# Indirect Methods for Nuclear Astrophysics

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**Astrophysics and Nuclear Structure**

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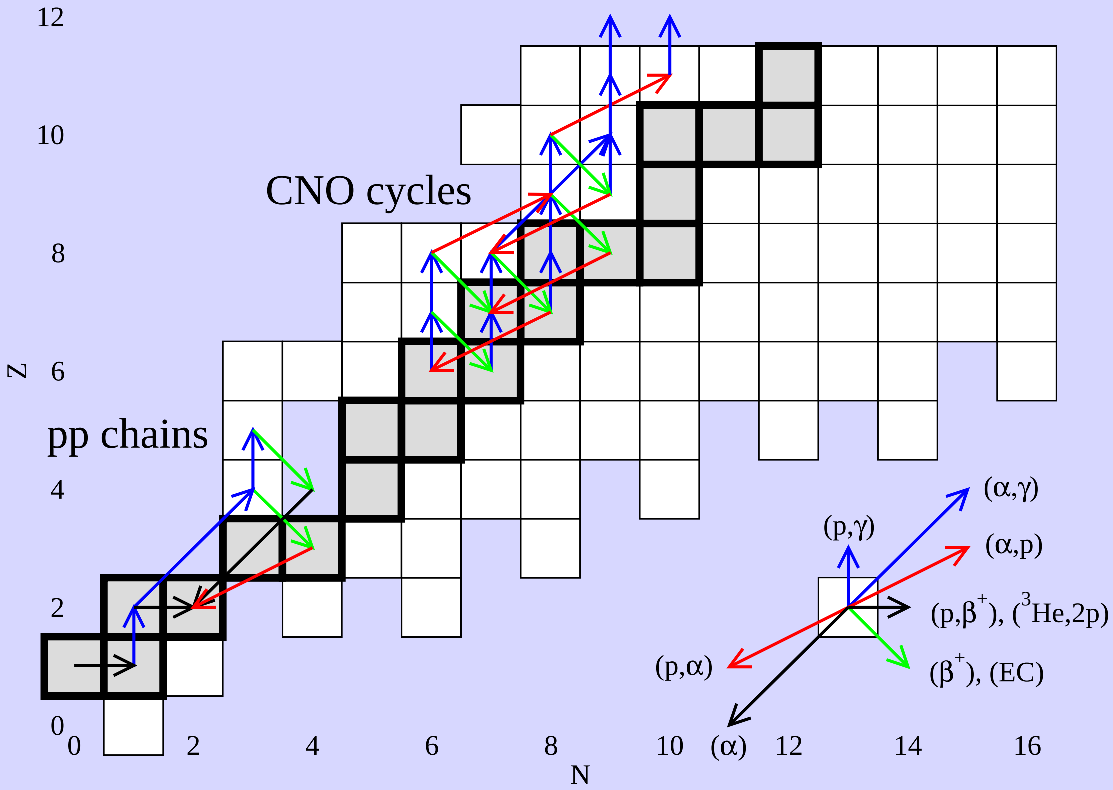
on Gross Properties of Nuclei and Nuclear Excitations

# Outline

- **Motivation**
  nuclear reactions of astrophysical interest,
  direct experiments
- **Indirect Methods**
  - general characteristics, theory
  - Coulomb Dissociation
  - ANC method
  - Trojan-horse method
- **Conclusions**

# Nuclear Reactions of Astrophysical Interest




- radiative capture/dissociation reactions with charged particles: $(p, \gamma)$, $(\alpha, \gamma)$, . . .
- direct nuclear reactions: $(p, \alpha)$, $(\alpha, p)$, . . .
- weak interaction reactions: $\beta^+$, $\beta^-$, EC

- **nuclear astrophysics**

nuclear reaction rates are basic input in many astrophysical models (primordial nucleosynthesis, stellar evolution, novae, supernovae, . . . ) for various processes (pp chains, CNO cycles, s, r, p, rp, . . . )

- **hydrostatic burning**

low temperatures of environment, cross sections of reactions with light, charged particles needed at small relative energies, dominated by non-resonant and only few resonant contributions

# Direct Experiments

**direct measurement of cross sections preferable, but . . .**

- Coulomb barrier for charged-particle reactions
  - ⇒ extremely small cross sections $\sigma(E)$ with strong energy dependence
  - ⇒ astrophysical relevant energies (Gamov peak) usually not accesible
  - ⇒ measurement at higher energies and extrapolation to low energies $E$

    with help of astrophysical S factor $S(E) = \sigma(E) E \exp\left(\sqrt{E_G/E}\right)$
  - ⇒ danger of extrapolation error, missed resonances, bound state tails
- often unstable nuclei involved
- suppression of background needed
- very demanding experiments, cf. talks by K.E. Rehm, F. Strieder
- electron screening in laboratory and in stellar plasma
  - ⇒ enhancement of low-energy cross sections, correction needed, theoretically well understood? cf. talk by M. Aliotta

**alternative: indirect methods**

# Indirect Methods I

## Coulomb dissociation

- study inverse of radiative capture reaction
  $b(x,\gamma)a \ \Leftrightarrow \ a(\gamma,x)b$
- use Coulomb field of target nucleus $A$ as source of photons
  $a(\gamma,x)b \ \Leftrightarrow \ A(a,bx)A$

$\Downarrow$

absolute S factors
as a function of energy

## ANC method

- extract asymptotic normalization coefficient of ground state wave function of nucleus $a$ from transfer reactions
- calculate matrix elements for radiative capture reaction $b(x,\gamma)a$

$\Downarrow$

S factor at zero energy

## Trojan-horse method

- study three-body reaction
  $A + a \rightarrow C + c + b$
  with Trojan horse
  $a = b + x$
  and spectator $b$
- extract cross section of two-body reaction
  $A + x \rightarrow C + c$

$\Downarrow$

energy dependence
of S factor

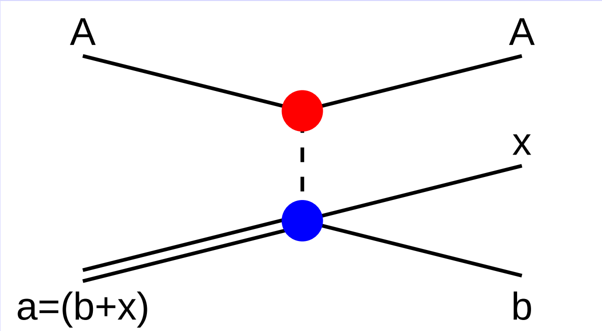
Relation of methods? Problems?

# Indirect Methods II

**general characteristics:**

- two-body reaction at low-energy is replaced by three-body reaction at "high-energy"
  - Coulomb dissociation $b(x,\gamma)a \Rightarrow A(a,bx)A$
  - ANC method $b(x,\gamma)a \Rightarrow A(a,B)b$
  - Trojan-horse method $A(x,c)C \Rightarrow A(a,Cc)b$
- transfer of virtual particle (photon $\gamma$ or nucleus $x$)
- relation of cross sections is found with the help of nuclear direct reaction theory
- theoretical approximations essential $\Rightarrow$ factorization of T-matrix elements
- study of peripheral reactions
  - asymptotics of wave functions relevant
  - selection of kinematical conditions

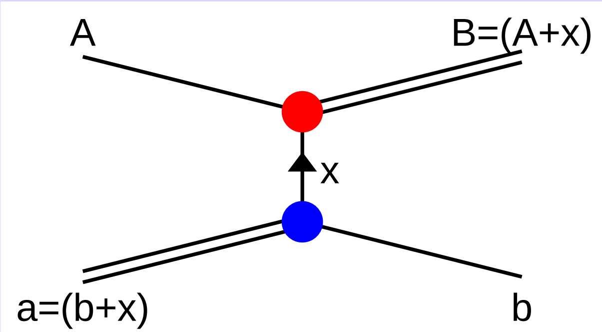
# T-Matrix Elements for Direct Reactions




**Coulomb Dissociation:** direct breakup reaction

prior-form $T_{fi} = \langle \Psi_f^{(-)} | V_{Aa}^{(i)} - U_{Aa} | \phi_A \phi_a \chi_{Aa}^{(+)} \rangle$

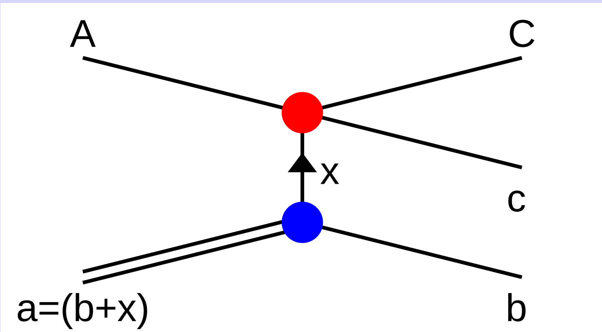
with bound-state wave function $\phi_a$




**ANC Method:** transfer reaction to bound state

post-form $T_{fi} = \langle \phi_B \phi_b \chi_{Bb}^{(-)} | V_{Bb}^{(f)} - U_{Bb} | \Psi_i^{(+)} \rangle$

with bound-state wave function $\phi_B$



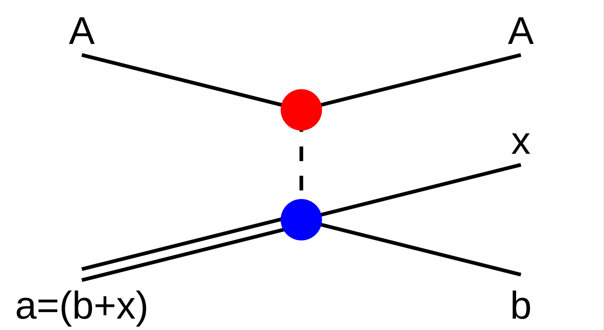

**Trojan-Horse Method:** transfer reaction to continuum

post-form $T_{fi} = \langle \phi_B \phi_b \chi_{Bb}^{(-)} | V_{Bb}^{(f)} - U_{Bb} | \Psi_i^{(+)} \rangle$

with scattering wave function $\phi_B = \Psi_{Cc}^{(-)}$

with exact scattering wave functions $\Psi_i^{(+)}, \Psi_f^{(+)}$, distorted waves $\chi_{Aa}^{(+)}, \chi_{Bb}^{(-)}$, full interactions $V_{Aa}^{(i)}, V_{Bb}^{(f)}$, and optical potentials $U_{Aa}, U_{Bb}$

# Coulomb Dissociation I




- distorted-wave Born approximation (DWBA)
- neglection of nuclear interaction
- multipole expansion of Coulomb potential in far-field approximation

$\Rightarrow$ **cross section** of Coulomb dissociation reaction

$$\frac{d^2\sigma}{dE_{bx}d\Omega_{Aa}} = \frac{1}{E_\gamma}\sum_{\pi\lambda} \sigma_{\pi\lambda}(a+\gamma \to b+x)\frac{dn_{\pi\lambda}}{d\Omega_{Aa}}$$

$\hat{=}$ first-order perturbation theory

- **photo dissociation cross section** $\sigma_{\pi\lambda}(a+\gamma \to b+x)$
- **virtual photon number** $\frac{dn_{\pi\lambda}}{d\Omega_{aA}}$ in quantal calculation or semiclassical approximation

**application in experiments**: cf. talk by T. Motobayashi

# Coulomb Dissociation II

contribution of **nuclear interaction** $\Rightarrow$ absorption, breakup

- quantal calculations (DWBA/Eikonal), optical potentials needed

**higher-order effects** of Coulomb interaction $\hat{=}$ multi-photon exchange

$\Rightarrow$ post-acceleration of fragments after breakup

- various theoretical approaches
  - higher-order perturbation theory / time-dependent dynamical calculations
  - full quantal approaches using three-body wave function with appropriate asymptotics
  - analytical result in certain limits
- effects on angular distributions, slope of extracted S factor $S(E)$
- need for reconciling slope difference between direct and CD experiments?

  H. Esbensen et al., Phys. Rev. Lett. 94, 042502 (2005)
- consistency of semiclassical and quantal (Q) calculations?

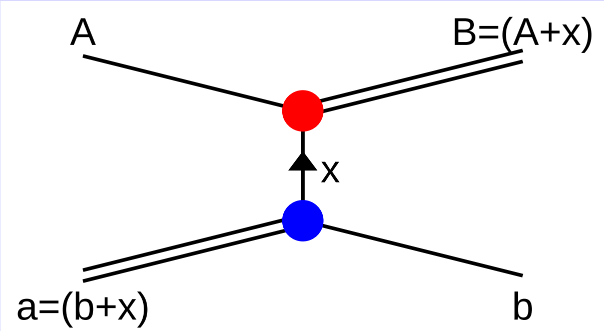
  Q: E. O. Alt et al., Phys. Rev. C 71, 024605 (2005)
- alternative analysis via ANC: consistent with direct results for $S(0)$

  K. Ogata et al., preprint nucl-th/0505007

$\Rightarrow$ select appropriate range of scattering angles and projectile energy

$\Rightarrow$ compare various theoretical approaches for the same experimental conditions

# ANC Method I




- distorted-wave Born approximation (DWBA)
- replace exact overlap functions by asymptotic form with asymptotic normalization coefficients (ANCs) and Whittaker functions

⇒ **overlap functions** ($\hat{=}$ wave function of transferred particle, neglecting spins)

$\langle \phi_b | \phi_a \rangle \approx \frac{C^a_{bx}}{r_{bx}} W_{-\eta_{bx}, l_a+1/2}(2 q_{bx} r_{bx}) Y_{l_a m_a}(\hat{r}_{bx}) \phi_x$ and $\langle \phi_A | \phi_B \rangle \approx \ldots$

⇒ **cross section** of transfer reaction

$\frac{d\sigma}{d\Omega_{Bb}} = \left| C^a_{bx} \right|^2 \left| C^B_{Ax} \right|^2 \frac{d\tilde{\sigma}}{d\Omega_{Bb}}$ with reduced DWBA cross section $\frac{d\tilde{\sigma}}{d\Omega_{Bb}}$

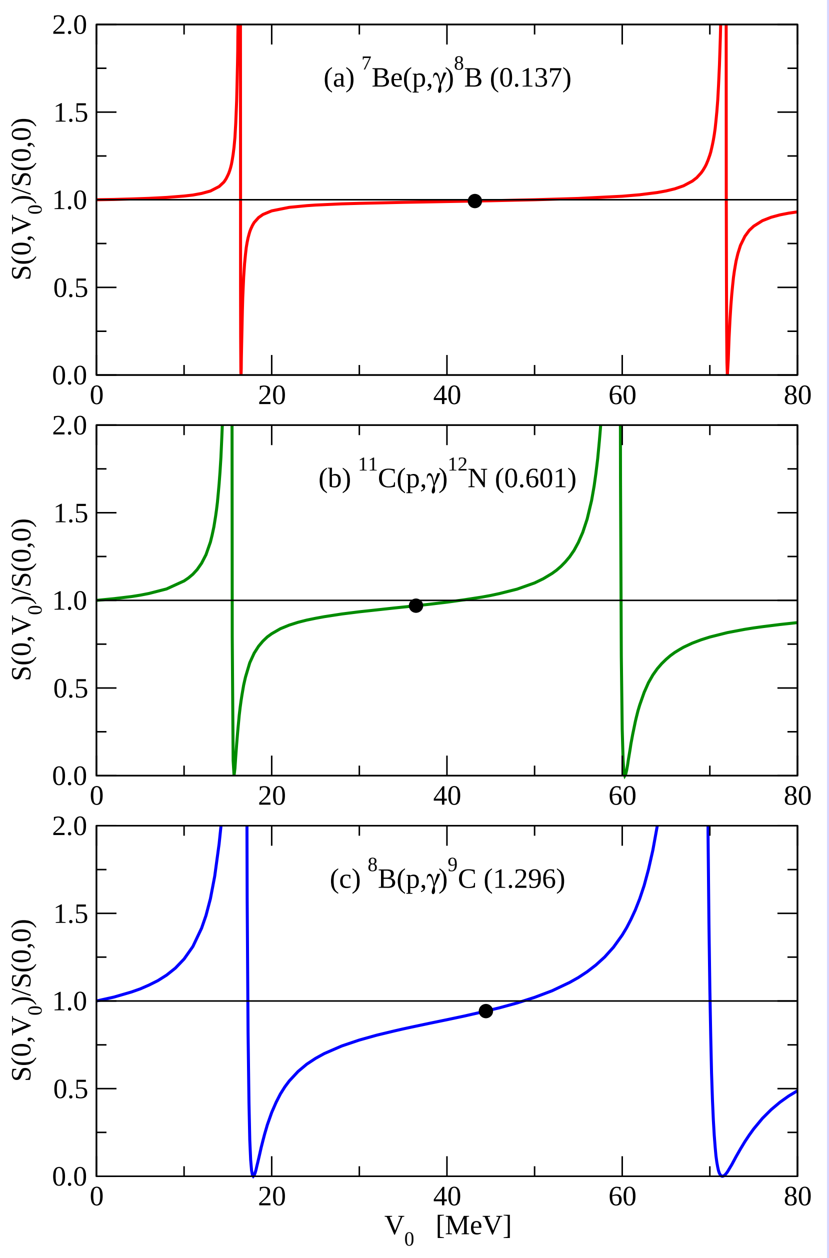
⇒ calculate S factor at zero energy of capture reaction $b(x,\gamma)a$ numerically

- $\left| C^a_{bx} \right|^2 \Leftrightarrow S(0)$ unique relation? effect of $b - x$ interaction?
- precise optical potentials and one additional ANC needed

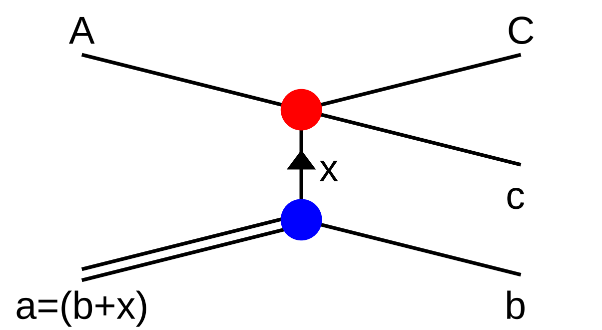
# Zero-Energy S Factor and Interaction in Continuum

- effects of interaction in continuum states
  - modification of shape of cross section, S factor (i.e. energy dependence)
  - change of $S(0)$ even though $\delta \to 0$
- calculation of zero-energy S factor $S(0)$ in single-particle model with Woods-Saxon potential with different depths $V_0$
  - $V_0 = 0 \Rightarrow$ no interaction in continuum
  - $V_0$ adjusted to reproduce bound state energy (black dot)
- example: $E1\ p \to s$ wave transitions for different nuclei with proton+core structure
  $\Rightarrow$ stronger variation of $S(0)$ with $V_0$ with larger proton separation energy

(S. Typel and G. Baur, Nucl. Phys. A 759 (2005), 245)



# Trojan Horse Method I




- distorted-wave Born approximation (DWBA)
- use asymptotic form of scattering wave function in reaction channel $C + c \to A + x$ (essential "surface approximation")

$\Rightarrow$ **overlap function** ($\hat{=}$ wave function of transferred particle, neglecting spins)

$$\langle \phi_A | \Psi^{(-)}_{Cc} \rangle \approx \frac{4\pi}{k_{Cc} r_{Ax}} \sqrt{\frac{v_{Cc}}{v_{Ax}}} \sum_{lm} \xi^*(r_{Ax}) i^l Y_{lm}(\hat{r}_{Ax}) Y^*_{lm}(\hat{k}_{Cc}) \phi_x$$

with $$\xi(r_{Ax}) = \frac{1}{2i} \left[ S^l_{AxCc} u_l^{(+)}(\eta_{Ax}; k_{Ax} r_{Ax}) - \delta_{AxCc} u_l^{(-)}(\eta_{Ax}; k_{Ax} r_{Ax}) \right]$$

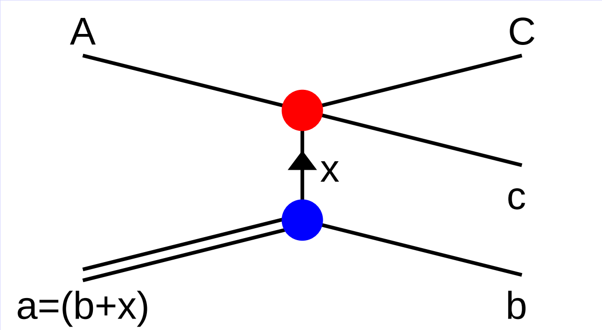
and S-matrix element $S^l_{AxCc}$ of reaction $C(c,x)A$

$\Rightarrow$ **cross section** of transfer reaction to continuum (single channel, $Ax \neq Cc$)

$$\frac{d^3\sigma}{d\Omega_{Bb} d\Omega_{Cc} dE_{Cc}} = \left| S^l_{AxCc} \right|^2 \frac{d^3\tilde{\sigma}_l}{d\Omega_{Bb} d\Omega_{Cc} dE_{Cc}}$$ with reduced DWBA cross section

**general theory:** S. Typel and G. Baur, Ann. Phys. (N.Y.) 305 (2003) 228

# Trojan Horse Method II




additional approximations
(not necessary in general, but convenient)

- potential $V_{Ab} + V_{xb} - U_{Bb} \approx V_{xb}$
- plane waves for distorted waves $\chi^{(-)}_{Bb}$, $\chi^{(+)}_{Aa}$

$\Rightarrow$ **cross section of transfer reaction to continuum**

$$\frac{d^3\sigma}{d\Omega_{Bb} d\Omega_{Cc} dE_{Cc}} = K\, W(\vec{Q}_{Bb}) \frac{d\sigma}{d\Omega}(Ax \to Cc)\, T_l(k_{Ax})$$ with kinematic factor $K$

- **momentum distribution** $W(\vec{Q}_{Bb}) = |\tilde{\Phi}^{a}_{bx}(\vec{Q}_{Bb})|^2$

  depending on momentum transfer to spectator $b \Rightarrow$ quasi-free scattering conditions

- **cross section** $\frac{d\sigma}{d\Omega}(Ax \to Cc)$ of two-body reaction

- **penetration factor** $T_l(k_{Ax}) \approx k^3_{Ax} \exp(2\pi\eta_{Ax})$

$\Rightarrow$ cancels suppression of two-body cross section by Coulomb barrier for $E_{Ax} \to 0$

# Trojan Horse Method III

- recent **applications**: (only in simple theoretical approximations)

| | | | |
|---|---|---|---|
| $^{2}$H(d,p)$^{3}$H | ⇒ | $^{2}$H($^{6}$Li,p t)$^{4}$He | A. Tumino et al., Eur. J. Phys. A 25, 649 (2005) |
| $^{3}$H(d,p)$^{4}$He | ⇒ | $^{6}$He($^{3}$He,p $\alpha$)$^{4}$He | M. La Cognata et al., Phys. Rev. C 71, 064301 (2005) |
| $^{6}$Li(p,$\alpha$)$^{3}$He | ⇒ | $^{2}$H($^{6}$Li,$\alpha$ $^{3}$He)n | A. Tumino et al., Phys. Rev. C 67, 065803 (2003) |
| $^{6}$Li(n,t)$^{4}$He | ⇒ | $^{2}$H($^{6}$Li,tp)$^{4}$He | A. Tumino et al., Eur. J. Phys. A 25, 649 (2005) |
| $^{6}$Li(d,$\alpha$)$^{4}$He | ⇒ | $^{6}$Li($^{6}$Li,$\alpha\alpha$)$^{4}$He | A. Musumarra et al., Phys. Rev. C 64, 068801 (2001) |
| $^{7}$Li(p,$\alpha$)$^{4}$He | ⇒ | $^{2}$H($^{7}$Li,$\alpha\alpha$)n | M. Lattuada et al., Astrophys. J. 562, 1076 (2001) |
| $^{9}$Be(p,$\alpha$)$^{6}$Li | ⇒ | $^{2}$H($^{9}$Be,$^{6}$Li $\alpha$)n | C. Spitaleri et al., Proceedings, Hanoi (2004) |
| $^{11}$B(p,$\alpha$)$^{8}$Be | ⇒ | $^{2}$H($^{11}$B,$\alpha$ $^{3}$He)n | S. Romano et al., Nucl. Phys. A 738, 406 (2004) |

- extracted S factor not affected by **electron screening**
  ⇒ determination of electron screening potential $U_e$ by comparison to direct data
  ⇒ consistent values for $U_e$, larger than adiabatic limit, challenge for theory
- extension to **radiative capture reactions** possible
  ⇒ additional approach independent from Coulomb dissociation and ANC methods
- study **elastic scattering** without Coulomb contribution
- application to reactions with **exotic nuclei** ⇒ large cross sections
- **full DWBA** calculations needed under quasi-free scattering conditions
  ⇒ numerically difficult because of scattering wave functions in final state

# Conclusions

- **Indirect methods** provide complementary information on reactions of astrophysical interest
  - Coulomb dissociation method
  - method of asymptotic normalization coefficients (ANC)
  - Trojan-Horse method
- similar characteristics and theoretical concepts
- importance of nuclear reaction theory
  - direct reactions with certain kinematical conditions
  - peripheral reactions, asymptotics of wave functions
  - approximations $\Rightarrow$ range of validity, accuracy
- still great potential for future applications (also beyond astrophysical applications)
- dedicated theoretical investigations needed