Kinetic and potential mechanisms for deuteron production in HICs

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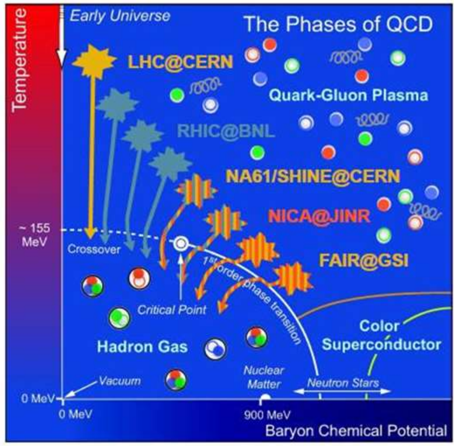
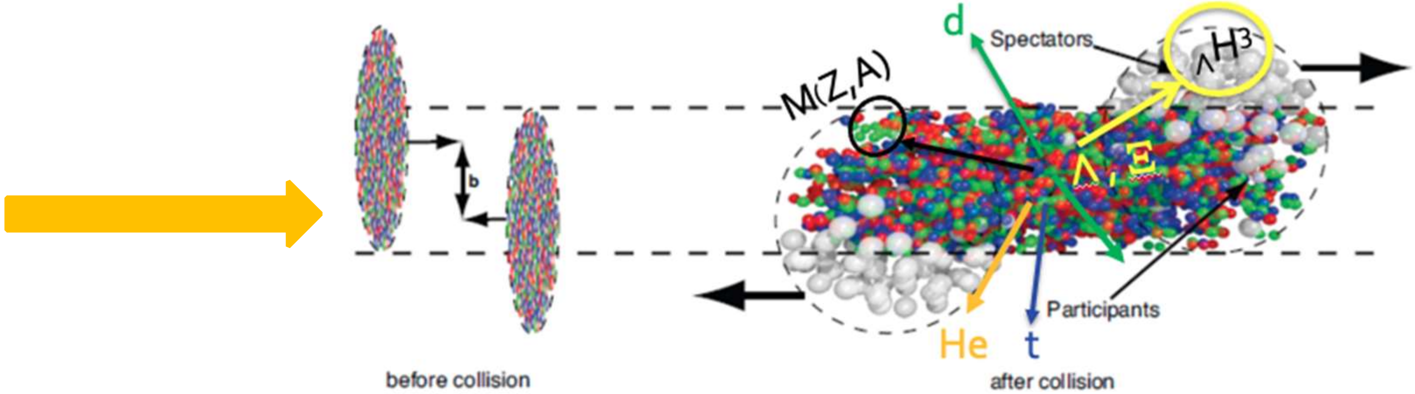
GSI Helmholtzzentrum für Schwerionenforschung

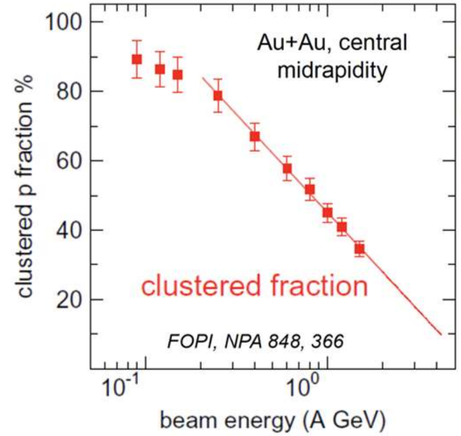
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

DFG Deutsche Forschungsgemeinschaft

Subatech

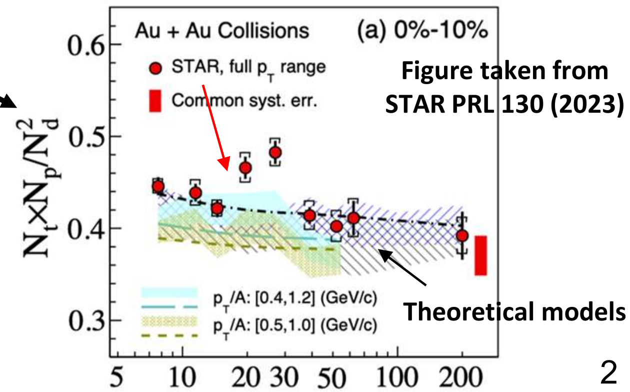






Observation of **light nuclei (d , t , ^{3}He , ^{4}He)** began with the first HICs facilities, then continued at AGS, GSI SIS, SPS, nowadays at RHIC, LHC and planned FAIR and NICA. However, **the mechanism of cluster formation in strongly interacting matter is not well understood.**

- At E_{LAB}∽1 AGeV about 40% of protons are bound → **p_T spectra** , **flow**.
- Yield ratios sensitive to density fluctuations near Critical Point (CP).
  [K. Sun, L. Chen, C.M. Ko PLB 774 103 (2017) , PLB 781 499 (2018)]
  [D. Oliinychenko at QM 2019 NPA 00 (2020) 1-9]



Production of hyper-nuclei ($_\Lambda H^3$, $_\Lambda H^4$, $_\Lambda He^3$, $_\Lambda He^4$) ⟸ Tom Reichert's talk, Apiwit Kittiratpattana's talk

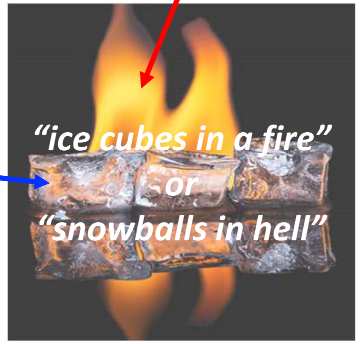
- At mid-rapidity by coalescence of Λ with participants during expansion
  → probe the phase space distributions of strange and non-strange baryons.
- At target/projectile by rescattering/absorption of Λ with spectators
  → carry information about NΛ interaction ('hyperon-puzzle' in neutron stars).

Existing models for cluster production:

❖ Statistical model: [Andronic, et al. PLB 697, 203 (2011), Nature 561, 321 (2018)] [Vovchenko et al. PLB 785, 171-174 (2018) , PLB 800, 135131 (2020)]

→ Assumption of a **globally equilibrated thermal source** at mid-rapidity $\left.\frac{dN_i}{dy}\right|_{y=0} = \frac{g_i V e^{\mu_i/T_f}}{2\pi^2} m_i^2 T_f K_2(m_i/T_f)$

→ Parameters (V , T_f , $\mu_i = B_i\,\mu_B + S_i\,\mu_s + I_{3i}\,\mu_{I3}$) fit to hadron multiplicities at **chemical freeze-out:** $T_f \backsim T_{CFO} \backsim$ **155 MeV** >> $|E_B(d)| \backsim$ **2 MeV**



❖ Coalescence models: [Butler, Pearson PRL 7 (1961)] → original nucleon coalescence for deuteron production
[Scheibl, Heinz PRC 59 (1999)]
[Oh, Lin, Ko PRC 80 (2009)] [Zhu, Ko, Yin PRC 92 (2015)]
[Sun, Chen, Ko et al. PRC 95 4 044905 (2017) , PLB 774 103 (2017) , PLB 781 499 (2018)]
[Sombun et al. PRC 99 (2019)] [Hillman et al. JPG 47 (2020) 5]

→ Spectra of light nuclei from phase-space distribution functions of nucleons $f_N(x,p)$ at **kinetic freeze-out**.
(differences: parameters (r_{coal} , p_{coal}) , inclusion of **deuteron Wigner function $W_d(r,p)$**)

$$\frac{dN_d}{d^3\mathbf{P}_d} = g_d \int d^3\mathbf{R} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int d^3\mathbf{r}\, f_p(\mathbf{R}+\mathbf{r}/2, \mathbf{P}_d/2+\mathbf{p}) f_n(\mathbf{R}-\mathbf{r}/2, \mathbf{P}_d/2-\mathbf{p}) W_d(\mathbf{r},\mathbf{p})$$

→ Experiments measure coalescence parameter B_A $\quad E_A\frac{dN_A}{d^3\mathbf{P}_A} = B_A\left(E_p\frac{dN_p}{d^3\mathbf{p}_p}\right)^Z \left(E_n\frac{dN_n}{d^3\mathbf{p}_n}\right)^{A-Z}\Bigg|_{p_p=p_n=P_A/A} \xrightarrow{f_n=f_p} E_A\frac{dN_A}{d^3\mathbf{P}_A} \approx B_A\left(E_p\frac{dN_p}{d^3\mathbf{p}_p}\right)^A\Bigg|_{p_p=P_A/A}$

$$B_A m_p^{A-1} \propto (1/V)^{A-1}$$

- Effect of $f_n = f_p$ approx. at low energy HICs [Kittiratpattana PRC 106 044905 (2022)]
- Model dependence → **important for DM observation in CRs** [Blum PRD 96 103021 (2017)]

- Both Coalescence and Thermal models provide good description of RHIC-STAR and LHC-ALICE exp. data.

- Spatial density fluctuations have been implemented in coalescence model.

[K.-J. Sun et al. PLB 774 103 (2017) , PLB 781 499 (2018)]

- However, cluster production is limited at some fixed time of HICs evolution, either chemical or kinetic freeze-out.

In order to understand the **microscopic origin** of cluster formation
a **realistic description of the dynamical evolution of HICs** is necessary → **TRANSPORT MODELS**

- In this talk:

Two **microscopic mechanisms** for deuteron production in HICs:

PHQMD

1) **"Potential"** → deuterons are identified as "bound" p-n pair by means of **clusterization algorithms**.

2) **"Kinetic"** → deuterons are produced/destroyed by **hadronic reactions** NN ↔ dπ , π **N N** ↔ π **d** , **N N N** ↔ **N d** ,
which can be implemented in Boltzmann-like collision integral.

SMASH (hydro + transport):
[D. Oliinychenko et al. PRC 99 (2019) 4, 044907 , PRC 103 (2021) 034913]
[J. Staudenmaier et al. PRC 104 (2021) 3, 034908]

AMPT (hydro + transport):
[K.-J. Sun et al. arxiv:2106.12742, R.-Q. Wang et al. PRC 108 (2023) 3]

Parton-Hadron Quantum Molecular Dynamics

- Model: A **unified n-body microscopic transport approach** for the description of HICs and **dynamical cluster formation** from low to ultra-relativistic energies.
- Realization: (**PHSD** + **QMD**) & **MST/SACA**.

[J. Aichelin et al. PRC 101 (2020) 044905]

Quantum Molecular Dynamics (QMD)
Initialization & Propagation of baryons

Elena Bratkovskaya's talk

Parton-Hadron-String-Dynamics (PHSD)
Propagation and interaction of partons and mesons
QGP phase by Dynamical Quasi-Particle Model (**DQPM**)

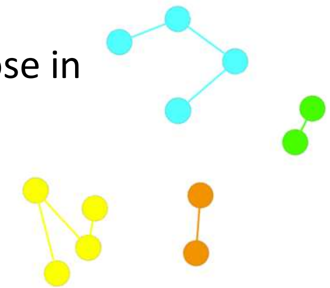
Olga Soloveva's talk
Ilia Grishmanovskii's talk

Cluster Identification
Simulated Annealing Clusterization Algorithm (SACA)
Minimum Spanning Tree → advanced MST (aMST)

Baryons described by *n*-body Wigner functions, preserve many-body correlations.
J. Aichelin Phys. Rep. 202, (1991) 233
C. Hartnack, Puri, Aichelin et al. EPJ A 1, (1998)

Collision Integral from PHSD
⍰ reactions of partons and hadrons
W. Cassing, E. Bratkovskaya, NPA 831, (2009)
P. Moreau, O. Soloveva, et al. PRC 100 (2019)
deuterons in this work [G. Coci et al. PRC 108 (2023) 014902]

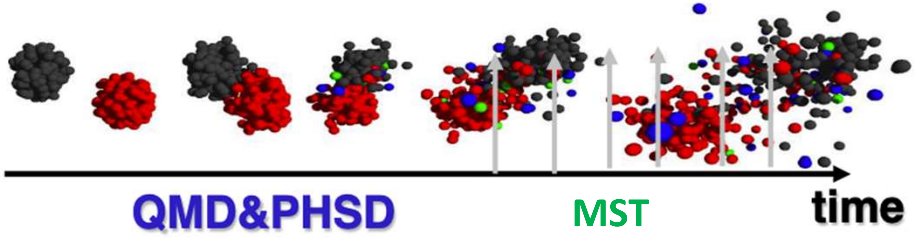
Identify clusters as baryons close in coordinate space.
S. Gläßel et al. PRC 105, (2022) 01498.
V. Kireyeu et al. PRC 105, (2022) 04909
V. Kireyeu et al. arxiv 2304.12019



Cluster identification via Minimum-Spanning Tree (MST)

The Minimum Spanning Tree (MST) is a cluster recognition algorithm which is applied in the asymptotic final state.

- At time snapshots MST searches for correlations of nucleons in coordinate space:



[Puri, Aichelin, J.Comp. Phys. 162 (2000) 245]

[J. Aichelin Phys. Rept. 202, 233 (1991)]

1. Two baryons are part of a cluster if their distance in the cluster rest frame fulfills: $|\vec{r_i} - \vec{r_j}| \leq 4$ fm

2. A baryon belongs to some cluster if it is “bound” at least to one baryon of that cluster.

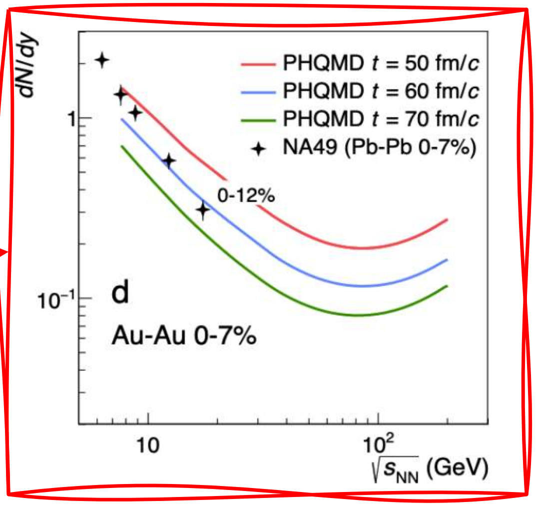
[S. Gläßel et al. PRC 105, (2022) 01498]

In previous PHQMD MST analysis cluster observables at “physical” time:

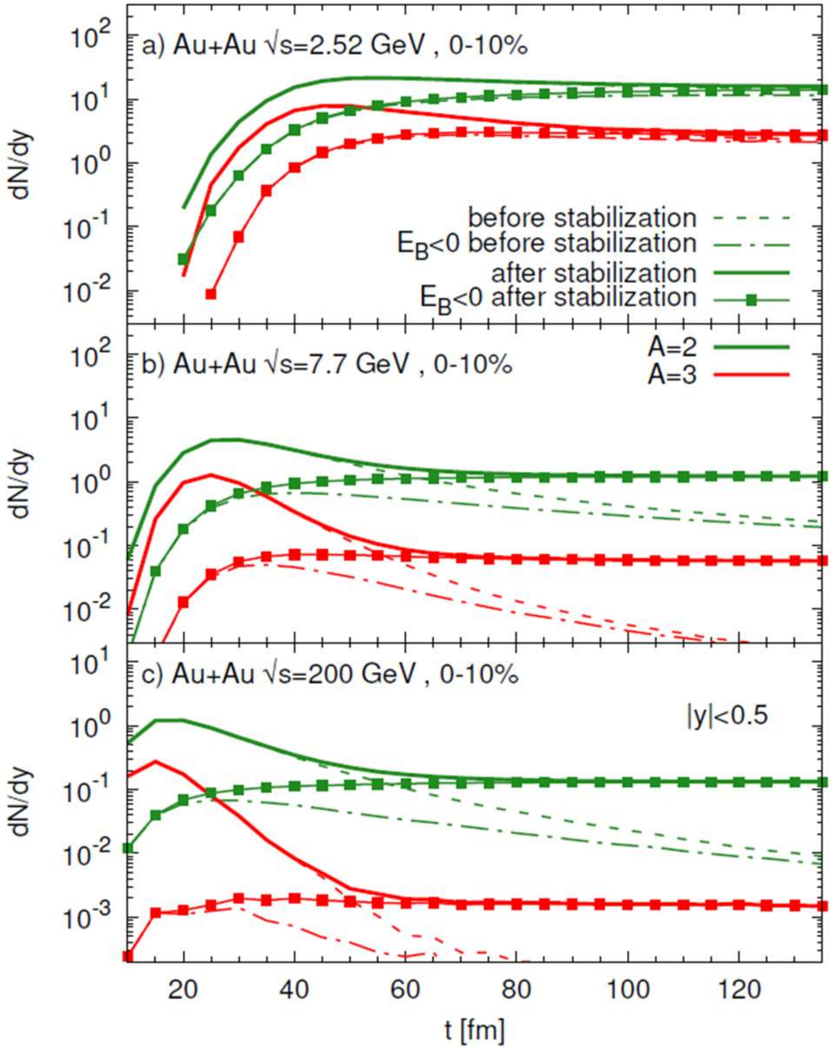
$$t = \cosh(y_{cm}) * t_0$$

Selected to give best description of exp. multiplicity at mid-rapidity

time dilatation between HICs c.m.s. and cluster rest-frame



Advanced Minimum-Spanning Tree (aMST)



[G.C. et al. PRC 108 (2023) 014902, V. Kireyeu et al. arxiv 2304.23019]

- In **semiclassical** approach (as QMD) a cluster which is "bound" at time *t* can **spontaneously** dissolve at $t + \Delta t$.

✓ This is a **numerical artifact**… we lose clusters at relativistic energies because
→. the sign of E_B changes: $E_B(t) < 0$ → $E_B(t + \Delta t) > 0$
→ a single energetic nucleon escapes

Solution through **Stabilization Procedure**

1. Nucleons entering MST can be part of a cluster only after their **last elastic or inelastic collision**.

2. **Only nucleons which belong to bound (E_B < 0) clusters stay together.**

3. **Recombine nucleons into a cluster by freezing its internal degrees of freedom.**

4. Applied after the full PHQMD "collision history" to preserve reaction dynamics.

Covariant Rate Formalism

[W. Cassing NPA 700 (2000) 618]
[E. Seifert, W. Cassing, PRC 97 (2018) 024913]

PHSD: multi-meson fusion reactions
$m_1+m_2+...+m_n \leftrightarrow$ B+Bbar
m=π,ρ,ω,... B=p,$\Lambda,\Sigma,\Xi,\Omega$ (>2000 channels)

- In Boltzmann Equation the Collision Integral accounts for all dissipative processes

$$p_{1,\mu}\partial_x^\mu f_i(x,p_1) = I^i_{coll} = \sum_n \sum_m I^i_{coll}[n \leftrightarrow m]$$

$$I^i_{coll}[n \leftrightarrow m] = \frac{1}{2}\frac{1}{N_{id}!}\sum_\nu \sum_\lambda \left(\frac{1}{(2\pi)^3}\right)^{n+m-1} \left(\prod_{j=2}^{n}\int \frac{d^3\vec{p}_j}{2E_j}\right)\left(\prod_{k=n+1}^{n+m}\int \frac{d^3\vec{p}_k}{2E_k}\right)$$

$$\times (2\pi)^4 \delta^4(p_1^\mu + \sum_{j=2}^{n} p_j^\mu - \sum_{k=1}^{n+m} p_k^\mu)\, W_{n,m}(p_1,p_j;i,\nu \mid p_k;\lambda)$$

$$\times \left[\prod_{k=n+1}^{n+m} f_k(x,p_k) - f_i(x,p_1)\prod_{j=2}^{n} f_j(x,p_j)\right]$$

(n-1) initial + m final integrations

Transition amplitude

Gain - Loss

- Collision rate for hadron "i" is the number of reactions in the covariant volume d^4x = dt*dV

$$\frac{dN_{coll}[n(i)\to m]}{dtdV} \propto \int \frac{d^3p_1}{2E_1} f_i(x,p_1) \int \left(\prod_{j=2}^{n}\frac{d^3p_j}{2E_j} f_j(x,p_j)\right) \int \left(\prod_{k=n+1}^{n+m}\frac{d^3p_k}{2E_k}\right)$$

$$\times (2\pi)^4 \delta^4(\sum_{j=1}^{n} p_j^\mu - \sum_{k=n+1}^{n+m} p_k^\mu)\, W_{n,m}(p_j;\tau(i),\nu \mid p_k;\lambda)$$

... similar for m → n(i)

Covariant Rate Formalism

[W. Cassing NPA 700 (2000) 618]
[E. Seifert, W. Cassing, PRC 97 (2018) 024913]

- With n=2 initial particles , the covariant rate can be expressed in terms of the reaction cross section

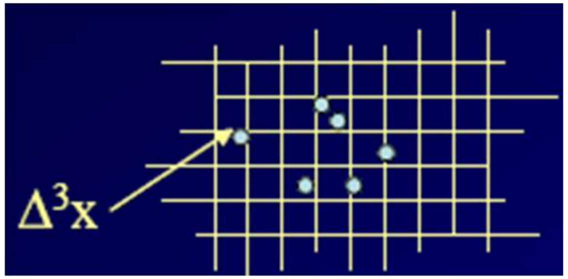
$$\frac{dN_{coll}[1(d)+2\rightarrow 3+4]}{dtdV} \propto \frac{1}{(2\pi)^6}\int\frac{d^3p_1}{2E_1}f_1(x,p_1)\int\frac{d^3p_2}{2E_2}f_2(x,p_2)\times$$

$$\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}W_{2,2}(p_1,p_2;p_3,p_4)(2\pi)^4\,\delta^4(p_1+p_2-p_3-p_4) \longrightarrow 4E_1E_2v_{rel}\sigma_{2,2}(\sqrt{s})$$

- Using test-particle ansatz for *f(x,p)* the collision integral is numerically solved dividing the coordinate space in cells of volume ΔV_{cell} where the reaction rate at each time step Δt are sampled **stochastically** with probability:

$$\frac{\Delta N_{coll}[1(d)+2\rightarrow 3+4]}{\Delta N_1 \Delta N_2} = P_{2,2}(\sqrt{s}) = v_{rel}\sigma_{2,2}(\sqrt{s})\frac{\Delta t}{\Delta V_{cell}}$$

Similarly… $$\frac{\Delta N_{coll}[1(d)+2\rightarrow 3+4+5]}{\Delta N_1 \Delta N_2} = P_{2,3}(\sqrt{s}) = v_{rel}\sigma_{2,3}(\sqrt{s})\frac{\Delta t}{\Delta V_{cell}}$$



[Lang, Babovsky, Cassing, Mosel, Reusch and Weber, J. Comp. Phys., vol. 106, no. 2, (1993)]
[Xu and Greiner PRC v. 71, (2005)]

- $\Delta t \rightarrow 0$, $\Delta v_{cell} \rightarrow 0$ convergence to exact solution

Covariant Rate Formalism for kinetic deuterons

- With n > 2 initial particles , the covariant rate cannot be expressed in terms of the reaction cross section

$$\frac{dN_{coll}[3+4+5 \to 1(d)+2]}{dtdV} = \int \left(\prod_{k=3}^{5} \frac{d^3p_k}{(2\pi)^3 2E_k} f_k(x,p_k) \right) \times$$
$$\int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} W_{3,2}(p_3,p_4,p_5;p_1,p_2)(2\pi)^4 \, \delta(p_1+p_2-p_3-p_4-p_5)$$

- With the ASSUMPTION for the TRANSITION AMPLITUDE: $W(\sqrt{s})$ **+ detailed balance**

the covariant rate can be still expressed in terms of the **collision probability**. With test-particle ansatz:

$$\frac{\Delta N_{coll}[3+4+5 \to 1(d)+2]}{\Delta N_3 \Delta N_4 \Delta N_5} = P_{3,2}(\sqrt{s})$$

[W. Cassing NPA 700 (2002) 618]

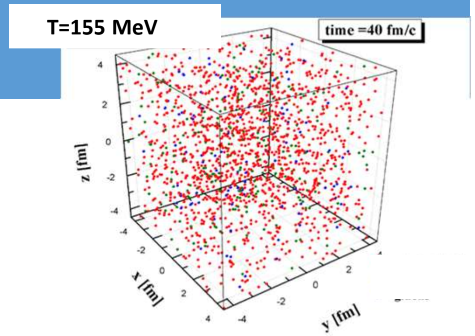
$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3E_4E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

$2 \longrightarrow 3$ cross section

Energy and momentum of final particles

2,3-body phase spaces [Byckling, Kajantie]

Deuteron reactions in the box (1)



π+p+n⟷ π+d , N+p+n ⟷ N+d , N+N ⟷ d+π , d+X elastic

- 2 ⟶ 2 and 2 ⟶ 3 either by geometric criterium or stochastic method.

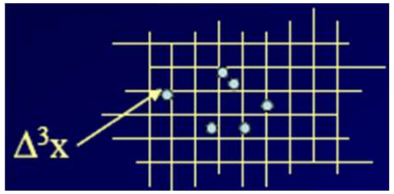
[Kodama et al. PRC 29 (1984)] **[W. Cassing NPA 700 (2002)]**

$$d_T < \sqrt{\frac{\sigma^{2,3}_{tot}(\sqrt{s})}{\pi}}$$

$$P_{2,3}\left(\sqrt{s}\right) = \sigma^{2,3}_{tot}(\sqrt{s})v_{rel}\frac{\Delta t}{\Delta V_{cell}}$$

- 2 ⟵ 3 realized via **covariant rate formalism** by stochastic method.

- Numerically tested in “static” box.



- Agreement with analytic solutions from rate eqs.

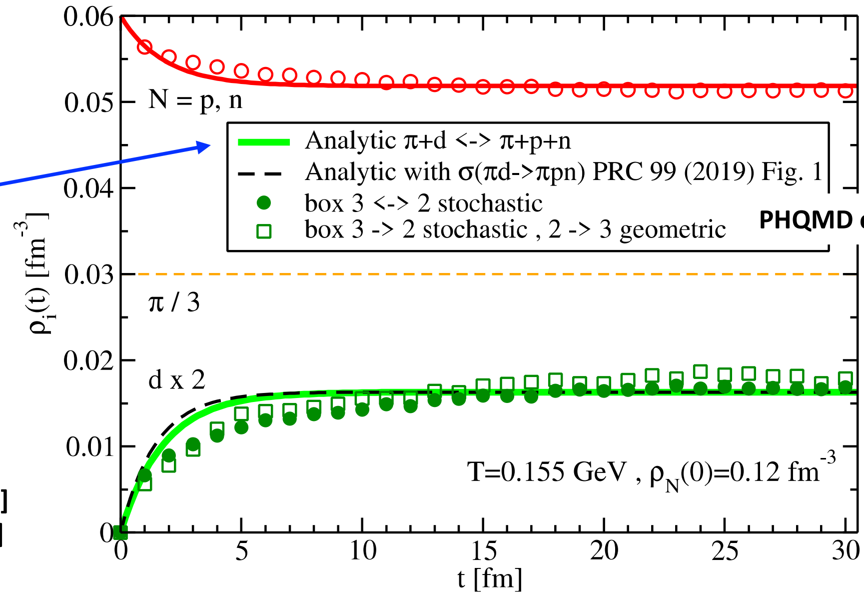
$$\begin{cases} \dot{\lambda}_d = \sum < v_{rel}\sigma_{\pi d} > \left(\frac{g_d g_\pi}{g_N^2 g_\pi}\lambda_N^2 - \lambda_d\right) n_\pi^{eq}\lambda_\pi \\ \dot{\lambda}_N = -\sum < v_{rel}\sigma_{\pi d} > \left(\frac{g_d g_\pi}{g_N^2 g_\pi}\lambda_N^2 - \lambda_d\right) n_\pi^{eq}\lambda_\pi \\ \dot{\lambda}_\pi = 0 \end{cases}$$

Particle densities in the box: $\rho_i = n^{eq}(T)*\lambda_i(t)$

[Y. Pan , S. Pratt PRC 89 (2014), 044911]
[T. Neidig et al. PLB 827 (2022), 136891]

Comparison with SMASH box results: **F_{iso} = 1**

J. Staudenmaier et al., PRC 104 (2021) 3, 034908



PHQMD coll. int.

Deuteron reactions in the box (2)

$$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$$

$$\pi^- + p + p \leftrightarrow \pi^0 + d$$
$$\pi^+ + n + n \leftrightarrow \pi^0 + d$$
$$\pi^0 + p + p \leftrightarrow \pi^+ + d$$
$$\pi^0 + n + n \leftrightarrow \pi^- + d$$

π+N+N⟷ π+d , N+p+n ⟷ d+N , N+N ⟷ d+π , d+X elastic

[G.C. et al. PRC 108 (2023) 014902]

Novel aspect in PHQMD:
N+N+π inclusion of all possible channels allowed by total isospin T conservation:

$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3 E_4 E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

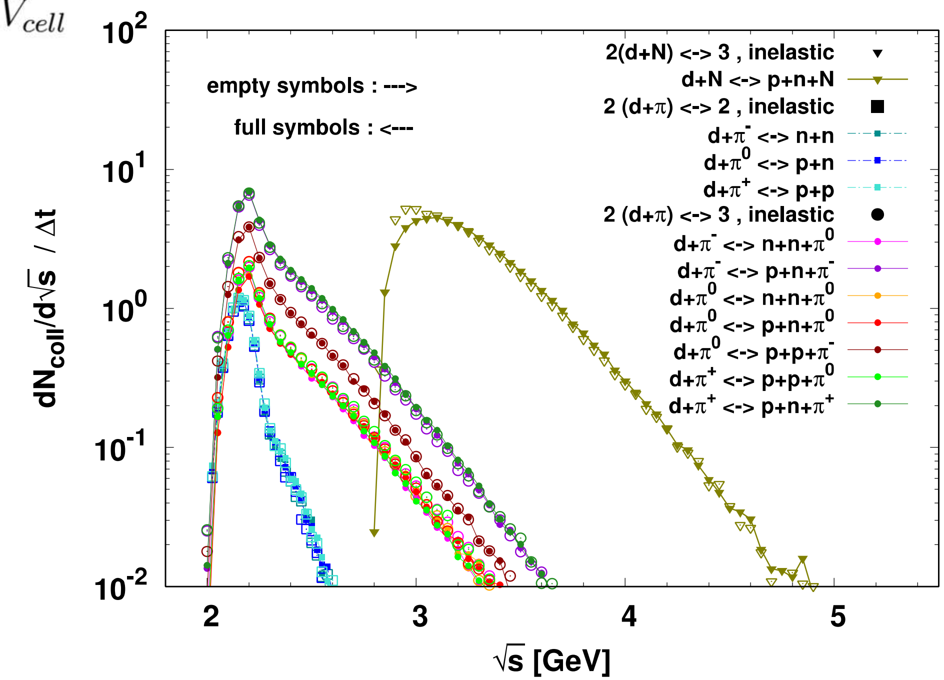
- NNπ expanded as superposition of eigenstates of total isospin T

$$|N, N, \pi\rangle = \sum_T \sum_{T_3=-T}^{-T} \langle T, T_3 | N, N, \pi\rangle \, |T, T_3\rangle$$

- Fourier coefficient of eigenstate of total isospin 1 (= T(d π)=T(π))

$$F_{iso} = |\langle N, N, \pi | T(d+\pi) = 1, T_3\rangle|^2$$

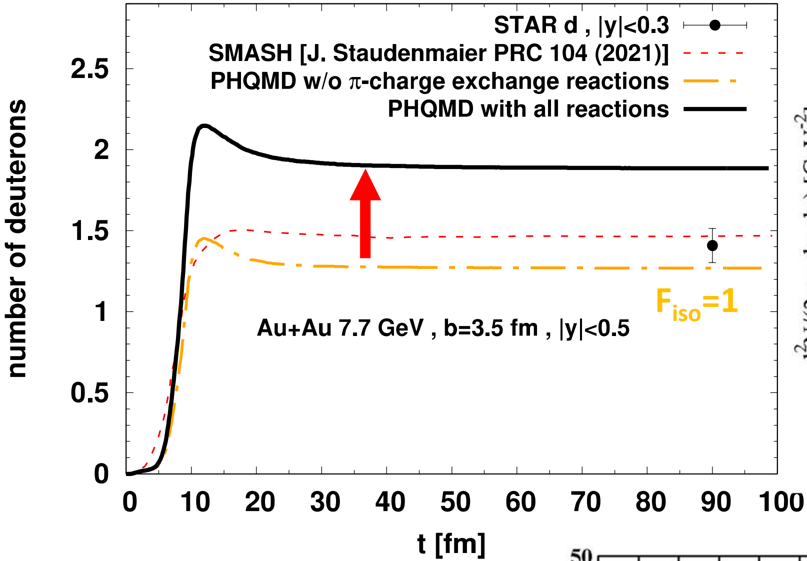
✓ **Detailed balance condition verified !**

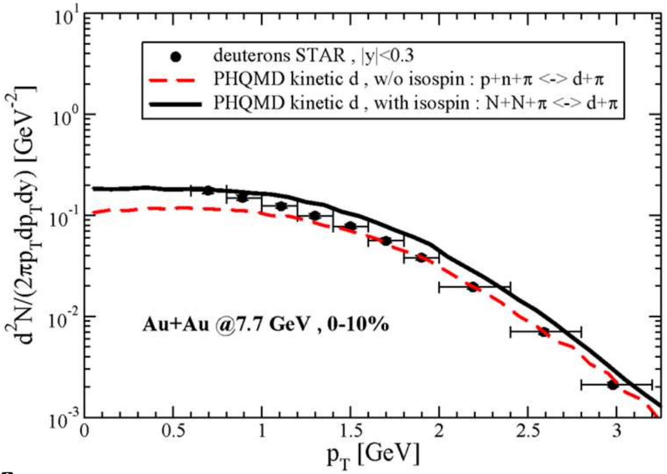


Kinetic deuterons in PHQMD

RHIC BES energy √s = 7.7 GeV:

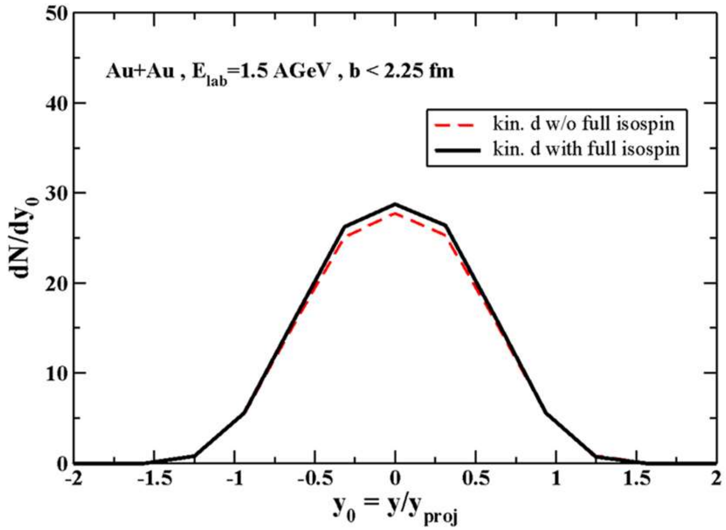
- Hierarchy due to large π abundance **π+N+N ⟷ π+d >> N+p+n ⟷ N+d**
- **π+N+N ⟷ π+d without charge-exchange (same as in SMASH)**
- **Inclusion of all channels enhances deuteron yield ~ 50%.**
- p_T slope is not affected.





GSI SIS energy √s < 3GeV :

- Baryonic dominated matter.
- **Enhancement due to inclusion of isospin channels is negligible.**



Modelling finite-size effects in kinetic mechanism

In QM the deuteron is a broad p-n bound system. It is reasonable to assume that, as soon as a deuteron is formed, it is immediately destroyed in high density regions.

We model this effect implementing an Excluded-Volume Condition:

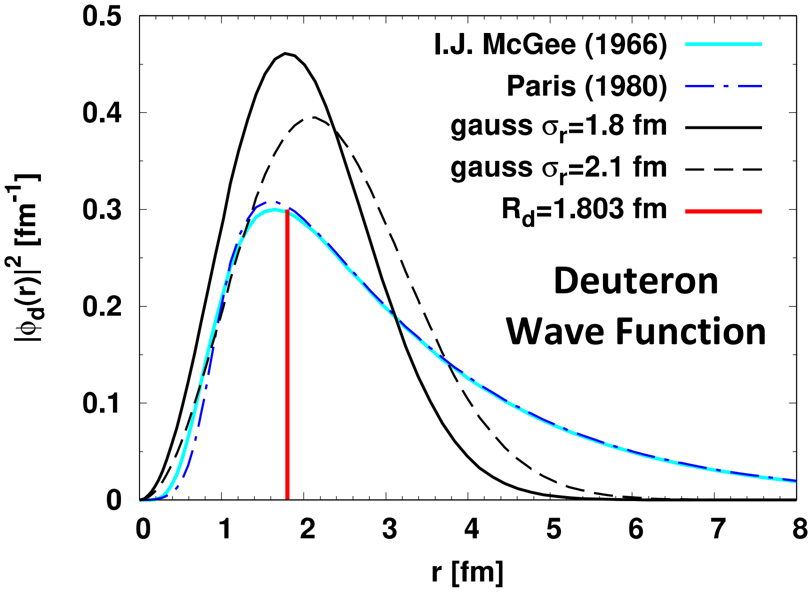
$$|\vec{r}(i)^* - \vec{r}(d)^*| > R_d$$

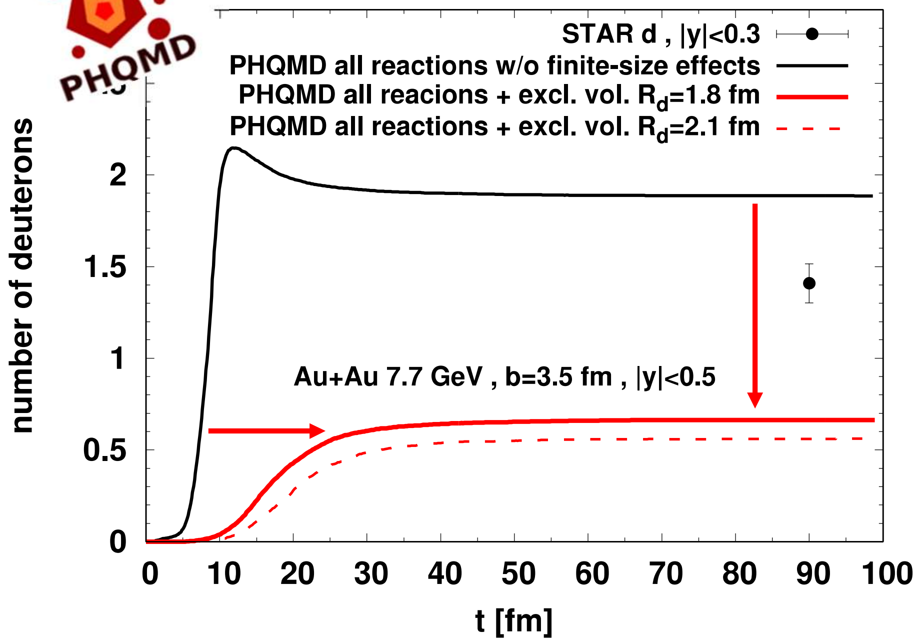
"i" is any particle not participating in πNN→ πd , NNN → Nd , NN → dπ

* means that positions are in the c.m.s. of pre-calculated "candidate" deuteron

The exclusion parameter R_d is tuned to the physical radius

$$R_d^2 \simeq < r_m^2 > = \int_0^\infty dr\, r^2 |\phi_d(r)|^2$$

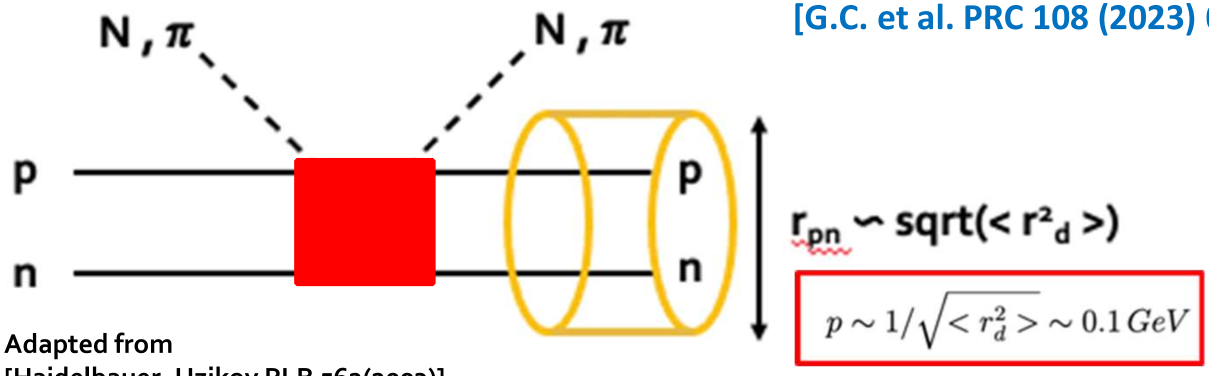




p_T slope is not affected!

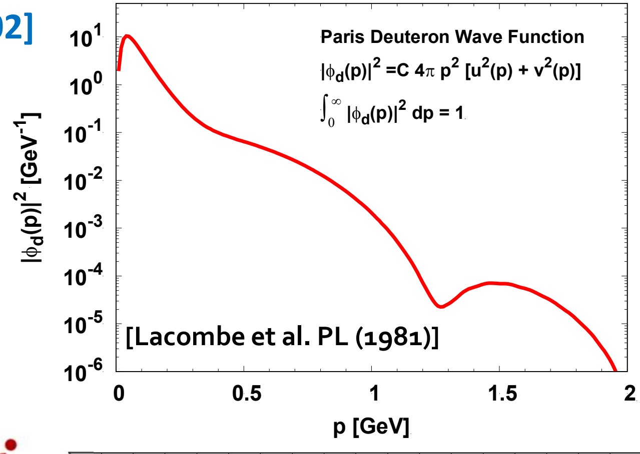
Modelling finite-size effects in kinetic mechanism

QM properties of deuteron must be also in momentum space → **momentum correlations of pn-pairs**



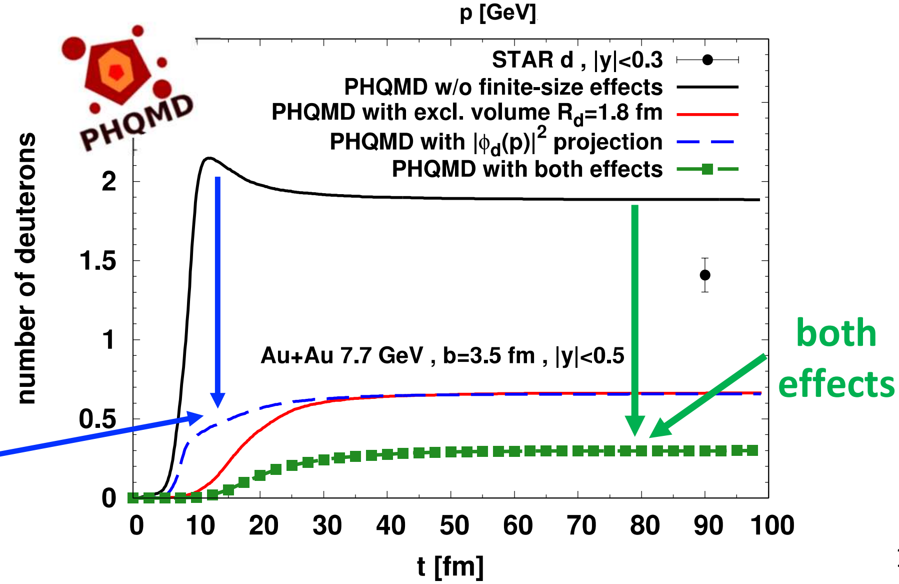
Adapted from
[Haidelbauer, Uzikov PLB 562(2003)]
[Hoftiezer et al. PRC23 (1981)]

[G.C. et al. PRC 108 (2023) 014902]



Similar to IA [Sun et al. arxiv:2106.12742] but fully in covariant rate formalism

- The probability of the pn-pair to bind into a final deuteron with momentum p is given by the DWF $|\phi_d(p)|^2$.
- For a "candidate" deuteron we calculate the relative momentum p of the interacting pn-pair in the deuteron rest frame.
- **We select bound pn-pairs in π+N+N↔ π+d and N+N+N ↔ N+d by projection on DWF $|\phi_d(p)|^2$.**



PHQMD results: combine two dynamical processes

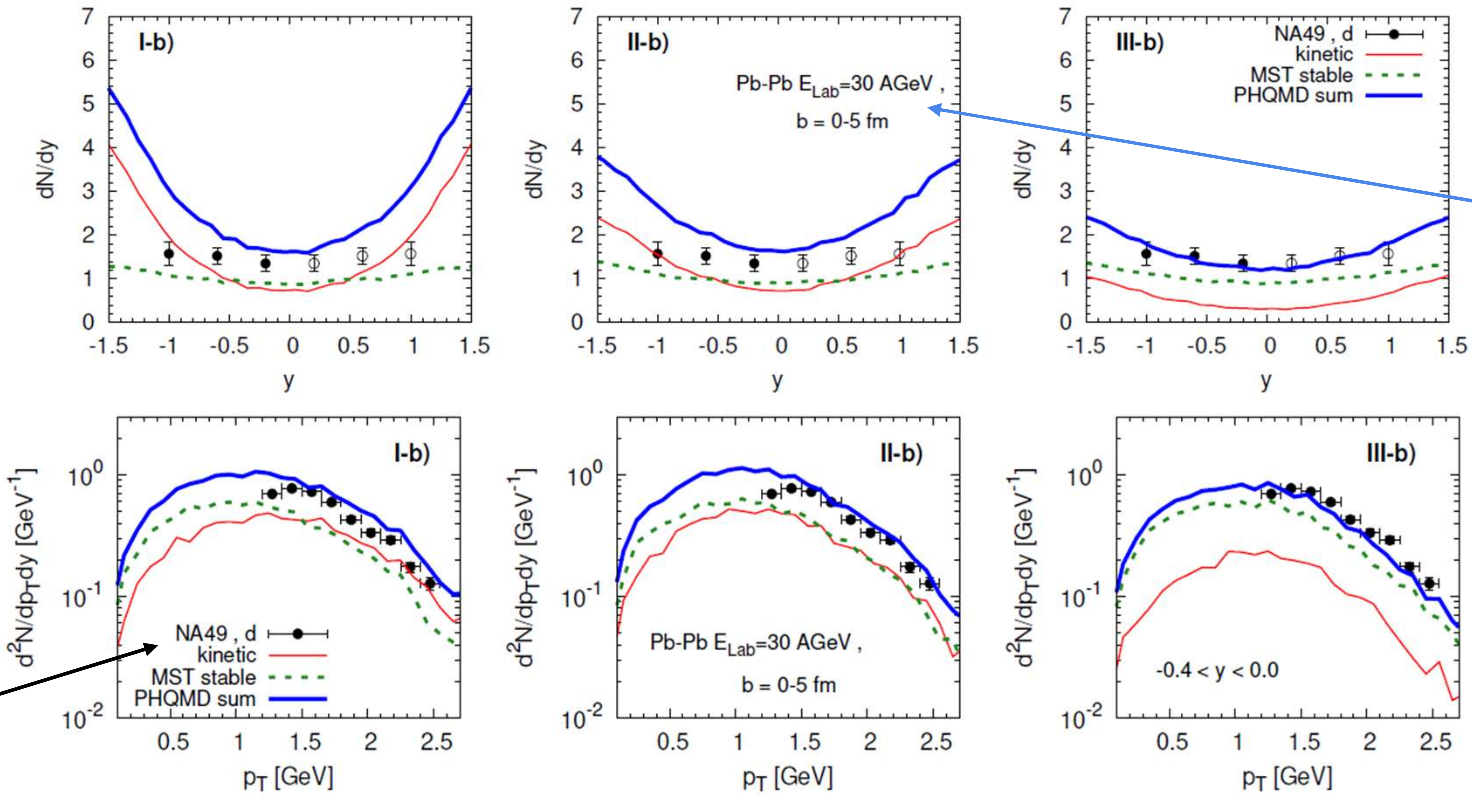
PHQMD

Kinetic with finite-size effects + **aMST bound (E_B<0) A=2 , Z=1 clusters** = **Total deuteron production**

- Avoid **double counting** → kinetic deuterons are not identified as MST clusters.
- Study the impact finite-size “scenarios” at different collision energies and compare with experimental data.

[G.C. et al. PRC 108 (2023)]

I) excluded-volume **II) Momentum projection** **III) both effects**

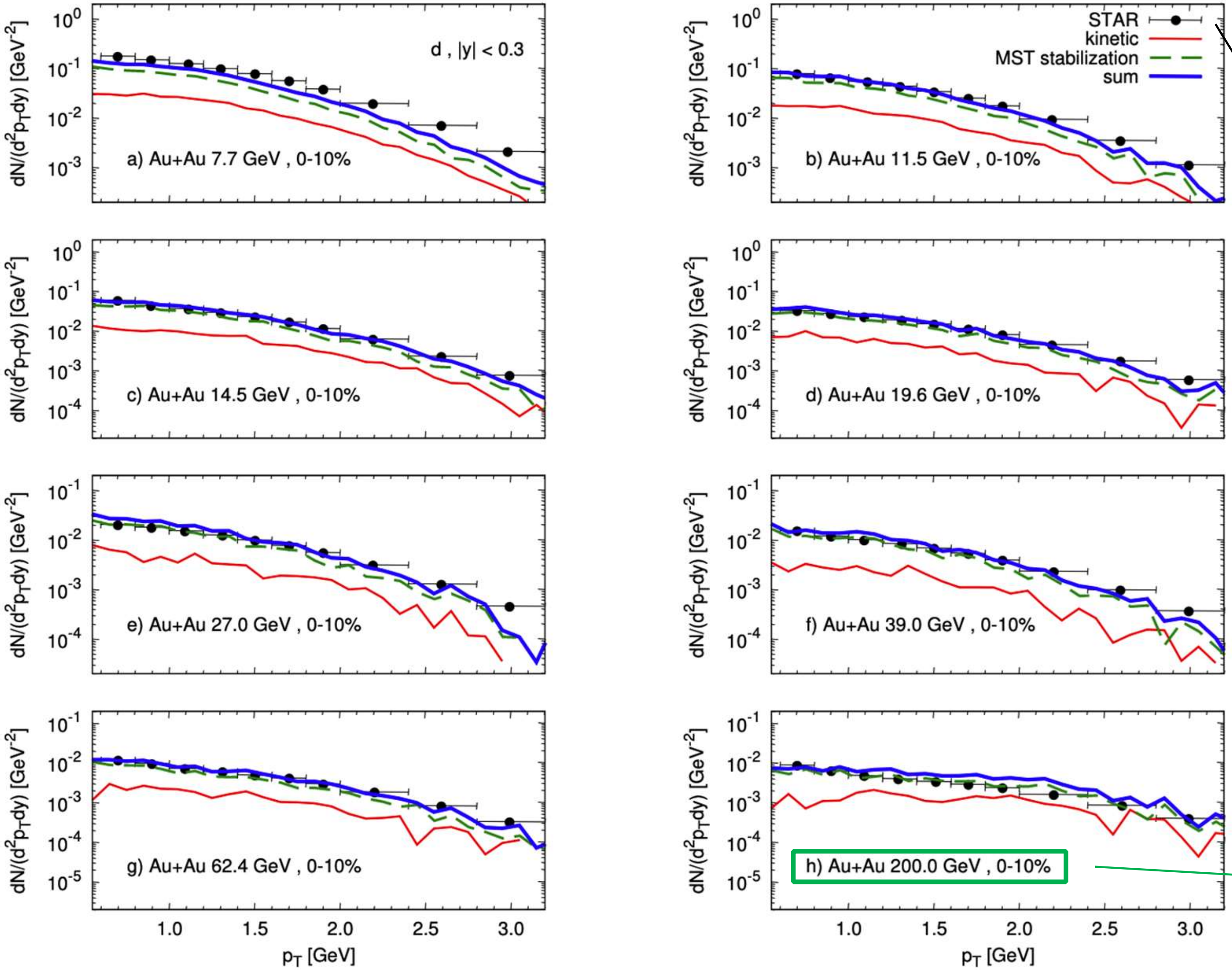


SPS central collisions

NA49 data
[PRC 94 (2016) 04490699]

PHQMD results: comparison with exp. data

[G.C. et al. PRC 108 (2023)]

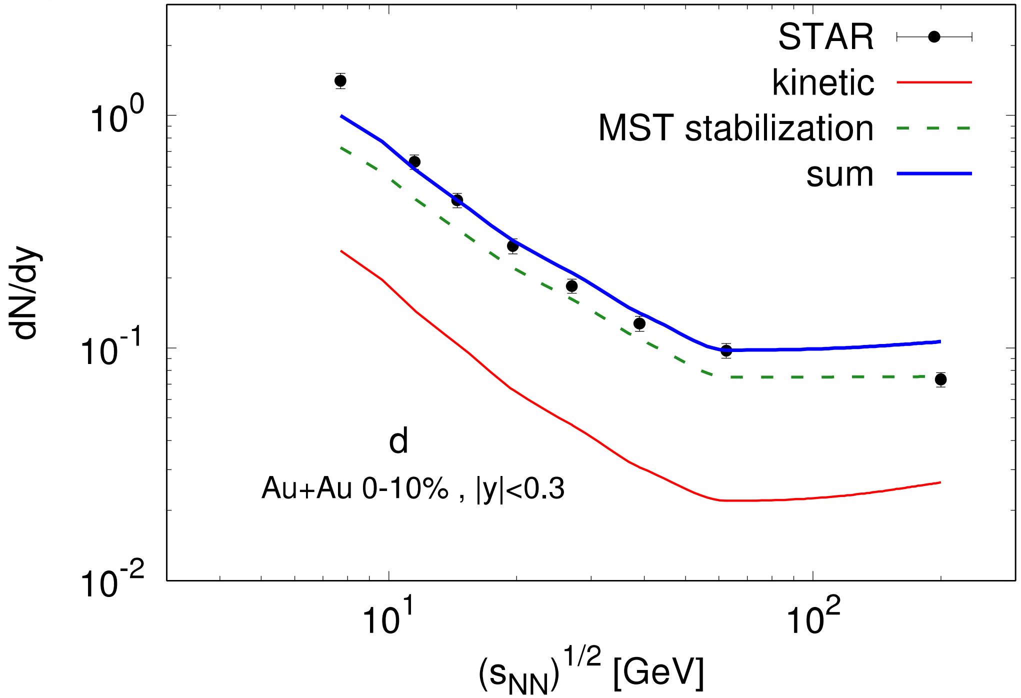


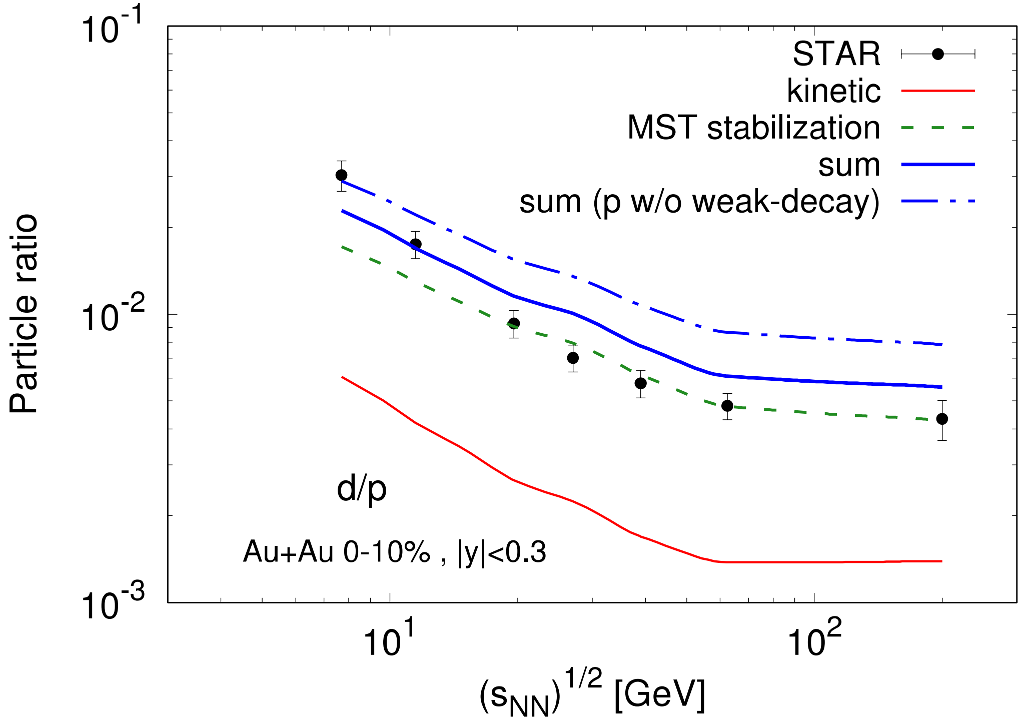
experimental data [PRC 99 (2019)]

top RHIC energy

Kinetic with both finite-size effects + **Potential** = **Total contribution** → Good description of mid-rapidity STAR data

PHQMD results: comparison with exp. data





- Comparison with d observables at SIS, AGS, SPS in [G.C. et al. PRC 108 (2023) 014902, V. Kireyeu et al. arxiv 2304.23019]
- **The potential mechanism is larger than the kinetic production at all energies !**

Summary:

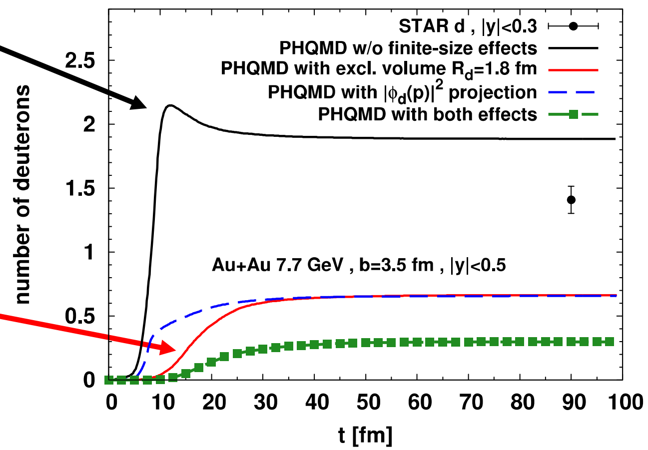
[G.C. et al. PRC 108 (2023) 014902]

"Kinetic" mechanism

- Hadronic reactions for deuteron formation/disintegration are implemented in PHQMD transport approach with inclusion of full *"isospin decomposition".*
  → **enhancement of d production at RHIC BES.**

$$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$$
$$\pi^{-} + p + p \leftrightarrow \pi^{0} + d$$
$$\pi^{+} + n + n \leftrightarrow \pi^{0} + d$$
$$\pi^{0} + p + p \leftrightarrow \pi^{+} + d$$
$$\pi^{0} + n + n \leftrightarrow \pi^{-} + d$$

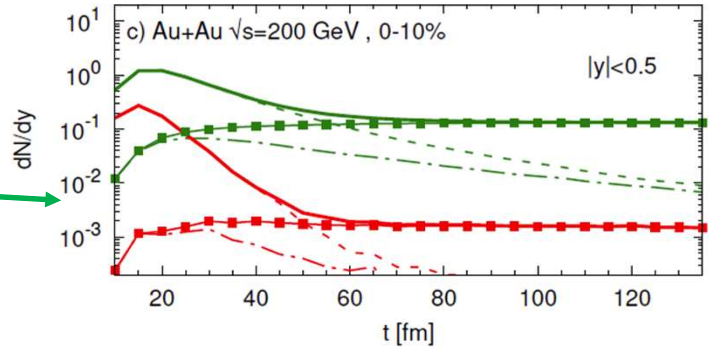
- **Quantum properties of the deuteron** can be captured by **finite-size effects**, modeled by the **excluded-volume condition** in coordinate space and by the **projection of the relative momentum of the interacting pn-pair on the Deuteron Wave-Function** in momentum space.
  → **kinetic production strongly reduced.**
  → target/projectile sensitive to different finite-size effects.



"Potential" mechanism

- In PHQMD clusters produced **dynamically by potential interaction** among nucleons are identified by **Minimum-Spanning-Tree (MST)** algorithm.

- Within the novel **advanced MST (aMST)** procedure "bound" (E_B <0) clusters are kept **stable** during the entire evolution of relativistic HICs.



Thank you for your attention!

Backup Slides

QMD propagation

Equation of Motions (EoM) derived from **generalized Ritz variational principle** **[Feldmeier NPA 515 (1990)]**

$$\delta \int_{t_1}^{t_2} dt \, \langle \psi(t) | \, i \frac{d}{dt} - H \, | \psi(t) \rangle = 0$$

ψ(t) is the quantum wavefunction for the N-particles system. **[Aichelin Phys. Rept. 202 (1991)]**

- *Assume* $\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r_i}, \mathbf{r_{i0}}, \mathbf{p_i}, \mathbf{p_{i0}}, t)$ *(neglect N-antisymmetrization)*
- *Ansatz* $\psi(\mathbf{r_i}, \mathbf{r_{i0}}, \mathbf{p_{i0}}, t) = Ce^{-\frac{1}{4L^2}\left(\mathbf{r_i} - \mathbf{r_{i0}}(t) - \frac{\mathbf{p_{i0}}(t)\, t}{m}\right)^2} e^{i\mathbf{p_{i0}}(t)\cdot(\mathbf{r_i} - \mathbf{r_{i0}}(t))} e^{-i\frac{\mathbf{p_{i0}}(t)^2}{2m}t}$

The single particle "trial" wavefunction has gaussian shape with constant width L ∽ 2 fm.

$$\dot{\mathbf{r}}_{\mathbf{i0}} = \frac{\partial \langle H \rangle}{\partial \mathbf{p_{i0}}} \qquad \dot{\mathbf{p}}_{\mathbf{i0}} = -\frac{\partial \langle H \rangle}{\partial \mathbf{r_{i0}}}$$

- EoM for the "classical" centers in coordinate and momentum space ($r_{i0}(t)$, $p_{i0}(t)$).
- Expectation value of the quantum Hamiltonian: $\langle H \rangle = \sum_i \langle H_i \rangle = \sum_i (\langle T_i \rangle + \sum_{j \neq i} \langle V_{i,j} \rangle)$

QMD interaction and EoS

$$\langle H\rangle = \sum_i \langle H_i\rangle = \sum_i \left(\sqrt{p_{i,0}^2+m_i^2}-m_i\right) + \sum_i \sum_{j\neq i} \langle V_{i,j}\rangle$$

- The two-body potential is composed by a Coulomb term + local Skyrme type interaction

$$\begin{aligned} V_{i,j} &= V_{Coul}(\mathbf{r_i},\mathbf{r_j}) + V_{Skyrme}(\mathbf{r_i},\mathbf{r_j}) \\ &= \frac{1}{2}\frac{Z_i Z_j e^2}{|\mathbf{r_i}-\mathbf{r_j}|} + \frac{t_1}{2}\delta(\mathbf{r_i}-\mathbf{r_j}) + \frac{t_2}{\gamma+1}\rho(\mathbf{r_i},\mathbf{r_{i,0}},\mathbf{r_j},\mathbf{r_{j0}},t) \end{aligned}$$

- The expectation value of the Skyrme term is replaced by a "static" density dependent expression

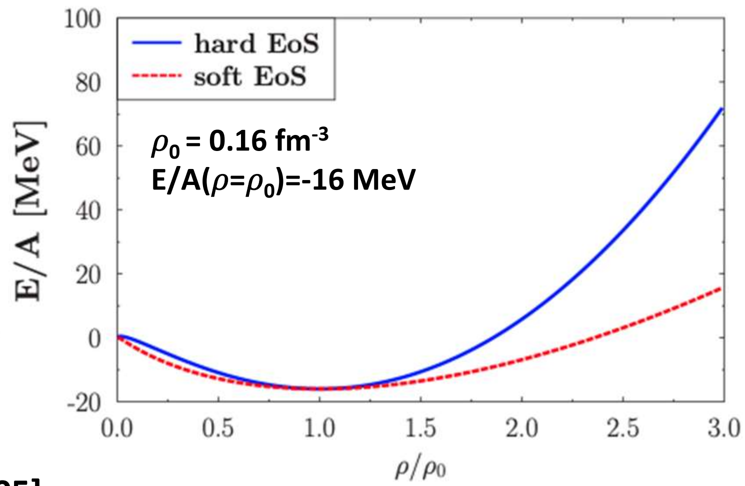
$$\sum_{j\neq i}\langle V_{Skyrme}(\mathbf{r_i},\mathbf{r_j},t)\rangle = \alpha\left(\frac{\rho_{int}(\mathbf{r_{i,0}},t)}{\rho_0}\right) + \beta\left(\frac{\rho_{int}(\mathbf{r_{i,0}},t)}{\rho_0}\right)^{\gamma}$$

- **Parameters tuned to EoS of infinite nuclear matter: E/A(T=0,ρ/ρ_0)**

	α (MeV)	β (MeV)	γ	K [MeV]
S	-390	320	1.14	200
H	-130	59	2.09	380

Compression modulus

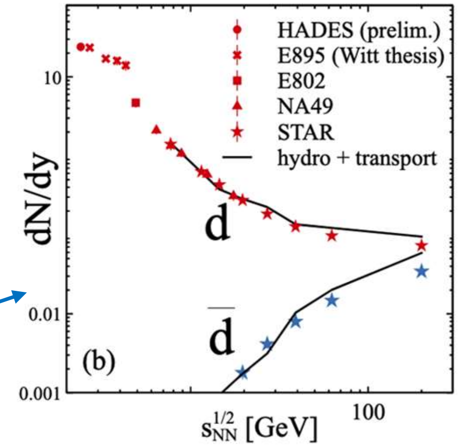
$$K = -V\frac{dP}{dV} = 9\rho^2 \frac{\partial^2[E/A(\rho)]}{(\partial\rho)^2}\bigg|_{\rho=\rho_0}$$



→ Implementation of p-dependent potential

→ Relativistic corrections to QMD **[J. Aichelin et al. PRC 101 (2020) 044905]**

Kinetic mechanism: what has been developed so far?



- $N+p+n \leftrightarrow N+d$ at low energy (P_{lab} < 1 AGeV) HICs

  [Kapusta PRC 21 4 , Siemens PRL 43 20 (1979)]
  [Gyulassy, Frankel, Remler NPA 402 (1982)]
  [Danielewicz, Bertsch NPA533 (1991) 712]

- **AMPT (blast wave + transport):** $N+N \leftrightarrow \pi+d$ → p_T-spectra , v_2 at top RHIC

  [Y. Oh, Z. Lin, C.M. Ko PRC80 (2009)]

- **SMASH (hydro + transport):**

  π**+p+n** ⟷ π**+d , N+p+n** ⟷ **N+d** realized in collision integral

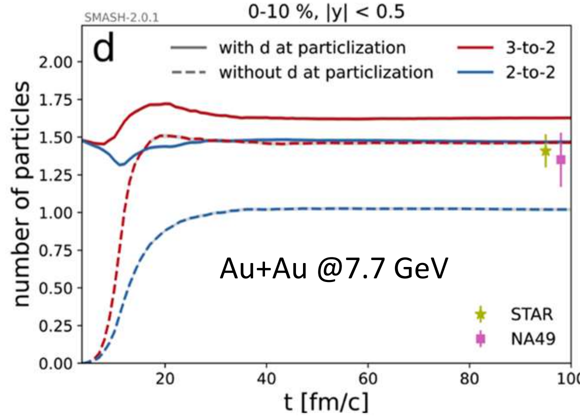
  1) via two-steps 2 ⟷ 2 using fictitious dibaryon d' resonance

  [D. Oliinychenko et al. PRC 99 044907 (2019) , PRC 103 034913 (2021)]

  2) via 3 ⟷ 2 based on covariant transition rate

  - **deuteron treated as point-like particle**
  - contribution from hydro has been investigated

  [J. Staudenmaier et al. PRC 104 034908 (2021)]



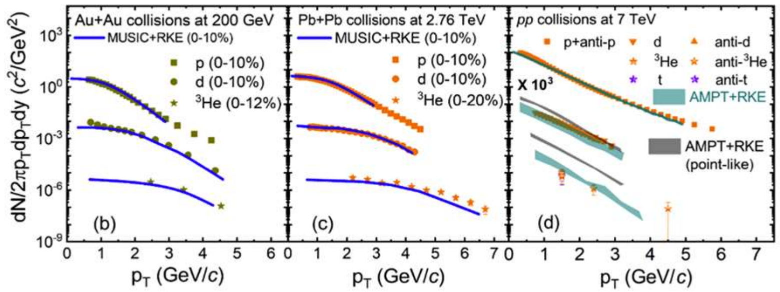
- **AMPT: Kinetic approach at relativistic HICs**
  - π**+p(d)+n→** π**+d(t) ,** π**+p+n+n →** π**+t** via Impulse Approximation

$$\overline{|\mathcal{M}_{\pi pn\to\pi d}|^2} \to 2m_d|\phi_d(\mathbf{p})|^2\left(\overline{|\mathcal{M}_{\pi n\to\pi n}|^2} + n \to p\right) \quad |\phi_d(\mathbf{p})|^2 = \int d^3\mathbf{r}(E_d/m_d)W_d(\mathbf{r},\mathbf{p})$$

  - π**+N →** π**+N$^{\text{off-shell}}$** followed by nucleon coalescence

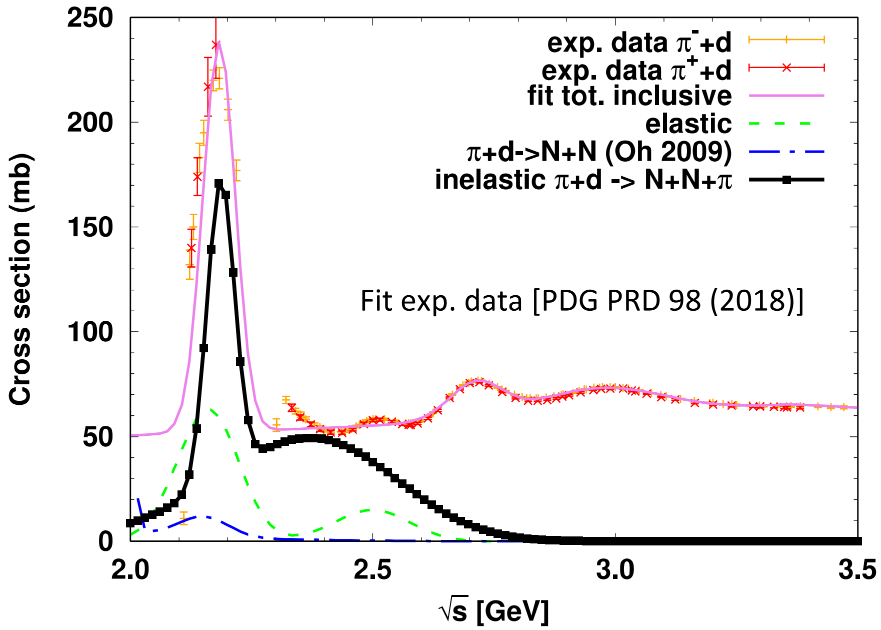
  → **finite-size effects modeled by Wigner function**

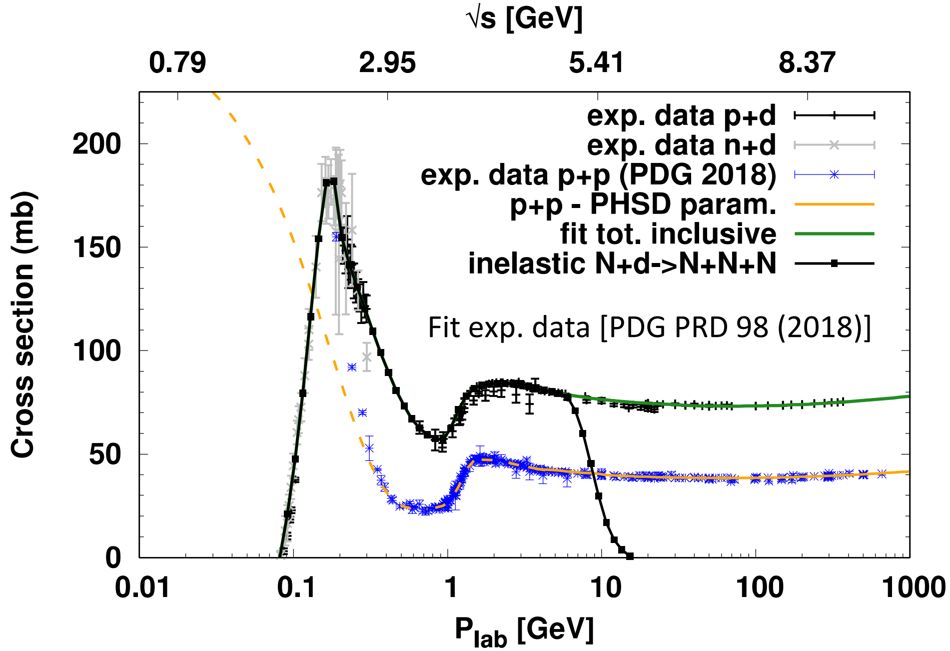
  [K. Sun et al. arxiv:2106.12742]



Kinetic mechanism: cross sections

- Hadronic reactions for π+d and N+d scattering characterized by inclusive cross sections $\sigma_{peak} \approx 200$ mb . [Kapusta PRC 21 4 (1979)]
- Inverse reactions X+N+N → X+d (X=π,N with X catalyzer) important for d formation in HICs .
- At relativistic HICs π-catalysis >> N-catalysis due to large π abundance . [Oliinychenko et al. PRC 99 (2019)]





$$\sigma_{tot}^{incl}(\pi d) = \sum_{n\geq 1} \sigma_{inel}(NN + n * \pi) + \sigma(\pi d \to NN) + \sigma_{el}(\pi d)$$

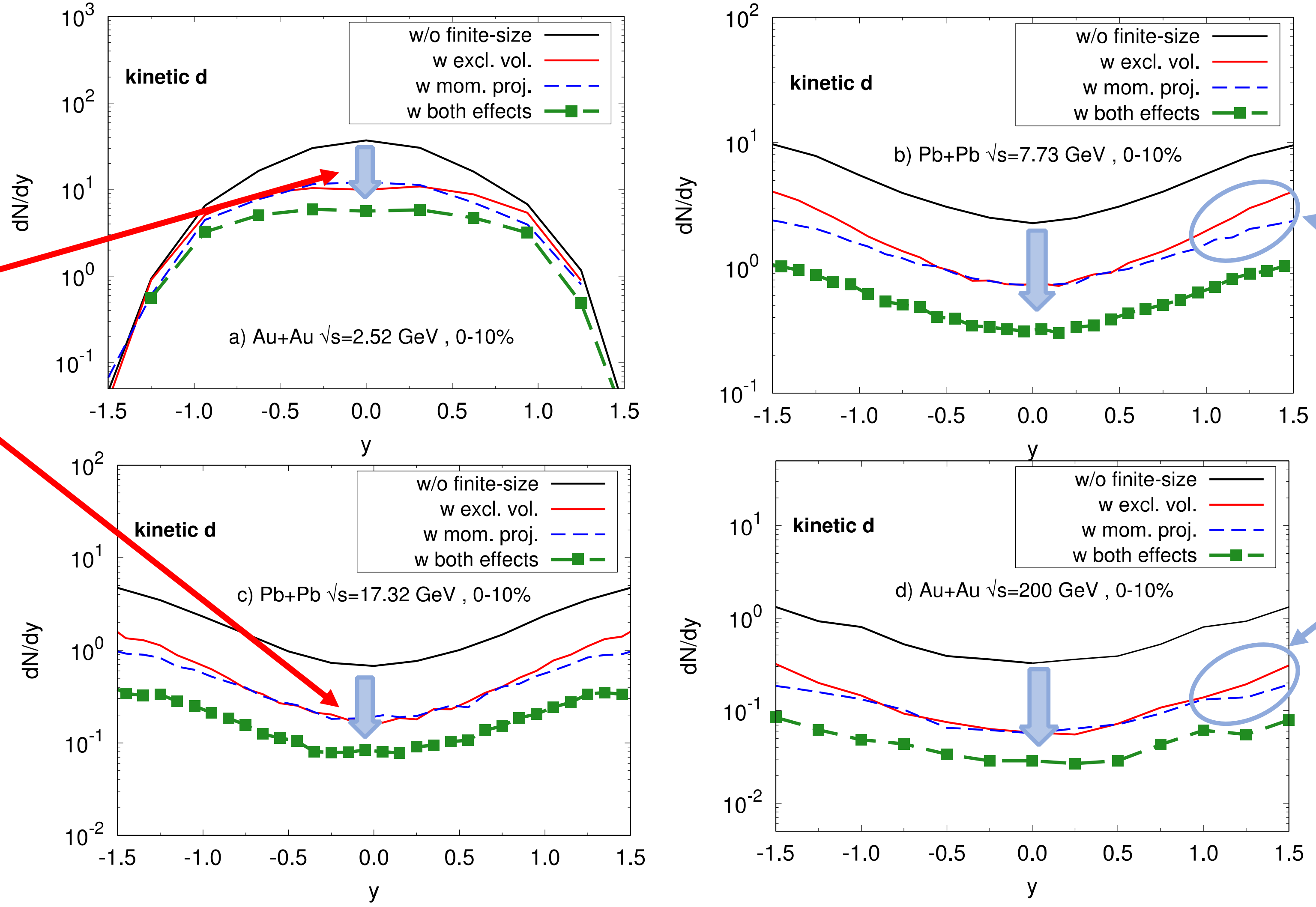
n=1

5% of total inclusive π+d cross section

$$\sigma_{tot}^{incl}(Nd) = \sum_{n\geq 1} \sigma_{inel}(NNN + n * \pi) + \sigma(Nd \to NNN) + \sigma_{el}(Nd)$$

Modelling finite-size effects in kinetic mechanism

[G.C. et al. PRC 108 (2023) 014902]

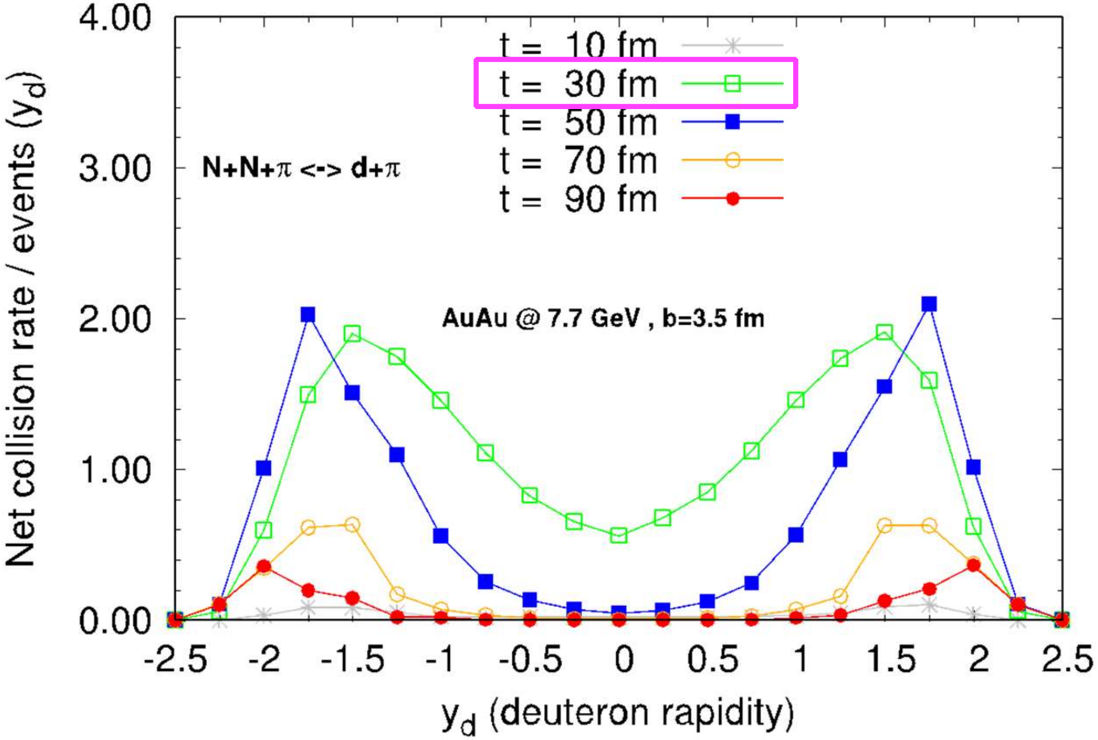


Strong suppression due to finite-size at all energies

Profile of dN/dy changes for different models of kinetic mechanism.

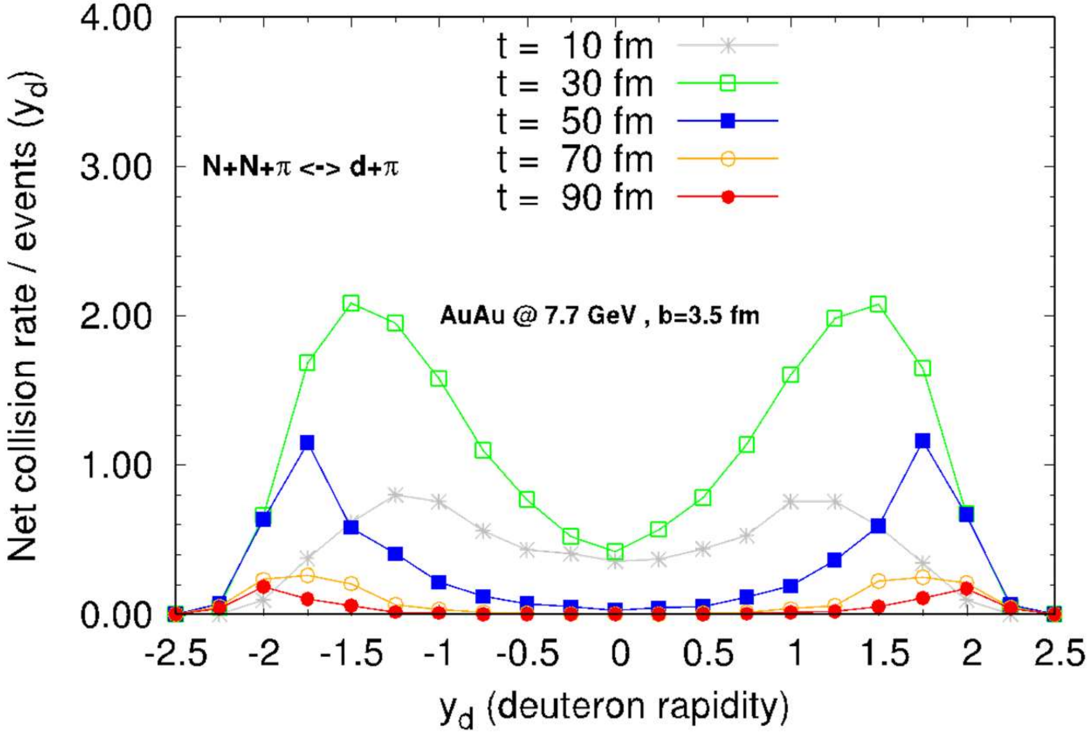
Model 1

kinetic mechanism + excluded-volume R_d=1.8 fm



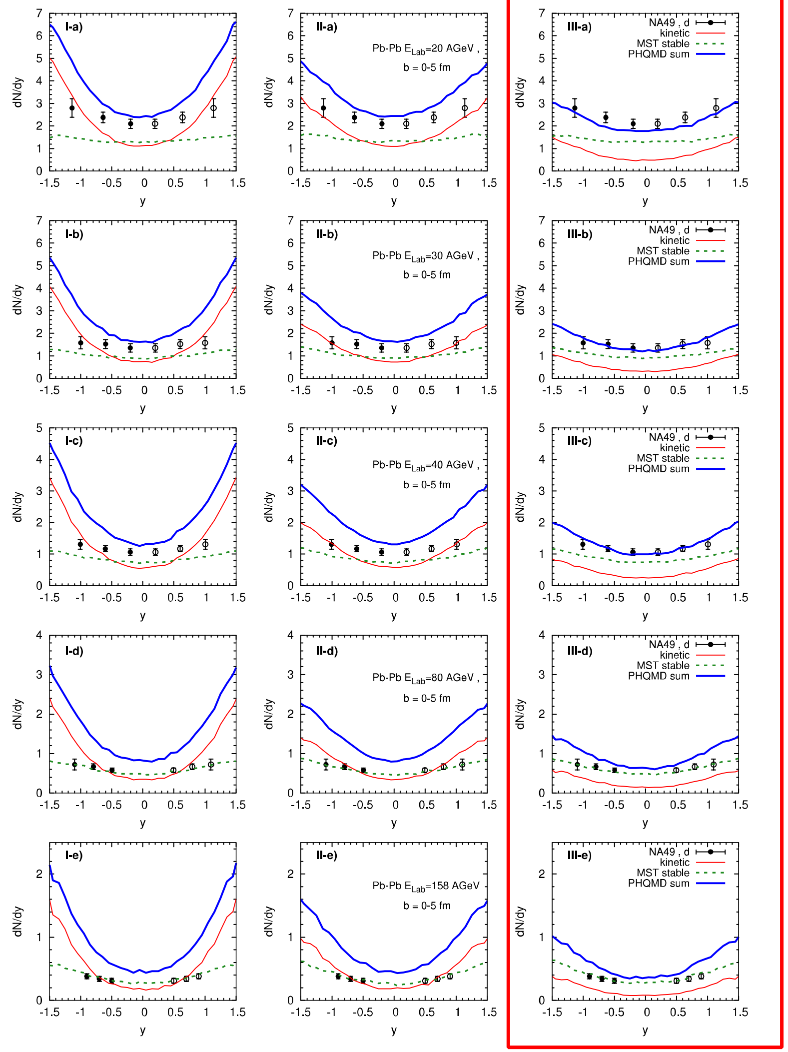
Model 2

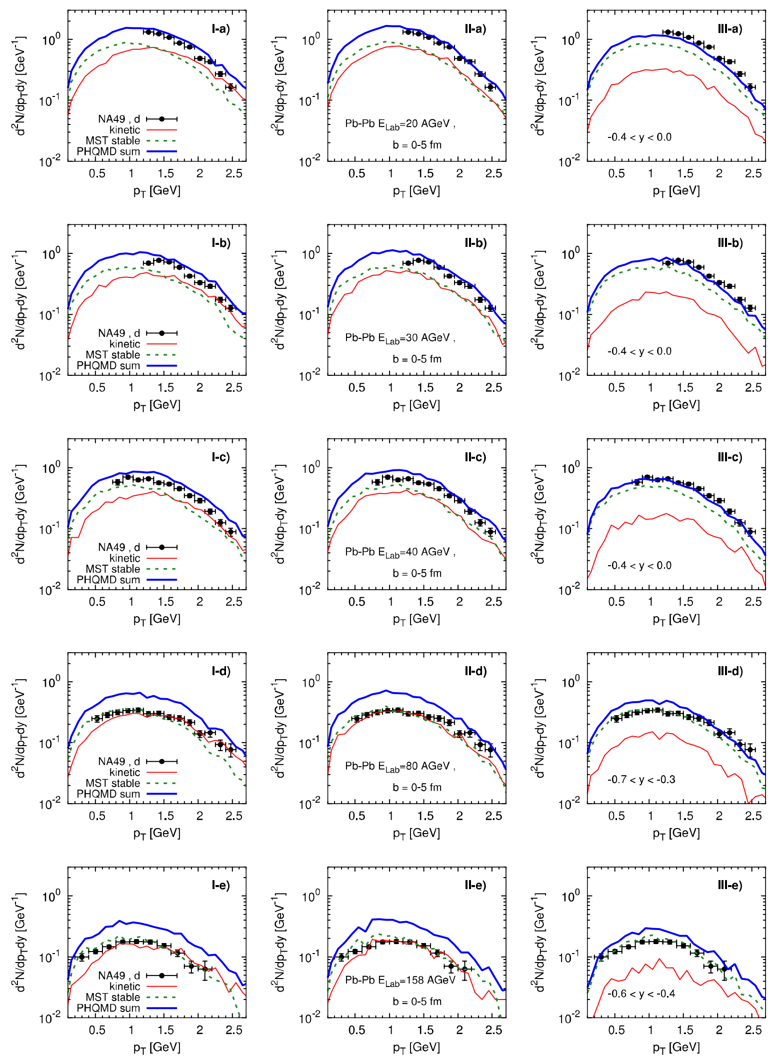
kinetic mechanism + momentum projection



- Deuteron production near target/projectile rapidity compared to mid-rapidity happens at **later time**.
- **Projection on pn-pair relative momentum** suppresses deuterons more effectively than **excluded-volume** at |y|>1
  → **Finite-size effects** are sensitive to **different phase-space regions** !

PHQMD results: comparison with exp. data [G.C. et al. PRC 108 (2023)]





Kinetic with finite-size effects + Potential = Total deuteron production → Good description of mid-rapidity NA49 data

N-body phase-space integrals

$$R_2\left(\sqrt{s}, m_1, m_2\right) = \frac{\sqrt{\lambda(s, m_1^2, m_2^2)}}{8\pi s} \qquad \lambda(s, m_1^2, m_2^2) = \left(s - m_1^2 - m_2^2\right)^2 - 4m_1^2 m_2^2$$

$$R_3\left(\sqrt{s}, m_3, m_4, m_5\right) =$$

$$\int_{(m_3+m_4)^2}^{(\sqrt{s}-m_5)^2} \frac{dM_2^2}{2\pi} R_2\left(\sqrt{s}, m_5, M_2\right) R_2(M_2, m_3, m_4) =$$

$$\int_{(m_3+m_4)^2}^{(\sqrt{s}-m_5)^2} \frac{dM_2^2}{2\pi} \frac{\sqrt{(s-m_5^2-M_2^2)^2 - 4M_2^2 m_5^2}}{8\pi s} \times \frac{\sqrt{(M_2^2-m_3^2-m_4^2)^2 - 4m_3^2 m_4^2}}{8\pi M_2^2}$$

→

$$f_3\left(t\right) = a_1 * t^{a_2} * \left(1 - \frac{1}{a_3 * t + 1 + a_4}\right)$$

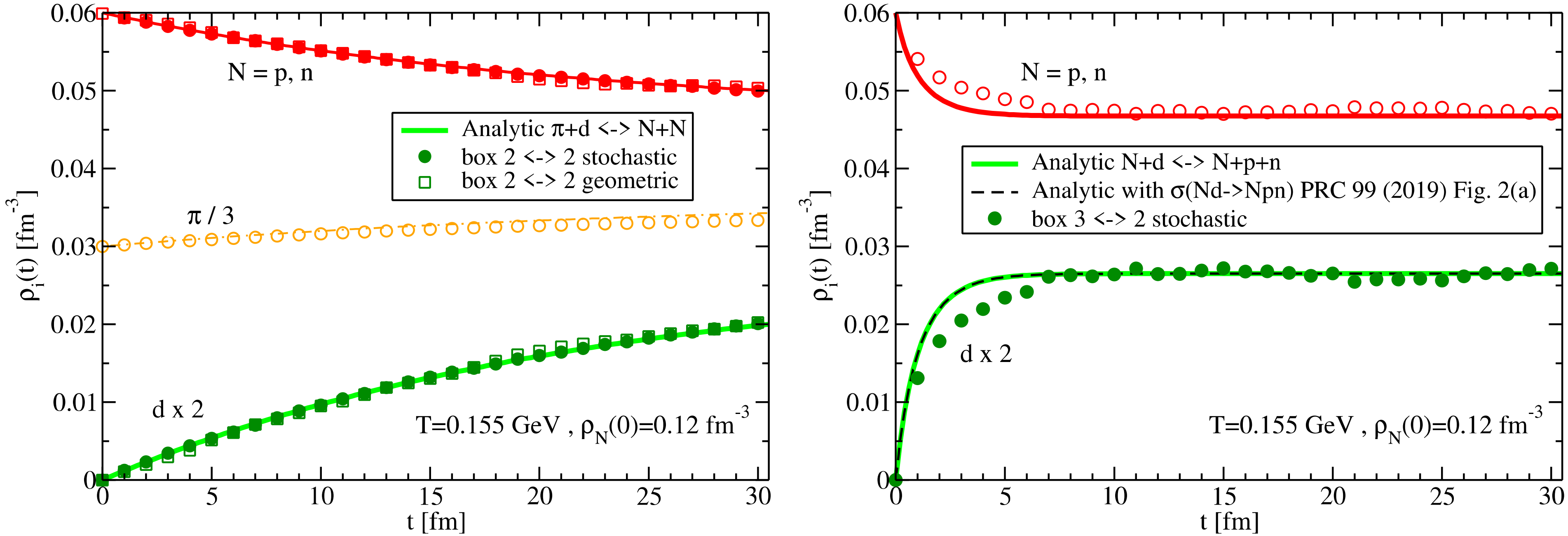
$$t = \sqrt{s} - m_3 - m_4 - m_5$$

[E. Seifert, W. Cassing, PRC 97 (2018) 024913, (2018) 044907]

$m_3\, m_4\, m_5$	a_1	a_2	$x = 2 - a_2$	a_3	a_4
$\pi\, N\, N$	0.000249	1.847779	0.152221	0.071509	9.973413
$N\, N\, N$	0.000350	1.781741	0.218259	0.052836	4.221995

[Byckling, Kajantie Particle Kinematics]

- Other deuteron reactions tested in the "box".
- **p+n+N ⟷ d+N** comparison with SMASH cross section. **[J. Staudenmaier et al. PRC 104 034908 (2021)]**
- Agreement with analytic solutions from corresponding rate equations.



[G.C. et al. PRC 108 (2023)]

Binding Energy Distribution dN/dE_B of potential deuterons as function of E_B/A with A=2:

- **<u>Before stabilization procedure</u>**
- **<u>After stabilization procedure</u>** → the average **$<E_B/A>$** reproduces $E_B(d)/A \backsim -1.1$ MeV
- **Select stable "bound" ($E_B < 0$) clusters**

  ⍰ tested also for t , ^3He , ^4He , ^4Li using Weizsäcker semi empirical mass formula for the expected E_B/A .