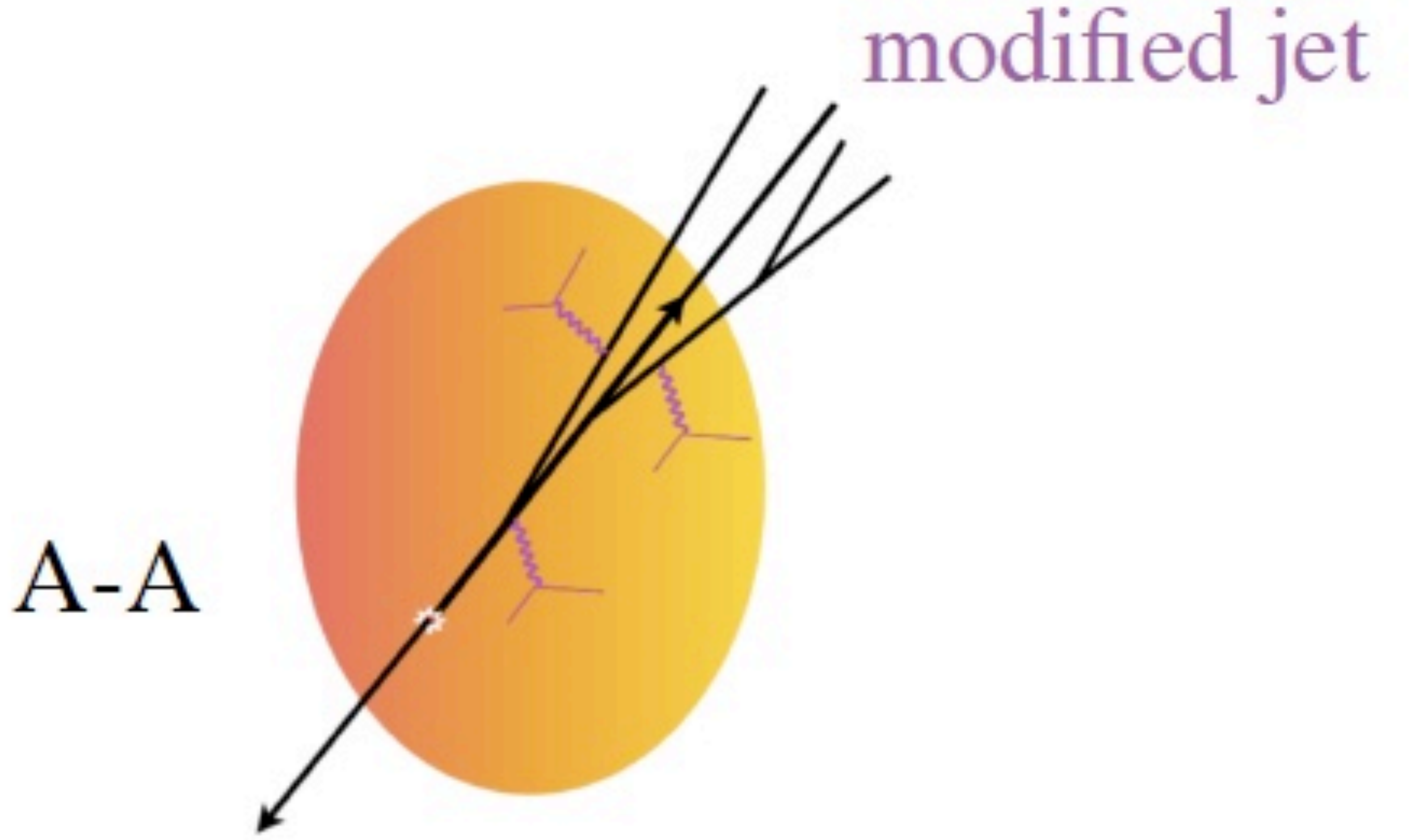
2nd Workshop of the Network NA7-HF-QGP of the European program "STRONG-2020" & 'HFHF Theory Retreat 2023'
28 September – 4 October 2023, Giardini Naxos, Italy

Outline

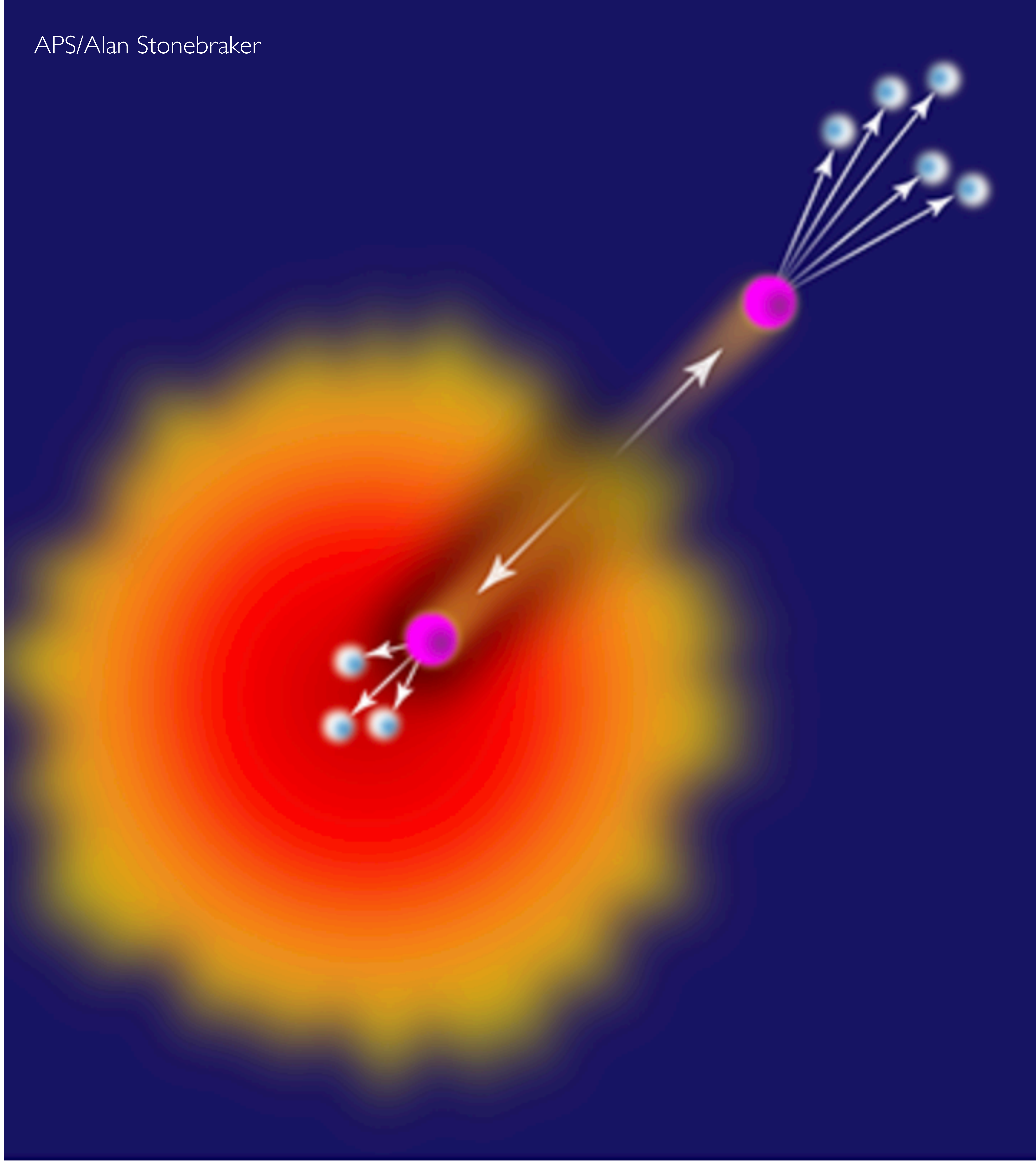
Structure of the lectures

- Lecture 1
  - learning to interpret "jet quenching" in experimental data
  - QCD jets in vacuum
- Lecture 2
  - theory of radiative parton energy loss
- Lecture 3
  - theory of full jet quenching



Lecture 2

theory of radiative parton energy loss in matter

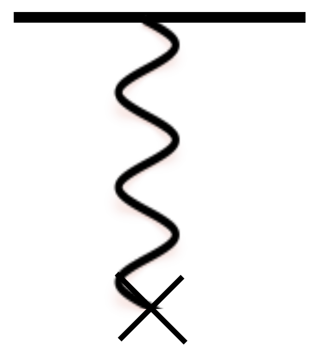


PARTON PROPAGATION IN MEDIUM

Baier, Dokshitzer, Mueller, Peigné, Schiff (1996); Zakharov (1996); Arnold, Moore, Yaffe (2003)

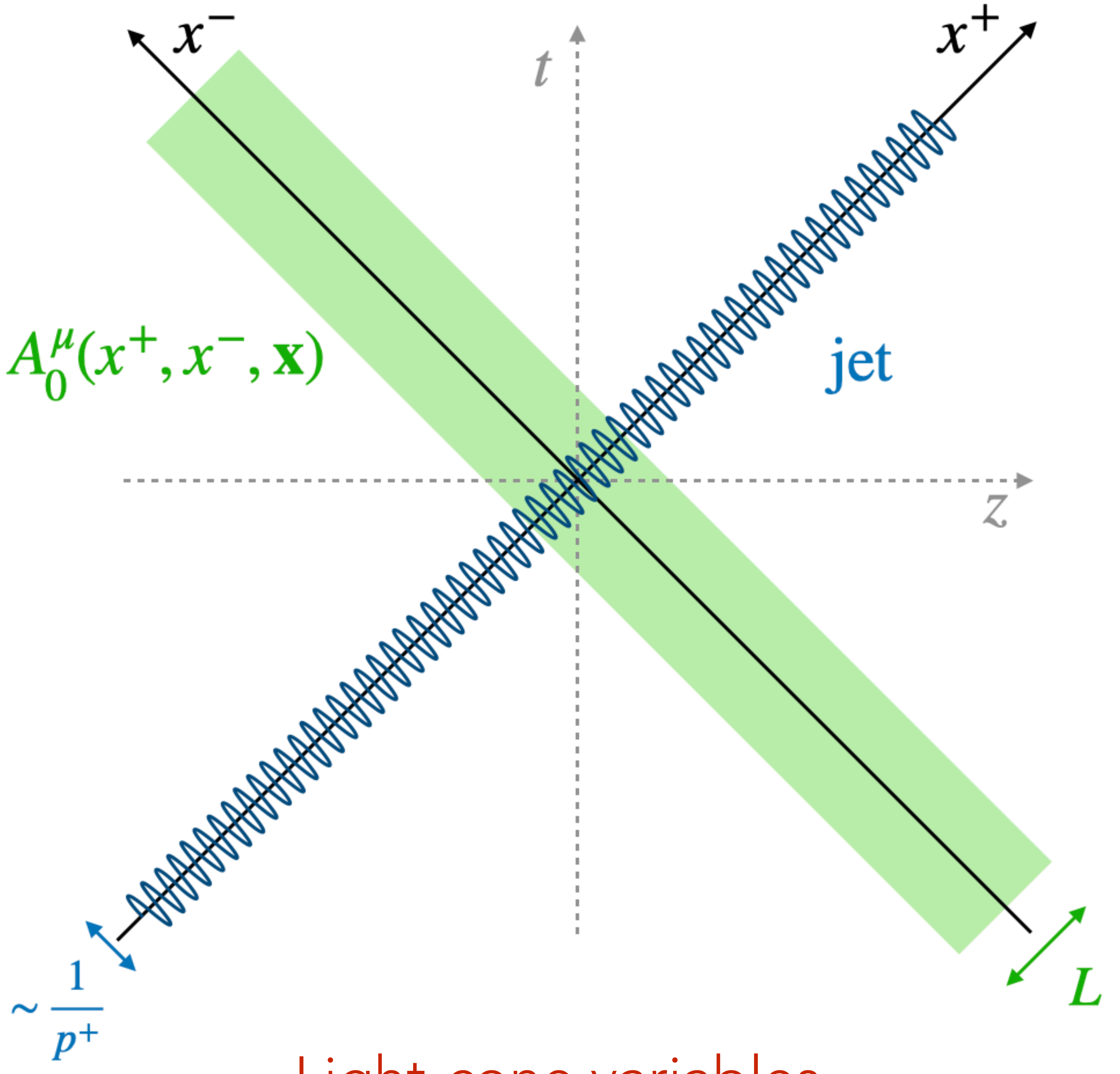
Setup: light-cone perturbation theory in A^- background field ($A^+ = 0$ gauge).

Interaction vertex: treated in the eikonal approximation



$$\simeq= ig\boldsymbol{T}^a\, 2p^+\, A^{-,a}(x)$$

We assume the medium potential does not have a extent in the x^- direction, i.e. $A(x^+, x^-, \boldsymbol{x}) \simeq A(x^+, 0, \boldsymbol{x})$. In Fourier space, this leads to $\delta(q^+)$ - no longitudinal momentum transfer & no elastic energy loss.

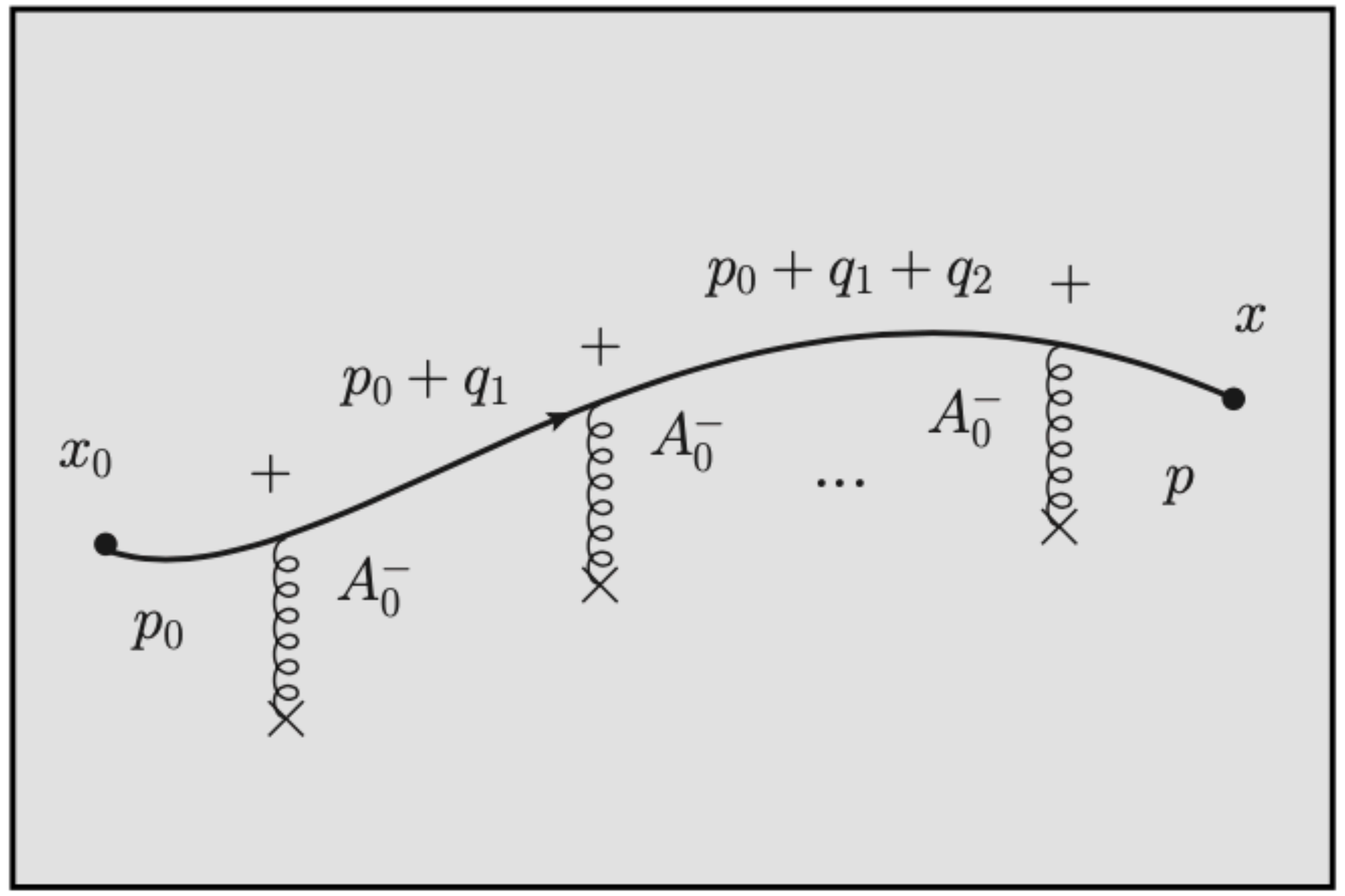


Light-cone variables

$$x^+ = \frac{1}{2}(x^0 + x^3)$$
$$x^- = x^0 - x^3$$

Parton propagation in the medium

Baier, Dokshitzer, Mueller, Peigné, Schiff (1996); Zakharov (1996); Arnold, Moore, Yaffe (2003)



Dressed (scalar) propagator:

$$(x|G_{\rm scal}|x_0) = (x|G_0|x_0) + 2p^+ \int_z (x|G_0|z)\, ig\mathcal{A}_0(z)\, (z|G_{\rm scal}|x_0)$$

Translational invariance in x^- components:

Conservation of large momentum component p^+.

$$(\boldsymbol{x}|\mathcal{G}(t,t_0)|\boldsymbol{x}_0) \equiv 2p^+ \int \mathrm{d}x^-\, \mathrm{e}^{ip^+(x-x_0)^-} (x|G_{\rm scal}|x_0)$$

$$\left[i\frac{\partial}{\partial t} + \frac{\partial_\perp^2}{2E} + g\mathcal{A}_0(t,\boldsymbol{x})\right](\boldsymbol{x}|\mathcal{G}(t,t_0)|\boldsymbol{x}_0) = i\delta(t-t_0)\delta(\boldsymbol{x}-\boldsymbol{x}_0)$$

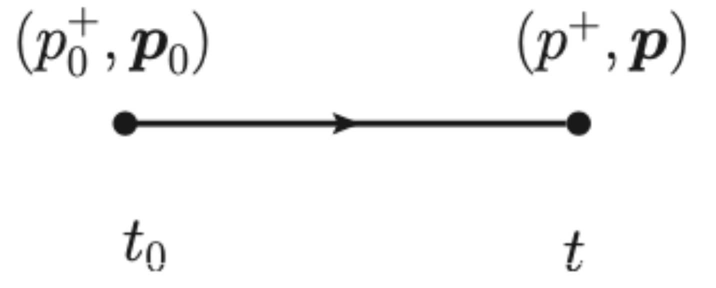
2+1D Schrödinger equation in transverse space with $m = E$.

FEYNMAN RULES

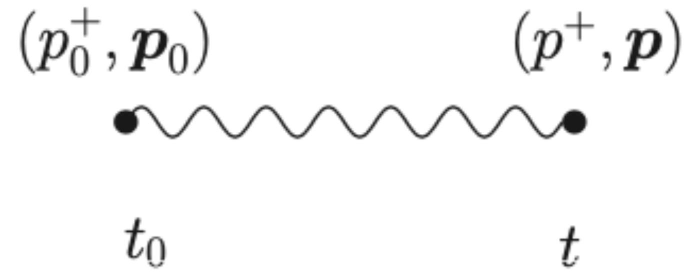
Propagators

[explicit time-dependence; typically not in LCPT]

Internal lines

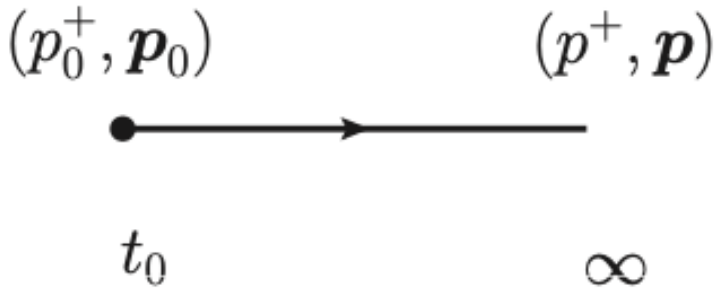


$$\frac{i\gamma^-}{4p^+}\,(\boldsymbol{p}|\,\mathcal{G}(t,t_0)|\boldsymbol{p}_0)\,(2\pi)\,\delta(p^+-p_0^+)$$

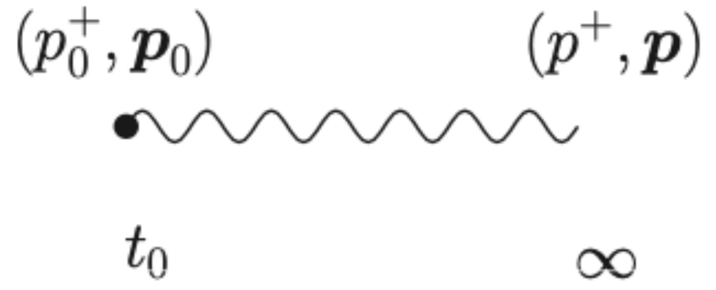


$$\frac{i}{2p^+}\,\delta^{ij}\,(\boldsymbol{p}|\,\mathcal{G}^{\rm adj}(t,t_0)|\boldsymbol{p}_0)\,(2\pi)\,\delta(p^+-p_0^+)$$

External lines



$$\bar{\xi}(s)\ \mathrm{e}^{i\frac{\boldsymbol{p}^2}{2p^+}t_\infty}\ (\boldsymbol{p}|\,\mathcal{G}_{\rm scal}(\infty,t_0)|\boldsymbol{p}_0)$$

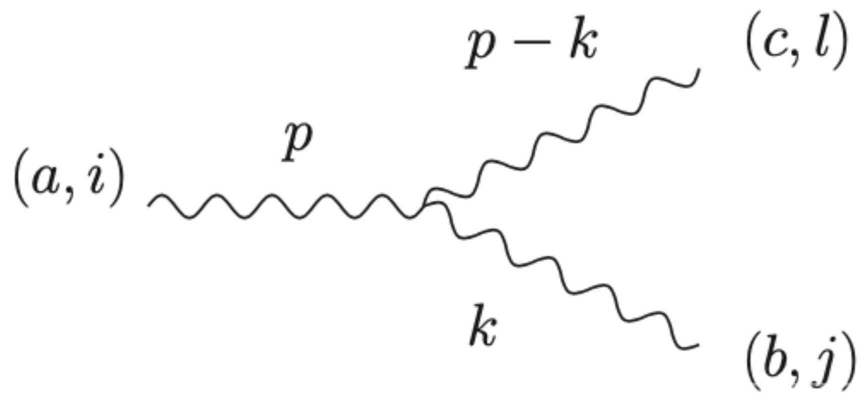


$$\epsilon_\lambda^{*,i}(p)\ \mathrm{e}^{i\frac{\boldsymbol{p}^2}{2p^+}t_\infty}\ (\boldsymbol{p}|\,\mathcal{G}^{\rm adj}_{\rm scal}(\infty,t_0)|\boldsymbol{p}_0)$$

Vertices

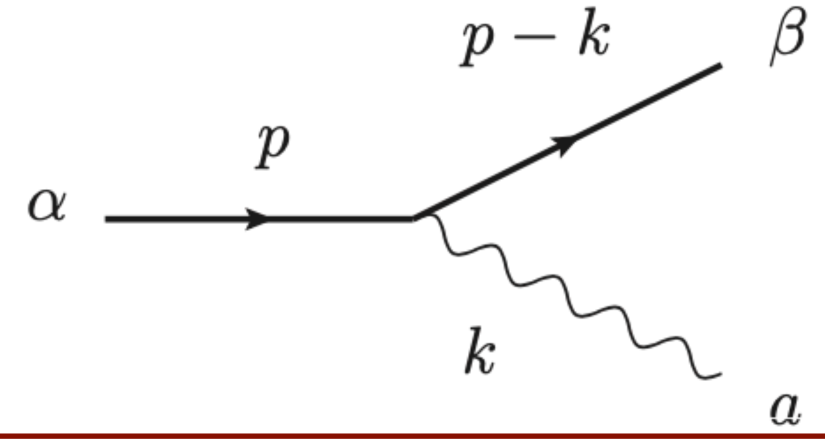
[identical to LCPT]

q→qg vertex



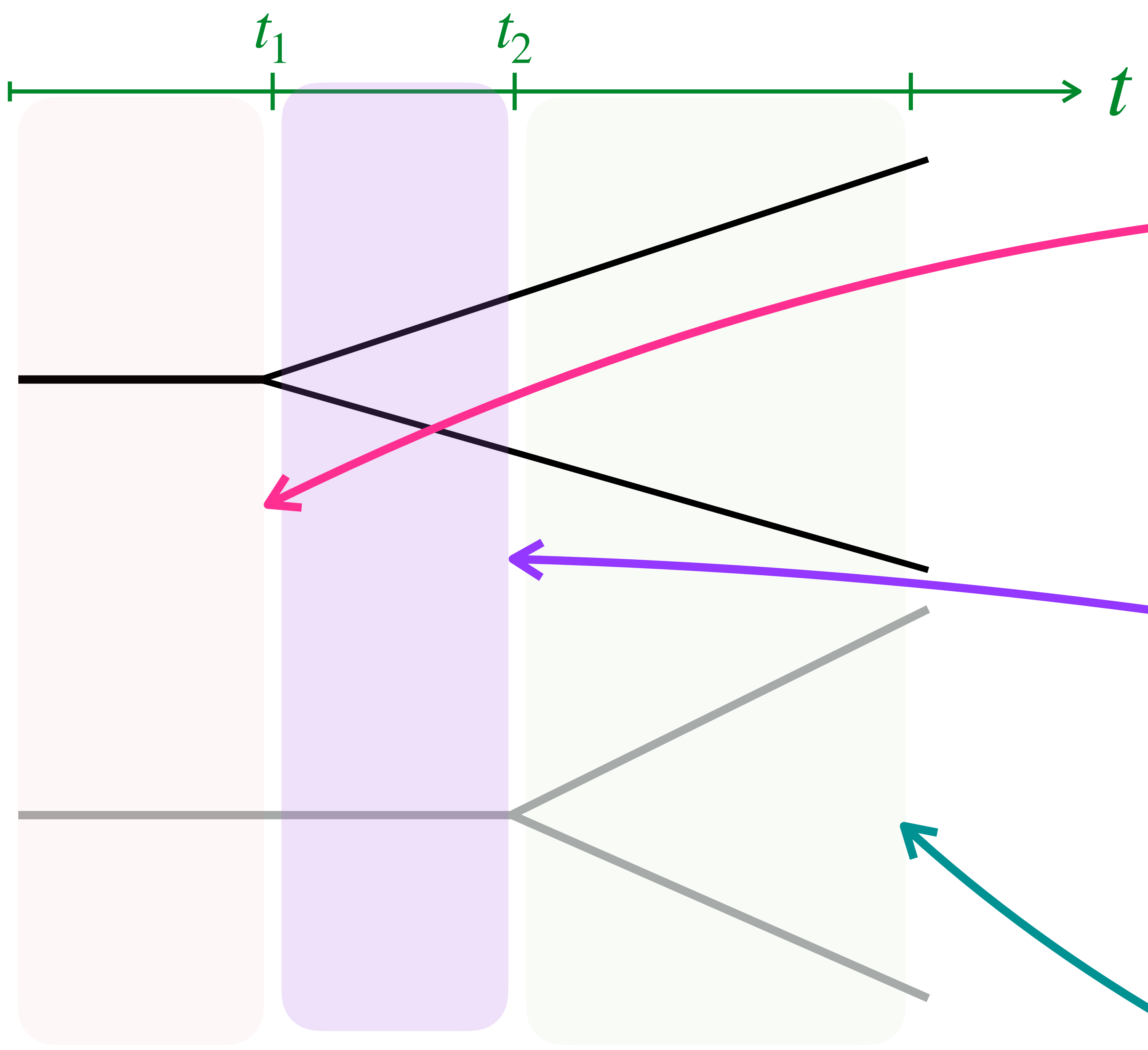
$$-2gf_{abc}\left\{-\frac{1}{1-z}(\boldsymbol{k}-z\boldsymbol{p})^l\delta^{ij}+(\boldsymbol{k}-z\boldsymbol{p})^i\delta^{jl}-\frac{1}{z}(\boldsymbol{k}-z\boldsymbol{p})^j\delta^{il}\right\}$$

g→gg vertex



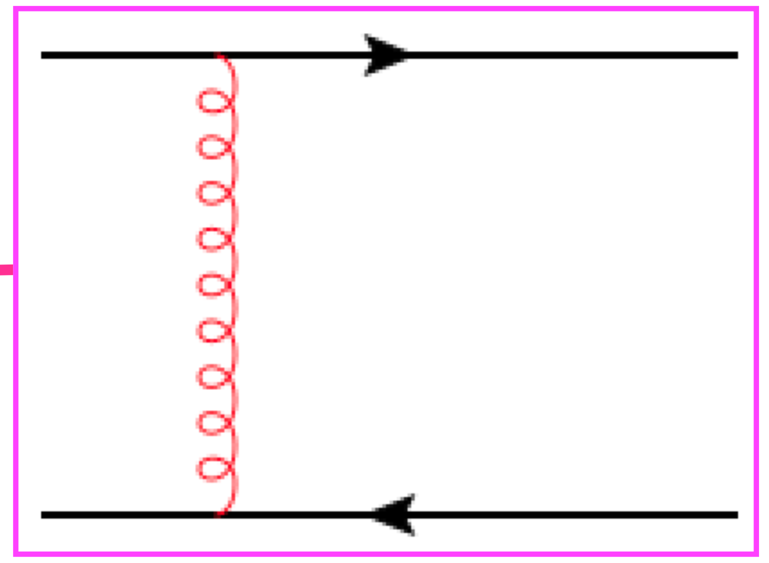
$$\gamma^+\frac{2igt^a}{z\sqrt{1-z}}\left[\left(1-\frac{z}{2}\right)(\boldsymbol{k}-z\boldsymbol{p})^i+i\frac{z}{2}S^3\epsilon^{ij}\,(\boldsymbol{k}-z\boldsymbol{p})^j\right]$$

Parton splitting in the medium



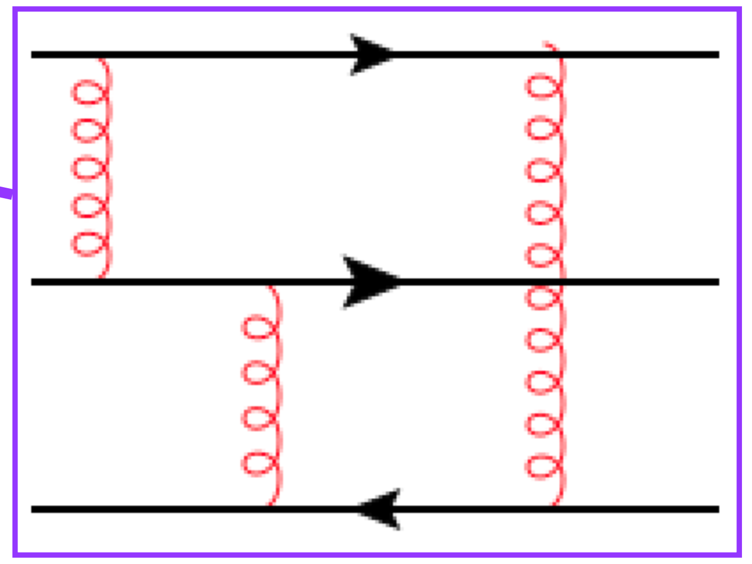
$S^{(2)}(t_1,0)$ $S^{(3)}(t_2,t_1)$ $S^{(4)}(\infty,t_2)$

2-point function



Molière (1948)
Liu, Rajagopal, Wiedemann hep-ph/0605178
d'Eramo, Rajagopal, Yin 1808.03250
Barata, Mehtar-Tani, Soto-Ontoso, KT 2009.13667

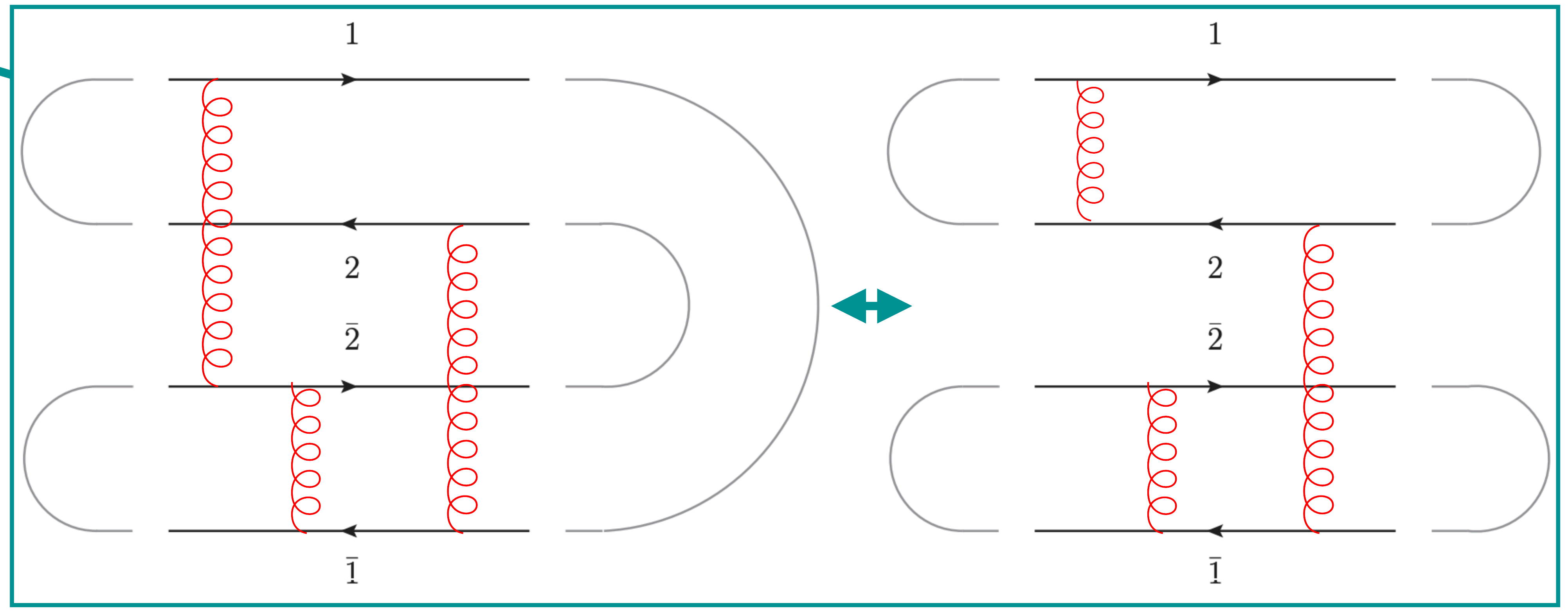
3-point function



Baier, Dokshitzer, Mueller, Peigné, Schiff (1996);
Zakharov (1996) (Arnold, Moore, Yaffe (2003))

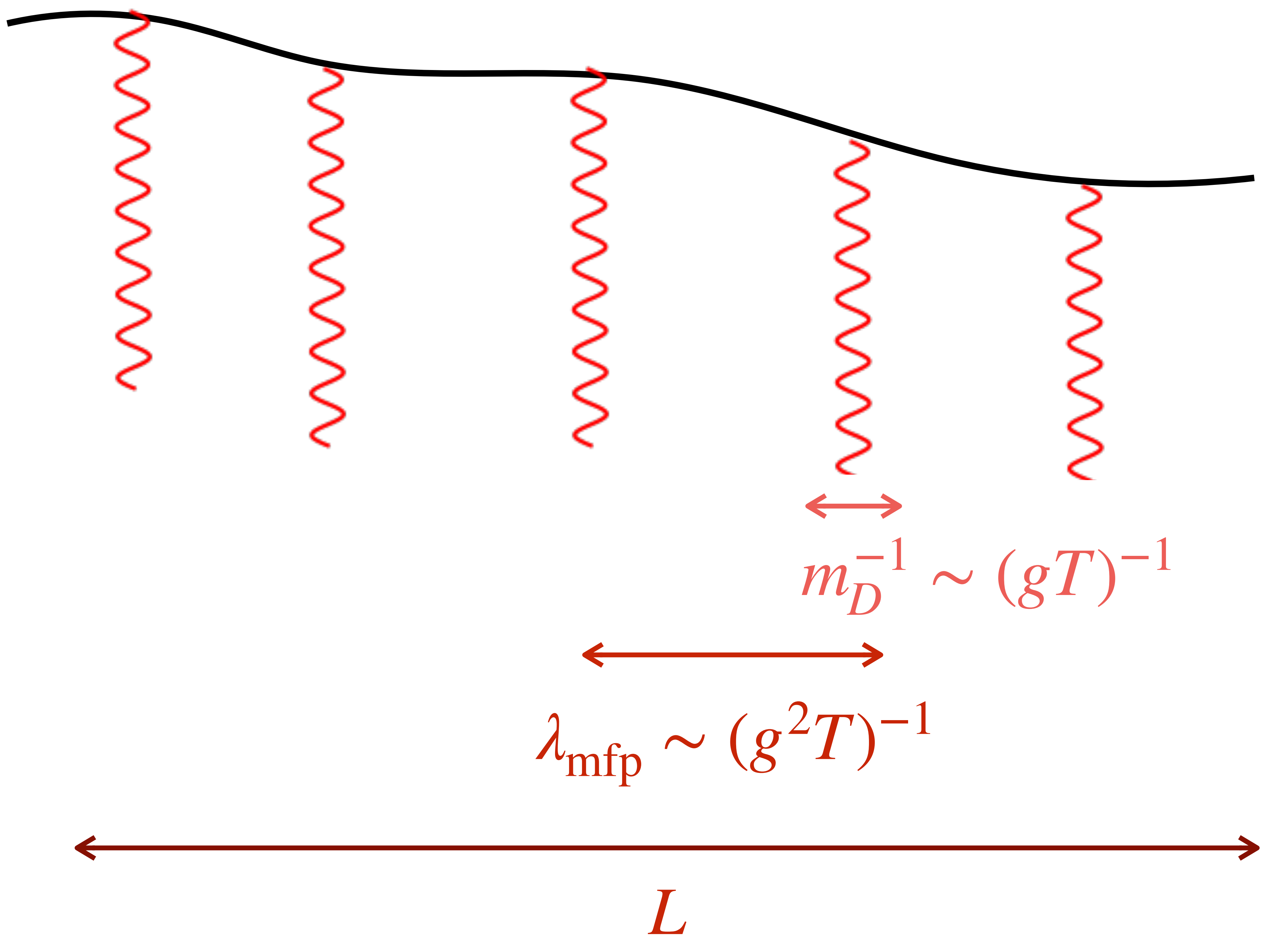
Blaizot, Dominguez, Iancu, Mehtar-Tani (2013)
Apolinario, Armesto, Milhano, Salgado (2015)
Isaksen, KT 2107.02542

4-point function



- decomposed into gauge-invariant objects (Wilson line correlators).
- Schwinger-Keldysh contour (real-time QFT)
- evaluate the correlators in the medium background

Scales of the medium

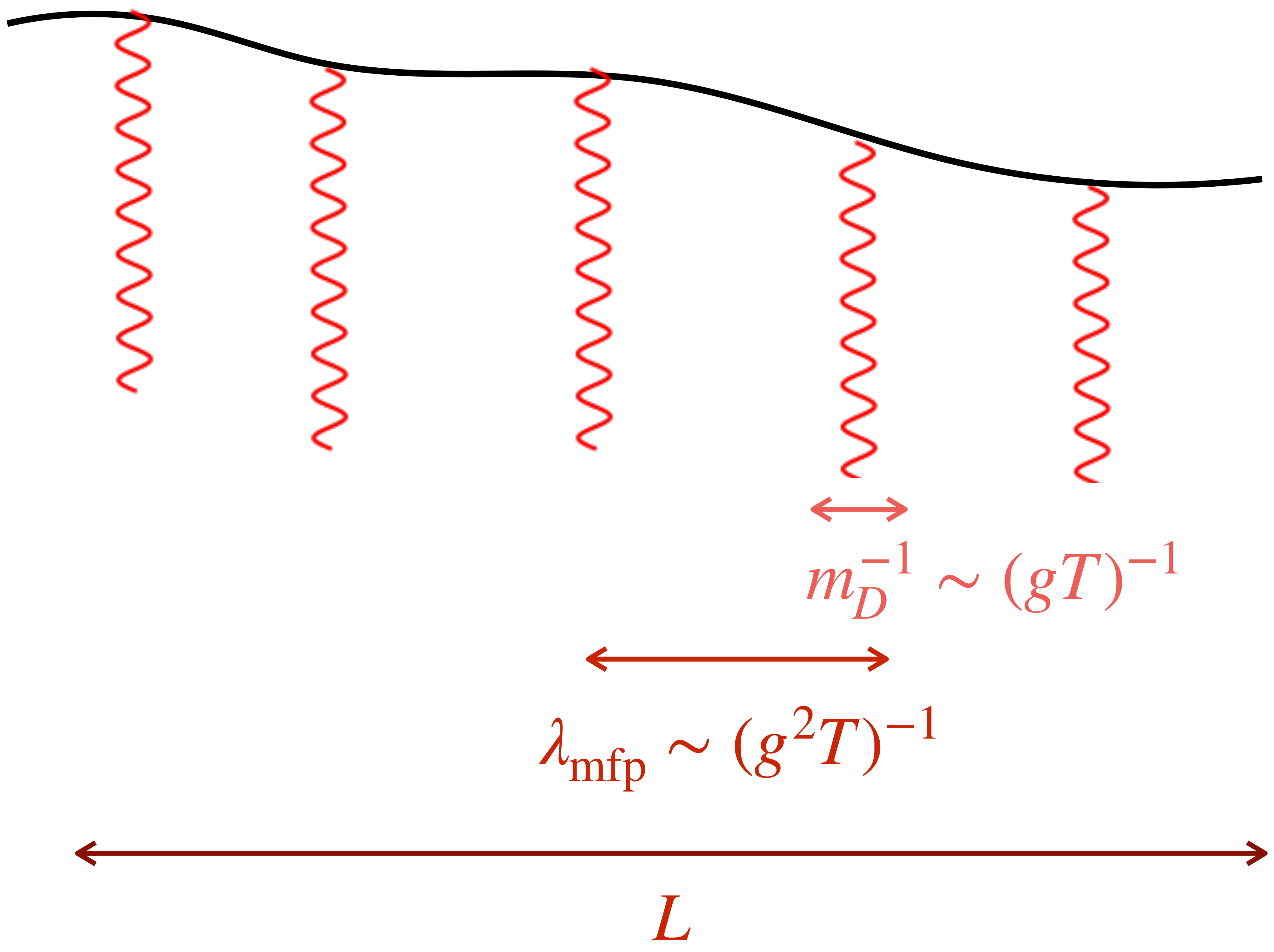


- at weak-coupling $g \ll 1$
- thermal distribution of medium scattering centers $n \sim T^3$
- separation of scales
  - how to push to strong coupling? NP evaluation of correlators (AdS/CFT,….)

$$\sigma_{\rm el} \sim \frac{g^4}{m_D^2}$$

$$\lambda_{\rm mfp} \sim \frac{1}{n\sigma_{\rm el}}$$

SCALES OF THE MEDIUM



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Markov approximation: $\langle A^{-,a}(t,\boldsymbol{x})A^{-,b}(t',\boldsymbol{x}')\rangle = \delta^{ab}\delta(t-t')\delta(\boldsymbol{x}-\boldsymbol{x}')\gamma(t,\boldsymbol{x})$

[instantaneous space-like interactions]

MEDIUM-AVERAGED CORRELATORS

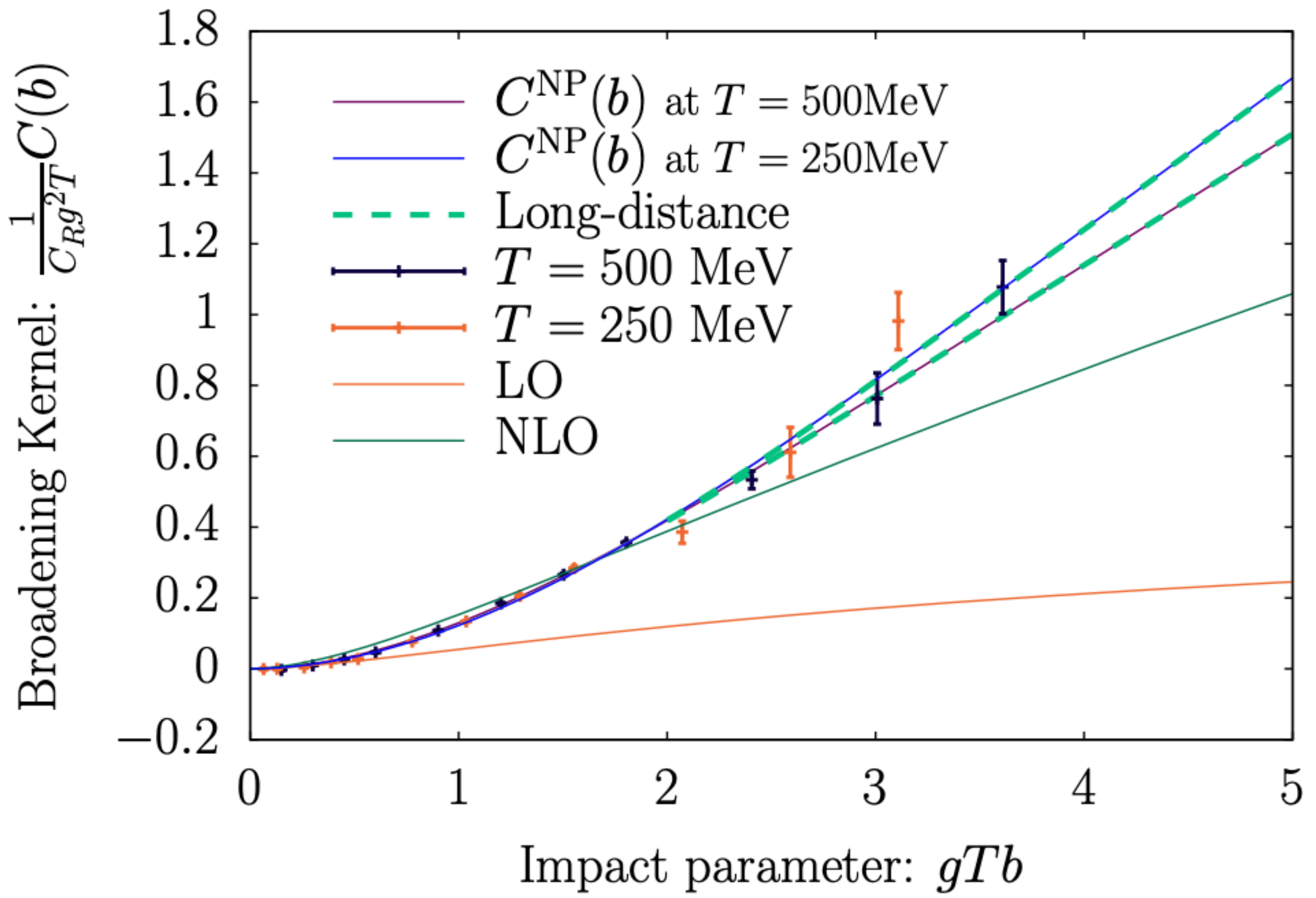
Resumming interactions via potential:

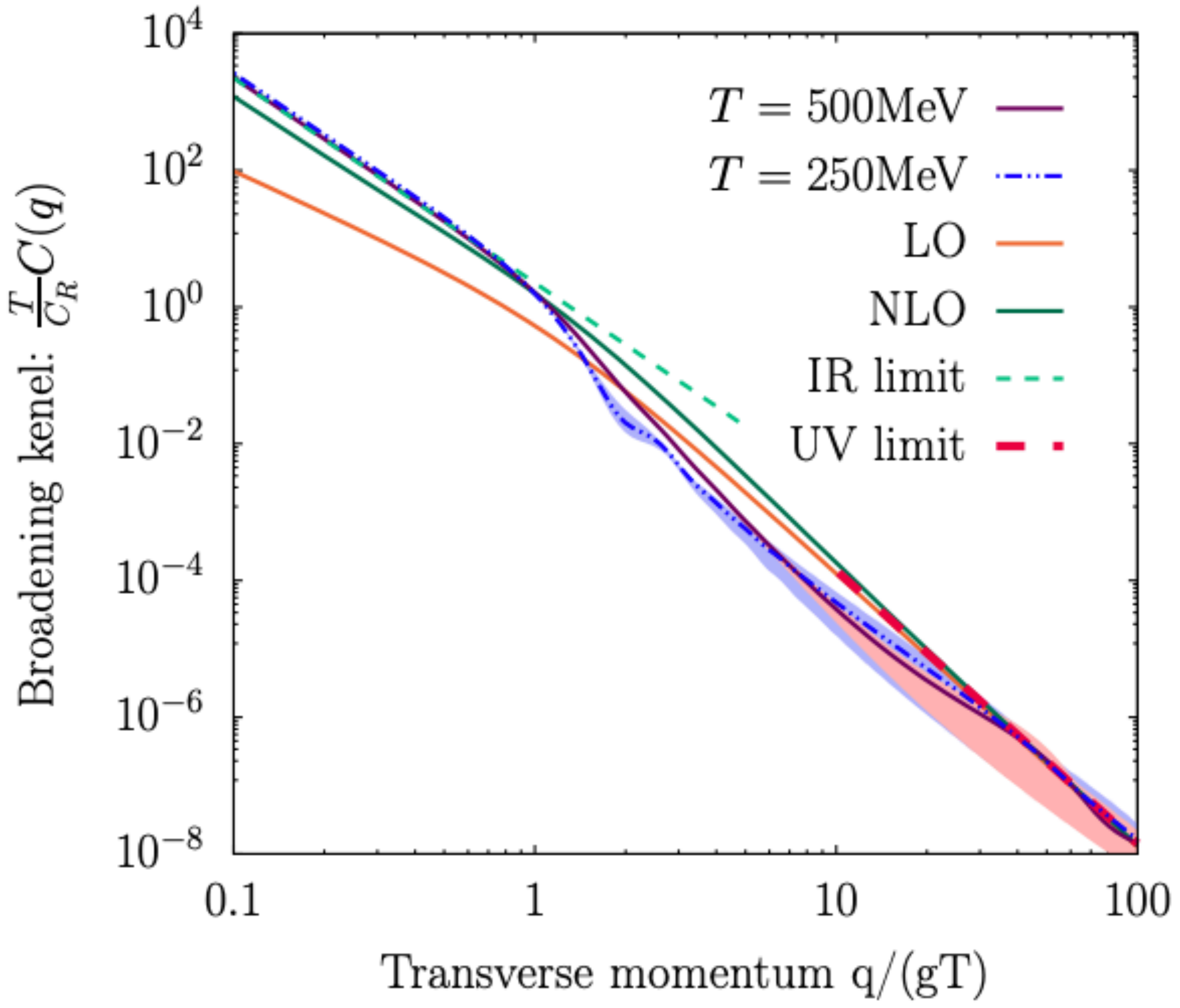
(Including real and virtual exchanges.)

$$v(t,\boldsymbol{x}) = \gamma(t,0) - \gamma(t,\boldsymbol{x}) = \int_{\boldsymbol{q}} \frac{\mathrm{d}^2\sigma_{\rm el}}{\mathrm{d}\boldsymbol{q}^2}\left(1 - \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right)$$
$$\simeq \frac{1}{4}\hat{q}_0 \boldsymbol{x}^2 \ln\frac{1}{\boldsymbol{x}^2\mu_*^2} + \mathcal{O}(\boldsymbol{x}^4\mu_*^2)$$

First term is universal:
$$\mu_*^2 = \begin{cases} \frac{1}{4}m_D^2\mathrm{e}^{-2+2\gamma_E} & \text{for HTL potential} \\ \frac{1}{4}m_D^2\mathrm{e}^{-1+2\gamma_E} & \text{for GW potential} \end{cases}$$

Schlichting, Soudi 2111.13731





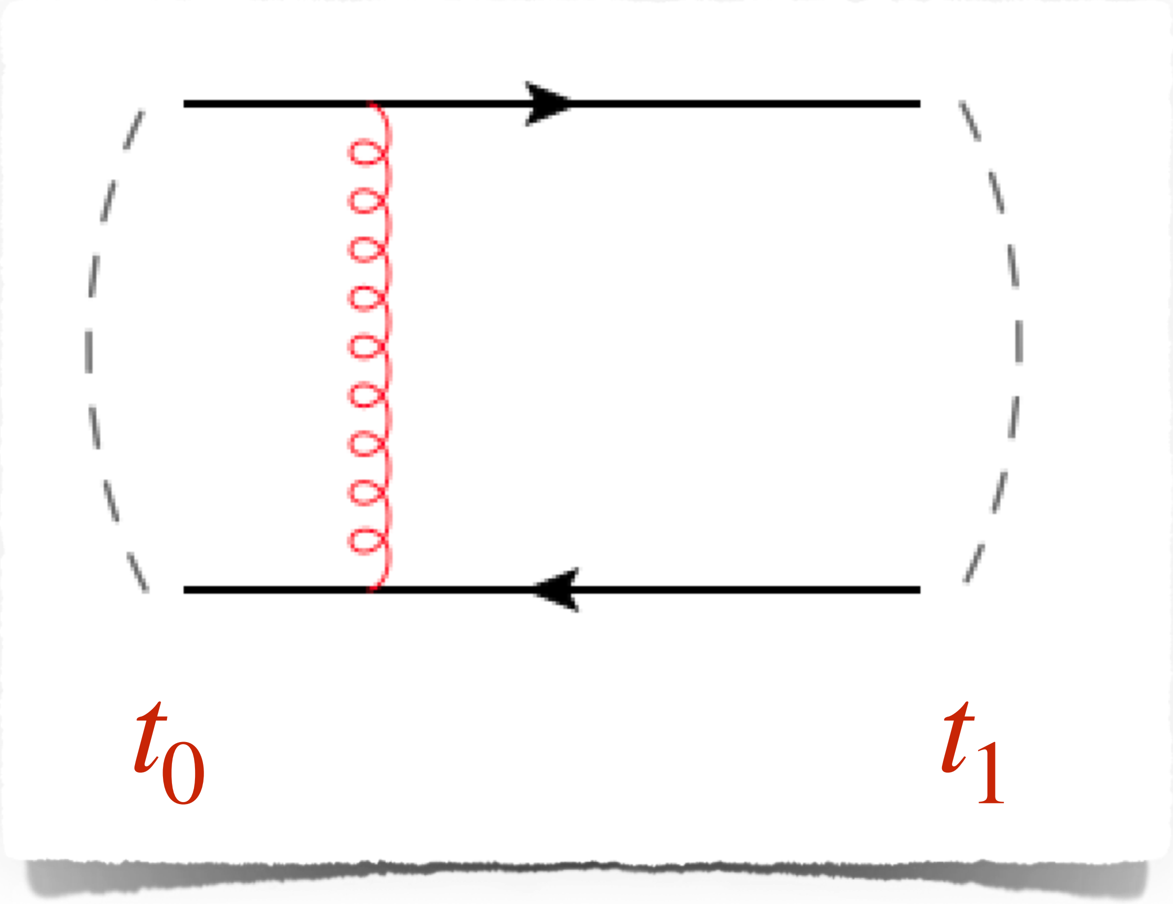
For space-like correlators non-perturbative contributions can be found!

significant impact on pheno!

S. Caron-Huot 0811.1603
Panero, Rummukainen, Schäfer 1307.5850
Moore, Schlusser 1905.09708, 1911.13127

BROADENING

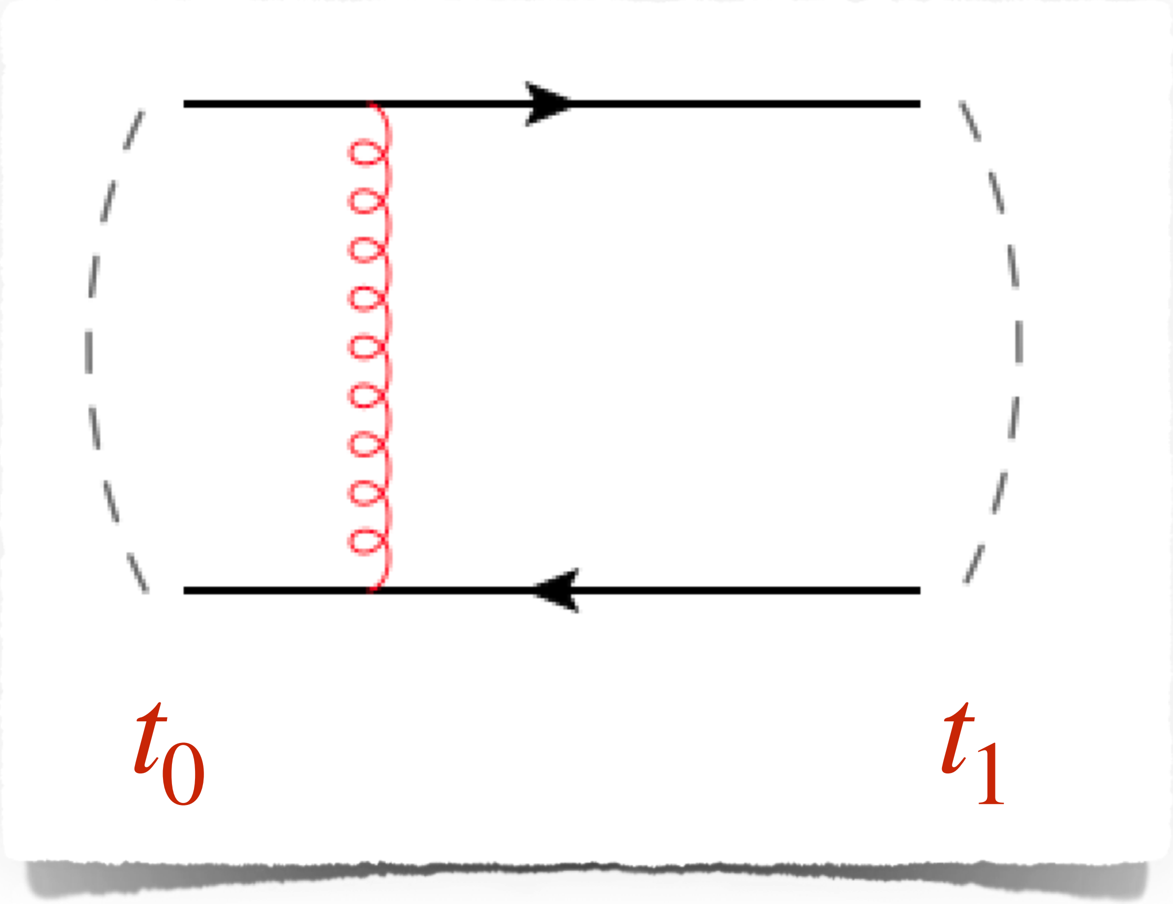
G. Molière, Zeitschrift Naturforschung Teil A 3, 78 (1948); Barata, Mehtar-Tani, Soto-Ontoso, KT 2009.13667



Transverse momentum broadening of a single parton $\langle \boldsymbol{k}^2 \rangle \sim \hat{q}t$

Broadening

G. Molière, Zeitschrift Naturforschung Teil A 3, 78 (1948); Barata, Mehtar-Tani, Soto-Ontoso, KT 2009.13667



Transverse momentum broadening of a single parton $\langle \boldsymbol{k}^2 \rangle \sim \hat{q}t$

Molière distribution (1948)

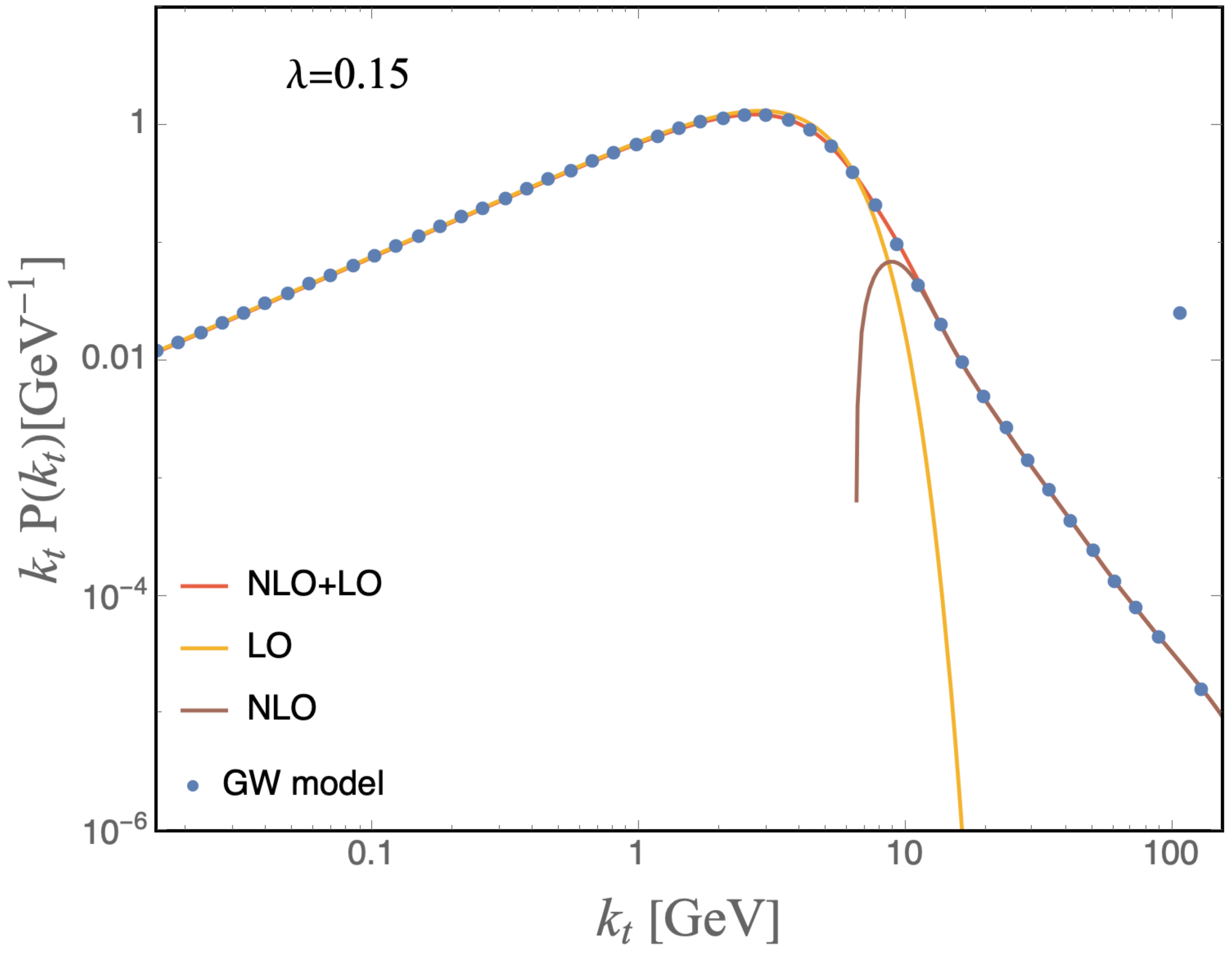
$x = \boldsymbol{k}^2/Q_s^2$, where $Q_s^2 = \hat{q}_0 L \log \frac{Q_s^2}{\mu_\star^2}$

$$\mathcal{P}^{\rm LO+NLO}(\boldsymbol{k}, L) = \frac{4\pi}{Q_s^2} e^{-x}\left\{1 - \lambda\left(e^x - 2 + (1-x)\,\left(\mathrm{Ei}\,(x) - \log(4x\,a)\right)\right)\right\}$$

Expansion parameter

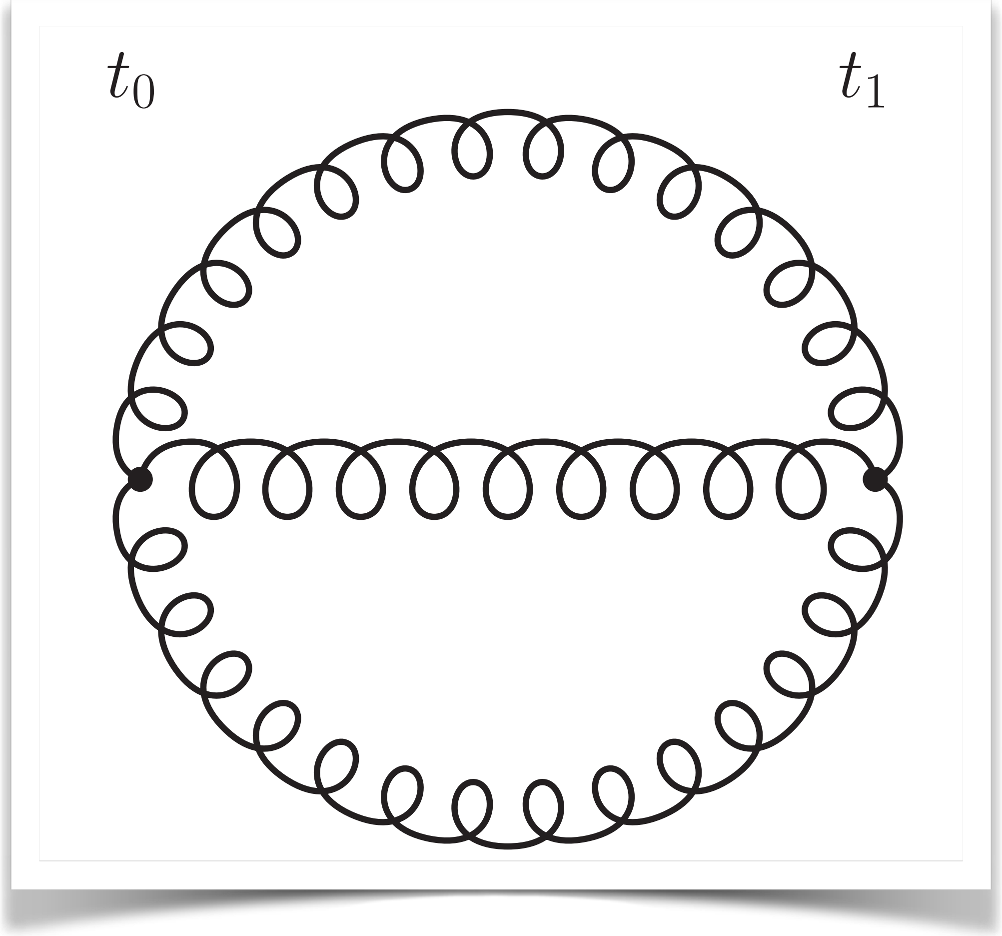
$$\lambda \equiv \frac{\hat{q}_0}{\hat{q}} = \frac{1}{\log(Q_s^2/\mu_\star^2)} \ll 1$$

Describes the distribution from diffusion dominated regime to higher-twist (HT) dominated regime.



Medium-induced radiation

Baier, Dokshitzer, Mueller, Peigné, Schiff (1996); Zakharov (1996); …



$$z\frac{\mathrm{d}I_{ba}}{\mathrm{d}z} = \frac{\alpha_s\, zP_{ba}(z)}{(z(1-z)E)^2}\, 2\mathrm{Re}\int_0^\infty \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1\; \mathrm{e}^{-\frac{m^2}{2z(1-z)E}(t_2-t_1)}$$
$$\boldsymbol{\partial}_{\boldsymbol{x}}\cdot\boldsymbol{\partial}_{\boldsymbol{y}}\Big[\mathcal{K}_{ba}(\boldsymbol{x},t_2;\boldsymbol{y},t_1)-\mathcal{K}_0(\boldsymbol{x},t_2;\boldsymbol{y},t_1)\Big]_{\boldsymbol{x}=\boldsymbol{y}=0}$$

3-point function: found from Schrödinger equation with potential $v(\boldsymbol{x},t)$

e.g. Casalderrey, Salgado 0712.3443

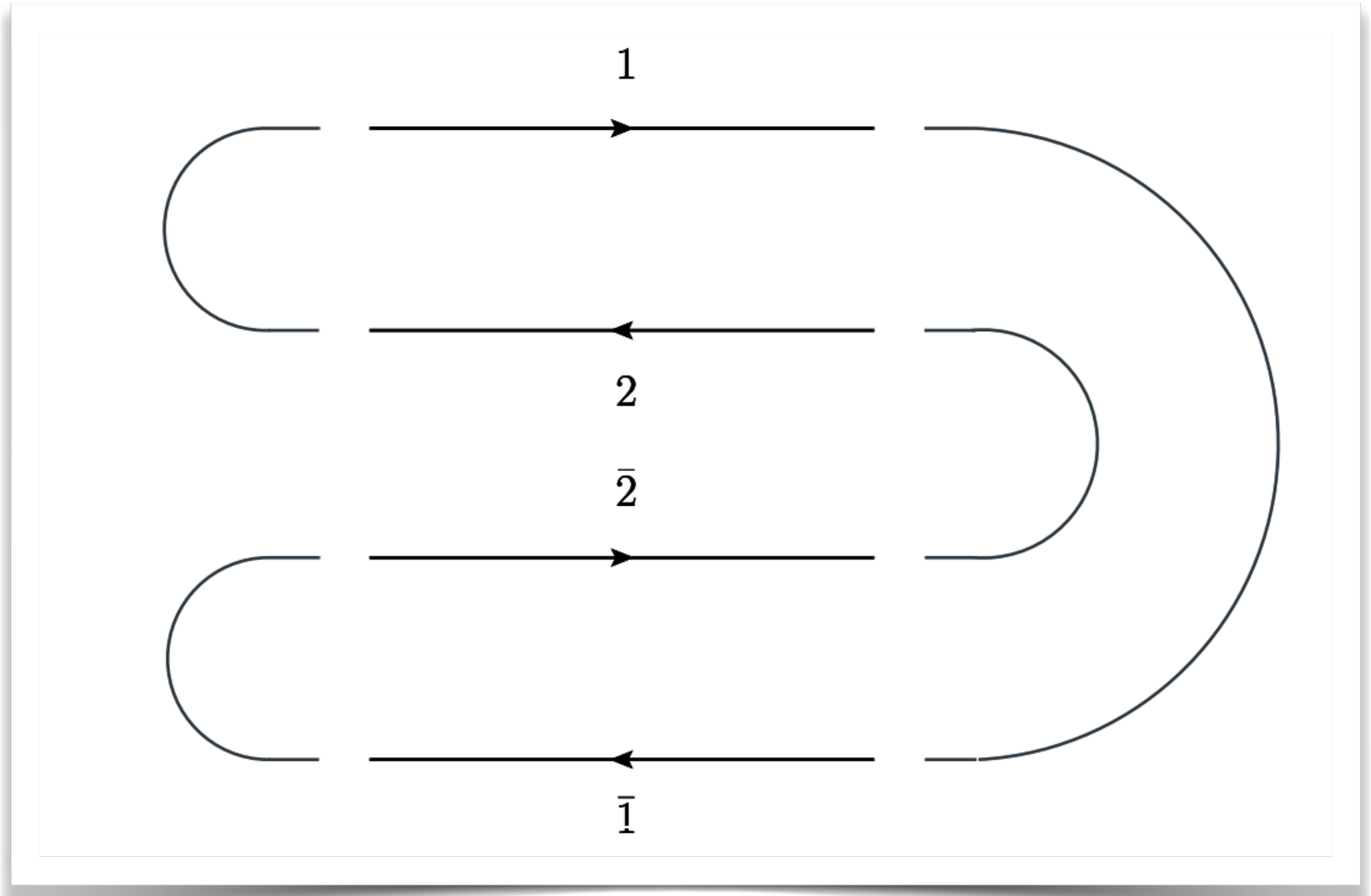
$$\left[i\frac{\partial}{\partial t}+\frac{\partial_{\boldsymbol{x}}^2}{2E}+iv(t,\boldsymbol{x})\right]\mathcal{K}(\boldsymbol{x},t;\boldsymbol{y},t_0) = i\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta(t-t_0)$$

- alternatively: rate from $dI/(dz\,dt)$
- numerical solutions and analytical control in the whole phase space
- currently more sophisticated treatment of medium (expanding, anisotropic,…)

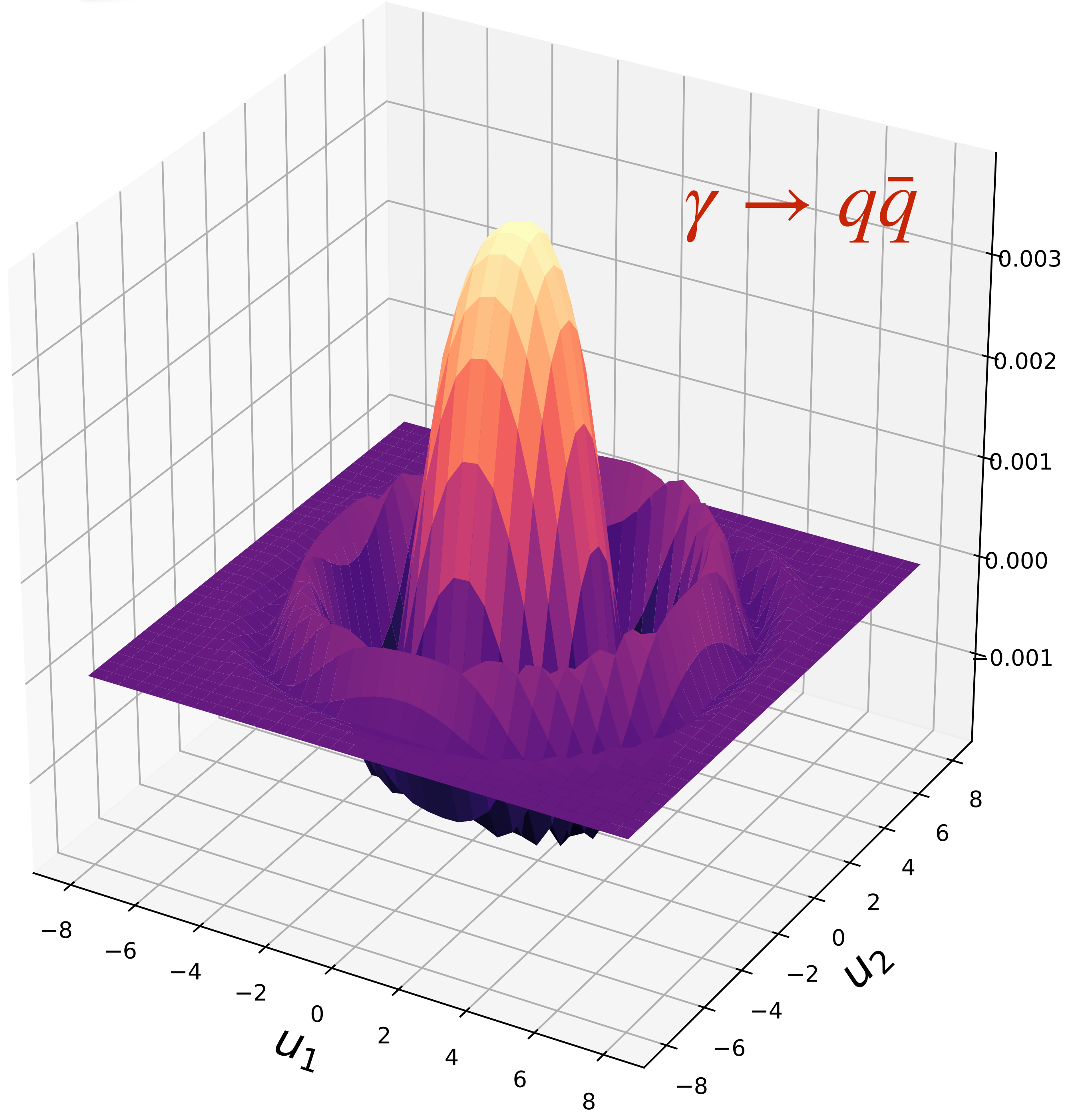
SPLITTING IN FULL KINEMATICS

Isaksen, KT 2107.02542; 2303.12119

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\,\mathrm{d}^2\boldsymbol{k}} = \frac{g^2\,P(z)}{2(2\pi)^3[z(1-z)E]^2}\,\mathrm{Re}\int_0^\infty \mathrm{d}t_1 \int_{t_1}^\infty \mathrm{d}t_2 \int_{\boldsymbol{x},\boldsymbol{u},\bar{\boldsymbol{u}}} \mathrm{e}^{-i(\boldsymbol{u}-\bar{\boldsymbol{u}})\cdot\boldsymbol{k}}$$
$$\times\, \partial_{\boldsymbol{y}}\cdot\partial_{\boldsymbol{z}}(\boldsymbol{u};\bar{\boldsymbol{u}}|\tilde{S}^{(4)}(L,t_2)|\boldsymbol{x};\boldsymbol{z})(\boldsymbol{x}|\tilde{S}^{(3)}(t_2,t_1)|\boldsymbol{y})\Big|_{\boldsymbol{y}=\boldsymbol{z}=0}$$



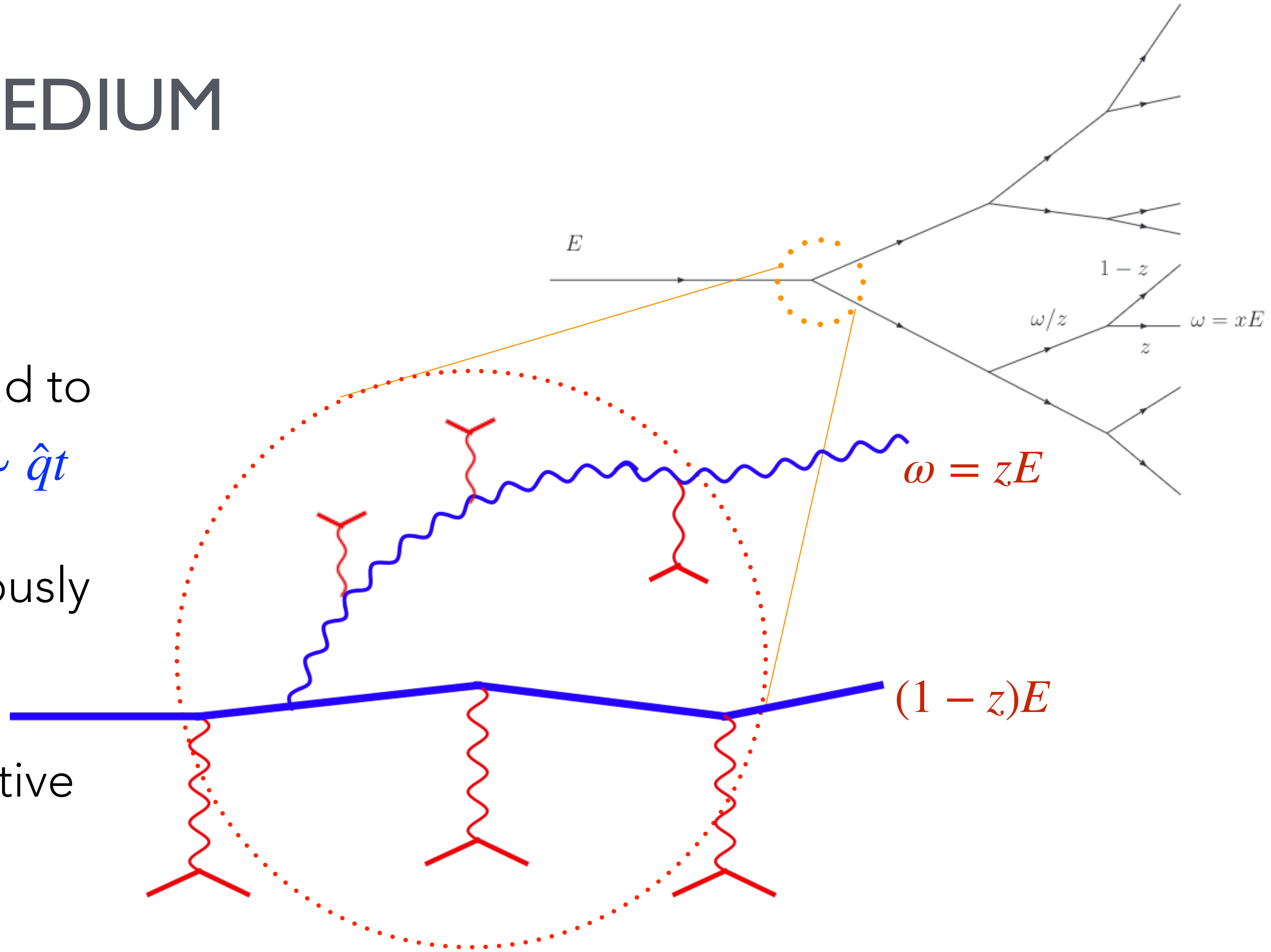
- numerical solutions of 1 or 2 body Schrödinger equations in 2+1D for n-level system in color space.
- toward full-kinematics precision calculation of splitting dynamics for all fundamental processes.



Two-body in-medium wave function.

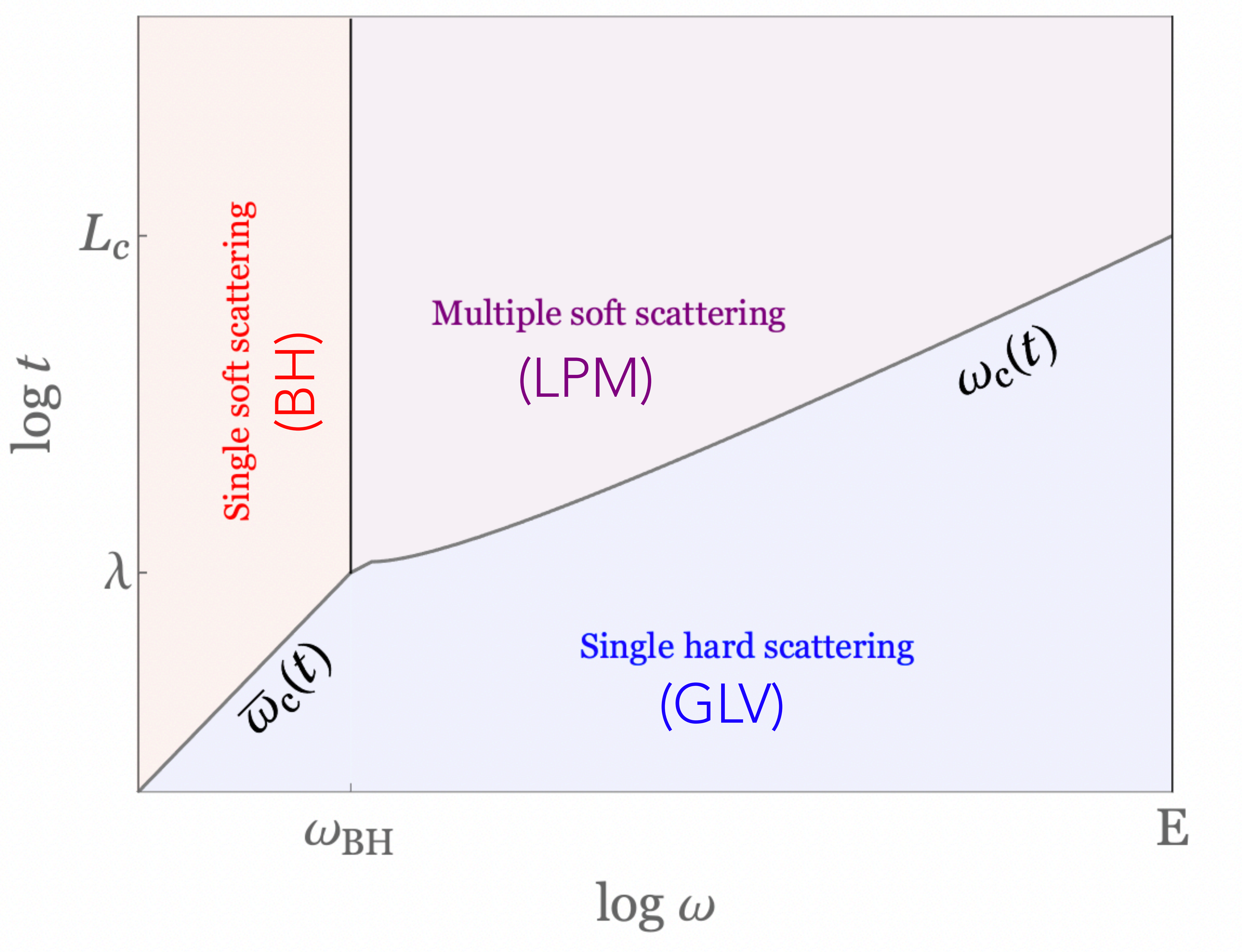
SPLITTING IN MEDIUM

- interactions with the medium lead to Gaussian broadening with $\langle k_t^2 \rangle \sim \hat{q} t$
- soft gluons are rapidly and copiously produced
- medium potential: non-perturbative input about medium transport properties.



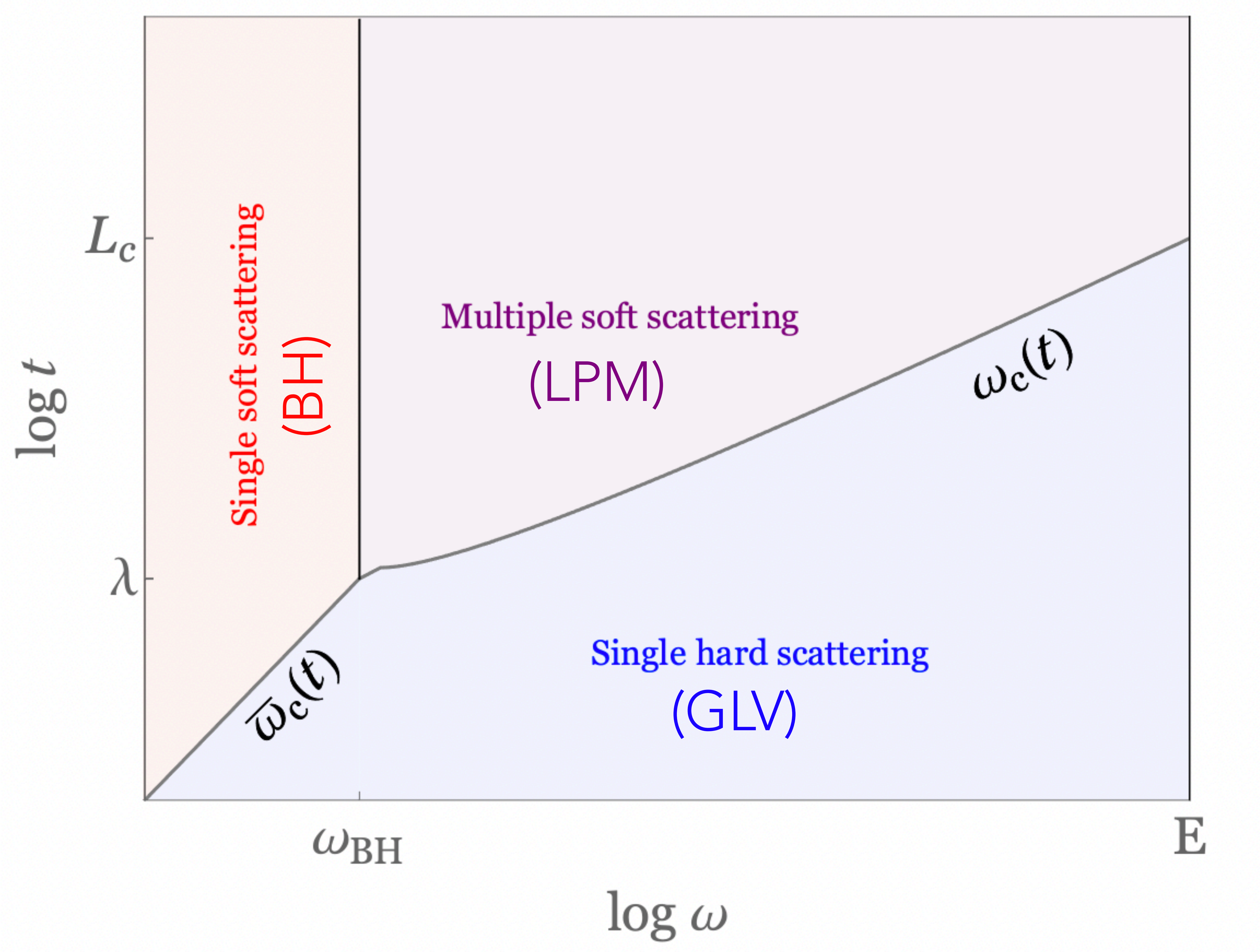
$$t_{\rm f} = \frac{\omega}{\langle k_t^2 \rangle} \sim \sqrt{\frac{\omega}{\hat{q}}}$$

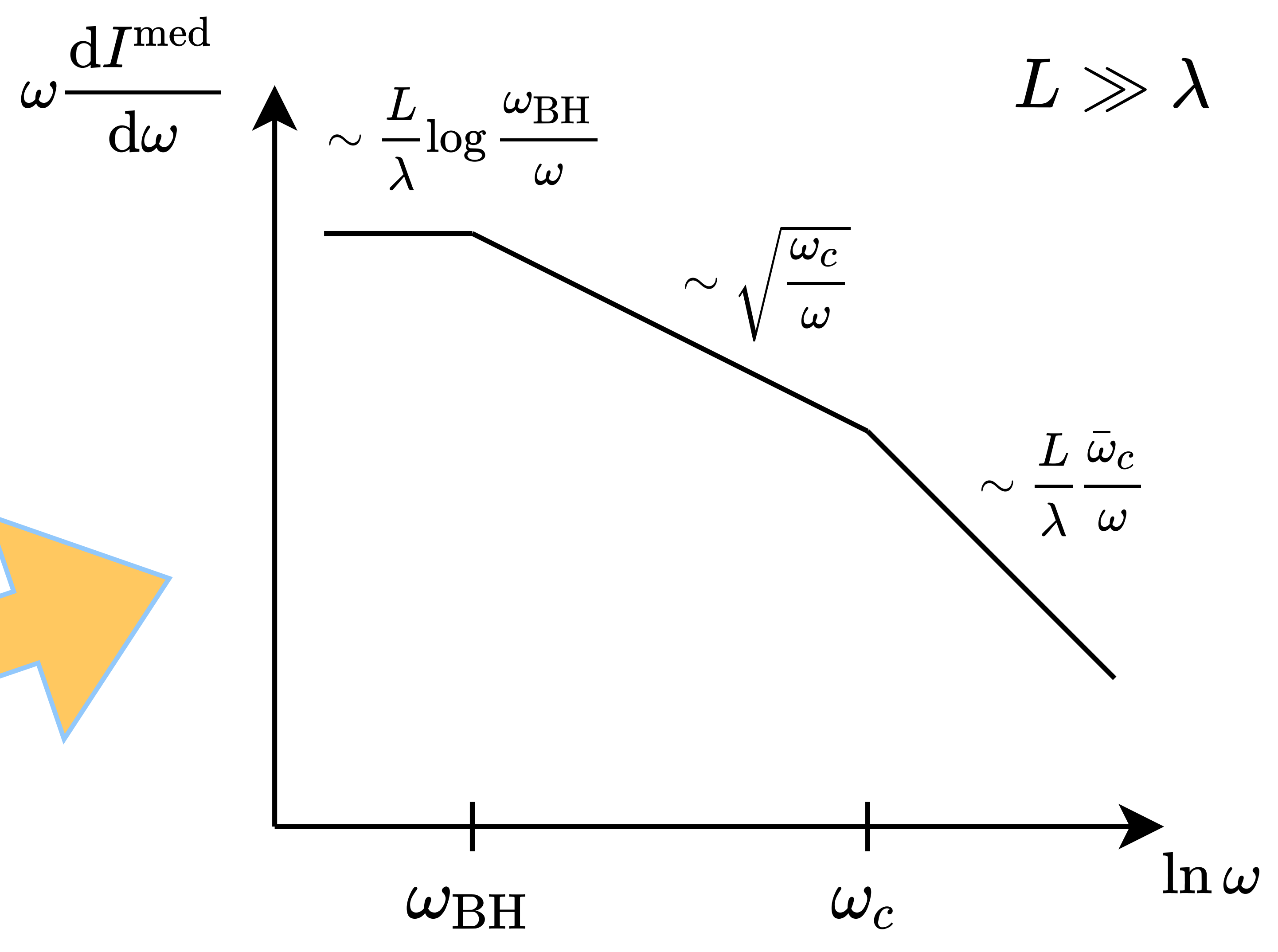
Full description of medium-induced radiation

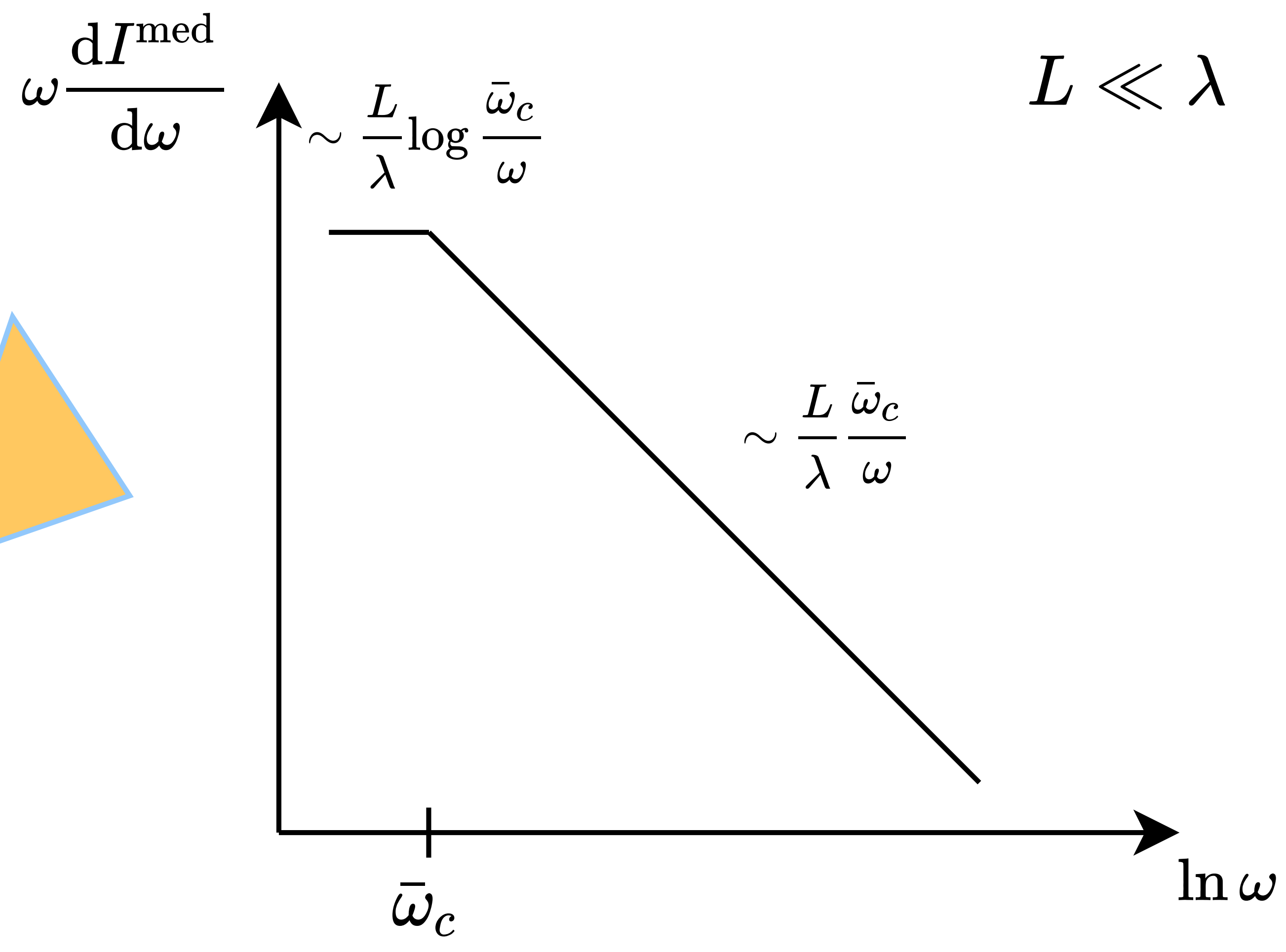


- picture breaks down in two limits:
  - when formation time becomes larger than the medium $t_{\rm f} \sim L$ (at $\omega > \hat{q}t^2/2$)
  - when formation time is of the order of mean free path $t_f \sim \lambda$ (at $\omega < \mu^2\lambda/2$)
- three expansion schemes cover the whole phase space
  - opacity expansion (in real+virtual momentum exchanges)
  - improved opacity expansion (around harmonic oscillator)
  - resummed opacity expansion (in real momentum exchanges)

Full description of medium-induced radiation







WHEN IS JET QUENCHING EFFECTIVE?

$$L \gg \lambda$$

- anticipating: the appearance of the LPM regime is extremely important for phenomenology
- leads to large multiplicity of emitted gluons &
- $1/\sqrt{\omega}$ spectrum gives rise to efficient transport of energy from leading particle to many soft particles
- all other regions lead to few ~$\mathcal{O}(\alpha_s)$ emissions

LPM REGIME: TWO LIMITS

$$t_f \sim \sqrt{\omega/\hat{q}} \quad \Rightarrow \quad k_{br}^2 \sim \sqrt{\omega \hat{q}} \quad \Rightarrow \quad \theta_{br} \sim (\hat{q}/\omega^3)^{1/4}$$

Multiplicity of emitted gluons

$$N(\omega) = \int_\omega^\infty \mathrm{d}\omega' \, \frac{\mathrm{d}I}{\mathrm{d}\omega'} = 2\sqrt{\frac{\bar{\alpha}^2 \hat{q} L^2}{\omega}}$$

Energy loss

BUT: average energy loss ≉ typical energy loss!

$$\Delta E = \int_0^\infty \mathrm{d}\omega' \, \omega' \frac{\mathrm{d}I}{\mathrm{d}\omega'} = 2\bar{\alpha}\hat{q}L^2$$

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rare, small-angle emission

$$\omega_c = \hat{q}L^2$$

$$\theta_\mathrm{br}(\omega_c) \sim \sqrt{\frac{1}{\hat{q}L^3}} \equiv \theta_c$$

LPM REGIME: TWO LIMITS

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$$N(\omega) = \int_\omega^\infty d\omega' \frac{dI}{d\omega'} = 2\sqrt{\frac{\bar{\alpha}^2\hat{q}L^2}{\omega}}$$

Energy loss

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BUT: average energy loss ≉ typical energy loss!

rare, small-angle emission

$$\omega_c = \hat{q}L^2$$

$$\theta_{br}(\omega_c) \sim \sqrt{\frac{1}{\hat{q}L^3}} \equiv \theta_c$$

copious, large-angle emissions

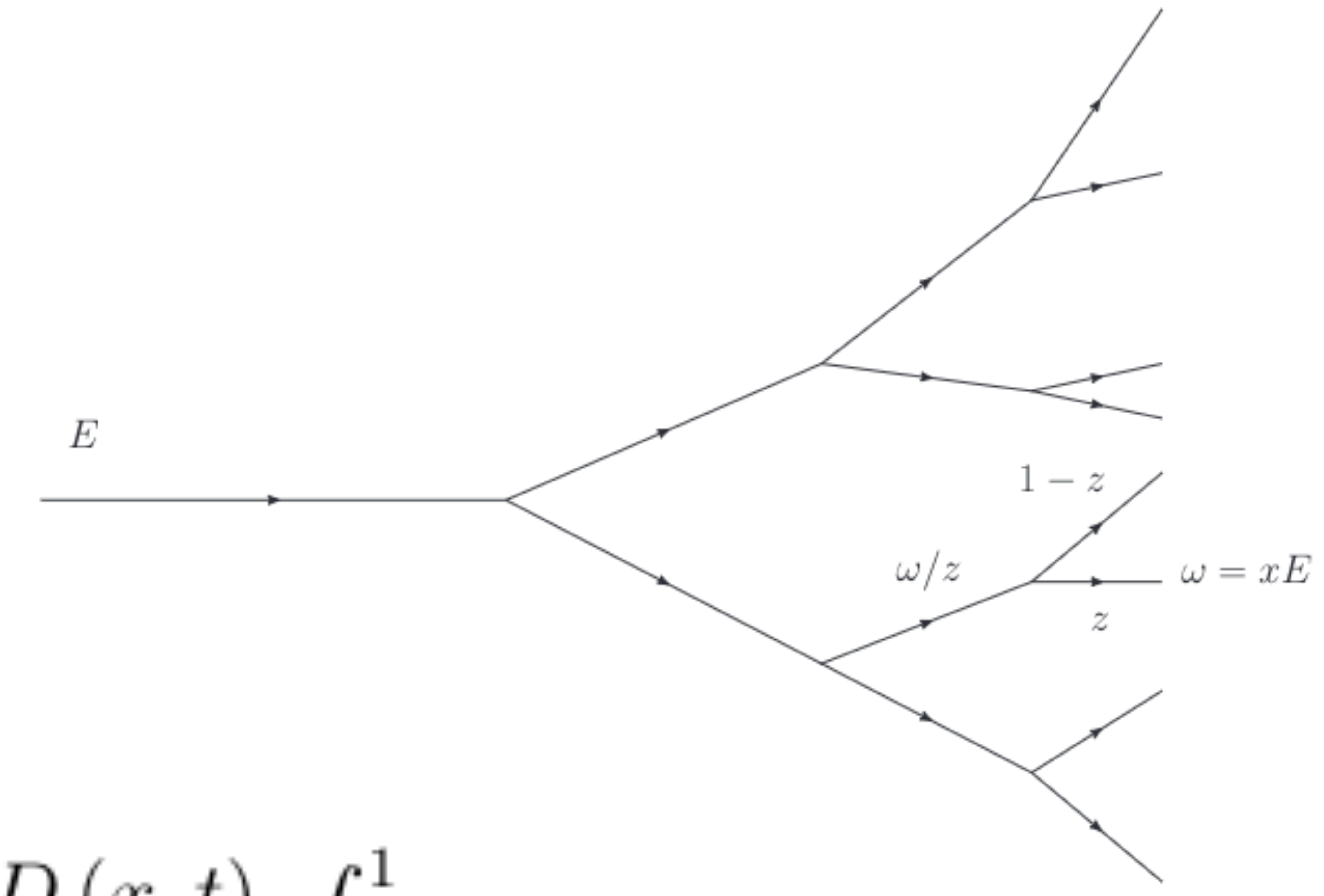
$$\omega_s = \bar{\alpha}^2\hat{q}L^2$$

$$\theta_{br}(\omega_s) \sim \frac{1}{\bar{\alpha}^{3/2}}\theta_c$$

EVOLUTION EQUATIONS

When multiplicity becomes large $N(\omega) \gg 1$, we need to take into account multiple splittings.

Can treat splittings as independent as long as the formation time is short $\omega \ll \omega_c$!



Evolution equation for energy distribution

$$\frac{\partial}{\partial t} D(x,t) = \int_x^1 \mathrm{d}z\, \mathcal{K}(z) \frac{D(x/z,t)}{t_*(x/z)} - \frac{D(x,t)}{t_*(x)} \int_0^1 \mathrm{d}z\, z\mathcal{K}(z)$$

evolution variable

splitting kernel

characteristic time-scale "stopping" time

In vacuum (DGLAP):

$$t = \ln \theta_0/\theta$$
$$t_*(x) = 1/\alpha_s$$

In medium:

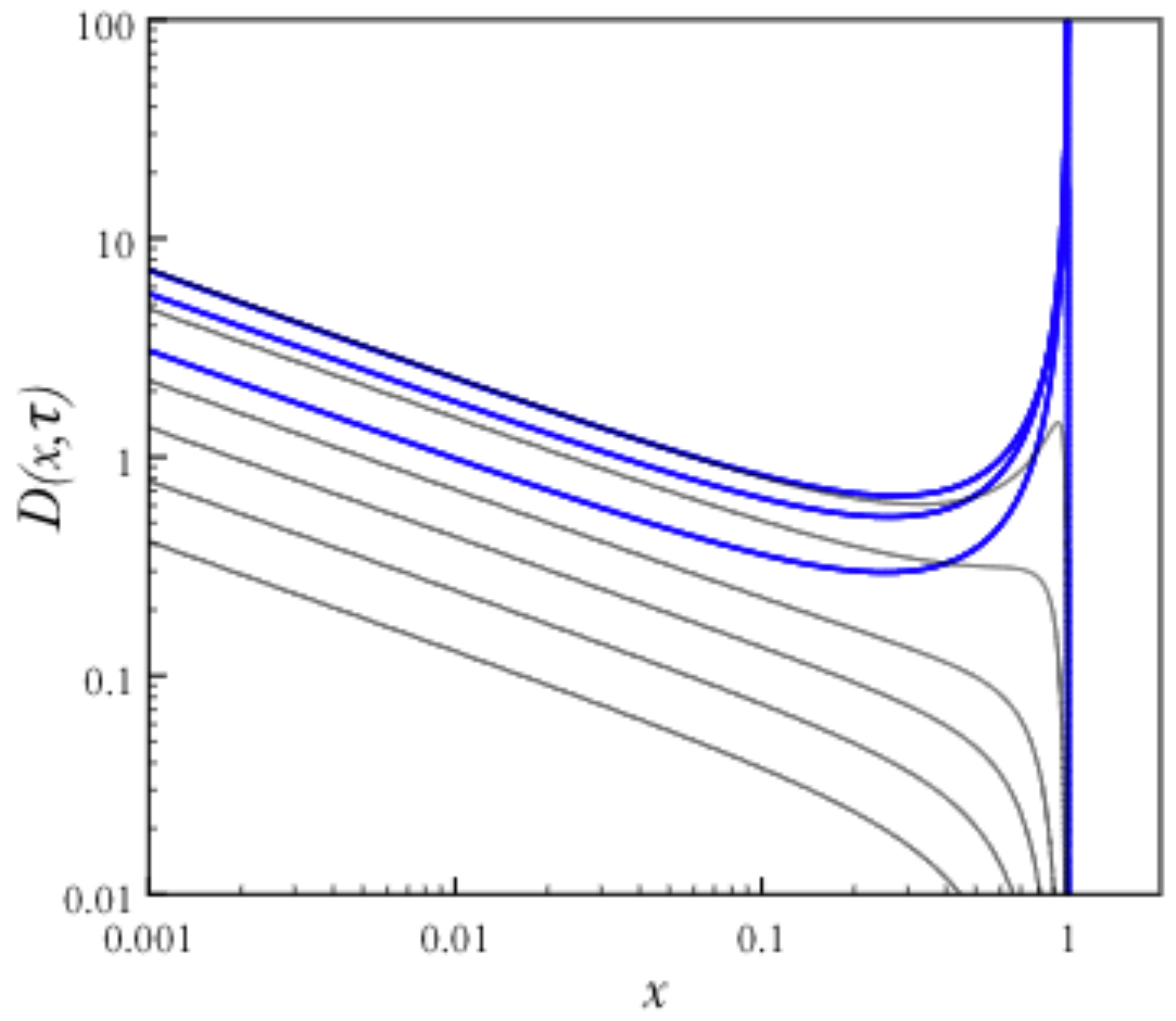
$$t = L$$
$$t_*(x) = \frac{1}{\alpha_s}\sqrt{\frac{xE}{\hat{q}}}$$

TURBULENCE IN JET QUENCHING

Blaizot, Mehtar-Tani 1501.03443

Medium evolution equation permits analytical solution:

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}} \mathrm{e}^{-\pi \frac{\tau^2}{1-x}} \qquad \tau = t/t_*$$

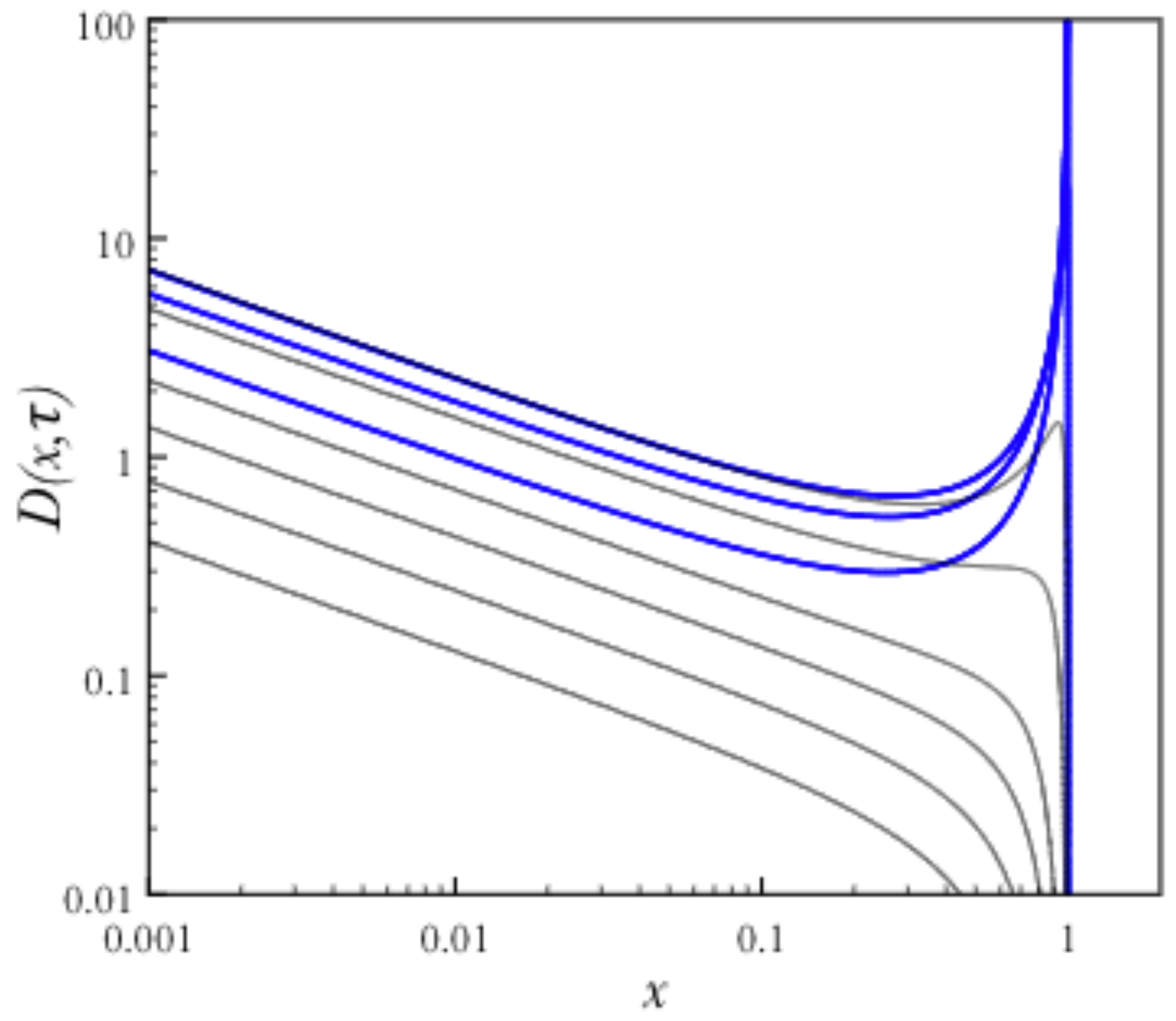


Turbulence in jet quenching

Blaizot, Mehtar-Tani 1501.03443

Medium evolution equation permits analytical solution:

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}} \mathrm{e}^{-\pi \frac{\tau^2}{1-x}} \qquad \tau = t/t_*$$



$$\mathcal{E}(x_0,\tau) = \int_{x_0}^{1} \mathrm{d}x\, D(x,\tau)$$

energy stored in particles with $x > x_0$

$$\mathcal{F}(x,\tau) = -\frac{\partial \mathcal{E}(x_0,\tau)}{\partial \tau}$$

flux of energy to modes at $x < x_0$

Turbulence in Jet Quenching

Blaizot, Mehtar-Tani 1501.03443

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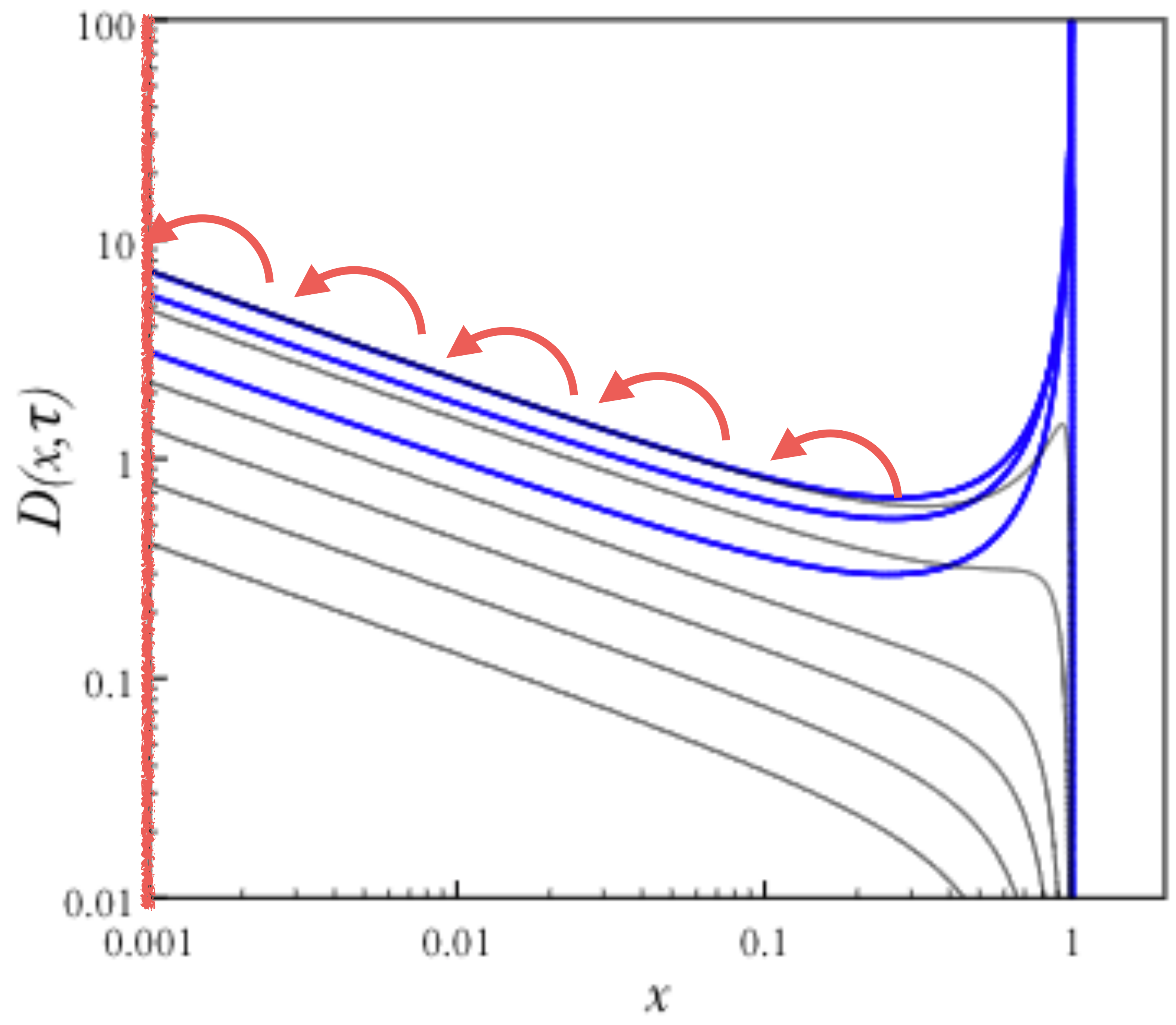


$$\mathcal{E}(x_0,\tau) = \int_{x_0}^{1} \mathrm{d}x\, D(x,\tau)$$ energy stored in particles with $x > x_0$

$$\mathcal{F}(x,\tau) = -\frac{\partial \mathcal{E}(x_0,\tau)}{\partial \tau}$$ flux of energy to modes at $x < x_0$

Energy flow all the way to zero energy:

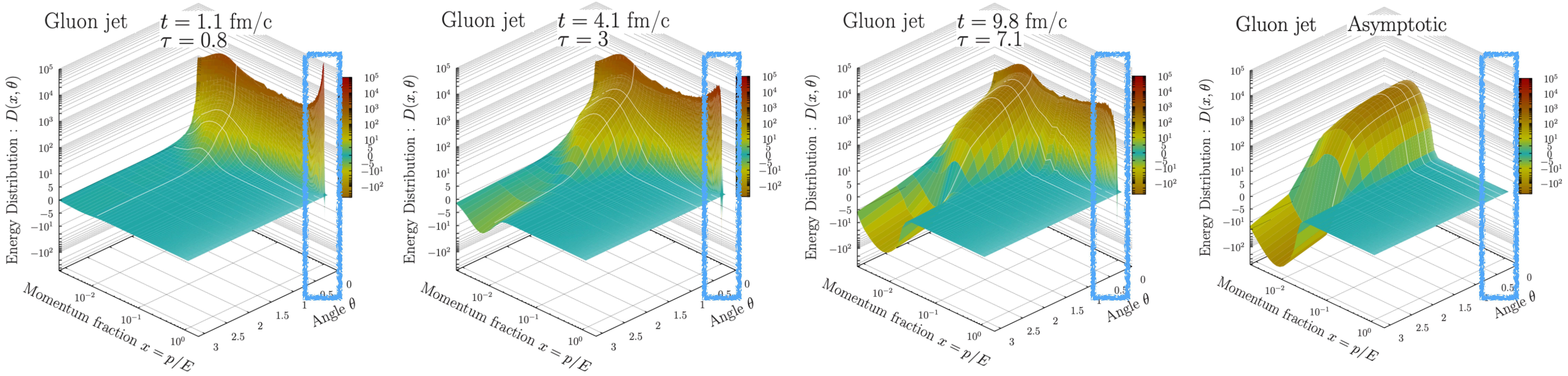
- "source" at $x = 1$ & "sink" at $x = 0$
- weak wave turbulence!

$$\mathcal{F}(0,\tau) = 2\pi\tau\, \mathrm{e}^{-\pi\tau^2}$$

$$\lim_{x_0 \to 0} \mathcal{E}(x_0,\tau) = \mathrm{e}^{-\pi\tau^2}$$

ENERGY DISTRIBUTION

Mehtar-Tani, Schlichting, Soudi 2209.10569



- evolution of leading particle: **energy loss**
- energy is located in soft peak & broadens to large angles
- jet thermalization leads to wake in the medium (medium response) - important for large-angle observables!

Summary
Lecture 2

- consistent framework to treat propagation & radiative processes of hard partons
- sensitivity to NP physics?
- LPM physics leads to rapid degradation of leading particle energy via turbulent cascade