From Imaginary to Real Chemical Potentials with DSEs and QCD Phase Transitions in the Light-Quark Chiral Limit

Julian Bernhardt

Institute for Theoretical Physics
Justus Liebig University Gießen

Based on:
JB, Fischer, EPJ A 59 (2023) 181
and
JB, Fischer, arXiv:2309.06737

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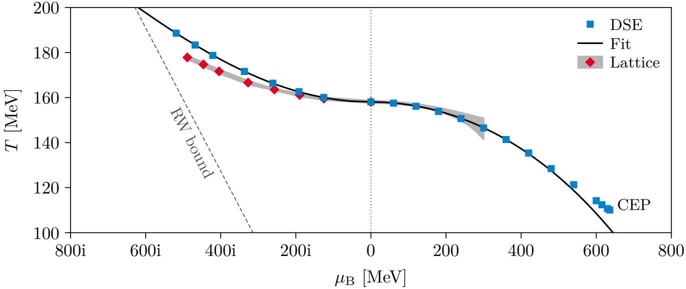
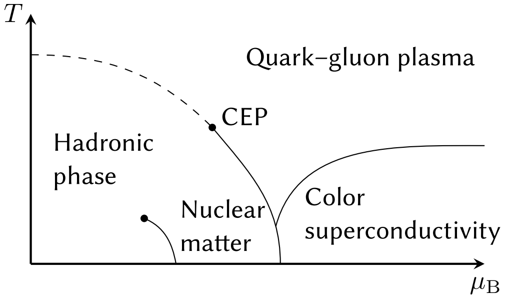
Outline

First Objective: Imaginary Chemical Potentials



Motivation: Why Imaginary Chemical Potentials?

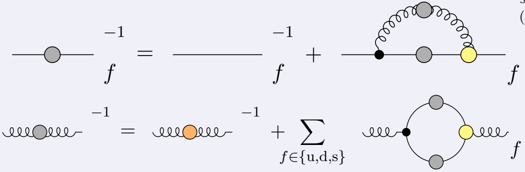


- Sign problem of lattice QCD prevents direct calculation of nonzero (real) chemical potentials
- Different methods to bypass/mitigate:
  - Reweighting, Taylor expansion around $\mu = 0$, ...
  - Calculation of imaginary chemical potentials and extrapolation/analytical continuation to real ones
- Functional methods (e.g., Dyson–Schwinger equations) can do real and imaginary $\mu \rightarrow$ possible to gauge quality of extrapolation

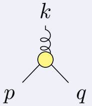
Functional Framework: Truncated Set of DSEs

Truncated DSEs for Quarks and Gluons

see Fischer, PPNP 105 (2019) 1
(and references therein)



Quark–Gluon Vertex Ansatz



$$\Gamma_\mu^f(k,p,q) = \Gamma(k,p,q)\Gamma_\mu^{f,\mathrm{BC}}(p,q)$$ (Information about quarks)

Quenched Gluon Propagator

$$D_{\mu\nu}^{\mathrm{que}}(k) = D_{\mu\nu}^{\mathrm{que}}(k;T)$$ (Temperature-dependent fit to lattice data)

reference for lattice data: Fischer, Maas, Müller, EPJ C 68 (2010) 165-181
Maas, Pawlowski, von Smekal, Spielmann, PRD 85 (2012) 034037

Inclusion of Temperature and Chemical Potential

Matsubara Frequencies

$$\omega_n = \begin{cases} 2n\pi T & \text{for bosons}\,, \\ (2n+1)\pi T & \text{for fermions}\,, \end{cases} \qquad n \in \mathbb{Z}$$

- At finite T, energy integral becomes sum over Matsubara frequencies

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}q_4}{2\pi} K(q_4) \to T \sum_{n=-\infty}^{\infty} K(\omega_n)$$

- Chemical potential corresponds to imaginary shift of energy

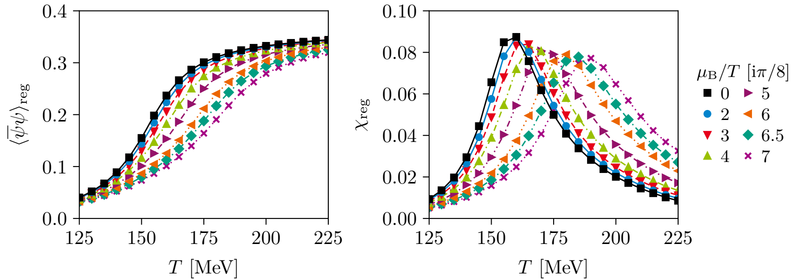
$$\omega_n \to \tilde{\omega}_n := \omega_n + \mathrm{i}\mu$$

  - Valid for $\mu \in \mathbb{C}$

Quantities of Interest

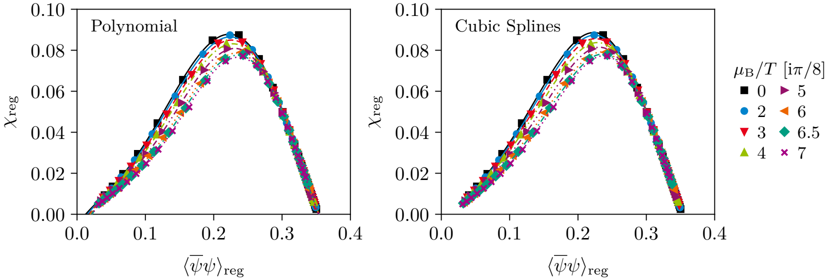
Quark Condensate and Susceptibility

$$\langle\overline{\psi}\psi\rangle(T) \sim T \sum_{\omega_n} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \operatorname{Tr}\left[S_{\mathrm{u}}(q)\right], \quad \chi(T) = \frac{\partial\langle\overline{\psi}\psi\rangle(T)}{\partial m_{\mathrm{u}}}$$



- Inflection point of $\langle\overline{\psi}\psi\rangle$ and maximum of χ move towards higher T for increasing $\mathrm{Im}(\mu_{\mathrm{B}})$
  - Qualitative agreement with the lattice

Determination of Pseudocritical Temperature



Procedure analogous to PRL 125 (2020) 052001:

1. Convert $\langle\overline{\psi}\psi\rangle(T)$ and $\chi(T)$ data to dependence $\chi\big(\langle\overline{\psi}\psi\rangle\big)$
2. Use either a fit to fifth-order polynomial or cubic-spline interpolation to determine peak position → defines $\langle\overline{\psi}\psi\rangle(T_c)$
3. Interpolate $\langle\overline{\psi}\psi\rangle(T)$ to extract T_c from $\langle\overline{\psi}\psi\rangle(T_c)$

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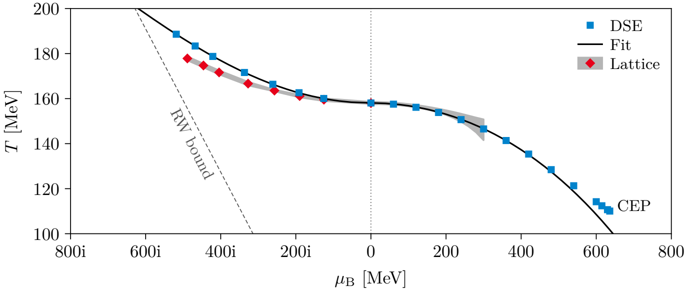
Quality of Extrapolation

Parametrization of the Crossover Line

$$\frac{T_c(\mu_B)}{T_c^0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c^0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c^0}\right)^4 , \quad T_c^0 = T_c(\mu_B = 0)$$



- Fit and extrapolation work very well up to $\mu_B \simeq 510$ MeV
  - Deviations in vicinity of CEP

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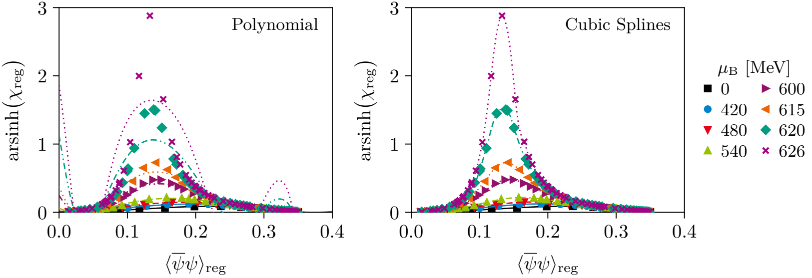
Behaviour in Vicinity of CEP



- Approximation of fifth-order polynomial breaks down
  - Expected since susceptibility has singularity at CEP

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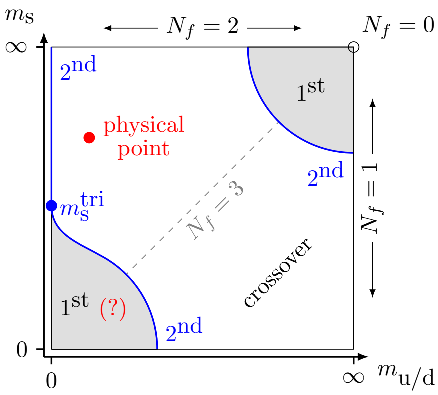
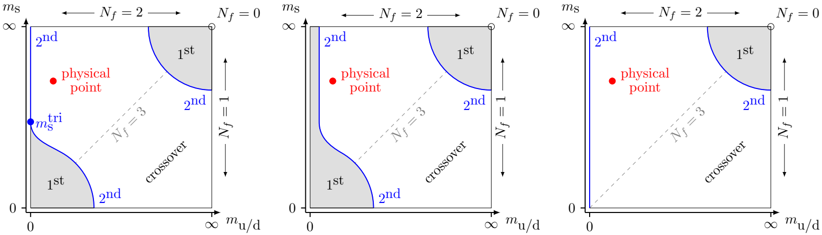
Second Objective: The Columbia Plot



Motivation: Columbia Plot(s)

for reference on upper right corner in DSE framework, see Fischer, Luecker, Pawlowski, PRD 91 (2015) 014024

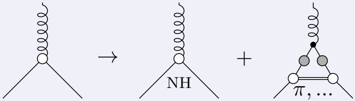


- Two different scenarios for Columbia plot: anomalously broken (left) or restored (middle) $U_A(1)$ symmetry
- Existence of first-order region in lower left corner (of left scenario) is not yet clear (right) see Cuteri, Philipsen, Sciarra, JHEP 11 (2021) 141
- Chiral limit is difficult for lattice QCD but no conceptual problem for our framework

Long-Range Correlations in Vertex

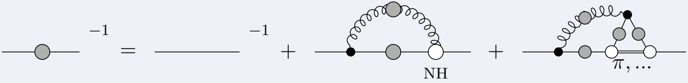
- In vicinity of second-order phase transitions, long-range correlations become important

Vertex Ansatz (from Expansion of Vertex DSE)



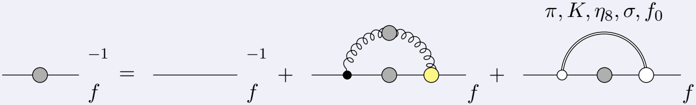
- Leads to modification of the quark self-energy → additional diagram

Resulting Quark DSE

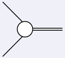


Meson-Backcoupling Setup

Quark DSE with Backcoupling Diagrams

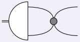


Bethe–Salpeter Amplitudes



(Goldberger–Treiman-like relations: quarks and decay constants)

Meson Decay Constants



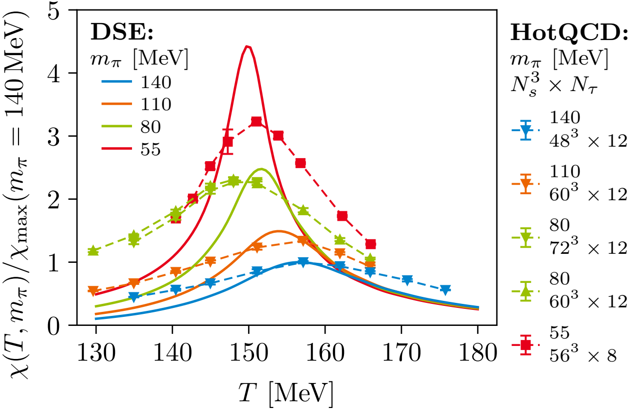
(Generalized Pagels–Stokar relation)

Free Meson Propagator

(Mass from Gell-Mann–Oakes–Renner fit)

details on meson backcoupling: Fischer, Müller, PRD 84 (2011) 054013; JB, Fischer, arXiv:2309.06737

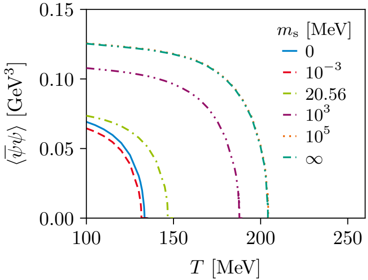
Towards the Chiral Limit

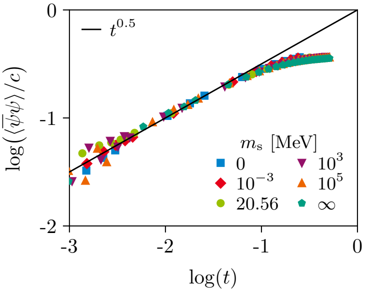


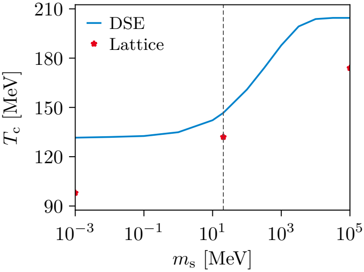
- Good qualitative agreement with lattice

JB, Fischer, arXiv:2309.06737

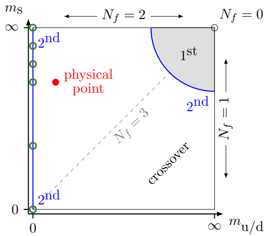
Results for Light-Quark Chiral Limit







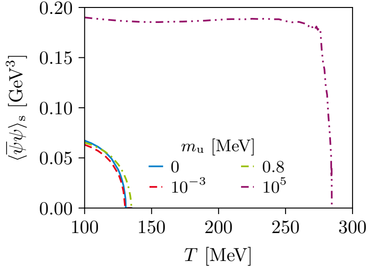
Resulting Columbia plot:



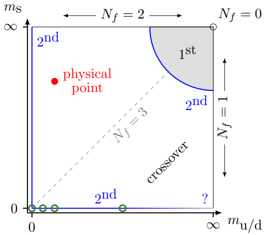
JB, Fischer, arXiv:2309.06737

Results for Strange-Quark Chiral Limit

- On bottom edge ($m_s \to 0$), strange-quark condensate $\langle\overline{\psi}\psi\rangle_s$ becomes order parameter:



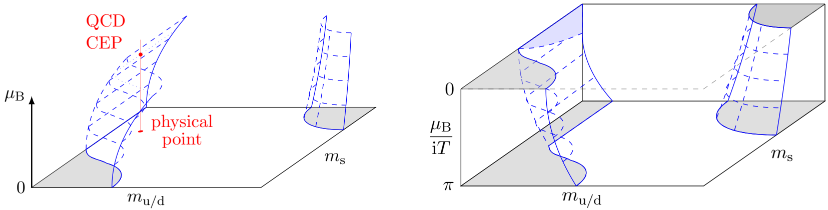
Resulting Columbia plot:



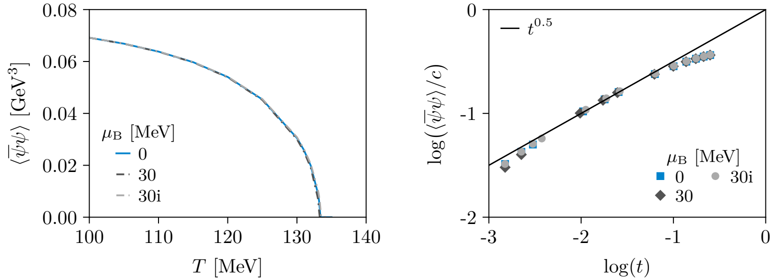
- For large range of light-quark masses ($m_u \lesssim 10^5$ MeV), also find second-order transition
  - Different symmetries in $N_f = 1$ corner $\to$ numerically inacessible at the moment

Three-Dimensional Extension of Columbia Plot

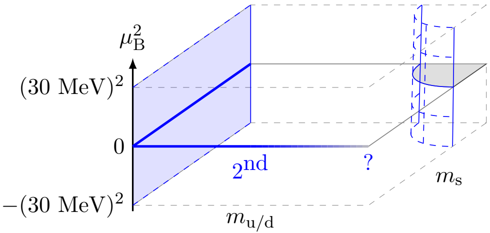
- Common extension of Columbia plot is to include (real and/or imaginary) chemical potential as third axis:



- Have only rough qualitative indications on behaviour of critical surface



- Resulting three-dimensional Columbia plot:



Conclusion and Outlook

Conclusion:

1. Studied QCD phase diagram at both real and imaginary chemical potentials using DSEs and gauged quality of extrapolation
   - Qualitative agreement with lattice
   - Extrapolation works almost perfectly across a very large portion of crossover line ($\sim 85\,\%$)
   - Naïve estimate of truncation error for position of CEP: $\sim 5 - 10\,\%$
2. Investigated impact of mesonic degrees of freedom on Columbia plot
   - See second-order phase transition across whole left edge, for both zero and small nonzero chemical potentials
   - See second-order transition also for large parts of bottom edge
   - No first-order region in $N_f = 3$ corner

Outlook:

- Investigate complex chemical potentials $\rightarrow$ Lee–Yang zeroes
- Study finite-volume effects with meson backcoupling

Backup Slides

Regularization of Quark Condensate and Susceptibility

Condensate and susceptibility are divergent for $m_\mathrm{u} > 0$, need to be regularized:

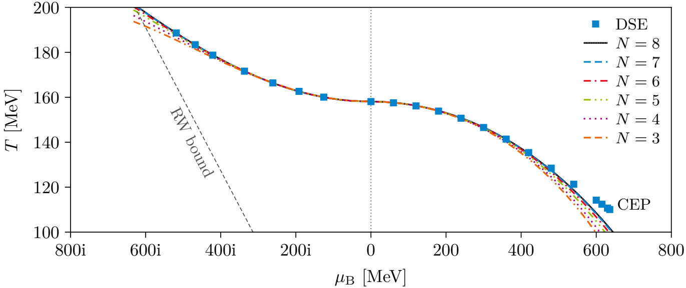
Imaginary-μ_B Analysis

$$\langle\overline{\psi}\psi\rangle_\mathrm{reg}(T) = \left[\langle\overline{\psi}\psi\rangle(T) - \langle\overline{\psi}\psi\rangle(0)\right]\frac{m_\mathrm{u}}{f_\pi^4}\,, \quad \chi_\mathrm{reg}(T) = \left[\chi(T) - \chi(0)\right]\frac{m_\mathrm{u}^2}{f_\pi^4}$$

Columbia-Plot Analysis (for Nonzero Quark Masses)

$$\langle\overline{\psi}\psi\rangle_\mathrm{reg}(T) = \langle\overline{\psi}\psi\rangle_\mathrm{u}(T) - \frac{m_\mathrm{u}}{m_\mathrm{s}}\langle\overline{\psi}\psi\rangle_\mathrm{s}(T)$$

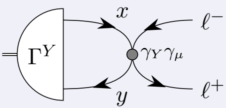
Sensitivity of Fit to Number of Input Points



- $N = 6$ points suffice → do not need to go all the way to RW bound

Meson Decay Constants

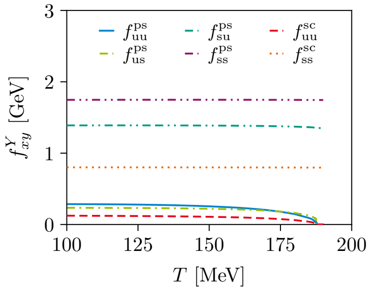
Generalized Pagels–Stokar Relation

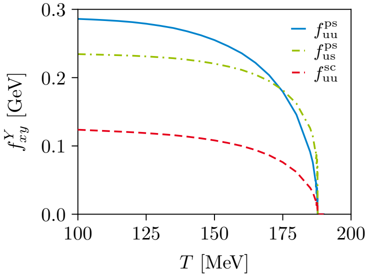
$P \to$ 

$= \mathrm{i}\tilde{P}_\mu \, f^Y_{xy}$

Decay constants in diagrams:

up	strange
π: $f^{\mathrm{ps}}_{\mathrm{uu}}$	
K: $f^{\mathrm{ps}}_{\mathrm{us}}$	K: $f^{\mathrm{ps}}_{\mathrm{su}}$
η_8: $f^{\mathrm{ps}}_{\mathrm{uu}}$	η_8: $f^{\mathrm{ps}}_{\mathrm{ss}}$
σ: $f^{\mathrm{sc}}_{\mathrm{uu}}$	f_0: $f^{\mathrm{sc}}_{\mathrm{ss}}$





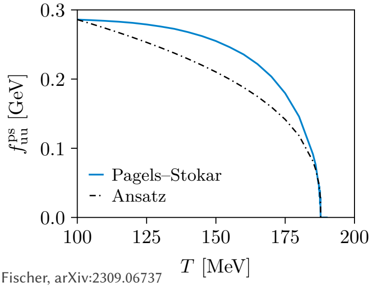
QCD Scaling

- We see mean-field critical exponents instead of $\mathrm{O}(4) \rightarrow$ can be bypassed: condensate inherits scaling behaviour from decay constants see Fischer, Müller, PRD 84 (2011) 054013

Scaling Ansatz

$$\tilde{f}^{Y}_{\mathrm{uu}}(T) = f^{Y}_{\mathrm{uu}}(T_0)\left(\frac{T_\mathrm{c} - T}{T_\mathrm{c} - T_0}\right)^{\beta}, \quad \tilde{f}^{\mathrm{ps}}_{\mathrm{us}}(T) = f^{\mathrm{ps}}_{\mathrm{us}}(T_0)\left(\frac{T_\mathrm{c} - T}{T_\mathrm{c} - T_0}\right)^{\beta/2},$$

$$\tilde{f}^{Y}_{xy}(T) = f^{Y}_{xy}(T_0)\,, \quad T_0 = 100\ \mathrm{MeV}\,, \quad \beta = 0.73/2$$



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