HADRONS WITH CHARMED QUARKS
IN THE NUCLEAR MEDIUM

W. Weise

Physik-Department, Theoretische Physik
Technische Universität München, D-85747 Garching, Germany

Abstract
A survey is given on expected changes of charmonium ($J/\psi$ and $\eta_c$) as well as charmed meson ($D$ and $\bar{D}$) masses in nuclear matter. The physical implications of such in-medium effects are discussed.

1 Introduction
Investigating the behaviour of heavy quark systems in a nuclear medium is of considerable interest, for several reasons. First, the ongoing discussion of $J/\psi$ suppression in ultrarelativistic heavy-ion collisions as a possible signal for quark-gluon plasma formation requires detailed knowledge about the in-medium interactions of a $J/\psi$ also under "normal", non-plasma conditions. Secondly, an important element of this discussion is the behaviour of open-charm $D$ and $\bar{D}$ mesons in matter. For example, a downward shift of the threshold for $D\bar{D}$ production may increase the width of the $\psi'$ and hence decrease the feeding of the $J/\psi$ channel.

Brodsky et al. [1] pointed out that multigluon exchange should generate an attractive potential between a $c\bar{c}$ meson and a nucleon. In their original paper, these authors predicted an attraction so strong that, for instance, the $\eta_c$ could form bound states with nuclei. More detailed calculations [2]-[5] indicate that the charmonium binding energy in nuclear systems is of the order of 10 MeV.

A phenomenological estimate [6] based on a simple extrapolation of kaon-nucleon interactions to the charm sector suggests an attractive $D^+$-nucleus potential with a depth of more than $-100$ MeV. A QCD sum rule estimate predicts a downward shift of the $D^+$ average mass by about the same magnitude [7]. Such substantial medium effects offer new and interesting perspectives, as we shall discuss.

The present paper concentrates in its larger part on the QCD sum rule approach to $J/\psi$, $\eta_c$ and $D$, $\bar{D}$ masses in nuclear matter [8, 9]. Our results imply

---

$^1$Work supported in part by BMBF and GSI
less pronounced attractions than those found in some of the previous, purely phenomenological estimates. The predictions nevertheless indicate interesting and potentially observable effects, such as the in-medium splitting of $D^+$ and $D^-$ masses.

2 Quark masses, scales and symmetry breaking patterns in QCD

Apart from a renormalization scale ($\Lambda_{QCD} \sim 0.2 GeV$), the quark masses are the only parameters in QCD. They implement a hierarchy of scales at which vastly different phenomena occur:

1. The sector of the lightest ($u$ and $d$) quarks with masses $m_{u,d} \sim 10^{-2} GeV$ is governed by chiral symmetry. In the QCD ground state this symmetry is spontaneously broken, with the pions representing the Goldstone bosons and the quark condensate $\langle \bar{q}q \rangle$ acting as order parameter (where $q$ stands for $u$ or $d$). The pion decay constant, $f_\pi \simeq 0.09 GeV$, determines the chiral "gap" $4\pi f_\pi \simeq (4\pi/m_\pi)(2m_q|\langle \bar{q}q \rangle|)^{1/2} \sim 1 GeV$. In the low energy limit QCD reduces to an effective field theory of weakly interacting Goldstone bosons (pions) coupled to heavy hadronic sources. A systematic power expansion in the parameter $Q/(4\pi f_\pi)$ is meaningful if the momentum or energy $Q$ is small compared to the chiral gap (chiral perturbation theory).

2. The heavy quark limit [10, 11] (applicable to $b$ and $t$ quarks) involves symmetries and spectroscopy governed by the almost static behaviour of the massive spin-$1/2$ particles. The dynamics is governed by the gluon field. Gluonic flux tubes connect the static quark sources and produce a confining potential. The relevant small expansion parameters are expressed in terms of $M^{-1}$, the inverse heavy quark mass.

3. The masses of strange and charmed quarks, $m_s \simeq 0.15 GeV \sim \Lambda_{QCD}$ and $m_c \simeq 1.3 GeV \sim 10 m_s$, are intermediate between the chiral symmetry and heavy quark limits. Strange quarks are part of the chiral $SU(3) \times SU(3)$ symmetry framework, but with large explicit symmetry breaking caused by $m_s$. Charmed quarks have many features in common with heavy quarks, given that $m_c \gg \Lambda_{QCD}$, but they are still far from the $M \to \infty$ limit. What makes the physics of strange and charmed quarks interesting is that they interpolate between the limiting scales of QCD.
3 QCD sum rules in vacuum and in-medium

The starting point for the study of in-medium spectral distributions of hadrons is the current-current correlation function

\[ \Pi_J(\omega, \vec{k}) = i \int dt \, e^{i\omega t - i\vec{k} \cdot \vec{x}} \langle T J(\vec{x}, t) J(0) \rangle, \]  

where the expectation value is either taken in the vacuum or in the ground state of nuclear matter, and \( T \) denotes the time-ordered product. The current \( J \) stands for

\[ J(x) = \bar{c}(x) \gamma_\mu c(x), \quad \bar{c}(x)i \gamma_5 c(x), \]  

with the charm-quark field \( c(x) \), in the case of \( J/\psi \) or \( \eta_c \), and

\[ J(x) = \bar{c}(x)i \gamma_5 q(x), \quad \bar{q}(x)i \gamma_5 c(x), \]  

with light quark fields \( q(x) = u(x) \) or \( d(x) \), in the case of \( D \) and \( \bar{D} \) mesons.

The basic idea of QCD sum rules [12] is to connect the spectrum, \((1/\pi) \text{Im} \Pi\), of the correlation function (1) with its operator product expansion (OPE),

\[ \Pi(k) = \sum_n C_n^{[k]} \langle O_n \rangle. \]  

This expansion in terms of expectation values of local operators \( O_n \) converges for large spacelike \( k^2 = \omega^2 - \vec{k}^2 = -Q^2 < 0 \). The (Wilson-) coefficients \( C_n \) can then be calculated using perturbative QCD, while non-perturbative QCD enters through the condensates. The leading non-trivial expectation values \( \langle O_n \rangle \) are the quark and gluon condensates, \( \langle \bar{q}q \rangle \) and \( \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle \), both of mass dimension 4. Condensates of higher dimension should be successively less important at large \( Q^2 \) since the corresponding coefficients \( C_n \) introduce increasing powers of \( 1/Q^2 \).

The spectrum of \( \Pi \) enters as part of the dispersion relation,

\[ \Pi(k) = \frac{1}{\pi} \int ds \frac{\text{Im} \Pi(s)}{s - k^2} + \text{subtractions}. \]  

The relevant information about the spectral distribution is expressed through its moments, \( \int_{s_0}^{s_\infty} ds \, s^n \text{Im} \Pi(s) \). On the other side, each of these moments is connected, via Eq. (4), with condensates of a given dimension. This implies sum rules for the moments, and we use this strategy in the present context (see also ref. [13]). Another (equivalent) approach uses Borel transforms.

In most practical applications one considers hadrons "at rest" (with \( \vec{k} = 0 \)) relative to the medium. For the vector current \( \bar{c} \gamma_\mu c \) in the \( J/\psi \) channel, the
longitudinal and transverse parts of the correlation tensor \( \Pi_{\mu\nu} \) coincide in this limit, and one can introduce the dimensionless quantity

\[
\Pi_V(\omega, \vec{k} = 0) = -\frac{1}{3\omega^2} \Pi^\prime_{\mu}(\omega, \vec{k} = 0),
\]

the vacuum spectrum of which is directly related to the cross section for \( e^+e^- \rightarrow c\bar{c} \rightarrow \text{hadrons} \):

\[
Im\Pi_V(s) = \frac{9s}{(8\pi\alpha)^2} \sigma(e^+e^- \rightarrow \text{charm}).
\]

In the pseudoscalar channels, one also often introduces the dimensionless quantity \( \Pi_P(\omega, \vec{k} = 0) = \Pi(\omega, \vec{k} = 0)/\omega^2 \). A schematic sketch of vacuum and in-medium spectral distributions is shown in Fig. 1. The leading moments of such distributions specify the vacuum and in-medium masses \( m_0 \) and \( m^*(\rho) \), respectively. For sufficiently narrow resonances, it is often convenient to parametrize the spectrum in the form of a \( \delta \)-function and a continuum, so that

\[
\Pi(k) = \frac{\text{const}}{m^2 - k^2 - i\varepsilon} + \Pi^c(k),
\]

with the condition that the continuum part \( \Pi^c \) asymptotically approaches the perturbative QCD result. When comparing spectra in vacuum with those in nuclear matter at baryon density \( \rho \) using QCD sum rules, the relationship between vacuum and in-medium condensates to leading order in \( \rho \) is:

![Figure 1: Schematic picture of typical spectral distributions \( Im\Pi/\omega^2 \) for a given meson channel in vacuum and in nuclear matter at some density \( \rho \). The vacuum and in-medium masses, \( m_0 \) and \( m^* \), are indicated.](image)
\[ \langle O \rangle_\rho = \langle O \rangle_o + \langle N | O | N \rangle \rho + \ldots \]  
\( \tag{9} \)
where the free nucleon states are normalized as \( \langle N(p) | N(p') \rangle = (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \).

For the quark condensate,
\[ \frac{\langle \bar{q} q \rangle_\rho}{\langle \bar{q} q \rangle_o} = 1 - \frac{\sigma_N}{m_\pi^2 f_\pi} \rho + \ldots, \]  
\( \tag{10} \)
with the pion-nucleon sigma term \( \sigma_N \simeq 0.05 \text{ GeV} \). For the gluon condensate \[ \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G_{\mu\nu} \rangle_\rho = \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G_{\mu\nu} \rangle_o - \frac{8}{9} M_N^0 \rho + \ldots, \]  
\( \tag{11} \)
where \( M_N^0 \simeq 0.75 \text{ GeV} \) is the nucleon mass in the chiral limit. In the nuclear medium, the twist-2 gluon operator \( G_{\alpha\mu} G_{\beta}^\mu \) can also have a non-vanishing expectation value, with
\[ \langle N(p) | \frac{\alpha_s}{\pi} G_{\alpha\mu} G_{\beta}^\mu | N(p) \rangle = -\frac{2\alpha_s}{\pi} \left( p_{\alpha\beta} - \frac{p^2}{4} g_{\alpha\beta} \right) A_G \]  
\( \tag{12} \)
where \( A_G = \int_0^1 dx x g(x) \simeq 0.5 \) is the fraction of the nucleon momentum carried by gluons. The correction from this term is, however, small compared to the leading density dependence (10).

We now turn to results for charmonium and \( D \) mesons in matter.

4 \textbf{J/}\psi \textbf{ and } \eta_c \textbf{ in nuclear matter}

In the charmonium case the light quark condensate plays no role since it does not couple to the \( c\bar{c} \) sector. Only the gluon condensates enter. From Eq. (11), their density dependence is relatively weak,
\[ \frac{\langle G^2 \rangle_\rho}{\langle G^2 \rangle_o} \simeq 1 - 0.06 \frac{\rho}{\rho_o} \]  
\( \tag{13} \)
in terms of \( \rho_o \simeq 0.16 \text{ fm}^{-3} \), the density of normal nuclear matter.

The spectrum of the current correlation function in the charmonium channels is parametrized in the form
\[ \text{Im} \Pi(s) = \sum_R \frac{9\pi m_R^2}{4g_R^2} \delta(s - m_R^2) + \text{continuum}, \]  
\( \tag{14} \)
with a sum over narrow resonances plus a continuum. Ratios of moments of
this spectral distribution are used to extract the mass of the lowest charmonium
state both in the vacuum and in nuclear matter.

The resulting mass shifts [8] of $J/\psi$ and $\eta_c$ at $\rho = \rho_0$ are small:
\[
\Delta m_{J/\psi} \simeq -7 \text{ MeV} \quad \text{and} \quad \Delta m_{\eta_c} \simeq -5 \text{ MeV}. \quad (15)
\]

When converted to charmonium-nucleon scattering lengths,
\[
a = \frac{\mu \Delta m}{2\pi \rho} \simeq 0.15 \text{ fm}, \quad (16)
\]
where $\mu$ is the reduced mass of the system. The corresponding low-energy
cross sections for $J/\psi$-nucleon scattering would be close to $3 \text{ mb}$, quite compatible with $\sigma_{\psi N} \simeq (3 - 4) \text{ mb}$ deduced from photoproduction. This should be
compared with $\sigma_{\psi N} \sim (6 - 7) \text{ mb}$ fitted to charmonium absorption in $p + A$
and $A + A$ collisions at high energies.

An attractive $J/\psi$ mass shift of less that $10 \text{ MeV}$ in nuclear matter is
consistent with the $J/\psi$-nuclear potential derived from the two-gluon exchange
$J/\psi$-nucleon interaction [3]. This potential can be computed exactly in the
limit $m_c \to \infty$.

Recently, corrections from condensates of mass dimension 6 have been esti-
\[
\text{mated [15] and found to reduce the attractive mass shift, calculated up to}
\text{dimension 4, by about 3 MeV. It appears that such corrections have alternating}
\text{signs and decrease properly with increasing dimension, so that the original}
\text{statement, about the $J/\psi$ and $\eta_c$ experiencing attractive mass shifts of less}
\text{than 10 MeV at $\rho = \rho_0$, remains valid. Corrections at finite momentum $\vec{k}$ are}
\text{small [9] (less than 1 MeV at $|\vec{k}| \sim 1 \text{ GeV}$).}
\]

5 \quad D \text{ mesons in matter}

The case of the heavy-light open charm systems is in several ways more in-
\[
teresting than the weakly interacting $J/\psi$ and $\eta_c$. The $D$ meson is reminis-
tent, by analogy, of a "hydrogen atom" in QCD (although the mass ratio $m_c/m_{u,d} \sim 200$ is not quite as large as $M_\pi/m_c \sim 2 \cdot 10^3$). The qualitative
picture is that of a light quark fluctuating around a heavy color source. Such
systems are unique in the sense that it is now a single light quark that probes
the QCD vacuum, while the heavy quark decouples almost completely. In the
following we concentrate on the $D^+(cd\bar{d})$ and $D^-(d\bar{c})$ in nuclear matter.

It is instructive to discuss in-medium effects for $D^\pm$ in comparison with
those for the $K^\pm$ mesons. When replacing $c$- by $s$-quarks and vice versa, the
$K^-$ corresponds to the $D^+$ and the $K^+$ to the $D^-$. The leading scalar and vector current interactions of those mesons with nucleons primarily involve the light quarks. Scalar forces are identical for $d$- and $\bar{d}$-quarks, whereas vector current couplings are repulsive for $d$- and attractive for $\bar{d}$-quarks interacting with $d$-quarks in the nucleon. The expected in-medium pattern is therefore a splitting such that the $D^+(K^-)$ mass is lowered while the $D^-(K^+)$ mass is raised. This is confirmed by the QCD sum rule approach for $D^\pm$ which we shall now discuss.

Consider first the vacuum correlation function (1) with the pseudoscalar current $J = \bar{d}i\gamma_5 c$ which annihilates a $D^+$ or creates a $D^-$. Choose the ansatz

$$\Pi(k) = \frac{2f_D^2m_D}{m_c^2} \frac{1}{m_D^2 - k^2 - i\varepsilon} + \Pi^c(k),$$

(17)

with the $D$-meson pole term (mass $m_D$) and a continuum part $\Pi^c$. The pole strength is determined by the matrix element $\langle 0|J(0)|D^+\rangle = \sqrt{2}f_Dm_D^2/(m_c + m_d)$, and we drop the small $d$-quark mass $m_d$. For the $D$-meson decay constant $f_D$, the one analogous to the pion decay constant $f_\pi$, detailed QCD sum rule analysis gives

$$f_D = (135 \pm 10) \text{ MeV.}$$

(18)

(Lattice QCD values for $f_D$ tend to be slightly larger but consistent within errors.)

Setting $Q^2 = -k^2$ large and positive and equating $\Pi$ of Eq. (17) with its OPE representation, the result is

$$\frac{2f_D^2m_D}{m_c^2(Q^2 + m_D^2)} = \frac{1}{\pi} \int_{m_c^2}^\infty ds \frac{Im(\Pi_{\text{pert}}(s) - \Pi^c(s))}{Q^2 + s} + \frac{-m_c\langle \bar{d}d \rangle + \langle \sigma_s G^2 \rangle}{Q^2 + m_c^2} + ...,$$

(19)

where the terms not shown explicitly include small corrections from condensates of higher dimension. The dispersion integral on the r. h. s. is taken over the perturbative $c\bar{d}$ loop, $\Pi_{\text{pert}}$, commonly evaluated to leading order in $\alpha_s$, minus the continuum part $\Pi^c$. Assuming that $\Pi^c(s)$ approaches $\Pi_{\text{pert}}(s)$ at some large $s = s_0$, the dispersion integral reduces to $\int_{m_c^2}^{s_0} ds Im\Pi_{\text{pert}}(s)/(Q^2 + s)$.

Note that the quark condensate term in Eq. (19) is multiplied by the heavy-quark mass $m_c$. The non-perturbative correction involving the chiral condensate is now an order of magnitude larger than the gluon condensate term, unlike the situation for light $q\bar{q}$ systems. The dispersion integral over $Im\Pi_{\text{pert}}$ still dominates over all, but the sensitivity to the chiral order parameter is considerably enhanced.
Let us now turn to the in-medium sum rules for $D$-mesons at rest ($k^\mu = (\omega, \vec{k} = 0)$) relative to the nuclear medium. In the vacuum correlation function, the $D^+$ and $D^-$ poles appear at $\omega = \pm m_D$. In matter we expect that these poles are shifted in separate ways. The in-medium correlation function has even and odd pieces in $\omega$, and Eq. (17) is replaced by:

$$\Pi(\omega, \vec{k} = 0; \rho) = \frac{F_+}{m_+^2 - \omega^2} (1 + \frac{\omega}{m_+}) + \frac{F_-}{m_-^2 - \omega^2} (1 - \frac{\omega}{m_-}) + \Pi_c(\omega; \rho), \tag{20}$$

where $m_{\pm}(\rho)$ denote the in-medium $D^+$ and $D^-$ masses, and $F_{\pm}$ are the corresponding in-medium pole strengths. The modification of the continuum part $\Pi_c(\omega; \rho)$ in matter requires an extra discussion.

On the OPE side, the primary new feature in matter is the appearance of the $d$-quark density, $\langle d^+ d \rangle = \frac{3}{2} \rho$, in the $\omega$-odd part of $\Pi$:

$$\Pi(\omega, \vec{k} = 0; \rho) = \frac{2}{\pi} \int_{m_c}^{\infty} d\omega \frac{\omega^2 \Pi_{\text{pert}}(\omega)}{\omega^2 - \omega^2} + \frac{1}{\omega^2 - m_c^2} [m_c \langle \bar{d} d \rangle \rho - \omega \langle d^+ d \rangle] + ..., \tag{21}$$

where we have neglected the small gluon condensate term and assumed that the perturbative QCD part will not change in matter. Next we separate the even and odd pieces, set $Q^2 = -\omega^2$ positive and large again and perform a systematic analysis of the moments of $Im \Pi$.

A key result of this analysis is the prediction that the $D^+$ and $D^-$ masses split in dense matter. One finds:

$$m_+(\rho) - m_-(\rho) = -\frac{3 \rho}{4 f_D^2} \left[ \frac{m_c^2}{m_D^2} (1 - \frac{m_c^2}{m_D^2}) + \frac{2 m_c^2}{3 \pi m_D^2} \int_{\omega_{th}}^{\omega_0} d\omega (\frac{\omega^2}{m_D^2} - 1) \text{Im} \mathcal{T}(\omega) \right], \tag{22}$$

where we have factorized the continuum part of the spectrum, $\Pi_c(\omega; \rho) \equiv \mathcal{T}(\omega) \rho$, to leading order in the density $\rho$. Note the driving term proportional to $\rho/f_D^2$ is incidentally reminiscent of the Weinberg-Tomozawa term in chiral $\pi N$ dynamics, with $f_\pi$ replaced by the $D$-meson decay constant $f_D$, but this term is now accompanied by factors which introduce the scale set by the large $c$-quark mass. The continuum integral includes inelastic processes such as $DN \to \pi \Lambda_c$ in the nuclear medium, i.e. the coupling of the $D^+$-mode to the $\Lambda_c$-particle-nucleon-hole continuum (accompanied by a pion in $s$-wave).

The integration extends from threshold, $\omega_{th} \simeq 1.5 GeV$, to the region where perturbative QCD presumably takes over, $\omega_0 \simeq 2.5 GeV$. A rough estimate for $\text{Im} \mathcal{T}(\omega)$ using the optical theorem and assuming an average $DN$ cross section of about $10 mb$ gives a number of order one for the bracket in Eq. (22), so
that a mass splitting \( m_+ - m_- \approx -60 \text{ MeV} (\rho/\rho_0) \) results. An effect of that magnitude should have observable consequences.

The overall mass shift \( \delta(m_+ + m_-) \) has larger uncertainties because it involves higher moments of the continuum spectrum, apart from the in-medium change of the chiral condensate \( \langle \bar{q}q \rangle \). Hayashigaki [7] finds a large downward shift of about \(-100 \text{ MeV} \) for the \( D \bar{D} \) threshold at \( \rho = \rho_0 \), but with very special model assumptions. Our predicted shifts are less extreme but still sizeable and lead to the qualitative picture shown in Fig. 2.

![Diagram](image)

**Figure 2:** Expected shifts and splitting of \( D^+ \) and \( D^- \) masses in nuclear matter at \( \rho = \rho_0 = 0.17 \text{ fm}^{-3} \), based on the QCD sum rule estimate [9].

## 6 Summary and perspectives

The in-medium modification of \( J/\psi \) and \( \eta_c \) masses are expected to be small (less than 10 MeV at nuclear matter density), reflecting the relatively weak gluonic van-der-Waals type forces between the charmonium ground states and the nucleons.

The situation is quite different for open-charm systems. Our QCD sum rule analysis predicts a splitting of \( D^+ \) and \( D^- \) masses of more than 50 MeV at \( \rho = \rho_0 \). At the same time we expect a lowering of the \( D \bar{D} \) threshold in matter, the amount of which is however subject to uncertainties at the level of the unknown \( DN \) coupling to the sector of charmed baryons plus pions.

The medium effects for \( D^\pm \) are sizeable. They should lead to observable phenomena, such as enhancements in the \( D^+ \) subthreshold production in heavy-ion collisions. If the \( D \bar{D} \) threshold is lowered, at sufficiently high density, so that \( \psi' \to D \bar{D} \) decay opens up, then this should have considerable impact on the subsequent branching into \( J/\psi \) in high-energy nuclear collisions.
The in-medium $D^+/D^-$ mass splitting can be viewed as a continuation of the symmetry breaking patterns already seen previously for $\pi^+/\pi^-$ and $K^+/K^-$. In isospin-asymmetric nuclear matter, the $\pi^-$ is shifted upward (by about $25 \text{ MeV}$) relative to the $\pi^+$ which remains almost unchanged, an effect seen in deeply bound pionic atom states in $Pb$. The $K^-$ is shifted downward with respect to the $K^+$, with a $K^-/K^+$ mass splitting of order $-100 \text{ MeV}$ at $\rho = \rho_0$, an effect that helps understanding the subthreshold $K^-$ and $K^+$ production rates in high-energy heavy-ion collisions. Apparently, the $D^\pm$ system with its single active light quark or antiquark still continues this pattern, although at quite a different mass scale, far removed from low-energy chiral dynamics. Investigating charm in dense nuclear matter is obviously an exciting challenge.

I am grateful to Philippe Morath and Su-Hyoung Lee whose work has contributed significantly to this presentation.

References