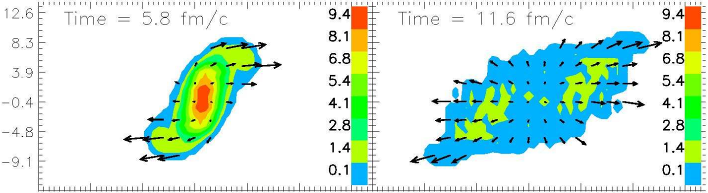
Continuous Decoupling of Dynamically Expanding Systems

J. Knoll



3 fluid hydro: Ivanov, Russkikh & Toneev

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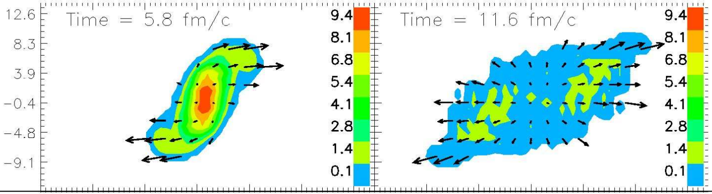
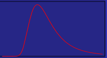
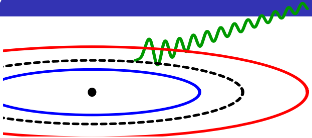


Au + Au at SPS	decoupl. period	volume growth
phase transition:	6 - 10 fm/c	> 5
chemical freeze-out:	> 5 fm/c	> 4
kinetic freeze-out:	> 8 fm/c	> 6
CMB early universe	$Z = [1300 - 800]$	$(13/8)^3 = 4.3$

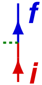


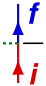
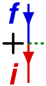
Outline

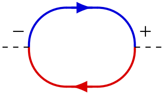
from Amplitude to Correlation Diagrams



Golden rule:

$$W = \sum_{if} n_i(1-n_f) \left| \right.$$  $$\left. \right|^2 (1+n_\omega)\delta(E_i - E_f - \omega_{\vec{q}})$$

$$= \sum_{if} n_i(1-n_f) \{$$  $$\} \times \{$$  $$\} (1+n_\omega)\delta(E)$$

$$=$$  $$(1+n_\omega)\delta(p^2-m^2)$$

$$\Downarrow$$

$$\left\langle j(x) j^\dagger(y) \right\rangle = -\mathrm{i}\,\Pi_{12}^{-+} = -i\Sigma_{12}^{-+}$$

current-current corr.fct

polarisation function
self energy

Exact Decoupling Rates

detector yield of particle *a*:

$$(2\pi)^4 \frac{dN_a(\vec{p}_A)}{d^3 p_A} = \frac{2\pi}{2p_A^0} \int d^4x \, d^4y \, \underbrace{\left\langle J_a^\dagger(x) J_a(y) \right\rangle_{\text{irred.}}}_{\text{source}} \, \underbrace{\psi_{\vec{p}_A}^\dagger(y) \psi_{\vec{p}_A}(x)}_{\text{dist. waves}}$$

$$= \frac{2\pi}{2p_A^0} \left\langle \psi_{\vec{p}_A} \right| \Pi_a^{\text{gain}} \left| \psi_{\vec{p}_A} \right\rangle$$

Gylassy '78, Danielewicz '92

Wigner transformation $(x, y) \rightarrow (X, p)$

local decoupling rate:

$$(2\pi)^4 \frac{dN_a(X,p)}{d^3X dt \, d^4p} = \Pi_a^{\text{gain}}(X,p) \, \underbrace{A(X,p)}_{\text{spectral function}} \, P_{\text{escape}}(X,p)$$

$$A(X,p) \xrightarrow{t \to \infty} 2\pi\delta(p^2 - m^2)$$

Semi-classical Rates

wave functions: built up by bundles of classical paths $(X(t), p(t))$
escape probability: $P_{\rm escape}(X,p) \approx \exp(-\chi(X,p))$ with

$$\chi(X,p) = \int_t^\infty dt' \Gamma(X(t'), p(t')) \quad \textit{with}$$

$$\Gamma(X,p) = -\frac{1}{p_0} \mathrm{Im}\, \Pi^R(X,t)$$

classical paths: given by $\mathrm{Re}\, \Pi^R(X,p)$

---

spectral funct.: 1^{st} ord. gradient expansion of Kadanoff-Baym Eq.

$$A(X,p) = \frac{2p^0\Gamma(X,p)}{(p^2 - m_a^2 - \mathrm{Re}\, \Pi^{\rm R}(X,p))^2 + (p^0\Gamma(X,p))^2}.$$

local decoupling rate:

$$(2\pi)^4 \frac{dN_a(X,p)}{d^3X dt\, d^4p} \approx \Pi_a^{\rm gain}(X,p)\; A(X,p)\; \exp(-\chi(X,p))$$

conserving scheme:

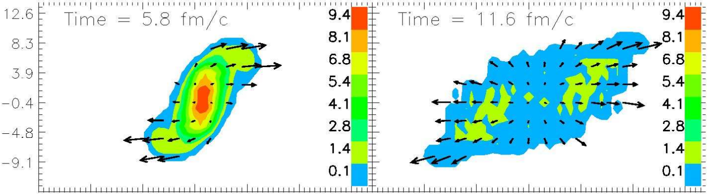
local rate: source × width × attenuation

$$(2\pi)^4 \frac{dN_a(X,p)}{d^3X dt\, d^4p} \approx \Pi_a^{\rm gain}(X,p)\, A_a(X,p)\, \exp(-\chi_a(X,p))$$

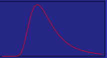
drain terms:

$$\partial_\mu j^\mu_{\alpha,\rm fluid}(X) = -\sum_a e_{a\alpha} \int d^4p \frac{dN_a(X,p)}{d^4p dt d^3X},$$

$$\partial_\mu T^{\mu\nu}_{\rm fluid} = -\sum_a \int d^4p\, p^\nu \frac{dN_a(X,p)}{d^4p dt d^3X} + \left\{ \begin{matrix}\text{interaction} \\ \text{terms}\end{matrix} \right\}$$



3 fluid hydro: Ivanov, Russkikh & Toneev



Local Equilibrium – Cooper-Frye limit

local equilibrium: $\Pi^{\rm gain}(X,p) = f_{\rm th}(p^0)\, 2p^0\, \Gamma(X,p)$

$$\int_{-\infty}^{\infty} dt\, \underbrace{\Gamma(t) \exp\{-\int_t^\infty dt' \Gamma(t')\}}_{P_t(t)} = 1.$$

maximum at: $\left[\dot\Gamma(t)+\Gamma^2(t)\right]_{t_{max}} = 0,$ with $P_t(t_{max}) \approx \Gamma(t_{max})/\mathrm{e}$

uncertainty relation: $\Delta t_{\rm dec} \approx \dfrac{\mathrm{e}}{\Gamma(t_{max})}$

$$(2\pi)^4 \frac{dN(p)}{d^3p} = 2\int p^0 dp^0 \underbrace{d^3\sigma_\mu dx^\mu}_{=d^4X} f_{\rm th}(p^0)\, A(X,p)\, \Gamma(X,p) \mathrm{e}^{-\int_t^\infty dt'\, \Gamma}$$

Cooper-Frye limit:

$$\underset{\text{instaneous limit}}{\Longrightarrow} \int_{\sigma_{\rm CFP}(p)} dp^0 d^3\sigma_\mu\, 2p^\mu\, f_{\rm th}(p^0)\, A^{\rm vac}(X,p)$$

(Cooper-Frye-Planck)

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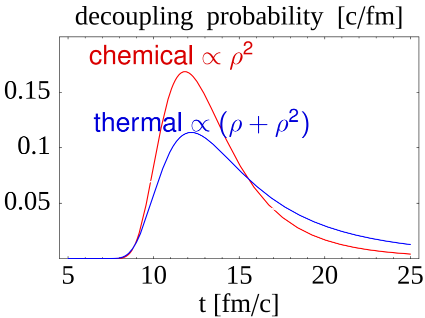
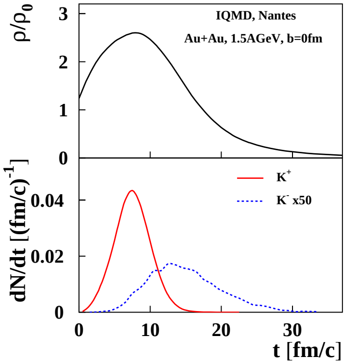
$$\underset{\text{instaneous limit}}{\Longrightarrow} \int_{\sigma_{\rm CFP}} dp^0 d^3\sigma_\mu\, 2p^\mu\, f_{\rm th}(p^0)\, A^{\rm vac}(X,p)\, \Theta(d\sigma_\mu p^\mu > 0)$$

(Cooper-Frye-Planck)

Expansion model

schematic expansion model:





Input: $V \propto t^3$

$R_{\text{freeze}} = 6$ fm

$v_{\text{flow}} = 0.5$ c

$\Rightarrow \quad \Gamma_{\text{chem}} = 100$ MeV

$\Delta t_{\text{chem}} \approx 5$ fm/c

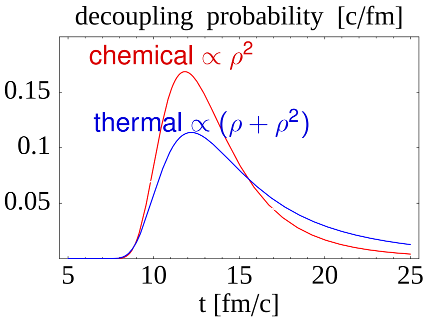
$\Delta t_{\text{th}} \approx 7$ fm/c

IQMD calc. of K^+ & K^-; Hartnack et al. 2007

$\Delta t_{\text{dec}} \approx 10$ fm/c

$\rho_i/\rho_f \approx 5$

schematic expansion model:



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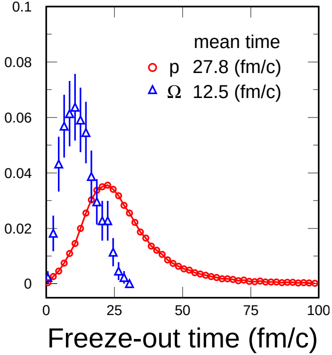
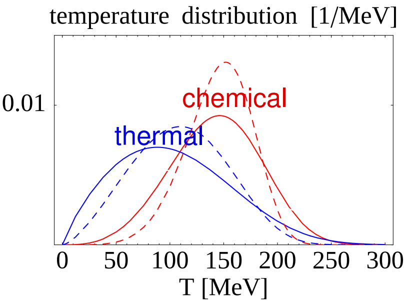


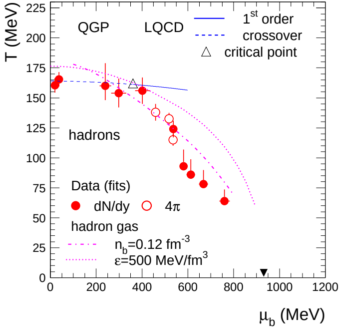
RQMD calc. of Ω & P;
van Hecke, Sorge, Xu '98

$\Delta t_{\rm dec} \approx 25$ fm/c
$\rho_i / \rho_f \approx 8$

Temperature distributions

temperature distributions: $P_{\rm dec}(T) = P_{\rm dec}(t)\left(\frac{dT(t)}{dt}\right)^{-1}$





dashed: $\kappa = 4/3$

· (mass-less ideal gas)

solid: $\kappa = 1.5$

· (half massive gas $m \approx T$)

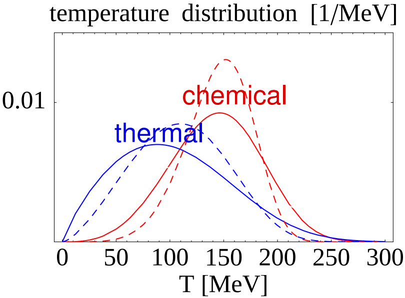
using: $TV^{\kappa-1}$ =const.

(Chemical freeze-out:
Andronic et al.)

(Dumitru et al. found sizable spreads in T & μ_B)

Phase transition

temperature distributions: $P_{\rm dec}(T) = P_{\rm dec}(t)\frac{dt}{dT}$



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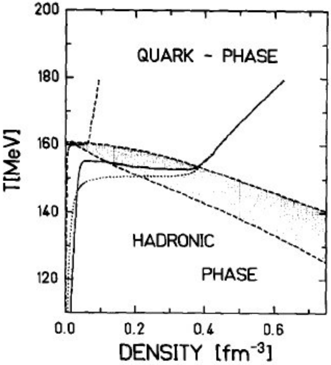
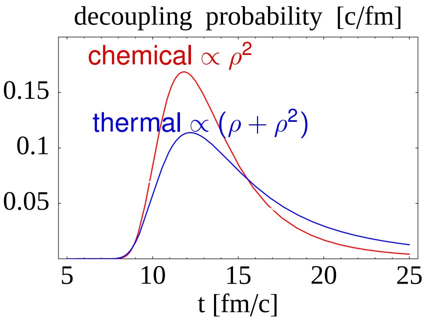
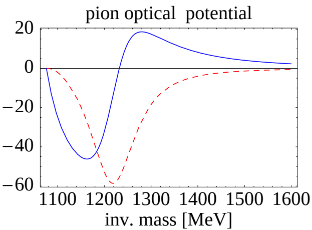


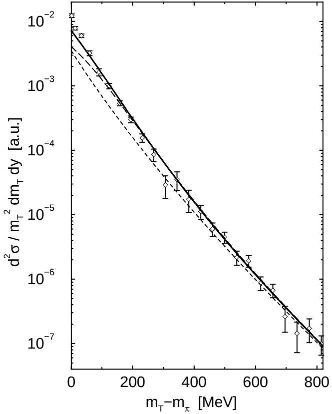
(Flavour kinetics:
Barz et al. (1988))

Finger prints of short lived resonances



pions from Δ resonance:





TAPS π^0 data

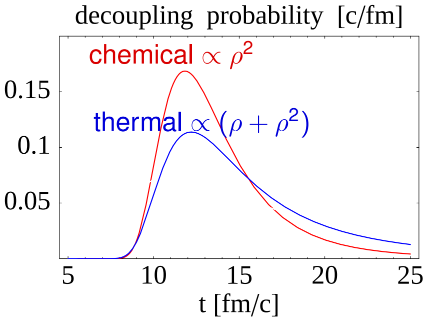
Theory: Weinhold - Friman

thermal pions

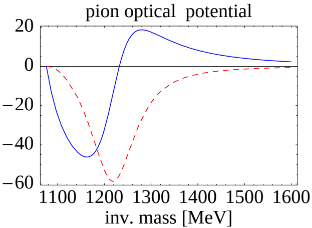
+ Delta decay

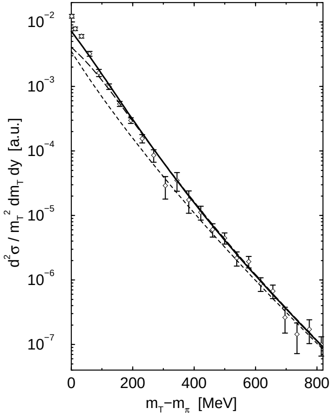
+ πN correlations

Finger prints of short lived resonances



pions from Δ resonance:

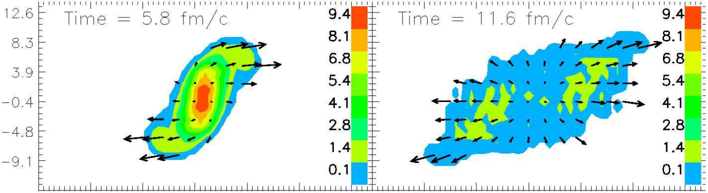




TAPS π^0 data
Theory: Weinhold - Friman
thermal pions
~~+ Delta decay~~
+ πN correlations
$\Rightarrow$ two slopes from ΔT?

Summary

nuclear collisions:	decoupling time	volume growth
phase transition:	6 - 10 fm/c	> 5
chemical freeze-out:	> 5 fm/c	> 4
kinetic freeze-out:	> 8 fm/c	> 6
CMB early universe:	$Z = [1300 - 800]$	$(13/8)^3 = 4.3$



3 fluid hydro: Ivanov, Russkikh & Toneev

- why is T_{chem} so sharply determined?
  $\Rightarrow$ signal for latent heat, phase transition?
- finger print of short lived resonances;
  (two slope behaviour: signal for spread in T?)
- HBT: the method determines the active emission zone