# Fluid dynamic calculations with HYDRA

## Network NA7-HF-QGP

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## overview


1. Theory
2. Code
3. Transport coefficients
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5. Summary, Outlook

# Fundamental equations of relativistic hydrodynamics

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

charge conservation of charge q

$$\partial_\mu N^\mu_{\boldsymbol{q}} = 0$$

normalised fluid 4-velocity

$$u_\mu u^\mu = 1$$

projector orthogonal to $u^\mu$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

# energy-momentum tensor and charge currents

## energy-momentum tensor

$$T^{\mu\nu} = eu^\mu u^\nu - 2W^{(\mu}u^{\nu)} - (P_{eq} + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

## charge 4-current of charge q

$$N_q^\mu = n_q u^\mu + V_q^\mu$$

## Landau matching

$$e = e_{eq} \quad n_q = n_{q,eq}$$

## choice of frame

$$T^{\mu\nu}u_\nu = eu^\mu \quad \Rightarrow \quad W^\mu = 0$$

# equations of motion

energy conservation equation

$$\dot{e} = -e\theta - (P_{eq} + \Pi)\theta + \pi^{\mu\nu}\sigma_{\mu\nu}$$

momentum conservation equations

$$(e + P_{eq} + \Pi)\dot{u}^{\mu} = \nabla^{\mu}(P_{eq} + \Pi) - \Delta^{\mu}_{\alpha}\partial_{\nu}\pi^{\alpha\nu}$$

charge conservation equations

$$\dot{n}_q = -n_q\theta - \partial_{\mu}V^{\mu}_q$$

# multiple conserved charges

## Navier-Stokes limit

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}, \quad V_q^\mu = \sum_{q'}^{\{B,Q,S\}} \kappa_{qq'} \nabla^\mu \frac{\mu_{q'}}{T}, \quad \Pi = -\zeta\theta$$

Diffusion matrix gains multiple entries in multi component description

## charge diffusion´ equation of motion

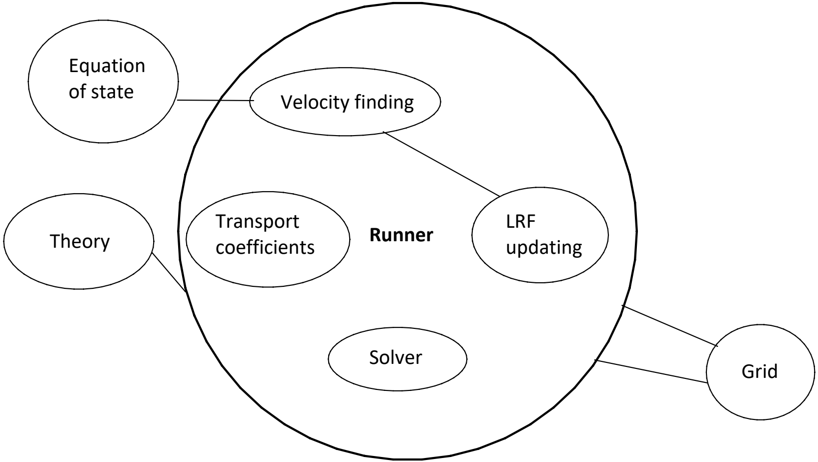
$$\sum_q^{\{B,Q,S\}} \tau_{q'q} \dot{V}_q^{\langle\mu\rangle} + V_{q'}^\mu = \sum_q^{\{B,Q,S\}} \kappa_{q'q} \nabla^\mu \frac{\mu_q}{T}$$

$$- \sum_q^{\{B,Q,S\}} \tau_{q'q} V_{q,\nu} \omega^{\nu\mu} - \sum_q^{\{B,Q,S\}} \delta_{VV}^{(q'q)} V_q^\mu \theta + [\ldots]$$

# concept

- HYDRA is written in C++20
- use of multiple conserved charges
- reduction of redundancies
- modularity of problem specific components
- eventually performance increase

# functionality



# transport coefficients

inverse temperature and thermal potential

$$\beta = \frac{1}{T} \quad \alpha_i = \frac{\mu_i}{T}$$

total pressure

$$P = \sum_{i=1}^{N_{spec}} P_i \quad P_i = \frac{g_i}{\pi^2} e^{\alpha_i} T^4$$

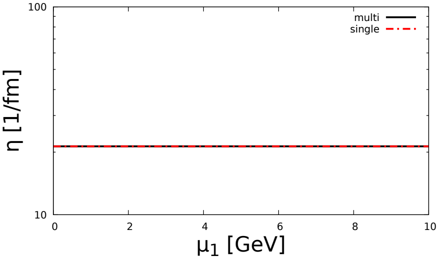
charge concentrations

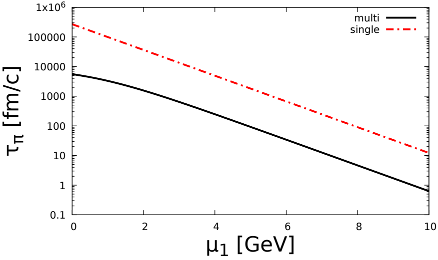
$$c_q = \sum_{i=1}^{N_{spec}} q_i \frac{P_i}{P} \quad c_{qq'} = \sum_{i=1}^{N_{spec}} q_i q_i' \frac{P_i}{P}$$

constant values for the following graphs

$$\mu_1 = \mu_2 = 0.4\mathrm{GeV} \quad T = 0.16\mathrm{GeV} \quad \sigma_{tot} = 0.01 \frac{1}{fm^2}$$

# transport coefficients

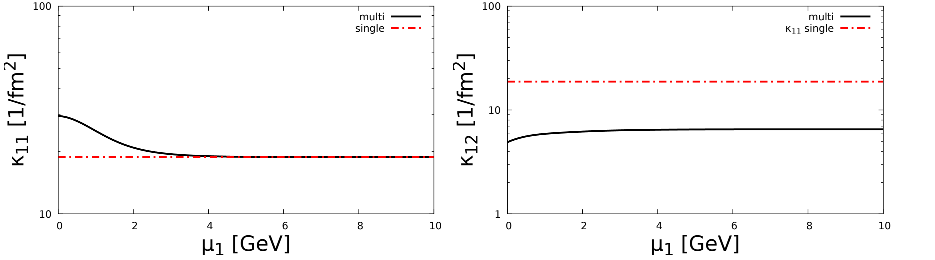



$$\eta = \frac{4}{3}\frac{1}{\sigma_{tot}\beta}$$

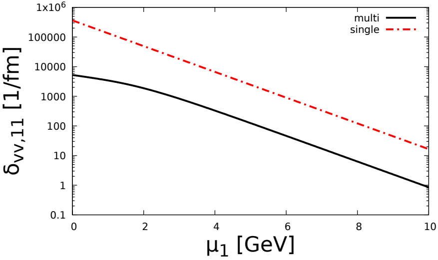
$$\tau_\pi = \frac{5}{3}\frac{1}{\sigma_{tot}\beta P}$$

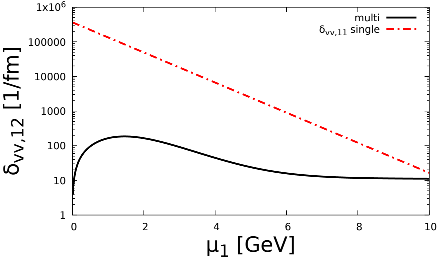
# transport coefficients




$$\kappa_{qq'} = \frac{8}{17\sigma_{tot}}\left(c_{qq'} - \frac{7}{128}c_q c_{q'}\right)$$

# transport coefficients

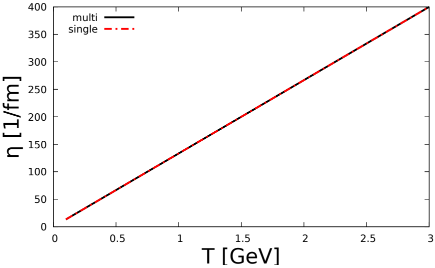


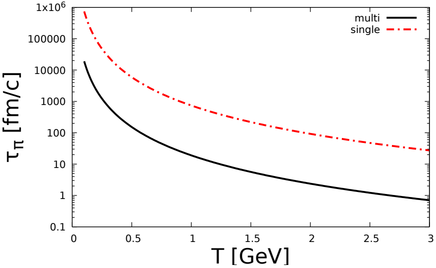




$$\delta_{vv}^{(qq')} = \frac{1}{\beta \sigma_{tot}^2 P} \sum_{q''}^{\{B,Q,S\}} \left(\kappa^{-1}\right)_{q''q} \left[ \frac{640}{867} c_{q'q''} - \frac{17551}{55488} c_{q'} c_{q''} + \frac{52}{289} \left( c_{q'q''} - c_{q'} c_{q''} \right) \right.$$

$$\left. \frac{4P}{\beta} \left( \tau_{00} + \frac{\beta}{4} \sum_{q'''}^{\{B,Q,S\}} \tau_{oq'''} c_{q'''} \right) \right] - \frac{4}{\beta \sigma_{tot}^2} \sum_{q''}^{\{B,Q,S\}} \left\{ \frac{\partial}{\partial \alpha_{q''}} \left[ \sum_{q'''}^{\{B,Q,S\}} \left(\kappa^{-1}\right)_{q'''q} \right] \right\} + [\ldots]$$

# transport coefficients




$$\eta = \frac{4}{3}\frac{1}{\sigma_{tot}\beta}$$

$$\tau_\pi = \frac{5}{3}\frac{1}{\sigma_{tot}\beta P}$$

# Bjorken test

boost inveriance

system is boost invariant along the $z$-direction

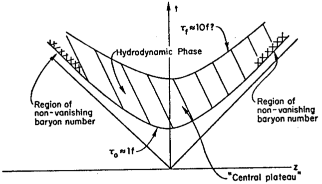
Milne coordinates

description in hyperbolic coordinates

$$ds^2 = -d\tau^2 + dx^2 + dy^2 + \tau^2 d\eta^2$$

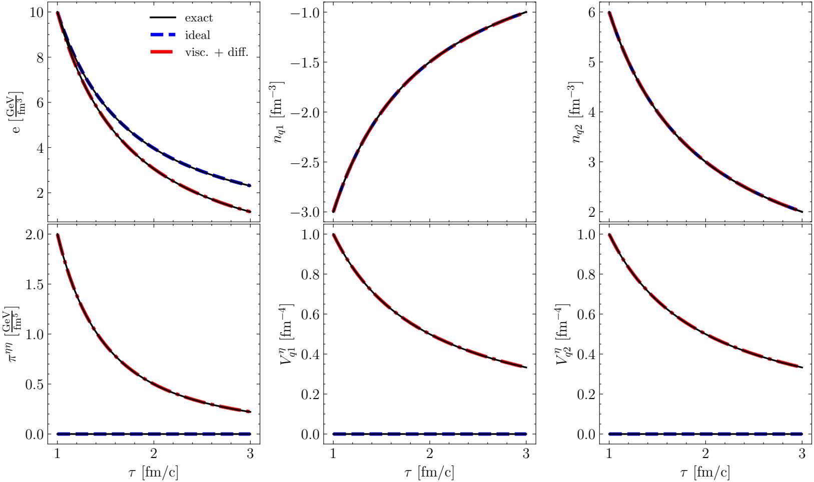
$$\tau = \sqrt{t^2 - z^2} \qquad \eta = \frac{1}{2} ln\left(\frac{t+z}{t-z}\right)$$

$$V_z = \frac{z}{t}$$




taken from J.D. Bjorken, Phys.Rev.D 27 (1983), 140-151

# Bjorken test



# Riemann test

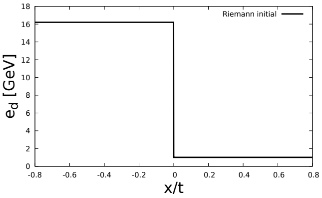
**two different mediums**

system consists of two initially seperated mediums

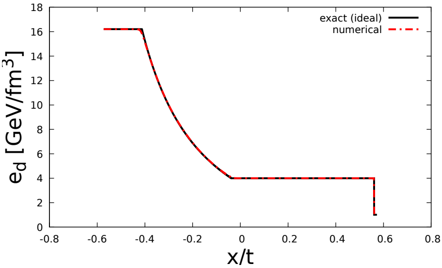
**1+1 dimensional**

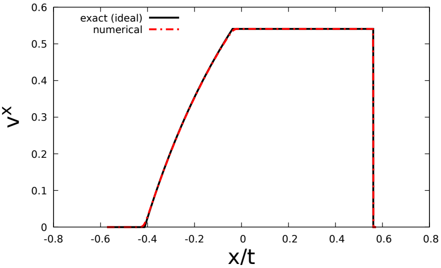
we only consider the x-direction of the system

$$T_1 = 0.4\mathrm{GeV} \quad T_2 = 0.2\mathrm{GeV}$$

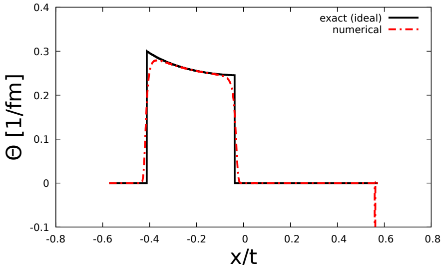


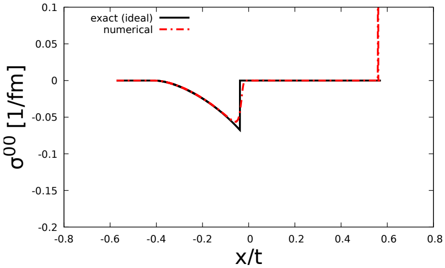
# Riemann test



# Riemann test



# summary

## Summary

- Implemented equations of motion from Fotakis et al [PRD D106, 036009 (2022)] for relativistic mixtures in (3+1)D
- HYDRA aims to be user-friendly through modularity, reduction of redundancy and tries keep or even improve performance
- Investigated transport coefficients of a simplistic ultrarelativistic, conformal mixture.

## Outlook

- Calculate transport coefficients for massive and realistic systems and make use of realistic EoS (e.g. lattice QCD) and transport coefficients
- Investigate the effects of baryon diffusion and coupled-charge transport

# kinetic theory

charge current

$$N_i^\mu = \int \frac{d^3\vec{k}_i}{(2\pi)^3 k_i^0} k_i^\mu f_{i,k} \qquad N_q^\mu = \sum_{i=1}^{N_{spec}} q_i N_i^\mu$$

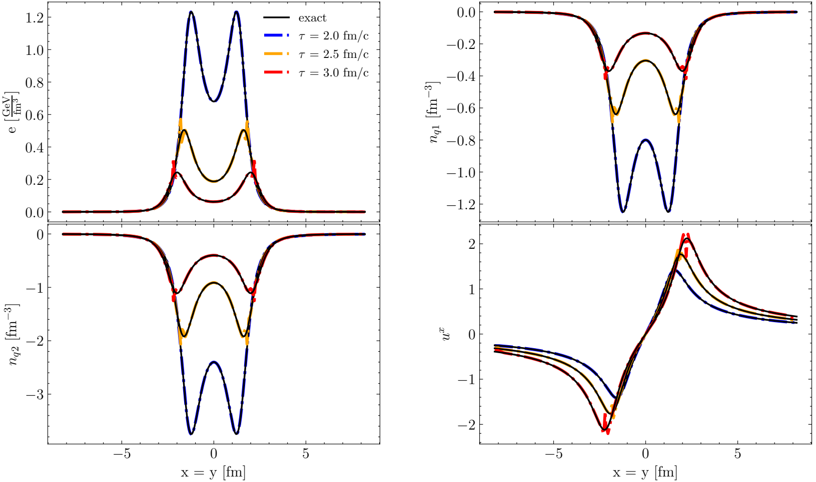
energy-momentum tensor

$$T_i^{\mu\nu} = \int \frac{d^3\vec{k}_i}{(2\pi)^3 k_i^0} k_i^\mu k_i^\nu f_{i,k} \qquad T^{\mu\nu} = \sum_{i=1}^{N_{spec}} T_i^{\mu\nu}$$

expansion in irreducible moments

$$f_k = f_{eq} \left[ 1 + \tilde{f_{eq}} \sum_{l=0}^{\infty} \sum_{n=0}^{N_l} \mathcal{H}_n^{(l)} \rho_n^{\mu_1 \ldots \mu_l} k_{\langle \mu_1 \ldots} k_{\mu_l \rangle} \right]$$

# Gubser test



exact
$\tau$ = 2.0 fm/c
$\tau$ = 2.5 fm/c
$\tau$ = 3.0 fm/c
e $\left[\frac{\text{GeV}}{\text{fm}^3}\right]$
$n_{q1}$ [fm$^{-3}$]
$n_{q2}$ [fm$^{-3}$]
$u^x$
x = y [fm]
x = y [fm]