Polarization, kinetic theory, and spin hydrodynamics

David Wagner
in collaboration with
Nora Weickgenannt, Enrico Speranza, and Dirk Rischke

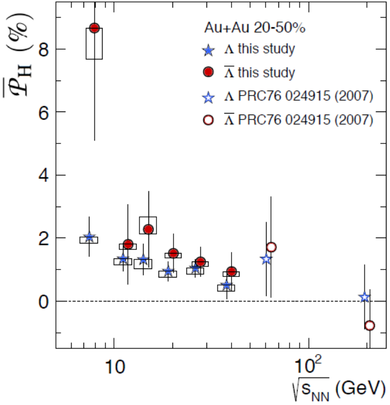
based mainly on

NW, ES, X.-L. Sheng, Q. Wang, DHR, Phys.Rev.D 104 (2021) 1, 016022
NW, DW, ES, DHR, Phys.Rev.D 106 (2022) 9, 096014
DW, NW, ES, Phys.Rev.Res. 5 (2023) 1, 013187
DW, NW, DHR, Phys.Rev.D 106 (2022) 11, 116021
DW, NW, ES, 2306.05936 (2023)

STRONG-NA7 Workshop & HFHF Theory Retreat | 02.10.2023

Global Λ-polarization

- Global polarization: polarization of Λ-hyperons along angular-momentum direction

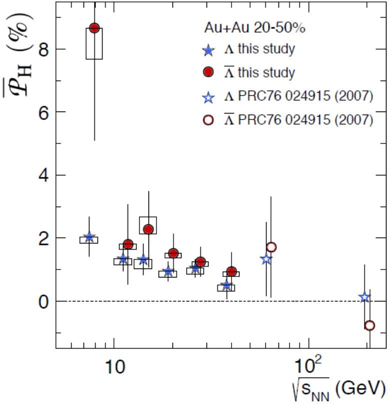


L. Adamczyk et al. (STAR), Nature 548 (2017) 62

Global Λ-polarization

- Global polarization: polarization of Λ-hyperons along angular-momentum direction
  - Can be well explained by considering **local equilibrium** on freeze-out hypersurface

$$S^\mu_\varpi = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0 (1-f_0) \varpi_{\alpha\beta}}{8m \int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$$



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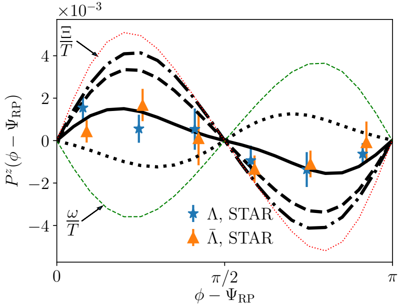
$$\varpi_{\mu\nu} := -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu),\ \beta^\mu := u^\mu / T,\ f_0 = [\exp(u^\mu k_\mu / T) + 1]^{-1}$$

- Local polarization:
  Angle-dependent polarization of Λ-hyperons along beam-direction

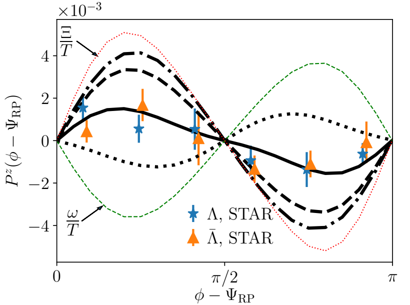


F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127 (2021) 272302

B. Fu, S. Y. F. Liu, L. Pang, H. Song, Y. Yin, PRL 127 (2021) 142301

- Local polarization: Angle-dependent polarization of Λ-hyperons along beam-direction
  - Could only be explained recently by incorporating shear effects (neglecting temperature gradients)

$$S^\mu_\xi = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0(1-f_0)\hat{t}_\alpha \frac{k^\gamma}{k^0} \Xi_{\gamma\beta}}{4mT\int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$$



F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127 (2021) 272302

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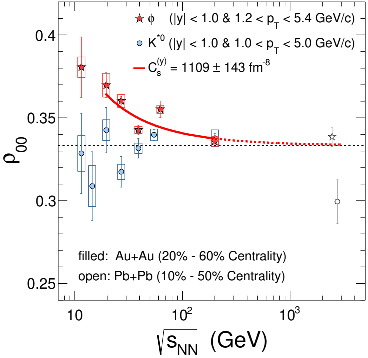
$$\omega_{\mu\nu} := \frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu),\ \Xi_{\mu\nu} := \frac{1}{2}(\partial_\mu u_\nu + \partial_\nu u_\mu),\ \Delta^{\mu\nu} := g^{\mu\nu} - u^\mu u^\nu$$

Alignment of ϕ-mesons

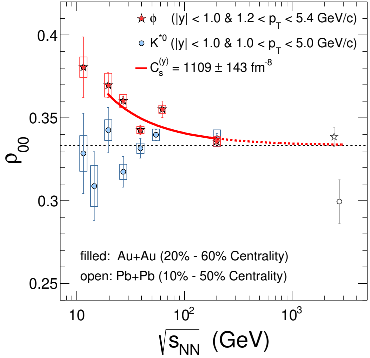
- Spin-1 particles feature tensor polarization ($\hat{=}$ alignment)



STAR collaboration, arXiv:2204.02302 (2022)

- Spin-1 particles feature tensor polarization ($\hat{=}$ alignment)
  - Larger than expected
  - Some theoretical developments, but no definitive answer yet

X.-L. Xia, H. Li, X-G. Huang, H.-Z. Huang, PLB 817 (2021) 136325
X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, X.-N. Wang, arXiv:2206.05868 (2022)
F. Li, S. Y. F. Liu, arXiv:2206.11890 (2022)
DW, NW, ES, 2207.01111 (2022)



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- Not yet clear
  - how valid the assumption of spin-equilibrium is,
  - how important dissipative effects are.
- Answers require a theory of **dissipative spin hydrodynamics**!

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- How to construct dissipative spin hydrodynamics?
    - Review conservation laws

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Spin Hydrodynamics: Basics

- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
    - $\rightarrow$ Should conserve energy-momentum and total angular momentum

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Conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \tag{1a}$$

$$\partial_\lambda J^{\lambda\mu\nu} =: \partial_\lambda S^{\lambda\mu\nu} + T^{[\mu\nu]} = 0 \tag{1b}$$

$A^{[\mu}B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

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- Additional information about dissipative quantities has to be provided
  - → Use **kinetic theory with spin** as effective microscopic model
- Rest of the presentation:
  - Construct such a kinetic theory
  - Perform hydrodynamic limit
  - Obtain expressions for observables

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- Independent components: scalar f_K, axial vector G^μ and traceless symmetric tensor $F_K^{\mu\nu}$

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- Determines a **quantum phase-space distribution function**
- Independent components: scalar f_K, axial vector G^μ and traceless symmetric tensor $F_K^{\mu\nu}$
- Equations of motion follow from field equations
  - Perform $\hbar$-expansion [small parameter: (Compton wavelength)/(macroscopic length scale)]

$f_K := (1/3)K_{\mu\nu}W^{\mu\nu}$, $G^\mu := -(i/2m)\epsilon^{\mu\nu\alpha\beta}k_\nu W_{\alpha\beta}$, $F_K^{\mu\nu} := K^{\mu\nu}_{\alpha\beta}W^{\alpha\beta}$
$K^{\mu\nu} := g^{\mu\nu} - k^\mu k^\nu/m^2$, $K^{\mu\nu}_{\alpha\beta} := (K^\mu_\alpha K^\nu_\beta + K^\mu_\beta K^\nu_\alpha)/2 - 1/3K^{\mu\nu}K_{\alpha\beta}$

Extending phase space

Boltzmann equations

- Not one, but nine equations in (x, k)-phase space

$$k \cdot \partial f_K(x,k) = \mathcal{C}_K \ , \quad k \cdot \partial G^\mu(x,k) = \mathcal{C}_G^\mu \ , \quad k \cdot \partial F_K^{\mu\nu}(x,k) = \mathcal{C}_K^{\mu\nu}$$

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- Measure $\mathrm{d}S := \frac{3m}{2\sigma\pi} \mathrm{d}^4\mathfrak{s}\,\delta[\mathfrak{s}^2 + \sigma^2]\delta(k \cdot \mathfrak{s})$

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Boltzmann equation in extended phase space

$$\mathfrak{f}(x,k,\mathfrak{s}):=f_K-\mathfrak{s}_\mu G^\mu+\frac{5}{4}\mathfrak{s}_\mu\mathfrak{s}_\nu F_K^{\mu\nu} \tag{2}$$

- Only on-shell parts $\mathfrak{f}(x,k,\mathfrak{s})=\delta(k^2-m^2)f(x,k,\mathfrak{s})$ contribute

$$k\cdot\partial f(x,k,\mathfrak{s})=\mathfrak{C}[f] \tag{3}$$

$\mathfrak{C}:=\mathcal{C}_K-\mathfrak{s}_\mu\mathcal{C}_G^\mu+\frac{5}{4}\mathfrak{s}_\mu\mathfrak{s}_\nu\mathcal{C}_K^{\mu\nu}$

Collision term

DW, NW, DHR, Phys.Rev.D 106 (2022) 11, 116021

Collision kernel

$$\mathfrak{C}[f] = \frac{1}{2}\int \mathrm{d}\Gamma_1 \mathrm{d}\Gamma_2 \mathrm{d}\Gamma' \mathrm{d}\bar{S}(k)\delta^{(4)}(k_1+k_2-k-k')\mathcal{W} \times \left[f(x+\Delta_1-\Delta, k_1, \mathfrak{s}_1) f(x+\Delta_2-\Delta, k_2, \mathfrak{s}_2) - f(x,k,\bar{\mathfrak{s}}) f(x+\Delta'-\Delta, k', \mathfrak{s}')\right] \tag{4}$$

$\mathrm{d}\Gamma := 2\mathrm{d}^4 k\,\delta(k^2-m^2)\mathrm{d}S(k)$

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- Contributions inside the collision term have gradient corrections

$$f(x, k, \mathfrak{s}) + \Delta^\mu \partial_\mu f(x, k, \mathfrak{s}) \approx f(x + \Delta, k, \mathfrak{s}) \tag{5}$$

- A (momentum- and spin-dependent) **spacetime shift** Δ^μ enters
  $\rightarrow$ Particles do not scatter at the same spacetime point!

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- A (momentum- and spin-dependent) **spacetime shift Δ^μ** enters
  $\rightarrow$ Particles do not scatter at the same spacetime point!
- This enables a conversion of orbital and spin angular momenta

$\mathrm{d}\Gamma := 2\mathrm{d}^4 k \delta(k^2 - m^2)\mathrm{d}S(k)$

Equilibrium

- Local equilibrium distribution function fulfills $\mathfrak{C}[f_{\mathsf{eq}}] = 0$
  - Specifically: Demand that the **local** part of the collision term vanishes!

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Local-equilibrium distribution function

$$f_{\text{eq}}(x,k,\mathfrak{s}) = \exp\left(-\beta_0 E_{\mathbf{k}} + \frac{\hbar}{2}\Omega_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu}\right) \tag{6}$$

$\Sigma_{\mathfrak{s}}^{\mu\nu} := -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha \mathfrak{s}_\beta$, $E_{\mathbf{k}} := k \cdot u$

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- Depends on Lagrange multipliers $\beta_0 u^\mu$, $\Omega^{\mu\nu}$ that determine ideal hydrodynamics

$\Sigma_{\mathfrak{s}}^{\mu\nu} := -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} k_\alpha \mathfrak{s}_\beta$, $E_{\mathbf{k}} := k \cdot u$

Moment expansion

- Split distribution function $f = f_{\text{eq}} + \delta f$

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Irreducible moments

$$\rho_r^{\mu_1\cdots\mu_\ell}(x) \quad := \quad \int \mathrm{d}\Gamma\, E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{7a}$$

$k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} := \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} k^{\nu_1} \cdots k^{\nu_\ell}$

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$$\tau_r^{\mu,\mu_1 \cdots \mu_\ell}(x) := \int \mathrm{d}\Gamma \, \mathfrak{s}^\mu E_{\mathbf{k}}^r k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} \delta f(x,k,\mathfrak{s}) \tag{7b}$$

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$$\psi_r^{\mu\nu,\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma K^{\mu\nu}_{\alpha\beta} \mathfrak{s}^\alpha \mathfrak{s}^\beta E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{7c}$$

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- Equations of motion can be derived from Boltzmann equation
- Knowing the evolution of all moments is equivalent to solving the Boltzmann equation

---

$k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle} := \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} k^{\nu_1}\cdots k^{\nu_\ell}$

Polarization observables in kinetic theory

Vector Polarization (Pauli-Lubanski Pseudovector)

$$S^\mu(k) := \mathrm{Tr}\left[\hat{S}^\mu\,\hat{\rho}(k)\right] = \frac{1}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k)\mathfrak{s}^\mu f(x,k,\mathfrak{s}) \qquad (8)$$

$N(k) := \int \mathrm{d}\Sigma_\gamma k^\gamma \int \mathrm{d}S(k) f(x,k,\mathfrak{s})$, $\quad \hat{S}^\mu := -(1/2m)\epsilon^{\mu\nu\alpha\beta}\hat{J}_{\nu\alpha}\hat{P}_\beta$

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Tensor Polarization

$$\rho_{00}(k) \quad = \quad \frac{1}{3} - \sqrt{\frac{2}{3}}\epsilon^{(0)}_\mu(k)\epsilon^{(0)}_\nu(k)\Theta^{\mu\nu}(k) \qquad (9a)$$

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CRC-TR 211

Vector Polarization (Pauli-Lubanski Pseudovector)

$$S^\mu(k) := \mathrm{Tr}\left[\hat{S}^\mu\,\hat{\rho}(k)\right] = \frac{1}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k)\mathfrak{s}^\mu f(x,k,\mathfrak{s}) \tag{8}$$

Tensor Polarization

$$\rho_{00}(k) = \frac{1}{3} - \sqrt{\frac{2}{3}}\epsilon^{(0)}_\mu(k)\epsilon^{(0)}_\nu(k)\Theta^{\mu\nu}(k) \tag{9a}$$

$$\Theta^{\mu\nu}(k) := \frac{1}{2}\sqrt{\frac{3}{2}}\mathrm{Tr}\left[\left(\hat{S}^{(\mu}\hat{S}^{\nu)} + \frac{4}{3}K^{\mu\nu}\right)\hat{\rho}(k)\right]$$

$$= \frac{1}{2}\sqrt{\frac{3}{2}}\frac{1}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k) K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta f(x,k,\mathfrak{s}) \tag{9b}$$

$N(k) := \int \mathrm{d}\Sigma_\gamma k^\gamma \int \mathrm{d}S(k) f(x,k,\mathfrak{s})$, $\quad \hat{S}^\mu := -(1/2m)\epsilon^{\mu\nu\alpha\beta}\hat{J}_{\nu\alpha}\hat{P}_\beta$

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- Goal: Express polarization observables through **fluid-dynamical** quantities

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Needed moments

$$\Pi := -\frac{m^2}{3}\rho_0 \,, \quad \pi^{\mu\nu} := \rho_0^{\mu\nu} \quad (T^{\mu\nu}) \tag{10a}$$

$$\mathfrak{p}^\mu := \tau_0^{\langle\mu\rangle} \,, \quad \mathfrak{q}^{\lambda\mu\nu} := \tau_0^{\langle\lambda\rangle,\mu\nu} \quad (J^{\lambda\mu\nu}) \tag{10b}$$

$$\psi_1^{\mu\nu} \,, \quad (\Theta^{\mu\nu}) \tag{10c}$$

CRC-TR 211

Dissipative Hydro: Evolution equations

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \text{h.o.t.} \tag{11a}$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \text{h.o.t.} \tag{11b}$$

$$\tau_{\mathfrak{p}} \dot{\mathfrak{p}}^{\langle\mu\rangle} + \mathfrak{p}^{\langle\mu\rangle} = \mathfrak{e}^{(0)}(\tilde{\Omega}^{\mu\nu} - \tilde{\varpi}^{\mu\nu})u_\nu + \text{h.o.t.} \tag{11c}$$

$$\tau_{\mathfrak{q}} \dot{\mathfrak{q}}^{\langle\lambda\rangle\langle\mu\nu\rangle} + \mathfrak{q}^{\langle\lambda\rangle\langle\mu\nu\rangle} = \mathfrak{d}^{(2)}\beta_0 \sigma_\alpha{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\beta + \text{h.o.t.} \tag{11d}$$

$$\tau_{\psi_1} \dot{\psi}_1^{\langle\mu\nu\rangle} + \psi_1^{\langle\mu\nu\rangle} = \xi\beta_0\pi^{\mu\nu} + \text{h.o.t.} \tag{11e}$$

$\varpi^{\mu\nu} := -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$, $\tilde{A}^{\mu\nu} := \epsilon^{\mu\nu\alpha\beta}A_{\alpha\beta}$

Results I: Dissipative Spin Hydro

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- Evaluate polarization and alignment in the **Navier-Stokes limit**

$\varpi^{\mu\nu} := -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$, $\tilde{A}^{\mu\nu} := \epsilon^{\mu\nu\alpha\beta}A_{\alpha\beta}$

Results II: Alignment

- Moments of spin-rank 2:

$$\psi_1^{\langle\mu\nu\rangle} \simeq \xi\beta_0\pi^{\mu\nu} \tag{12}$$

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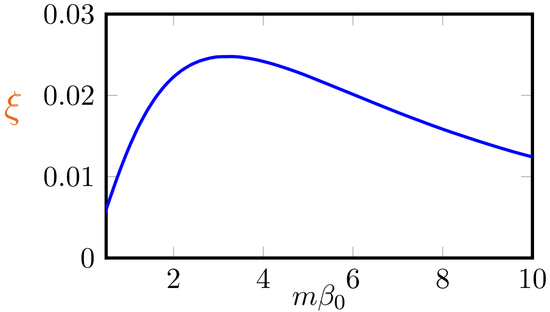

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Pauli-Lubanski pseudovector (spin 1/2)

$$S^\mu(k) = \frac{1}{2\mathcal{N}} \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) \mathfrak{s}^\mu f(x,k,\mathfrak{s}) \tag{13a}$$

$$\simeq \int \mathrm{d}\Sigma_\lambda k^\lambda \frac{f_0}{2\mathcal{N}} \Bigg\{ -\frac{\hbar}{2m}\tilde{\Omega}^{\mu\nu}k_\nu + \left(\delta^\mu_\nu - \frac{u^\mu k_{\langle\nu\rangle}}{E_\mathbf{k}}\right) \times \left[\mathfrak{e}\chi_\mathfrak{p}\left(\tilde{\Omega}^{\nu\rho} - \tilde{\varpi}^{\nu\rho}\right)u_\rho - \chi_\mathfrak{q}\mathfrak{d}\beta_0 \sigma_\rho{}^{\langle\alpha}\epsilon^{\beta\rangle\nu\sigma\rho}u_\sigma k_{\langle\alpha}k_{\beta\rangle}\right]\Bigg\} \tag{13b}$$

$\mathcal{N} := \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) f(x,k,\mathfrak{s})$

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$$\left.\times\left[\mathfrak{e}\chi_\mathfrak{p}\left(\tilde{\Omega}^{\nu\rho}-\tilde{\varpi}^{\nu\rho}\right)u_\rho - \chi_\mathfrak{q}\mathfrak{d}\beta_0{\sigma_\rho}^{\langle\alpha}\epsilon^{\beta\rangle\nu\sigma\rho}u_\sigma k_{\langle\alpha}k_{\beta\rangle}\right]\right\} \tag{13b}$$

- Contains novel contributions from fluid shear
  - Only sourced by **nonlocal** collisions

$\mathcal{N} := \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) f(x,k,\mathfrak{s})$

Summary & Outlook

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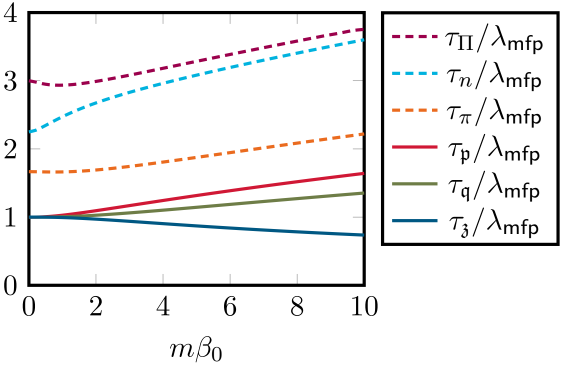
- Future perspectives:
  - Evaluate expressions for polarization and alignment with hydrodynamic simulations
  - Implement full spin hydrodynamics numerically
  - Include electric and magnetic fields

Appendix

Relevant time scales: An estimation



- Simplest interaction: constant cross section
- Spin-related relaxation times shorter than standard dissipative time scales, but not much

Nonlocal collisions

DW, NW, ES, 2306.05936 (2023)

Spacetime shifts

$$\Delta^\mu := \frac{1}{3}\frac{1}{\mathcal{W}}\frac{(2\pi\hbar)^3}{32}\frac{i\hbar}{m^2} M^{\gamma_1\gamma_2\delta_1\delta_2} M^{\zeta_1\zeta_2\eta_1\eta_2} h_{1,\gamma_1\eta_1} h_{2,\gamma_2\eta_2} h'_{\zeta_2\delta_2} \times \left(H^\mu{}_{\delta_1} k_{\zeta_1} - k_{\delta_1} H_{\zeta_1}{}^\mu\right) \tag{14}$$

- Depend on the transfer-matrix elements

$$\langle 11'|\,\widehat{t}\,|22'\rangle = \epsilon^*_{1,\alpha}\epsilon^*_{1',\beta}\epsilon_{2,\gamma}\epsilon_{2',\delta} M^{\alpha\beta\gamma\delta} \tag{15}$$

- Manifestly covariant
  - $\rightarrow$ no "no-jump" frame

$$h^{\mu\nu} := \frac{1}{3}K^{\mu\nu} + \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}k_\alpha\mathfrak{s}_\beta + K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta,$$
$$H^{\mu\nu} := \frac{1}{3}K^{\mu\nu} + \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}k_\alpha\mathfrak{s}_\beta + \frac{5}{8}K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta$$

Moment equations: Spin-rank 0

- Moments follow relaxation-type equations

Moment equation for $\ell = 0$

$$\dot{\rho}_r - \mathfrak{C}_{r-1} = [(1-r)I_{r1} - I_{r0}]\theta - I_{r0}\dot{\alpha}_0 + I_{r+1,0}\dot{\beta}_0 + (r-1)\rho^{\mu\nu}_{r-2}\sigma_{\mu\nu} + r\rho^{\mu}_{r-1}\dot{u}_\mu - \nabla_\mu \rho^\mu_{r-1} - \frac{1}{3}\left[(r+2)\rho_r - (r-1)m^2\rho_{r-2}\right]\theta \tag{16}$$

- Depend both on equilibrium and dissipative quantities
- Not a closed system
- Blue terms will become Navier-Stokes values

$\dot{A} := u \cdot \partial A$, $\nabla^\mu := \Delta^{\mu\nu}\partial_\nu$
$\theta := \nabla \cdot u$, $\sigma^{\mu\nu} := \nabla^{\langle\mu}u^{\nu\rangle}$, $E_k := k \cdot u$
$I_{nq} := [(2q+1)!!]^{-1} \int \mathrm{d}\Gamma E_k^{n-2q}(-k^{\langle\alpha\rangle}k_\alpha)^q$

Results II: Alignment

Alignment: Explicit expression

$$\rho_{00}(k) = \frac{1}{3} - \frac{4}{15}\left[\int \mathrm{d}\Sigma_\lambda k^\lambda f_{0\mathbf{k}}\left(1 - 3\mathcal{H}^{(0,0)}_{\mathbf{k}0}\Pi/m^2 + \mathcal{H}^{(0,2)}_{\mathbf{k}0}\pi^{\mu\nu}k_\mu k_\nu\right)\right]^{-1} \times \int \mathrm{d}\Sigma_\lambda k^\lambda \mathcal{H}^{(2,0)}_{\mathbf{k}1}\xi\beta_0 f_{0\mathbf{k}}\epsilon^{(0)}_\mu \epsilon^{(0)}_\nu K^{\mu\nu}_{\alpha\beta}\Xi^{\alpha\beta}_{\gamma\delta}\pi^{\gamma\delta} \tag{17}$$

$f_{0\mathbf{k}} \coloneqq \exp(-\beta_0 E_\mathbf{k})$
$\Xi^{\mu\nu}_{\alpha\beta} \coloneqq \frac{1}{2}\Xi^{(\mu}_{\alpha}\Xi^{\nu)}_{\beta} - \frac{1}{\Xi^2}\Xi^{\mu\gamma}\Xi^{\nu}_{\gamma}\Xi_{\alpha\delta}\Xi^{\delta}_{\beta}$
$\Xi^{\mu\nu} \coloneqq \Delta^{\mu\nu} + k^{\langle\mu\rangle}k^{\langle\nu\rangle}/E^2_\mathbf{k}$

Moment equations: Spin-rank 1

- Same procedure as for the moments of spin-rank 0

Moment equation for $\ell = 0$

$$
\begin{aligned}
\dot{\tau}_r^{\langle\mu\rangle} - \mathfrak{C}_{r-1}^{\langle\mu\rangle} \quad = \quad & \frac{\hbar}{2m}\left\{[I_{r+1,0} + rI_{r+1,1}]\theta + I_{r+1,0}\dot{\alpha}_0 - I_{r+2,0}\dot{\beta}_0\right\}\omega_0^\mu \\
& -\frac{\hbar}{4m} I_{r+1,1}\Delta^\mu_\lambda \nabla_\nu \tilde{\Omega}^{\lambda\nu} - \frac{\hbar}{4m} I_{r+1,0}\epsilon^{\mu\nu\alpha\beta} u_\nu \dot{\Omega}_{\alpha\beta} \\
& -\frac{\hbar}{4m}\tilde{\Omega}^{\langle\mu\rangle\nu}\left[I_{r+1,1}\nabla_\nu \alpha_0 - I_{r+2,1}\left(\nabla_\nu \beta_0 + \beta_0 \dot{u}_\nu\right)\right] \\
& + r\,\dot{u}_\nu \tau_{r-1}^{\langle\mu\rangle,\nu} + (r-1)\sigma_{\alpha\beta}\tau_{r-2}^{\langle\mu\rangle,\alpha\beta} - \Delta^\mu_\lambda \nabla_\nu \tau_{r-1}^{\lambda,\nu} \\
& -\frac{1}{3}\left[(r+2)\tau_r^{\langle\mu\rangle} - (r-1)m^2\tau_{r-2}^{\langle\mu\rangle}\right]\theta \qquad (18)
\end{aligned}
$$

- Determine the **(vector) polarization** of particles

$\tilde{\Omega}^{\mu\nu} := \epsilon^{\mu\nu\alpha\beta}\Omega_{\alpha\beta}$, $\Omega^{\mu\nu} = u^{[\mu}\kappa_0^{\nu]} + \epsilon^{\mu\nu\alpha\beta}u_\alpha\omega_{0,\beta}$

Moment equations: Spin-rank 2

Moment equation for $\ell = 0$

$$\dot{\psi}_r^{\langle\mu\nu\rangle} - \mathfrak{C}_{r-1}^{\langle\mu\nu\rangle} = -\frac{\theta}{3}\left[(r+2)\psi_r^{\langle\mu\nu\rangle} - (r-1)m^2\psi_{r-2}^{\langle\mu\nu\rangle}\right] + r\psi_{r-1}^{\langle\mu\nu\rangle,\alpha}\dot{u}_\alpha - \Delta^{\mu\nu}_{\alpha\beta}\nabla_\gamma\psi_{r-1}^{\alpha\beta,\gamma} + (r-1)\psi_{r-2}^{\langle\mu\nu\rangle,\alpha\beta}\sigma_{\alpha\beta} \quad (19)$$

- No dependence on equilibrium quantities appears because moments of spin-rank 2 do not appear in any conserved current
- Nonetheless, they determine the **tensor polarization** of spin-1 particles

$\Delta^{\mu\nu}_{\alpha\beta} := (\Delta^{(\mu}_{\alpha}\Delta^{\nu)}_{\beta})/2 - (1/3)\Delta^{\mu\nu}\Delta_{\alpha\beta}$

Truncation

- Spin-1: Which moments are contained in the *total* tensor polarization?

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Total tensor polarization

$$\bar{\Theta}^{\mu\nu} := \int \mathrm{d}K N(k) \Theta^{\mu\nu}(k) = \frac{1}{2}\sqrt{\frac{3}{2}} \int \mathrm{d}\Sigma_\lambda \left(u^\lambda \psi_1^{\mu\nu} + \psi_0^{\mu\nu,\lambda} \right) \tag{20}$$

$\mathrm{d}K := \mathrm{d}^3\mathbf{k}/[(2\pi\hbar)^3 2k^0]$

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- Lowest-order approximation: Keep only these moments in the employed basis, i.e.,

$$\delta f(x,k) \hat{=} \delta f \left(\Pi \, , \; \pi^{\mu\nu} \, , \; \mathfrak{p}^{\mu} \, , \; \mathfrak{z}^{\mu\nu} \, , \; \mathfrak{q}^{\lambda\mu\nu} \, , \; \psi_1^{\mu\nu} \, , \; \psi_0^{\mu\nu,\lambda}, k \right) \tag{21}$$

$\mathrm{d}K := \mathrm{d}^3\mathbf{k}/[(2\pi\hbar)^3 2k^0]$