Electromagnetic transitions between giant resonances within a continuum-RPA approach

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Abstract

A general continuum-RPA approach is developed to describe electromagnetic transitions between giant resonances. Using a diagrammatic representation for the three-point Green's function, an expression for the transition amplitude is derived which allows one to incorporate effects of mixing of single and double giant resonances as well as to take the entire basis of particle-hole states into consideration. The radiative widths for E1 transition between the charge-exchange spin-dipole giant resonance and Gamow-Teller states are calculated for ^{90}Nb and ^{208}Bi.

1 Introduction

In recent years significant theoretical efforts have been devoted to describe intensities of the radiative transitions between charge-exchange giant resonances (GR) in nuclei [1]-[2]. The interest in the problem is related to the possibility to obtain corresponding experimental data from the (^{3}He,$t\gamma$)-reaction cross sections. A confrontation of the data with the calculation results would be especially interesting in view of the theoretical speculations on the possible enhancement of the transition intensities [1].

It should be mentioned that conclusions on the enhancement have been based upon the consideration of the relevant sum rules. However, the main contribution to the sum rules seems to come from the nuclear states of 2 particle - 2 hole type (double GR). For instance, the dipole sum rule for the Gamow-Teller resonance (GTR) should be mainly exhausted by the giant dipole resonance (GDR) built on the top of the GTR, i.e. a double GR, but not by the spin-dipole resonance (SDR), a single GR. In this connection the problem of a correct description for the mixing of single and double GR arises, because it might change significantly the transition intensities.

In the present work a general expression for the amplitude of the radiative transitions between different GRs is obtained for closed-shell nuclei within the continuum-RPA using a diagrammatic representation for the three-point Green's function. The expression allows us to take the entire basis of particle-hole states into consideration as well as to incorporate the important effects of mixing of single and double GR. We apply the general approach developed in this paper to calculate the intensity of E1 transition between the SDR and GT states for ^{90}Nb and ^{208}Bi nuclei.

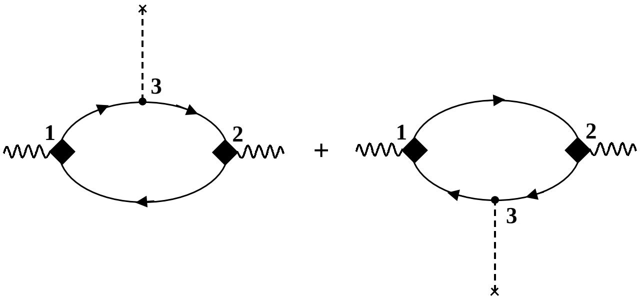
2 The amplitude of electromagnetic transitions between giant resonances

The partial width corresponding to a radiative transition between initial $|1\rangle$ and final $|2\rangle$ nuclear states with the excitation energies E_1 and E_2, respectively, is:

$$\Gamma_\gamma^{(1\to 2)} = C_\gamma \sum_{m_2 \mu} \left| \langle 2, m_2 | \hat{\mathcal{Q}}_{k\mu} | 1, m_1 \rangle \right|^2 = C_\gamma \frac{\left| \langle 2 \| \hat{\mathcal{Q}}_k \| 1 \rangle \right|^2}{2J_2 + 1}. \qquad (1)$$

Here, $\hat{\mathcal{Q}}_{k\mu} = \sum_a Q_{k\mu}(x_a)$ is a single-particle multipole operator, $\langle 2 \| \hat{\mathcal{Q}}_k \| 1 \rangle$ is the reduced matrix element, x is the nucleon coordinate, m is the angular momentum projection, C_γ is an appropriate dimensional coefficient. In the case of E1 transitions, considered in the present paper to apply the general approach (see Sect. 3), $Q_{1\mu} = D_\mu = -\frac{1}{2} e\, r\, Y_{1\mu}(\vec{n})\, \tau^{(3)}$ is the isovector electric dipole operator and $C_\gamma = \frac{16\pi}{9} \left(\frac{E_\gamma}{\hbar c} \right)^3$ with $E_\gamma = E_1 - E_2$ being the gamma-quantum energy.

Let $|1\rangle$ and $|2\rangle$ be states of the particle-hole (p-h) type so that their structure can be described within the RPA. Expression for the transition amplitude $\langle 2 \| \hat{\mathcal{Q}}_k \| 1 \rangle$ can be obtained in Green function technique used in the theory of finite Fermi-systems [4] to describe the Fermi-system response to a single-particle probing operator. Let us consider the diagrams for the 3-point Green function (3-point vertex function) describing the transition amplitude of the system under the action of the external field $\hat{\mathcal{Q}}$:

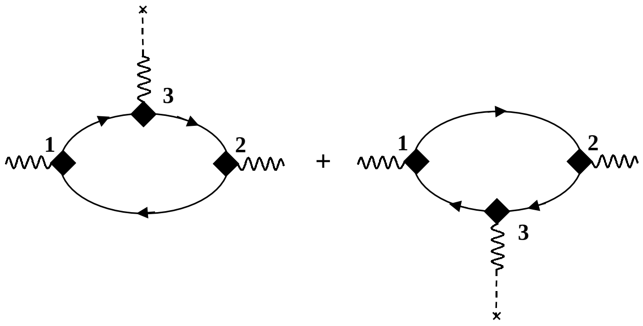


The p-h structure of the state 1 (2) enters these graphs by means of the transition fields $v_{1(2)}(x) = \int F(xx')\rho_{1(2)}(x')\,dx'$ with $\rho_{1(2)}(x)$ and $F(xx')$ being the transition densities and the particle-hole interaction (depicted by the the wavy lines and full diamonds, respectively; see Ref. [4] for the details). The solid line presents the nucleon, the dashed one the single-particle external field $Q_k(x)$. The matrix element corresponding to the graphs is:

$$
\begin{aligned}
\langle 2\|\hat{\mathcal{Q}}_k\|1\rangle = {} & (-)^{J_1-k}\sum_{345}\begin{Bmatrix} J_2 & k & J_1\\ j_3 & j_4 & j_5\end{Bmatrix}\Big[(-)^{J_1+J_2+k}\langle 4\|v_2^+\|5\rangle\langle 5\|Q_k\|3\rangle\langle 3\|v_1\|4\rangle \\
& \times\left(\frac{n_4(1-n_5)(1-n_3)}{(\varepsilon_4+E_2-\varepsilon_5)(\varepsilon_4+E_1-\varepsilon_3)}-\frac{(1-n_4)n_5n_3}{(\varepsilon_4+E_2-\varepsilon_5)(\varepsilon_4+E_1-\varepsilon_3)}\right) \\
& -\langle 4\|v_1\|3\rangle\langle 3\|Q_k\|5\rangle\langle 5\|v_2^+\|4\rangle\left(\frac{(1-n_4)n_5n_3}{(\varepsilon_4-E_2-\varepsilon_5)(\varepsilon_4-E_1-\varepsilon_3)}-\frac{n_4(1-n_5)(1-n_3)}{(\varepsilon_4-E_2-\varepsilon_5)(\varepsilon_4-E_1-\varepsilon_3)}\right)\Big],
\end{aligned}
\tag{2}
$$

with n_λ and ε_λ being occupation numbers and energies of the single-particle states, respectively (λ is the set of the single-particle quantum numbers, $(\lambda) = \{lj\}$). The definition of the reduced single-particle matrix elements $\langle\lambda\|Q_k\|\mu\rangle$ by the Wigner-Eckart theorem is taken in accordance with Ref. [5].

One sees in Fig. 1 that there is an asymmetry between the vertex 1,2 (”dressed”) on the one hand and vertex 3 (”undressed”) on the other. This is because we have not incorporated effects of particle-hole interaction in channel 3, or, in other words, we neglected the effect of the virtual excitation of the corresponding giant multipole resonance. This additional contribution which takes into account virtual excitation of the giant resonance can be represented by the following diagrams:



The wavy line in the channel 3 depictures the total p-h propagator $\tilde{A}$ describing off-shell excitation of all p-h states (including the corresponding GR) with appropriate spin and parity. Using the fact that $\tilde{A}Q = A\tilde{Q}$ [4], one can see that the sum of the graphs of Figs.1-2 is equivalent to the graphs of Fig.1 only if one replaces the external field $Q_k(x)$ by the corresponding effective field $\tilde{Q}_k(x, \omega)$. Then Eq.(2) with such a substitution can be transformed further with the use of the spectral decomposition for the Green's function of the single-particle radial Schrödinger equation to include explicitly the single-particle continuum (see Ref. [7] for the final expression) and to get the general expression for the amplitude of the electromagnetic transitions within the CRPA containing also contribution from the double phonon configurations.

3 E1 transitions between the spin-dipole and Gamow-Teller resonances.

As an application of the approach described in the previous section, we calculate intensities and branching ratios for E1 decay of the charge-exchange spin-dipole resonance to some Gamow-Teller states (GTR and low-lying satellites) identified in Ref. [8]. To calculate the GT and SD strength distribution, we use the CRPA approach of Refs. [2, 10]. The approach has allowed one to reproduce very well the experimental GTR energies in ^{90}Nb and ^{208}Bi nuclei. The details of the calculation of the effective fields for the SD and GT cases can be found in Refs. [2, 10], and for the isovector dipole one - in Ref. [9].

We calculate the radiative widths Γ_γ and Γ_γ^0 (with and without taking into account the virtual GDR excitation, respectively). The calculated widths for the E1 decay of the above-mentioned SD states to the GT states at the excitation energy E_{GT} are listed in Table 1. The corresponding branching ratios are calculated as the ratio $\Gamma_\gamma/\Gamma^\downarrow$ with $\Gamma^\downarrow$ being the mean doorway-state spreading width chosen to reproduce the experimental total SDR width in the strength-function calculations ($\Gamma^{\downarrow} = 4.7$ MeV for ^{208}Bi [10], $\Gamma^{\downarrow} = 4.$ MeV for ^{90}Nb).

Table 1: Calculated radiative widths and branching ratios for the E1 transitions between SDR and GT states in ^{90}Nb and ^{208}Bi nuclei without $(\Gamma^0_\gamma, b^0_\gamma)$ and with $(\Gamma_\gamma, b_\gamma)$ taking into account the virtual GDR excitation, respectively.

Nucleus	E_{GT}, MeV $/J^\pi_{SDR}$	Γ^0_γ, keV (b^0_γ, $\times 10^{-4}$)			Γ_γ, keV (b_γ, $\times 10^{-4}$)		
		0^-	1^-	2^-	0^-	1^-	2^-
^{90}Nb	8.9	3.40 (8.5)	1.03 (2.6)	0.10 (0.3)	3.68 (9.2)	0.02 (0.05)	0.008 (0.02)
	0.0	0.21 (0.5)	0.10 (0.2)	0.08 (0.2)	1.62 (4.0)	0.13 (0.3)	0.37 (0.9)
^{208}Pb	15.5	2.28 (4.9)	0.70 (1.5)	0.09 (0.2)	6.49 (13.8)	0.03 (0.05)	0.008 (0.02)
	9.3	0.15 (0.3)	0.03 (0.05)	0.14 (0.3)	0.80 (1.7)	0.43 (0.9)	0.22 (0.5)
	8.1	0.24 (0.5)	0.02 (0.03)	0.04 (0.08)	1.05 (2.3)	0.26 (0.5)	0.16 (0.3)
	4.9	1.05 (2.2)	0.0002 (0.0005)	0.002 (0.003)	2.36 (5.0)	0.003 (0.007)	0.01 (0.03)
	3.8	0.58 (1.2)	0.004 (0.01)	0.006 (0.01)	1.19 (2.5)	0.02 (0.03)	0.02 (0.04)

As can be seen from the calculation results listed in the Table 1, taking into account of the virtual GDR excitation affects the results drastically. In the cases where the transition energy is well below the GDR energy, a destructive interference between the direct and semidirect parts of the amplitude always occurs. In the cases when the transition energy is close to the GDR energy or exceeds it, the opposite situation takes place and one can see a significant enhancement of the corresponding amplitudes. This circumstance along with the steep E^3_γ dependence of the E1 radiative width leads to the fact that the widths for SDR decay to some low-lying GT states can be comparable to those for SDR → GTR E1 transitions. Some of the calculated branching ratios are of order of 10^{-3}, that seem to be accessible in experiments.

4 Conclusions and Acknowledgments

A general continuum-RPA approach has been developed in the present paper to describe the intensities of electromagnetic transitions between different giant resonances. We have used a diagrammatic representation for the three-point Green's function to derive the expression for the transition amplitude. The expression has allowed us to take the entire basis of particle-hole states into consideration as well as to incorporate important effects of mixing of single and double giant resonances. The radiative widths for E1 transition between the SDR and GT states (including GTR) have been calculated for ^{90}Nb and ^{208}Bi nuclei. Some of the widths seem to be large enough to be accessible in experiments.

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