INFN CATANIA
Istituto Nazionale di Fisica Nucleare
Sezione di Catania

Heavy quark interaction in hot QCD matter: open charm and bottom dynamics in relativistic heavy-ion collisions

Maria Lucia Sambataro

In collaboration with: V. Minissale, S. Plumari, V. Greco

Dipartimento di Fisica e Astronomia 'E.Majorana'- Università degli Studi di Catania

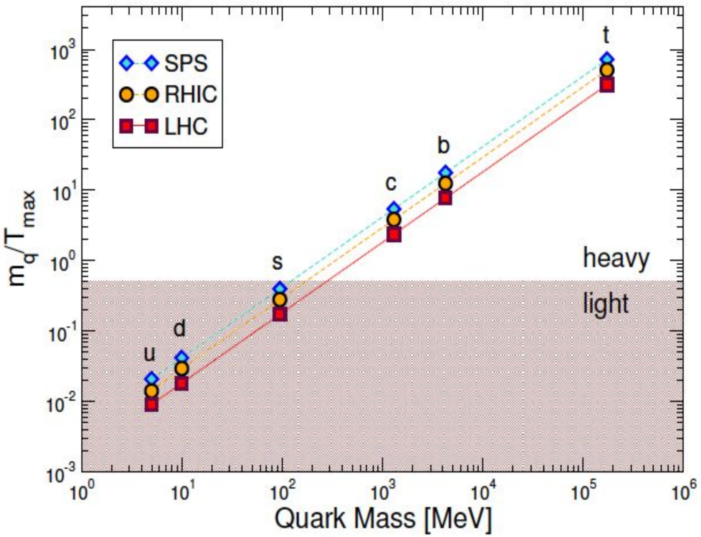
INFN -Laboratori Nazionali del Sud (LNS)

Outline

- **Catania Quasi-Particle Model for charm quark dynamics:**
  R_{AA} , v_n and their correlations $\rightarrow$ Spatial diffusion coefficient D_s (T) of charm.
- **Predictions for bottom quark**
  R_{AA} , v_2 and v_3 of electrons from semileptonic B-meson decay.
- **Spatial diffusion coefficient D_s (T): charm vs bottom and the infinite mass limit**
- **Preliminary**: Extension to momentum dependent QPM
- Conclusions and new perspectives

Basic scales of charm and bottom quarks

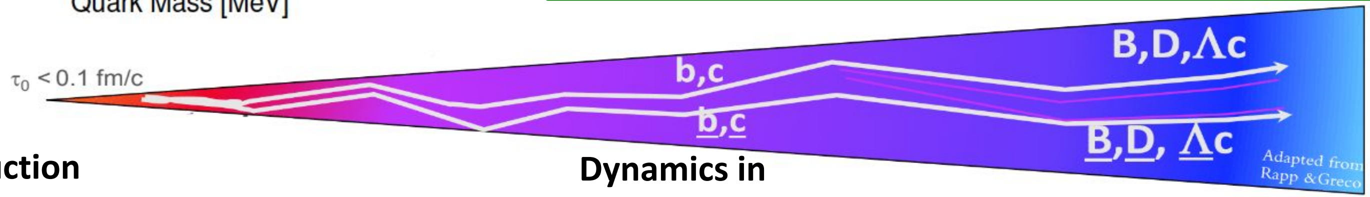
Charm M_c≈1.3 GeV and Bottom M_b≈4.2 GeV



- $m_{c,b} >> \Lambda_{QCD}$
  pQCD initial production
- $m_{c,b} >> T_{RHIC,LHC}$
  negligible thermal production
- τ_0 < 0,08 fm/c << τ_{QGP}
- $\tau_{th} \approx \tau_{QGP} >> \tau_{g,q}$

They experience the full evolution of the QGP.

They carry more informations with respect to their light counterparts.



Initial production

Dynamics in QGP

Hadronization:
Final hadron Spectra and observables

Reviews:

1. X.Dong, V. Greco Prog. Part. Nucl. Phys. 104 (2019),
2. A.Andronic EPJ C76 (2016), 3) R.Rapp, F.Prino J.Phys. G43 (2016)

CATANIA MODEL: QUASI-PARTICLE MODEL AND TRANSPORT THEORY

R_{AA}, v_n and v_n -v_m correlations in charm sector

Quasi Particle Model (QPM) fitting lQCD

Non perturbative dynamics → M scattering matrices (q,g → Q) evaluated by Quasi-Particle Model fit to **lQCD thermodynamics**

N_f=2+1
Bulk:
u,d,s

$$m_g^2(T) = \frac{2N_c}{N_c^2-1} g^2(T)\,T^2$$
$$m_q^2(T) = \frac{1}{N_c} g^2(T)\,T^2$$

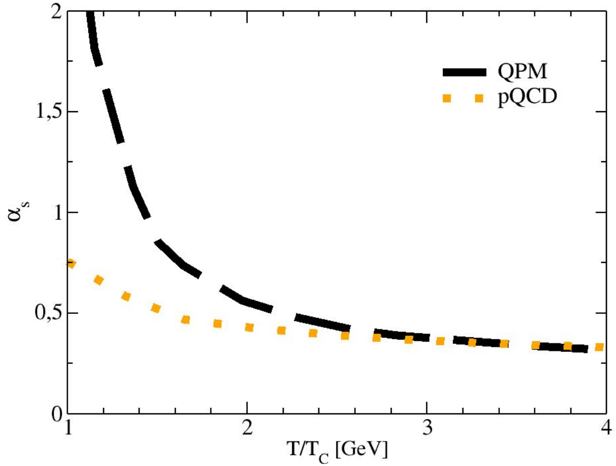
→ **Thermal masses of gluons and light quarks**

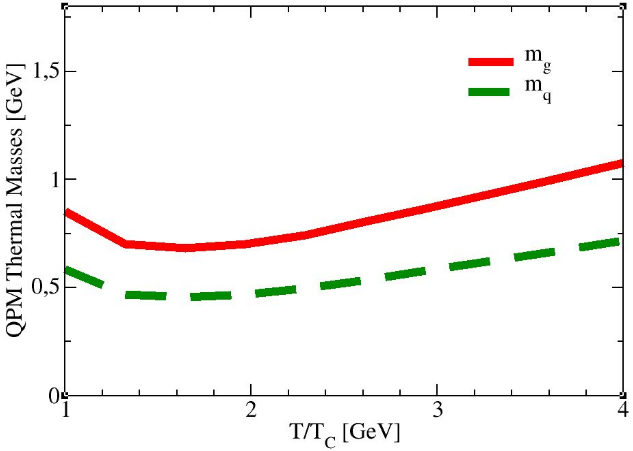
g(T) from a fit to ε from lQCD data → good reproduction of P, ε-3P

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

λ=2.6
T_s=0.57 T_c

Larger than pQCD especially as T → T_c





S. Plumari et al, *Phys.Rev.D* 84 (2011) 094004
H. Berrehrah,, PHYSICAL REVIEW C **93**, 044914 (2016)

Relativistic Boltzmann equation at finite η/s

Bulk evolution

$$p^{\mu}\partial_{\mu}f_q(x,p)+m(x)\partial^{x}_{\mu}m(x)\partial^{\mu}_{p}f_q(x,p)=C[f_q,f_g]$$
$$\underbrace{p^{\mu}\partial_{\mu}f_g(x,p)}_{\text{Free-streaming}}+\underbrace{m(x)\partial^{x}_{\mu}m(x)\partial^{\mu}_{p}f_g(x,p)}_{\text{field interaction}}=\underbrace{C[f_q,f_g]}_{\text{Collision term}}$$

Free-streaming

field interaction
$\varepsilon - 3p \neq 0$

Collision term
gauged to some η/s≠ 0

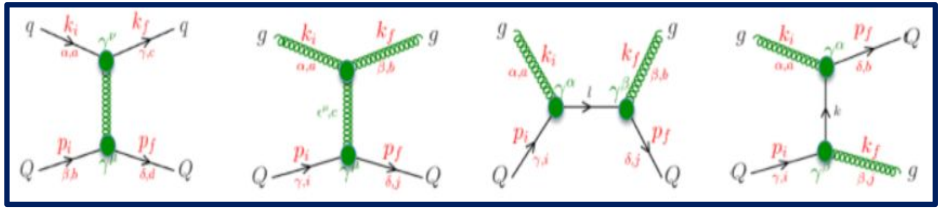
Equivalent to
viscous hydro at η/s ≈ 0.1

HQ evolution

$$p^{\mu}\partial_{\mu}f_Q(x,p)=C[f_q,f_g,f_Q]$$

$$C[f_q,f_g,f_Q]=\frac{1}{2E_1}\int\frac{d^3p_2}{2E_2(2\pi)^3}\int\frac{d^3p_1'}{2E_1'(2\pi)^3}$$
$$\times[f_Q(p_1')f_{q,g}(p_2')-f_Q(p_1)f_{q,g}(p_2)]$$
$$\times|M_{(q,g)\to Q}(p_1p_2\to p_1'p_2')|$$
$$\times(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')$$

Feynman diagrams at first order pQCD for HQs-bulk interaction:

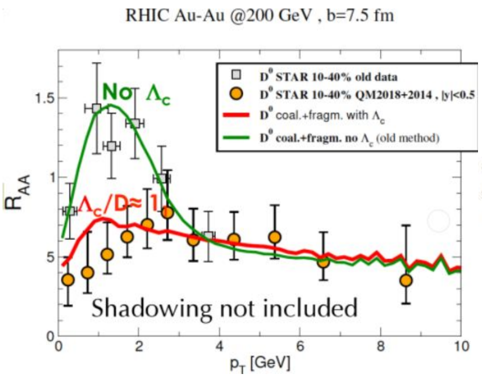


Scattering matrices $M_{g,q}$ by QPM fit to lQCD thermodynamics

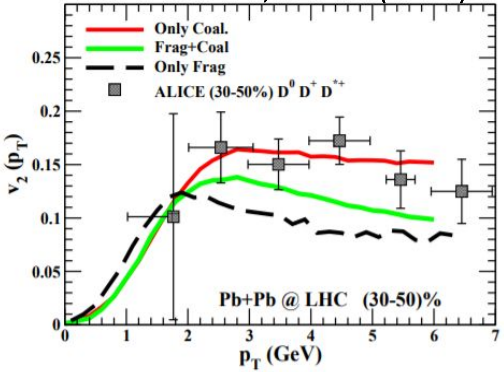
HADRONIZATION: hybrid Coalescence + fragmentation

For details: S. Plumari talk [29 Sept at 11.15]

Catania QPM: some prediction for charm…



Scardina et al., PRC 97(2017)

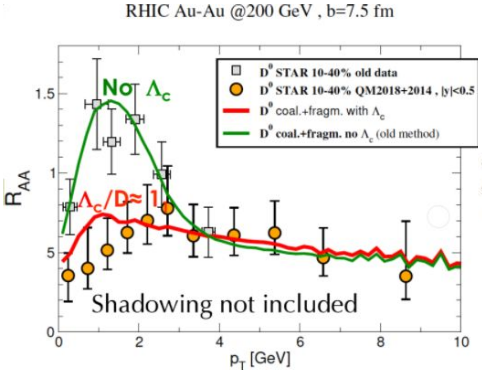


Good description of

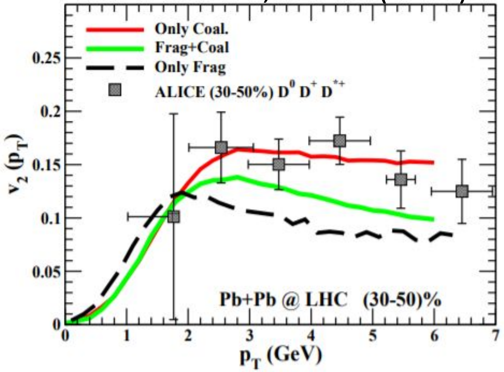
R_{AA}, v_2 at RHIC & LHC energies

within error bars

Catania QPM: some prediction for charm…



Scardina et al., PRC 97(2017)



Good description of

R_{AA}, v_2 at RHIC & LHC energies

within error bars

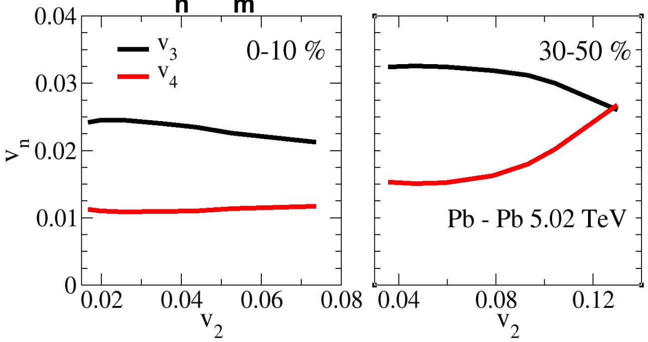
Monte Carlo Glauber for initial condition of partons

S.Plumari et al, *Phys.Rev.C* 92 (2015) 5

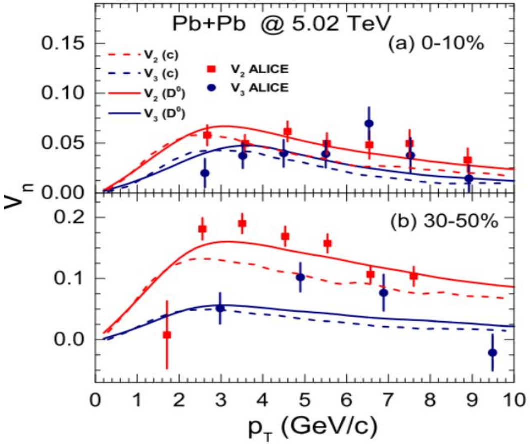
Predictions for D mesons

- **Event-Shape Engeenering Technique:**
  Prediction for similar correlation for hard particles wrt bulk

v_n - v_m correlations



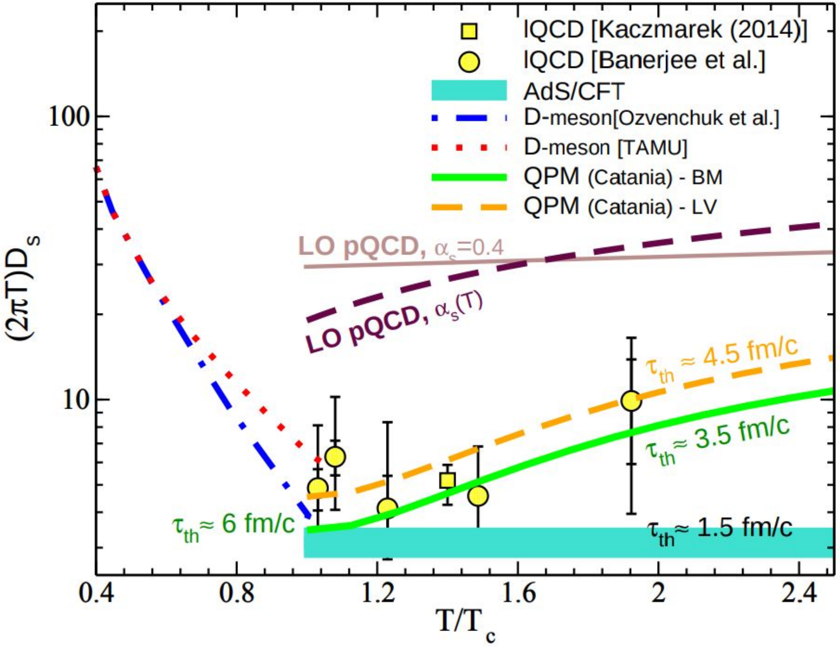
Triangular flow v_3



ALICE collaboration, *Phys.Lett.B* 813 (2021) 136054

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

Spatial diffusion coefficient of charm quark



Not a model fit to lQCD data, but D_s estimate that comes from results of R_{AA} (p_T) and v_2 (p_T)

We have a probe with $\tau_{therm} \approx \tau_{QGP}$

$$\tau_{th} = \frac{M}{2\pi T^2}(2\pi T D_s) \cong 1.8 \frac{2\pi T D_s}{(T/T_c)^2} \text{ fm/c}$$

FUTURE:
-Access low p and precision data (detector upgrade)
-Better insight into hadronization
-New observables
-Bottom → **Main focus of this talk**

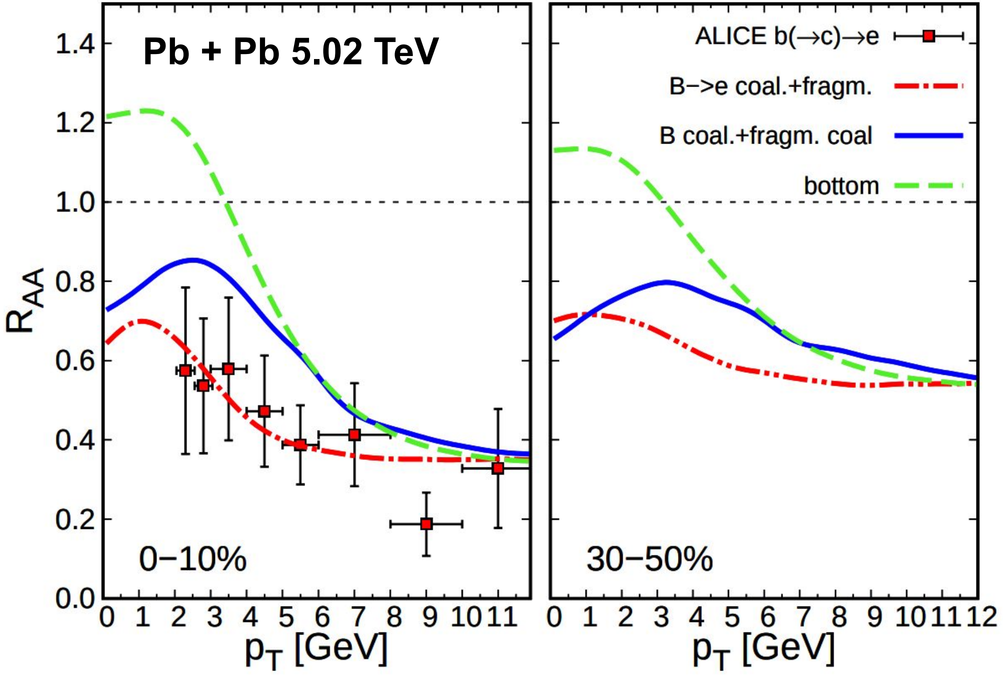
Reviews:
- F. Prino and R. Rapp, JPG(2019)
- X. Dong and V. Greco, Prog.Part.Nucl.Phys. (2019)
- Jiaxing Zhao et al., arXiv:2005.08277

CHARM VS BOTTOM:
Ds IN THE INFINITE MASS LIMIT

Extension to bottom dynamics: R_{AA}

Hadronization with coalescence + fragmentation model

- ➢ Prediction for B meson R_{AA}
- ➢ R_{AA} of electrons from semileptonic B meson decay



No parameters changed with respect to charm dynamics → same interaction

- Shift of the peak to higher momenta ➔ smaller with respect to the one for D mesons in the same model.

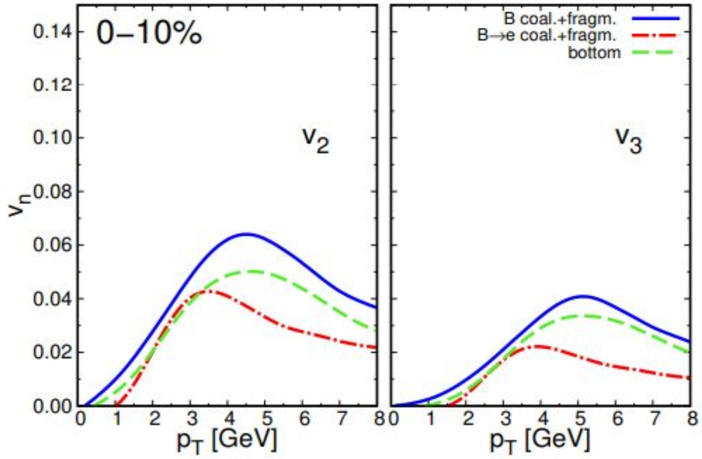
Data from: ALICE coll., arxiv:2211.13985

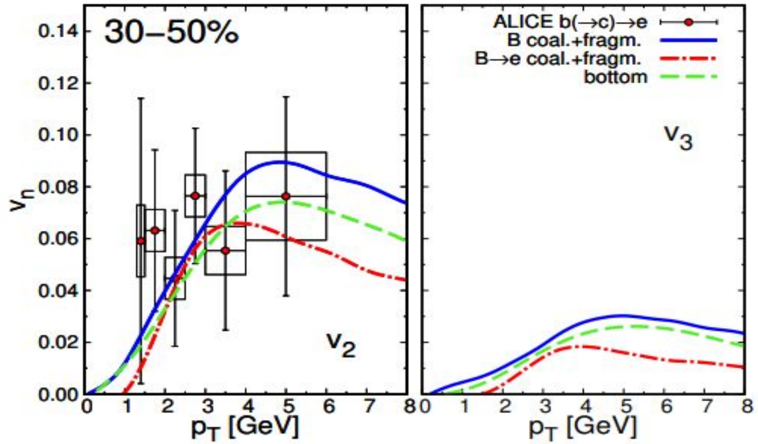
M.L. Sambataro et al., e-Print: 2304.02953

Extension to bottom dynamics: $v_{(n=2,3)}$

Data from ALICE, PRL 126, 162001 (2021)

- ➢ Prediction for B meson
- ➢ electrons from semileptonic B meson decay within a coal + fragm model





Compared to charm quark:

- Efficiency of conversion of ε_2 :
  - ➔ 15% smaller for v_2 in most central collisions.
  - ➔ 40% smaller for v_2 at 30−50% centrality.
- Efficiency of conversion of ε_3 :
  - ➔ 30% smaller for v_3 at both 0-10% and 30-50% centralities.

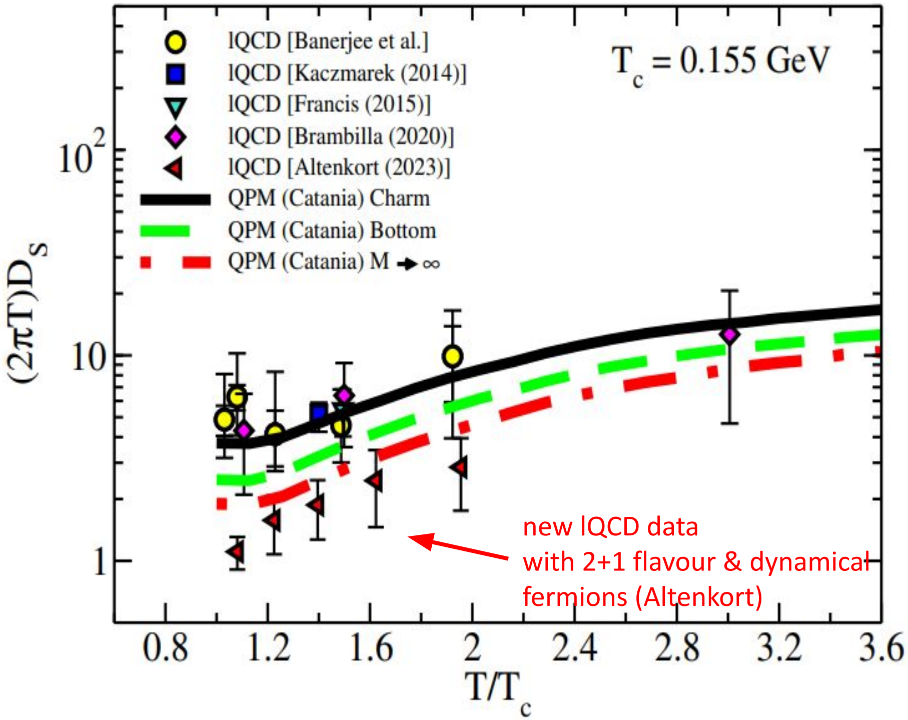
No parameters changed with respect to charm dynamics

From central to peripheral:

- enhancement of v_2 (ε_2(0-10%)≃0.13 and ε_2(30-50%)≃0.42).
- SImilar v_3 (ε_3(0-10%)≃0.11 and ε_3(30-50%))≃0.21).

M.L. Sambataro et al., e-Print: 2304.02953

$(2\pi T)D_s$: Charm quark vs Bottom quark



- lQCD data are in $M_Q \rightarrow \infty$, so the D_s evaluated is mass independent + quenched medium
- QPM use finite mass and includes dynamical fermions

$$D_s = \frac{T}{M\,\gamma} = \frac{T}{M}\,\tau_{th}$$

From kinetic theory is expected that:

$$\tau_{th}(b)/\,\tau_{th}(c) \approx \gamma_c/\,\gamma_b \approx M_b/\,M_c$$

In QPM approach → D_s(c) is 30-40% larger than D_s(b) (no mass independence)

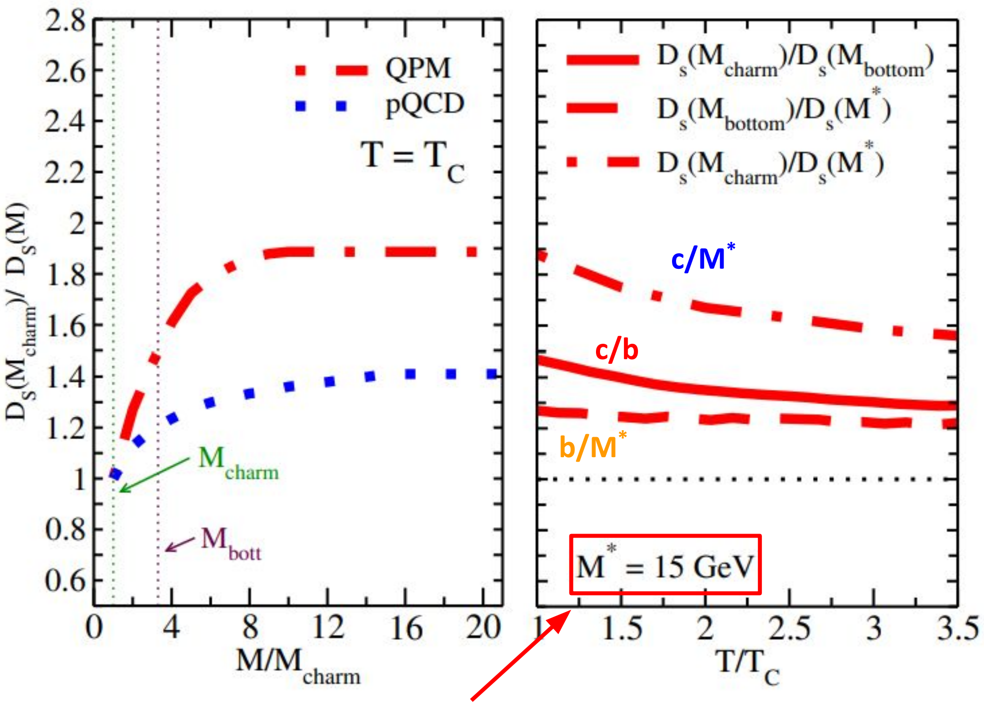
M→ ∞ limit is not reached for charm

From D_s we obtain (in the 1-2T_c range):
- τ_{th}(c) ~ 5 fm/c
- τ_{th}(b) ~ 11 fm/c breaking w.r.t. the relation:

  $\tau_{th}(b) = (M_b/M_c)\tau_{th}(c) \sim 3.3\ \tau_{th}(c) \sim 16.5$ fm/c

M.L. Sambataro et al., e-Print: 2304.02953

$(2\pi T)D_s$ ratios: Charm quark vs Bottom quark



fictitious super-heavy quark staying in the $M_Q \rightarrow \infty$ limit

- $D_s(M_{charm})/D_s(M)$ as a function of M/M_{charm} at T_C:

Saturation scale of Ds for M_Q ~ 8 M_{charm} $\gtrsim$ 10 GeV
$Ds(M_{charm})/Ds(M \rightarrow \infty)$ = 1.9 for QPM.

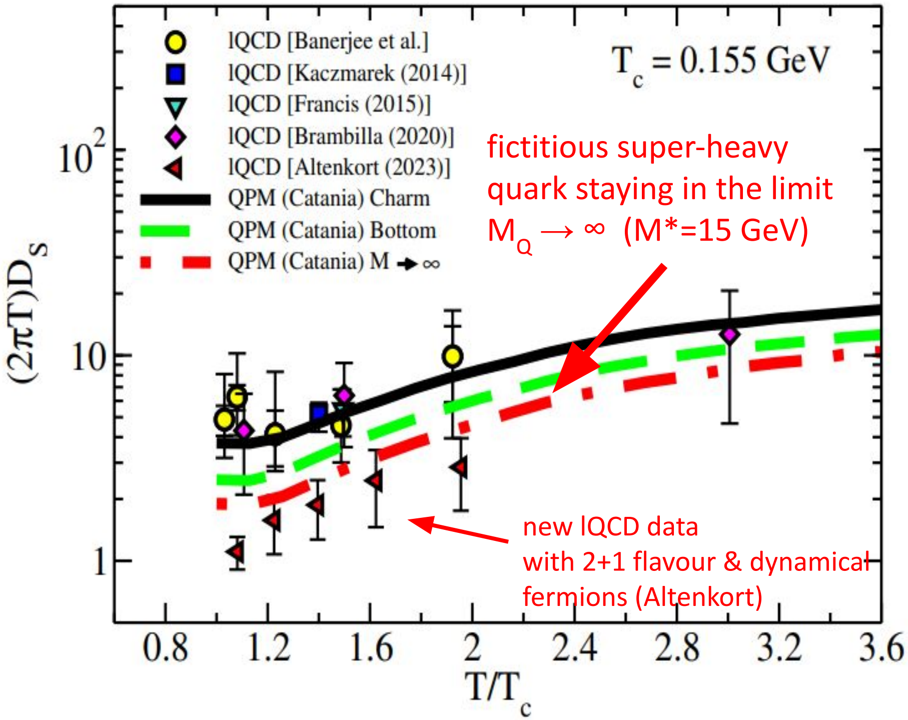
$Ds(M_{charm})/Ds(M \rightarrow \infty)$ ≃1.4 for pQCD.

- Ratios at fixed mass as a function of T:

  - **b/M^*: about 25% in all T range**
  - **c/b: about 50% at T_C and not smaller than 30%**
  - **c/M^*: factor 1.5-2**

M.L. Sambataro et al., e-Print: 2304.02953

$(2\pi T)D_s$: Charm quark vs Bottom quark



- lQCD data are in $M_Q \rightarrow \infty$ so D_s is mass independent

$$D_s = \frac{T}{M\,\gamma} = \frac{T}{M}\,\tau_{th}$$

- QPM use finite mass and includes dynamical fermions

From kinetic theory is expected that:

$$\tau_{th}(b)/\,\tau_{th}(c) \approx \gamma_c/\,\gamma_b \approx M_b/M_c$$

$D_s(T)$ from QPM in the infinite mass limit is the more pertinent to compare to lQCD simulations evaluated taking into account dynamical fermions

From D_s we obtain (in the 1-2T_c range):

- $\tau_{th}(c)$ ~ 5 fm/c
- $\tau_{th}(b)$ ~ 11 fm/c   breaking w.r.t. the relation:

$\tau_{th}(b) = (M_b/M_c)\tau_{th}(c)$ ~ 3.3 $\tau_{th}(c)$ ~ 16.5 fm/c

M.L. Sambataro et al., e-Print: 2304.02953

PRELIMINARY:

MOMENTUM DEPENDENT QPM

Going back to Quasi Particle Model (QPM)…

Equation of State and Susceptibilities

Non perturbative dynamics → M scattering matrices (q,g → Q) evaluated by Quasi-Particle Model fit to **lQCD thermodynamics**

N_f=2+1
Bulk:
u,d,s

QPM Standard

no momentum dependence

$$m_g^2(T) = \frac{2N_c}{N_c^2-1} g^2(T)\, T^2$$
$$m_q^2(T) = \frac{1}{N_c} g^2(T)\, T^2$$

→ **Thermal masses of gluons and light quarks**

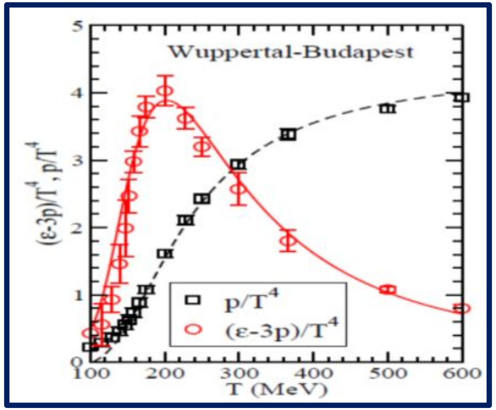
g(T) from a fit to ε from lQCD data → good reproduction of P, ε-3P

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

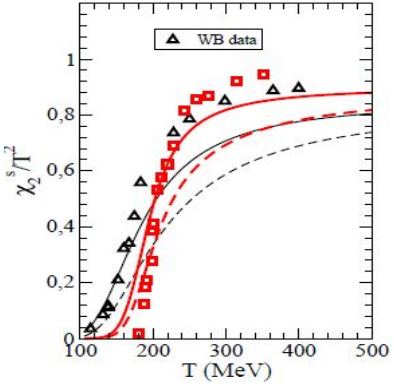
λ=2.6
T_s=0.57 T_c

Larger than pQCD especially as T → T_c

S. Plumari et al, *Phys.Rev.D* 84 (2011) 094004
H. Berrehrah,, PHYSICAL REVIEW C **93**, 044914 (2016)



Standard QPM understimates the **quark susceptibilities**



QPM extended – momentum dependence

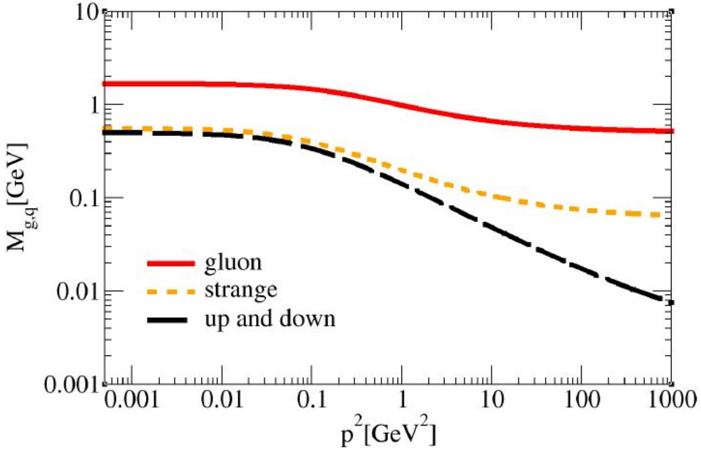
Dyson-Schwinger studies in the vacuum → following the model developed by PHSD group

H. Berrehrah, W. et al., Phys.Rev.C 93, 044914 (2016).
C. S. Fischer, J. Phys. G 32, R253 (2006).

$$M_g(T,\mu_q,p) = \left(\frac{3}{2}\right)\left(\frac{g^2(T^\star/T_c(\mu_q))}{6}\left[\left(N_c+\frac{1}{2}N_f\right)T^2+\frac{N_c}{2}\sum_q\frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_g(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2}+m_{\chi g}$$

$$M_{q,\bar{q}}(T,\mu_q,p) = \left(\frac{N_c^2-1}{8N_c}g^2(T^\star/T_c(\mu_q))\left[T^2+\frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_q(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2}+m_{\chi q}$$

Momentum dependent factors



we have also extended our quasi-particle model approach for **Nf = 2+1** to **Nf = 2 + 1 + 1** where the **charm quark is included**

QPM extended – momentum dependence

Dyson-Schwinger studies in the vacuum → following the model developed by PHSD group

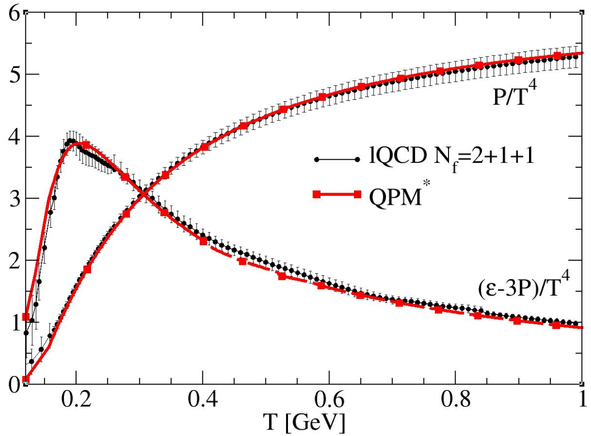
H. Berrehrah, W. et al., Phys.Rev.C 93, 044914 (2016).
C. S. Fischer, J. Phys. G 32, R253 (2006).

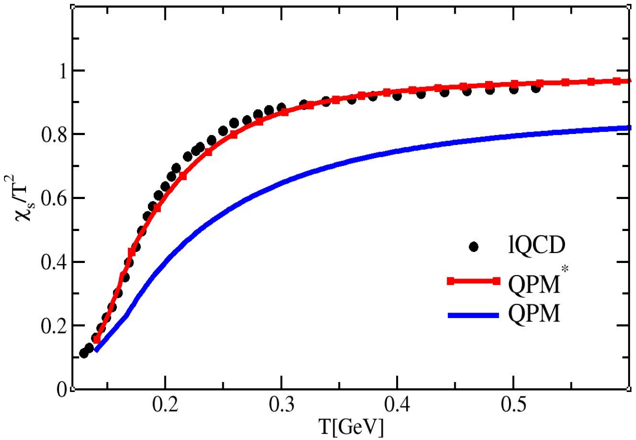
$$M_g(T,\mu_q,p) = \left(\frac{3}{2}\right)\left(\frac{g^2(T^\star/T_c(\mu_q))}{6}\left[\left(N_c+\frac{1}{2}N_f\right)T^2+\frac{N_c}{2}\sum_q\frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_g(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2}+m_{\chi g}$$

$$M_{q,\bar{q}}(T,\mu_q,p) = \left(\frac{N_c^2-1}{8N_c}g^2(T^\star/T_c(\mu_q))\left[T^2+\frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_q(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2}+m_{\chi q}$$

Momentum dependent factors

We correctly reproduce both **EoS** and **quark susceptibilities** which are understimated in the standard QPM approach.

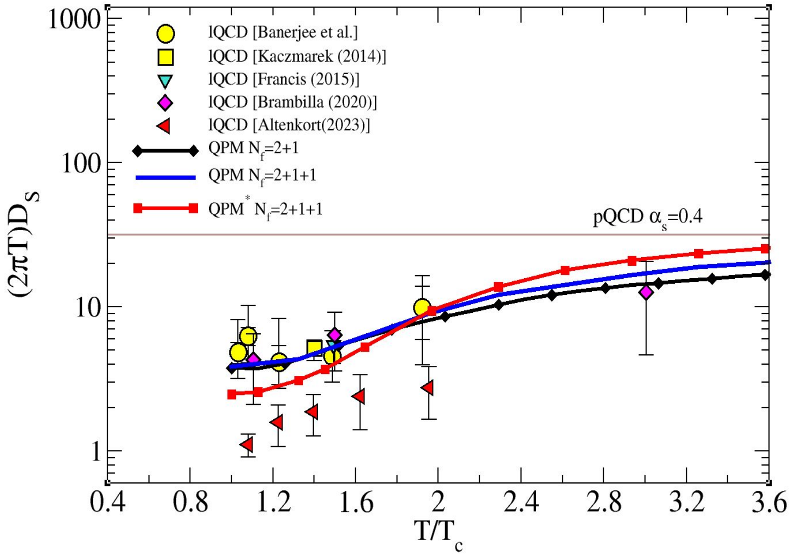




$$\chi_s = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial \mu_s^2}$$

QUARK SUSCEPTIBILITIES

QPM extended – Preliminary D_S and R_{AA}



Spatial diffusion coefficient D_s → standard QPM

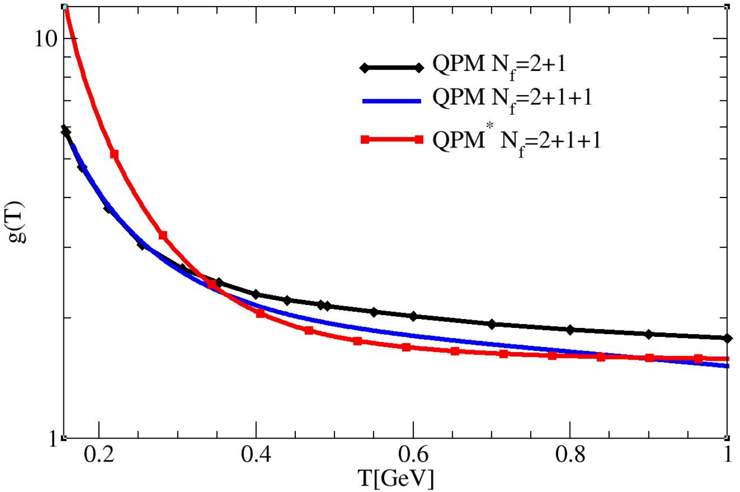
standard QPM including charm

extended QPM

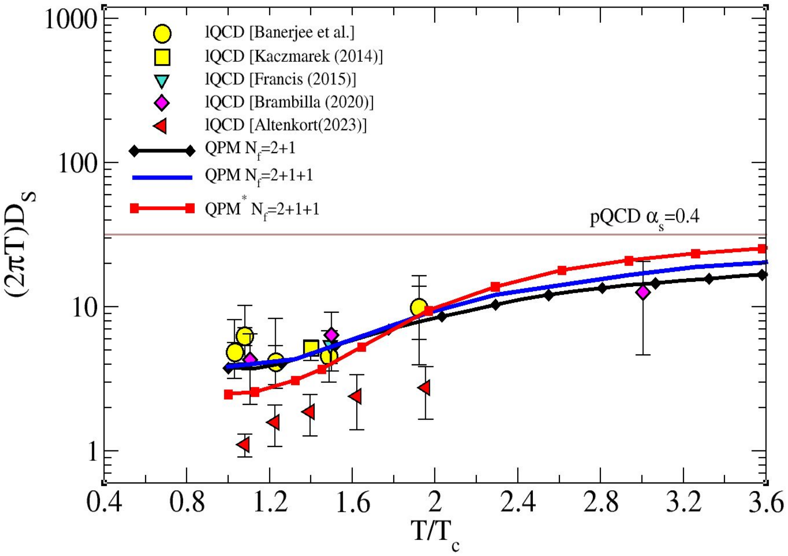
$T/T_c < 2$ → strong non-perturbative behaviour near to T_c.

high T region → the D_s reaches the pQCD limit quickly than the standard QPM.

coupling g(T)



QPM extended – Preliminary D_S and R_{AA}



Spatial diffusion coefficient D_s → standard QPM
standard QPM including charm
extended QPM

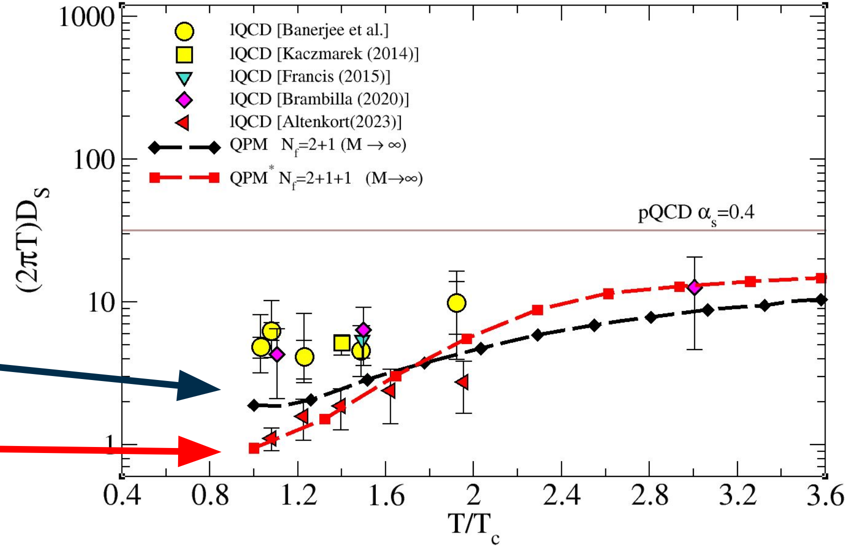
$T/T_c < 2$ → strong non-perturbative behaviour near to T_c.

high T region → the D_s reaches the pQCD limit quickly than the standard QPM.

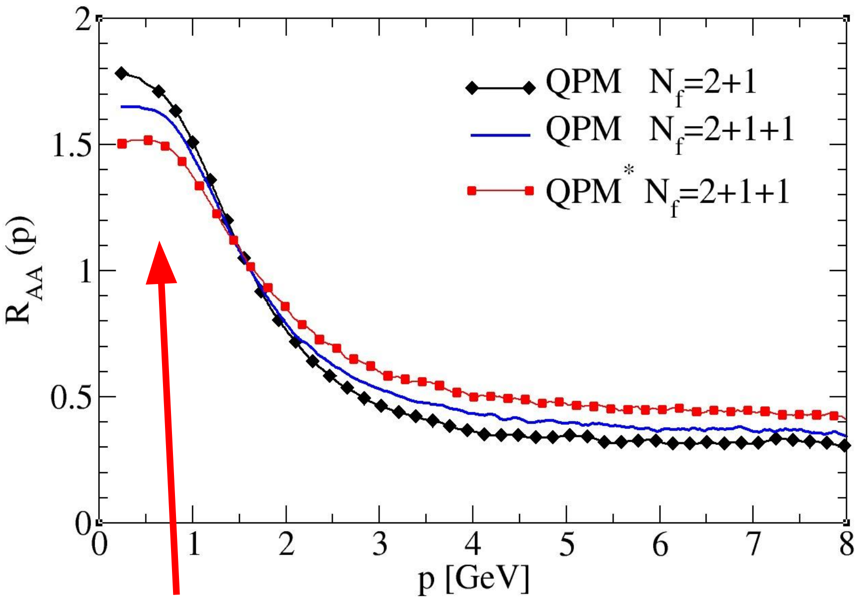
Extended QPM + infinite mass limit?

standard QPM

extended QPM



QPM extended – Preliminary D_s and R_{AA}



Decrease at low p

In a static box

Initial momentum distribution function
→ FONLL for charm quark

$$R_{AA} \doteq f_C(p, t_f)/f_C(p, t_0)$$

Momentum dependent QPM approach

- Better description of recent lQCD data.
- Effects on the global χ^2coming from the comparison to the experimental data of R_{AA}, v_n?

Conclusions

- Extension to bottom quark dynamics: good description of R_{AA} and v_2 of electrons from semileptonic B meson decay and prediction for v_3

- **Spatial diffusion coefficient D_s(T): charm vs bottom and the infinite mass limit**
  - ➔ D_s(c)/ D_s(b) ratio of about a factor of 1.5 at T ~ Tc and 1.3 at higher temperatures (T ~ 3 − 4 Tc)
  - ➔ For the **charm mass scale:** the infinite mass limit used in lQCD is not yet reached;
    For the **bottom mass scale**: discrepancy of only about a 20% w.r.t. the infinite mass limit
  - ➔ Taking into account mass scale dependence in QPM, we have satisfactory agreement with the most recent lQCD calculations that include dynamical fermions, differently from previous lQCD data in quenched approximation.
  - ➔ Thermalization time for bottom quark: τ_{th} ~ 10 − 12 fm/c which is about a factor of 2 larger than charm and so quite smaller than 3.3 as suggest by a simple M_Q/T scaling.

- **Extended QPM**

  Good reproduction of both EoS and susceptibilities.

  Preliminary for a static medium: decrease of D_s at small T and decrease of R_{AA} at low p

  Perspectives: Effect on observables for realistic simulations?

Thanks for the attention!

Back up slides

Relativistic Boltzmann equation at finite η/s

Bulk evolution

$$p^{\mu}\partial_{\mu}f_{q}(x,p)+m(x)\partial_{\mu}^{x}m(x)\partial_{p}^{\mu}f_{q}(x,p)=C[f_{q},f_{g}]$$
$$\underbrace{p^{\mu}\partial_{\mu}f_{g}(x,p)}_{\text{Free-streaming}}+\underbrace{m(x)\partial_{\mu}^{x}m(x)\partial_{p}^{\mu}f_{g}(x,p)}_{\substack{\text{field interaction} \\ \varepsilon\,-\,3p\,\neq\,0}}=C[f_{q},f_{g}]$$

Collision Integral gauged to reproduce viscous Hydro at fixed η/s by means of Chapman-Ensksog

$$\boxed{\sigma(n(\vec{x}),T)=\frac{1}{15}\frac{\langle p\rangle_0}{g(a)n(\vec{x})}\frac{1}{\eta/s}}$$

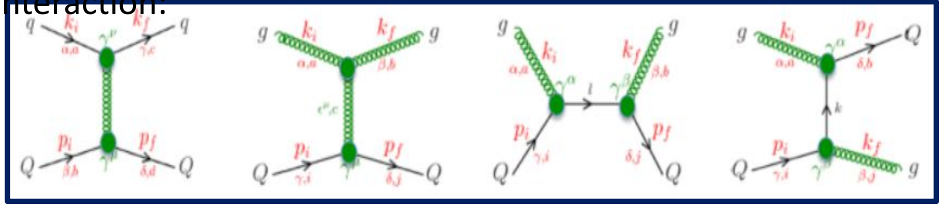
Equivalent to viscous hydro at η/s ≈ 0.1

HQ evolution

$$p^{\mu}\partial_{\mu}f_{Q}(x,p)=C[f_{q},f_{g},f_{Q}]$$

$$C[f_{q},f_{g},f_{Q}]=\frac{1}{2E_1}\int\frac{d^3p_2}{2E_2(2\pi)^3}\int\frac{d^3p_1{}'}{2E_1{}'(2\pi)^3}$$
$$\times[f_Q(p_1{}')f_{q,g}(p_2{}')-f_Q(p_1)f_{q,g}(p_2)]$$
$$\times|M_{(q,g)\to Q}(p_1p_2\to p_1{}'p_2{}')|$$
$$\times(2\pi)^4\delta^4(p_1+p_2-p_1{}'-p_2{}')$$

Feynmann diagrams at first order pQCD for HQs-bulk interaction:

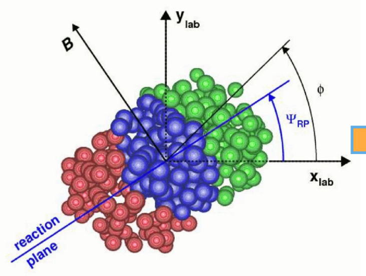


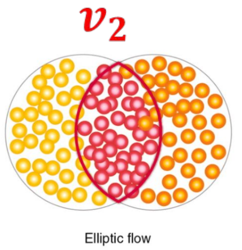
Scattering matrices $M_{g,q}$ by QPM fit to lQCD thermodynamics

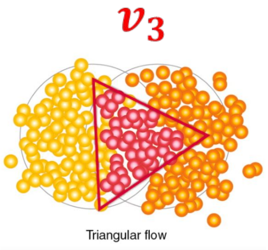
Extension to higher order anisotropic flows $v_n(p_T)$

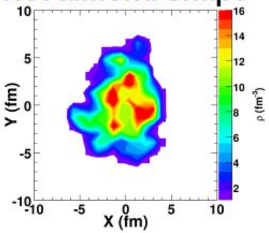
$$E\frac{d^3N}{dp_T} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left\{1+\sum_{i=1}^{\infty} v_n\cos[n(\varphi-\Psi_n)]\right\}$$

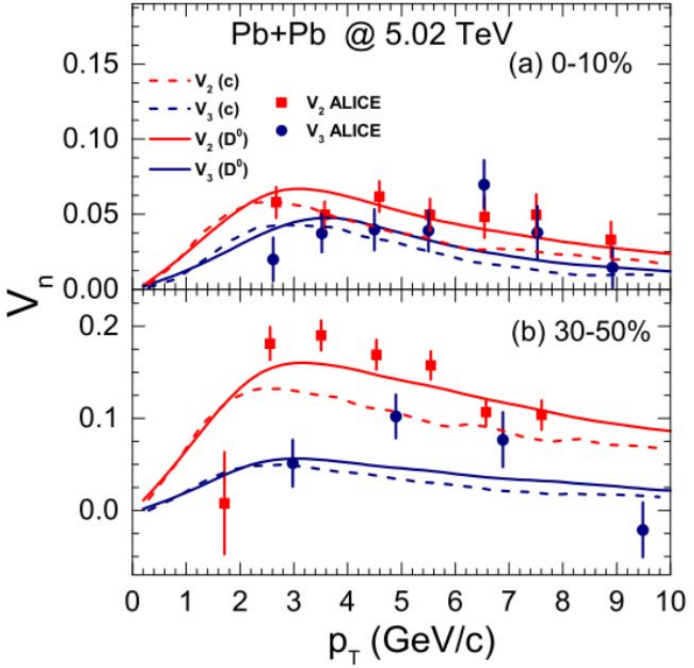
Sambataro et al., Eur.Phys.J.C 82 (2022) 9, 833











Monte Carlo Glauber for initial condition of partons
S.Plumari et al, *Phys.Rev.C* 92 (2015) 5

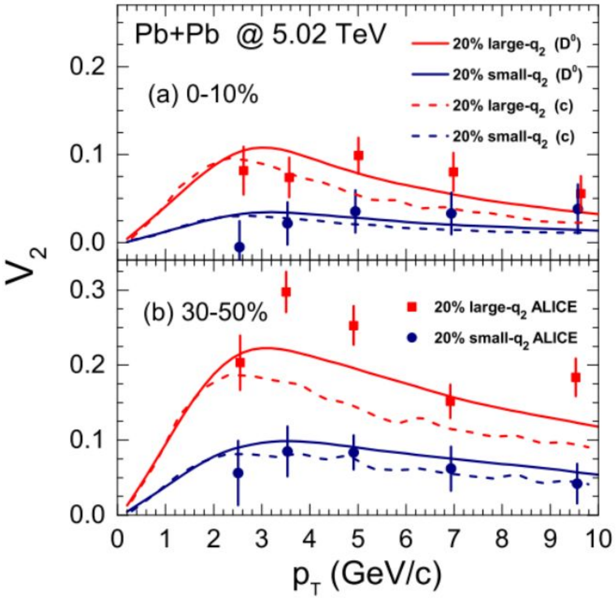
Data taken from ALICE collaboration: *Phys.Lett.B* 813 (2021) 136054

In the more peripheral collision (30-50 % centrality class) → larger v_2 and comparable v_3

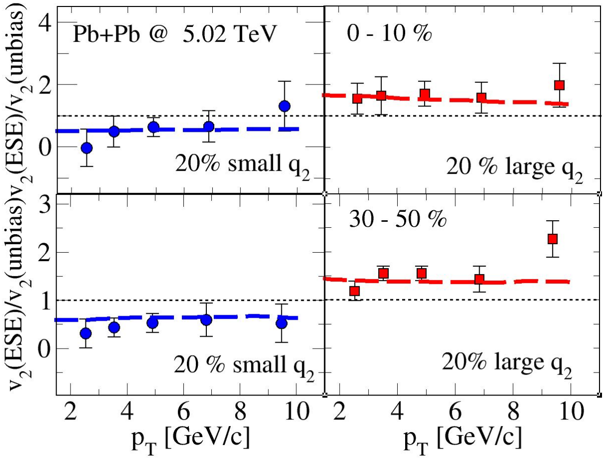
- ➢ v_2 mainly generated by the geometry of overlapping region in larger centrality collision
- ➢ v_3 mainly driven by the fluctuation of the triangularity of overlap region at all centrality

ESE: v_2 and spectra (20% small/large q_2)

$q_2$selected $v_2(p_T)$



$q_2$selected $v_2(p_T)$ ratio



Data taken from ALICE collaboration:
Phys.Lett.B 813 (2021) 136054

- v_2 (large- q_2 /small- q_2) $\gtrless v_2$ (unbiased) of about 50% in both 0-10% and 30-50% centrality.

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

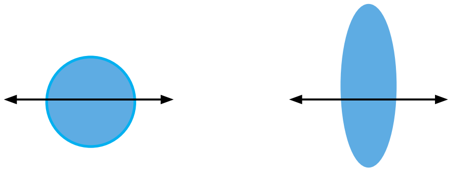
Extension to higher order anisotropic flows $v_n(p_T)$

ESE technique and v_n correlations

Selection of events with the same centrality but different initial geometry on the basis of the magnitude of the second-order harmonic reduced flow vector q_2.

$$q_2 = |\vec{Q}_2|/\sqrt{M}$$

$$\vec{Q}_2 = \sum_{j=1}^{M} e^{i2\varphi_j}$$



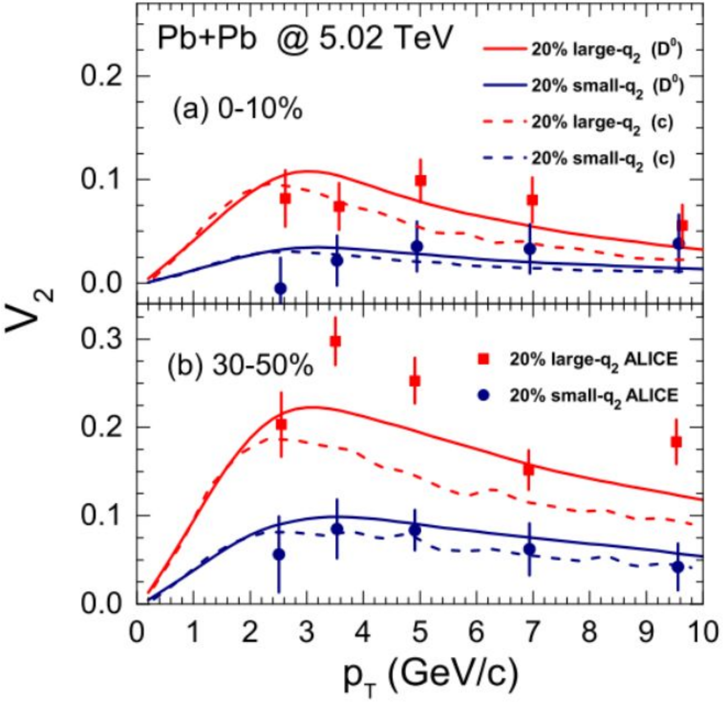
20 % small q_2 20 % large q_2

Large q_2 → large ε_2

$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\varphi - \Phi_n)]\rangle}{\langle r_\perp^n \rangle} \qquad \Phi_n = \frac{1}{n}\arctan\frac{\langle r_\perp^n \sin(n\varphi)\rangle}{\langle r_\perp^n \cos(n\varphi)\rangle}$$

$$r_\perp = \sqrt{x^2 + y^2}, \qquad \varphi = \arctan(y/x)$$

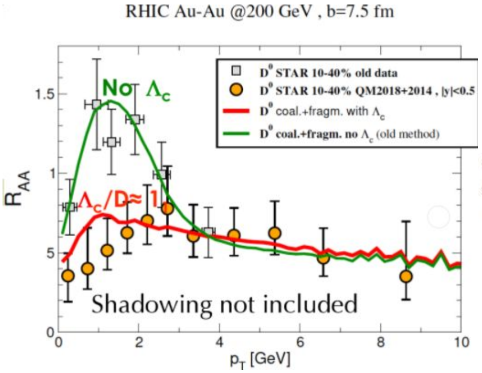
q_2 selected $v_2(p_T)$



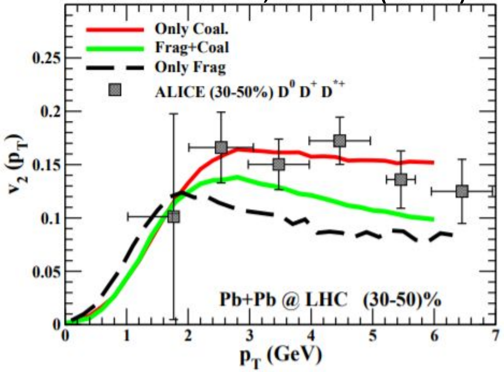
Data from ALICE collaboration: *Phys.Lett.B* 813 (2021) 136054

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

Catania QPM: some prediction for charm…



Scardina et al., PRC 97(2017)



Good description of

R_{AA}, v_2 at RHIC & LHC energies within error bars

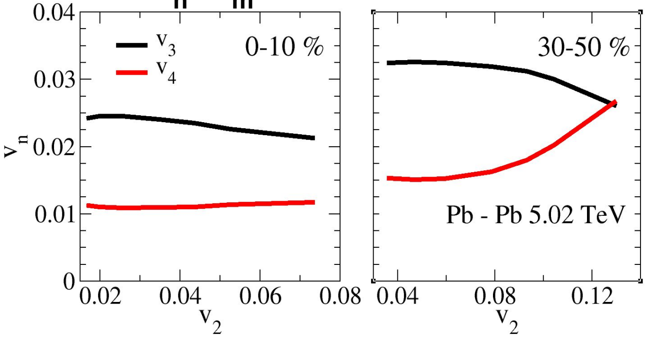
Monte Carlo Glauber for initial condition of partons

S.Plumari et al, *Phys.Rev.C* 92 (2015) 5

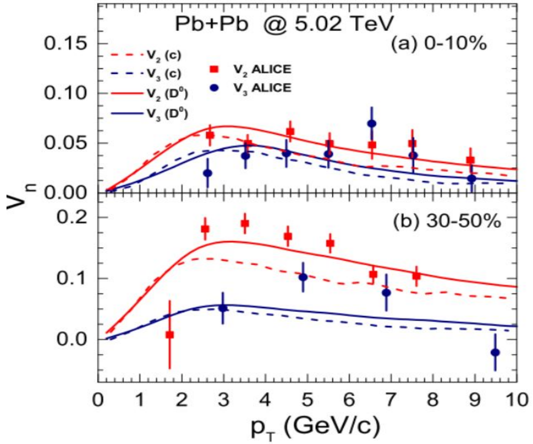
Predictions for D mesons

- **Event-Shape Engeenering Technique:**
  Prediction for similar correlation for hard particles wrt bulk

v_n -v_m correlations



Triangular flow v_3



ALICE collaboration, *Phys.Lett.B* 813 (2021) 136054

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

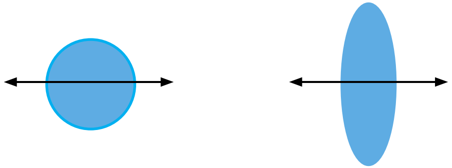
Extension to higher order anisotropic flows $v_n(p_T)$

ESE technique and v_n correlations

Selection of events with the same centrality but different initial geometry on the basis of the magnitude of the second-order harmonic reduced flow vector q_2.

$$q_2 = |\vec{Q}_2| / \sqrt{M}$$

$$\vec{Q}_2 = \sum_{j=1}^{M} e^{i2\varphi_j}$$



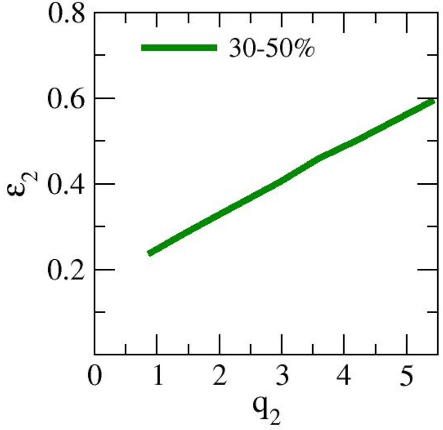
20 % small q_2 20 % large q_2

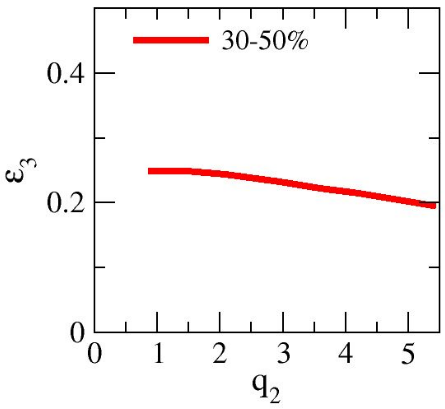
Large $q_2 \rightarrow$ large ε_2

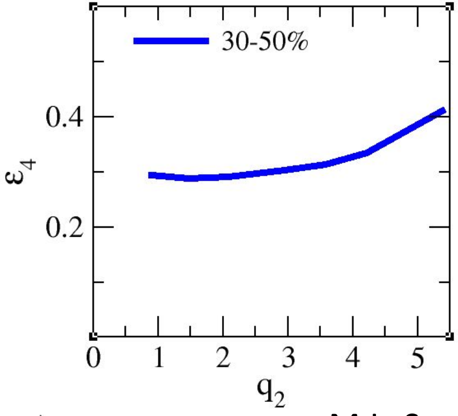
Monte Carlo Glauber for initial condition of partons

S.Plumari et al, *Phys.Rev.C* 92 (2015) 5

Pb-Pb 5,02







Anti-correlation between ε_2 and ε_3

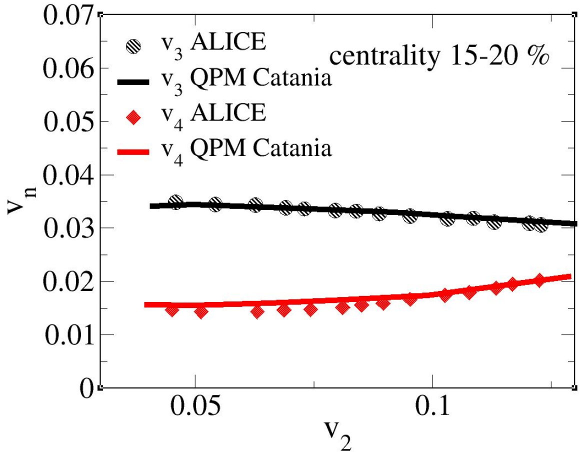
Non-linear correlation between ε_2 and ε_4

M.L. Sambataro, et al., *Eur.Phys.J.C* 82

ESE: $v_n - v_m$ correlations

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

Charged particles

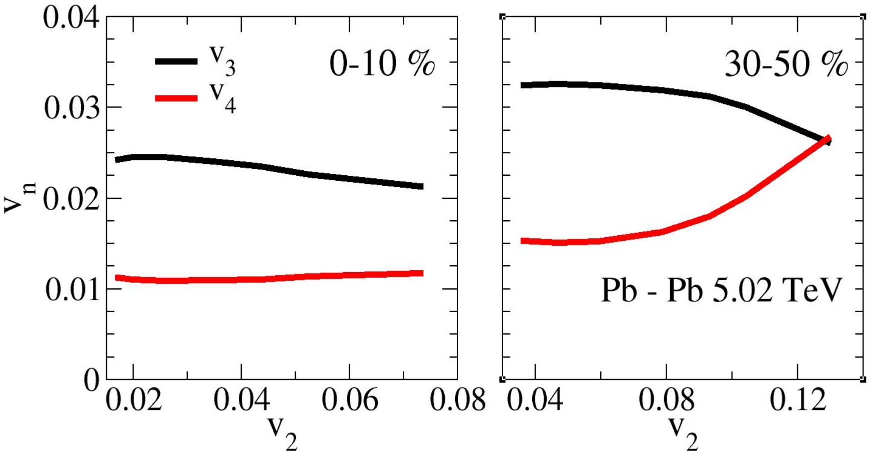


Correlations between the ε_n and ε_m present in the initial geometry → correlations between flow harmonics different orders, i.e. correlations v_n and v_m

- **Good description of $v_{n\text{-}m}$ correlation for bulk**
- **Prediction for similar correlation for hard particles**
- **Correlation for D mesons provide insights on the interaction**

Plumari et al, Phys.Lett.B 805 (2020) 135460

Predictions for D mesons

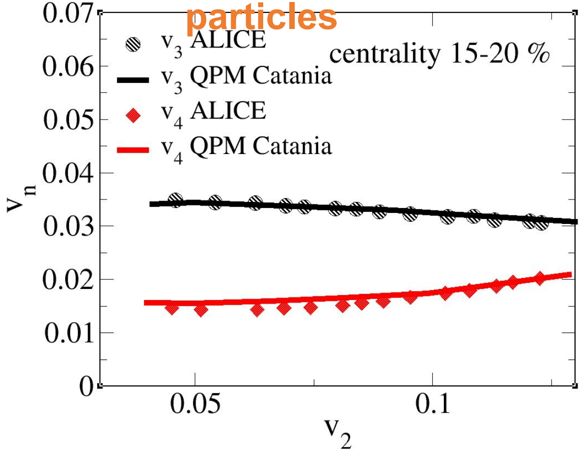


Data taken from: S. Mohapatra *Nucl.Phys.A* 956 (2016) 59-66

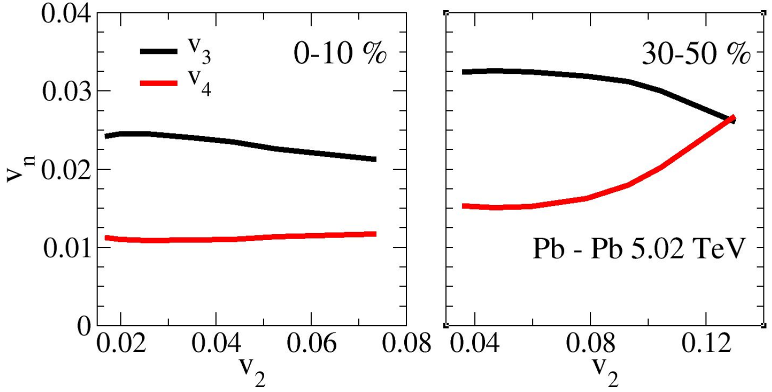
ESE: $v_n - v_m$ correlations

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

Charged particles



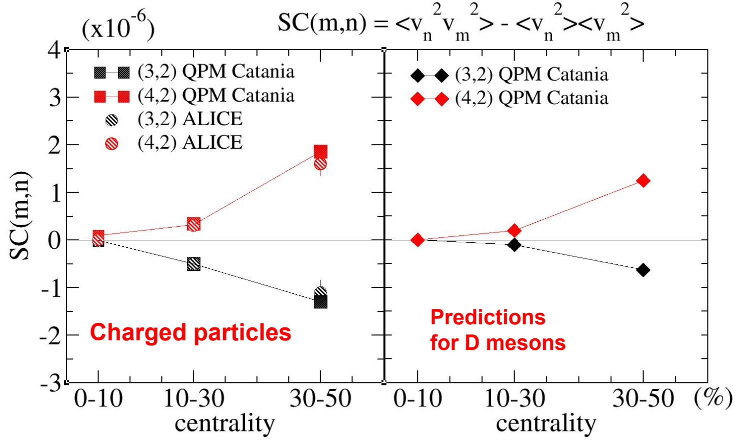
Predictions for D



Correlations between the ε_n and ε_m present in the initial geometry → correlations between flow harmonics different orders, i.e. correlations v_n and v_m

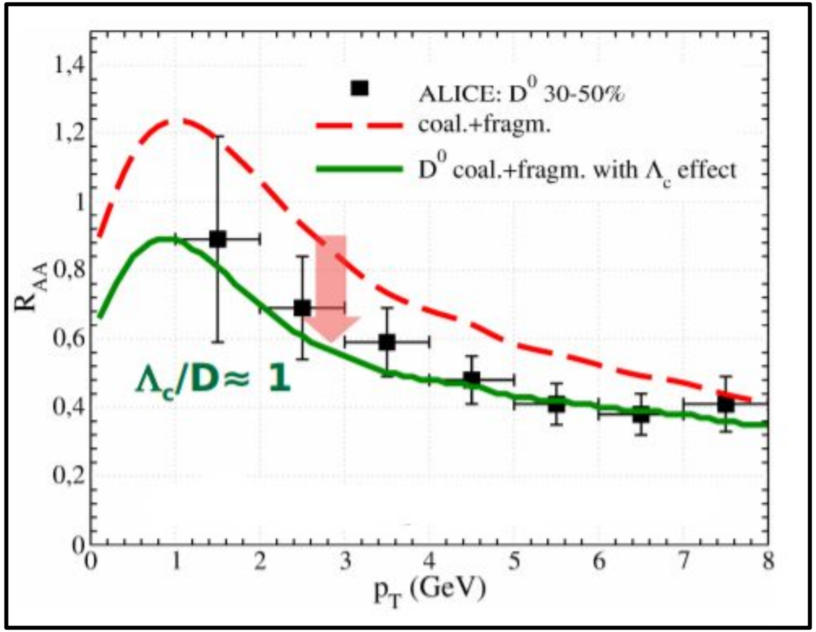
- **Good description of $v_{n\text{-}m}$ correlation for bulk**
- **Prediction for similar correlation for hard particles**
- **Correlation for D mesons provide insights on the interaction and its temperature dependence**

Plumari et al, Phys.Lett.B 805 (2020) 135460



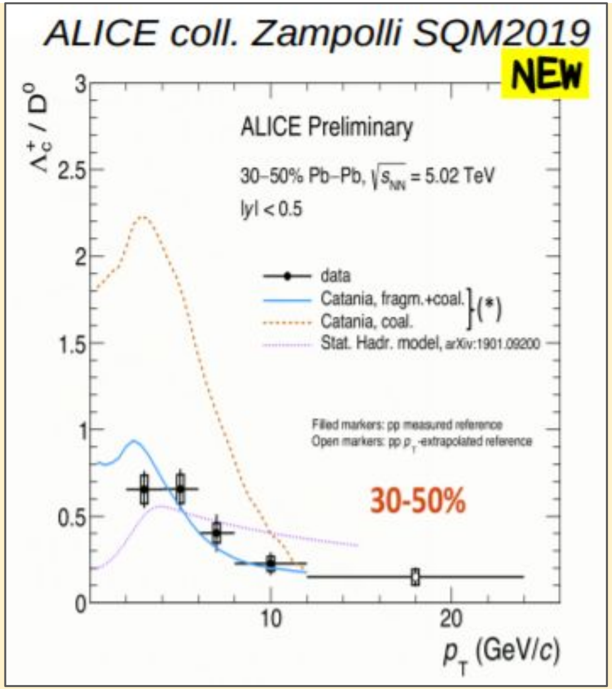
Data taken from: S. Mohapatra *Nucl.Phys.A* 956 (2016) 59-66

D meson: Impact of large Λ_c production on R_{AA}



$D_s(\mathrm{T})$ of charm quark that reproduces R_{AA} and v_2 gives good description of

- Impact of $\Lambda_C/\ D^0$
- Triangular flow $v_3(p_T)$.
- q_2 selected anisotropic flow and spectra.



- With the same coalescence plus fragmentation model we describe the $\Lambda_C/\ D^0$

S. Plumari, et al.,
Eur. Phys. J. C78 no. 4, (2018) 348

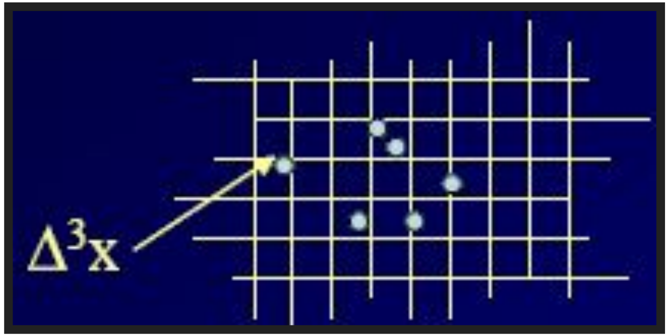
Numerical solution of Boltzmann Equation

- **Use Test-Particle Method to sample the phase space distribution function**

$$f(\vec{x},\vec{p},t)=\omega\sum_{i=1}^{N_{test}}\delta^{(3)}(\vec{x}-\vec{r}_i(t))\delta^{(3)}(\vec{p}-\vec{p}_i(t))$$

F_i solution of Boltzmann eq.
→ Test particles solve classical Hamilton eq. of motion

$$\begin{cases}\vec{p}_i(t+\Delta t)=\vec{p}_i(t-\Delta t)+2\Delta t\cdot\left(\frac{\partial\vec{p}_i}{\partial t}\right)_{coll}\\ \vec{r}_i(t+\Delta t)=\vec{r}_i(t-\Delta t)-2\Delta t\cdot\left[\frac{\vec{p}_i(t)}{E_i(t)}\right]\end{cases}$$



- **Collision Integral mapped through a Stochastic Al**

Δt □ 0 and Δ^3x □ 0 : exact solution

$$P_{22}=\frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1\Delta N_2}=v_{rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

Final phase-space of HQ + bulk parton scattering sampled according to $|M_{QCD}|^2$ □ code test through simulations in a "box"

[Scardina, Colonna, Plumari, and Greco PLB v.724, 296 (2013)]
[Xu and Greiner PRC v. 71, (2005)]

Hybrid Hadronization Model for HQs

✔ **COALESCENCE**: Formula developed for the light sector **[Greco, Ko, Levai PRL 90 (2003)]**

$$\frac{dN_H}{d^2\boldsymbol{P}_T} = g_H \int \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i \, f_{q_i}(x_i, p_i) \, f_W(x_1 \ldots x_n; p_1 \ldots p_n) \, \delta\left(\boldsymbol{P_T} - \sum_i^n p_{T,i}\right)$$

Statistical Factor
Color-spin-isospin

Parton Distribution Functions
(after Boltzmann evolution)

Hadron Wigner Function
(parameters fix according to quark model)
C.-W. Hwang, EPJ C23, 585 (2002)
C. Albertus et al., NPA 740, 333 (2004)

✔ **FRAGMENTATION**: HQs that do not undergo to Coalescence

$$\frac{dN_H}{d^2\boldsymbol{P}_T} = \sum_f \int dz \frac{dN_f}{d^2 p_T} \frac{D_{f\to H}(z)}{z^2}$$

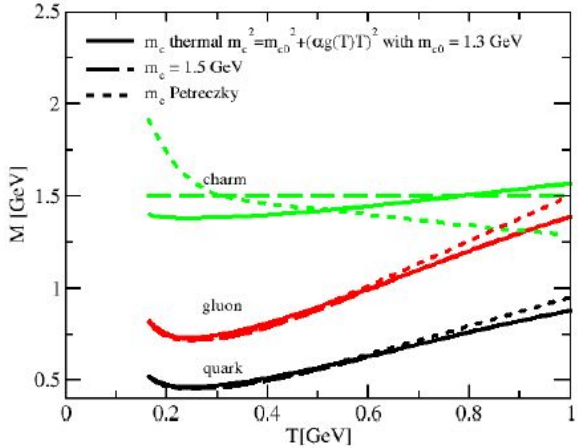
We use Peterson parametrization: $D_H(z) \propto \left[z \left(1 - \frac{1}{z} - \frac{\epsilon_c}{1-z} \right)^2 \right]^{-1}$ *Peterson et al. PRD 27 (1983) 105*

Parameter ε_c tuned to reproduce *D* and *B* meson spectra in pp collisions.

Plumari, Minissale, Das, Coci, Greco, EPJ C 78 (2018) no.4

QPM with $N_f = 2 + 1 + 1$ including charm

Recently, new lattice results for the equation of state of QCD with 2+1+1 dynamical flavors have become available. Therefore, we extend our QPM approach for $N_f = 2 + 1$ to $N_f = 2 + 1 + 1$ where the charm quark is included.



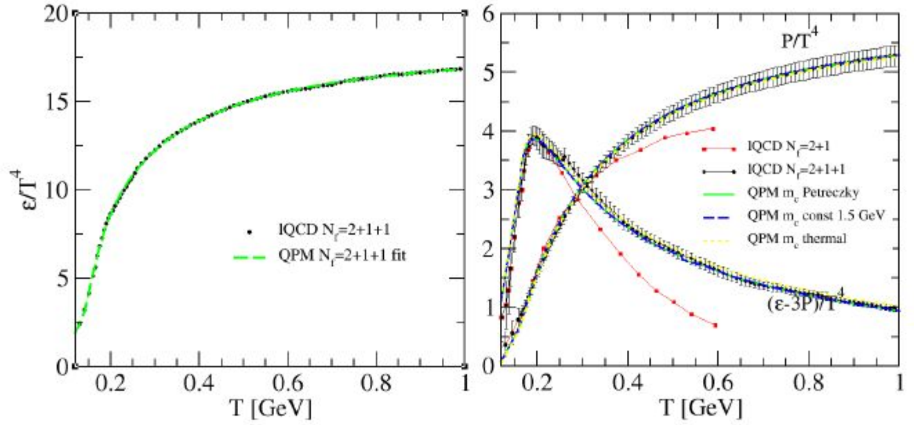
Three cases:

Constant mass $m_c = 1.5\ GeV$

Thermal mass $m_c^2 = m_{c0}^2 + \frac{N_c^2-1}{8N_c} g^2 T^2$

Charm Mass constrained by quark susceptibility from lQCD m_c [Petreczky]

- **Energy density** fit to lQCD data
- Good reproduction of **pressure** and **interaction measure** for three different charm quark masses.



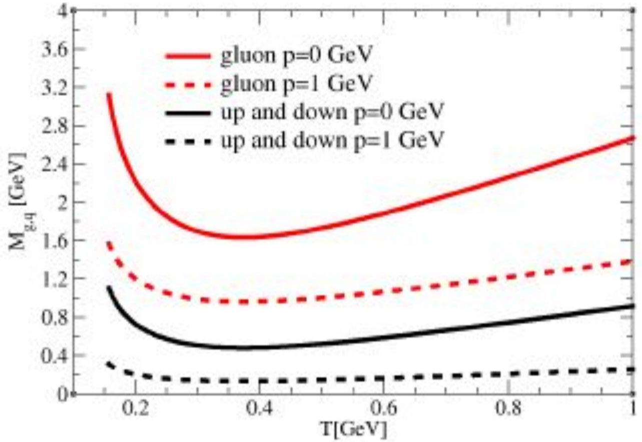
Petreczky, Heavy-Quark Transport from Lattice-QCD, Heavy flavor transport in QCD matter workshop (2021).

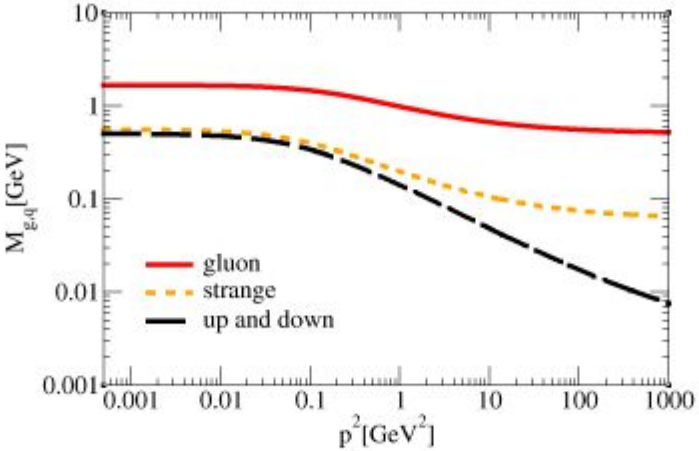
QPM extended – momentum dependence

Dyson-Schwinger studies in the vacuum → following the model developed by PHSD group

$$\boxed{\begin{aligned} M_g(T,\mu_q,p) &= \left(\frac{3}{2}\right)\left(\frac{g^2(T^\star/T_c(\mu_q))}{6}\left[\left(N_c+\frac{1}{2}N_f\right)T^2+\frac{N_c}{2}\sum\frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_g(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2}+m_{\chi g} \\ M_{q,\bar{q}}(T,\mu_q,p) &= \left(\frac{N_c^2-1}{8N_c}g^2(T^\star/T_c(\mu_q))\left[T^2+\frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_q(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2}+m_{\chi q} \end{aligned}}$$

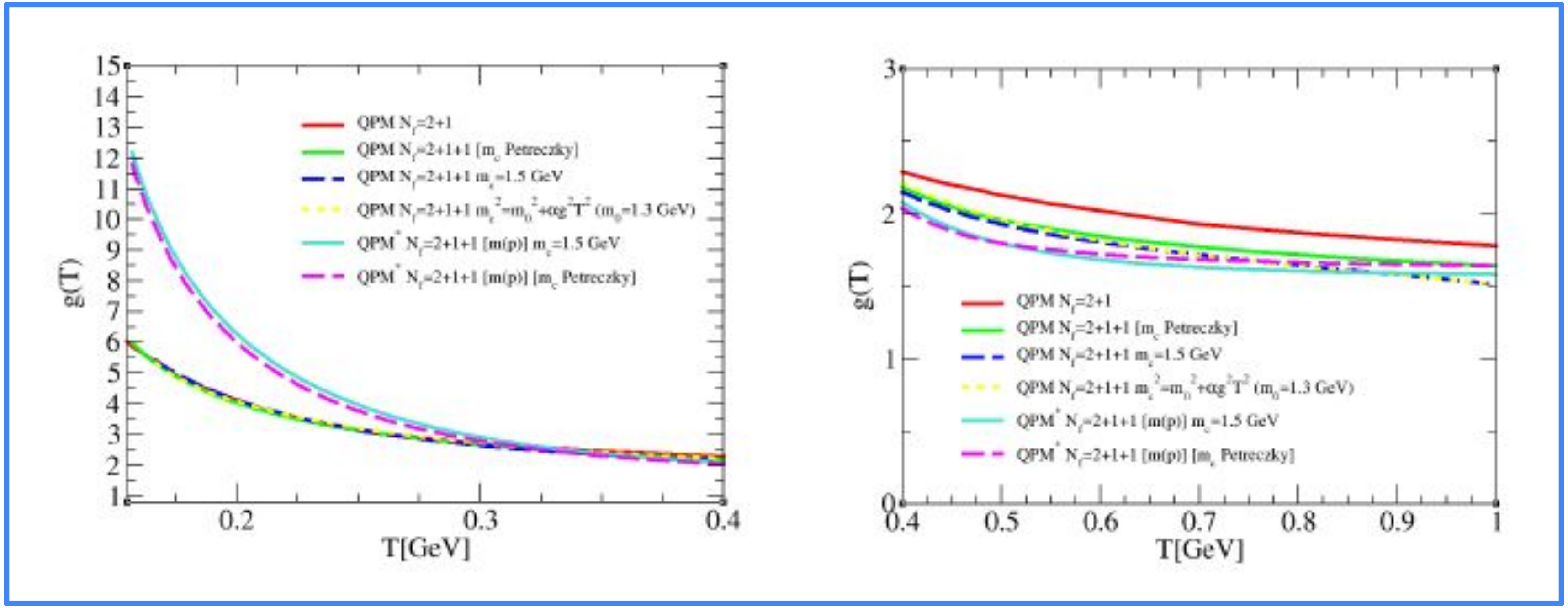
Momentum dependent factors





QPM extended – coupling and drag coefficient

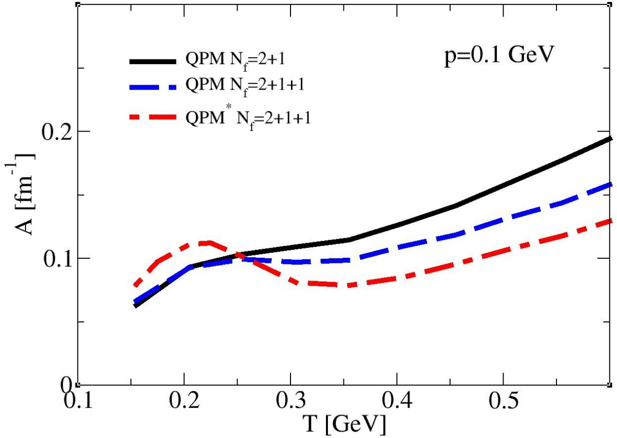
$0{,}155 < T < 0{,}4\ GeV$ $\qquad$ $0{,}4 < T < 1\ GeV$



Coupling g(T) → standard QPM vs extended QPM

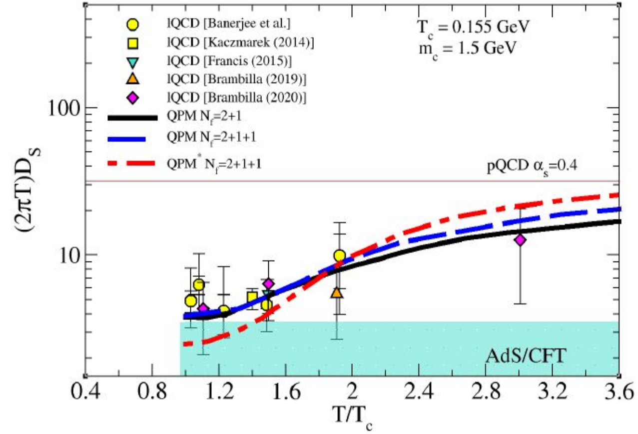
- Large **enhancement** at low T
- **Comparable value** at high T with a discrepancy

Drag and D_s in QPM extended



Drag coefficient → standard QPM
standard QPM including charm
extended QPM

- **Increase** at low T consistent with the large enhancement of the coupling in the same T region
- **Decrease** at high T



Spatial diffusion coefficient D_s

$T/T_c < 2$ → strong non-perturbative behaviour near to T_c.

high T region → the D_S reaches the pQCD limit quickly than the standard QPM.

DQPM Dynamical Quasi-particle Model

In order to determinate the parton masses and widths within the DQPM approach, the analytical expression of the dynamical quasi-particle entropy density s_{DQP} is fitted to the lQCD entropy density s_{lQCD} determined numerically:

$$s^{DQP} = -\sum_{i=g,q,\bar{q}} \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n_{B/F,F}}{\partial T} \times (\mathcal{I}ln(-\Delta_i^{-1}) + \mathcal{I}\Pi_i \mathcal{R}\Delta_i)$$

Quasi-particle contribution interaction contribution

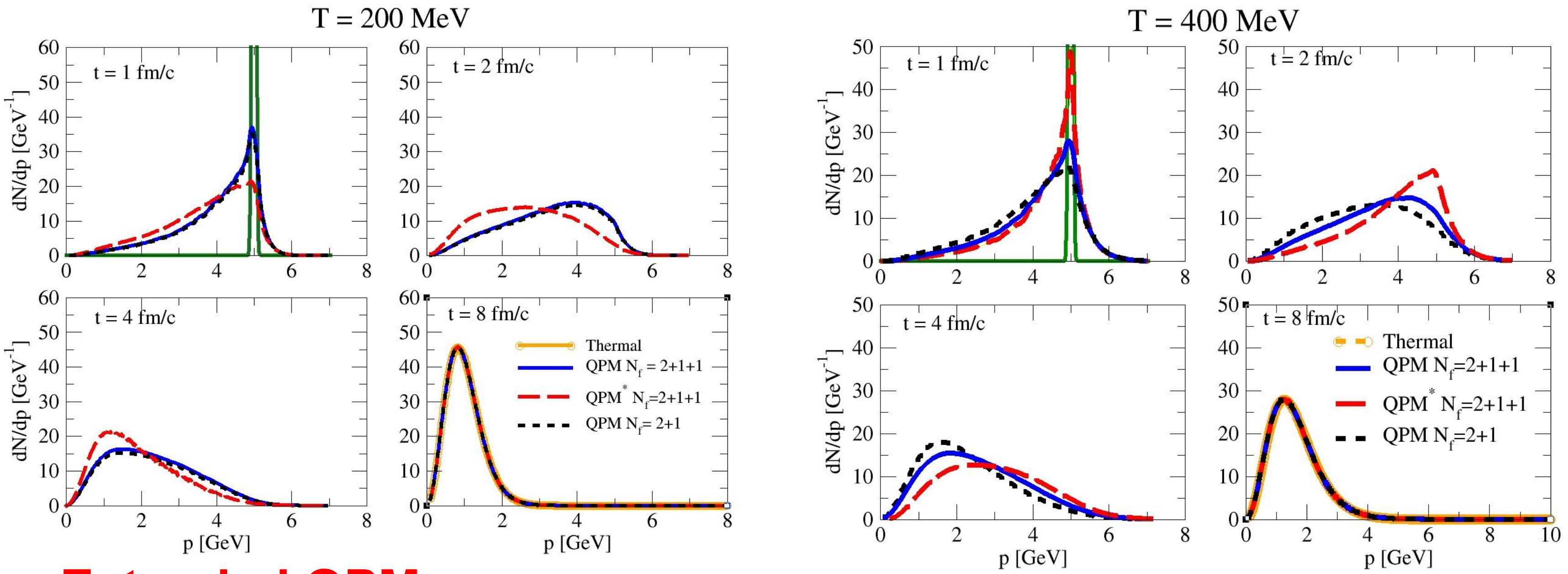
The **spectral functions** within DQPM approach describe the variation of parton masses as a function of the medium properties with result that these spectral functions are no longer δ functions in the invariant mass squared (as in the case for bare masses) but related to the imaginary part of the trace of the effective propagator $D_{\nu\mu}$ and to the partonic self-energies $\Sigma_{\nu\mu}$ as:

$$S(p) \propto \mathcal{I}D^{\mu}_{\mu}(p) \propto \frac{\mathcal{I}\Sigma^{\mu}_{\mu}(p)}{[p^2 - \mathcal{R}\Sigma^{\mu}_{\mu}(p)]^2 + [\mathcal{I}\Sigma^{\mu}_{\mu}(p)]^2}$$

Σ Self energy

Preliminary: Off-shell
evolution of charm quark distribution function

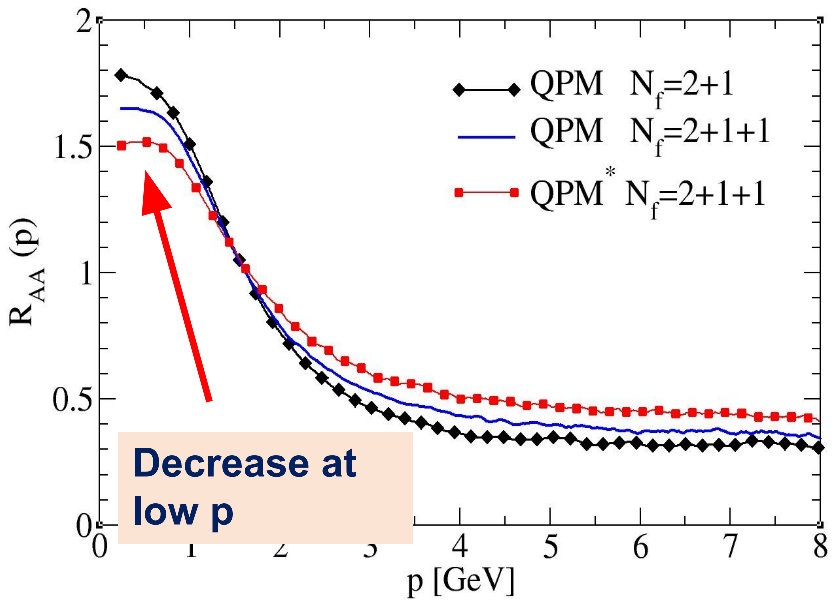
BOX CALCULATION FOR CHARM → static medium



Extended QPM:

- $T > T_c$ slower dynamics
- $T \rightarrow T_c$ the increase in the drag coefficient leads to a faster evolution

Preliminary:
Nuclear modification factor R_{AA}



$$R_{AA} \doteq f_C(p, t_f)/f_C(p, t_0)$$

Initial momentum distribution function
→ FONLL for charm quark

Momentum dependent QPM approach

- Better description of recent lQCD data.
- Effects on the global χ^2 coming from the comparison to the experimental data of R_{AA}, v_n?