# Shell structure from N=Z ($^{100}$Sn)to N≫Z ($^{78}$Ni)

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**Abstract**

The shell structure of $^{100}$Sn shows striking resemblance to $^{56}$Ni one major shell below. Large-scale shell model calculations employing realistic interactions derived from effective NN potentials and allowing for up to $4p4h$ excitations of the $^{100}$Sn core account very well for the spectroscopy of key neighbours $^{102,103}$Sn, $^{98}$Cd and $^{94}$Ag, as inferred from level energies, isomerism, E2 strengths and Gamow-Teller (GT) decay of high-spin states. Recent $\beta$- decay studies of $^{101-104}$Sn using the sulphurisation ISOL technique open the perspective to study the $^{100}$Sn GT resonance. At N≫Z the persistence of the N=50 and the weakness of the N=40 shells are traced back to the monopole interaction in S=0 proton-neutron ($\pi\nu$) pairs of nucleons, a scenario which can be generalised to account for the new N=6,16(14),34(32) magicity in light neutron-rich nuclei.

## 1 Introduction

The shell structure of $^{100}$Sn has been inferred by a detailed shell model (SM) analysis from abundant experimental data from in-beam and decay spectroscopy (see [1] for a review). In analogy to $^{56}$Ni one major shell lower $l = 2$ and spin-flip core excitations are expected, which can be treated in modern SM codes [2]. The high-spin intruder orbital $\pi\nu g_{9/2}$ gives rise to spin-gap isomerism and the $\pi g_{9/2} \rightarrow \nu g_{7/2}$ GT conversion, which towards $^{100}$Sn develops into the super-GT resonance [3], which sets the stage for a stringent test of SM theory.

The change of shell structure in neutron-rich nuclei has been subject of numerous theoretical and experimental studies (see [1, 4] for a recent review). The scenario is characterized by the sequential *shell quenching* and *reordering*, a transition *spin-orbit (SO) to harmonic oscillator (HO)* gap, *smooth* evolution with $N/Z$ and *reduced* SO splitting. None of these signatures applies to the new shell structure observed in light and medium-heavy nuclei. Based on the decisive role of monopole driven shell structure for the N=40 and 50 shell strength, an alternative approach [4] will be discussed in Sec. 3 for the observed new magicity in light neutron-rich nuclei.

## 2 The $^{100}$Sn region and N≃Z nuclei

As the single particle (hole) neighbours are not yet accessible to experimental studies, the shell structure of $^{100}$Sn is inferred from shell model studies of less exotic neighbouring nuclei as described in Ref. [5]. The latest results are reviewed in Refs. [1, 6] and are found to be in excellent agreement with recent experimental evidence for single particle energies [6] and shell gaps [7]. The shell structure shows an almost one to one correspondence to $^{56}$Ni, situated at N=Z one major shell lower.

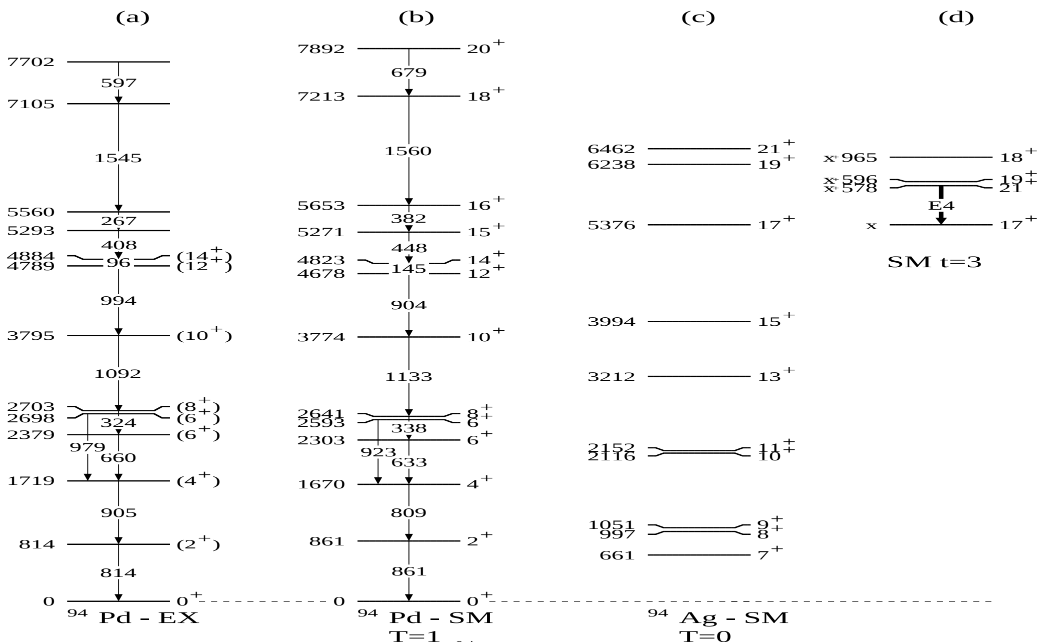


Fig. 1: Experimental level scheme for $^{94}$Pd as inferred from in -beam spectroscopy and $\beta$ decay of $^{94}$Ag (a) and SM results (b) in the $\pi\nu$ $(p_{1/2}, g_{9/2})$ model space using an empirical interaction [9]. The SM results for the T=0 states in $^{94}$Ag (c) are given together with a partial level scheme calculated in the $\pi\nu$ $(s, d, g)$ space using a realistic interaction and up to $3p3h$ excitations (t=3) of the $^{100}$Sn core [2].

Ever since the first observation of $^{98}$Cd and $^{102}$Sn in-beam and the measurement of the respective B(E2;$8^+ \rightarrow 6^+$) and B(E2;$6^+ \rightarrow 4^+$) values the small proton and the large neutron polarization charges needed in pure proton hole and neutron particle model spaces have been a puzzle [6]. Large-scale SM calculations in the (s,d,g) model space allowing for up to $4p4h$ excitations of the $^{100}$Sn core have solved the problem recently with effective E2 polarisation charges of 0.1$e$ and 0.6$e$ for protons and neutrons, respectively [2, 6]. The importance of core excitations for spin-gap isomerism is stressed by the fact that the SM approach in analogy to $^{54}$Fe predicts an $I^{\pi}$=12$^{+}$ E4 isomer in $^{98}$Cd, which is presently under experimental analysis, and an $I^{\pi}$=6$^{+}$ isomer in $^{100}$Sn ! First experimental confirmation has been found in a recent $\beta$-decay study of $^{94}$Ag [8]. A presumably $I^{\pi}$=(21$^{+}$) spin-gap isomer was established which was not predicted in SM calculations in the valence hole space [9] (Fig. 1c) in spite of the good predictive power of this approach (Fig. 1a,b). Inclusion of up to $3p3h$ excitations in the (s,d,g) model [2] accounts well for the isomerism (Fig. 1d) enforcing the confidence in the $^{98}$Cd and $^{100}$Sn predictions.

Recently the $\beta$ decay of the Sn isotopes down to $^{101-103}$Sn was studied employing the sulphurisation technique [10], which is Z selective to $\leq 10^{-4}$ against isobaric contamination, and both a high-resolution germanium detector array and a total absorption spectrometer (TAS) [11]. A preliminary summary of experimental cross sections and rates extrapolated down to $^{100}$Sn opens the challenge to study the $^{100}$Sn GT resonance at a rate of 4 atoms/h at the GSI ISOL facility .

## 3 The $^{78}$Ni region and N≫Z nuclei

The propagation of single particle energies with increasing occupation of a major shell is governed by the monopole part of the residual interaction [12] as defined by

$$V^{m}_{jj'} = \Sigma_J (2J+1) \langle jj' | V | jj' \rangle_J / \Sigma_J (2J+1) \qquad (1)$$

It has been pointed out that the $(\sigma \cdot \sigma)(\tau \cdot \tau)$ part of the in-medium NN interaction provides the schematic explanation for the enhancement of $V^m$ for $\pi\nu$ pairs relative to T=1 pairs and for spin-orbit partners in the long-range limit [13]. A closer inspection of experimental data in the N=51 [6] (Fig. 2a), N=29 [4] isotones, the Z=29 [4] (Fig. 2b) and Z=51 [14] isotopes, and of realistic interactions derived from effective NN potentials and fitted to experimental data reveals $V^m$ to be strong in $\pi\nu$ pairs with total spin S=0 and preferably identical number of nodes in their radial wave functions. In fact the selected examples yield an approximate factor of 2 between S=0 and S=1 configurations [4].

The effect is demonstrated best in the evolution of the N=40 and 50 shell gaps for N≫Z. The persistence of the N=50 shell gap at $^{78}$Ni is discussed since long [1, 17]. As the last measured value is known for Se (Z=34), removal of the last $\pi f_{5/2}$ protons in the light of monopole driven shell structure is essential, as the $\pi f_{5/2} \nu g_{9/2}$ monopole is known to be strong which is reproduced by realistic interactions (Fig. 2c). Starting from $^{84}$Se for N=50 and $^{68}$Ni for Z=28 the $^{78}$Ni shell gaps can be determined. The gaps are determined by $V^m$

in the configurations $\pi f_{5/2}\nu g_{9/2}$ and $\pi f_{5/2}\nu d_{5/2}$ for N=50 and $\pi f_{5/2}\nu g_{9/2}$ and $\pi f_{7/2}\nu g_{9/2}$ for Z=28, respectively. Assuming the empirical factor of 2 between S=0 and S=1 monopoles, the relative monopole shift of two orbitals $j_1', j_2'$ as given by

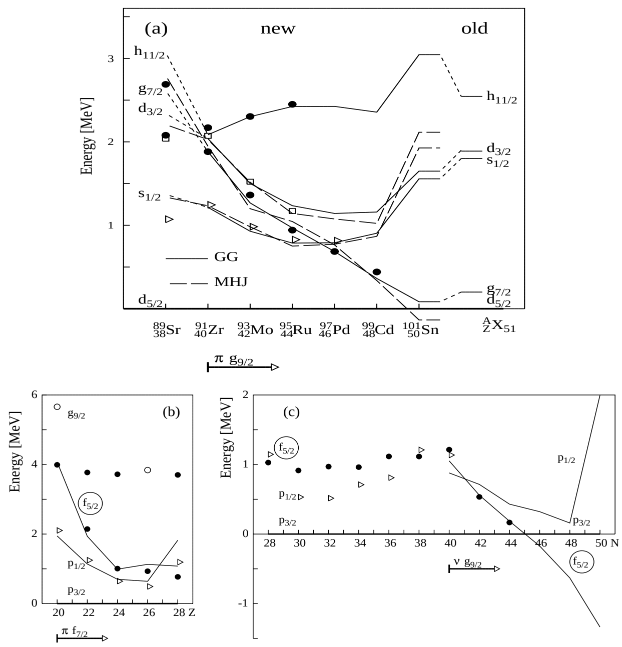


Fig. 2: Monopole migration of neutron single particle states in (a) N=51 isotones, (b) N=29 isotones and (c) in Z=29 (Cu) isotopes. The lines refer to SM calculations with realistic interactions GG [6], MHJ [7] (a), FPD6 [15] (b) and S3V [16] (c).

$$\Delta(j_1', j_2') = (V_{jj_1'}^m - V_{jj_2'}^m) \cdot (2j+1) \tag{2}$$

with $(j_1', j_2')$=$(\nu g_{9/2}, \nu d_{5/2})$ and $(\pi f_{5/2}, \pi f_{7/2})$ and j=$\pi f_{5/2}$ and $\nu g_{9/2}$, respectively, can be estimated. The results for various interactions yield a substantially reduced but still appreciable N=50 gap of 2.0-2.6 MeV, while the Z=28 gap (4.5 MeV) is well preserved. It should be noted, that these estimates do not include cross-shell interactions and mutual enhancement of proton and neutron shell and therefore represent lower limits only. Therefore it is expected that the $I^{\pi} = 8^{+}$ isomerism observed at the beginning of the $\nu g_{9/2}$ shell in $^{70}$Ni and in $^{78}$Zn [4, 6] is preserved for $^{76}$Ni. In a recent experiment evidence for the latter has been found [18].

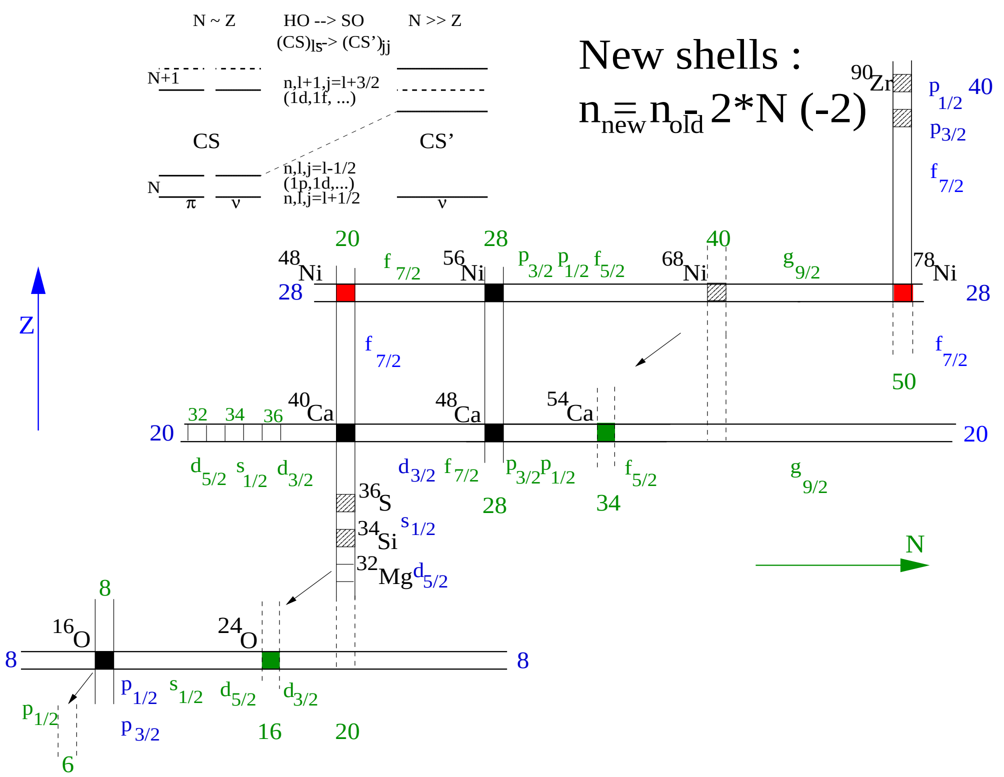


Fig. 3: Schematic chart of known and expected shell structure in N≫Z nuclei. The insert shows the changing scenario from N∼Z to N≫Z .

The HO closed shell N=40 in $^{68}$Ni is weak and isolated and looses its strength already at two particles/holes distance [1, 19]. Removing $\pi f_{7/2}$ protons from $^{68}$Ni prompts the $\nu f_{5/2}$ orbit to move into the (small) N=40 shell gap, so that Fe isotopes feature deformation. This was proven recently by assigning the Nilsson configuration $5/2^{+}[422]$ to the ground state of $^{67}$Fe [20]. Correlated to this upward shift of the $\nu f_{5/2}$ orbit a N=34 gap opens above the $\nu p_{3/2}, p_{1/2}$ levels as also exhibited by the N=29 single particle states at Z=20 (Fig. 3b). The presence of the $p_{1/2}$ orbit introduces the N=34(32) ambiguity. Experimentally a large $E_{2+}$ is observed in $^{52}$Ca [1] and shell gaps are established in the yrast spectrum of the $^{52,54}$Ti isotopes[21].

The effect can be generalised as a change of a HO shell closure with magic number $n_m = 8, 20, 40$ to $n_m - 2 \cdot N = 6, 16(14), 34(32)$, with $N$ counting the

HO quanta. The ambiguity for $N > 1$ is due to the presence of j=1/2 orbits as e.g. $s_{1/2}$ or $p_{1/2}$, which strongly mix by pair scattering with the neighbouring higher-spin orbitals. The scenario is characterized by the following signature, which substantially deviates from the mechanism described in Sec. 1 :

- a HO ($ls$-closed) shell changes to a SO ($jj$-closed) shell;
- the change is rapid with subshell occupation, and highly localized;
- the apparent SO splitting is increased.

The according shell structure change is shown in the chart of Fig. 3.

## References


[1] H. Grawe and M. Lewitowicz, Nucl. Phys. **A693** (2001) 116

[2] F. Nowacki, Nucl. Phys. **A704** (2002) 223c, and private communication

[3] E. Roeckl, Nucl. Phys. **A704** (2002) 200c

[4] H. Grawe, Proc. XXXVII Zakopane School of Physics, Acta Physica Polonica **B**, (2003), in print

[5] H. Grawe et al., Physica Scripta **T56** (1995) 71

[6] H. Grawe et al., Nucl. Phys. **A704** (2002) 211c

[7] M. Lipoglavšek et al., Phys. Rev. **C65** (2002) 021302(R)

[8] M. La Commara et al., Nucl. Phys. **A708** (2002) 167

[9] R. Gross and A. Frenkel, Nucl. Phys. **A267** (1976) 85

[10] R. Kirchner et al., Nucl. Instr. and Meth. **B**, in print

[11] M. Karny et al., GSI Ann. Rep. 2002, in print

[12] E. Caurier et al., Phys. Rev. **C60** (1994) 225

[13] T. Otsuka et al., Phys. Rev. Lett. **87** (2001) 082502

[14] A. Covello, private communication

[15] W.A. Richter et al., Nucl. Phys. **A523** (1991) 325

[16] J. Sinatkas et al., J. Phys. **G18** (1992) 1377 and 1401

[17] J. Dobaczewski et al., Phys. Rev. Lett. **72** (1994) 981

[18] M. Sawicka et al., Proc. NS2002, Padova, Italy, Eur. Phys. J. **A**, in print

[19] H. Grawe et al., Proc. Tours 2000, AIP **CP561** (2001) 287

[20] M. Sawicka et al., Eur. Phys. J. **A16** (2003) 51

[21] R.V.F. Janssens et al., Phys. Lett. **B546** (2002) 55