

# Percolation and conformal limits in neutron stars

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# Quark Matter in Neutron Stars?

## Solid Constraints

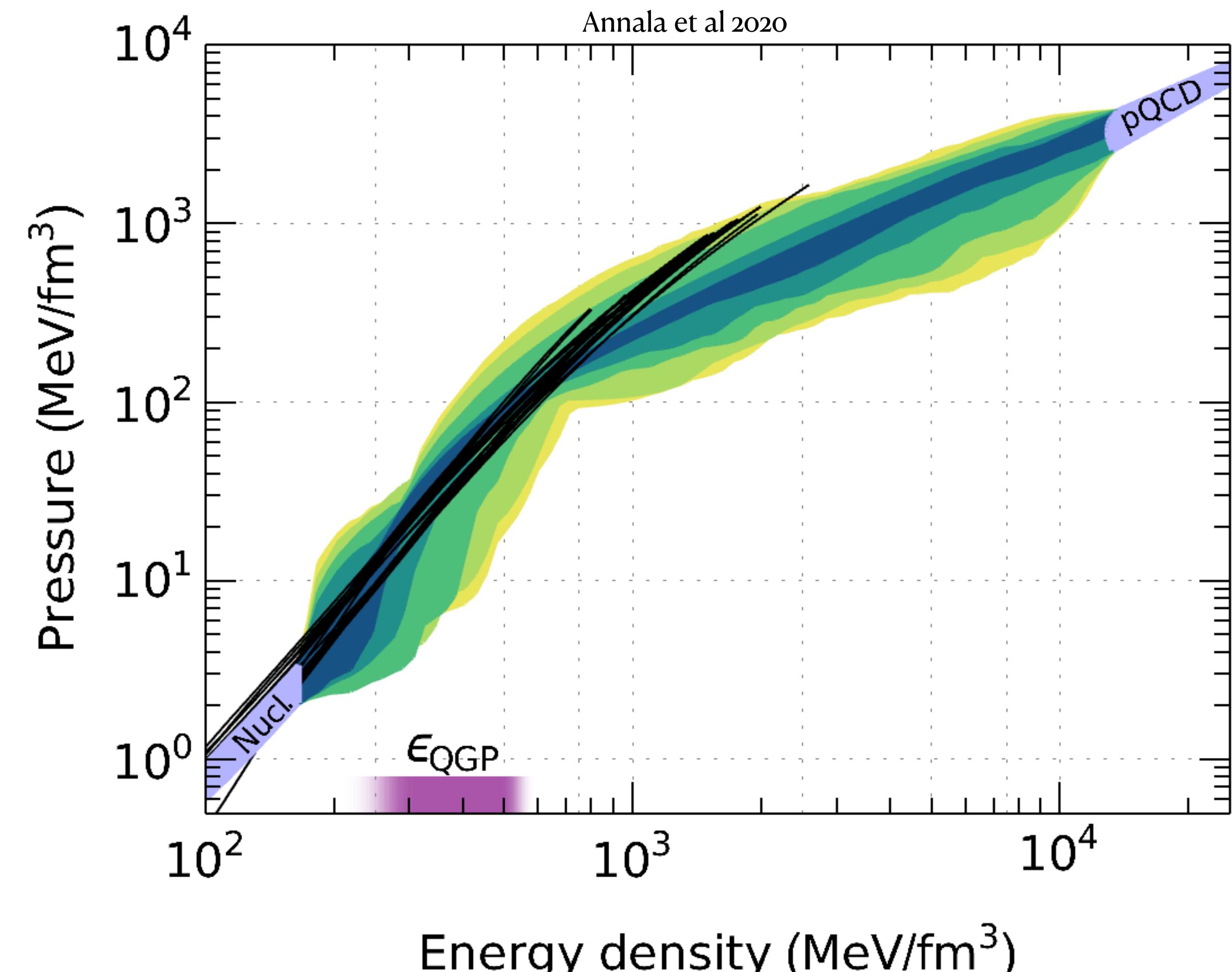
- Low density:  $\chi$ EFT ( $n \lesssim 1.1n_0$ ) Tews et al, 2013
- High density pQCD ( $n \gtrsim 40n_0$ ) Gorda et al, 2018

## Interpolation methods

- Polytropes, CSS, Linear Speed of Sound  
eg. Annala et al, 2018, 2020; Alford et al 2013, 2017, Li et al 2021

## Deconfinement by polytropic index

$$\gamma = \frac{d \log p}{d \log \epsilon} \rightarrow \begin{cases} \gamma > 1.75 \rightarrow \text{Hadrons} \\ \gamma < 1.75 \rightarrow \text{Quarks} \end{cases}$$



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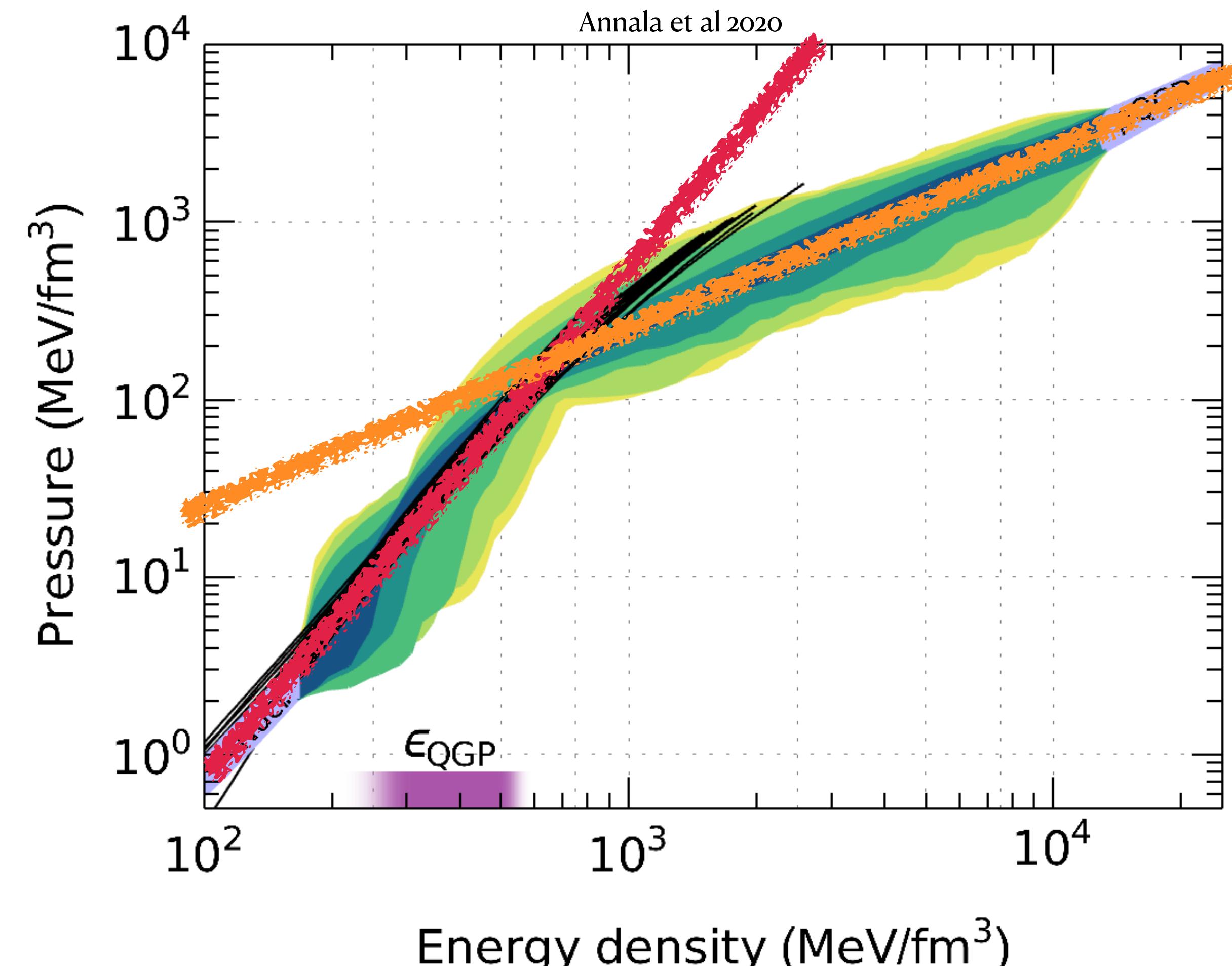
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# Methodology: Piecewise-linear speed of sound

Annala et al 2020

$$c_s^2 = \frac{n}{\mu} \frac{d\mu}{dn}$$

$$c_{s,i}^2 = \frac{(\mu_i - \mu)c_{s,i}^2 + (\mu - \mu_i)c_{s,i+1}^2}{\mu_{i+1} - \mu_i}$$

+

$\chi$ EFT + pQCD

+

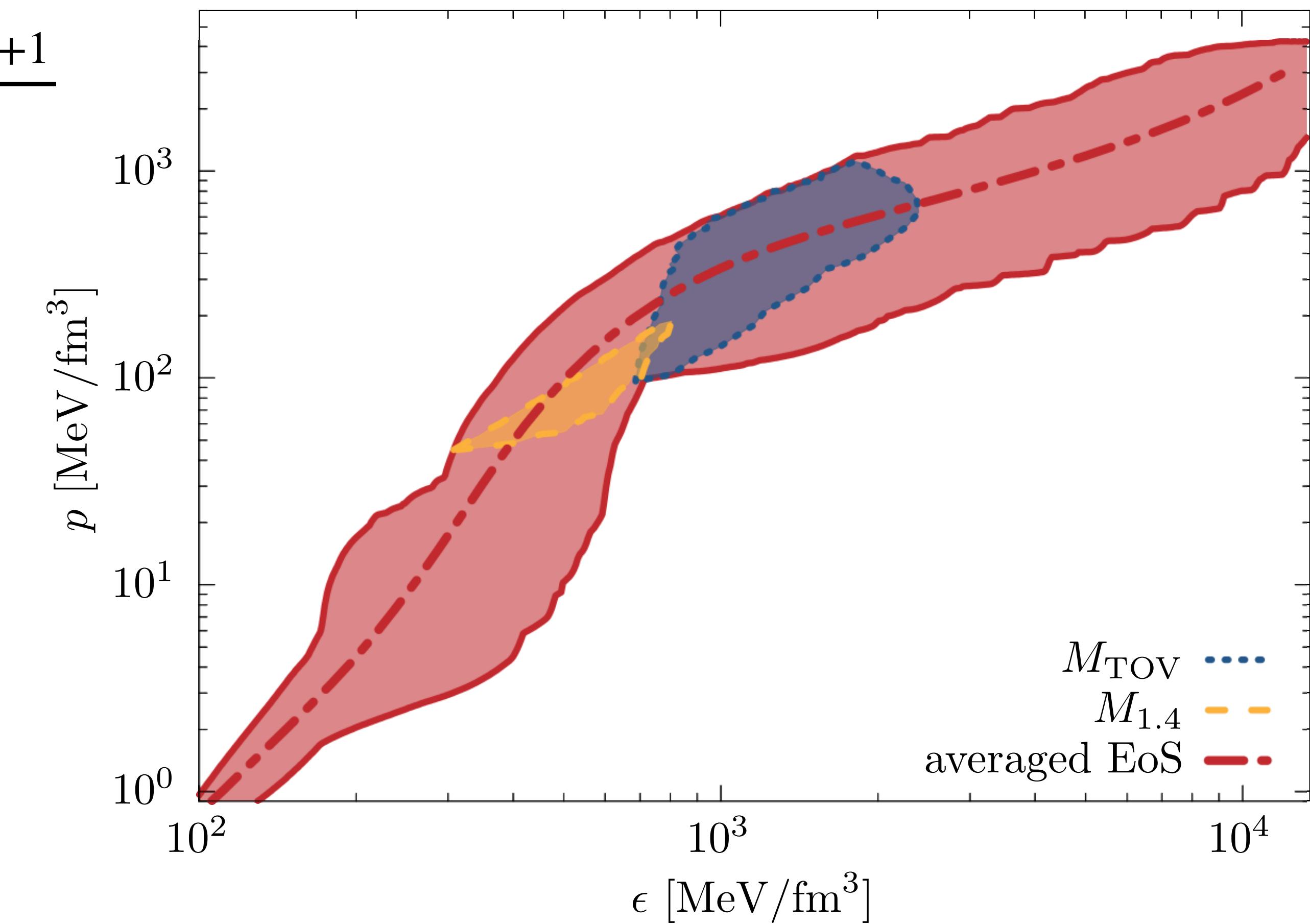
Mass measurement of J0740+6620

$M_{\text{TOV}} \geq (2.08 \pm 0.07) M_\odot$  Fonseca et al 2021

+

Tidal Deformability from GW170817

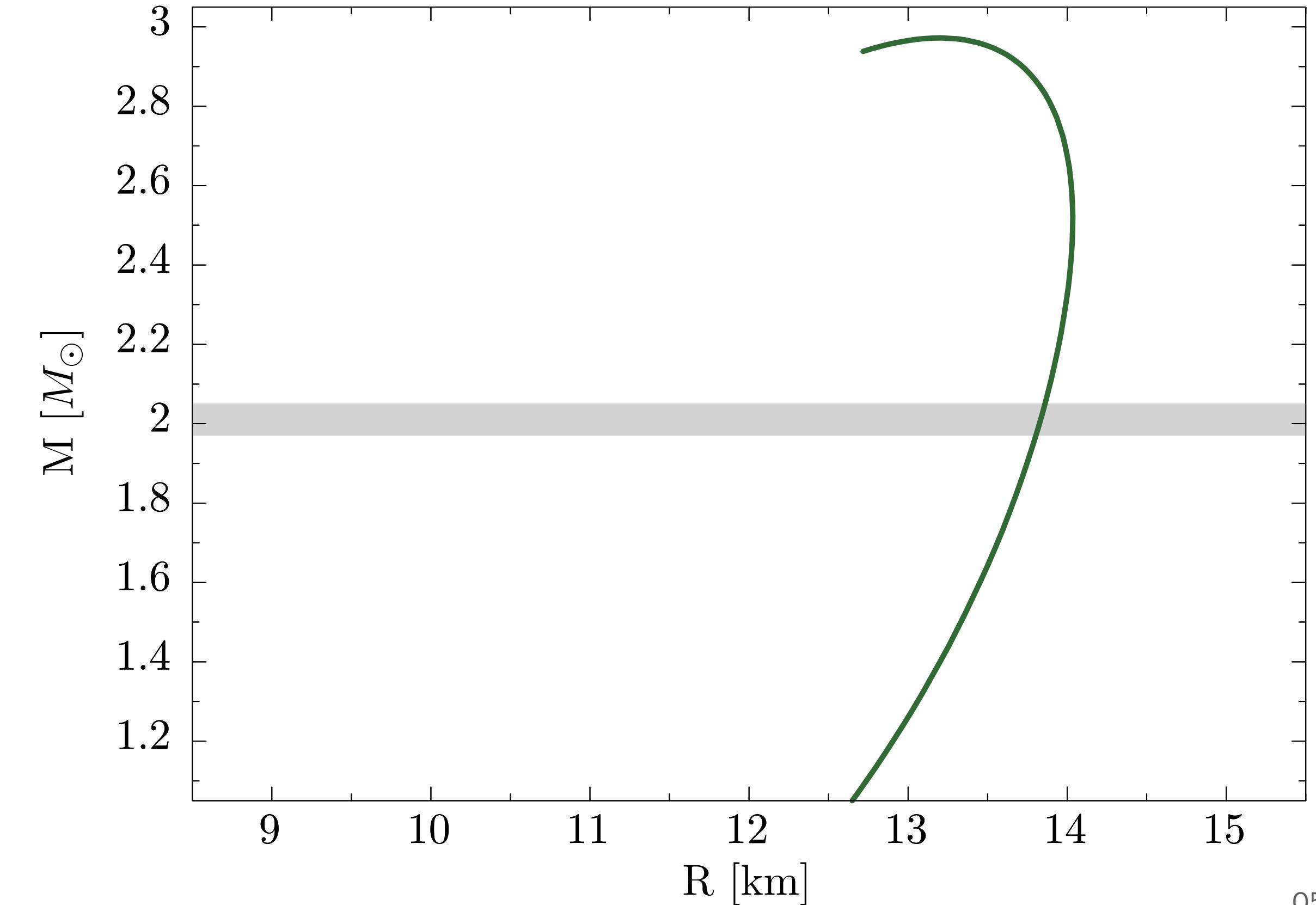
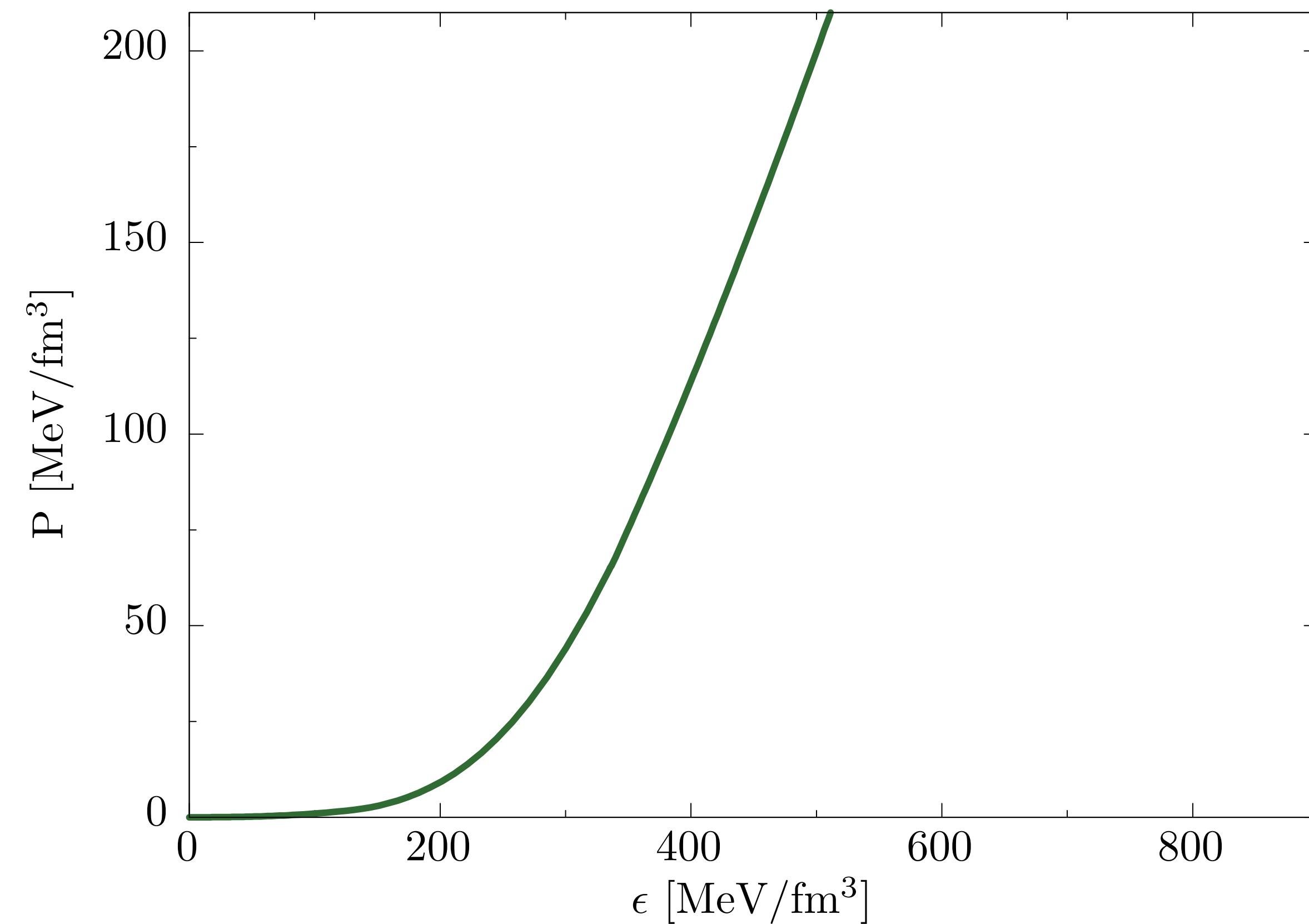
$\Lambda_{1.4M_\odot} = 190^{+380}_{-120}$  Abbott et al 2018



$5 \times 10^5$  viable Equations of State

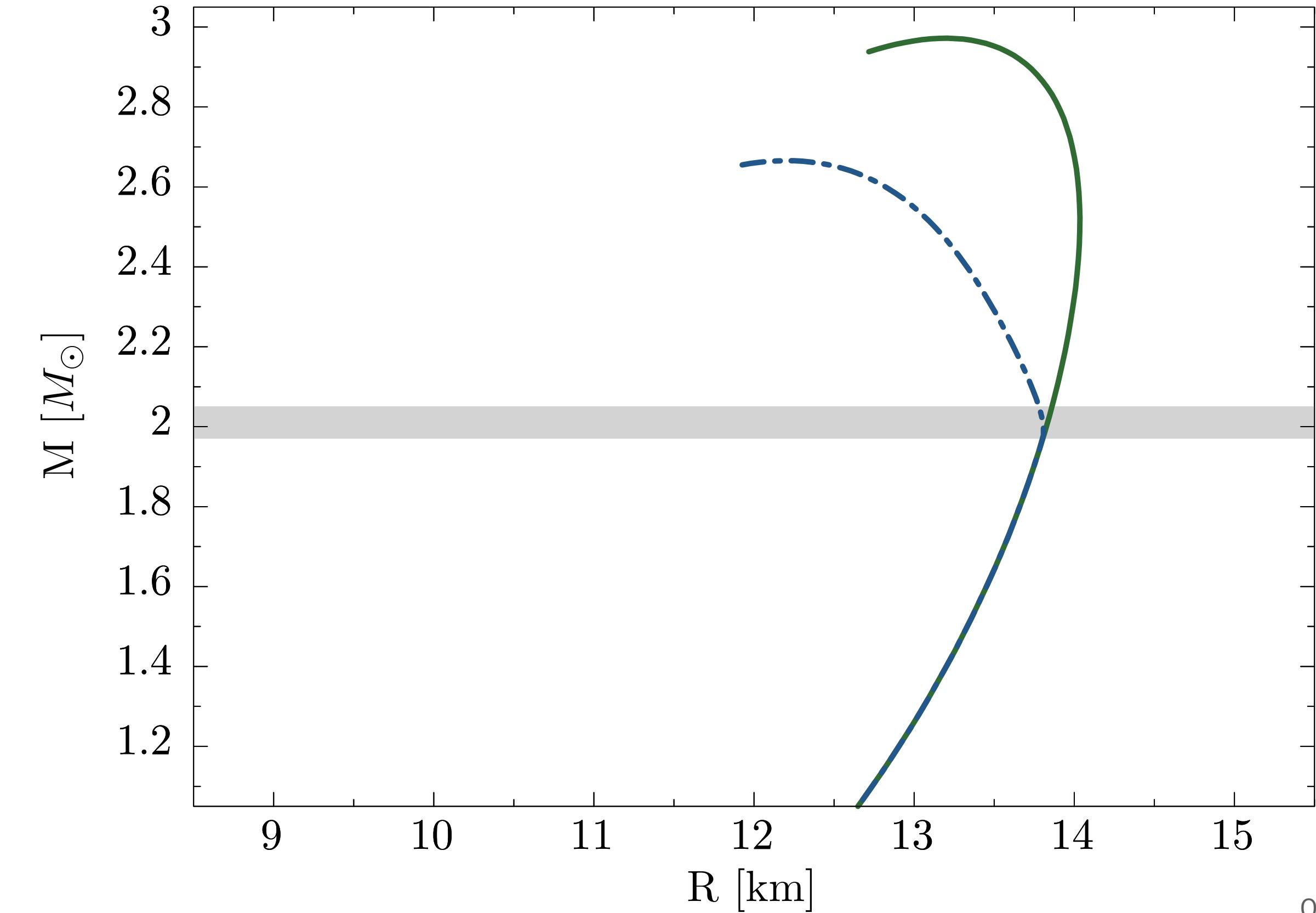
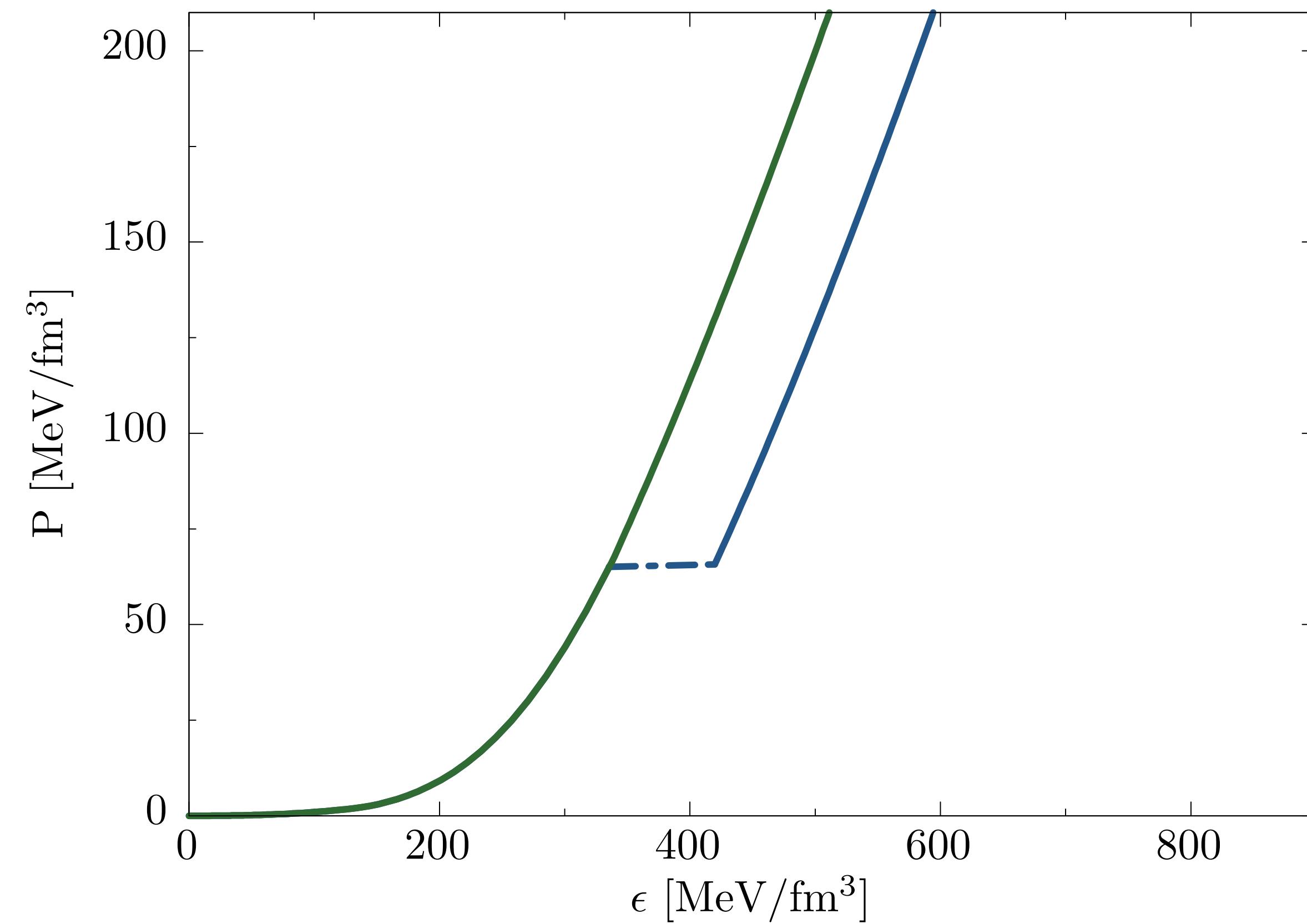
# Neutron Star Structure

$P(\epsilon)$   TOV Equations  $M(R)$



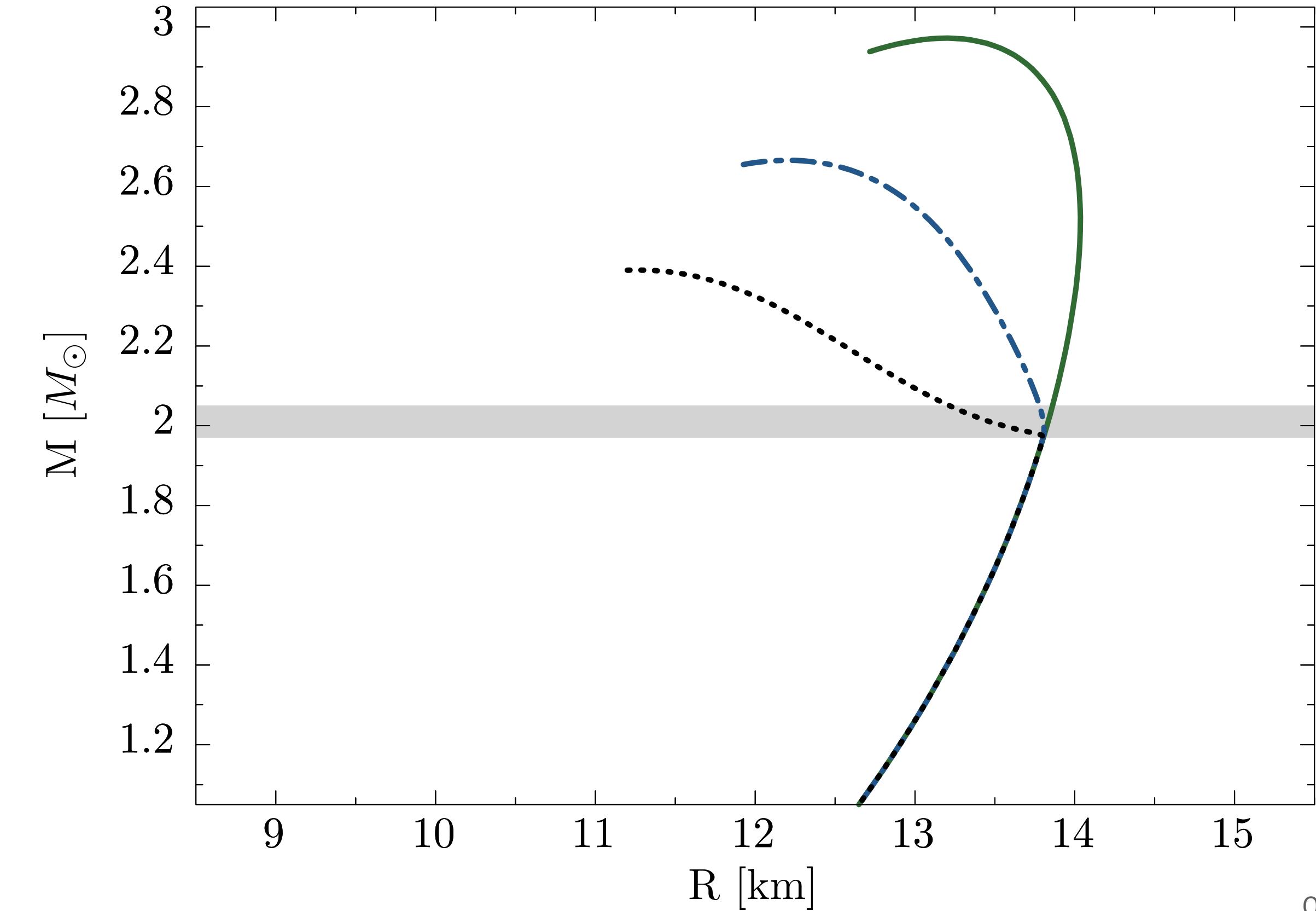
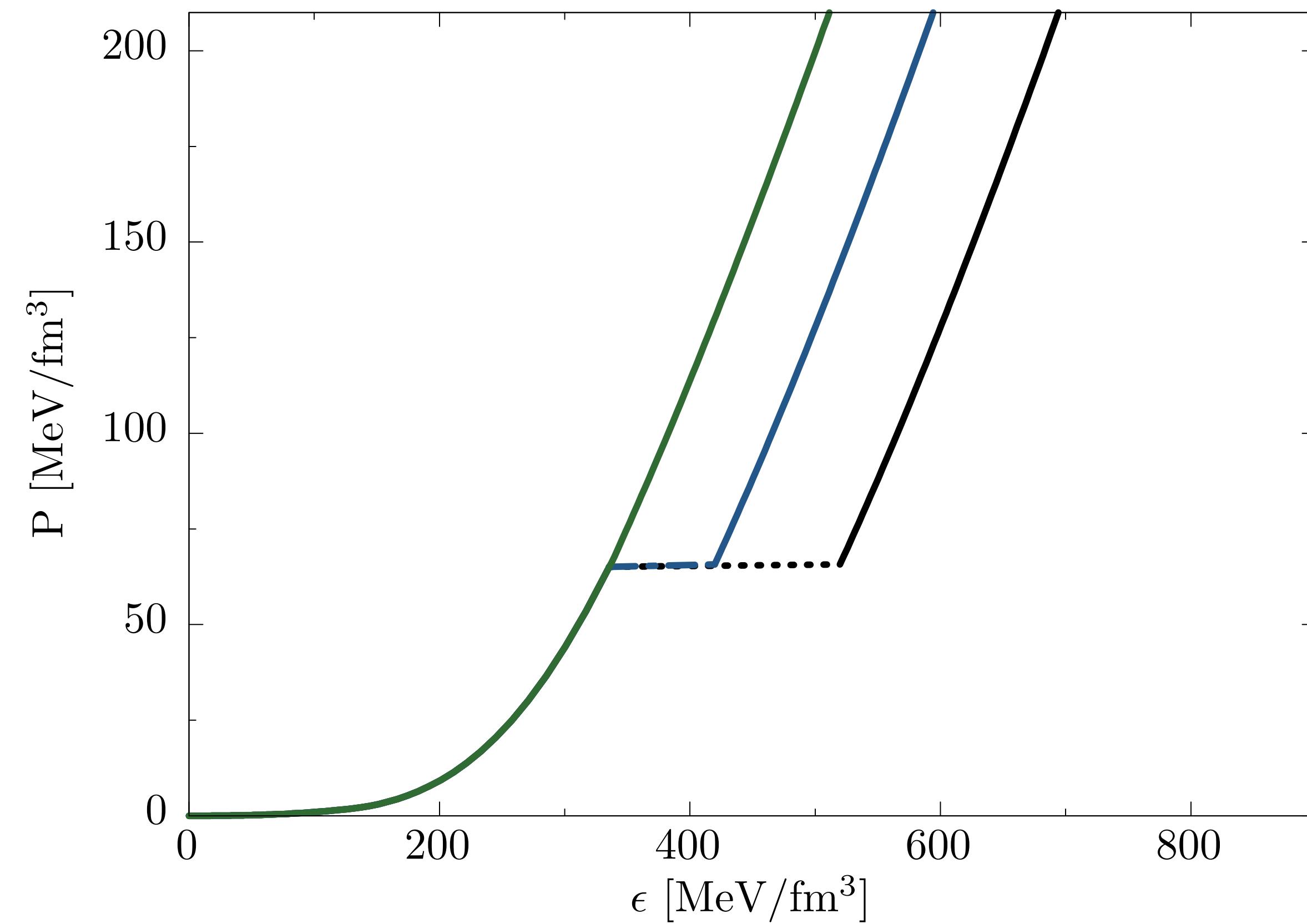
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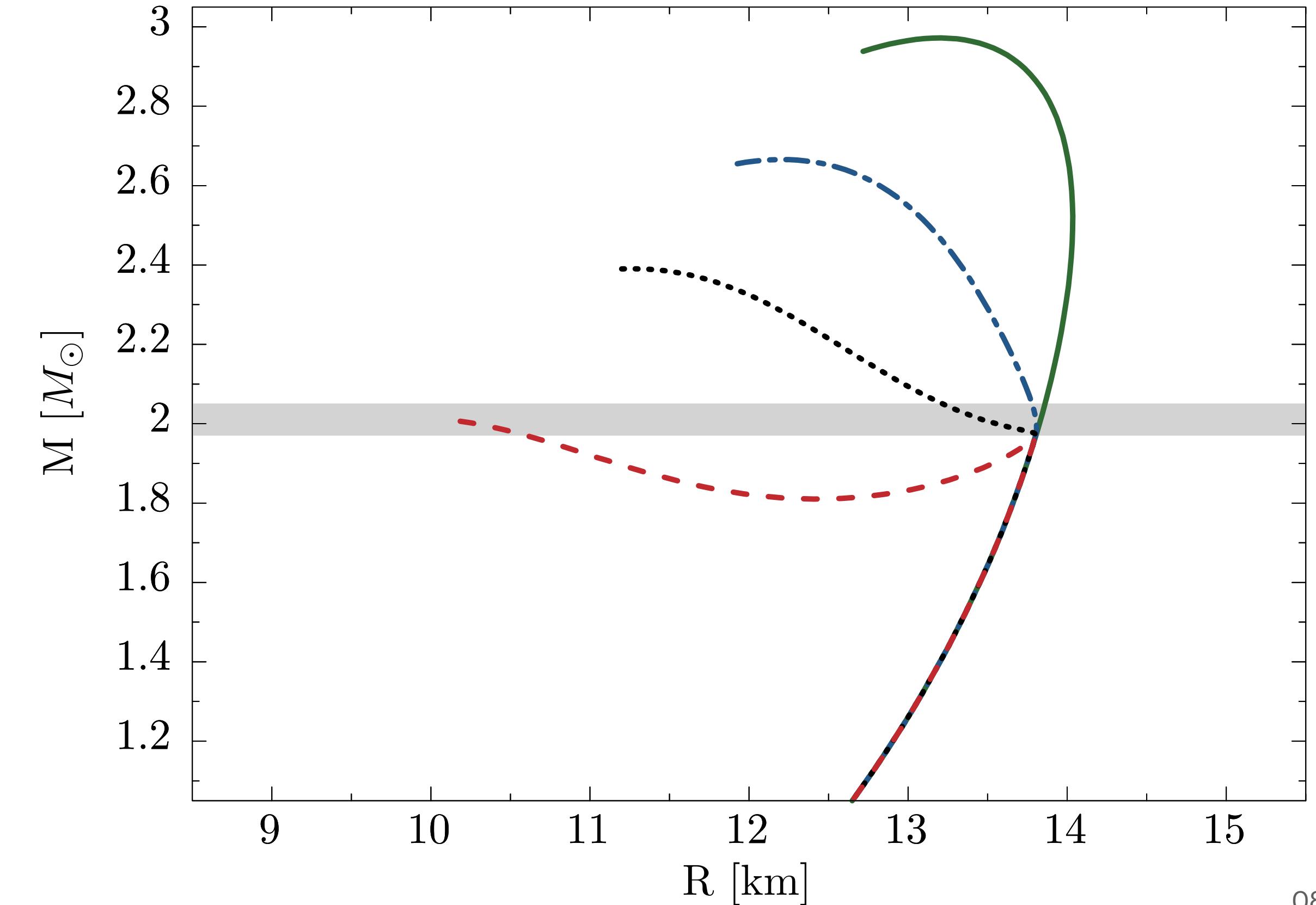
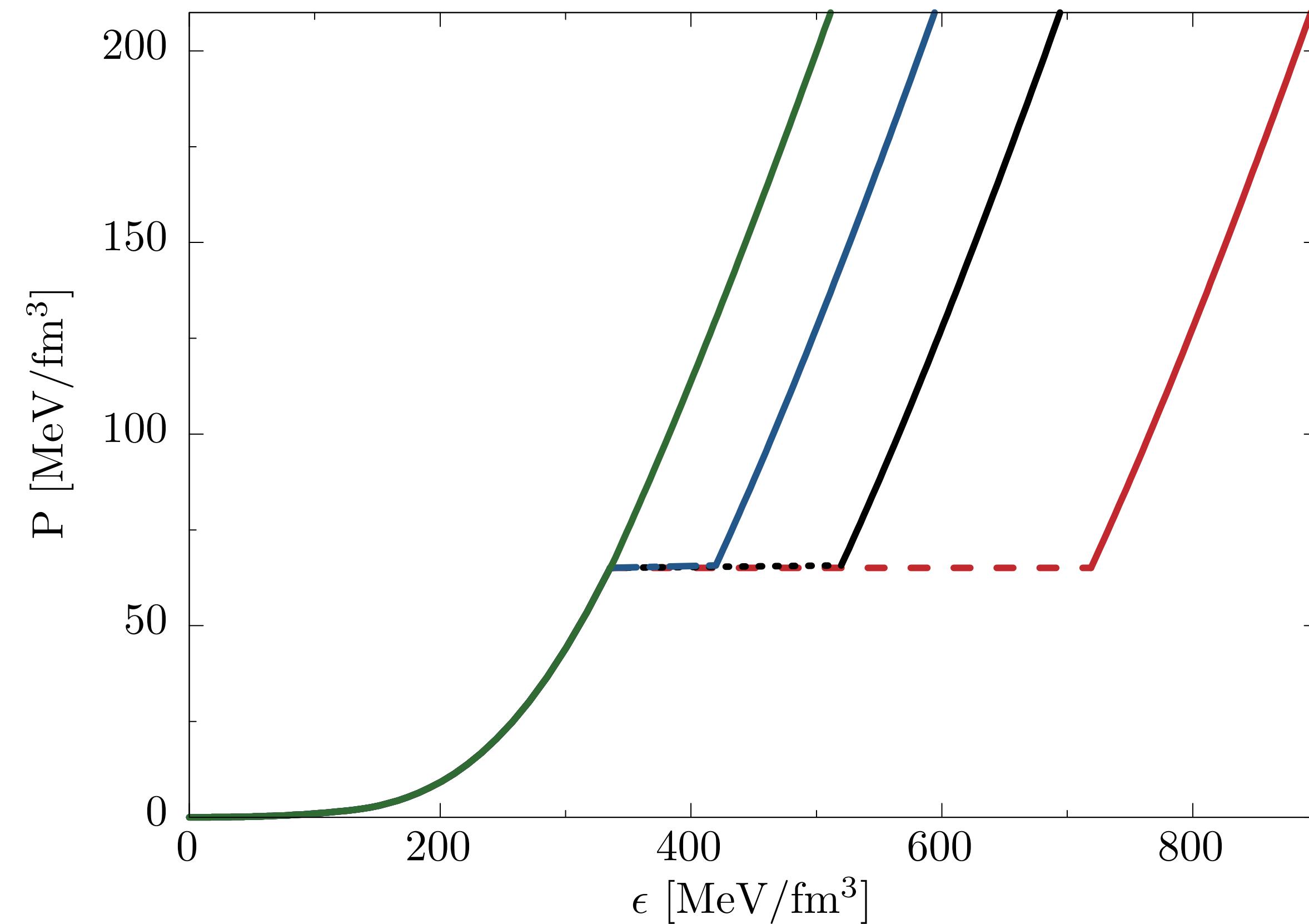
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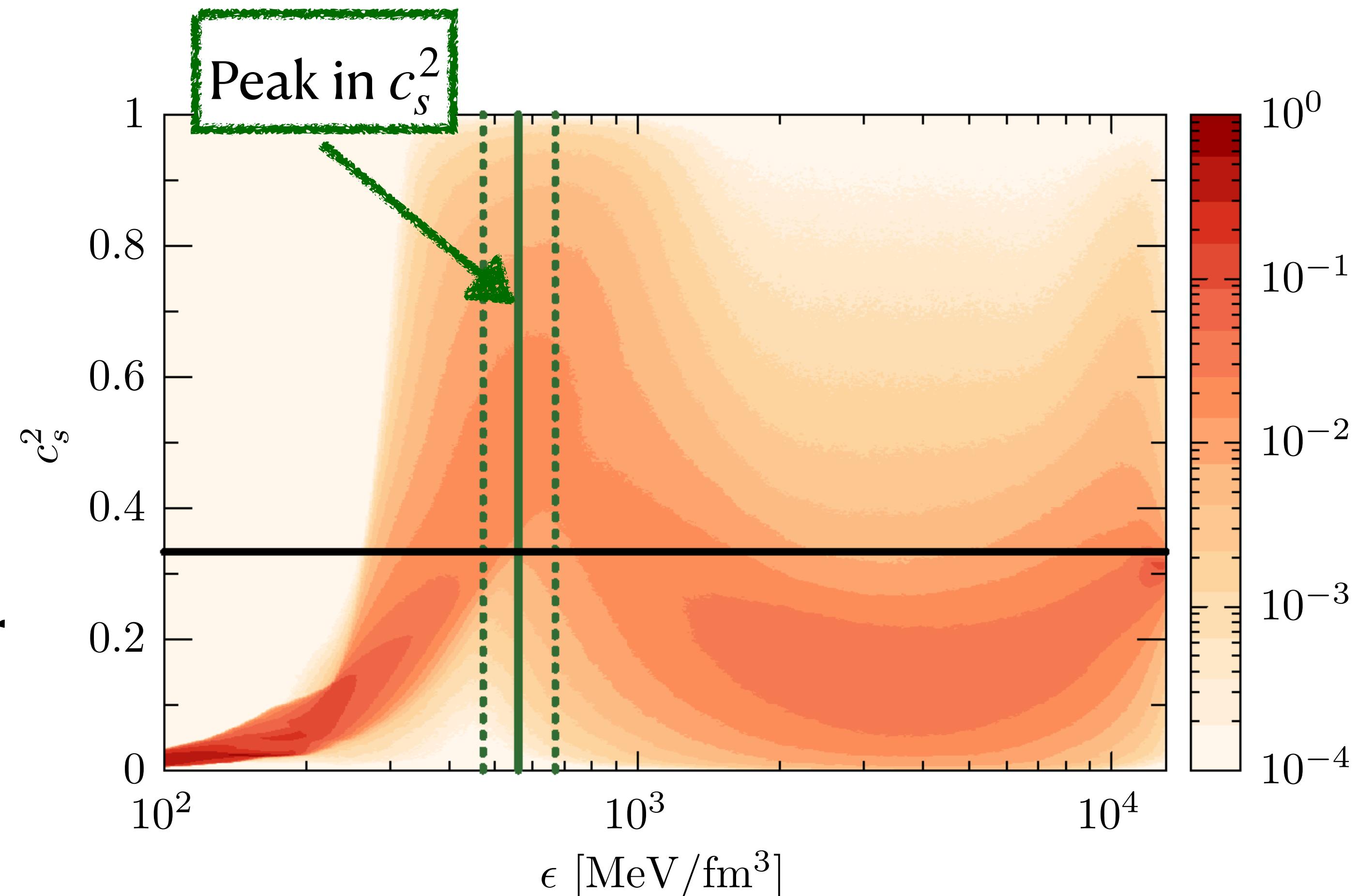
# General Structure of Speed of Sound

- General peak-dip structure

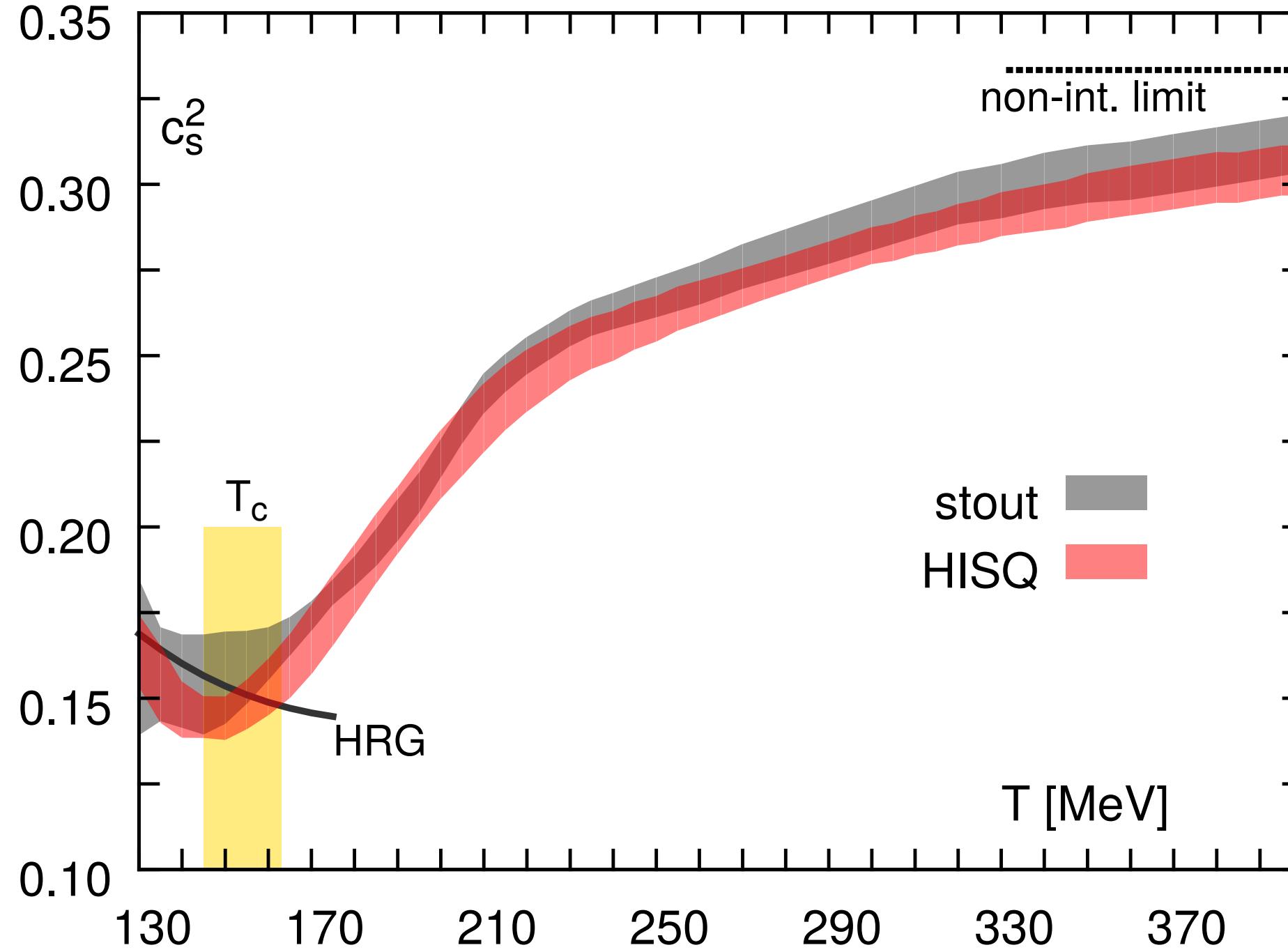
Altiparmak et al, 2022

- Peak similar to quarkyonic matter

McLerran, Reddy, 2019; Pang et al, 2023



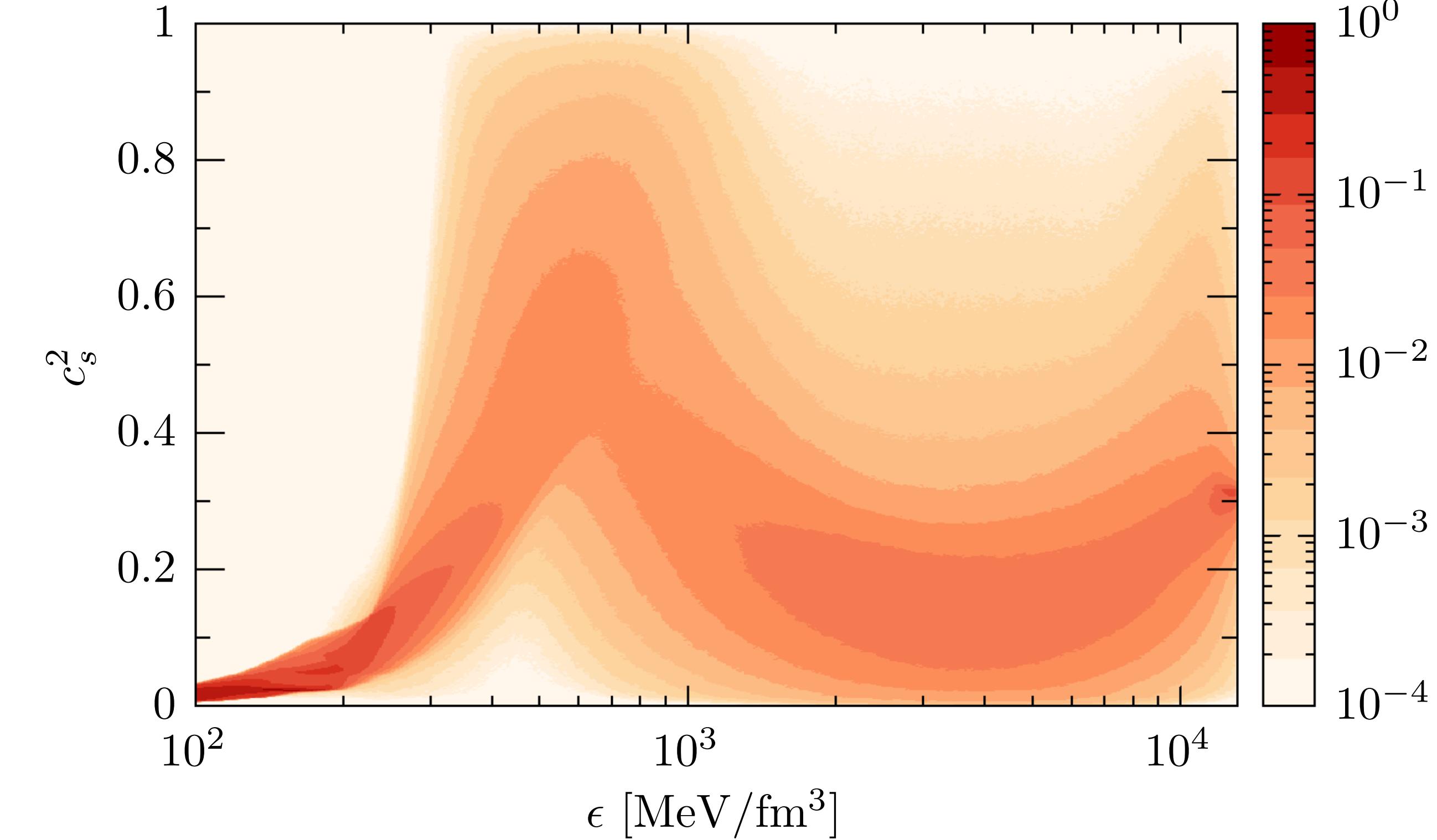
Local maximum at  $\epsilon_{\text{peak}} = 0.56^{+0.11}_{-0.09} \text{ GeV}/\text{fm}^3$  with  $c_s^2 = 0.82 \pm 0.08$



$$c_s^2 = \frac{S}{T} \frac{dT}{dS} < \frac{1}{3}$$

- Attractive interactions with resonance formation
- Chiral symmetry restoration and deconfinement

Non-monotonicity



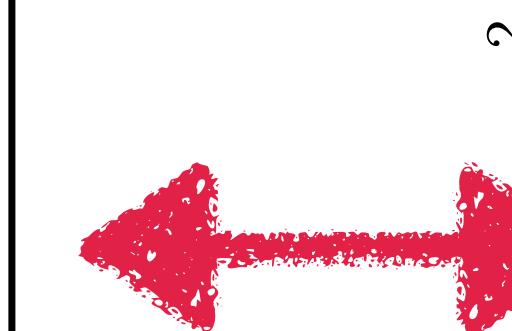
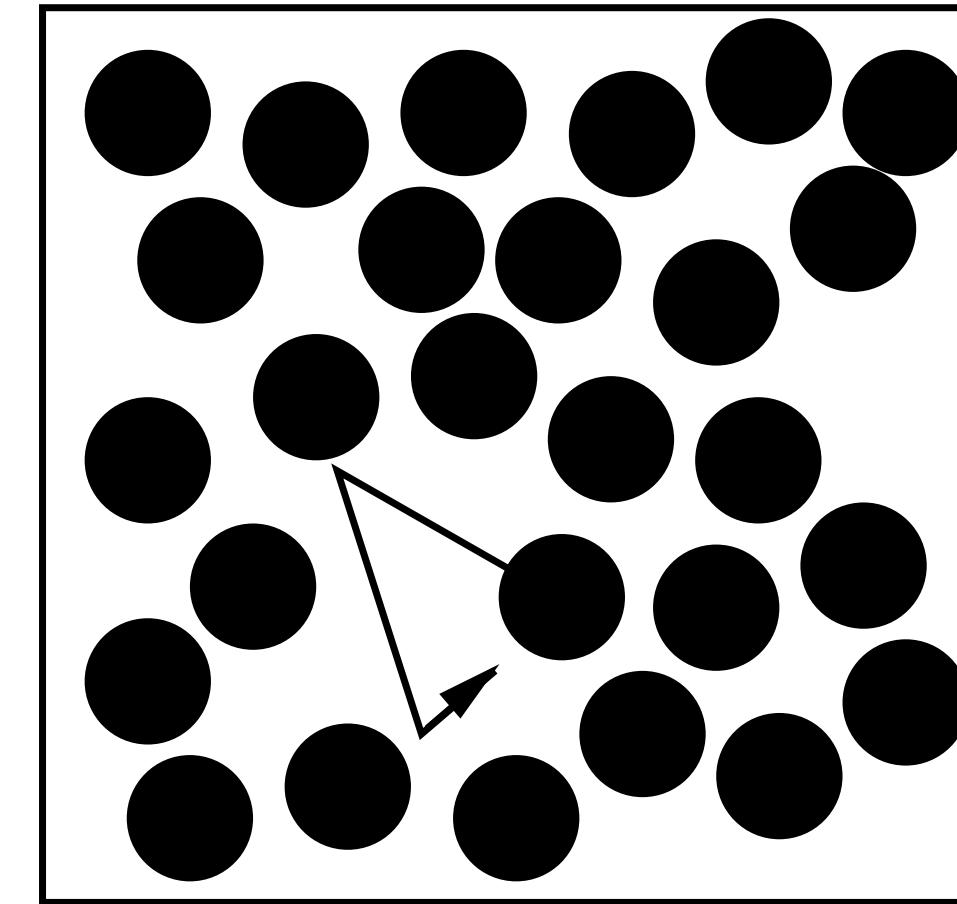
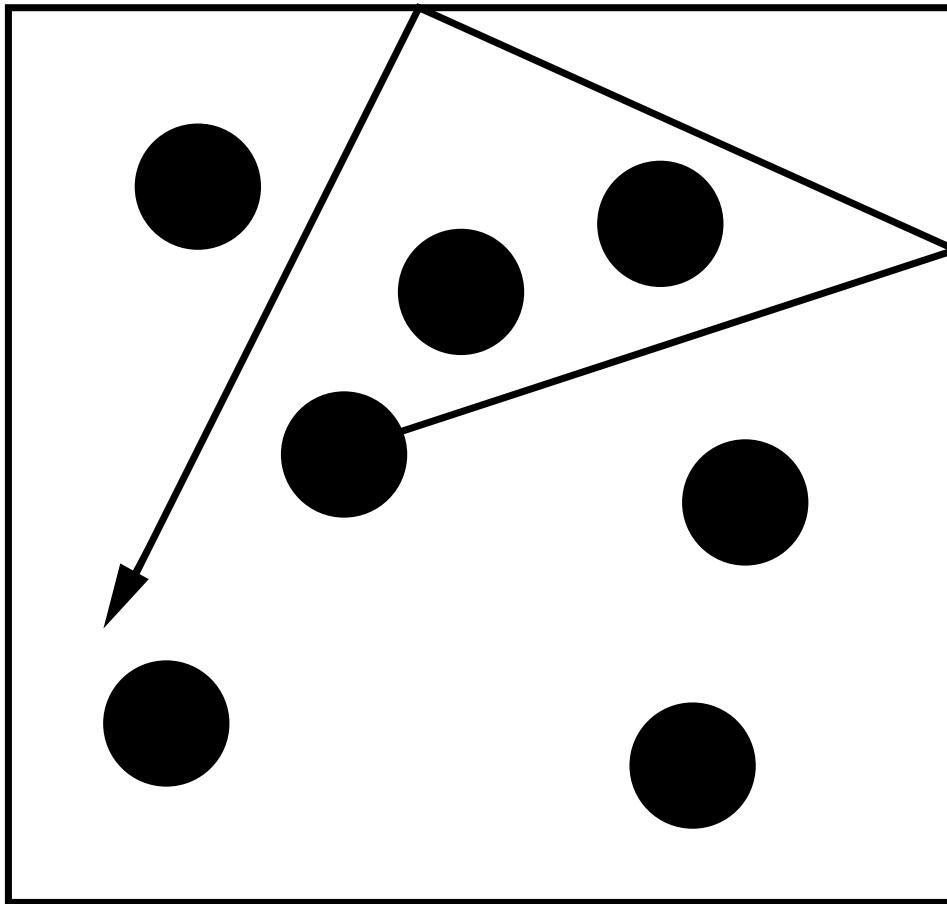
$$c_s^2 = \frac{n}{\mu} \frac{d\mu}{dn} > \frac{1}{3}$$

- Dominance of repulsive interactions
- Onset of quark or quarkyonic matter?

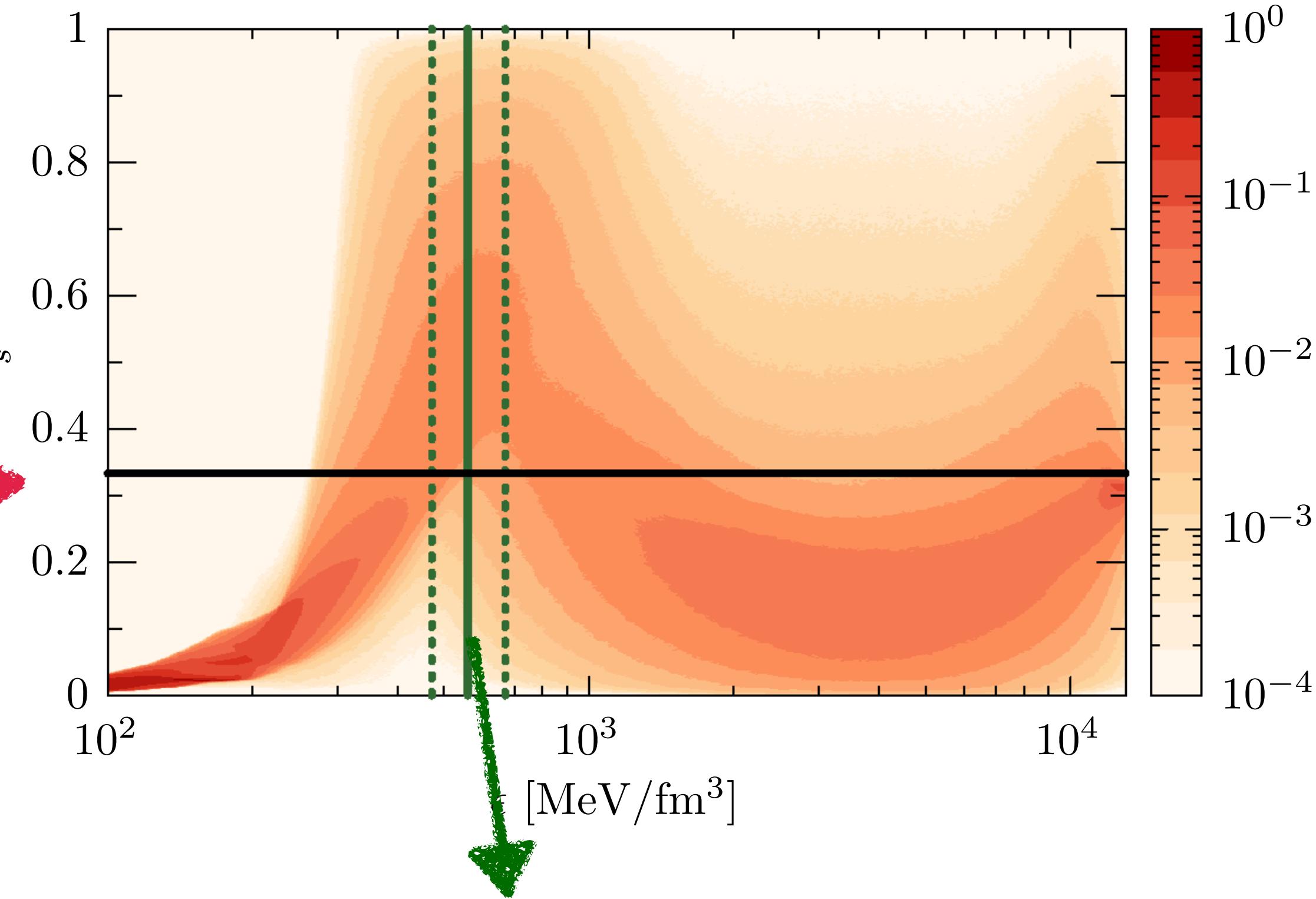
Change of phase

# Percolation theory vs speed of sound

see e.g. Satz, 1998; Castorina et al, 2009; Fukushima, 2020



$$\text{Percolation theory: } n_c = 1.22/V_0$$



$$n_{\text{peak}} = 0.54^{+0.09}_{-0.07} \text{ fm}^{-3}$$

$$\text{Avg. proton radius: } R_0 = 0.80 \pm 0.05 \text{ fm}$$

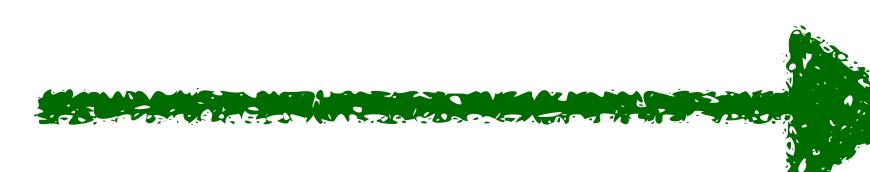
Wang et al 2022



$$n_c = 0.57^{+0.12}_{-0.09} \text{ fm}^{-3}$$

$$\text{Pb-Pb collisions at } \sqrt{s} = 2.76 \text{ TeV}$$

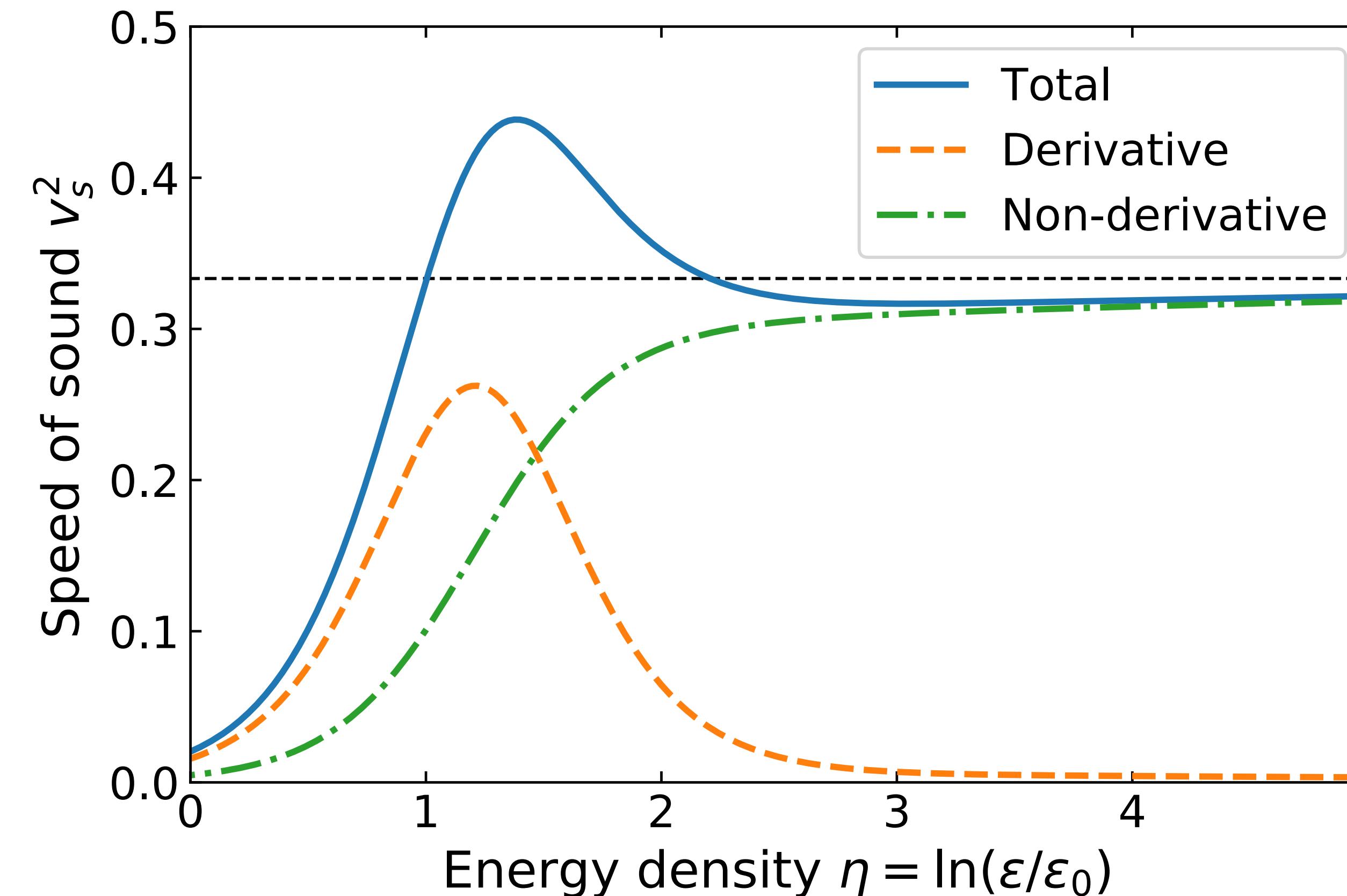
Andronic et al 2018



$$n_c = 0.60 \pm 0.07 \text{ fm}^{-3}$$

# Speed of Sound as Trace Anomaly

Fujimoto et al 2022



$$\text{Trace Anomaly}$$
$$\Delta \equiv \frac{\epsilon - 3p}{3\epsilon} = \frac{1}{3} - \frac{p}{\epsilon}$$

$$c_s^2 = \frac{d\left(\epsilon \frac{p}{\epsilon}\right)}{d\epsilon} = \frac{1}{3} - \Delta - \epsilon \frac{d\Delta}{d\epsilon}$$

Trace anomaly more informative than speed of sound

# Measure of conformality

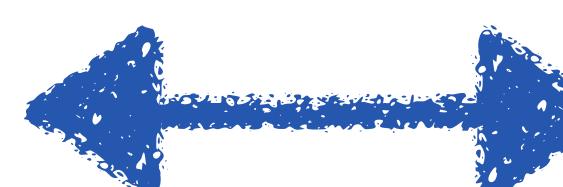
$\Delta$  monotonic up to  $\simeq \epsilon_{\text{TOV}}$

$$c_s^2 = \frac{1}{3} - \Delta - \epsilon \frac{d\Delta}{d\epsilon}$$

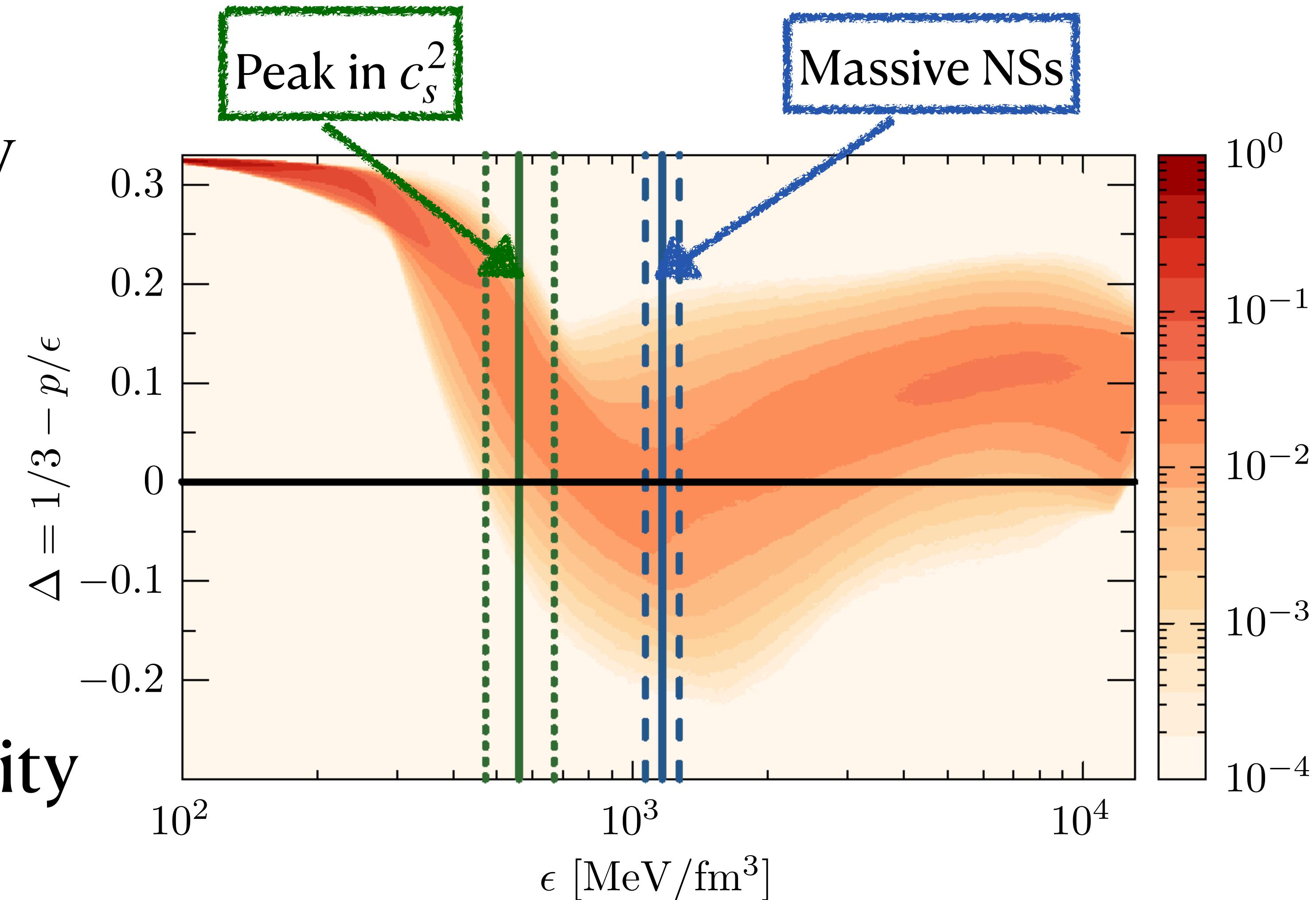
Maximum in  $c_s^2$

Fast approach to conformality

$\Delta \simeq 0$  at  $\epsilon \simeq 1 \text{ GeV/fm}^3$



$\epsilon_{\text{TOV}} = 1.16 \pm 0.01 \text{ GeV/fm}^3$



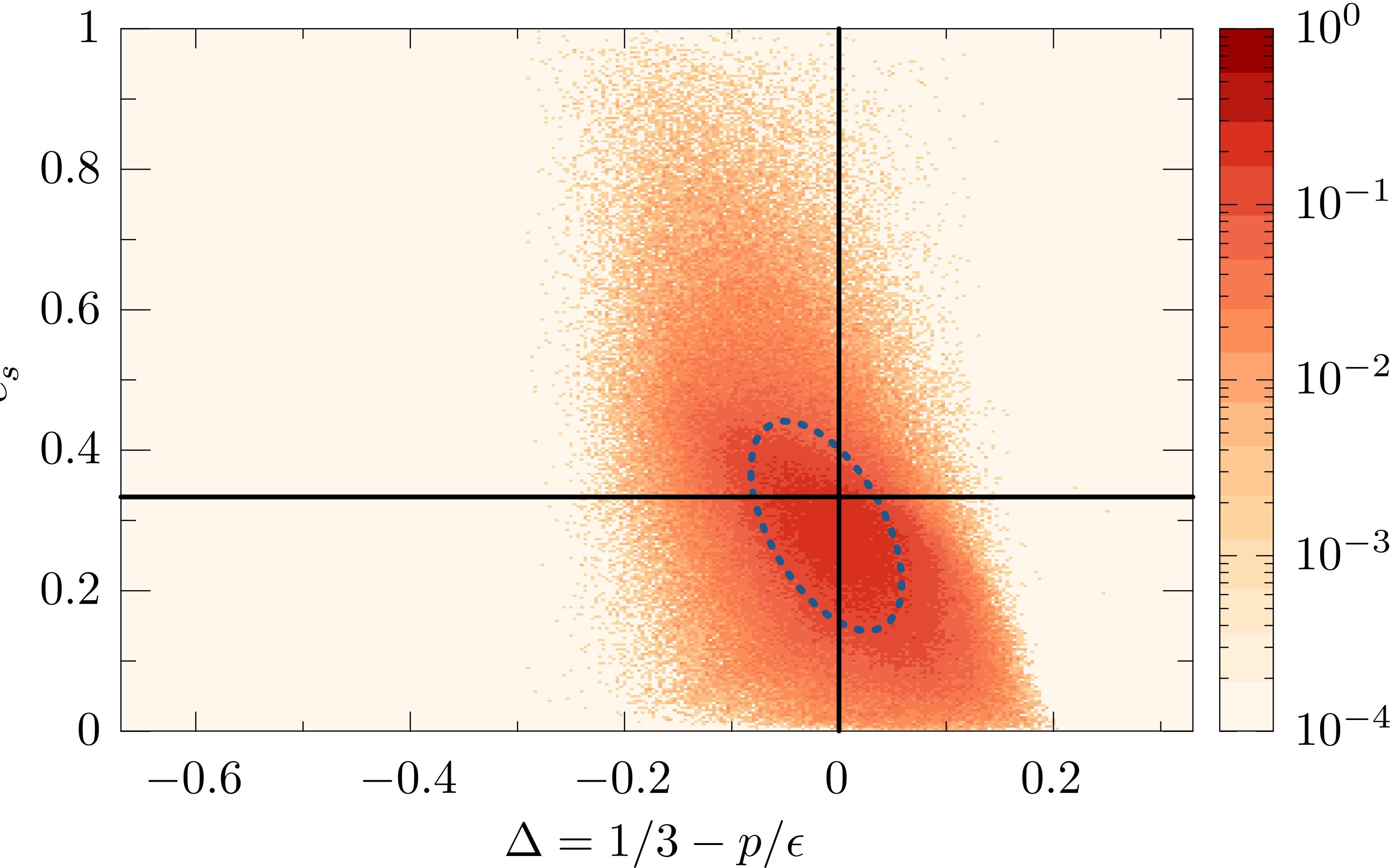
# $c_s^2$ and $\Delta$ in Heavy Neutron Stars

Conformality:

$$c_s^2 = \frac{1}{3} \text{ and } \Delta = 0$$

$$c_{s, \text{TOV}}^2 = 0.28 \pm 0.16 \approx \frac{1}{3}$$

$$\Delta_{\text{TOV}} = -0.01 \pm 0.03 \approx 0$$



Matter almost conformal in the cores of maximally massive NSs

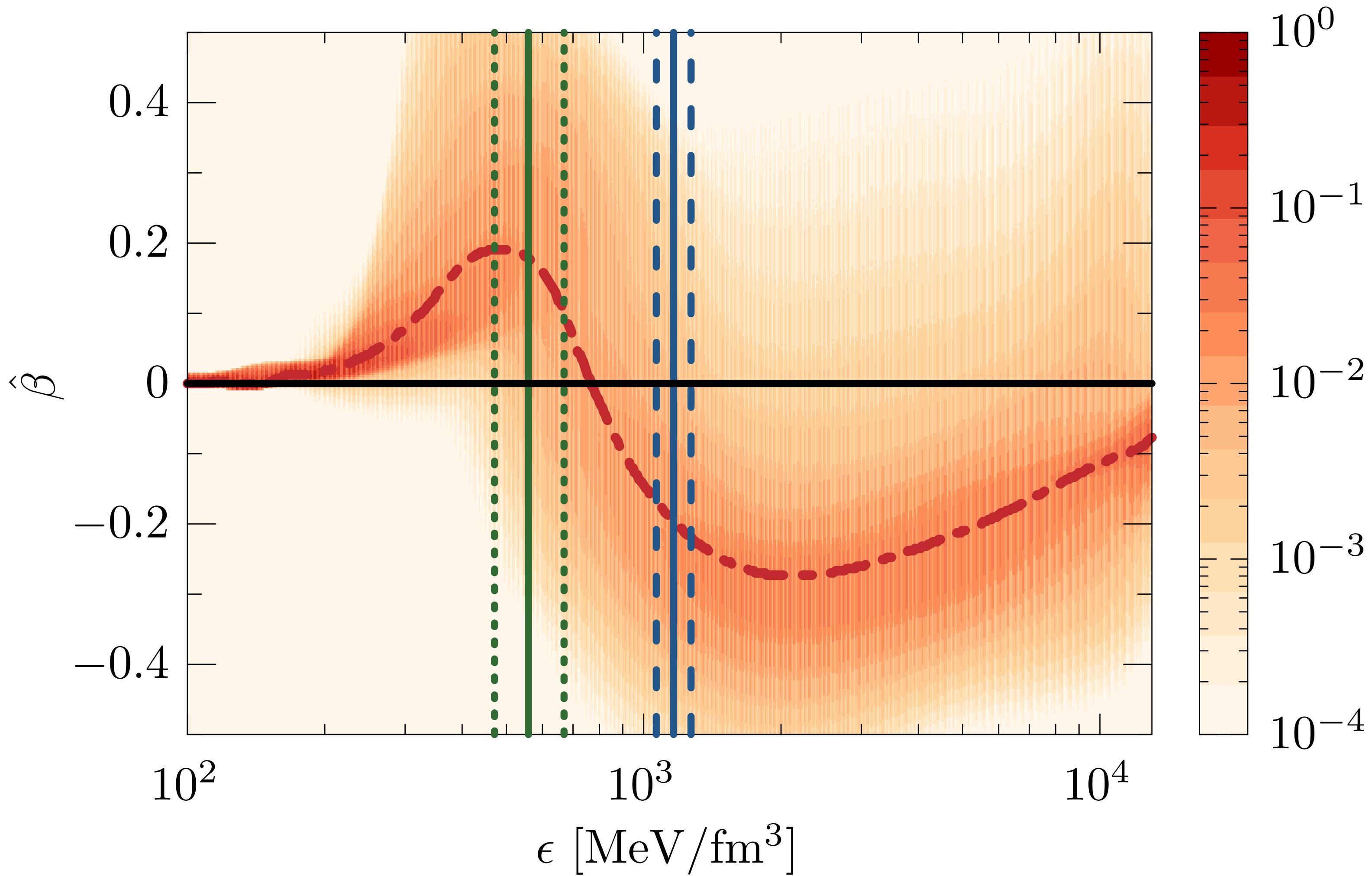
# Incompressibility & curvature of binding energy

MM, K. Redlich, C. Sasaki, to appear

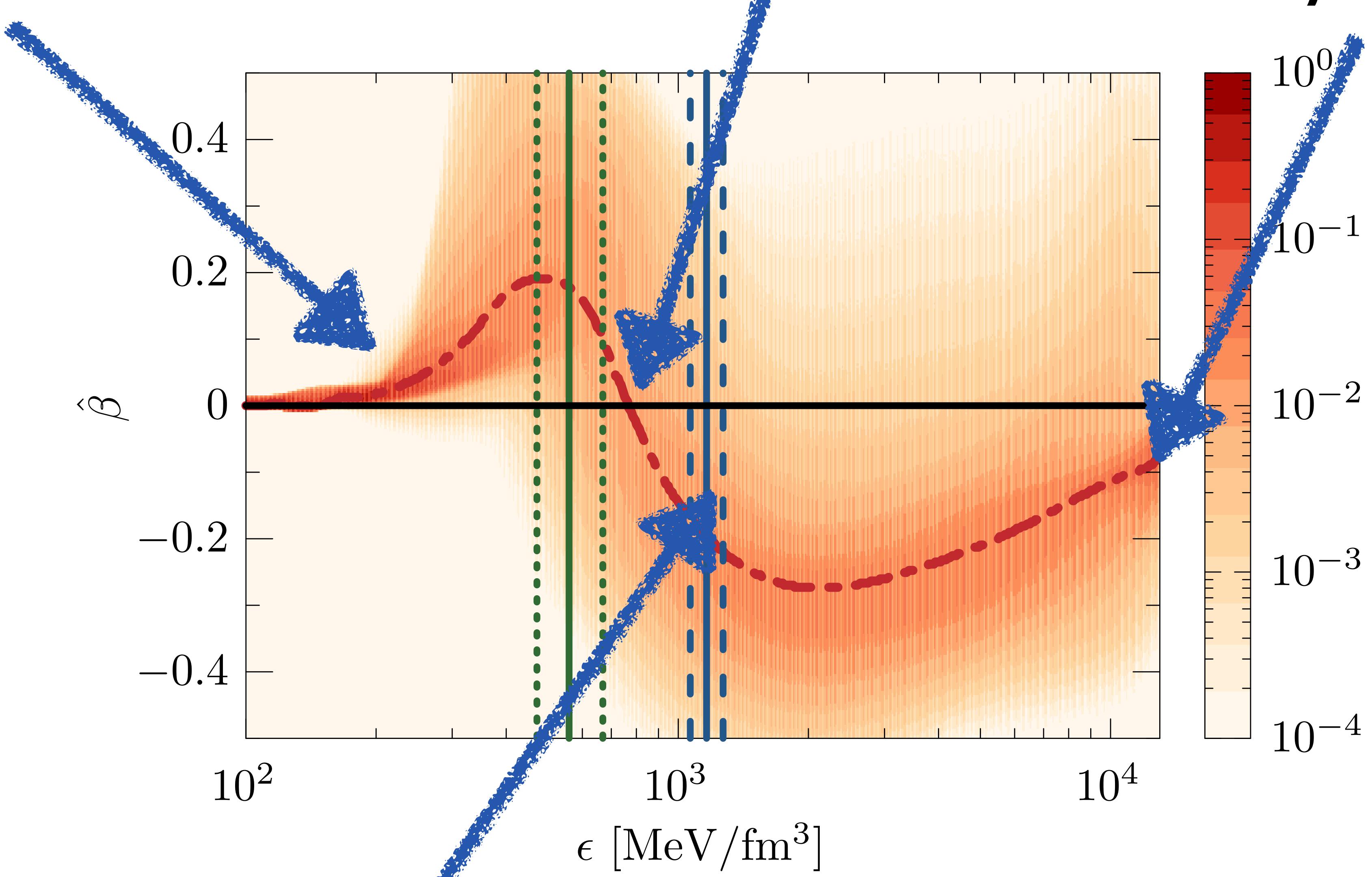
$$K \equiv \frac{dP}{dn} = 9\mu(\hat{\alpha} + \hat{\beta})$$

$$\hat{\alpha} \equiv 2 \frac{n}{\mu} \frac{d\epsilon/n}{dn} = 2 \frac{1/3 - \Delta}{4/3 - \Delta}$$

$$\boxed{\hat{\beta} \equiv \frac{n^2}{\mu} \frac{d^2\epsilon/n}{dn^2} = c_s^2 - \hat{\alpha}}$$



$\hat{\beta} < 0 \rightarrow$  changeover to conformal regime

$\hat{\beta} \approx c_s^2 > 0$  $\hat{\beta} = 0 \text{ at } \epsilon \lesssim \epsilon_{\text{TOV}}$  $\hat{\beta} \approx -1/6$ 

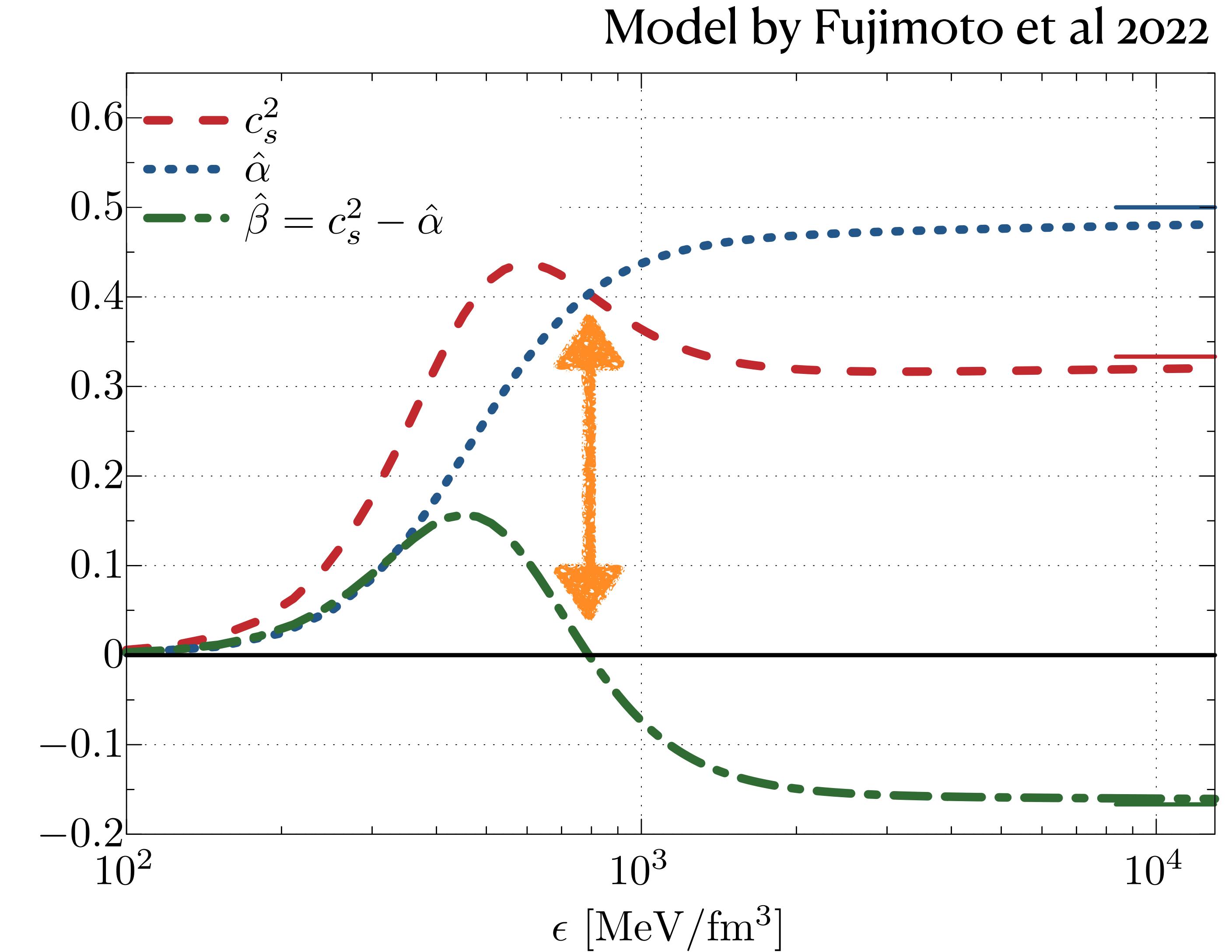
$$\hat{\beta}_{\text{TOV}} = 0.22 \pm 0.05$$

# Changeover consistent with percolation

$$K \equiv \frac{dP}{dn} = 9\mu(\hat{\alpha} + \hat{\beta})$$

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$$\hat{\beta} \equiv \frac{n^2}{\mu} \frac{d^2\epsilon/n}{dn^2} = c_s^2 - \hat{\alpha}$$



# Summary

Maximum of  $c_s^2$  consistent with percolation threshold

Matter seems to be conformal in the cores of massive NSs

Thank You