

# Nuclear structure at the drip-line : the continuum-QRPA

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## Abstract

The QRPA response is derived starting from the time-dependent Hartree-Fock-Bogoliubov (HFB) equations. The residual interaction between the quasiparticles is determined consistently from the two-body force used in the HFB equations, and the continuum coupling is treated exactly. Dipole and quadrupole excitations as well as two-neutron transfer processes are discussed for the case of neutron-rich oxygen isotopes.

## 1 Introduction

The collective excitations of atomic nuclei in the presence of pairing correlations are usually described by the quasiparticle-Random Phase Approximation (QRPA) [1]. Recently there is a renewed interest in this subject, generated mainly by the studies of unstable nuclei close to the drip line. In these nuclei characterized by a small nucleon separation energy, the excited states are strongly influenced by the coupling with the quasiparticle (qp) continuum configurations. In order to describe such excited states within QRPA one needs a proper treatment of the continuum coupling, which is missing in the usual QRPA calculations based on a discrete qp spectrum.

In nuclei close to the drip lines one expects also a strong connection between the excitations of the system and the properties of the ground state, which may present such specificities as neutron skins. Therefore, in addition to the qp spectrum, the residual interaction used in QRPA should be determined from the same two-body force as it is done in the self-consistent continuum RPA calculations [2, 3, 4].

We present the first continuum QRPA calculations with the single-particle spectrum and the residual interaction determined from the same effective two-body force [5]. The ground state is calculated using the continuum HFB approach [6] with the mean field and the pairing field described by a Skyrme

interaction and a density dependent delta force, respectively. Based on the same HFB energy functional we derive the QRPA response function in coordinate space.

## 2 Continuum-QRPA equations in coordinate space

In this section we summarize the derivation of the continuum-QRPA response, presented in Ref. [5], using Green's function formalism. The QRPA response is obtained from the time-dependent HFB (TDHFB) equations [1]:

$$i\hbar\frac{\partial\mathcal{R}}{\partial t} = [\mathcal{H}(t) + \mathcal{F}(t), \mathcal{R}(t)] \quad (1)$$

where  $\mathcal{R}$ ,  $\mathcal{H}$  are the time-dependent generalized density and HFB hamiltonian respectively, and  $\mathcal{F}$  the external oscillating field. In the small amplitude limit the TDHFB equations become:

$$\hbar\omega\mathcal{R}' = [\mathcal{H}', \mathcal{R}^0] + [\mathcal{H}^0, \mathcal{R}'] + [F, \mathcal{R}^0] \quad (2)$$

where ' stands for the perturbed quantity.

The variation of the generalized density  $\mathcal{R}'$  is expressed in term of 3 quantities, namely  $\rho'$ ,  $\kappa'$  and  $\bar{\kappa}'$ , which are written as a column vector :

$$\rho' = \begin{pmatrix} \rho' \\ \kappa' \\ \bar{\kappa}' \end{pmatrix} \quad (3)$$

In the following we will denote  $\rho'$  in bold face the column vector defined in Eq. 3. Thus, in variance with the RPA, where one needs to know only the change of the p-h density ( $\rho'$ ), in QRPA one should calculate the variation of three basis quantities. It should be noted that in the three dimensional space introduced above, the first dimension represents the particle-hole (ph) subspace, the second the particle-particle (pp) one, and the third the hole-hole (hh) one. The response matrix has 9 coupled elements in QRPA, compared to one in the RPA formalism.

The variation of the HFB hamiltonian is expressed in terms of the second derivatives of the HFB energy functional  $\mathcal{E}[\rho, \kappa, \bar{\kappa}]$  with respect to the densities :

$$\mathcal{H}' = \mathbf{V}\rho' \quad (4)$$

where  $\mathbf{V}$  is the residual interaction matrix, namely :

$$\mathbf{V}^{\alpha\beta}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \frac{\partial^2 \mathcal{E}}{\partial \rho_\beta(\mathbf{r}'\sigma') \partial \rho_{\bar{\alpha}}(\mathbf{r}\sigma)}, \quad \alpha, \beta = 1, 2, 3. \quad (5)$$

Here, the notation  $\bar{\alpha}$  means that whenever  $\alpha$  is 2 or 3 then  $\bar{\alpha}$  is 3 or 2.

The quantity of interest is the QRPA Green's function  $\mathbf{G}$ , which relates the perturbing external field to the density change:

$$\boldsymbol{\rho}' = \mathbf{G}\mathbf{F}. \quad (6)$$

Replacing the above three equations in Eq. 2, yields the so-called Bethe-Salpeter equation :

$$\mathbf{G} = (\mathbf{1} - \mathbf{G}_0\mathbf{V})^{-1} \mathbf{G}_0 = \mathbf{G}_0 + \mathbf{G}_0\mathbf{V}\mathbf{G} \quad (7)$$

which is a set of 9x9 coupled equations.

In Eq. 7 the unperturbed Green's function  $\mathbf{G}_0$  is defined by :

$$\mathbf{G}_0^{\alpha\beta}(\mathbf{r}\sigma, \mathbf{r}'\sigma'; \omega) = \sum_{ij} \frac{\mathcal{U}_{ij}^{\alpha 1}(\mathbf{r}\sigma) \bar{\mathcal{U}}_{ij}^{*\beta 1}(\mathbf{r}'\sigma')}{\hbar\omega - (E_i + E_j) + i\eta} - \frac{\mathcal{U}_{ij}^{\alpha 2}(\mathbf{r}\sigma) \bar{\mathcal{U}}_{ij}^{*\beta 2}(\mathbf{r}'\sigma')}{\hbar\omega + (E_i + E_j) + i\eta} \quad (8)$$

where  $E_i$  are the single qp energies and  $\mathcal{U}_{ij}$  are 3 by 2 matrices calculated from the U and V HFB wave functions [5].

In the case of transitions from the ground state to excited states within the same nucleus, only the (ph,ph) component of  $\mathbf{G}$  is acting. If the interaction does not depend on spin variables the strength function is thus given by :

$$S(\omega) = -\frac{1}{\pi} \text{Im} \int F^{11*}(\mathbf{r}) \mathbf{G}^{11}(\mathbf{r}, \mathbf{r}'; \omega) F^{11}(\mathbf{r}') d\mathbf{r} d\mathbf{r}' \quad (9)$$

In the case of transitions from the ground state of a nucleus with A nucleons to a state of a nucleus with A+2 nucleons, the (pp,pp) component of  $\mathbf{G}$  is used instead.

### 3 The QRPA response for neutron-rich oxygen isotopes

The calculations are performed assuming spherical symmetry. The ground states are calculated within the continuum HFB approach [6] where the continuum is treated exactly. The HFB equations are solved in coordinate space with a step of 0.25 fm for the radial coordinate. In the HFB calculations the mean field quantities are calculated by using the Skyrme interaction SLy4 [7],

while for the pairing interaction we take a zero-range density-dependent interaction. Due to its zero-range this force should be used in the HFB calculations with a cutoff in qp energy. This cutoff is determined from the strength of the pairing interaction, in order to keep the averaged gap value unchanged [5]. In the HFB calculation the qp cutoff energy is equal to 50 MeV. In the QRPA calculations the residual interaction is derived from the interaction used in HFB. The residual interaction corresponding to the velocity-dependent terms of the Skyrme force is approximated in the (ph,ph) subspace by its Landau-Migdal limit[8]. All the qp states are included until an energy cutoff of 50 MeV, allowing pairs of qp energy until 100 MeV. The strength distribution is calculated until  $\omega_{Max}=50$  MeV, with a step of 100 keV and an averaging width  $\eta=150$  keV. In a fully consistent calculation the spurious center-of-mass state should come out at zero energy. Because of the Landau-Migdal form of the interaction adopted here the consistency between mean field and residual qp interaction is broken and the spurious state becomes imaginary. We cure this defect by renormalizing the residual interaction by a factor  $\alpha$ . We find that in all cases the spurious state  $J^\pi = 1^-$  comes out at zero energy for  $\alpha=0.80$ .

### 3.1 Quadrupole response

The quadrupole response for oxygen isotopes is discussed in detail in Ref. [5]. As an example we show here (see Fig 1) the comparison between box discretization calculations (in a box size of 22.5 Fermi) and exact continuum calculations for  $^{22}\text{O}$  nucleus.

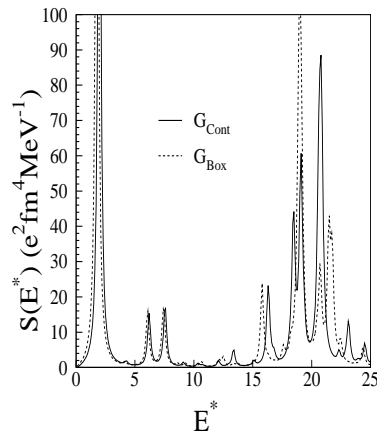


Fig. 1: Isoscalar Quadrupole strength function calculated in continuum QRPA (solid line) and with a box discretization (dashed line) for the  $^{22}\text{O}$  nucleus

As one can see from Fig.1, the quadrupole response is sensitive to the way the continuum is treated, especially in the giant quadrupole resonance (GQR) region.

### 3.2 Dipole response

The dipole response of  $^{18-24}\text{O}$  isotopes is shown in Fig. 2. In the left side of this figure we can see an enhancement of the low-lying strength of the giant dipole resonance (GDR) with the increase of the neutron number. On the right panel, the comparisons with measured strength [9] below 15 MeV is displayed. The predicted strength is in good agreement with the data for  $^{18,20}\text{O}$ , but overestimated in the case of  $^{22}\text{O}$ . For  $^{24}\text{O}$  the QRPA predicts a very strong increase of the strength, which is at variance with the shell-model calculations [10]. The measurement of the low-lying strength of the GDR in  $^{24}\text{O}$  is therefore relevant.

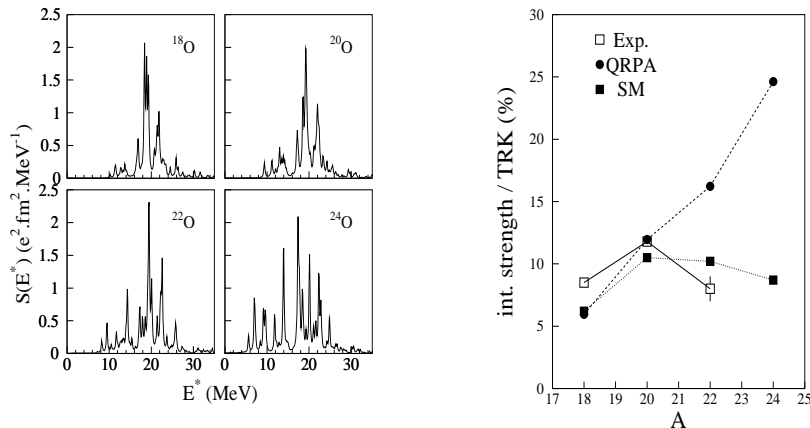


Fig. 2: Left : Isovector dipole strength function calculated in continuum QRPA  
Right : integrated energy weighted strength below 15 MeV

### 3.3 Two-neutron transfer response

The QRPA allows also to calculate two-particle transfer processes since 2 qp excitations contain two-particle and two-hole components. As an example, in Fig. 3 we display the monopole strength function for two neutrons transfer from  $^{22}\text{O}$  to  $^{24}\text{O}$ . The strength  $G_{cont}$  shown in Fig.3 is calculated by using in Eq.(8) an averaging factor  $\eta= 1$  MeV for excitations energies above 11 MeV. The lowest peak in Fig.3 corresponds to the transition towards the ground state of the  $^{24}\text{O}$  nucleus. The collective nature of this state makes it sensitive

to the way the continuum is treated. The strength of the state located just below 10 MeV is overestimated by 10% in the box type calculations. The calculations also predict a high energy mode. At present we are investigating if this mode is related or not to the giant pairing vibration [11].

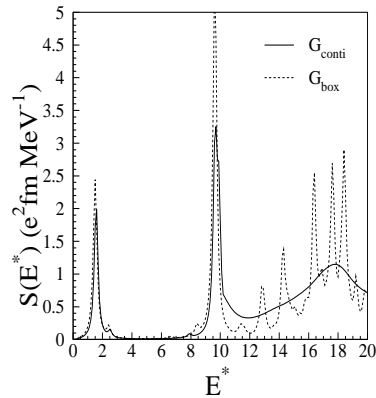


Fig. 3:  $^{22}\text{O}$  pairing response ( $L=0$ ) for the two-neutron addition mode.

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