Critical fluctuations in heavy-ion collisions

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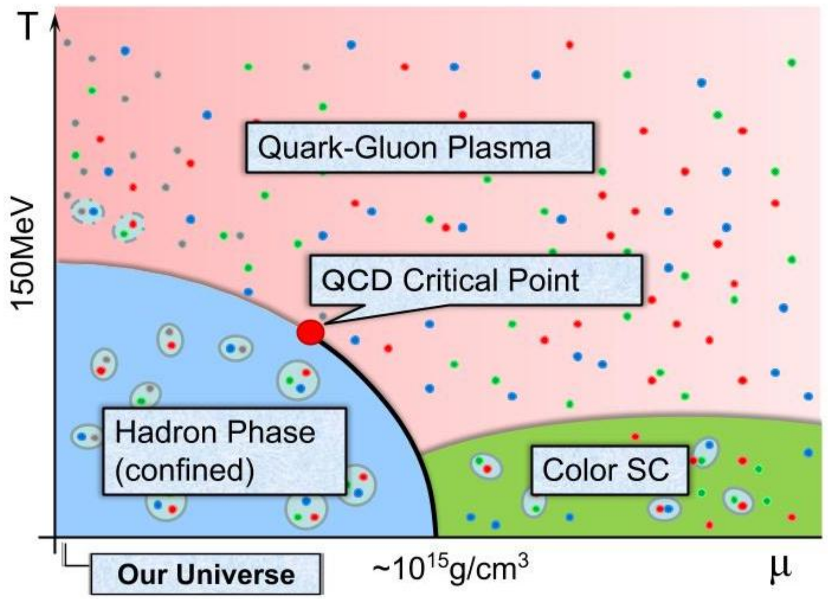
STRONG-HFHF-2023 Workshop

Extreme QCD in heavy-ion collisions

Understanding the dynamics of the strong interaction under extreme conditions of temperature and density!

Important questions:

- Onset of deconfinement and chiral symmetry restoration?
- Properties of the strongly coupled QGP?
- Existence of a phase transition with critical end point?
- What are the dof in the core of compact stars?



Connect first-principle QCD calculations with experimental observables via a realistic dynamical modeling of heavy-ion collisions and astrophysical events**!**

How to observe the critical point in HIC

- At a critical point, the correlation length ξ diverges and so do the fluctuations.

- Observable in higher-order cumulants of net-baryon number.

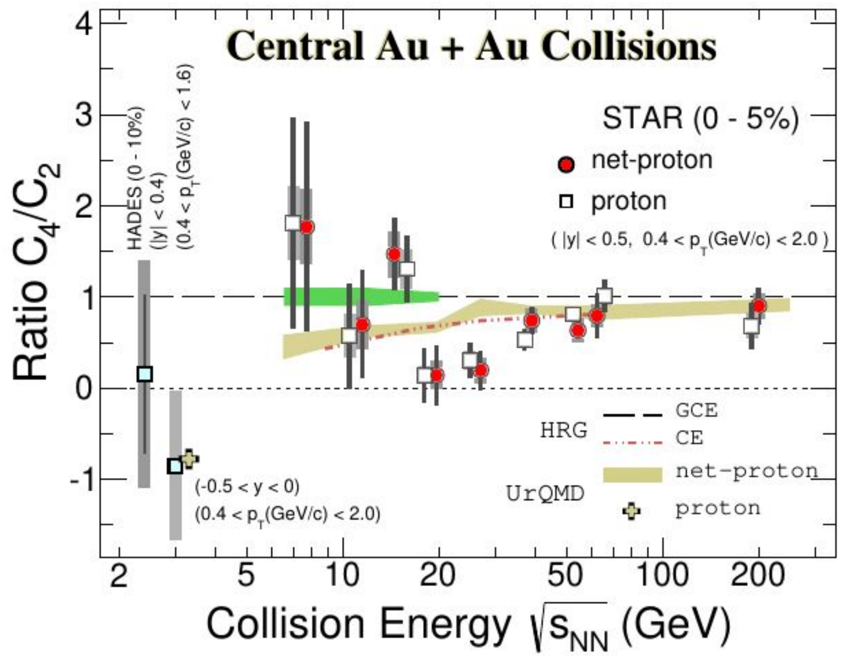
$$\chi_1 = \frac{1}{VT^3}\langle N \rangle\,, \quad \chi_2 = \frac{1}{VT^3}\langle (\Delta N)^2 \rangle\,, \quad \chi_3 = \frac{1}{VT^3}\langle (\Delta N)^3 \rangle\,,$$
$$\chi_4 = \frac{1}{VT^3}\langle (\Delta N)^4 \rangle_c \equiv \frac{1}{VT^3}\left(\langle (\Delta N)^4 \rangle - 3\langle (\Delta N)^2 \rangle^2\right)\,.$$

- To 0th order in V fluctuations:

$$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = S\sigma \qquad \frac{\chi_4}{\chi_2} = \kappa\sigma^2$$

variance | Skewness | Kurtosis

- At a CP the τ_{relax} diverges with ξ which leads to critical slowing down



STAR, PRL 128 (2022)

Interesting deviations from the baseline in the experimental data... are they due to the critical point of QCD?

Need a dynamical model!

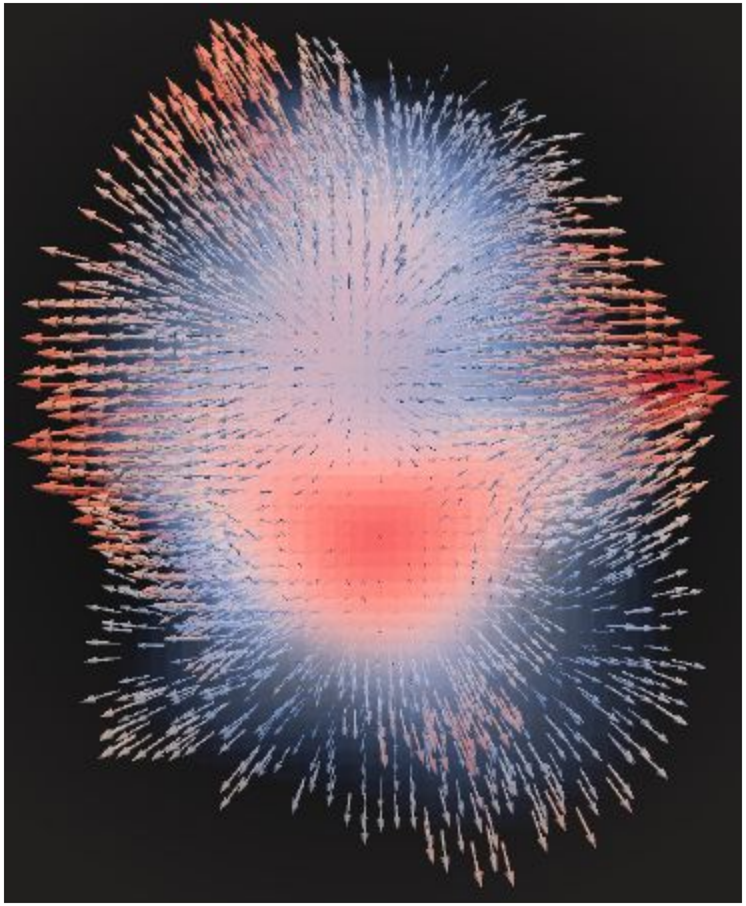
Importance of dynamical modeling

In a grand-canonical ensemble the system is...

- in thermal equilibrium (= long-lived)
- in equilibrium with a particle heat bath
- spatially infinite
- and static

Systems created in a heavy-ion collision are

- short-lived
- spatially small
- inhomogeneous
- and highly dynamical!

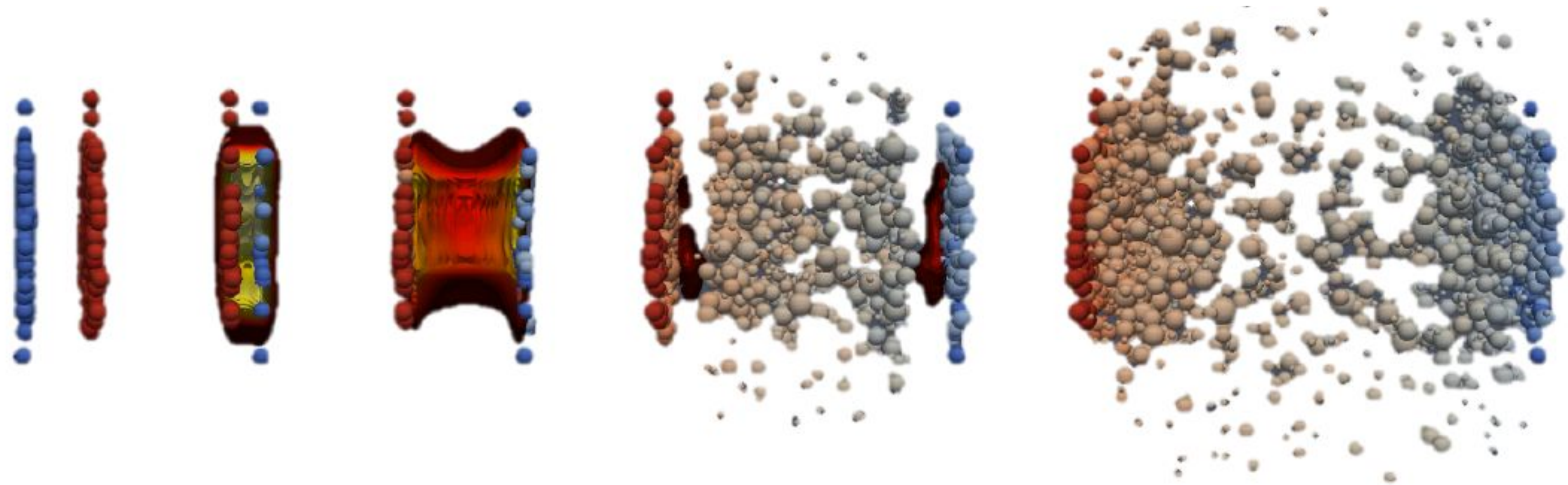


madai.us

Solution: Develop dynamical models to describe the phase transition in heavy-ion collisions
Event-by-event dynamical modeling allows us in addition to study different particle species, experimental cuts, hadronic final interactions, etc.

EMMI Rapid Reaction Task Force: M. Bluhm, A. Kalweit, MN et al. NPA 1003 (2020), 2001.08831

Fluctuations all along the way



- Initial state fluctuations due to quantum mechanical fluctuations and multiplicity fluctuations
- Thermal fluctuations, including the formation of critical fluctuations
- Fluctuations due to the hadronization process
- Fate of fluctuations in the hadronic phase
- Imperfect detection efficiency and finite acceptance

Approaches to fluid dynamical fluctuations

There are two main approaches of describing fluid dynamics with noise:

Hydro-kinetics

- Set of deterministic kinetic equations for n-point functions of fluid dynamical fields
- Renormalization (perturbatively) performed during the derivation
- Statistical average performed in the derivation of deterministic equations

A.Andreev, Sov. Phys. JETP 32 no. 5 (1971) and 48 no. 3 (1978); Y. Akamatsu et al., PRC 95 no. 1 (2017) and 97 no. 2 (2018); M. Martinez et al., PRC 99 no. 5 (2019); X. An et al., PRC 100 no. 2 (2019), PRL 127 (2021); L. Du et al., PRC 102 (2020); K. Rajagopal, NPA 1005 (2021)

Stochastic/fluctuating fluid dynamics

- Numerical implementation of the fluid dynamical equations with stochastic conservation law:

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + T^{\mu\nu}_{\text{viscous}} + S^{\mu\nu}_{\text{noise}}\,,$$
$$\partial_\mu J^{\mu} = 0, \quad J^{\mu} = J^{\mu}_{\text{ideal}} + J^{\mu}_{\text{viscous}} + I^{\mu}_{\text{noise}}\,.$$

- Sample discretized noise event-by-event
- Observables are calculated from statistical averaging over events.
- Can easily be integrated in standard event generators of HIC!
- Many challenges....

Fluctuating Dissipative Fluid Dynamics

The correlators of the thermal noise terms in the energy momentum tensor and the conserved currents :

$$\langle S^{\mu\nu}(x_1)S^{\alpha\beta}(x_2)\rangle = 2T\begin{bmatrix}\eta\left(\Delta^{\mu\alpha}\Delta^{\nu\beta}+\Delta^{\mu\beta}\Delta^{\nu\alpha}\right)\\ +\left(\zeta-\frac{2}{3}\eta\right)\Delta^{\mu\nu}\Delta^{\alpha\beta}\end{bmatrix}\delta^{(4)}(x_1-x_2).$$

$$\langle I^{\mu}(x_1)I^{\nu}(x_2)\rangle = 2T\sigma\Delta^{\mu\nu}\delta^{(4)}(x_1-x_2)\,.$$

Several issues arise from the discretization of the Dirac delta function in the noise

- Stochastic noise introduces a lattice spacing dependence.
- Correction terms due to renormalization become large for small lattice spacings.
- Large noise contributions can locally lead to negative energy densities.
- Large gradients introduced by the uncorrelated noise is a problem for PDE solvers.

EMMI Rapid Reaction Task Force: M. Bluhm, A. Kalweit, MN et al. NPA 1003 (2020), 2001.08831

Fluctuating Dissipative Fluid Dynamics

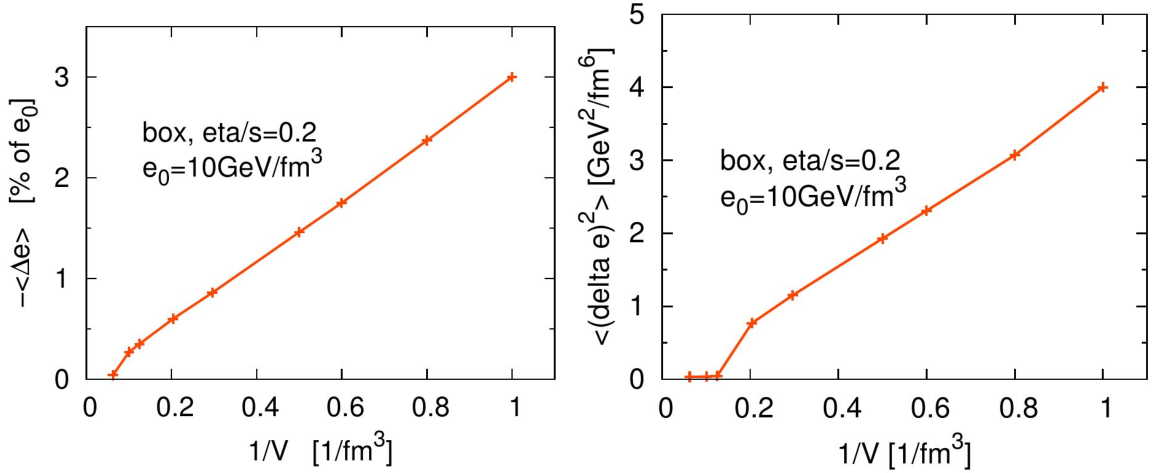
First implementations of FDFD have shown: need to limit the resolution scale; simulate noise down to a particular filter length scale, for which:

$$l_{\rm grid} < l_{\rm filter} \lesssim l_{\rm noise} \ll l_{\rm hydro}$$

Murase et al.: noise is smeared by Gaussians with widths of 1-1.5 fm (choice not discussed), large enhancement of flow observed. K. Murase et al, NPA 956 (2016);

Nahrgang et al.: noise is coarse-grained over distances of approx. 1fm, lattice spacing dependence of the energy density and its fluctuations observed.

M. Nahrgang et al, Acta Phys. Polon. 10 (2017);



Singh et al.: high-mode Fourier filter with a coarse graining scale of > 1fm, multiplicities and flow are little affected by the inclusion of fluctuations

M. Singh se et al, NPA 982 (2019);

Diffusive dynamics of net-baryon density

- In the long-time, equilibrium limit the net-baryon density is the slowest mode near the CP.
- For baryonic matter that decouples from the energy flow of the system (model B of Hohenberg, Halperin), the diffusive dynamics follows the minimization of the free energy F

$$\partial_t n_B(t,x) = \kappa \nabla^2 \left(\frac{\delta \mathcal{F}[n_B]}{\delta n_B} \right) + \nabla \mathbf{J}(\mathbf{t},\mathbf{x})$$

with the stochastic current (Gaussian, white noise)

$$J(t,x) = \sqrt{2T\kappa}\,\zeta(t,x), \quad \kappa = \frac{Dn_c}{T}$$

MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

First: static box with fixed temperature

Critical energy density from 3d Ising Model

Ginzburg-Landau

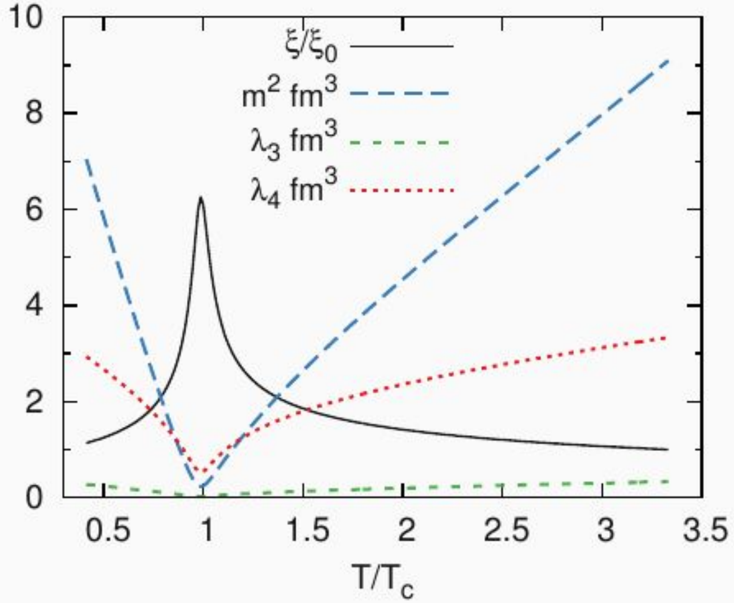
$$\mathcal{F}[n_B] = T\int d^3r \left(\frac{m^2}{2n_c^2}\Delta n_B^2 + \frac{K}{2n_c^2}(\nabla\Delta n_B)^2 + \frac{\lambda_3}{3n_c^3}\Delta n_B^3 + \frac{\lambda_4}{4n_c^4}\Delta n_B^4 + \frac{\lambda_6}{6n_c^6}\Delta n_B^6 \right)$$

Gauss + surface

The couplings depend on temperature via the correlation length $\xi(T)$:

$$m^2 = 1/(\xi_0\xi^2)$$
$$K = \tilde{K}/\xi_0$$
$$\lambda_3 = n_c\,\tilde{\lambda}_3\,(\xi/\xi_0)^{-3/2}$$
$$\lambda_4 = n_c\,\tilde{\lambda}_4\,(\xi/\xi_0)^{-1}$$
$$\lambda_6 = n_c\,\tilde{\lambda}_6$$

M. Tsypin PRL 73 (1994); PRB 55 (1997)



parameter choice: $\Delta n_B = n_B - n_c$

$\xi_0 \sim 0.5$ fm, $T_c = 0.15$ GeV, $n_c = 1/3$ fm^{-3}

$K = 1/\xi_0$ (surface tension)

$\tilde{\lambda}_3, \tilde{\lambda}_4, \tilde{\lambda}_6$ (universal, but mapping to QCD)

in this Fig.: $\tilde{\lambda}_3 = 1$, $\tilde{\lambda}_4 = 10$

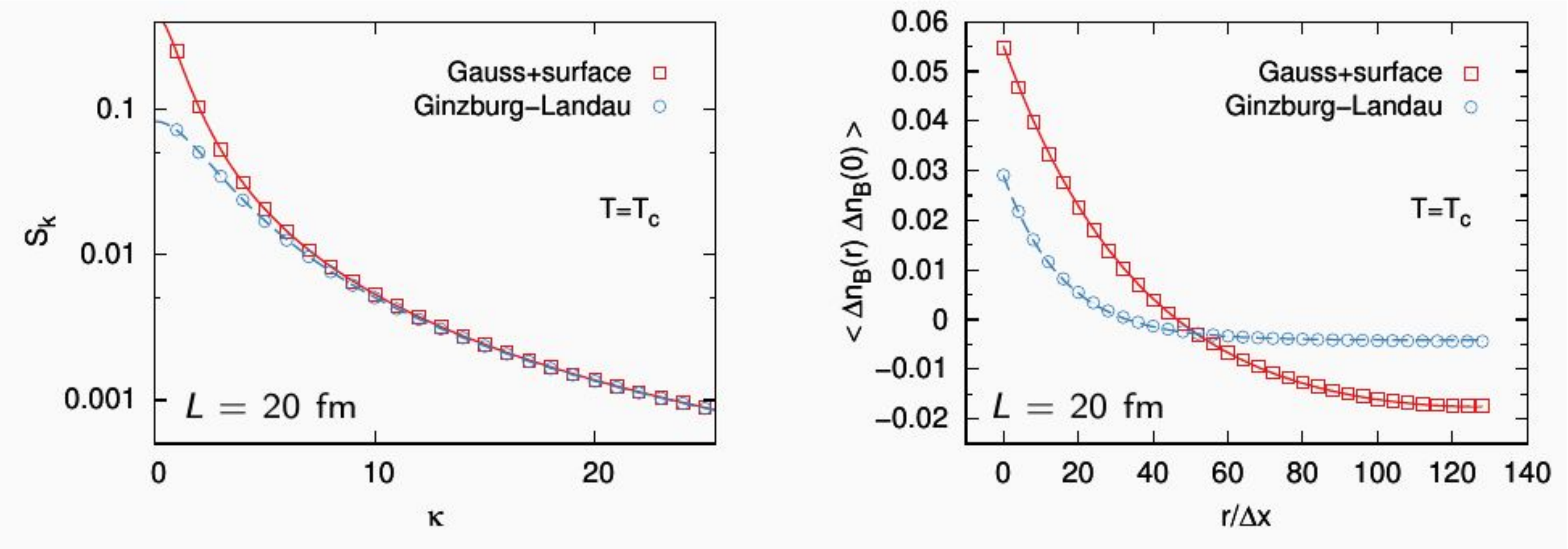
MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

Studied in a static and cooling box of QGP

Validated in the equilibrium limit for the Gauss + surface model:

- Structure factor and equal-time correlation function are well reproduced
- Approaches continuum as resolution is increased
- Baryon conservation effects under control

Important step for all fluctuating codes!



MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

Scaling of equilibrium cumulants

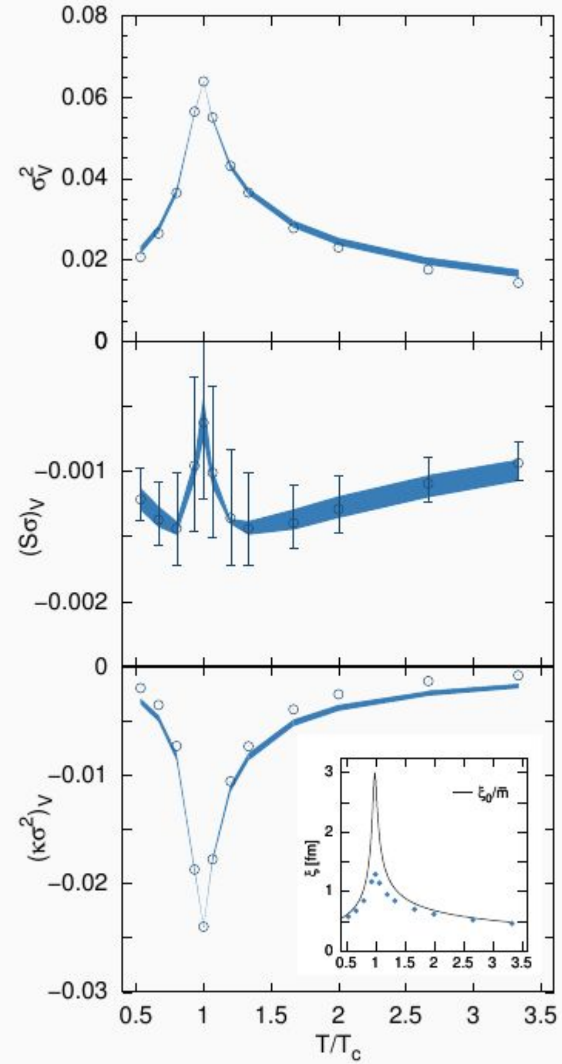
- Expected scaling in an infinite system ($\xi \ll V$): M. Stephanov PRL 102 (2009)

  $\sigma_V^2 \propto \xi^2, \quad (S\sigma)_V \propto \xi^{2.5}, \quad (\kappa\sigma^2)_V \propto \xi^5$

- Here, a finite system with exact baryon conservation ($\xi \lesssim V$)! Can be systematically studied in $\xi/V \Rightarrow$ affects equilibrium scaling!

- E.g. for the skewness terms $\propto \lambda_3\lambda_4$ and $\propto \lambda_3\lambda_6$ contribute with opposite sign.

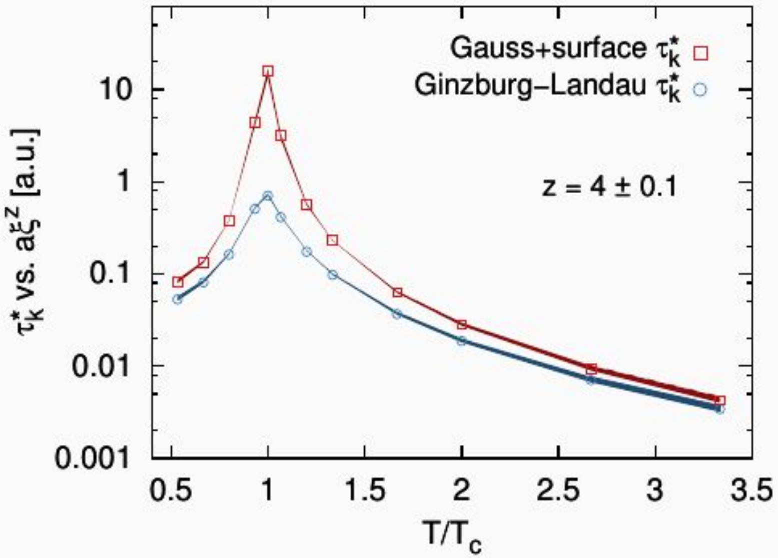
$$\sigma_V^2 \propto \xi^{1.3\pm0.05}$$
$$(S\sigma)_V \propto -\#\xi^{1.47\pm0.05} + \#\xi^{2.4\pm0.05}$$
$$(\kappa\sigma^2)_V \propto \xi^{2.5\pm0.1}$$



MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

Dynamical critical scaling

- Dynamical structure factor for Gaussian model in continuum:
  $S(k,t) = S(k)\exp(-t/\tau_k)$ with $\tau_k^{-1} = \frac{Dm^2}{n_c}\left(1 + \frac{K}{m^2}k^2\right)k^2$
- Analyze ξ-dependence of relaxation time for modes with $k^* = 1/\xi$:



for both models: $\tau_k^* = a\,\xi^z$ with

$$z = 4 \pm 0.1$$

$$a = \frac{n_c \xi_0}{D(1+\tilde{K})}$$

$\Rightarrow$ Simulations reproduce scaling of model B!

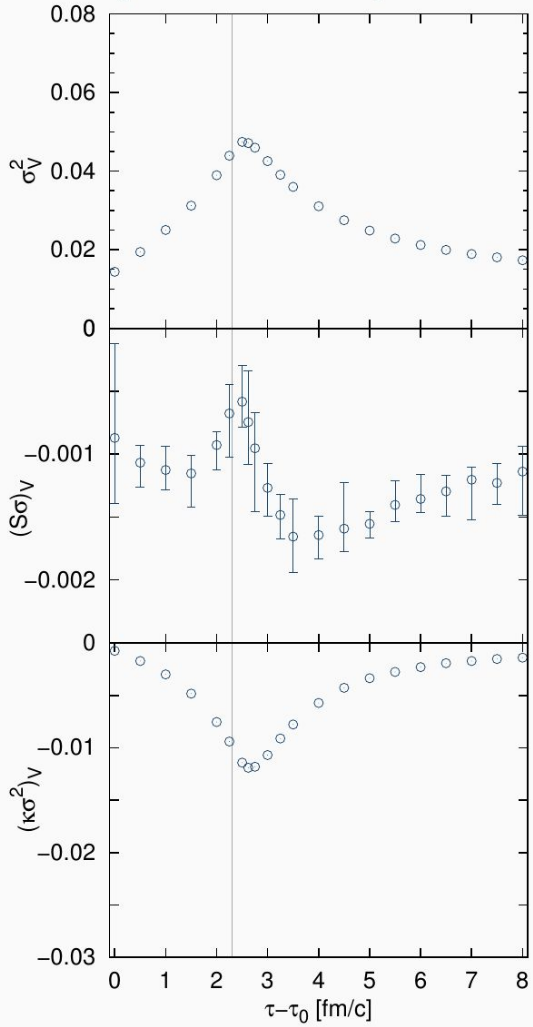
For the full dynamics of a HIC, couple to fluctuations in $T^{\mu\nu}$ → model H

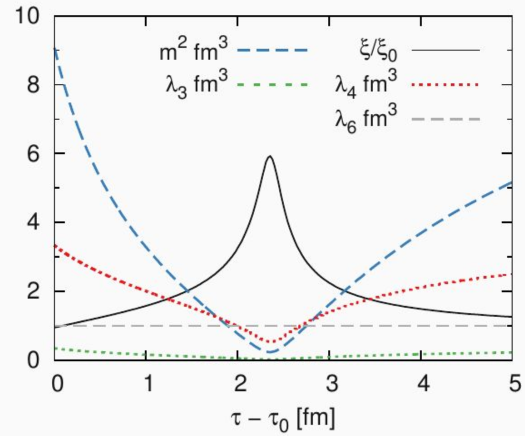
According to Hohenberg, Halperin, Rev. Mod. Phys. 49 (1977)

MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

Time evolution of critical fluctuations

For a Bjorken-like temperature evolution:





- shift of extrema for variance/kurtosis (retardation effects) to later times corresponding to $T(\tau) < T_c$
- |extremal values| in dyn simulations < equilibrium values (nonequilibrium effects):

$$(\sigma_V^2)^{\mathrm{max}}_{\mathrm{dyn}} \approx 0.75\,(\sigma_V^2)^{\mathrm{max}}_{\mathrm{eq}}$$
$$((\kappa\sigma^2)_V)^{\mathrm{min}}_{\mathrm{dyn}} \approx 0.5\,(\kappa\sigma_V^2)^{\mathrm{min}}_{\mathrm{eq}}$$

- expected behavior with varying D and c_s^2 (expansion rate)

MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

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$$J(t,x) = \sqrt{2T\kappa}\,\zeta(t,x), \quad \kappa = \frac{Dn_c}{T}$$

MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

- Apply the evolution in a 1+1 dimensional, boost-invariant Bjorken expansion

Fluctuations in an expanding background, e.g. J. Kapusta et al, PRC 85 (2012); Y. Akamatsu et al. PRC 95 (2017), M. Martinez et al, PRC 99 (2019)

- The nonlinear stochastic diffusion equation transforms as:

$$\partial_\tau n_B = \frac{Dn_c}{\tau \chi_2(\tau)} \partial_y^2 n_B - \frac{Dn_c K(\tau)}{\tau} \partial_y^4 n_B + \frac{Dn_c}{6\,\tau \chi_4(\tau)} \partial_y^2 {n_B}^3 - \partial_y \xi \,.$$

In Gauss limit: M. Sakaida et al PRC 95 (2017); nonlinear (only critical): M. Kitazawa, G.Pihan, N. Touroux, M. Bluhm, MN NPA 1005 (2021)

Singular and regular susceptibilities

- Parametrize the susceptibilities $\chi_2(\tau)$ and $\chi_4(\tau)$ with a regular part using the argument in

M. Asakawa, U. Heinz, B. Müller, PRL 85 (2000)

$$\chi_n(\tau) = \frac{\langle \Delta N_B^n \rangle}{S}\bigg|_{\mathrm{QGP/HRG}} = \frac{\chi_{\mathrm{B}}^n}{s/T^3}\bigg|_{\mathrm{QGP/HRG}}$$

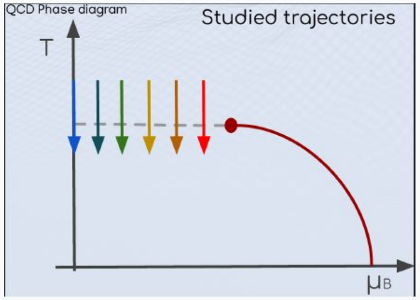
With χ_B^n and the entropy fixed to lattice results at T=280 MeV for the QGP and T=130 MeV for the HRG, matched via tanh function.

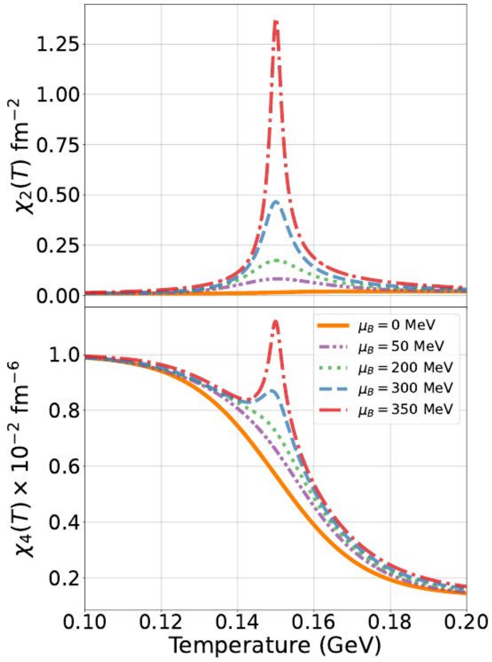
- Couple with the singular contribution (3D Ising) via

$$\chi_n(T) = \chi_n^{sing}(T) + \chi_n^{reg}(T)$$

- Match to the coefficients in the expansion of the free energy density functional.

$$\chi_n(\tau) = \tau \left(\frac{\delta^n \mathcal{F}}{\delta n_B^n}\bigg|_{\Delta n_B = 0} \right)^{-1}$$





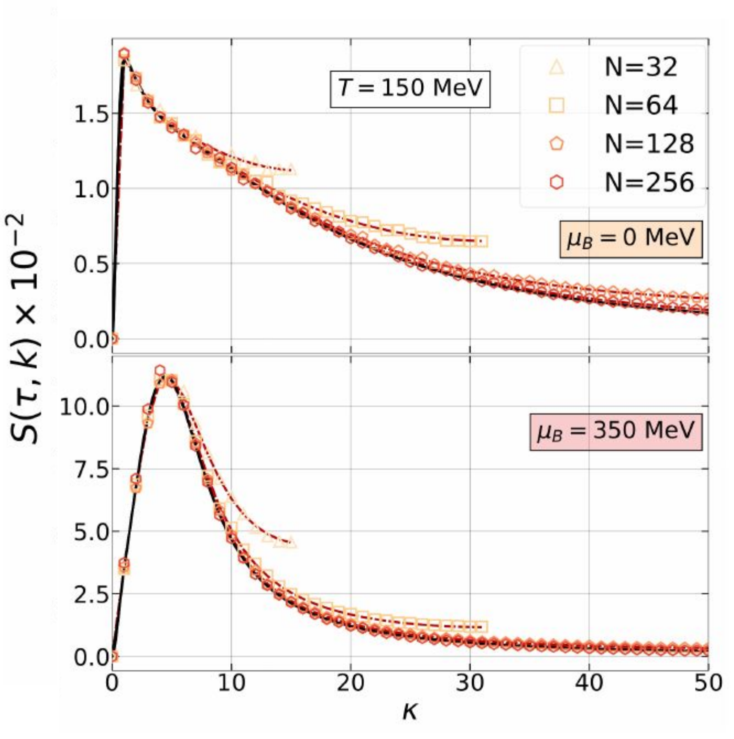
- Investigate several trajectories in the QCD phase diagram.

G. Pihan, M. Bluhm, M. Kitazawa, T. Sami, MN, PRC 107 (2023)

Validating the linear model

only here
$\lambda_4 = 0$!

Structure factor Sk



G. Pihan, M. Bluhm, M. Kitazawa, T. Sami, MN, PRC 107 (2023)

Important step for all fluctuating codes: validation of the appropriate linear model:

- Structure factor and equal-time correlation function are well reproduced
- Approach to continuum as resolution is increased.
- Lower wavenumbers well described with the maximal resolution chosen for this work.
- Enhancement of fluctuations with low wavenumbers at T_c = 150 MeV.
- Discretization and baryon conservation effects under control.

Anticorrelations as a signal for the critical point

$$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = S\sigma \qquad \frac{\chi_4}{\chi_2} = \kappa\sigma^2$$

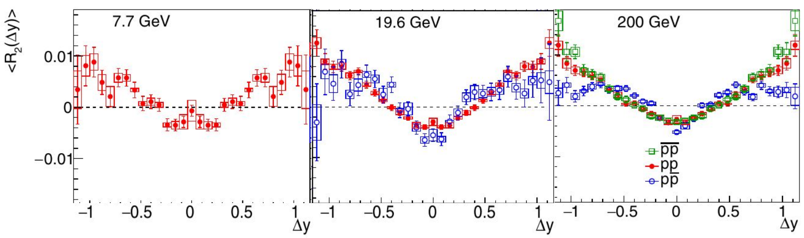
variance Skewness Kurtosis

Higher-order moments are more sensitive

- to the divergence of the correlation length
- and to any other noncritical aspect of HIC...

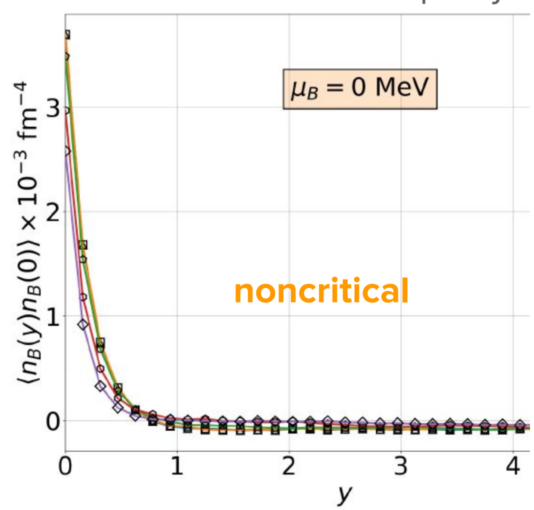
Here: dynamical fluctuations of net-baryon density

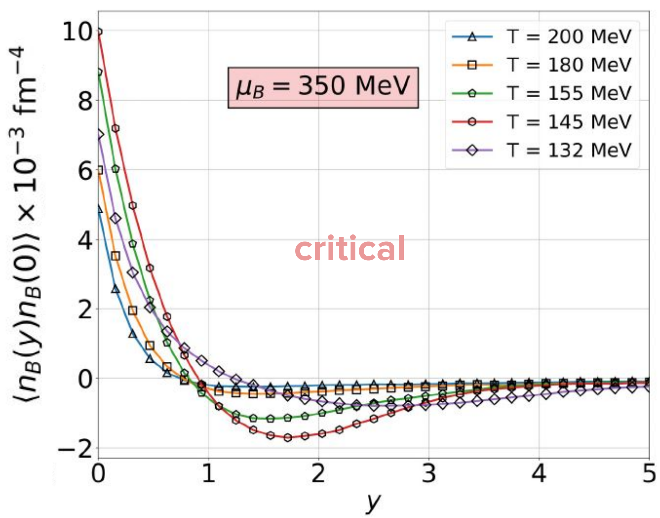
- Large fluctuations are balanced by large anti-correlations (net-baryon conservation)
- Due to the dynamics these anti-correlations cannot diffuse fast enough
- Approaching Tc they are visible at y ~ 1-2
- At lower T the minimum becomes smaller and moves to larger y
- → Possible detection depends crucially on T_{FO}
- Interesting experimental data:
  (STAR AuAu, 30-40% most central, 0.4<pT<2GeV)



STAR Collab, PRC 101 (2021)

correlation function in rapidity



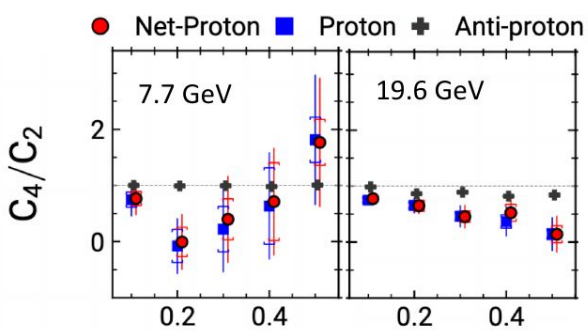


G. Pihan, M. Bluhm, M. Kitazawa, T. Sami, MN, PRC 107 (2023)

Word of caution: not yet an apple-to-to apple comparison possible

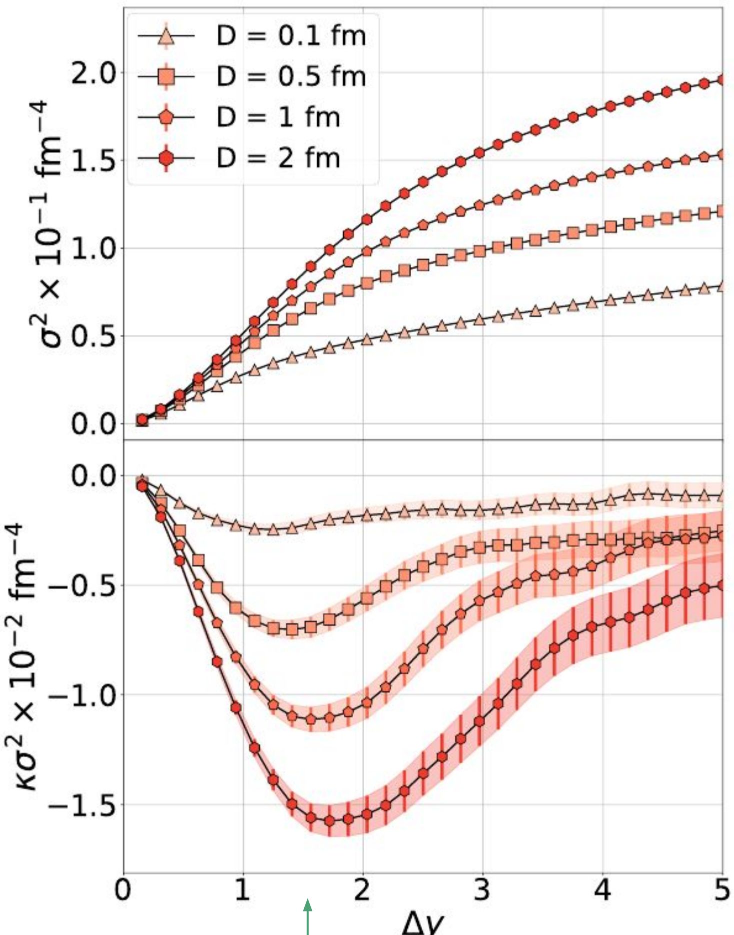
Non-monotonic kurtosis as a signal for the critical point

- Monotonic increase in the variance
- Non-monotonic Kurtosis only for the trajectories with critical point.
- This non-monotonic behavior of the kurtosis survives the rapid expansion for a diffusion length D = 1 fm
  → strong indication for the presence of the critical point.
- For increasing D the minimum moves to larger distances in rapidity.
  → Essential for the experiment to cover a wide range in rapidity to see the non-monotonicity.
- Interesting experimental data:
  (STAR AuAu, 0-5% most central, 0.4<pT<2GeV, |y|<ymax)



STAR Collab, PRC 104 (2021)

rapidity dependence of fluctuation observables



G. Pihan, M. Bluhm, M. Kitazawa, T. Sami, MN, PRC 107 (2023)

STAR iTPC

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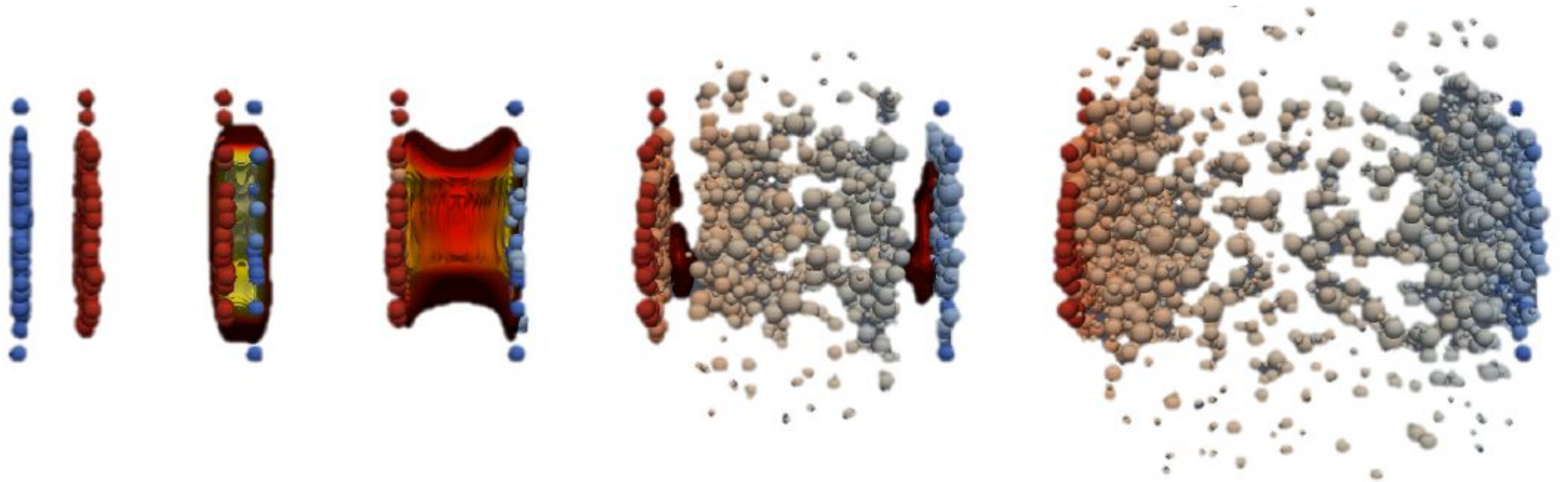
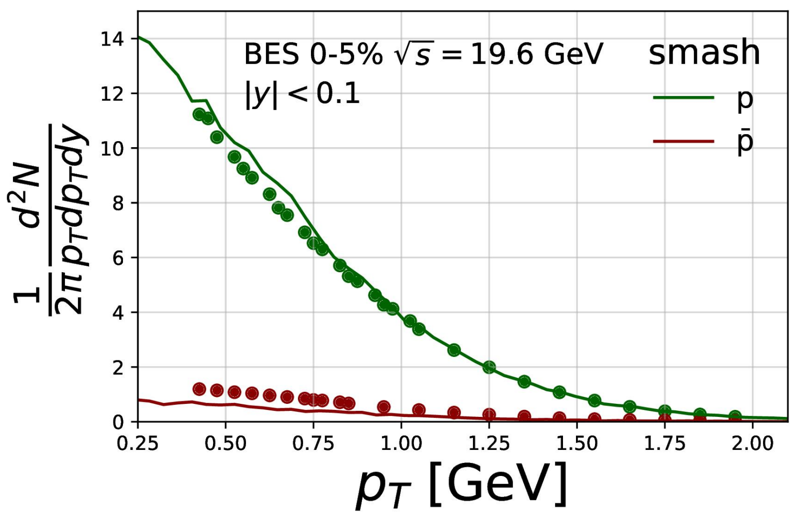
Fluctuations all along the way

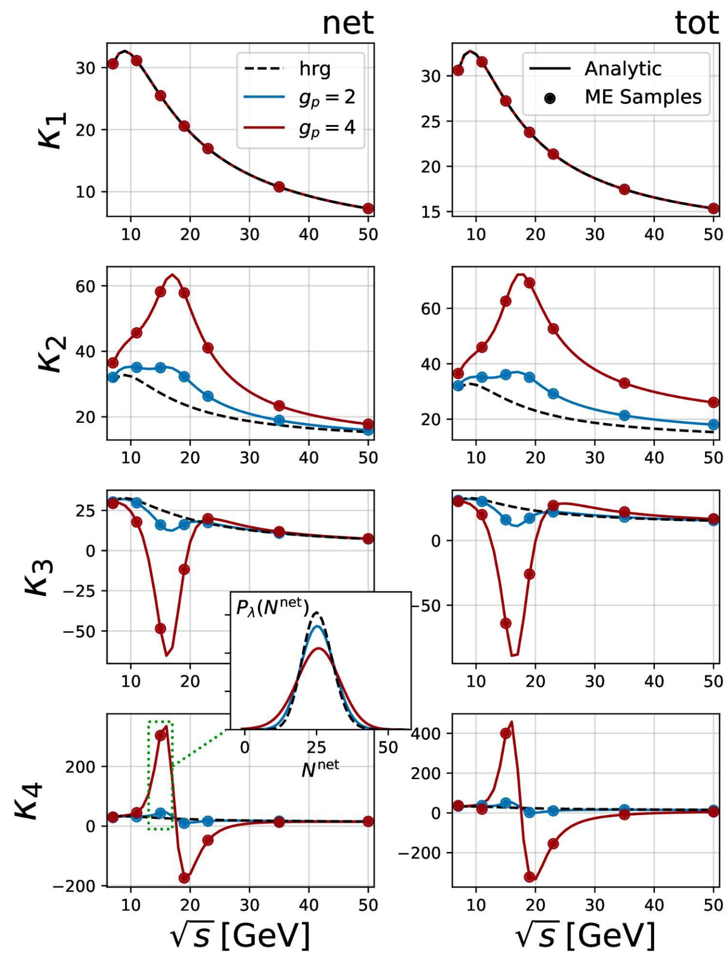


- Initial state fluctuations due to quantum mechanical fluctuations and multiplicity fluctuations
- Thermal fluctuations, including the formation of critical fluctuations
- Fluctuations due to the hadronization process
- Fate of fluctuations in the hadronic phase
- Imperfect detection efficiency and finite acceptance

Fate of critical fluctuations in the hadronic phase

- Calculate up to 4th order cumulants of critical fluctuations from an 3d Ising model mapping to QCD and couple it to HRG cumulants (g_c=2,4).
- Reconstruct the particle distributions from the cumulants + maximum entropy constraint
- Assume simple geometry at particlization: uniform spatial distribution in a sphere (R=9fm)
  Momentum distribution $f_{i,k} = e^{-u \cdot k_i / T}$
  velocity $\vec{u}(r) = \vec{e}_r u_0 r/R$ with u_0=0.5



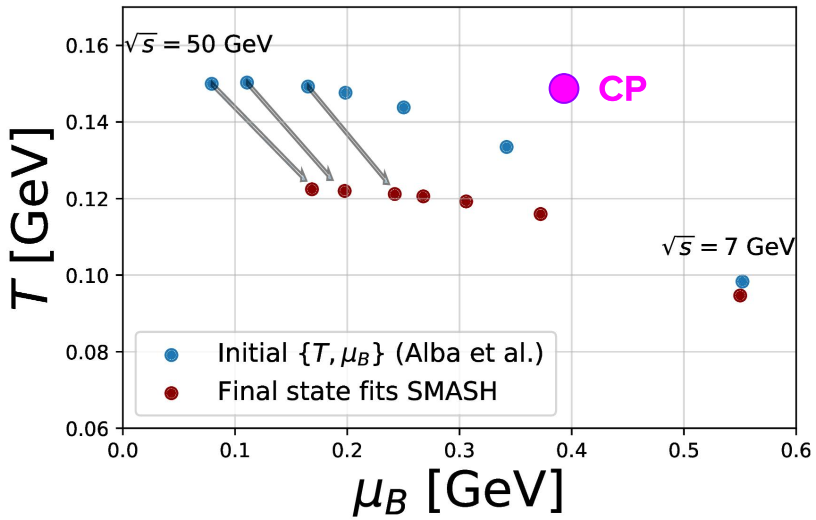


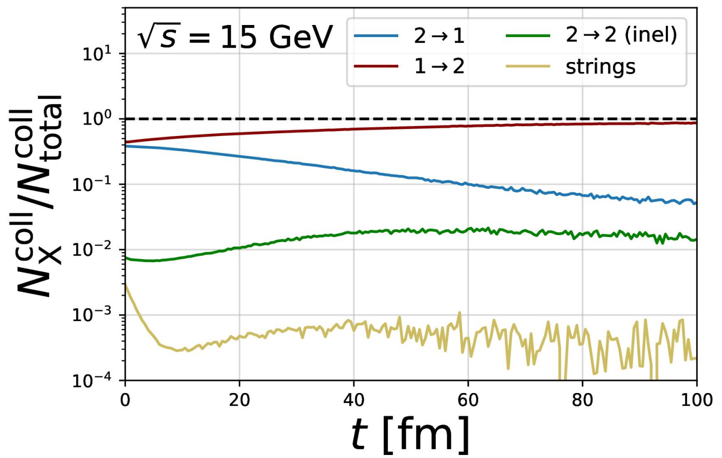
J. Hammelmann,, M. Bluhm, MN, H. Elfner, in preparation

Fate of critical fluctuations in the hadronic phase

- Apply smash (https://smash-transport.github.io) to the final hadronic interactions of the initialized particles.
- Resonance decay and regeneration are the dominant processes during the hadronic expansion.

Particle	Mass [GeV/c^2]	Degenercy
π	0.138	3
ρ	0.776	6
K	0.494	4
$K^\star(892)$	0.892	8
N	0.938	8
Δ	1.232	32
Λ	1.116	2
Σ	1.189	12





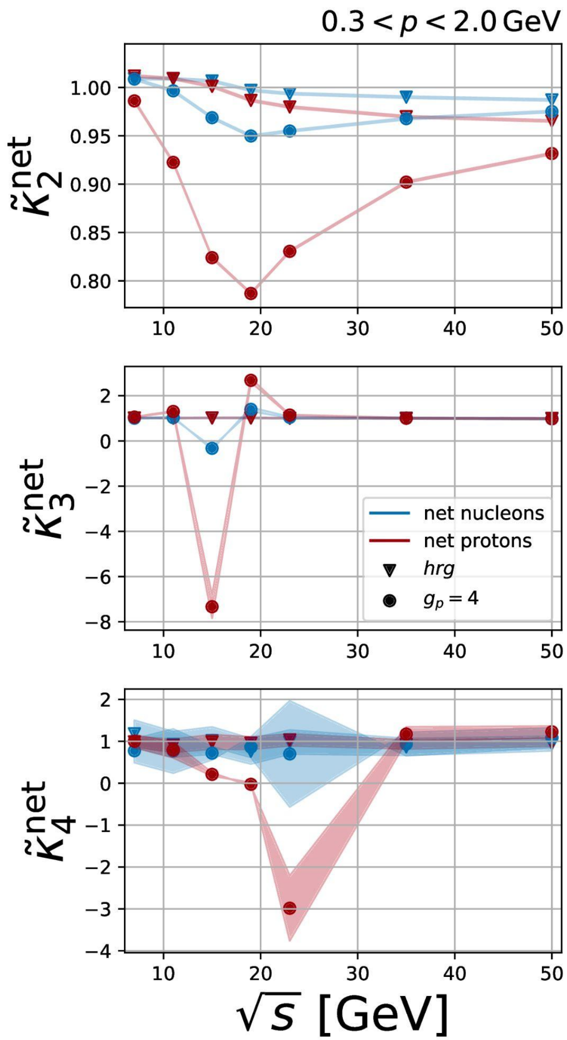
J. Hammelmann,, M. Bluhm, MN, H. Elfner, in preparation

Impact of resonance dynamics vs decay

- Compare the dynamical effect of the resonance decay and regeneration compared to only resonance decay:

$$\tilde{\kappa}_n = \frac{\kappa_n^{\mathrm{dynamical}}}{\kappa_n^{\mathrm{decays}}}$$

- Net-proton cumulants are strongly impacted by the hadron dynamics compared to the net-nucleons -> importance of isospin randomization processes.

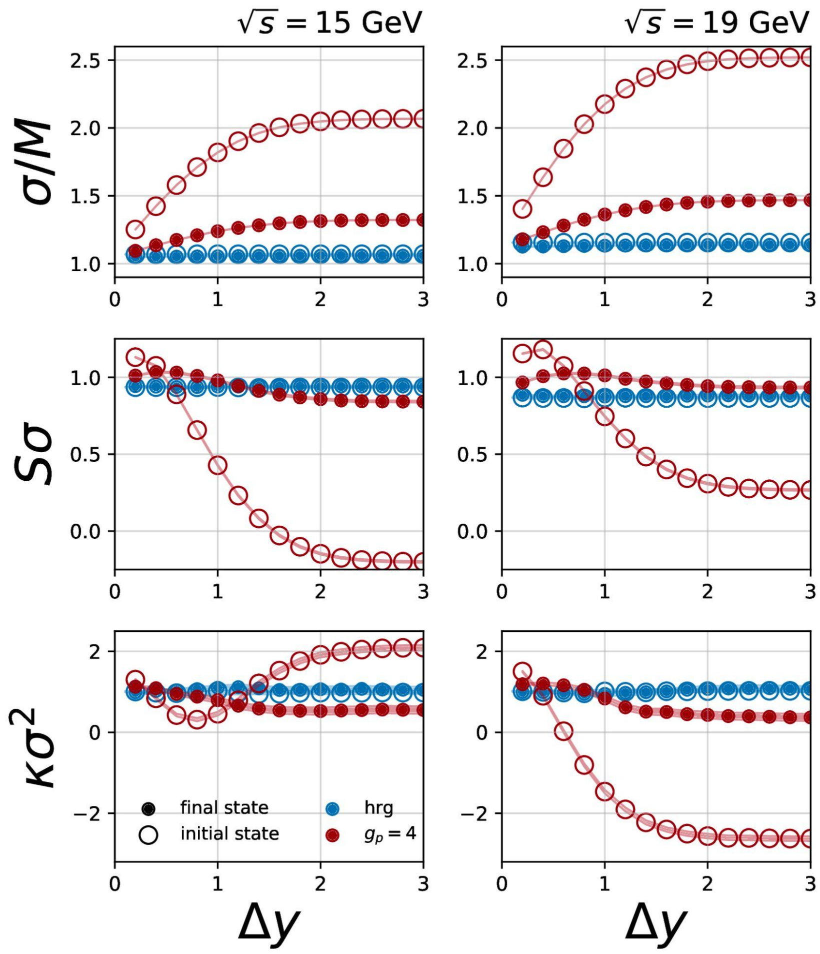


M. Bluhm, MN, S. Bass, T. Schaefer, Eur.Phys.J.C 77 (2017); MN, M. Bluhm, P. Alba, R. Bellwied, C. Ratti Eur.Phys.J.C 75 (2015); M. Kitazawa, M. Asakawa, Phys.Rev.C 85 (2012)

J. Hammelmann,, M. Bluhm, MN, H. Elfner, in preparation

Final proton and nucleon cumulant ratios

- For $g_c = 2$, no critical signal is seen in the net-proton variance and skewness, a very small signal in the kurtosis survives.
- For $g_c = 4$, the net-proton variance shows critical features -> not compatible with experiment.
- The nucleon critical signal is significantly more pronounced than for protons only.
- Signal depends strongly on the rapidity acceptance and can even change sign in the kurtosis.



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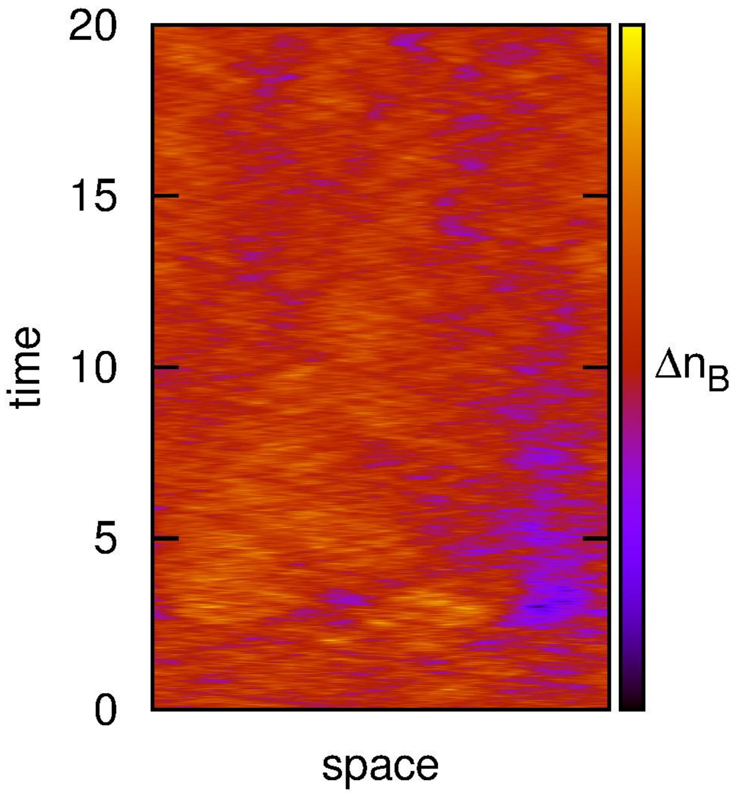
Conclusions

Treating the dynamics of fluctuations near the Critical Point is important for quantitative statements about its existence based on heavy-ion collision data!

- Net-baryon density fluctuations are strongly impacted by the expansion dynamics.
- Anticorrelations of baryons can signal the CP.
- Non-monotonic dependence of the kurtosis on the rapidity window near the CP.
- Resonance decay and regeneration strongly affects the critical fluctuations.

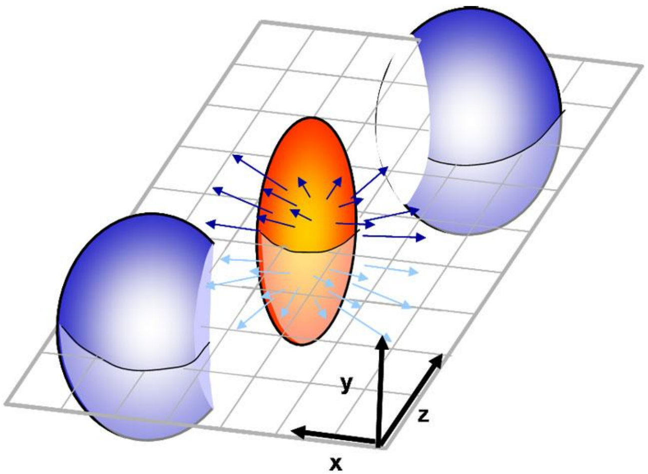


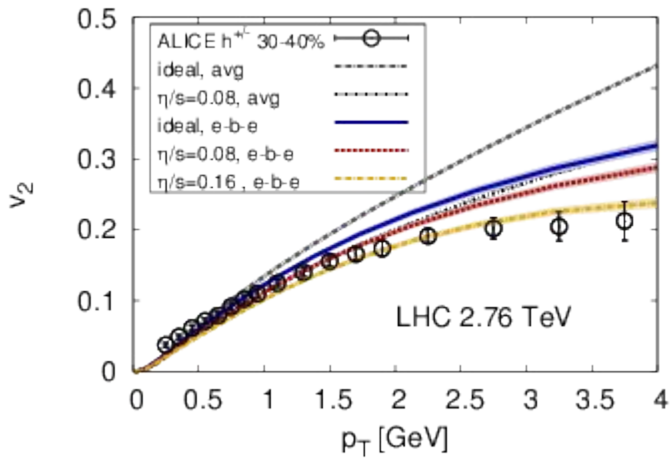
Currently explored:

- Renormalizing (chiral) fluid dynamics
- Implementing (renormalized) fluctuating fluid dynamics

APPENDIX

Fluid dynamical simulations of HIC





B. Schenke et al, PLB 702 (2011)

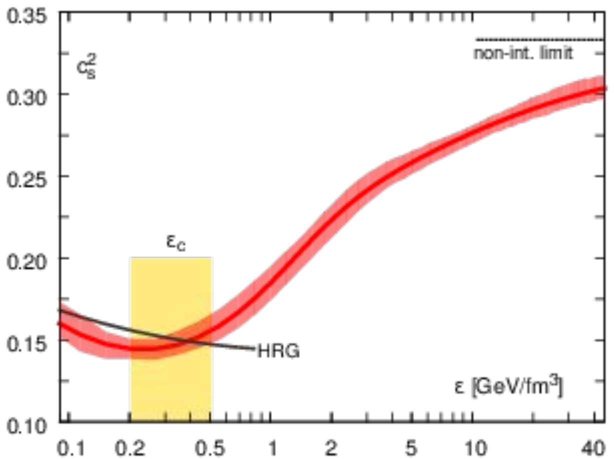
- Fluid dynamical simulations of heavy-ion collisions describe successfully, particle spectra and anisotropic flow coefficients v_n
- Many improvements in recent years have started:
  - viscosities and coupling terms between viscosities - done
  - initial state fluctuations - done
  - anisotropic fluid dynamics - preliminary
  - external fields (magnetic field, order parameters, etc) - preliminary

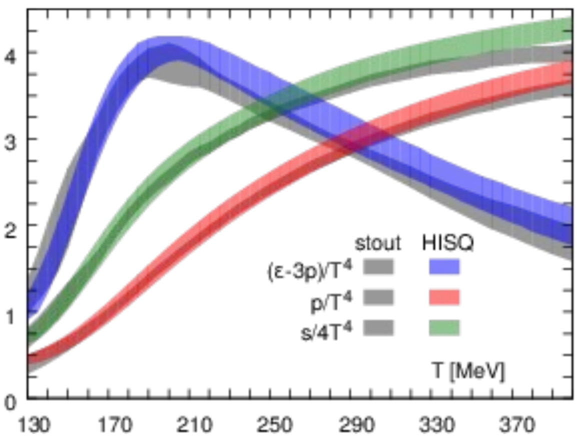
No code has it all (yet)!

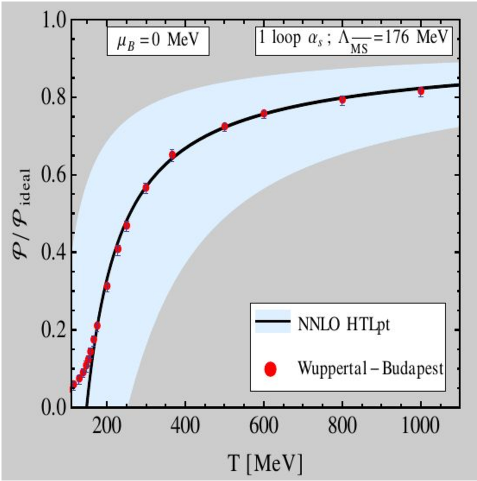
Equation of state (EoS)

Success story!
At μ_B = 0 the QCD EoS is known!

- Lattice QCD results on the EoS are far from the Stefan-Boltzmann-limit of a non-interacting gas => strongly coupled system
- Thermodynamic quantities change characteristically at the phase transition.
- Energy density, pressure and entropy increase at the crossover transition due to the liberation of color degrees of freedom -> quark-gluon plasma (QGP)
- Speed of sound has a minimum at the phase transition/ crossover
- **Naive pQCD poorly convergent => find more convergent gauge-invariant schemes (e.g. HTLpt, resummed DR)**







HotQCD Coll. PRD90 (2014); WB Coll. PLB730 (2014)

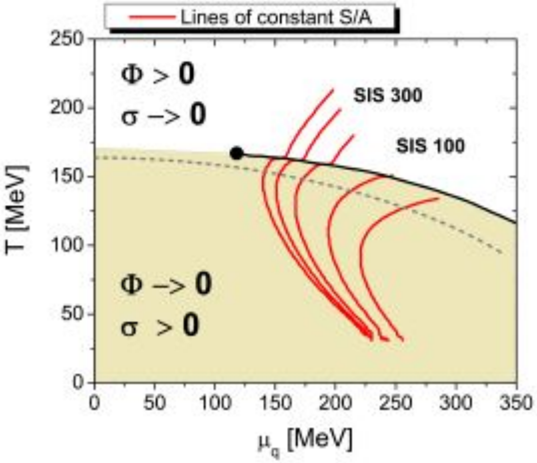
N. Haque et al, PRD89 (2014)

EoS at finite density

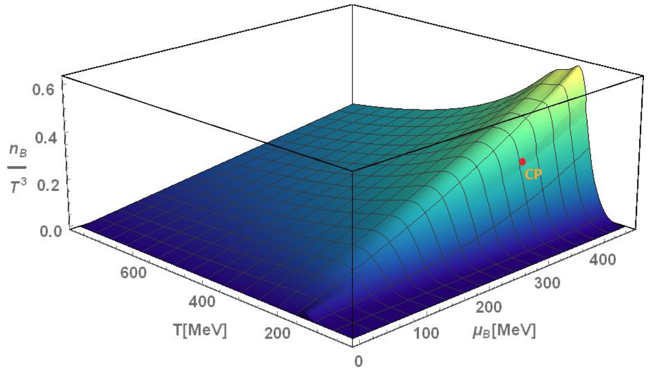
At $\mu_B \neq 0$ there are many unknowns in the QCD EoS...

- Toward finite net-baryon density the phase transition is supposed to change from crossover to a first-order phase transition via a critical end point.
- Beyond the onset density the hadronic gas description is not valid anymore => nuclear liquid or a mixed phase.
- The correct inclusion of the hadronic degrees of freedom is crucial and constraints from the nuclear ground state need to be taken into account.
- Constrain parameters by experimental results from HIC and astrophysical observations.

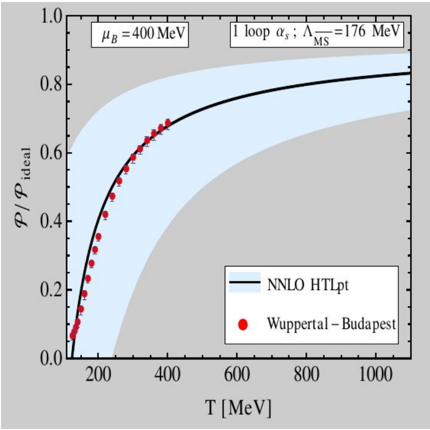
Need to extend the validity of the approaches to the QCD EoS to $\mu_B \neq 0$!



V. Dexheimer et al, PRC81 (2010)

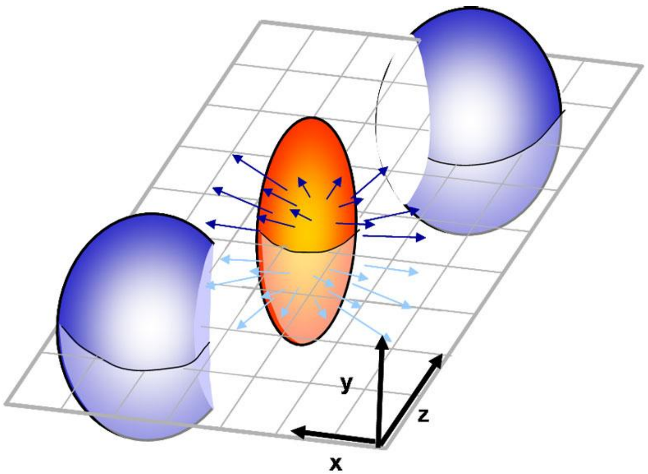


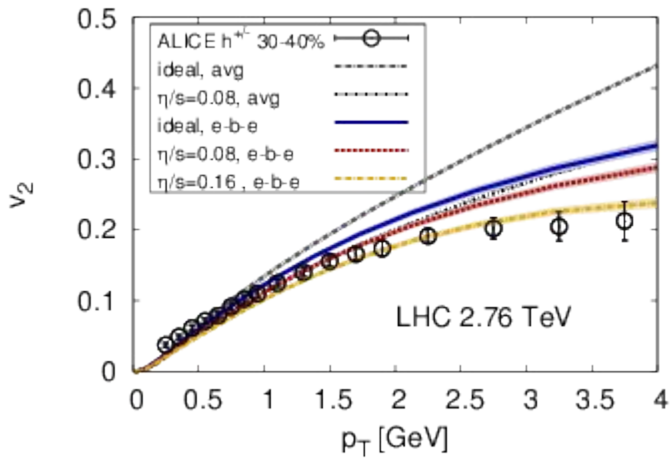
P. Parotto, MN et al, accepted in PRC



N. Haque et al, PRD89 (2014)

Fluid dynamical simulations of HIC





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No code has it all (yet)!

- **NOT included so far: thermal fluctuations!**

Importance of fluctuations for transport coefficients

$$\eta \sim \int d^3x\,dt \, \langle T^{ij}(x,t)\, T^{ij}(0,0) \rangle$$

Included in fluid dynamics

NOT included in fluid dynamics

- symmetrized correlator:

$$G_S^{xyxy}(\omega, \mathbf{0}) = \int \mathrm{d}^3x\mathrm{d}t\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})} \left\langle \frac{1}{2}\{ T^{xy}(t, \mathbf{x}), T^{xy}(0, \mathbf{0})\} \right\rangle$$

- for the shear-shear contribution $\Rightarrow$

$$G_{R,\mathrm{shear-shear}}^{xyxy}(\omega, \mathbf{0}) = -\frac{7T}{90\pi^2}\Lambda^3 - i\omega\frac{7T}{60\pi^2}\frac{\Lambda}{\gamma_\eta} + (i+1)\omega^{3/2}\frac{7T}{90\pi^2}\frac{1}{\gamma_\eta^{3/2}}$$

cutoff-dependent fluctuation contribution to the pressure

cutoff-dependent correction to η

frequency-dependent contribution to η and τ_π

P. Kovtun et al, PRD84 (2011); C.Chafin, T. Schäfer, PRA87 (2013); P. Romatschke et al, PRA87 (2013)

Nonequilibrium chiral fluid dynamics

Idea: Couple the explicit propagation of the chiral order parameter to a fluid dynamical evolution!

- Relaxational equation for the critical mode: damping and noise from the interaction with the fermions/fast modes

$$\partial_\mu \partial^\mu \sigma + \frac{\delta V_{\rm eff}(\sigma)}{\delta \sigma} + \eta \partial_t \sigma = \xi$$

- Phenomenological dynamics for the Polyakov-loop

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\rm eff}(\ell)}{\partial \ell} = \xi_\ell$$

- Fluid dynamical expansion = heat bath, including energy-momentum exchange

$$\partial_\mu T^{\mu\nu}_{\rm fluid} = S^\nu = -\partial_\mu T^{\mu\nu}_\sigma \,, \quad \partial_\mu N^\mu_{\rm q} = 0$$

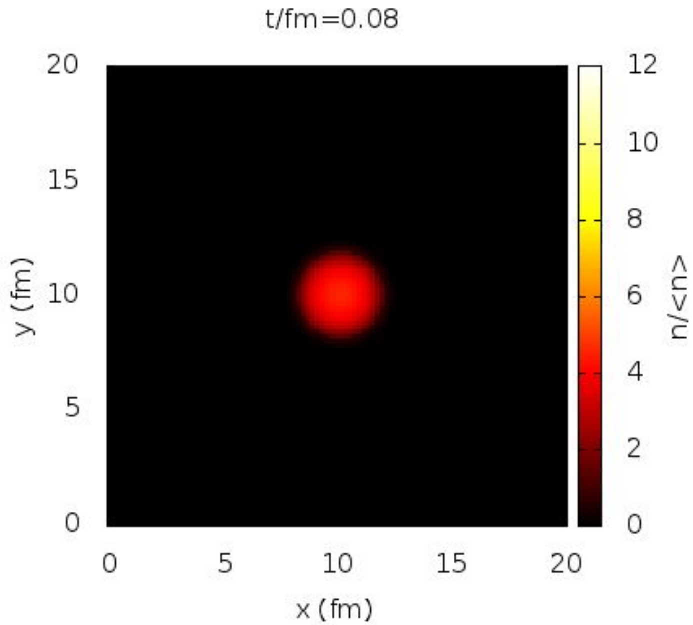
$\Rightarrow$ includes a stochastic source term!

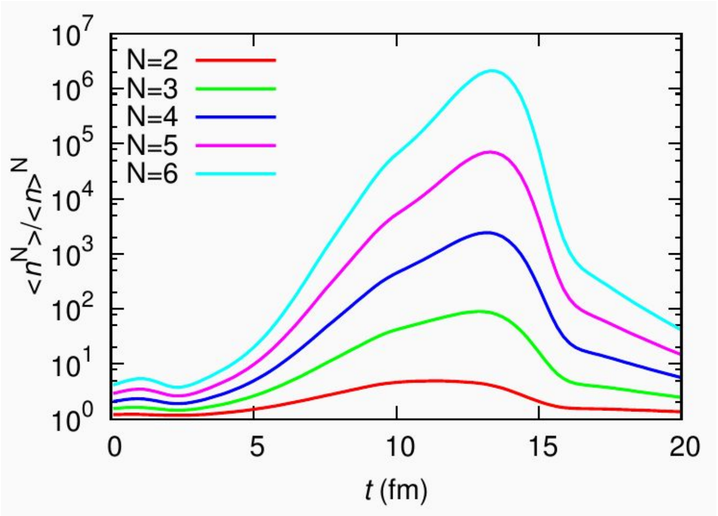
- Nonequilibrium equation of state $p = p(e, \sigma)$

MN, S. Leupold, I. Mishustin, C. Herold, M. Bleicher, PRC 84 (2011); PLB 711 (2012); JPG 40 (2013); C. Herold, MN, I. Mishustin, M. Bleicher PRC 87 (2013); NPA 925 (2014), C. Herold, MN, Y. Yan, C. Kobdaj JPG 41 (2014); MN and C. Herold, EPJA 52 (2016); C. Herold, MN, Y. Yan and C. Kobdaj, PRC93 (2016) no.2, PLB790 (2019)

Droplet formation & decay at the QH phase transition

- Chiral effective model with correct low-temperature degrees of freedom.
  V. Dexheimer, S. Schramm, PRC 81 (2010); M. Hempel, V. Dexheimer, S. Schramm, I. Iosilevskiy PRC 88 (2013)



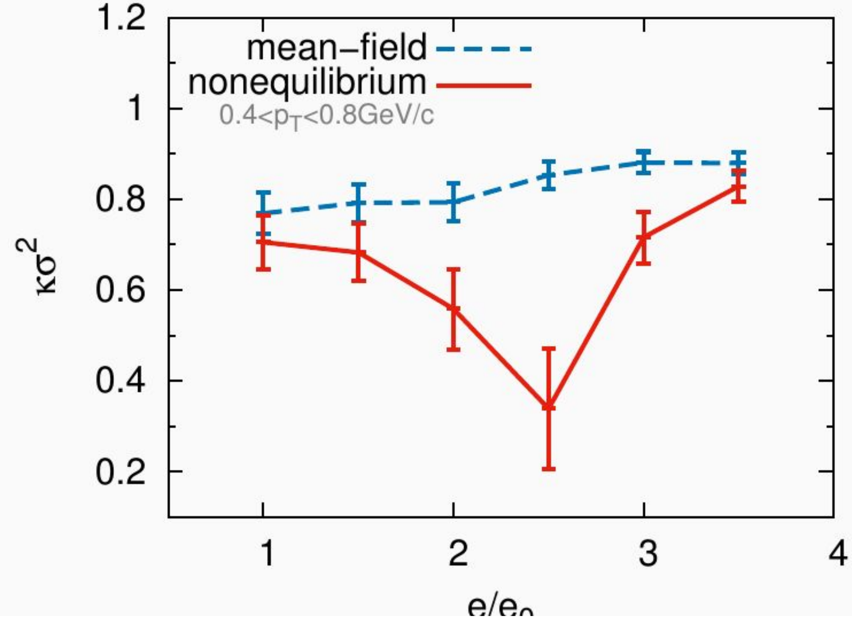


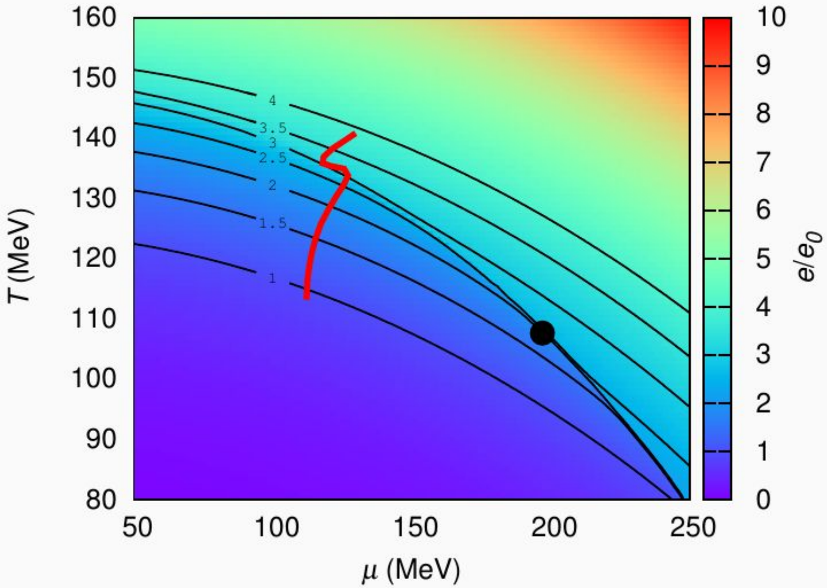
- Droplets of quark density form dynamically at the phase transition.
- Droplets of quark density decay in the hadronic phase due to non-vanishing large pressure.

MN and C. Herold, EPJA 52 (2016); cf, but no dynamical fluctuations: J. Steinheimer et al. PRC 89 (2014)

Net-proton fluctuations near the critical point

- UrQMD initial conditions rescaled to the EoS of the effective model.
- From densities to particles via Cooper-Frye particlization.
- At particlization: densities of the sigma field coupled to the FD densities.





C. Herold, MN, Y. Yan and C. Kobdaj, PRC 93 (2016) no.2

- No non-monotonic behavior in pure mean-field equilibrium calculations.
- Clear signal for criticality in net-proton fluctuations at transition energy density!
- Overall decreasing trend probably due to net-baryon number conservation

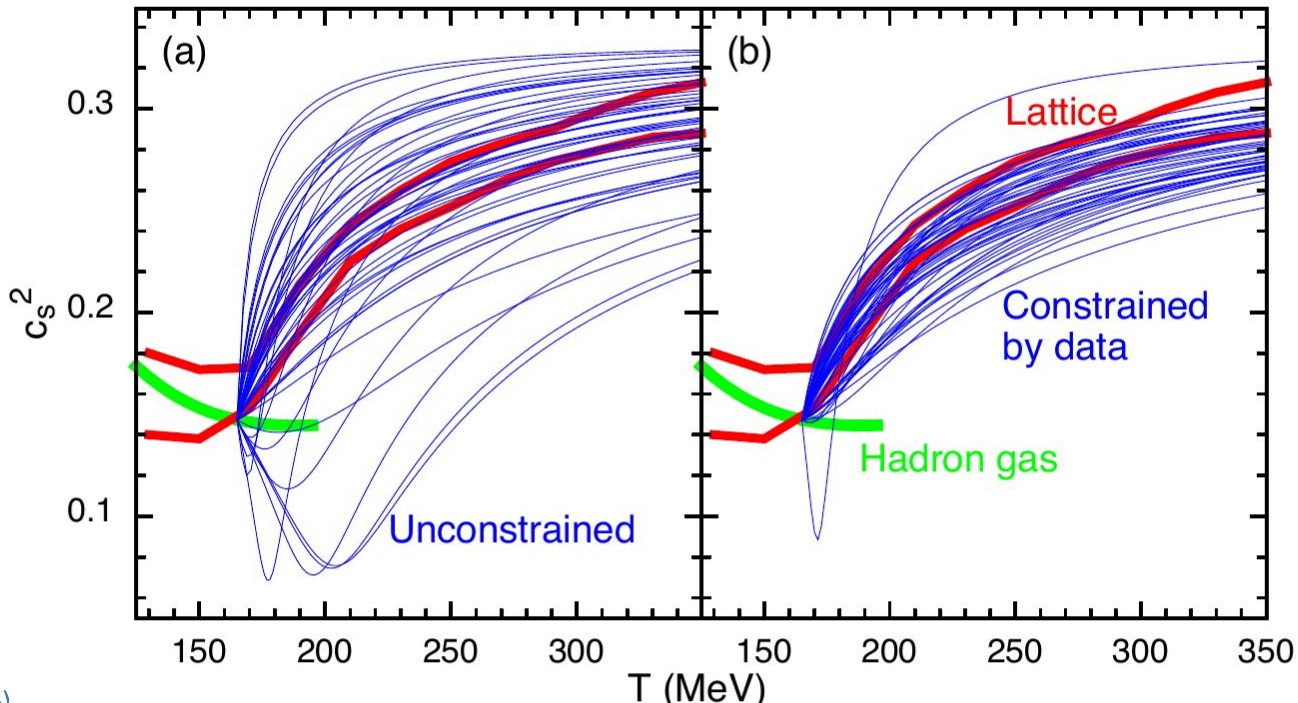
MN et al, EPJC72 (2012), 0903.2911

Equation of state (EoS)

Success story!
At μ_B = 0 the QCD EoS is known!

- Not only is the EoS precisely known from lattice QCD calculations.
- It has also been validated in a model-to-data statistical analysis.
- This combined (Bayesian analysis) tunes model and physics parameters simultaneously!

An UNBIASED model-to-data comparison should be standard for all simulations of HIC!



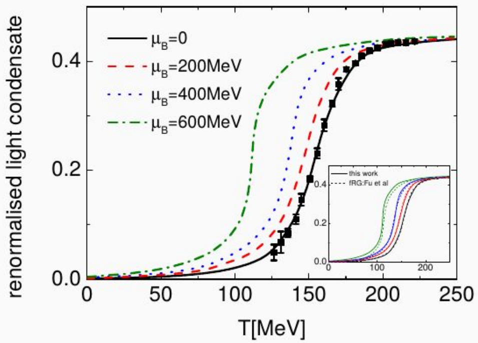
S. Pratt et al, PRL114 (2015)

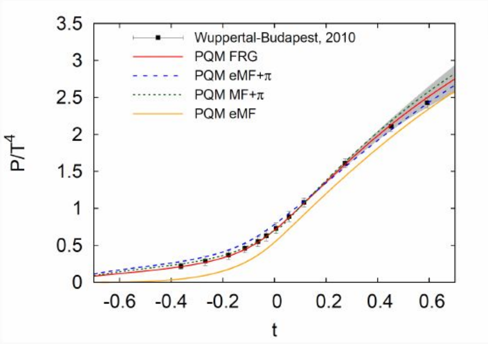
NchiFD + FRG ⟶ QCD assisted transport

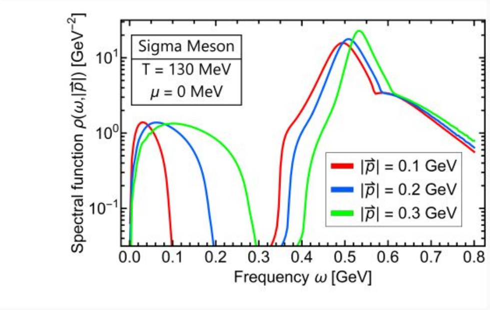
- Include effective potential beyond mean field, momentum dependent equilibrium sigma spectral function ⇒ linear response regime of QCD.

First-principle approach to QCD from the Functional Renormalization Group (FRG)

Cyrol, Mitter, Pawlowski, Strodthoff PRD97 (2018)

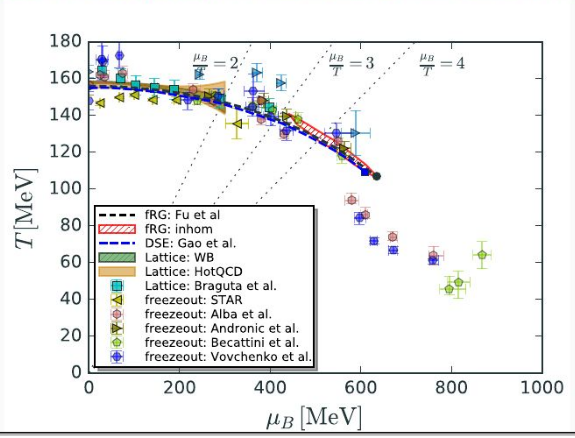






F. Gao, J. Pawlowski, 2010.13705; T. Herbst et al, PLB731 (2014); T. Herbst PRD88 (2013); F. Rennecke, J. Pawlowski, N. Wink

- Excellent description of phase structure at vanishing chemical potential.
- Phase structure qualitatively similar to the conjectured QCD phase diagram.
- Obtain spectral functions from analytical continuation.



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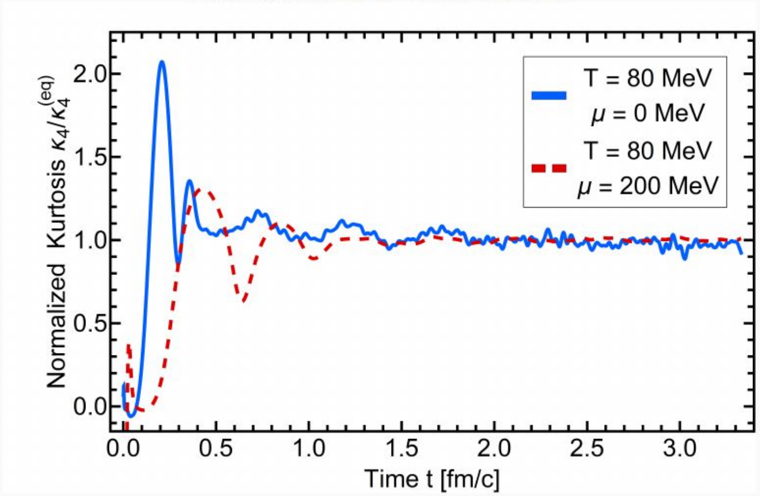
NchiFD + FRG ⟶ QCD assisted transport

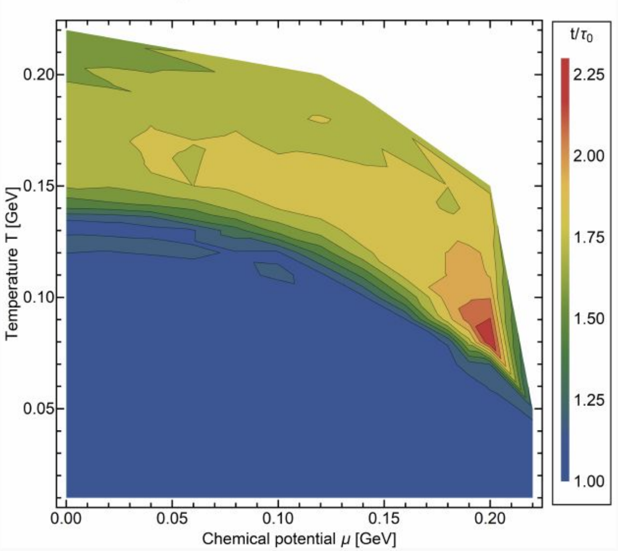
M. Bluhm et al., NPA982 (2019)

Transport equation: $\frac{\delta\Gamma}{\delta\sigma} = \xi$, where $\{\Re\Gamma^{(2)}_\sigma(\omega,\vec{p}), \Im\Gamma^{(2)}_\sigma(\omega,\vec{p}), U\} \in \Gamma$

Normalized Kurtosis:

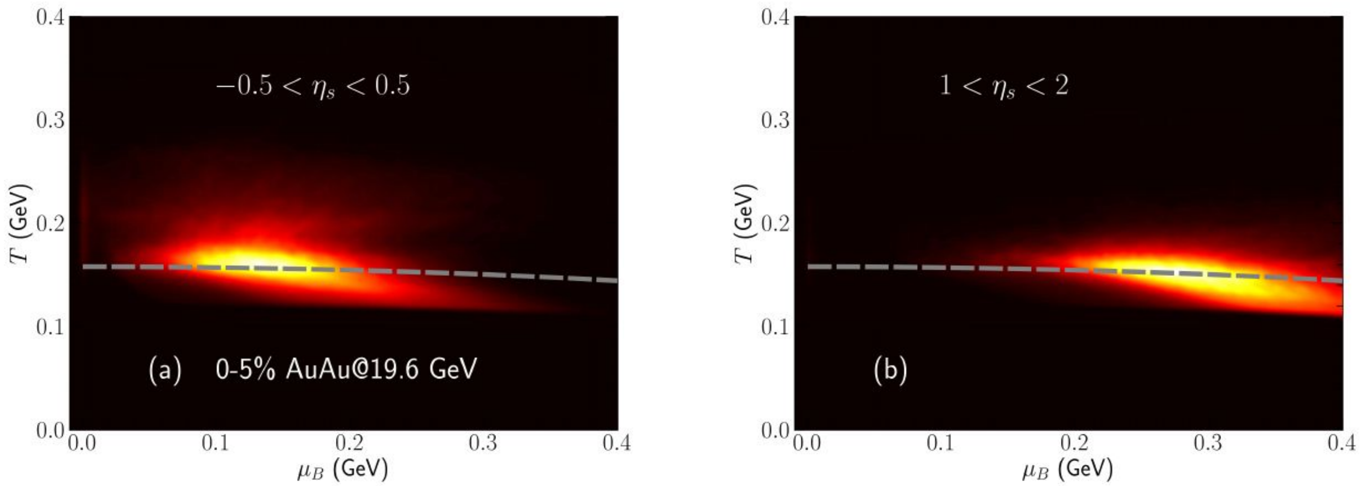


Equilibration time:



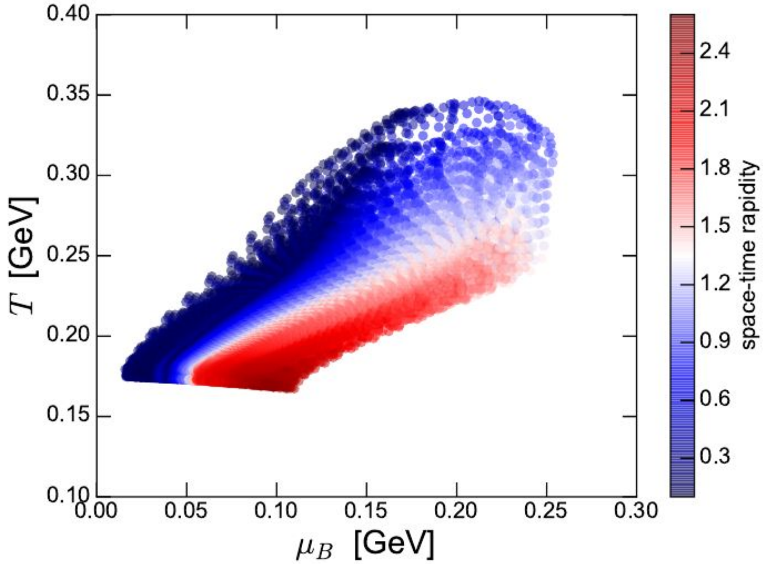
- Critical end point and the phase structure are clearly identifiable.
- Critical slowing down in the vicinity of the critical point, but no dramatic enhancement of $\tau_{\rm relax}$ in a dynamic setup!

Heavy-ion collisions at finite density



plots by Chun Shen

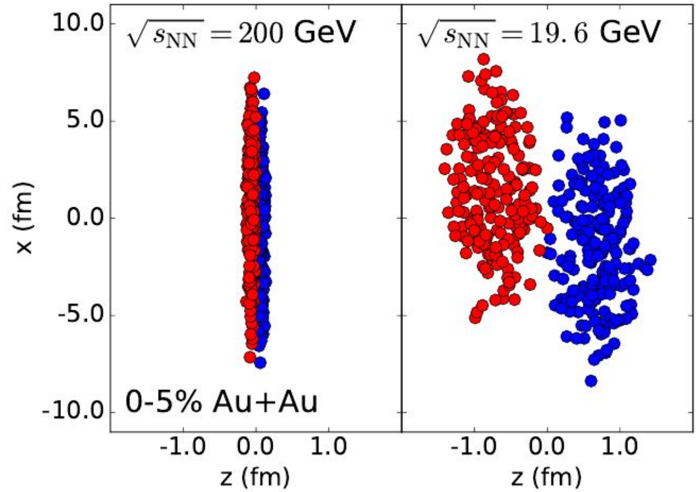
- Especially at low densities HIC span over large regions in the phase diagram.
- Already in $\sqrt{s_{NN}}$=72GeV rather large $\mu_B / T > 1$ are probed.

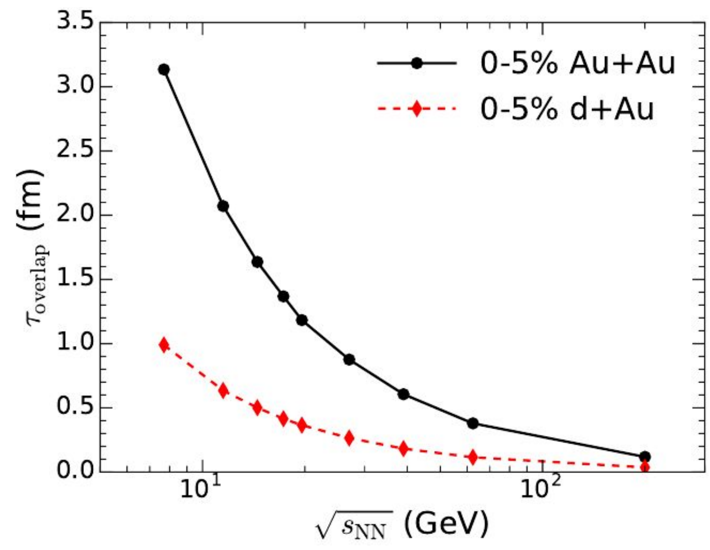


plot by Iurii Karpenko

The initial state of HIC at finite density

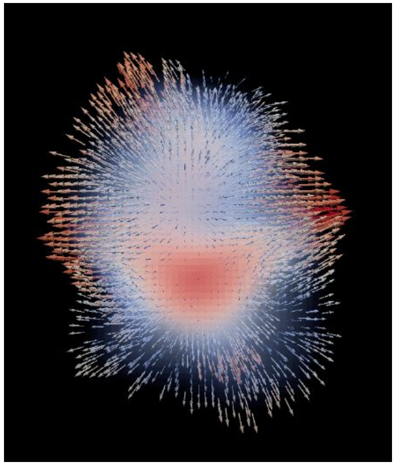
- Boost invariance is not a good approximation anymore -> thick pancakes!
- Increasing penetration time for colliding nuclei. J. Auvinen and H. Petersen, PRC88 (2013)
- CGC picture does not work anymore.
- Dynamical fluidization: Y. Akamatsu et al, PRC98 (2018)
  Need to evolve pre-fluid and fluid in parallel!
- Multi-fluid approach: fluid dynamical description from the initial state on.
  P. Batyuk, MN et al, PRC94 (2016)
  Need to couple the fluid description dynamically to the corona formation!
- **EPOS: currently all collisions happen in parallel, for application at finite density a cascade with sequential collisions needs to be implemented!**





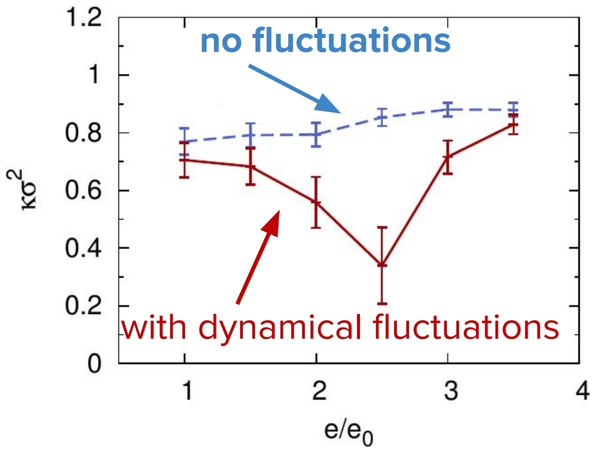
C. Shen and B. Schenke, PRC97 (2018)

Role of fluctuations & the phase transition



- Fluctuation observables, in particular higher-order cumulants of net-baryon number, are sensitive to critical phenomena at the phase transition.
- So far predictions are based on grand-canonical thermodynamics, but HIC are highly dynamical!

=> Need to include the fluctuation dynamics into descriptions of HIC!



C. Herold, MN, et al, PRC93 (2016)

- **Propagate fluctuations of the chiral order parameter and fluid dynamical fields coupled to the regular evolution**
- **Highly non-trivial due to conceptual challenges, e.g. renormalization of the EoS and the transport coefficients.**
- **Numerical implementation requires new algorithms!**

So far, largely unexplored territory!

The role of numerics and computational resources

Time for two heavy nuclei to collide and produce particles:

~ 10^{-23} seconds

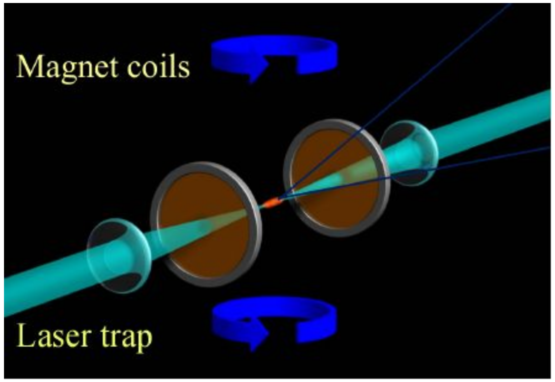
Time for a simulation of two colliding heavy nuclei and particle production:

~ 1 hour

Example: even with a Gaussian Process Emulator the Bayesian analysis of the temperature dependence of bulk and shear viscosity costs 100 Mio CPU hours.

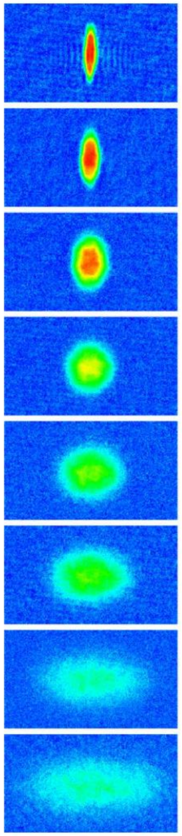
This assumes $O(10^4)$ events per point, fluctuation studies easily require $O(10^8)$ events per point...

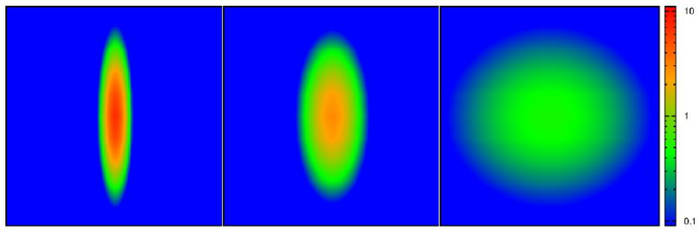
Connections to related systems



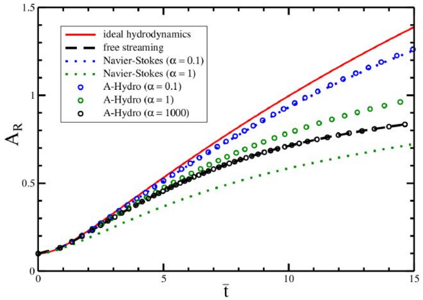
K.M. O'Hara et al. Science 298 (2002)

- The QGP shares features with other strongly coupled quantum systems, like **ultracold atomic gases**.
- UAG: unique experimental playground to study fermionic many-body problems: variable density, temperature and interatomic interaction strength
- Elliptic flow measurements: release the UAG from the trapping potential and take pictures of the expansion.
- Can theoretically be well described by anisotropic fluid dynamics:





M. Bluhm et al, PRA92 (2015)



- **Include fluctuations in anisotropic fluid dynamics and study the BCS-BEC crossover and the second-order superfluid phase transition - in equilibrium and dynamically!**
- **Connect to further research groups in France: e.g. IPN Orsay (superfluidity in UAG and neutron stars) or IP2I Lyon!**