MESON STRUCTURE IN THE NEXT-to-LEADING ORDER

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Abstract

We present here a general scheme for accessing informations about the Meson structure. The idea is based on calculating the structure of a constituent quark, which is common to all hadrons. The method is applied to pion and kion as an example. While there is only a scarce data on the ratio of up-quark valence distributions of the \(K^+\) and \(\pi^+\) mesons, our model calculation nicely agrees with the data.

1 Introduction

Experimentally, it is very difficult to have first hand knowledge about the mesonic structure, therefore our knowledge of meson structure at best is an inference from the nucleon structure, mainly proton for which there are ample data in Deep Inelastic Scattering kinematics. Information on hadronic structure on the other hand is intimately connected with the presence of large number of partons. Although, various evolution equations provide guidance to achieve such a task but still they are problematic in which an initial distribution needs to be introduced on \textit{ad. hoc.} basis. Moreover, the color charge of quark field in QCD Lagrangian is ill-defined and not gauge invariant in an interacting theory reflecting the color of gluons, whereas color associated with a Constituent Quark (CQ) is a well defined concept. It is shown [1] that dressing of a valence quark or gluon can be carried out successfully to all orders in perturbation theory in conformity with confinement, to represent a CQ. Thus, a CQ can be regarded as a quasi particle with structure, participating in all hadrons. This means that the structure of a CQ is universal and common to all hadrons. Therefore, the aim of this paper is to calculate the structure of a CQ in the Next-to-Leading Order (NLO) in QCD and with that knowledge to evaluate the structure of mesons as well as baryons. Such a program recently is carried out for proton [2]. We now extend it to include mesons as well. Unfortunately there is not much of experimental data available on mesons and
none will be available in the near future. In that respect we think it is useful to elude on the structure of mesons. We take as an example pion and kion here.

2 CQ STRUCTURE FUNCTION

In DIS scattering with high enough $Q^2$ the structure function of a CQ can be written in terms of its parton distributions. For a U-type CQ one have:

$$F_2^U(z, Q^2) = \frac{4}{9} z (G_g + G_g) + \frac{1}{9} z (G_g + G_g + G_g + G_g) + ...$$

(1)

where all the functions on the right hand side are the probability functions for quarks having momentum fraction $z$ of a the parent CQ. Going to the N-moment space and defining the moments as:

$$M_i(N, Q^2) = \int_0^1 x^{N-1} G_i(x, Q^2) dx$$

(2)

the subscription $i$ stands for Singlet (S) and Nonsinglet (NS) distributions. In the NLO approximation these moments are given as follows:

$$M^{NS}(N, Q^2) = [1 + \frac{\alpha_s(Q^2)}{4\pi} \frac{\alpha_s(Q^2)}{2\beta_0} \left( \frac{\gamma_{11}^{(1)}_N}{2\beta_0} - \frac{\beta_1\gamma_{22}^{(0)}_N}{2\beta_0^2} \right) + \frac{\alpha_s(Q^2)}{4\pi} \frac{\sum^{(0)}_N}{2\beta_0}$$

(3)

The moments of singlet and gluon sector $\left( \begin{array}{c} M^S(N, Q^2) \\ M^G(N, Q^2) \end{array} \right)$ is governed by anomalous dimension matrices $\gamma^{(0,1)}_N$:

$$\gamma^{(0,1)}_N = \left( \begin{array}{cc} \gamma_{11}^{(0,1)}_N & \gamma_{12}^{(0,1)}_N \\ \gamma_{21}^{(0,1)}_N & \gamma_{22}^{(0,1)}_N \end{array} \right)$$

(4)

and given by:

$$\left( \begin{array}{c} M^S(N, Q^2) \\ M^G(N, Q^2) \end{array} \right) = \left\{ \begin{array}{c} \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q^2_0)} \right)^{\frac{\sum^N}{2\beta_0}} [p_-^N - \frac{1}{2\beta_0} \frac{\alpha_s(Q^2_0)}{4\pi} \frac{\alpha_s(Q^2) - \alpha_s(Q^2)}{4\pi} p_-^N \gamma^N p_-^N - \\
\left( \frac{\alpha_s(Q^2)}{4\pi} - \frac{\alpha_s(Q^2)}{4\pi} \right) \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q^2_0)} \right)^{\frac{\sum^N}{2\beta_0}} \frac{p_-^N \gamma^N p_+^N}{2\beta_0 + \lambda^N - \lambda_+^N} ] \right\}$$

(5)

We have taken our initial scale $Q^2_0 = 0.283$ GeV$^2$ and $\Lambda = 0.22$ GeV. Of course these moments are expressed completely in terms of $Q^2$ through the evolution parameter $s$ defined as:

$$s = \ln \frac{Q^2}{Q^2}$$

(6)
Evaluation of these moments are straight forward. Now the moments of valence and sea quarks in a CQ are:

\[ M_{v,\text{val}}^{\text{CQ}} = M^{NS}(N, Q^2) \]  
\[ M_{s,\text{sea}}^{\text{CQ}} = \frac{1}{2f}(M^S - M^{NS}) \]

![Graphs showing moments of partons in a CQ at various Q^2 values.](image)

**Fig. 1:** Moments of partons in a CQ at \( Q^2 = 8.5, 25, 120 \text{ GeV}^2 \).

These moments are shown in Fig. 1. For every value of \( s \) we fit the moments by a beta function that is the moment of parton distribution in the form:

\[ z q_{\text{val}}(z, Q^2) = az^b(1 - z)^c \]  
\[ z q_{\text{sea}}(z, Q^2) = \alpha z^\beta(1 - z)^\gamma[1 + \eta z + \xi z^{0.5}] \]

the parameters \( a, b, c, \alpha, \beta, \gamma, \eta, \xi \) are functions of \( Q^2 \). The functional form of them are given in Ref. [2], here we give there values for \( Q^2 = 20 \text{ GeV}^2 \) only because at this value we will check our model against the data. \( a = 0.716, b = 0.608, c = 0.221, \alpha = 0.033, \beta = -0.344, \gamma = 1.696, \eta = 0.919, \xi = -1.765 \).
3  MESONIC STRUCTURE

Having determined the parton distributions in the CQ, we are now in a position to evaluate the meson structure function. We will use the convolution theorem and write the structure function of any meson as follows:

\[ F_2^M(x, Q^2) = \sum_{CQ} \int_x^1 \frac{dy}{y} G^CQ_M(y) F_2^CQ(y, Q^2) \]

where summation runs over the number of CQ’s in a particular meson. \( G^CQ_M(y) \) is the probability function for finding a CQ in the meson with momentum fraction \( y \). Also notice that this function is independent of the nature of the probe and its \( Q^2 \) value. In effect \( G^CQ_M(y) \) describes the wave function of meson in CQ representation containing all the complications due to confinement. From the theoretical point of view this function cannot be evaluated accurately. To facilitate phenomenological analysis, following [2] we assume a simple form for the exclusive CQ distribution in pion as follows:

\[ G_{CD/p}(y_1, y_2) = q y_1^\mu y_2^\nu \delta(y_1 + y_2 - 1) \]

\( l \) and \( q \) are normalization parameters. After integrating over unwanted momenta, we can arrive at inclusive distribution of individual CQ:

\[ G_{\bar{C}/p}(y) = \frac{1}{B(\mu + 1, \nu + 1)} y^\mu (1 - y)^\nu \]

similar expression for \( G_{D/p} \) is obtained with the interchange of \( \mu \leftrightarrow \nu \). In the above equations \( B(i, j) \) is Euler Beta function and its arguments, as well as \( l \) and \( q \) are fixed using the number and momentum sum rules:

\[ \int_0^1 G^CQ_M(y)dy = 1 \quad \sum_{CQ} \int_0^1 y G^CQ_M(y)dy = 1 \]

where \( CQ = U, D, S(U, D, S) \) and \( M = anymeson \). Numerical values are: \( \mu = 0.01, \nu = 0.06 \) for pion. Similar relations can be obtained for kion, only we chose the corresponding \( \mu \) and \( \nu \) such that the ration of average momenta carried by CQ in the mesons to be proportional to their masses. For the case of kion where a strange type CQ is present, we require that:

\[ \frac{m_{\pi}}{m_s} = \frac{y_1}{y_2} = \frac{\mu}{\nu} = \frac{330}{500} \approx \frac{2}{3} \]
where \( y_1 \) and \( y_2 = (1 - y_1) \) are the momentum fractions carried by each of the two CQ in a meson, respectively. For kion we have \( \mu = 0.21 \) and \( \nu = 0.32 \). We have also checked other values while maintaining the ratio but we found very little sensitivity. Now we can determine various parton distributions in any meson and in particular for kion and pion.

\[
q_{\text{val}/M}(x, Q^2) = 2 \int_x^1 \frac{dy}{y} G_{CQ/M}(y) q_{\text{val}/CQ}(x, Q^2)
\]

(16)

\[
q_{\text{sea}/M}(x, Q^2) = \int_x^1 \frac{dy}{y} G_{CQ_1/M}(y) q_{\text{sea}/CQ_1}(x, Q^2) + \int_x^1 \frac{dy}{y} G_{CQ_2/M}(y) q_{\text{sea}/CQ_2}(x, Q^2)
\]

(17)

![Graph showing the Q^2-evolution of the valence distribution.](image)

The above equation represents the contribution of constituent quarks 1,2 to the meson sea and valence distributions. It should be noticed that \( \bar{u}^+ = d^- = \bar{d}^+ = u^+ \). Similarly for kion we have: \( u^K = \bar{u}^K \) and \( \bar{u}^K = \lambda^K \). In Figures (2) the CQ contribution to valence distributions in \( \pi^- \) at some value of \( Q^2 \) is shown. In Figure (3) we present the ratio of valence quark distributions of \( K^+ \) and \( \pi^+ \) at \( Q^2 = 20 \text{ GeV}^2 \) and compare it with the data from NA3 Drell-Yan data of [3].
Fig. 3: The ratio of valence quark distributions of $K^+$ and $\pi^+$ meson at $Q^2 = 20\text{GeV}^2$.

References