Isospin symmetry in medium-light nuclei

Silvia M. Lenzi

Dipartimento di Fisica dell'Università and INFN
Via F. Marzolo 8, I-35131 Padova, Italy

Abstract

Recent experimental data on high spin states in the mirror nuclei ^{50}Fe and ^{50}Cr are presented. Through the differences in excitation energies along the rotational bands, the mechanism of the backbending is explained in terms of the successive alignment of pairs of protons and neutrons in the shell model framework. The systematic study of energy differences between analogue states in mirror nuclei and isobaric analogue triplets in the middle of the $f_{7/2}$ shell indicates that these measurements greatly expand our understanding of the interplay between the Coulomb potential and the isospin breaking nuclear interaction.

1 Introduction

Isospin symmetry is a consequence of the (approximate) charge invariance of nucleon-nucleon forces. Though the symmetry is already broken, to some extent, at the level of strong interaction and—to a much larger extent—by electromagnetic forces, the isospin formalism remains a very powerful tool to relate the properties of corresponding levels in different nuclei, from which complementary information can be derived on the structure of the nuclear wave function. The most important component of the symmetry breaking interaction, i.e., the Coulomb force, is certainly the best known part of the Hamiltonian, and its effects can be calculated as a perturbation series if the structure of unperturbed (charge symmetric) states is assumed to be known.

During the last decade, the latest generation of large γ-ray spectrometers and the development of new experimental techniques have allowed for the first time the exploration of the isospin symmetry in nuclei above ^{40}Ca at high rotational frequency. In a typical fusion-evaporation reaction aimed at the spectroscopy of N$\sim$Z nuclei, tens of channels are opened and those of interest in these studies account for a very small fraction of the total cross section. For this purpose, very efficient and dedicated ancillary devices are needed mainly when weak reaction channels—neutron deficient channels—shall be studied. Parallel to these experimental progress, high performance shell model codes have been developed which give an excellent description of the rotational bands found in nuclei in the middle of the $f_{7/2}$ shell [1, 2, 3].

There is a variety of effects related to isospin symmetry that can be studied in nuclei. Due to the Coulomb force (and other non charge-invariant effects) the isospin is not—strictly speaking—a good quantum number. Every nuclear state is expected to contain, in addition to the main component of isospin T, minor components of different isospin. The degree of isospin mixing in the wave function can be taken as an indicator of the size of the symmetry violation. In N=Z nuclei, the isospin mixing can be deduced from measuring forbidden E1 transitions. In recent experiments performed at Euroball, the degree of isospin mixing has been studied in ^{48}Cr and ^{64}Ge [4]. In these pages I will skip the description of these results, even if they were presented at the Conference, and will concentrate on the second subject which is related to the isospin symmetry in isobaric multiplets at high spin, studied by means of the Coulomb energy differences (CED). In Sect. 2 it will be shown that the mechanism of the backbending in rotating nuclei can be understood when looking at CED of mirror pairs (we call them mirror energy differences (MED)). Moreover, when extending these studies to energy differences in isobaric T=1 triplets (TED), it turns out that the isospin non-conserving part of the nuclear interaction is of the same magnitude of the Coulomb contribution (Sect. 3). Conclusions and perspectives close the presentation in Sect. 4.

2 Backbending in A=50 mirror nuclei

Coulomb energy differences between *excited* analogue states in mirror nuclei are a very delicate probe of nuclear structure. CED amount to *tens of keV*, while huge Coulomb effects (*tens of MeV*) are seen in the displacement energies between the ground state energy of mirror nuclei. Given that large Coulomb effects have been eliminated in CED and that the wave functions remain essentially unperturbed, isospin non-conserving effects can be treated in first order perturbation theory. Recently, data for T=1/2 mirror nuclei and T=1 isobaric multiplets at high spin in the $f_{7/2}$ shell have become available [5, 6, 7, 8, 9]. One can then calculate the difference in excitation energy (E_I) in mirror nuclei ($MED_I = E_I(Z > N) - E_I(Z < N)$); and the energy differences among the isobaric analogue members of the triplets ($TED_I = E_I(Z > N) + E_I(Z < N) - 2E_I(Z = N)$) at every spin I along the rotational bands.

From MED data, the alignment of pairs of nucleons at the backbending region can be inferred [10]. The role of the Coulomb force is clear: repulsion is weakest for aligned protons, because the overlap of wave functions is smallest. Thus, when a pair of identical nucleons is broken due to the alignment, we expect a jump in MED at backbending, whose sign will depend on which fluid (neutrons or protons) aligns first. These studies were so far limited to odd-mass (T = 1/2) nuclei and their extension to even-mass nuclei (T = 1) became feasible only very recently.

An interesting case is the A=50 mirror pair. ^{50}Cr, one of the best rotors in the $f_{7/2}$ shell, presents a rotational band showing a backbending at I=10^+. The yrast band together with the excited 10_2^+ and 12_2^+ states are shown in Fig. 1 in comparison with full *pf* shell model calculations, which give a very good description of the data [2, 3].

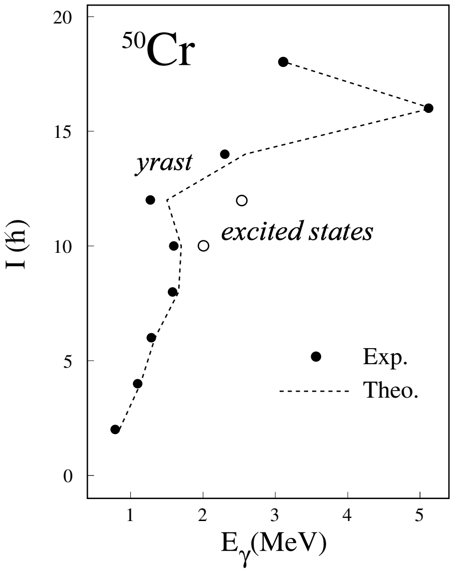

Fig. 1: Backbending plot of ^{50}Cr. Full circles correspond to the yrast states, open circles to the excited 10_2^+ and 12_2^+ states. The dashed-line corresponds to full *pf* shell model calculations.

Different interpretations have been suggested for the backbending mechanism in this nucleus in terms of band crossing with an oblate band, a high-k band, etc.. To apply the above method for studying of the mechanism of backbending in ^{50}Cr, an experiment was performed at Euroball to look for high spin states in the N=Z-2 mirror nucleus ^{50}Fe, in which no γ-rays were known so far. The Euroball γ-array was coupled to the charged particle detector ISIS and the Neutron-wall. ^{50}Fe was produced in the reaction ^{28}Si+ ^{28}Si at 110 MeV bombarding energy, after the evaporation of one α-particle and two neutrons [6].

In Fig. 2a) the rotational band observed in ^{50}Fe is displayed and compared with its mirror ^{50}Cr (in the case of ^{50}Cr only the analogues to the states observed in ^{50}Fe are shown). An estimated ratio of the cross sections for the present experiment is $\sigma(^{50}\mathrm{Fe})/\sigma(^{50}\mathrm{Cr})= 10^{-4}$ [6].

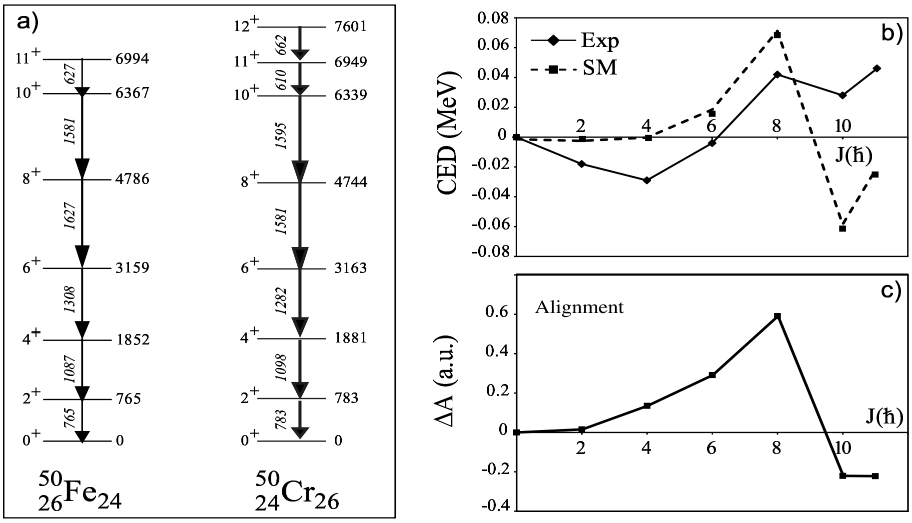

Fig. 2: a) Rotational bands in ^{50}Fe and ^{50}Cr; b) experimental and theoretical (SM) mirror energy differences (MED); c) Difference of expectation values for the operator which counts the number of aligned pairs (see text for details)

The experimental and shell model MED for the A=50 mirrors are reported in Fig. 2b). A rapid increase of the MED is observed at I=8 followed by a decrease at I=10. The trend is reproduced but grossly emphasised by shell model calculations in the full *pf* shell (dashed line) performed with the KB3 interaction. Coulomb matrix elements are calculated in the harmonic oscillator basis except for those involving only the $f_{7/2}$ orbits where empirical Coulomb matrix elements were used. The alignment process can be deduced by comparing Fig. 2b) and c) where the difference in the expectation value of the operator $A = [(a^+a^+)^{J=6}(aa)^{J=6}]^0$ which "counts" the number of maximally aligned pairs in the $f_{7/2}$ shell is shown. In this plot, the difference is between the number of aligned protons minus the number of aligned neutrons in ^{50}Cr (the opposite happens in ^{50}Fe). The similarity of these curves clearly shows the connection between backbending and alignment. One can thus deduce that a pair of protons in ^{50}Cr (neutrons in ^{50}Fe) aligns first, followed by the alignment of a pair of neutrons (protons) at I=10 [6].

3 Evidence of the isospin non-conserving terms in the nuclear interaction

Large shell model calculations for nuclei in the middle of the $f_{7/2}$ shell give a very good description of the observed level energies and electromagnetic properties. When coming to MED, the description is not very accurate as results from Fig. 2b). To improve the quantitative description of MED data, in Ref. [6] it was pointed out that one has to consider the effects associated to the monopole (m) part of the Coulomb field $V_{Cm} = 3e^2 Z(Z-1)/5R_\pi$, ($R_\pi$ is the proton radius). V_{Cm} produces large bulk and displacement energies that cancel approximately in the MED. A small, but significant contribution remains, due to changes in yrast states radii. In the pf shell they originate in differences between the $1f$ and $2p$ orbits. The latter have a larger radial extension, which leads to the Thomas-Ehrmann shift in ^{41}Sc where the excitation energy of the $p_{3/2}$ level is 200 keV below its analogue in ^{41}Ca. The deformed (low spin) yrast states have large $p_{3/2}$ admixtures, and hence larger radius and smaller Coulomb energies than their aligned counterparts. Within the shell model, orbital occupancies are translated into changes in radii. Taking into account this effect it was shown in Ref. [6] that the shell model description of the observed data in Fig. 2b) significantly improves, mainly in the low spin regime.

To further improve the theoretical description, the need of a renormalisation of the multipole (M) component of the Coulomb interaction was suggested in Ref. [6]. (This is the part of the Coulomb interaction related to the matrix elements in the valence space.) This prescription gave a quantitative good agreement for the MED not only in the case of A=50 but also for all the T=1/2 mirror pairs measured in the $f_{7/2}$ shell [11]. The physical meaning of the renormalisation remained, however, obscure: The large differences between the empirical Coulomb matrix elements and those calculated in the harmonic oscillator basis were difficult to explain. The conundrum was cleared when data for the T_z=0 members of the T=1 isobaric multiplets became available [7, 9]. The good fits obtained with the proposed renormalisation of the multipole Coulomb interaction for the MED between mirror pairs could not be obtained for the TED in the isobaric triplets. It was clear then that the isospin non-conserving nuclear interaction had to be considered [12]. The key point was the analysis of the MED and TED in the A=42, T=1 isobaric triplet data.

In the triplet ^{42}Ti, ^{42}Ca, ^{42}Sc, one can assume that the observed states are essentially $f_{7/2}^2$ configurations on top of the ^{40}Ca core. (Even if this is a strong approximation, more exact considerations do not affect the following results.)

The observed spectra thus defines an interaction in the $f_{7/2}$ subshell.

Table I. Coulomb (V_C), MED-V_C and TED-V_C energies (kEV) in $A = 42$. V_C is calculated in the harmonic oscillator basis.

	$J = 0$	$J = 2$	$J = 4$	$J = 6$
$V_C \equiv V_{C\mathbf{f_{7/2}}}^{ho}$	81.60	24.60	6.40	-11.40
$E_J[^{42}\mathrm{Ti} - {}^{42}\mathrm{Ca}] - V_C$	5.38	92.55	4.57	-47.95
$E_J[^{42}\mathrm{Ti} + {}^{42}\mathrm{Ca} - 2\,{}^{42}\mathrm{Sc}] - V_C$	116.76	80.76	2.83	-42.15

In Table I, the different contributions for A=42 are reported. For clarity their centroids —$V = \sum_J(2J+1)V^J/\sum_J(2J+1)$—have been subtracted. In the first line, the bare Coulomb terms V$_C$ are calculated in the harmonic oscillator basis in the $f_{7/2}$ space. Subtracting the bare V$_C$ from the observed data, the corresponding MED-V$_C$ and TED-V$_C$ terms are obtained (second and third lines). If the effects in MED and TED are only due to the Coulomb contribution one would expect the same and small numbers in the latter two lines. The corresponding numbers are, however, neither equal nor small. To explain these features, the charge independence breaking [13] of the nuclear interaction has to be invoked. Moreover, one can say that the $A = 42$ data indicate that the role of isospin non-conserving nuclear forces is at least as important as that of the Coulomb potential in the observed MED and TED [12].

We can now extend this analysis to the data in heavier, rotational nuclei in the $f_{7/2}$ shell. For this purpose we use the exact (isospin-conserving) shell model wave functions obtained with the KB3G interaction and single particle energies of ^{41}Ca. We then express the theoretical MED and TED in terms of the expectation values of the monopole (V_{Cm}) and multipole (V_{CM}) Coulomb and nuclear isospin symmetry breaking (V_B) contributions:

$$\begin{aligned} MED_I^{theo} &= \; < V_{Cm} >_I + < V_{CM} >_I + < V_B >_I \qquad (1) \\ TED_I^{theo} &= \; < V_{CM} >_I + < V_B >_I, \end{aligned}$$

The monopole Coulomb contribution (V$_{Cm}$) that takes into account the drift in radii is obtained from the relative occupation numbers of the $p_{3/2}$ orbit in the shell model wave functions. The strength of this term is obtained from the data in the mirrors A=41. Note that this contribution cancels in TED. The matrix elements for V$_{CM}$ are calculated in the harmonic oscillator basis for the whole pf shell. Finally, the nuclear isospin breaking interaction V$_B$ is deduced from the A=42 data in Table I (see Ref. [12] for further details). It is important to note that this is a parameter-free calculations, no adjustment of parameters is performed for the different nuclei. The results for the three

T=1/2 mirror pairs A=47, 49 and 51 and for the T=1, A=50 case are reported in Fig. 3, where the single contributions are shown. It can be seen that all three contributions are essential to get the correct description of the data.

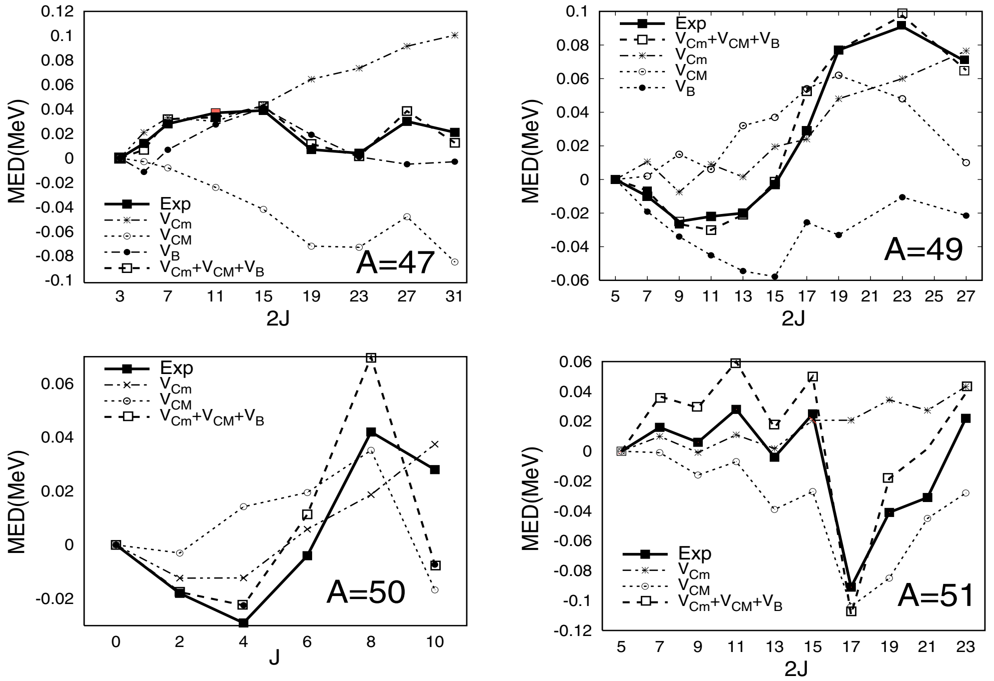

Fig. 3: Experimental [5, 6, 8] and theoretical [12] MED for the pairs ^{47}Cr-^{47}V, ^{49}Mn-^{49}Cr, ^{50}Fe-^{50}Cr and ^{51}Fe-^{51}Mn

In Fig. 4, the experimental TED for the T=1 A=50 and A=46 isobaric analogue triplets are reported in comparison with the present theoretical results. The agreement is very good for the A=50 triplet. For A=46, shell model calculations do not give a very precise description of the spectroscopic data and, consequently, TED are not very well reproduced.

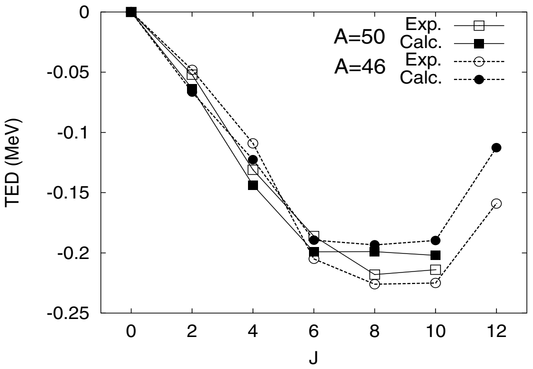

Fig. 4: Experimental [6, 7, 9] and theoretical [12] TED for A=46 and A=50

4 Conclusions and perspectives

In this contribution I have focused on one particular subject related to the isospin symmetry. Many other interesting studies are being performed in this nuclear field both from the theoretical and experimental sides. In the present work, by comparing the excitation energies in mirror nuclei, we have unveiled the mechanism of the backbending in even-even rotating nuclei. Moreover, they give us information on the evolution of radii along the yrast bands and provide a direct evidence for charge symmetry breaking of the nuclear field. In this respect, it should be noted that direct evidence for charge symmetry breaking has been confined, so far, to very light systems.

A very exciting physics, but we are at the beginning. With high intensity stable beams and radioactive proton rich beams we will be able in the near future to extend these studies to analogue bands up to the limit of nuclear stability and to heavier nuclei where the effects of isospin symmetry breaking should be stronger.

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