Renormalization Group Methods for the Nuclear Many-Body Problem

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Abstract

The application of renormalization group (RG) methods to microscopic nuclear many-body calculations is discussed. We present the solution of the RG equations in the particle-hole channels for neutron matter and the application to S-wave pairing. Furthermore, we point out that the inclusion of tensor and spin-orbit forces leads to spin non-conserving effective interactions in nuclear matter.

The solution to the many-body problem for systems of strongly interacting particles, such as finite nuclei and nuclear matter, can often be facilitated, by making a judicious use of the separation of low- and high-energy scales. One introduces a truncated Hilbert space (model space), where the particles are restricted to low energies. These are the so-called "slow" modes. The operators and the degrees of freedom in the truncated space must be renormalized to account for intermediate excitations to states outside the model space, the "fast" modes. The operators of interest include the effective interaction and various transition operators, e.g., the axial current. This procedure defines an effective theory, which is equivalent to the full theory at low energies. The RG method provides a systematic way to compute the effective operators of particles in the truncated space.

There are several advantages of working in a truncated space. By integrating out the high-energy modes, the strong short-range repulsion of realistic nucleon-nucleon forces is tamed, and the resulting effective interaction is model independent, if it acts only at energies that are constrained by the scattering data [1, 2, 3, 4]. Due to the separation of scales, it can usually be achieved that the effective operators in the model space are energy-independent. For finite nuclei, the model space concept has been used for many years in the derivation of effective shell model interactions, where the truncated space contains the low-lying shells near the Fermi energy.

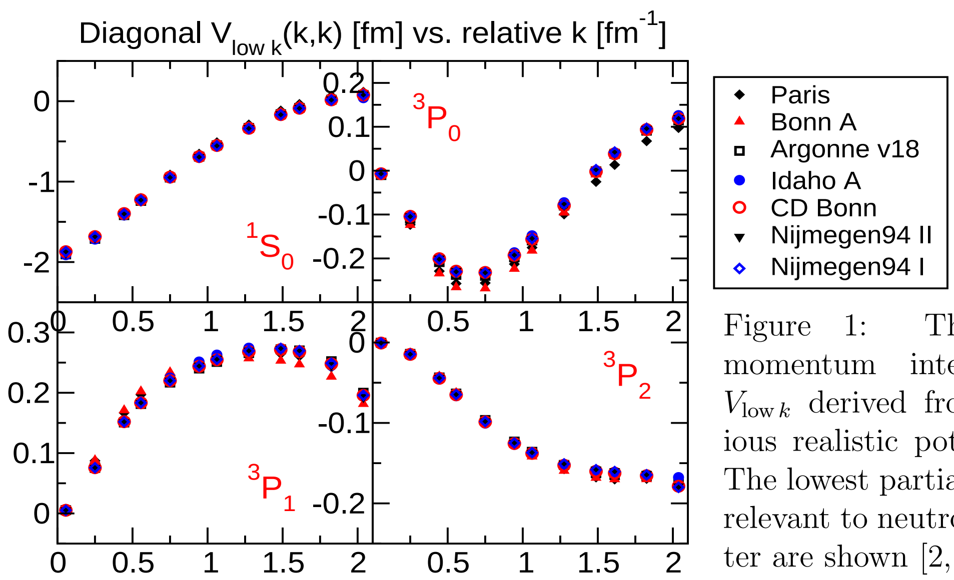

Figure 1: The low momentum interaction $V_{\mathrm{low}\,k}$ derived from various realistic potentials. The lowest partial waves relevant to neutron matter are shown [2, 3, 4].

In this first application of RG methods to effective nuclear interactions, we consider infinite neutron matter. The model space consists of the single particle states near the Fermi surface. In neutron matter, the effects of tensor as well as three-body forces are reduced compared to symmetric nuclear matter, since they do not act in the S-wave. Therefore, as a first approximation we neglect these forces in this calculation. In Section 4, we briefly discuss some novel consequences of tensor and spin-orbit forces in a nuclear medium.

1 Two-body low momentum interaction

As input to the many-body calculation we take the two-body low momentum potential $V_{\mathrm{low}\,k}$ [1, 2, 3, 4], which is derived from realistic nucleon-nucleon interaction models by integrating out the high momentum modes in the sense of the RG. Here, we briefly summarize the properties of $V_{\mathrm{low}\,k}$ and refer to the original papers and Ref. [5] for details. The RG decimation to low momenta is constructed such that $V_{\mathrm{low}\,k}$ is energy-independent and reproduces the half-on-shell T matrix for momenta below a cutoff Λ. For $k', k < \Lambda$, we thus have

$$T(k',k;k^2) = V_{\mathrm{low}\,k}(k',k) + \frac{2}{\pi}\,\mathcal{P}\int_0^\Lambda \frac{V_{\mathrm{low}\,k}(k',p)\;T(p,k;k^2)}{k^2-p^2}\,p^2dp. \qquad (1)$$

Consequently, $V_{\mathrm{low}\,k}$ is phase shift equivalent to the original nucleon-nucleon interaction. For values of the cutoff, $\Lambda \sim 2.1\,\mathrm{fm}^{-1}$ (the scale up to which the potential models are fitted to empirical phase shifts), one finds the same low momentum potential for the various realistic nucleon-nucleon interactions, i.e., the resulting potential is model independent, as demonstrated in Fig. 1.

In $V_{\mathrm{low}\,k}$, the model-dependent high momentum (short distance) modes have been integrated out. This procedure tames the strong short-range repulsion in the original interaction. Thus, the low momentum interaction $V_{\mathrm{low}\,k}$ can, in contrast to the bare interaction, be used directly in many-body calculations. Furthermore, since the states above the cutoff Λ are included in $V_{\mathrm{low}\,k}$ and those below are not, the cutoff acts much like the Pauli-blocking operator in the Brueckner G matrix [6]. By analyzing the angle-averaged Pauli blocking operator, one finds an effective cutoff slightly larger than the Fermi momentum. Here we employ a density-dependent cutoff for $V_{\mathrm{low}\,k}$, $\Lambda_{V_{\mathrm{low}\,k}} = \sqrt{2}\,k_{\mathrm{F}}$. Due to the separation of short distance and long distance scales, $V_{\mathrm{low}\,k}$ is almost independent of the cutoff in the $T = 1$ channel [2] for $0.7\ \mathrm{fm}^{-1} \lesssim \Lambda \lesssim 2.1\ \mathrm{fm}^{-1}$. Thus, the exact value of the cutoff is not crucial for pure neutron matter.

2 Renormalization group approach to Fermi liquids

We now discuss the RG approach to the many-fermion problem. As originally proposed by Shankar [7], one separates the slow modes from the fast ones in a many-fermion system by introducing a momentum cutoff relative to the Fermi momentum. At zero temperature, the phase space for fast modes is then characterized by the occupation factors

$$n_{\mathbf{p}}(\Lambda) = \Theta(k_{\mathrm{F}} - \Lambda - |\mathbf{p}|) \quad \text{and} \quad 1 - n_{\mathbf{p}}(\Lambda) = \Theta(|\mathbf{p}| - (k_{\mathrm{F}} + \Lambda)). \tag{2}$$

The slow modes that make up the model space are then located in the complementary shell of thickness 2Λ around the Fermi surface. Starting at a large cutoff ($\Lambda = k_{\mathrm{F}}$), where the effective interaction in the model space is assumed to be given by the free-space low momentum interaction $V_{\mathrm{low}\,k}$, it is successively renormalized by including the contributions of fast intermediate states lying in a thin shell between $\Lambda - \delta\Lambda$ and Λ. For $\delta\Lambda \to 0$, one obtains a differential (RG) equation for the renormalization of the effective interaction as a function of the cutoff. As Λ is decreased, more and more modes are shifted from the model space into the effective interaction, in such a way that the low energy scattering amplitude remains invariant. At the same time the single-particle degrees of freedom are gradually converted into quasiparticles. As $\Lambda \to 0$, the effective interaction equals the scattering amplitude for quasiparticles on the Fermi surface, since all modes have been integrated out. To one-loop order, the RG equation in the particle-hole channels, which play a special role in Fermi

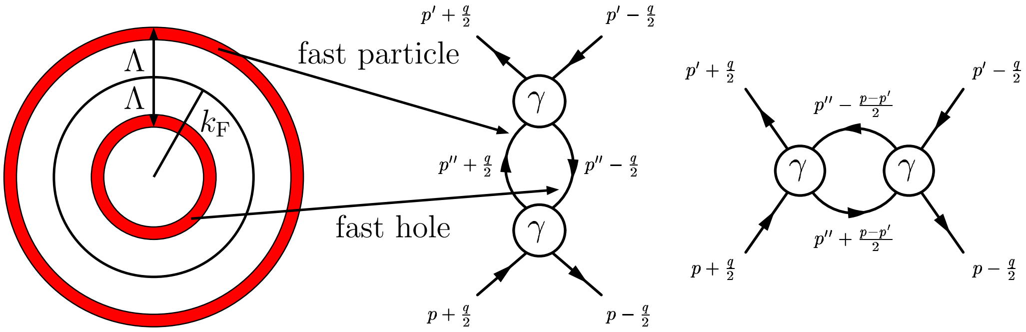

Figure 2: The one-loop contributions to the RG equation, where $\gamma(\Lambda)$ denotes the running four-point vertex. For particles on the Fermi surface, the four-point vertex at zero energy transfer ω is given by $a(\Lambda) = \gamma(\omega = 0, \Lambda)$. On the left, the momentum shells which are integrated out at every step Λ are shown.

liquid theory, is at zero temperature given by [6]

$$\begin{aligned} \frac{d}{d\Lambda} a(\mathbf{q}, \mathbf{q}'; \Lambda) = z_{k_F}^2 \, \frac{d}{d\Lambda} \biggl\{ \, g \int \frac{d^3\mathbf{p}''}{(2\pi)^3} \, \frac{n_{\mathbf{p}''+\mathbf{q}/2}(\Lambda) - n_{\mathbf{p}''-\mathbf{q}/2}(\Lambda)}{\varepsilon_{\mathbf{p}''+\mathbf{q}/2} - \varepsilon_{\mathbf{p}''-\mathbf{q}/2}} \biggr\} \\ \times \, a\bigl(\mathbf{q}, \frac{\mathbf{p}+\mathbf{p}'}{2} + \frac{\mathbf{q}'}{2} - \mathbf{p}''; \Lambda\bigr) \; a\bigl(\mathbf{q}, \mathbf{p}'' - \frac{\mathbf{p}+\mathbf{p}'}{2} + \frac{\mathbf{q}'}{2}; \Lambda\bigr) \\ - \text{exchange channel} \biggl\{ \mathbf{q} \leftrightarrow \mathbf{q}' \biggr\}. \end{aligned} \tag{3}$$

The contributions from the direct and the exchange channel are shown diagrammatically in Fig. 2. When only one channel is considered, the one-loop RG equation is exact in the sense that it is equivalent to the corresponding scattering equation. Within the RG method, the scattering amplitude remains antisymmetric at any-loop order when both particle-hole channels are included.

We choose a scattering geometry, where the external particles are restricted to the Fermi surface, $|\mathbf{p}^{(\prime)} \pm \mathbf{q}/2| = k_F$. Then, the scattering amplitude depends only on two angles, or equivalently the magnitude of the two momenta $|\mathbf{q}|$ and $|\mathbf{q}'| = |\mathbf{p} - \mathbf{p}'|$. The direct and exchange channels couple in the RG equation, Eq. (3). Microscopically the RG in the particle-hole channels includes particle-hole bubbles and ladders as well as vertex corrections. At this point we include only the high-momentum ladder diagrams summed in $V_{\text{low}\,k}$ in the particle-particle channel. Finally, the effective interaction of particles on the Fermi surface is obtained by setting $|\mathbf{q}| = 0$ in Eq. (3) [8]. The resulting effective interaction includes the so-called induced interaction.

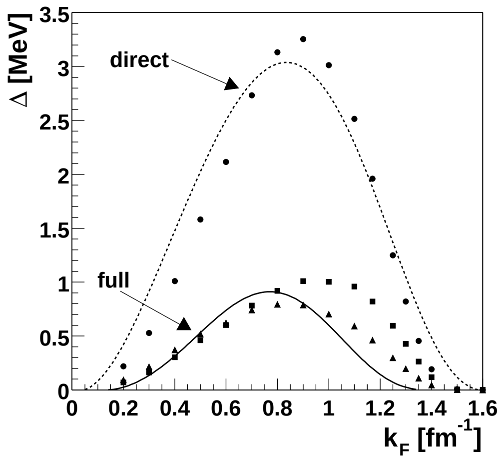

Figure 3: The $^1S_0$ superfluid gap versus the Fermi momentum k_F. The dots denote the gap obtained using $V_{\rm low\,k}$ only, whereas the squares and the triangles are computed with the full RG solution using different approximations for the quasiparticle strength [8]. The dashed line is obtained by solving the BCS equation with various bare interactions [9], while the solid line includes particle-hole polarization effects [10].

3 Neutron matter and S-wave pairing

We solve the RG equation, Eq. (3), for neutron matter (for details see Ref. [8]). The evolution of the effective mass as well as an approximate treatment of the renormalization of the quasiparticle strength is included in the flow. We note that the RG approach does not require a truncation in Landau l of the Fermi liquid parameters. The resulting Landau parameters are given in Ref. [8].

While the Fermi liquid parameters are defined in the forward scattering limit, the RG procedure yields the quasiparticle scattering amplitude also for finite scattering angles. The latter can be used to compute the S-wave superfluid gap. In weak coupling BCS theory,

$$\Delta = 2\,\varepsilon_F \exp\left(\frac{1}{\langle \mathcal{A} \rangle}\right), \qquad (4)$$

where $\langle \mathcal{A} \rangle$ denotes the angle averaged scattering amplitude on the Fermi surface, we find the pairing gap shown in Fig. 3. The S-wave gap is strongly suppressed, from 3.3 MeV to 0.8 MeV at maximum, due to particle-hole screening in the many-body medium. This agrees well with the results obtained in the polarization potential model by Wambach *et al.* [10].

At higher densities, neutrons form a P-wave ($^3P_2$–$^3F_2$) superfluid. For fermions interacting by means of a delta function force, the second order particle-hole contributions to the P-wave scattering amplitude are attractive for back-to-back configurations on the Fermi surface, and thus one expects the P-wave gap to increase. For a quantitative analysis of pairing in this channel, however, the effects of tensor and spin-orbit forces in the medium must be included.

4 Tensor and spin-orbit forces in the nuclear medium

Here we briefly discuss the noncentral parts of the effective two-nucleon interaction. In a nuclear medium, new spin-dependent interactions are induced by the presence of other particles nearby. The polarization of the Fermi sea leads to contributions, which depend on the two-body cm momentum $\mathbf{P}$ in the rest frame of the many-body system. In the long-wavelength Landau limit, two of these new operators survive, a modified spin-orbit term $(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \cdot (\mathbf{q}' \times \mathbf{P})$, which does not conserve two-particle spin, and a tensor term $S_{12}(\mathbf{P})$. Furthermore, relativistic corrections related to the transformation from the two-body cm frame to the rest frame of the many-body system also contribute to such nonstandard operators [11]. Neither of these effects are included in conventional many-body calculations, where the two-body interaction is treated in the independent pair approximation. A calculation of the effective tensor and spin-orbit interactions in nuclear matter is in progress [12].

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