The relativistic quasi-particle random phase approximation and applications in exotic nuclei

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Abstract

Collective Vibrations of nuclei far from the valley of stability are investigated within the quasi-particle random phase approximation based on relativistic Hartree-Bogoliubov theory with non-linear meson couplings. Pairing correlations are described in by a finite range effective particle-particle interaction of Gogny type. The quasi-particle random phase equations are solved in the canonical basis and collective strength distributions are discussed.

1 Introduction

In recent years the investigation of nuclei far from stability has gained worldwide interest as well on the experimental as on the theoretical side. These nuclei are characterized by unique structure properties: the weak binding of the outermost nucleons and the effects of the coupling between bound states and the particle continuum. On the neutron rich side, in particular, the modification of the effective nuclear potential leads to the formation of nuclei with very diffuse neutron densities, to the occurrence of the neutron skin and halo structures. These phenomena will also affect collective vibrations of unstable nuclei, in particular the electric dipole and quadrupole excitations, and new modes of excitations might arise in nuclei near the drip line.

A quantitative description of ground-states and properties of excited states in nuclei characterized by the closeness of the Fermi surface to the particle continuum, necessitates a unified description of mean-field and pairing correlations, as for example in the framework of the Hartree-Fock-Bogoliubov (HFB) theory. In order to describe transitions to low-lying excited states in weakly bound nuclei, in particular, the two-quasiparticle configuration space must include states with both nucleons in the discrete bound levels, states with one nucleon in a bound level and one nucleon in the continuum, and also states with both nucleons in the continuum. This cannot be accomplished in the framework of the BCS approximation, since the BCS scheme does not provide a correct description of the scattering of nucleonic pairs from bound states to the positive energy particle continuum. Collective low-lying excited states in weakly bound nuclei are best described by the quasiparticle random phase approximation (QRPA) based on the HFB framework.

In this talk we concentrate on the relativistic QRPA in the canonical single-nucleon basis of the relativistic Hartree-Bogoliubov (RHB) model. The RHB model is based on the relativistic mean-field theory and on the Hartree-Fock-Bogoliubov framework. It has been very successfully applied in the description of a variety of nuclear structure phenomena, not only in nuclei along the valley of β-stability, but also in exotic nuclei with extreme isospin values and close to the particle drip lines.

The RRPA with nonlinear meson interaction terms, and with a configuration space that includes the Dirac sea of negative-energy state, has been very successfully employed in studies of nuclear compressional modes [1, 2, 3], of multipole giant resonances and of low-lying collective states in spherical nuclei [4], of the evolution of the low-lying isovector dipole response in nuclei with a large neutron excess [5, 6], and of toroidal dipole resonances [7].

2 The relativistic quasiparticle random phase approximation

The equations of motion of relativistic mean field theory can be derived starting from a density functional $E_{RMF}[\hat{\rho}, \phi_m]$, which depends on the density matrix $\hat{\rho}$ and the meson fields $\phi_m = \sigma$, ω, ρ and A. From the classical time-dependent variational principle

$$\delta \int_{t_1}^{t_2} dt \left\{ \langle \Phi | i\partial_t | \Phi \rangle - E_{RMF}\left[\hat{\rho}, \phi_m\right] \right\} = 0 \tag{1}$$

one derives the equations of motion for the Dirac spinors and for the meson fields. The equation of motion for the density matrix reads

$$i\partial_t \hat{\rho} = \left[\hat{h}(\hat{\rho}, \phi_m), \hat{\rho} \right] \ .$$

The single particle Hamiltonian $\hat{h}$ is the functional derivative of the energy E_{RMF} with respect to the single particle density matrix $\hat{\rho}$.

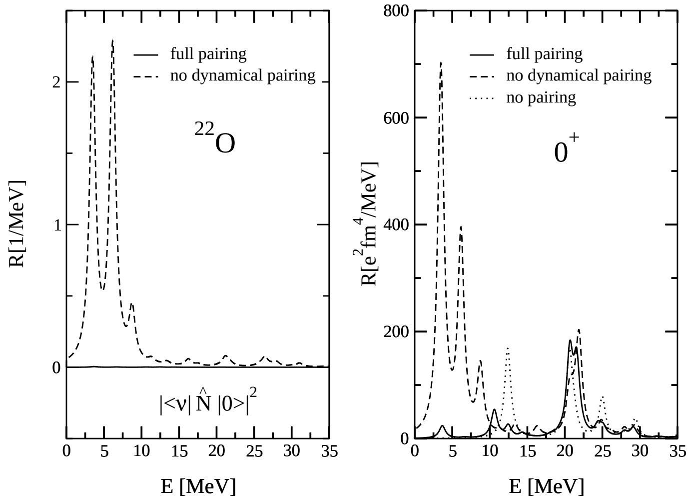

Figure 1: The strength function for the neutron number operator (left), and the isoscalar strength function for the monopole operator (right) in ^{22}O. The curves correspond to the RMF+RRPA calculation without pairing (dotted), with pairing correlations included in the RHB calculation of the ground state, but not in the RRPA residual interaction (dashed), and to the fully self-consistent RHB+RQRPA calculation (solid).

Pairing correlations can be easily included in the framework of density functional theory, by using a generalized Slater determinant $|\Phi\rangle$ of the Hartree-Bogoliubov type. It is characterized by the single particle density matrix $\hat{\rho}$ and pairing tensor $\hat{\kappa}$. The energy functional depends not only on the density matrix $\hat{\rho}$ and the meson fields ϕ_m, but in addition also on the pairing tensor. It has the form

$$E[\hat{\rho}, \hat{\kappa}, \phi_m] = E_{RMF}[\hat{\rho}, \phi_m] + E_{pair}[\hat{\kappa}], \tag{2}$$

The pairing energy $E_{pair}[\hat{\kappa}]$ is given by

$$E_{pair}[\hat{\kappa}] = \frac{1}{4}\mathrm{Tr}\left[\hat{\kappa}^* V^{pp} \hat{\kappa}\right]. \tag{3}$$

V^{pp} is a general two-body pairing interaction. In the framework of Relaivistic Hartree Bogoliubov Theory (RHB) we use the finite range Gogny interaction in the pairing channel . Finally, the total energy can be written as a functional of the generalized density matrix $\mathcal{R}$ which obeys the equation of motion

$$i\partial_t \mathcal{R} = \left[\mathcal{H}(\mathcal{R}), \mathcal{R}\right]. \tag{4}$$

The generalized Hamiltonian $\mathcal{H}$ is a functional derivative of the energy with respect to the generalized density $\mathcal{R}$.

By eliminating the meson degrees of freedom the generalized Hamiltonian $\mathcal{H}$ can be expressed as a functional of the generalized density $\mathcal{R}$ only. In the linear approximation the generalized density matrix is expanded

$$\mathcal{R} = \mathcal{R}_0 + \delta\mathcal{R}(t), \tag{5}$$

where $\mathcal{R}_0$ is the stationary ground-state generalized density. The linearized equation of motion (4) reduces to the relativistic quasi-particle RPA- (RQRPA) equations is

$$\left(\begin{array}{cc} A & B \\ -B^* & -A^* \end{array} \right) \left(\begin{array}{c} X^\nu \\ Y^\nu \end{array} \right) = \omega_\nu \left(\begin{array}{c} X^\nu \\ Y^\nu \end{array} \right). \tag{6}$$

the RQRPA matrix elements A and B are given as second derivatives of the energy functional with respect to the generalized densities $\mathcal{R}$. We solve these equations in the canonical basis, the basis where the single particle density matrix ρ is diagonal.

3 Applications in the Oxygen-region

In order to illustrate the RHB+RQRPA approach and to test the numerical implementation of the RQRPA equations we discuss the isoscalar monopole, isovector dipole and isoscalar quadrupole response of ^{22}O.

In the left panel of Fig. 1 we display the monopole strength function of the neutron number operator in ^{22}O. There should be no response to the number operator since it is a conserved quantity, i.e. the Nambu-Goldstone mode associated with the nucleon number conservation should have zero excitation energy. The dashed curve (no dynamical pairing) represents the strength function obtained when the pairing interaction is included only in the RHB calculation of the ground state, but not in the residual interaction of the RQRPA. The solid line (zero response) corresponds to the full RHB+RQRPA calculation, with the pairing interaction included both in the RHB ground state, and in the RQRPA residual interaction.

The isoscalar strength functions of the monopole operator $\sum_{i=1}^{A} r_i^2$ in ^{22}O, shown in the right panel of Fig. 1, correspond to three different calculations: a) the RMF+RRPA calculation without pairing, b) pairing correlations are included in the RHB calculation of the ground state, but not in the RQRPA residual interaction (no dynamical pairing), and c) the fully self-consistent RHB+RQRPA calculation. Just as in the case of the number operator, by including pairing correlations only in the RHB ground state a strong spurious response is generated below 10 MeV. The Nambu-Goldstone mode is found

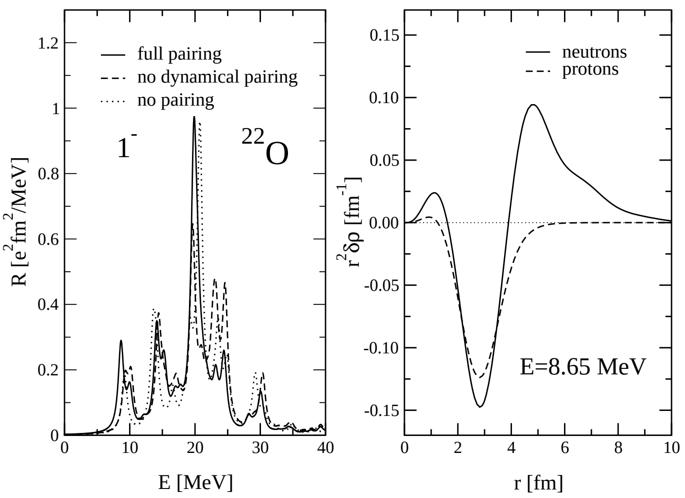

Figure 2: The isovector strength function of the dipole operator in ^{22}O (left). The fully self-consistent RHB+RQRPA response (solid line) is compared with the RMF+RRPA calculation without pairing (dotted line), and with the RHB+RRPA calculation that includes pairing correlations only in the ground state (dashed line). The proton and neutron transition densities for the peak at $E = 8.65$ MeV are shown in the right panel.

at zero excitation energy (in this particular calculation it was located below 0.2 MeV) only when pairing correlations are consistently included also in the residual RQRPA interaction. When the result of the full RHB+RQRPA is compared with the response calculated without pairing, one notices that, as expected, pairing correlations have relatively little influence on the response in the region of giant resonances above 20 MeV. A more pronounced effect is found at lower energies. The fragmentation of the single peak at ≈ 12.5 MeV reflects the broadening of the Fermi surface by the pairing correlations.

The isovector strength function ($J^{\pi} = 1^{-}$) of the dipole operator for ^{22}O is displayed in the left panel of Fig. 2. In this example we also compare the results of the RMF+RRPA calculations without pairing, with pairing correlations included only in the RHB ground state (no dynamical pairing), and with the fully self-consistent RHB+RQRPA response. A large configuration space enables the separation of the zero-energy mode that corresponds to the spurious center of mass motion. In the present calculation for ^{22}O this mode is found at $E = 0.04$ MeV.

The strength functions shown in Fig. 2 illustrate the importance of including pairing correlations in the calculation of the isovector dipole response. Pairing is, of course, particularly important for the low-lying strength below

10 MeV. The inclusion of pairing correlations in the full RHB+RQRPA calculation enhances the low-energy dipole strength near the threshold. For the main peak in the low-energy region (≈ 8.65 MeV), in the right panel of Fig. 2 we display the proton and neutron transition densities. In contrast to the well known radial dependence of the IVGDR transition densities (proton and neutron densities oscillate with opposite phases, the amplitude of the isovector transition density is much larger than that of the isoscalar component), the proton and neutron transition densities for the main low-energy peak are in phase in the nuclear interior, there is no contribution from the protons in the surface region, the isoscalar transition density dominates over the isovector one in the interior, and the strong neutron transition density displays a long tail in the radial coordinate.

Summarizing, the relativistic QRPA formulated in the canonical basis of the RHB model represents a significant contribution to the theoretical tools that can be employed in the description of the multipole response of unstable weakly bound nuclei far from stability.

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