Asymmetric nuclear matter with hyperons

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Astrophysics and Nuclear Structure

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Outline

Introduction

Infinite nuclear matter

- only nucleons, no nuclei
- no surface effects
- beta-stable ($e^- + p \rightarrow \nu_e + n$)
- charge neutral, spin-unpolarized matter

Nuclear matter (infinite) properties

- saturation density $\rho_0 = 0.16 \pm 0.02\ fm^{-3}$
- energy per nucleon $\mathcal{E} = \frac{E}{A} = -15.6 \pm 0.2\ MeV$
- incompressibility $K \approx 220 \pm 30\ MeV$
- asymmetry energy $a_t = \frac{1}{2} \frac{\partial^2 \mathcal{E}}{\partial \eta_t^2} \approx 30\ MeV$
  (η_t isospin-asymmetry parameter)

Introduction

Motivation

- astrophysics
- neutron stars, proto-neutron stars, dense nuclear systems
- central density of neutron starts in the range of 3-10 ρ_0
- neutrino trapping in proto-neutrons stars

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Hyperons

- rapid increase of the nucleon chemical potentials with density
- hyperons (Λ,Σ^-,Σ^0,Σ^+ and others) expected to appear

HF calculation of nuclear matter with hyperons

Energy of infinite nuclear matter

$$E = \langle M \rangle + \langle T \rangle + \langle V \rangle + \langle V_{3BF} \rangle$$

HF calculation of nuclear matter with hyperons

Energy of infinite nuclear matter

$$
\begin{aligned}
E &= \langle M \rangle + \langle T \rangle + \langle V \rangle + \langle V_{3BF} \rangle \\
&= \sum_{i=1}^{A} \langle \phi_i | M | \phi_i \rangle + \sum_{i=1}^{A} \langle \phi_i | T | \phi_i \rangle \\
&+ \frac{1}{2} \sum_{i,j=1}^{A} \left\langle \phi_i \phi_j \right| V \mathcal{A}_{12} \left| \phi_i \phi_j \right\rangle \\
&+ \frac{1}{6} \sum_{i,j,k=1}^{A} \left\langle \phi_i \phi_j \phi_k \right| V_{3BF} \mathcal{A}_{123} \left| \phi_i \phi_j \phi_k \right\rangle
\end{aligned}
$$

HF calculation of nuclear matter with hyperons

- ϕ_i plane wave states
- normalized to a box of volume Ω
- $\mathcal{A}_{12} = 1 - P_{12}$;
  $\mathcal{A}_{123} = 1 - P_{12} - P_{13} - P_{23} + P_{12}P_{23} + P_{13}P_{23}$;
  P-exchange operator

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Mass term and kinetic energy

$$\langle M \rangle = \langle M_\Lambda \rangle + \langle M_N \rangle + \langle M_\Sigma \rangle = \Omega \sum_{t_1 m_{t_1}} M_{t_1 m_{t_1}} \rho_{t_1 m_{t_1}}$$

$$\langle T \rangle = \langle T_\Lambda \rangle + \langle T_N \rangle + \langle T_\Sigma \rangle = \frac{\Omega}{2\pi^2} \sum_{t_1 m_{t_1}} \frac{p^5_{F_{t_1 m_{t_1}}}}{5 M_{t_1 m_{t_1}}}$$

HF calculation of nuclear matter with hyperons

Two-body potential energy

$$\langle V \rangle = \langle V_{\Lambda\Lambda} \rangle + 2 \langle V_{\Lambda N} \rangle + 2 \langle V_{\Lambda\Sigma} \rangle + \langle V_{NN} \rangle + 2 \langle V_{\Sigma N} \rangle + \langle V_{\Sigma\Sigma} \rangle$$

HF calculation of nuclear matter with hyperons

Two-body potential energy

$$\langle V \rangle = \langle V_{\Lambda\Lambda} \rangle + 2 \langle V_{\Lambda N} \rangle + 2 \langle V_{\Lambda\Sigma} \rangle + \langle V_{NN} \rangle + 2 \langle V_{\Sigma N} \rangle + \langle V_{\Sigma\Sigma} \rangle$$

- $V_{low\ k}$ used for NN and YN interaction;
  YY interaction is neglected
- transformation form plane wave basis to spherical wave basis
- couple single particle spins into total spin
- couple the total spin and angular momentum into total angular momentum

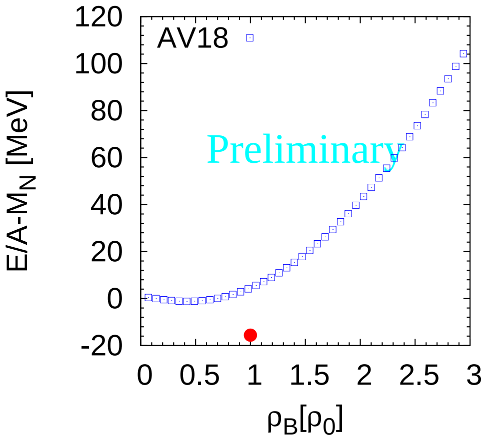
HF calculation of nuclear matter with hyperons

Three-body potential energy

$$\langle V_{3BF} \rangle = \langle V_{NNN} \rangle$$

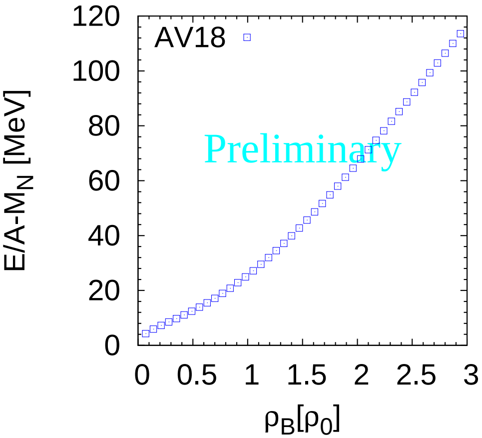
- only NNN interaction used
- all other YNN, YYN and YYY are neglected
- NNN interaction coming from chiral perturbation theory
  S.K.Bogner, A. Schwenk, R.J.Furnstal, A.Nogga,NPA 763, (2005), 59.
- interaction used directly in the operator form
- expectation value evaluated for asymmetric nuclear matter
- introduce relative coordinates and integrate for various combinations of nucleons

Results of symmetric nuclear matter



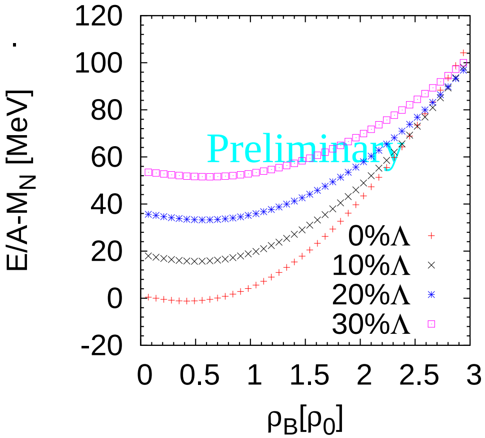
- symmetric nuclear matter with nucleons
- $E \sim \langle T \rangle + \langle V_{NN} \rangle + \langle V_{NNN} \rangle$
- saturation not reproduced correctly
- higher order contributions needed

Results of neutron nuclear matter



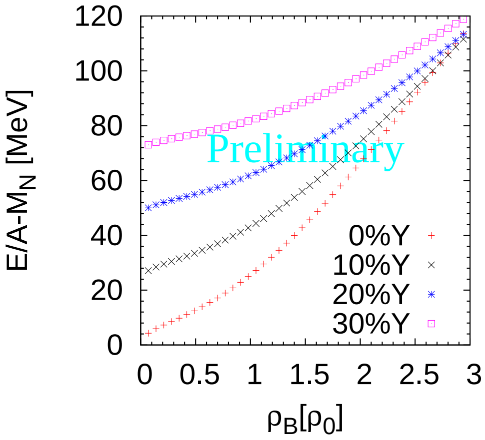
- neutron nuclear matter with nucleons
- $E \sim \langle T \rangle + \langle V_{NN} \rangle + \langle V_{NNN} \rangle$
- neutron nuclear matter means $\frac{\rho_n}{\rho_N} \sim 1$

Results of symmetric nuclear matter with Λ



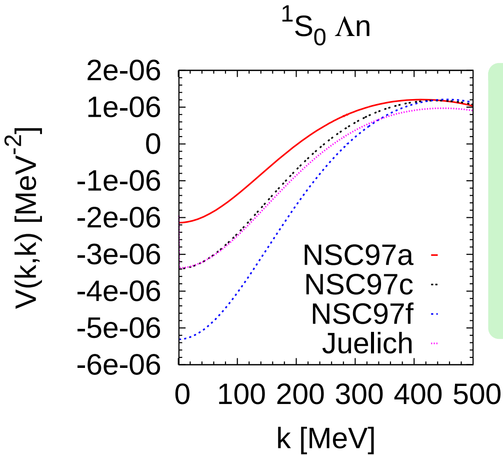
- symmetric nuclear matter with nucleons and the Λ particle
- $E \sim \langle T \rangle + \langle V_{NN} \rangle + \langle V_{\Lambda N} \rangle + \langle V_{NNN} \rangle$
- density of Σ is zero $\Rightarrow$ no contribution
- 0%Λ curve goes above 10%Λ curve for $\rho_B > 2.5\rho_0$

Results of neutron nuclear matter with hyperons



- equal densities of all hyperons
- $E \sim \langle T \rangle + \langle V_{NN} \rangle + \langle V_{\Lambda N} \rangle + \langle V_{\Sigma N} \rangle + \langle V_{NNN} \rangle$
- more hyperon $\Rightarrow$ softer equation of state

Model comparison



- YN $V_{low\ k}$ constructed from Nijmegen a-f and Juelich potentials
- relatively different YN $V_{low\ k}$ potential produce similar energy per particle results

Model comparison

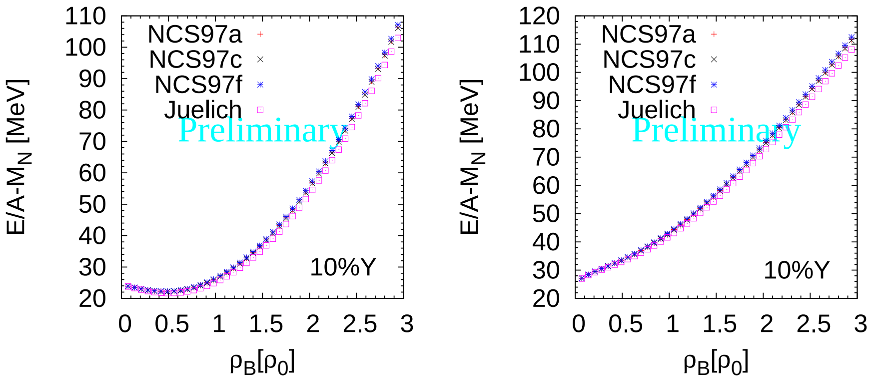


Figure: Symmetric nuclear matter(left) and neutron nuclear matter(right) with 10%Y

Summary and Outlook

Summary

- microscopic studies of nuclear matter with hyperons can be done by combining $V_{low\ k}$ for NN,$V_{low\ k}$ for YN and chiral perturbation V_{NNN}
- hyperons can have a profound effect on the EOS
- different models of YN interaction will result in similar EOS

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- microscopic studies of nuclear matter with hyperons can be done by combining $V_{low\ k}$ for NN,$V_{low\ k}$ for YN and chiral perturbation V_{NNN}
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Outlook

- calculate a EOS of beta-stable nuclear matter
- include higher order corrections and temperature effects
- calculate neutrino in medium cross-sections, etc.

Extra:Two-body potential energy

$$\langle V \rangle = \frac{\Omega}{2(2\pi)^3} \frac{2}{\pi} \sum_{t_1 m_{t_1}} \sum_{t_2 m_{t_2}} \int d^3\vec{p}_1 \int d^3\vec{p}_2 \sum_{SM_S} \sum_{LM_L} \sum_{J} \left(\begin{array}{cc|c} L & S & J \\ M_L & M_S & M_L + M_S \end{array} \right)^2 Y_{LM_L}(\hat{\vec{p}}) Y^*_{LM_L}(\hat{\vec{p}}) \left[V^{direct}_{(LS)Jt_1 m_{t_1} t_2 m_{t_2}}(p) + (-1)^{L+S} V^{exchange}_{(LS)Jt_1 m_{t_1} t_2 m_{t_2}}(p) \right]$$

Extra:Three-body potential energy

$$\frac{\langle V_c \rangle}{A} = \frac{2}{3(2\pi)^7}\left(\frac{g_A}{f_\pi}\right)^2 \frac{1}{f_\pi^2}\frac{1}{\rho_B}$$
$$[F_c(p,p,p) + F_c(p,p,n) + F_c(p,n,p) + F_c(n,p,p)$$
$$+ \; F_c(p,n,n) + F_c(n,p,n) + F_c(n,n,p) + F_c(n,n,n)]$$

Extra:Three-body potential energy

$$
\begin{aligned}
F_c(p,p,p) &= \int P^2 dP \int k^2 dk \int q^2 dq \int d(\cos(\theta_k)) \int d(\cos(\theta_q)) \int d\phi \\
& \left\{ -2c_1 m_\pi^2 \left[\left(\frac{\vec{p}_{31}\cdot\vec{p}_{12}}{(p_{31}^2 m_\pi)(p_{12}^2 m_\pi)} + \frac{\vec{p}_{31}\cdot\vec{p}_{23}}{(p_{31}^2 m_\pi)(p_{23}^2 m_\pi)} + \frac{\vec{p}_{12}\cdot\vec{p}_{23}}{(p_{12}^2 m_\pi)(p_{23}^2 m_\pi)} \right) \right.\right. \\
+ & \left.\left(\frac{\vec{p}_{21}^2}{(p_{21}^2 m_\pi)^2} + \frac{\vec{p}_{31}^2}{(p_{31}^2 m_\pi)^2} + \frac{\vec{p}_{32}^2}{(p_{32}^2 m_\pi)^2} \right) \right] \\
+ & c_3 \left[\left(\frac{\vec{p}_{31}^2 \vec{p}_{12}^2}{(p_{31}^2 m_\pi)(p_{12}^2 m_\pi)} + \frac{\vec{p}_{31}^2 \vec{p}_{23}^2}{(p_{31}^2 m_\pi)(p_{23}^2 m_\pi)} + \frac{\vec{p}_{12}^2 \vec{p}_{23}^2}{(p_{12}^2 m_\pi)(p_{23}^2 m_\pi)} \right) \right. \\
- & \left.\left.\left(\frac{\vec{p}_{21}^4}{(p_{21}^2 m_\pi)^2} + \frac{\vec{p}_{31}^4}{(p_{31}^2 m_\pi)^2} + \frac{\vec{p}_{32}^4}{(p_{32}^2 m_\pi)^2} \right) \right] \right\} f_R^2(p, 2q/3)
\end{aligned}
$$