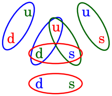
Color-Superconducting Phases in Dense Matter

STRONG-NA7 & HFHF Theory Retreat 2023

Marco Hofmann, in collaboration with Hosein Gholami and Michael Buballa
TU Darmstadt



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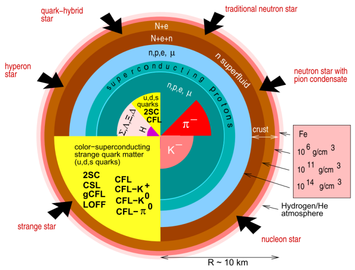
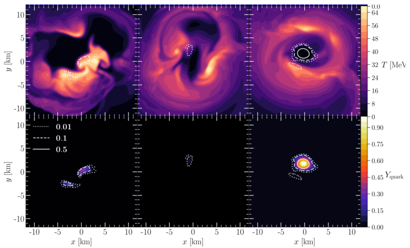
CRC-TR 211
Strong-interaction matter under extreme conditions

DFG

Quark production in NS merger simulation



[Weber (1999)]



[Tootle et al. (2022)]

Merger Simulations: Densities produced in merger remnant might be sufficient to produce quark matter

Expected implications:

- Accelerated collapse to Black Hole
- Modified post-merger frequency spectrum [Bauswein et al. (2019)]
- Details depend on the nature of the transition

This Talk: Color superconductivity in neutron star cores?

Phase Diagrams of HFHF 2023

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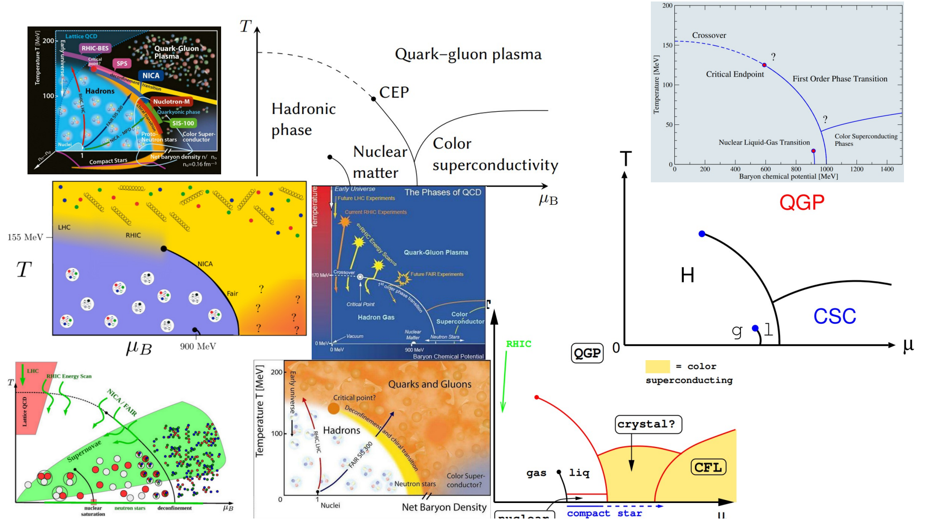

- High densities, low and medium Temperatures: Quarks form Cooper pairs via the strong interactions
- Dominant channel: Spin 0, antisymmetric in flavor and color

$$\mathcal{L}_D = \sum_{A,A'=2,5,7} (\bar{\psi} i\gamma_5 \tau_A \lambda_{A'} \psi^c)(\bar{\psi}^c i\gamma_5 \tau_A \lambda_{A'} \psi)$$

with antisymmetric Gell-Mann matrices λ_A, τ_A in flavor and color space

▷ Zoo of possible pairings

Phase	Pairing Pattern	Gap
χSB	-	-
NQM	-	-
2SC	u-d	Δ_2
2SCus	u-s	Δ_5
2SCds	d-s	Δ_7
uSC	u-d,u-s	Δ_2, Δ_5
dSC	u-d,d-s	Δ_2, Δ_7
sSC	u-s,d-s	Δ_5, Δ_7
CFL	u-d,u-s,d-s	$\Delta_2, \Delta_5, \Delta_7$


Model
CRC-TR211
Nambu Jona-Lasinio (NJL)-type model
kinetic term
scalar NJL interaction
't Hooft (KMT) interaction
diquark interaction
with charge conjugated spinor
Regularization: sharp 3-momentum cutoff
fitted to vacuum meson spectrum
choose
(Fierz value is
Hosein Gholami
Sept 29, 2023

NJL-type model

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi + G_S \sum_a \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2 \right] \qquad \text{NJL}$$

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi + G_S \sum_a \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right] && \text{NJL} \\
&- K\left[\mathrm{det_f}(\bar{\psi}(\mathbb{1}+\gamma_5)\psi) + \mathrm{det_f}(\bar{\psi}(\mathbb{1}-\gamma_5)\psi)\right] && \text{KMT int.}
\end{aligned}
$$

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$$\begin{aligned} \mathcal{L} =& \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi + G_S \sum_a \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2 \right] && \text{NJL} \\ & - K \left[\mathrm{det_f}(\bar{\psi}(\mathbb{1} + \gamma_5)\psi) + \mathrm{det_f}(\bar{\psi}(\mathbb{1} - \gamma_5)\psi) \right] && \text{KMT int.} \\ & + G_D \sum_{A,A'=2,5,7} (\bar{\psi}i\gamma_5\tau_A\lambda_{A'}\psi^c)(\bar{\psi}^c i\gamma_5\tau_A\lambda_{A'}\psi) && \text{Diquark int.} \end{aligned}$$

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$$- K\left[\mathrm{det_f}(\bar{\psi}(\mathbb{1}+\gamma_5)\psi) + \mathrm{det_f}(\bar{\psi}(\mathbb{1}-\gamma_5)\psi)\right] \qquad \text{KMT int.}$$
$$+ G_D \sum_{A,A'=2,5,7} (\bar{\psi}i\gamma_5\tau_A\lambda_{A'}\psi^c)(\bar{\psi}^c i\gamma_5\tau_A\lambda_{A'}\psi) \qquad \text{Diquark int.}$$

- Choose $\eta_D = G_D/G_S$ as a free parameter
- Mean field approximation and **RG-consistent treatment** (Hosein's Talk)

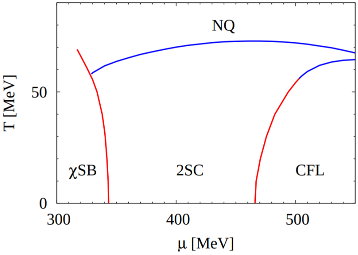
$$\Omega_{\mathrm{RG}} = \Omega^\Lambda - \Omega^\Lambda_{\mathrm{vac},\Lambda'} - \sum_{i,j} \frac{1}{2}\mu_{ij} \frac{\partial^2\Omega}{\partial\mu_{ij}^2}\bigg|_{\hat{\mu},T=0} \qquad , \Lambda \gg \Lambda'$$

- Require charge and color neutrality and include leptons in $\beta-$equilibrium

I. Phase Diagram of the RG-consistent NJL diquark model

From low to high chemical potentials:

- Chiral broken phase.
- $\mu < M_s$: Only u-d-pairing possible (2SC phase). Melting to NQM with increasing T.
- $\mu > M_s$: Strange quarks participate in pairing (CFL phase).



Oertel, Buballa (2002)

plus complications from charge neutrality requirement.

What is the melting pattern of the CFL phase in neutral matter?

Ginzburg-Landau anlaysis around T_c [Iida et al. (2004)]:
Pairing of flavor (i, j) with largest average fermi momentum $p_F^{ij} = \frac{1}{2}(p_F^i + p_F^j)$ favored

- $M_s \gg M_{u,d}$ favors ud-pairing (Δ_2)
- Charge neutrality favors ds-pairing (Δ_7), but smaller effect

▷ $p_F^{ud} > p_F^{ds} > p_F^{us}$

▷ Melting pattern CFL → dSC → 2SC

Phase	Pairing
2SC	u-d
uSC	u-d,u-s
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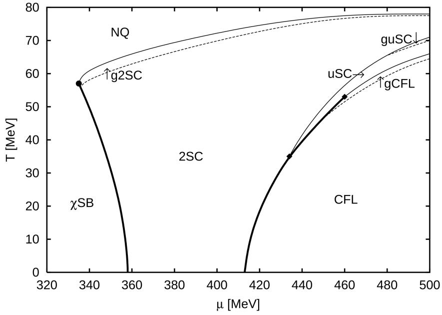
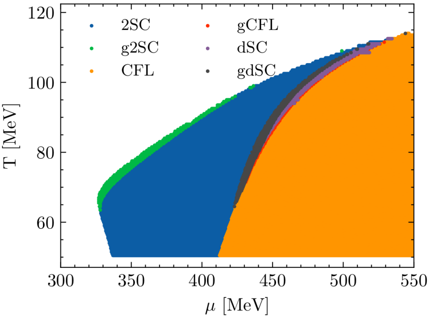


[Rüster et al. (2005)]



RG-consistent calculation

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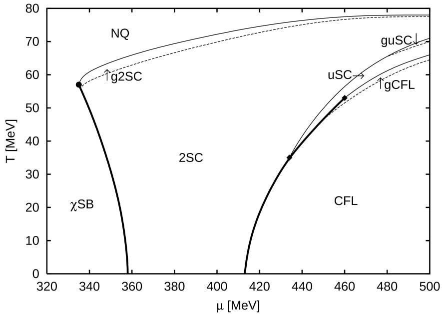

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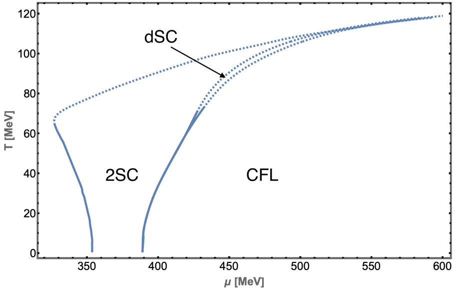

RG-consistent calculation

II. Hybrid Stars with a Color-Superconducting Core

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi + G_S \sum_a \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2 \right] \\
& - K \left[\mathrm{det_f}(\bar{\psi}(\mathbb{1} + \gamma_5)\psi) + \mathrm{det_f}(\bar{\psi}(\mathbb{1} - \gamma_5)\psi) \right] \\
& + G_D \sum_{A=2,5,7} (\bar{\psi}i\gamma_5\tau_A\lambda_{A'}\psi^c)(\bar{\psi}^c i\gamma_5\tau_A\lambda_{A'}\psi) \\
& \boldsymbol{- G_V (\bar{\psi}\gamma^\mu\psi)^2}
\end{aligned}
$$

- Provides stiffening of the equation of state at high temperatures to reach $2M_\odot$ hybrid stars [Klähn et al (2007, 2013), Alaverdyan (2022)]
- Chiral to 2SC-transition becomes 2nd order
- 2 free parameters η_D, η_V can be constrained by observational constraints on the static EoS of isolated hybrid stars

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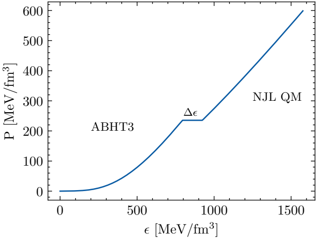
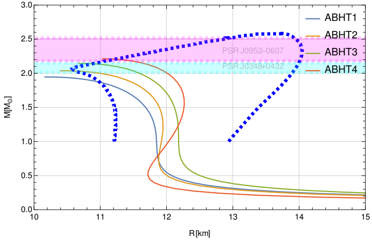
Procedure:

- Choose EoS for Hadronic Matter (HM) at low densities that satisfies all observational constraints
- Calculate Maxwell construction: $\{P, \mu_B, T\}_{\text{HM}} = \{P, \mu_B, T\}_{\text{QM}}$ in $\beta-$equilibrium at the point of the phase transition
  By construction, this gives a first order phase transition from HM to QM
- Calculate M-R-relation to test if all observational constraints are satisfied

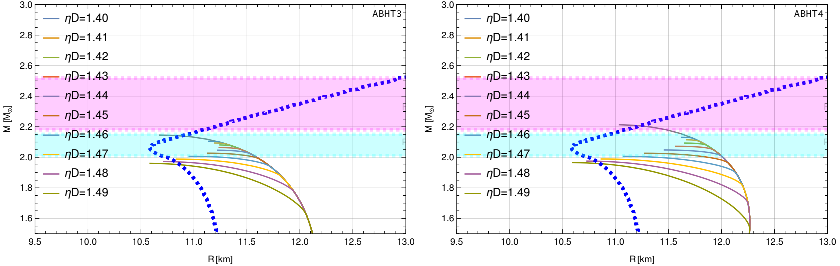
Hadronic EoS: ABHT relativistic mean-field model consistent with observational constraints [Alford et al, 2022]





[Altiparmak, Ecker, Rezzolla, 2019]

Variation of the diquark coupling at constant $\eta_V = 0.8$



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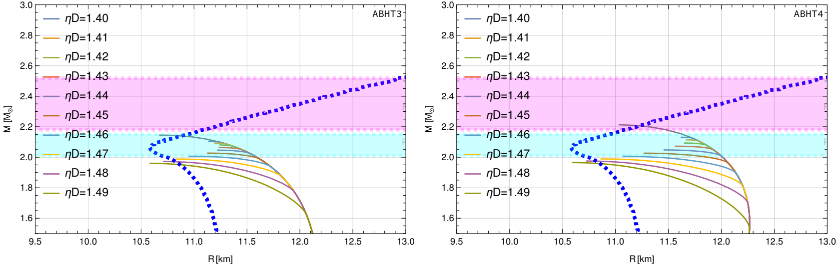
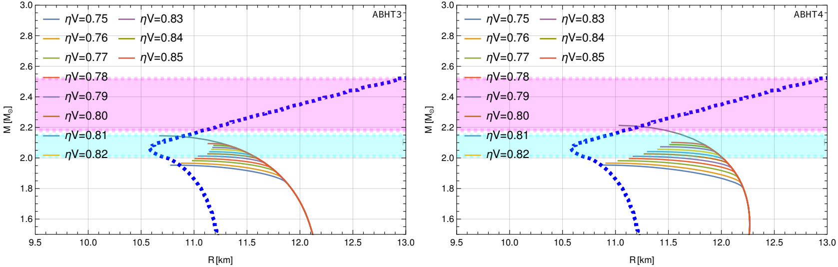


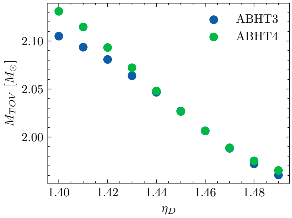
Variation the vector coupling at constant $\eta_D = 1.45$

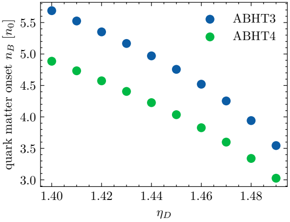


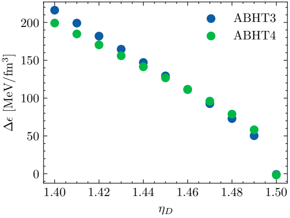
With increasing the diquark coupling:

- Matching pressure and density decrease
- Max. mass M_{TOV} decreases
- Latent heat $\Delta\epsilon$ decreases

▷ Earlier onset of deconfinement transition and smoother transition



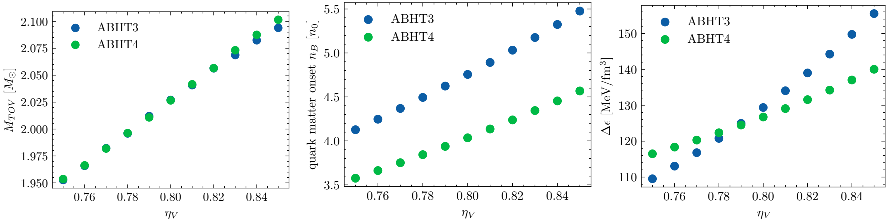




With increasing the vector coupling:

- Matching pressure and density increase
- Max. mass M_{TOV} increases
- Latent heat $\Delta\epsilon$ increases

▷ Later onset of deconfinement transition and stronger transition



- Too much latent heat $\Delta\epsilon$ renders the star configuration unstable

▷ Diquark coupling and vector coupling have to be increased/decreased simultaneously to obtain stable $2M_\odot$ hybrid stars

Summary

- RG-consistent treatment of the model reproduces CFL melting pattern as predicted by Ginzburg-Landau theory
- Construction of hybrid RMF-NJL models with color superconducting cores consistent with observational constraints possible
- Quark matter onset and maximum mass strongly depend on parameters of the model

Thank You.

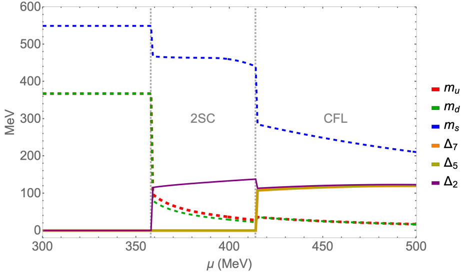
Appendix

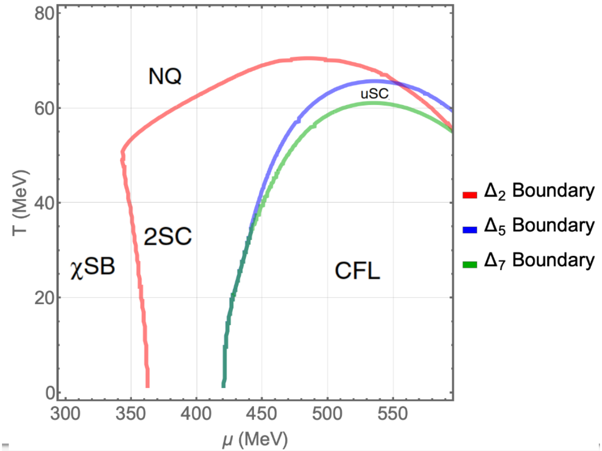
Mean field approximation: Linearise theory around condensates

$$\begin{aligned}\phi_f &= \langle \bar{\psi}_f \psi_f \rangle & f = u, d, s \\ \Delta_A &= -2G\eta_D \langle \bar{\psi}^c \gamma_5 \tau_A \lambda_A \psi \rangle & A = 2, 5, 7 \\ n &= \langle \bar{\psi} \gamma_0 \psi \rangle\end{aligned}$$

and find charge and color neutral ground state

- $\eta_D = 8/9$, $\eta_V = 0$ at $T = 0$

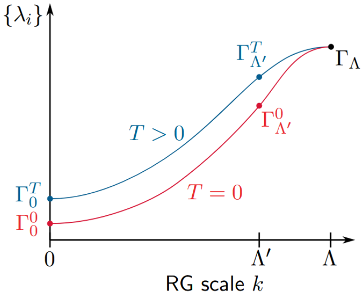




- Cutoff artefacts: Gaps and phase boundary to normal phase bend downwards for $\mu \sim \Lambda'$

RG-consistency: Full quantum effective action is cutoff independent $\Lambda \frac{\mathrm{d}\Gamma}{d\Lambda} = 0$

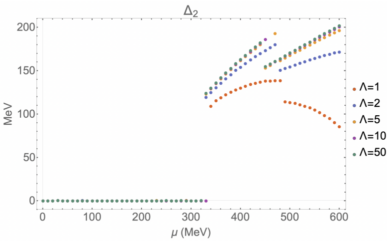
- Idea: Flow up to higher scale Λ that is much larger than external parameters and gaps $(\Lambda \gg \mu, T, \Delta, M)$ [Braun, Leonhardt, Pawlowski, 2019]

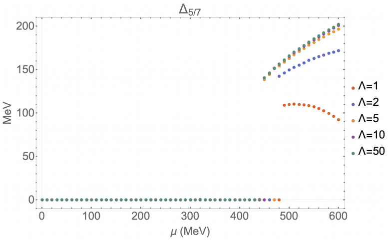


$$\Omega_{\text{RG}} = \Omega_{\Lambda'} + \Omega^{\Lambda}_{\text{med}} - \Omega^{\Lambda'}_{\text{med}} - (\Omega^{\Lambda}_{\text{vac}} - \Omega^{\Lambda'}_{\text{vac}})$$

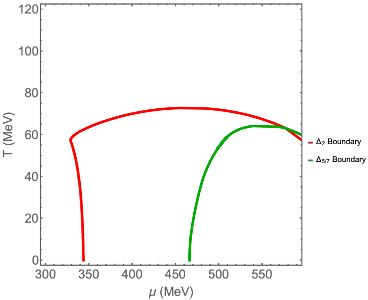
$$-\frac{1}{2}\mu^2 \left.\frac{\partial^2\Omega}{\partial\mu^2}\right|_{\mu,T=0}$$

Condensates in RG-consistent model

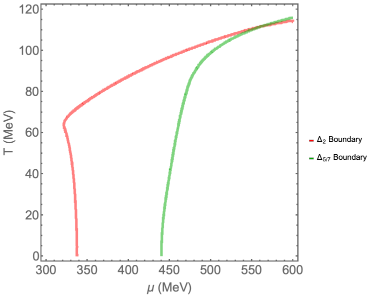




- At $\Lambda \approx 10\Lambda^{'}$, results become independent of Λ
- Gap values become enlarged for constant diquark coupling
- Phase boundaries to CSC phases move to lower μ



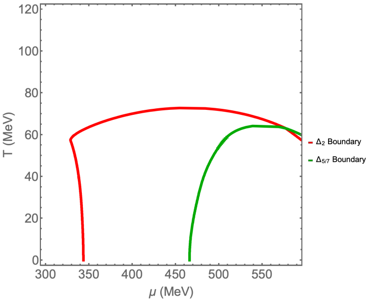
Non-neutral, without RG $\Lambda = \Lambda'$



Non-neutral, with RG $\Lambda = 10\Lambda'$

- Cutoff artefacts are removed: Phase boundary rising in μ, T-plane
- Critical temperature increases by factor 1.5-2 (for constant diquark coupling)

Neutral Phase Diagram in RG-consistent model



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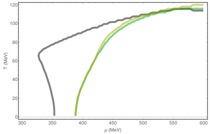
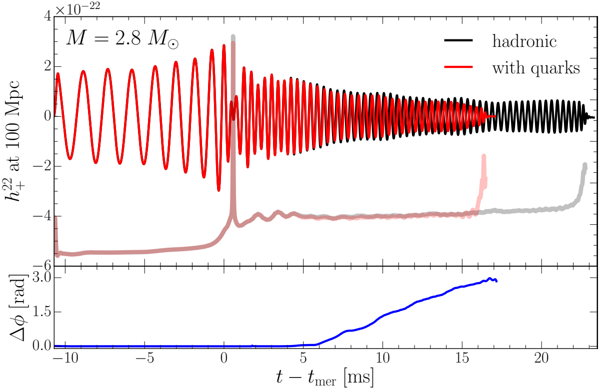


Non-neutral, with RG $\Lambda = 10\Lambda'$

Possible imprints of a Phase transition to quark matter in Gravitational Wave Signals from Neutron Star Mergers



[Rezzolla et al., 2019]

- Phase transition might be detected in data of postmerger signal

- Chemical potential matrix in color-flavor space:

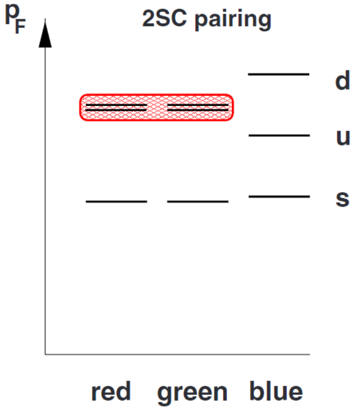
$$\mu_{f,c} = \mu + Q_f \mu_Q + \lambda_{3,c}\mu_3 + \lambda_{8,c}\mu_8$$
$$\text{e.g. } \mu_{u,r} = \mu + \frac{2}{3}\mu_Q + \mu_3 + \frac{1}{\sqrt{3}}\mu_8$$

- Neutron star: Enforce charge and color-neutrality locally, i.e. for every phase:

$$\frac{\partial\Omega}{\partial\mu_Q} = \frac{\partial\Omega}{\partial\mu_3} = \frac{\partial\Omega}{\partial\mu_8} = 0$$

- Leptonic contribution: e^- and μ^- in $\beta-$equilibrium $\mu_e = \mu_\mu = -\mu_Q$
- Optimization problem with nonlinear constraints

Neutral system: Mismatch of Fermi momenta for up and down quarks



- Eletric charge neutrality suppresses pairing in the 2SC phase

RG consistency with gaps

Quasiparticle spectra with gaps lead to divergence in the medium contribution, e.g.

$$\int_0^\Lambda \frac{\mathrm{d}^3p}{2\pi^2}(\omega_+ + \omega_-) \sim \mu^2\Delta^2\log(\Lambda)$$

where $\omega_\pm = \sqrt{(\sqrt{p^2+M^2}\pm\mu)^2+\Delta^2}$.

- Remove divergence through counterterm $\frac{1}{2}\mu^2\ \frac{\partial^2\Omega}{\partial\mu^2}\Big|_{\mu,T=0}$

$$\Omega_{\mathrm{RG}} = \Omega_{\Lambda'} + \Omega_{\mathrm{med}}^{\Lambda} - \Omega_{\mathrm{med}}^{\Lambda'} - (\Omega_{\mathrm{vac}}^{\Lambda} - \Omega_{\mathrm{vac}}^{\Lambda'}) - \frac{1}{2}\mu^2\ \frac{\partial^2\Omega}{\partial\mu^2}\bigg|_{\mu,T=0}$$

- Cooper theorem: Fermi surface unstable against finite attractive interaction of particles
- Cooper pairing and Gapped modes in excitation spectrum below critical Temperature $T_c \simeq 0.57\Delta(T = 0)$
- Strong interactions: Attractive Diquark interaction in color-, flavor antitriplet channel
- Pairing of particular color-flavor combinations

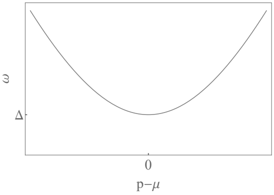


Figure: Most simple case: $\omega = \sqrt{(E-\mu)^2 + \Delta^2}$

NJL-type model

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\sum_{a=0}^{8}\left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right] \\
&+ H \sum_{A,A'=2,5,7} (\bar{\psi}i\gamma_5\tau_A\lambda_{A'}\psi^c)(\bar{\psi}^c i\gamma_5\tau_A\lambda_{A'}\psi) \\
&- K\left[\mathrm{det_f}(\bar{\psi}(\mathbb{1}+\gamma_5)\psi) + \mathrm{det_f}(\bar{\psi}(\mathbb{1}-\gamma_5)\psi)\right]
\end{aligned}
$$

with NJL coupling G, scalar diquark coupling H, and $U_A(1)$ breaking Kobayashi-Maskawa-'t Hooft interaction K

- Regularization: sharp 3-momentum cutoff $\Lambda^{'}$
- $\Lambda^{'}, G, K$ fitted to vacuum meson spectrum, choose $H \sim G$
- $\Lambda = 602.3\,\mathrm{MeV}$, $G\Lambda^2 = 1.835$, $H\Lambda^2 = 1.739$, $K\Lambda^2 = 12.36$, $m_{u/d} = 5.5\,\mathrm{MeV}$, $m_s = 140.7\,\mathrm{MeV}$

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Linearize theory around condensates

$$\phi_f = \langle \bar{\psi}_f \psi_f \rangle \qquad f = u, d, s$$
$$s_{AA} = \langle \bar{\psi}^c \gamma_5 \tau_A \lambda_A \psi \rangle$$

,i.e. neglect perturbations around expectation value of 2nd order and higher

$$\mathcal{L}_{\mathsf{MF}} = \bar{\psi}(i\partial\!\!\!/ - M + \gamma_0 \hat{\mu})\psi + \sum_{A=2,5,7} \left(\frac{\Delta_A}{2} \bar{\psi}\gamma_5 \tau_A \lambda_A \psi^c - \frac{\Delta_A^*}{2} \bar{\psi}^c \gamma_5 \tau_A \lambda_A \psi \right)$$
$$- 2G(\phi_u^2 + \phi_d^2 + \phi_s^2) + 4K\phi_u \phi_d \phi_s - \frac{1}{4H} \sum_{A=2,5,7} \Delta_A^2$$

- Relation to quark masses and gap parameters:

$$M_u = m_u - 4G\phi_u + 2K\phi_d\phi_s$$
$$M_d = m_d - 4G\phi_d + 2K\phi_u\phi_s$$
$$M_s = m_s - 4G\phi_s + 2K\phi_u\phi_d$$
$$\Delta_A = -2H s_{AA}$$

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- Linearize theory around condensates

$$\phi_f = \langle \bar{\psi}_f \psi_f \rangle \qquad f = u, d, s$$
$$s_{AA} = \langle \bar{\psi}^c \gamma_5 \tau_A \lambda_A \psi \rangle \qquad A = 2, 5, 7$$
$$n = \langle \bar{\psi} \gamma_0 \psi \rangle$$

- Relation to quark masses and gap parameters:

$$M_u = m_u - 4G\phi_u + 2K\phi_d\phi_s \qquad \text{(and other combinations)}$$
$$\Delta_A = -2G\,\eta_D\; s_{AA}$$

- Matsubara formalism to go to finite temperature $\rightarrow \Omega(\mu, T)$

$$\tilde{\mu} = \mu - 2G\eta_V n$$

- Self-consistent solution of diquark Gaps, quark masses and $\tilde{\mu}$:

$$\frac{\partial \Omega}{\partial M_f} = \frac{\partial \Omega}{\partial \Delta_i} = \frac{\delta \Omega}{\delta \tilde{\mu}} = 0$$

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- In momentum Nambu Gorkov space $\mathcal{L}_{\text{MF}} = \frac{1}{2}\bar{\psi}_{\text{NG}} S^{-1}(p)\psi_{\text{NG}} - \mathcal{V}$ with

$$S^{-1}(p) = \begin{pmatrix} \not{p} - M + \mu\gamma^0 & \sum_{A=2,5,7} \Delta_A \gamma_5 \tau_A \lambda_A \\ -\sum_{A=2,5,7} \Delta_A^* \gamma_5 \tau_A \lambda_A & \not{p} - M - \mu\gamma^0 \end{pmatrix}$$

- Finite T $\Rightarrow$ Matsubara Formalism

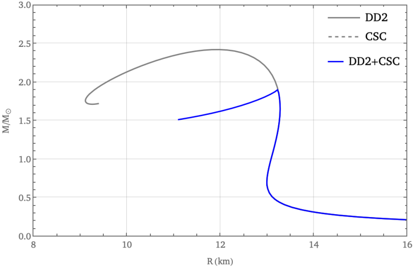
$$\Omega(\mu, T) = -\frac{T}{2}\int \frac{\mathrm{d}^3 p}{(2\pi^3)} \sum_n \ln \det \left(\frac{S^{-1}(i\omega_n, \vec{p})}{T} \right) + 2G(\phi_u^2 + \phi_d^2 + \phi_s^2) - 4K\phi_u\phi_d\phi_s + \frac{1}{4H}(\Delta_2^2 + \Delta_5^2 + \Delta_7^2)$$

- Minimization of Ω provides self-consistent solution of Gaps and Quark masses:

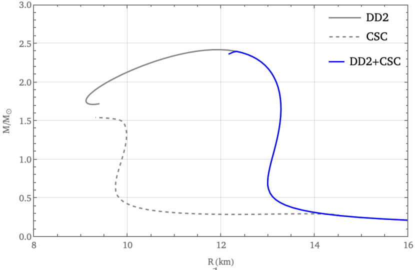
$$\frac{\partial \Omega}{\partial M_f} = \frac{\partial \Omega}{\partial \Delta_i} = 0 \quad \textit{Gap Equations}$$

Hybrid Equation of State

- Hybrid EOS with DD2

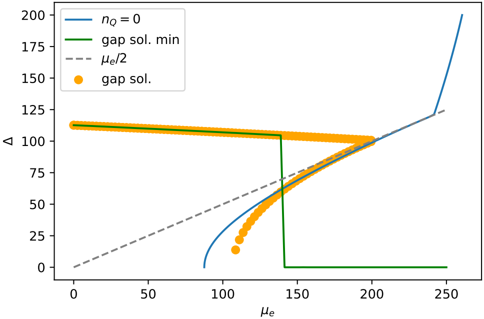


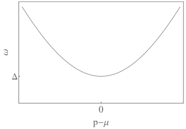
$\eta_V = 0$ (No vector interation)

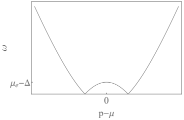


$\eta_V = \frac{1}{2}$

- Measurements of PSR J0952–0607 $(2.35 \pm 0.17 M_\odot)$ and PSR J0348+0432 $(2.01 \pm 0.04 M_\odot)$
- Repulsive vector channel increases stiffness [Klähn et al, 2007] [Pagliara, Schaffner-Bielich, PRD 77, 2007] [G. B. Alaverdyan, 2022]







- Solution of Gap eq. fall into two branches: gapped ($\Delta > \delta\mu$) and ungapped ($\Delta < \delta\mu$)