

Short versus long range interactions and the size of two-body weakly bound objects

R.J. Lombard^{a)} and C. Volpe^{a),b)}

^{a)} *Groupe de Physique Théorique, Institut de Physique Nucléaire,
F-91406 Orsay Cedex, France*

^{b)} *Institut für Theoretische Physik der Universität,
Philosophenweg 19, D-69120 Heidelberg, Germany*

Abstract

Very weakly bound systems may manifest intriguing “universal” properties, independent of the specific interaction which keeps the system bound. An interesting example is given by relations between the size of the system and the separation energy, or scaling laws. So far, scaling laws have been investigated for short-range and long-range (repulsive) potentials. We report here on scaling laws for weakly bound two-body systems valid for a larger class of potentials, i.e. short-range potentials having a repulsive core and long-range attractive potentials. We emphasize analogies and differences between the short- and the long-range case. In particular, we show that the emergence of halos is a threshold phenomenon which can arise when the system is bound not only by short-range interactions but also by long-range ones, and this for any value of the orbital angular momentum ℓ ! These results enlarge the image of halo systems we are accustomed to.

1 Introduction

Weakly bound systems have attracted a lot of attention in different domain of physics. Halo nuclei have been studied for several years using various probes [1]. Diffuse Van-der-Waals dimers and trimers represent another interesting example, such as $({}^4\text{He})_2$ whose existence has been established only recently [2, 3, 4]. All these systems are characterized by very large spatial extensions on one hand and very small separation energies on the other hand. For example, the one-neutron halo ${}^{11}\text{Be}$ has a separation energy of about 0.5 MeV, the average separation energy being of 6 MeV, while the average distance between the halo neutron and the core is about 7 fm. The $({}^4\text{He})_2$ dimer is a paragon: its predicted dissociation energy is 0.1 μeV , whereas the ionization energy is of about 27 eV and the measured average distance between the two atoms is

of about 52 \AA [2, 5]. This is by far the largest dimer ever observed !

One of the intriguing features of very weakly bound systems is that they can reveal “universal” behaviors, i.e. independent of the specific interaction which keeps together the particles. An example is given by the relation between the size of the system, which can be characterized through different moments of the wave function, like the mean square radius or $\langle r^2 \rangle$, and the (separation or dissociation) energy E_S necessary to break the system, that is

$$\langle r^2 \rangle = f(E_S) \quad E_S \rightarrow 0. \quad (1)$$

This relation is often called (asymptotic) scaling law.

Several works exist on the topic of scaling laws for two- and three-body systems obtained with short-range and repulsive (mainly Coulomb) long-range interactions [6, 4, 7]. For the short-range case, so far only potentials defined by two parameters, the depth and the range, have been considered.

Recently these investigations have been widened by the study of a larger class of potentials [8], namely : *i*) short range potentials having a repulsive core to simulate the Pauli exclusion principle; *ii*) long-range attractive potentials. Here we report on this investigation and describe how scaling laws for short-range potentials are modified by the presence of the core. We present scaling laws for long-range interactions and emphasize analogies and differences between the short- and the long-range case.

Finally, we come to a question which has been extensively discussed in the literature [4, 6], namely : What is a halo system ? The image we are now accustomed to is the one in which the halo particle has a very large probability of being outside the classically allowed region. This image needs to be enlarged if one wants to include the case of long-range potentials, as we will discuss.

2 Theoretical Framework

Scaling laws for two-body systems (1) can be treated in the context of the two-body problem in quantum mechanics¹. A bound state of the system is then identified as a bound state of the potential $V(r)$ ² acting between the two atoms or the core and the halo nucleon in a nucleus. The separation energy of the system E_S is taken equal to the energy of the bound state E_ℓ .

¹In the case of halo nuclei, such a description has a validity as far as the degrees of freedom of the halo nucleons can be separated from those of the core.

²Here we will consider that the system has spherical symmetry so that the potential is a function of the relative distance between the two bodies only. A dependence on the spin degrees of freedom does not change our conclusions.

There are (at least) three possible ways to tackle this problem [8] :

- * Using very general arguments, without specifying the potential $V(r)$. These arguments are based, for example, on inequalities like the Bertlmann-Martin inequality, or on properties of the Schrödinger equation³ ($\hbar = 2m = 1$) :

$$\left[-\frac{\partial^2}{\partial r^2} - \frac{2}{r} \frac{\partial}{\partial r} + \frac{\ell(\ell+1)}{r^2} + \lambda V(r)\right]\psi_\ell(r) = E_\ell \psi_\ell(r), \quad (2)$$

such as its invariance under the transformation⁴ $x = r/R_0$ and $\epsilon = E_\ell R_0^2$, where R_0 characterizes the range of the potential.

- * Once the range of the potential is defined, in the limit of very small binding one can make the following approximations : *i*) for short-range potentials the wave function is mainly given by the tail outside the potential, i.e. by the Hankel wave function $\psi_\ell(r) \approx e^{-\mu r}/r^{\ell+1}$ with $\mu(E_\ell)$; *ii*) for long-range potentials the wave function, mainly inside the potential, can be taken as the spherical Bessel function of the first kind $\psi_\ell(r) \approx j_\ell(kr)$ with $k(E_\ell)$, cutting the integrals at the first zero.
- * Specific potentials can be used to get quantitative estimates.

3 What happens to Scaling Laws for Short-Range Potentials having a Repulsive Core ?

For the class of short-range potentials going to zero faster than $1/r^2$, the scaling laws are [4, 6] :

$$\langle r^2 \rangle_0 \approx \frac{c_0}{|E_0|}, \quad \langle r^2 \rangle_1 \approx \frac{c_1 R_0}{\sqrt{|E_1|}}, \quad \langle r^2 \rangle_2 \approx c_2 R_0^2, \quad (3)$$

for s- ($\ell = 0$), p- ($\ell = 1$) and d-states⁵ ($\ell = 2$). As we can see, the mean square radius diverges as $1/|E_0|$ and as $1/\sqrt{|E_1|}$ for s- and p-states respectively, but no divergence is present for states having $\ell \geq 2$. If we take the divergence of the second moment of the wave function as a reference for the appearance of

³Since we consider only the lowest states of each angular momentum (the wave function has no node), they are simply labeled by ℓ .

⁴This transformation is valid for potentials defined by two parameters only.

⁵There is no energy dependence in the scaling laws of states having $\ell > 2$ as well.

halos, we can say that halos may occur for s- and p-states, in the limit of very weak binding, but that such phenomenon cannot appear for states of higher ℓ .

The relations (3) have been obtained using the behavior of the tail of the wave functions [6]. This automatically implies that the constants c_ℓ are independent of the potential V . Numerical studies have also been performed in the past to check the sensitivity of $c_\ell[V]$ to the specific shape of the potentials; but so far potentials defined by their depth and their range (like the square well and the Gaussian potential) have been considered.

However, strictly speaking the scaling law is completely independent of the shape of the short-range potential for the s-state only, since $c_0 = 1/2$ [9]. Can physical systems, like halo nuclei or diffuse dimers, attain this region of very small binding, where the scaling law (3) become completely independent of the potential? This can be easily checked by looking if $\langle r^2 \rangle_0 |E_0| = 1/2$. We show results for $^{11}\text{Be}^6$ and the $(^4\text{He})_2$ dimer⁷, as examples (Fig.1). We can see that if nuclear halo systems are located in a region which is still sensitive to the specific choice of the potential, diffuse dimers attain the region of very weak binding [7, 8].

Concerning the other coefficients c_ℓ ($\ell > 0$), if one considers a larger class of short-range potentials, defined by more than two-parameters, like those including a repulsive core, the independence of (3) from the potential is no more strictly true [8]. To illustrate how c_ℓ depend on V , we compare the results obtained with a two parameters' potential and with a potential including a repulsive core (Fig.1).

We can see that the higher ℓ is, the stronger is the dependence of c_ℓ and therefore of the corresponding scaling law (3) on the shape of the potential. In fact, the role played by the centrifugal barrier is the larger, the higher is ℓ . As a consequence, the wave function is pushed more and more inside the potential and becomes sensitive to its specific shape.

4 What are Scaling Laws for Long-Range Potentials?

Using both inequalities and the behavior of the wave functions we have shown that the scaling laws for the class of long-range potentials going to zero slower than $1/r^2$ (such as the attractive Coulomb interaction, the confining potentials like the linear potential and the harmonic oscillator) are [8] :

$$\langle r^2 \rangle_\ell \approx \frac{c_\ell[V]}{|E_\ell|} \quad \forall \ell \quad E_\ell \rightarrow 0 . \quad (4)$$

⁶The same is true for the other halo nuclei [7].

⁷We use the $\langle r^2 \rangle$ from calculations in [3] which well reproduce the measured $\langle r \rangle$ of $(^4\text{He})_2$.

For the s-states, the scaling law (4) has the same dependence on $1/|E_0|$ as for short-range potentials. However, the coefficients c_0 here (as well as the other c_ℓ with $\ell > 0$) will always depend on the potential V because the wave function is mainly inside the potential and feels its specific shape. For example, $c_0 = 3$ for the Kratzer potential, the Coulomb attractive potential and the harmonic oscillator.

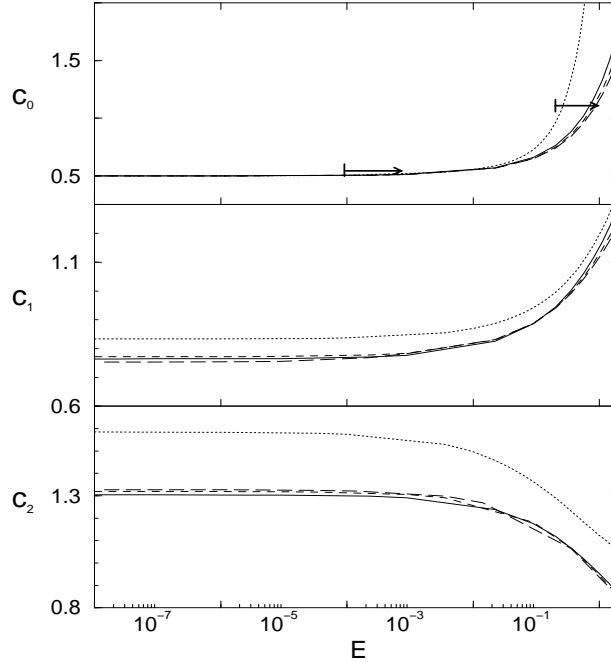


Fig. 1: Scaling laws (3) obtained with short-range potentials for s- (top), p- (middle) and d-states (bottom) [8] : The coefficients c_ℓ are shown as a function of the energy E_ℓ of the state. The full, the short-dashed and long-dashed lines correspond to a potential including a repulsive core with three different ranges R_0 . As an illustration the cut Kratzer potential is taken: $V_{cur}(r) = -2(\frac{a}{r} - \frac{a^2}{2r^2})\Theta(R_0 - r)$. For comparison the results obtained with a potential without a repulsive core (the square well) are shown (dotted line). For c_0 , the arrows indicate the range of the energies corresponding to halo nuclei (right) and to diffuse Van-der-Waals dimers (left).

For the states having $\ell > 0$, the mean square radius always diverges as $1/|E_\ell|$, contrary to the short-range case (3). Therefore systems kept together by a long-range interaction can develop very large extensions, and this for states of any angular momentum ℓ ! This difference with the known short-range case results can be intuitively understood. In fact, in the long-range case the centrifugal barrier is not playing a major role in confining the halo particle inside the potential.

An example of very extended systems bound by a long-range interaction is already known : Rydberg atoms. However, there is a major difference. Here we are predicting that there may exist very weakly bound two-body systems bound by long-range interactions having large spatial extensions in their lowest energy states.

5 What is a Halo System?

How to define a halo system has been the object of extensive discussions in the literature. Nowadays we think of halo systems as systems in which the halo particle has a very high probability of being outside the classical turning point [4, 6]. In the case of long-range potentials the particle is mainly in the classical allowed region. The appearance of halos can still be characterized in a common way for both the short- and the long-range cases : the $\langle r^2 \rangle_\ell^{1/2}$ has to be much larger than a typical physical length of the system (such as for example the size of the core).

These new results tell us that the appearance of halos is a threshold phenomenon, independent of the range, short or long, of the interaction and therefore also of the time spent by the halo particle inside or outside the classical region.

References

- [1] K. Riisager's Proceeding; J. Al-Khalili's Proceeding.
- [2] F. Luo, C.F. Giese and W.R. Gentry, J. Chem. Phys. **104** (1996) 1151; W. Schöllkopf and J.P. Toennies, idem p.1155 (and refs. therein).
- [3] U. Kleinekathöfer *et al.*, Phys. Rev. Lett. **83** (1999) 4717.
- [4] K. Riisager, D.V. Fedorov and A.S. Jensen, Europhys. Lett. **49** (2000) 547 (and refs. therein).
- [5] R.E. Grisenti, Phys. Rev. Lett. **85** (2000) 2284 (and refs. therein).
- [6] K. Riisager, A.S. Jensen and P. Möller, Nucl. Phys. **A 548** (1992) 393 ; D.V. Fedorov, A.S. Jensen and K. Riisager, Phys. Lett. **B 312** (1993) 1; K. Riisager, Rev. Mod. Phys. **66** (1994) 1105.
- [7] R.J. Lombard, Yad. Fiz. **64** (2001) 1315.
- [8] R.J. Lombard and C. Volpe, Phys. Rev. Lett. **88** (2002) 190402.
- [9] M. Lassaut, M. Keriaki and R.J. Lombard, J. Phys. A : Math. Gen **31** (1998) 1621.