

Equation of state constraints from postmerger gravitational-wave emission

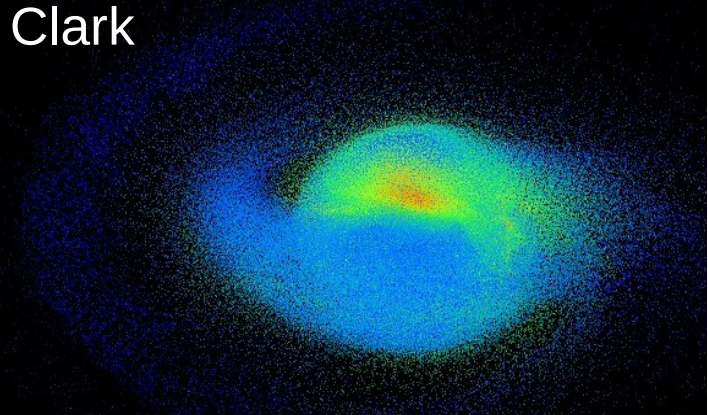
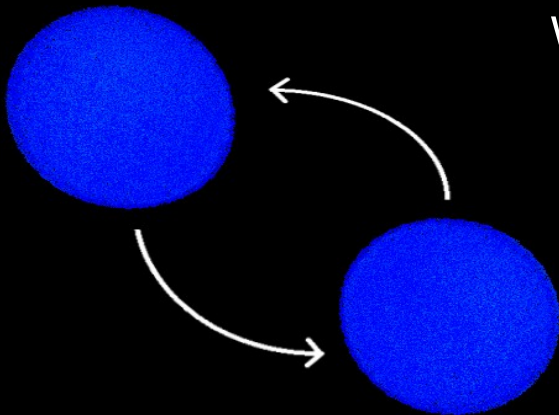
Neutron star mergers: From gravitational waves to
nucleosynthesis

16/01/2017

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(Heidelberg Institute for Theoretical Studies)

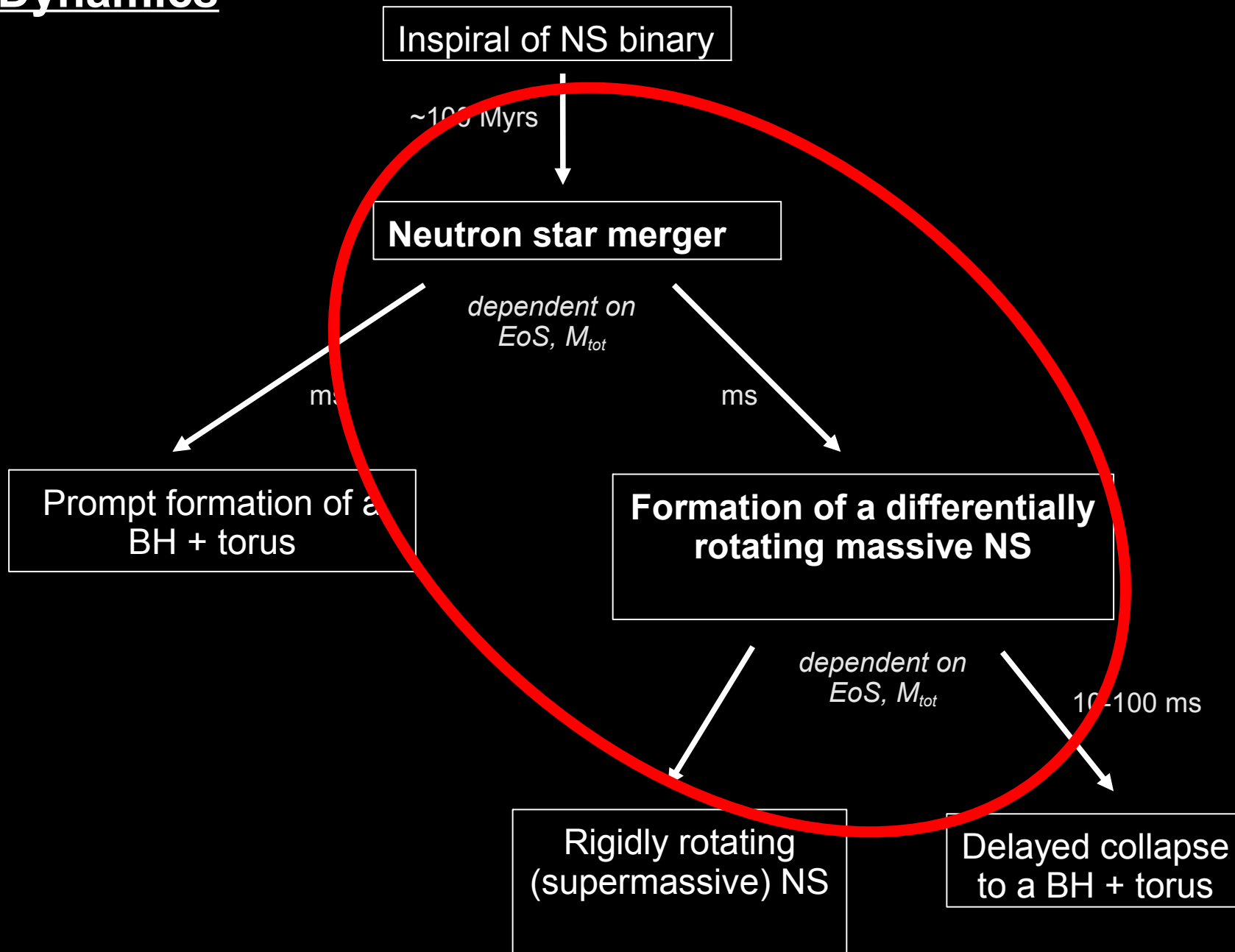
with N. Stergioulas, H.-T. Janka, J. Clark



Outline: Neutron-star mergers

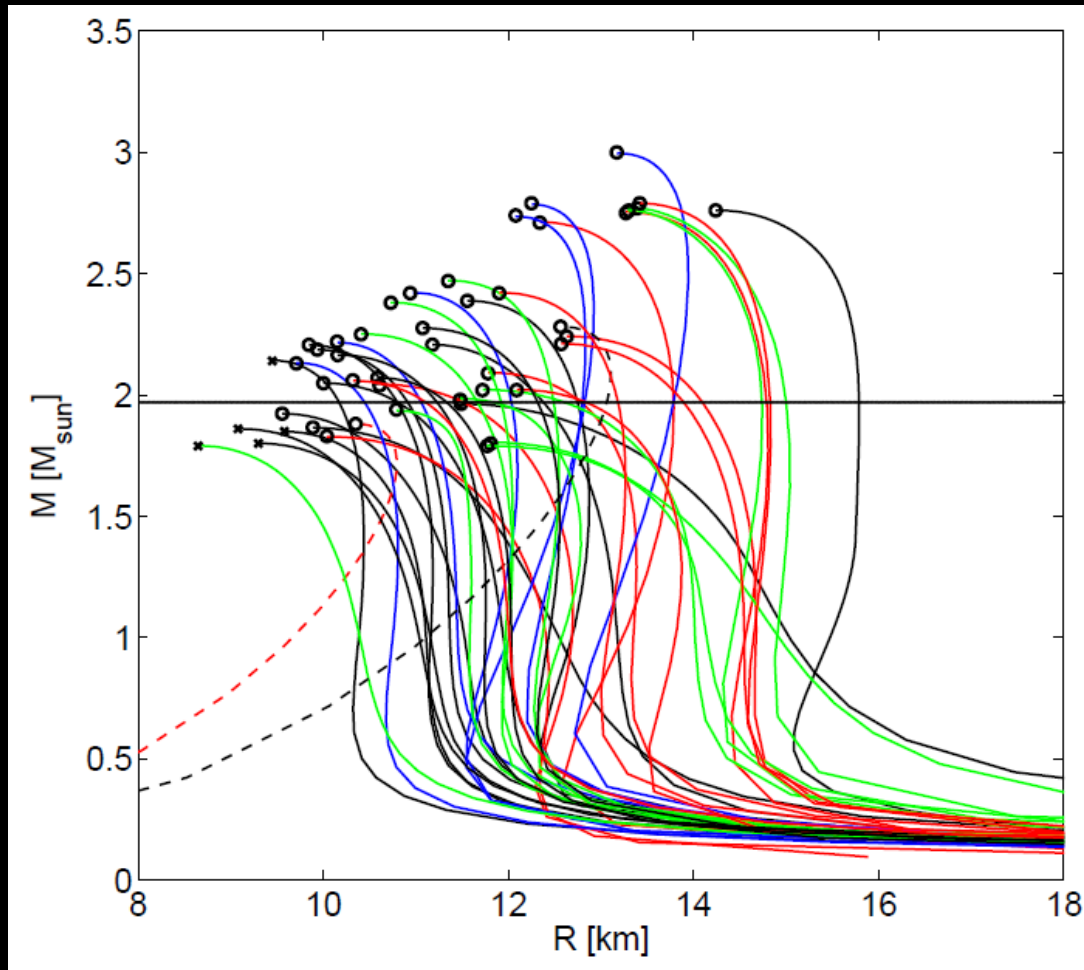
- ▶ Overview
- ▶ Binary mass measurements
- ▶ Dominant postmerger oscillation
- ▶ Radius measurements
- ▶ Data analysis
- ▶ Estimates of the maximum mass – collapse behavior of merger remnants
- ▶ Secondary features of the postmerger spectrum

Dynamics



General outcome

for $1.35\text{-}1.35 M_{\text{sun}}$ binaries

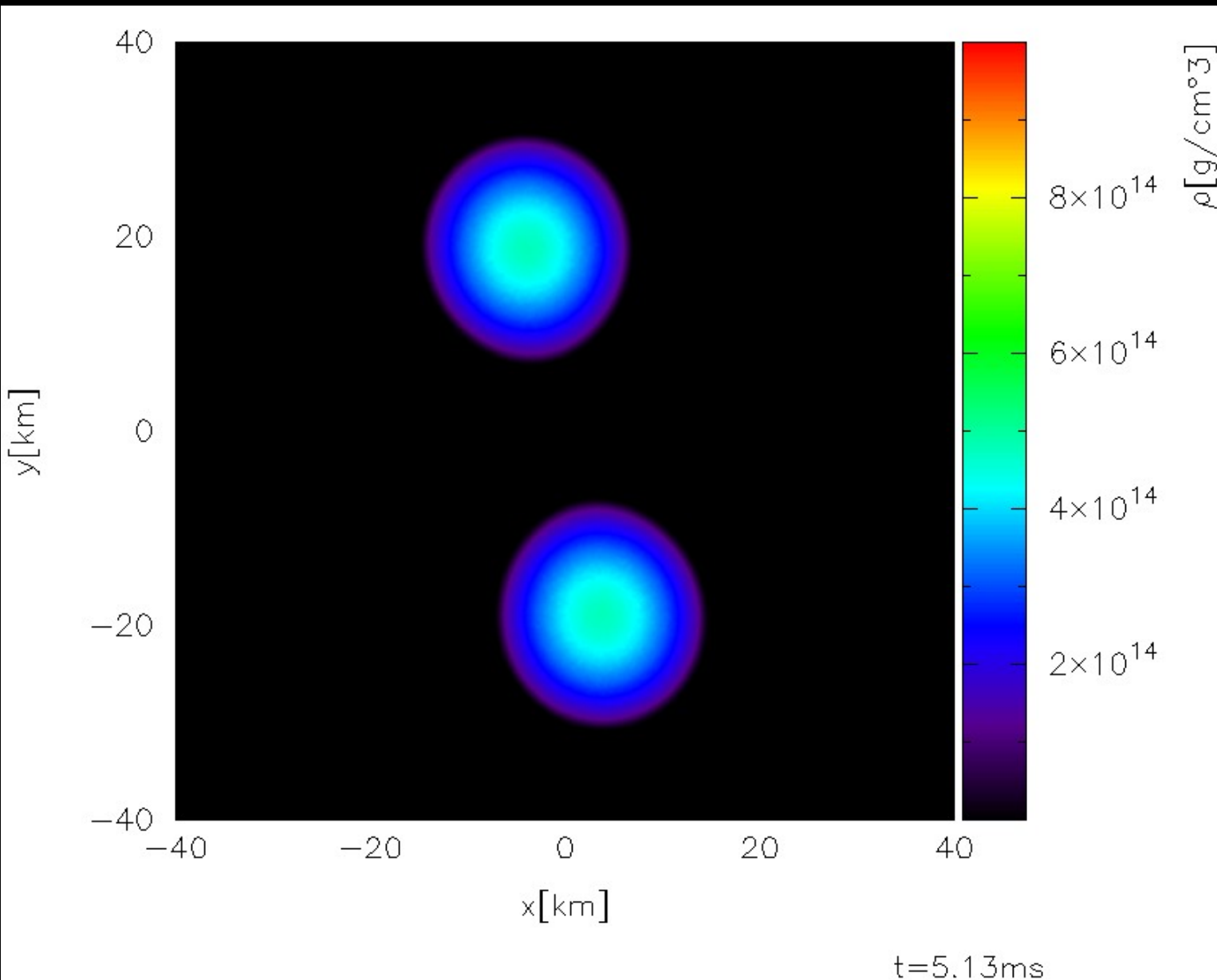


M_{max} with circle:
rotating NS merger
remnant

M_{max} with cross:
direct BH formation

42 out of 47 models lead to the formation of
a differentially rotating NS

(only one accepted EoS leads to prompt collapse)



1.35-1.35 Msun, Shen EoS, rest-mass density in equatorial plane
Smooth Particle Hydrodynamics, conformally flat spatial metric

Binary mass measurements

Masses from inspiral

- ▶ Accurately measured “**chirp mass**”

$$M_{chirp} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$

- ▶ **Mass ratio** with larger error

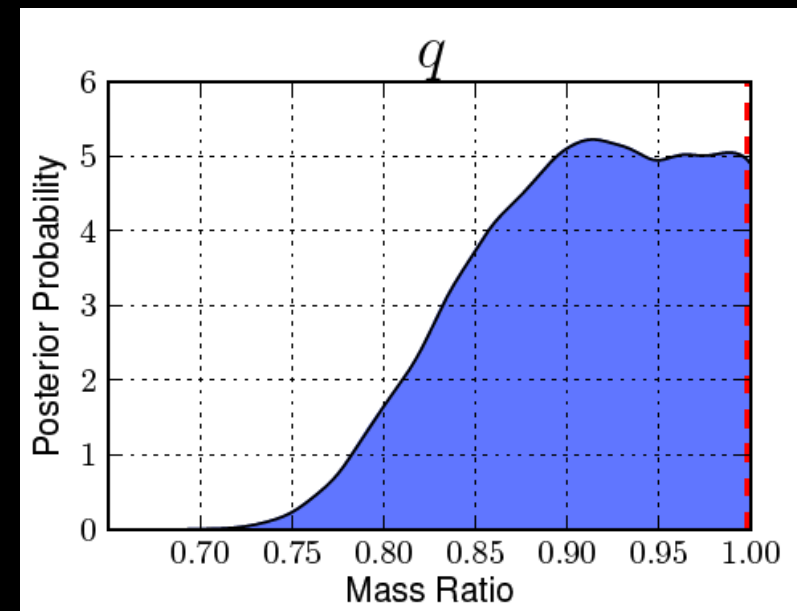
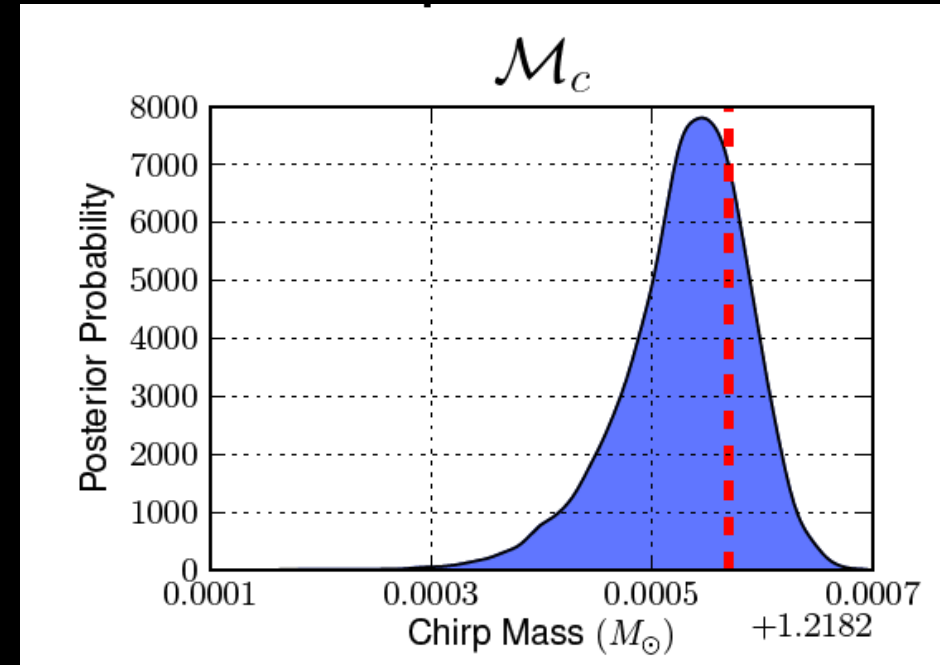
$$q = M_1/M_2$$

- ▶ i.e. q only for near-by mergers
- ▶ the closer the more accurate

Dashed red line = injected signal

Distribution function of recovered signals in blue

Rodriguez et al 2014 – injected at 100 Mpc

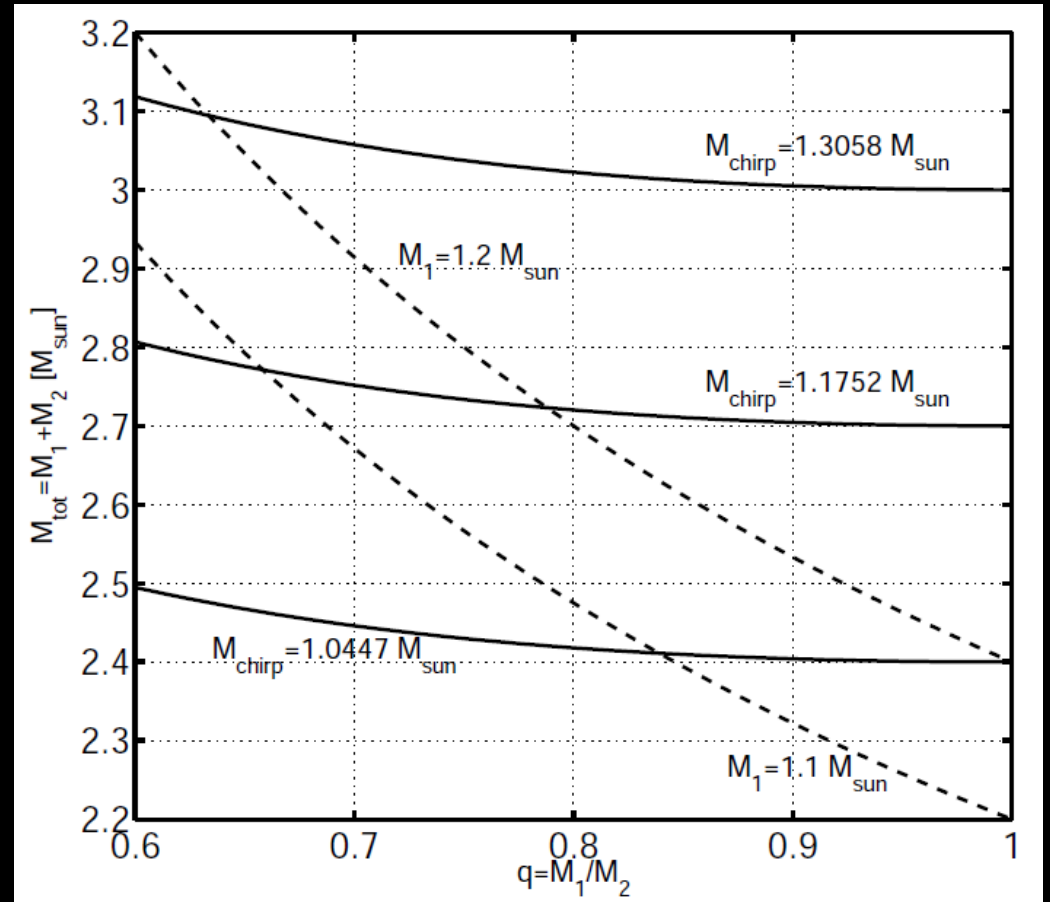


Total mass from chirp

$$M_{chirp} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$

$$M_{tot} = M_1 + M_2$$

→ Chirp mass determines M_{tot} quite well



Minimum NS mass 1.1 - 1.2 Msun (e.g. Ertl et al. 2015 or measured masses)

EoS from gravitational waves

GWs: an oversimplified picture

Two complementary approaches to infer EoS properties:

- GW inspiral:

strong signal - weak EoS effect

(Jim's talk, Sanjay's talk; e.g. Read et al. 2013 → ~1 km @ 100 Mpc)

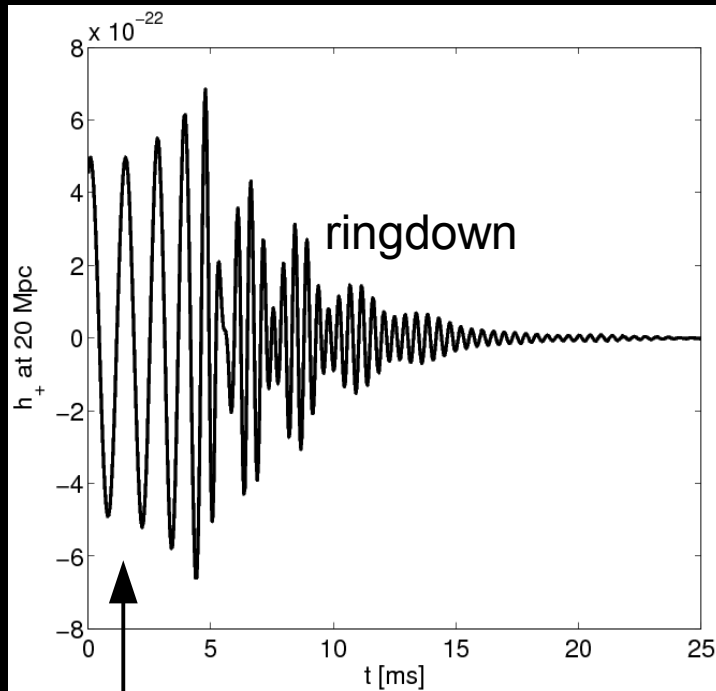
- Postmerger oscillations:

weak signal – robust strong EoS effect

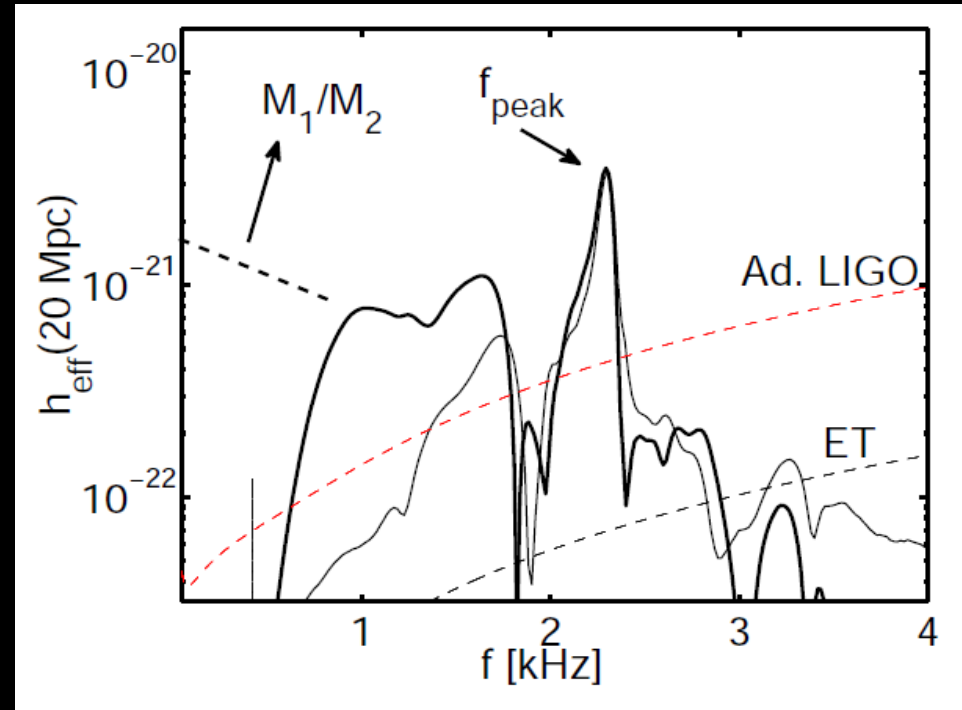
Postmerger gravitational-wave emission

Gravitational-wave spectrum

1.35-1.35 M_{sun} TM1 equation of state (EoS), 20 Mpc

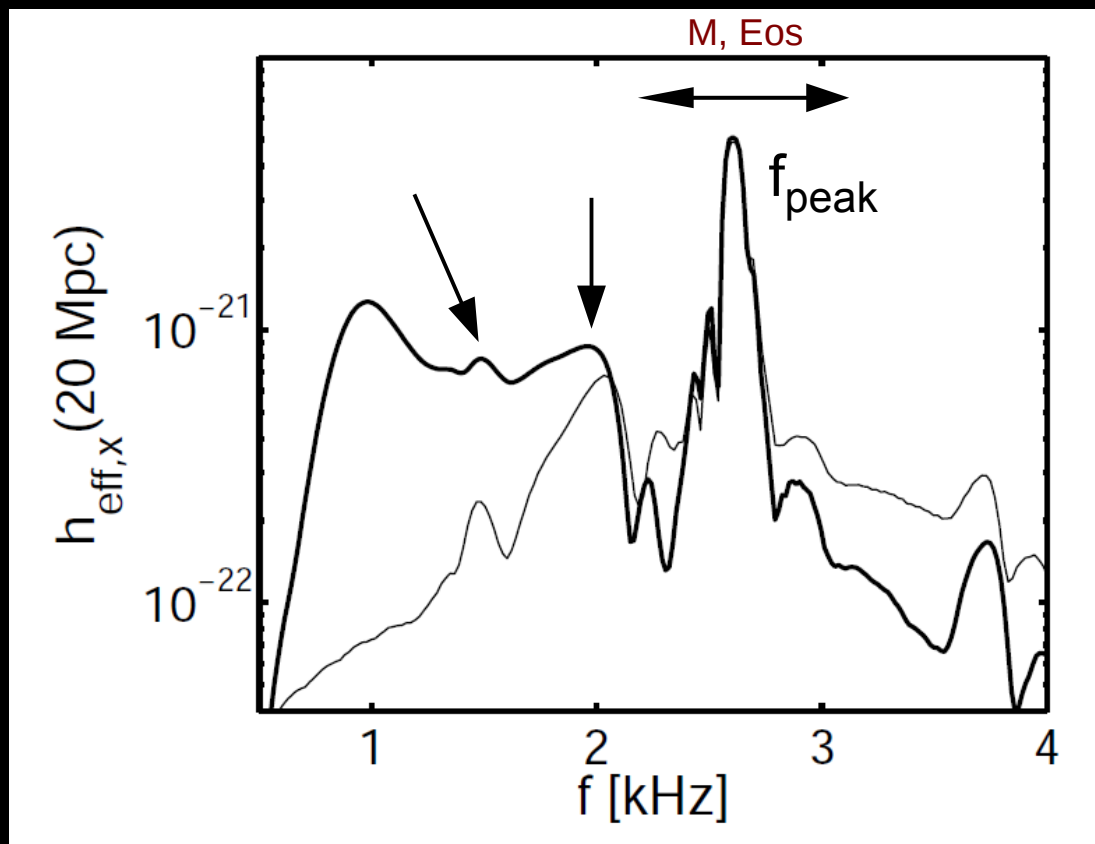


inspiral



- Pronounced **peak in the kHz** range as a **robust feature** of all models forming a differentially rotating NS
- **Characteristic GW feature: f_{peak}**
- Binary masses M_1/M_2 are measurable from GW inspiral signal (most of the inspiral not covered by simulation)

Generic GW spectrum



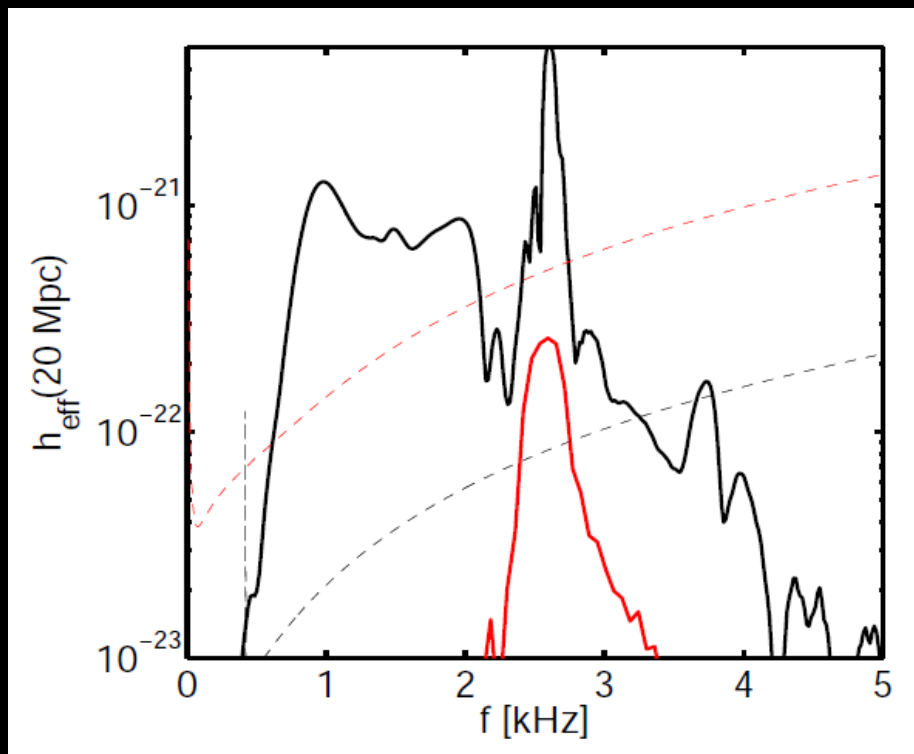
Thin line
postmerger only

Note: no unique nomenclature in the literature, e.g. f_{peak} is also called f_2 ...

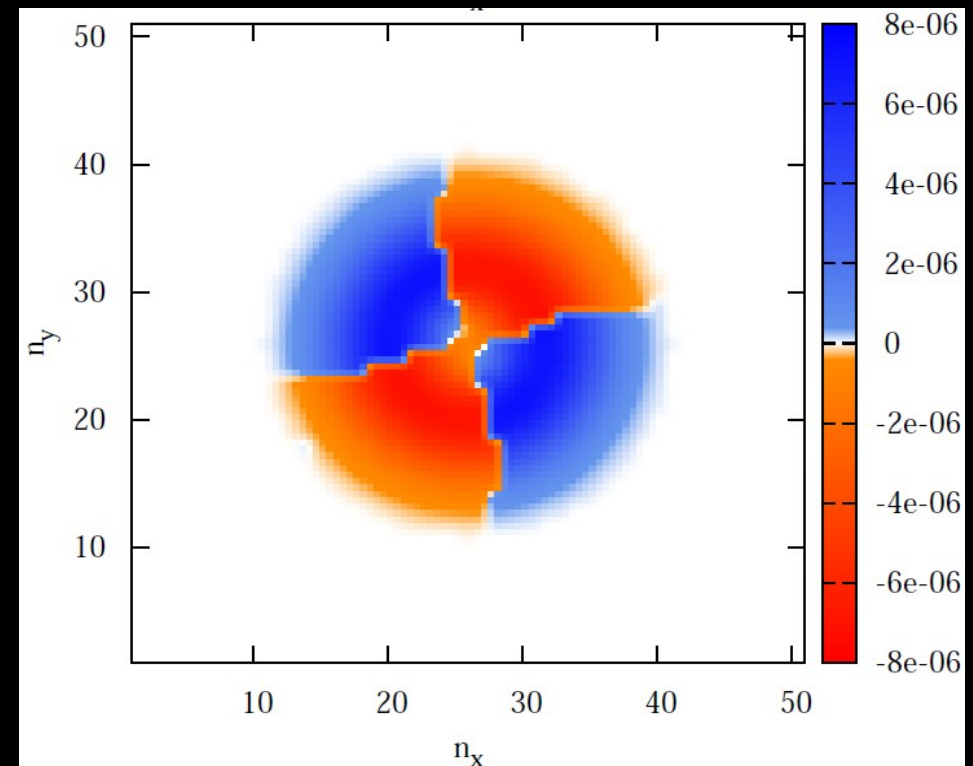
- **Up to two secondary features at lower frequencies** in the postmerger spectrum (subdominant wrt to sensitivity curve) + structure at higher frequencies
- Simulation: $1.35-1.35 M_{\text{sun}}$ DD2 EoS, Smooth Particle Hydro, Conformal Flatness
- Generic in the sense that not all secondary peaks are necessarily present
- **Classification scheme of postmerger emission and dynamics** depending on presence/strength of secondary peaks (Bauswein et al. 2015)

Dominant oscillation frequency

- Robust feature, which occurs in all models (which don't collapse promptly to BH)
- f_{peak} = fundamental quadrupolar fluid mode of the remnant



Re-excitation of f-mode ($l=|m|=2$) in late-time remnant (Bauswein et al. 2016)

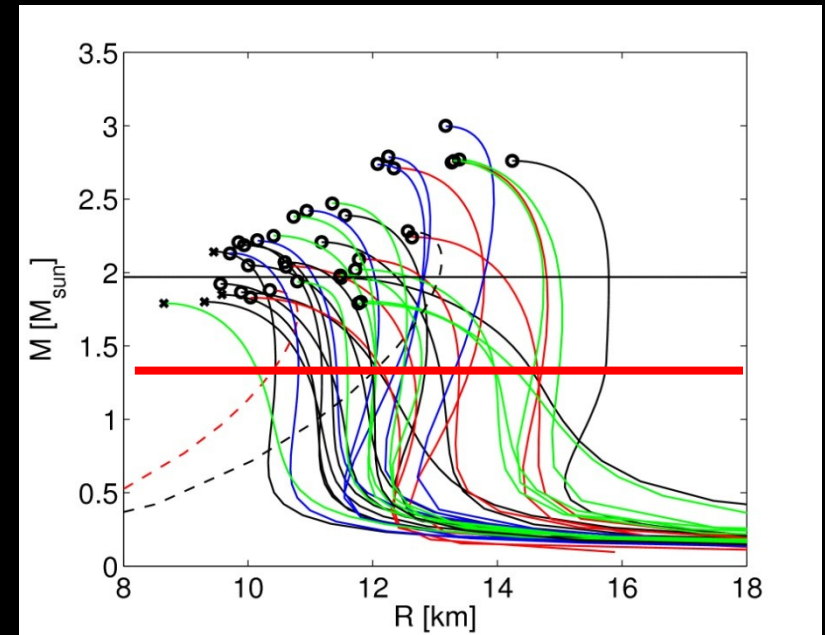
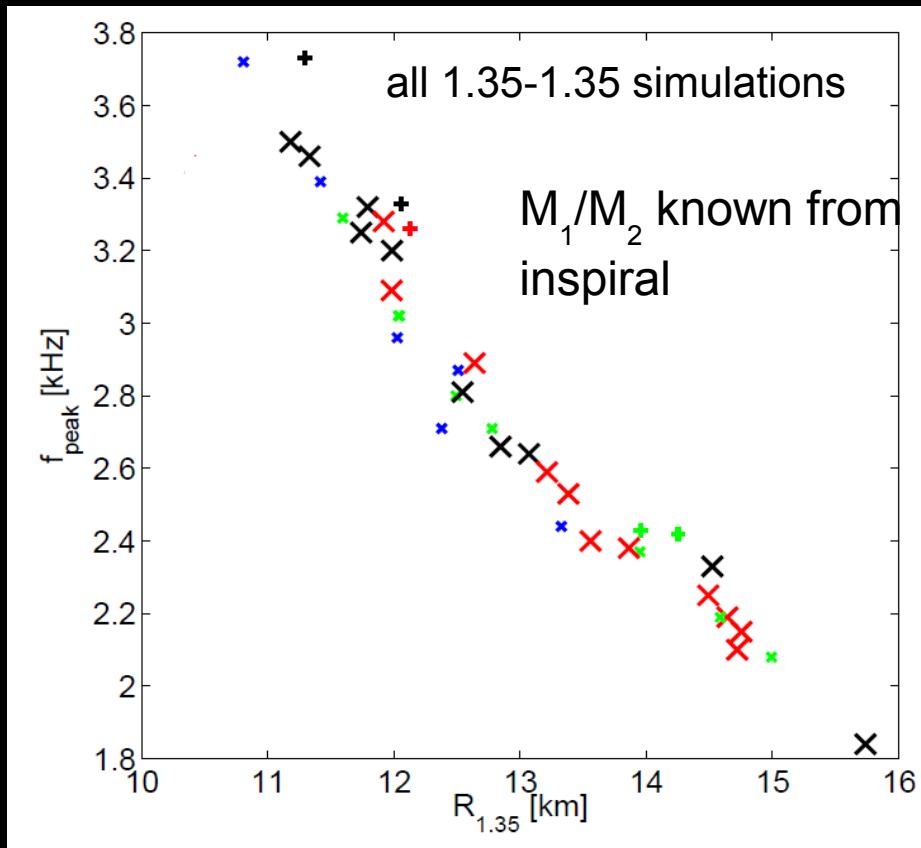


Mode analysis at $f=f_{\text{peak}}$
Stergioulas et al. 2011

EoS dependence of f_{peak}

- we consider relations with fixed binary masses
- masses can be well measured from inspiral
- in particular for distances that allow measurement of postmerger emission

Gravitational waves – EoS survey

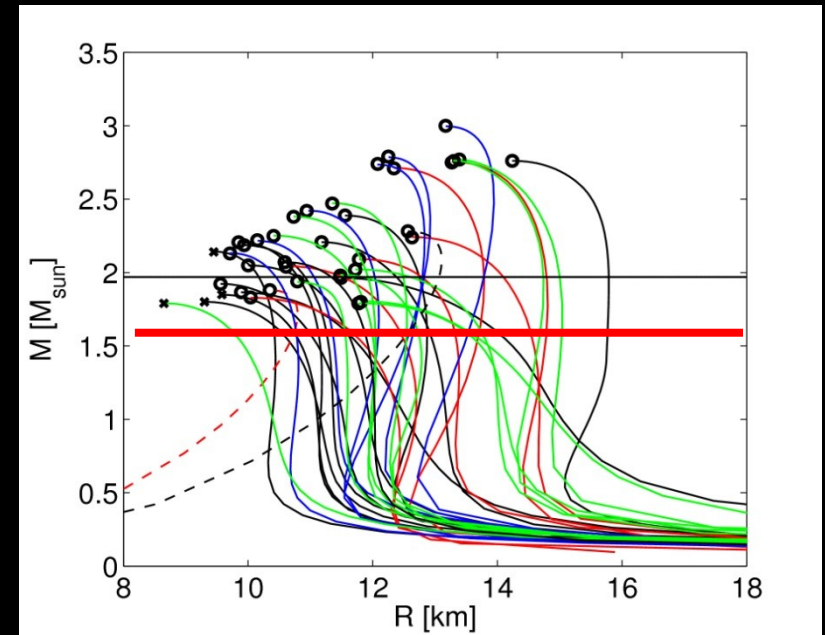
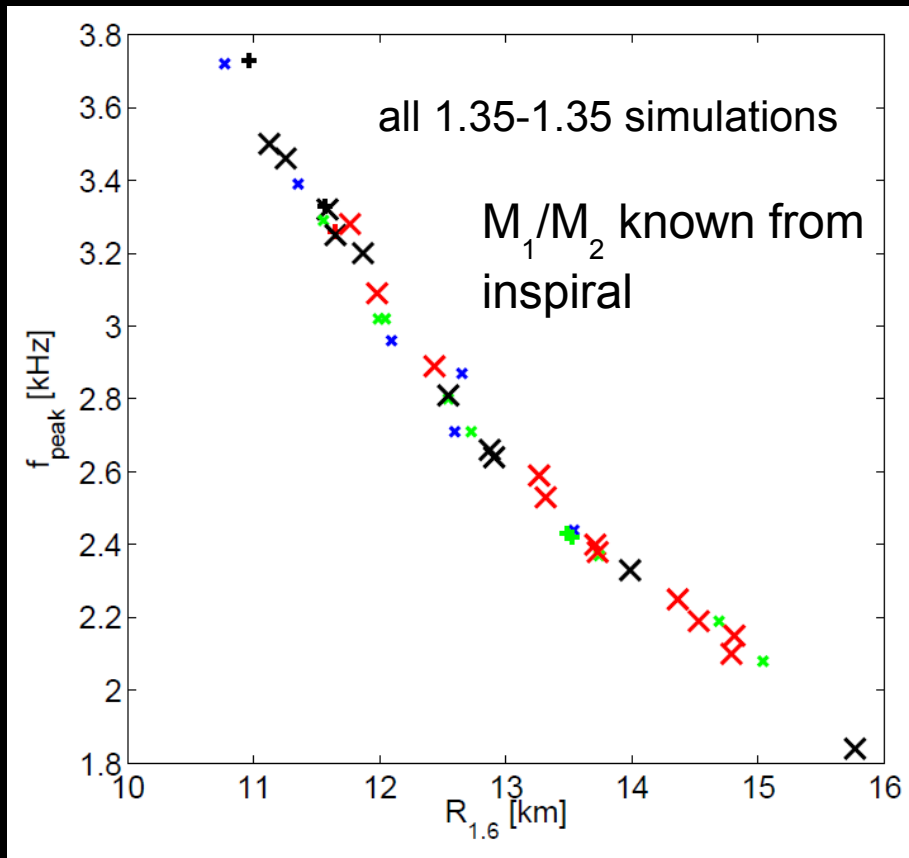


characterize EoS by radius of nonrotating NS with $1.35 M_{\text{sun}}$

Pure TOV/EoS property => **Radius measurement** via f_{peak}

Important: Simulations for the same binary system, just with varied EoS

Gravitational waves – EoS survey



characterize EoS by radius of nonrotating NS with $1.6 M_{\text{sun}}$

Bauswein et al. 2012

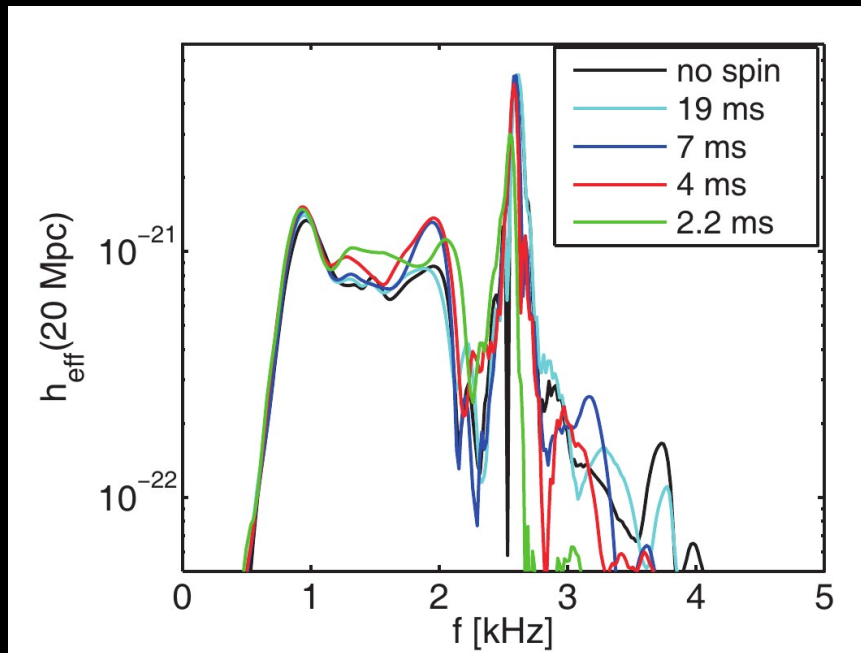
Pure TOV/EoS property => **Radius measurement** via f_{peak}

Important: Simulations for the same binary system, just with varied EoS

Note: R of $1.6 M_{\text{sun}}$ NS scales with f_{peak} from 1.35-1.35 M_{sun} mergers (density regimes comparable)

Remarks

- ▶ Similar relations for asymmetric binaries and other binary masses
- ▶ (mass will be measured by inspiral)
- ▶ Initial intrinsic NS rotation unimportant
- ▶ Agreement with different groups (Kyoto, Caltech, Frankfurt, Parma, e.g. Hotokezaka et al 2013, Bernuzzi et al. 2015, Takami et al. 2015, De Pietri et al. 2015, Foucart et al. 2016)

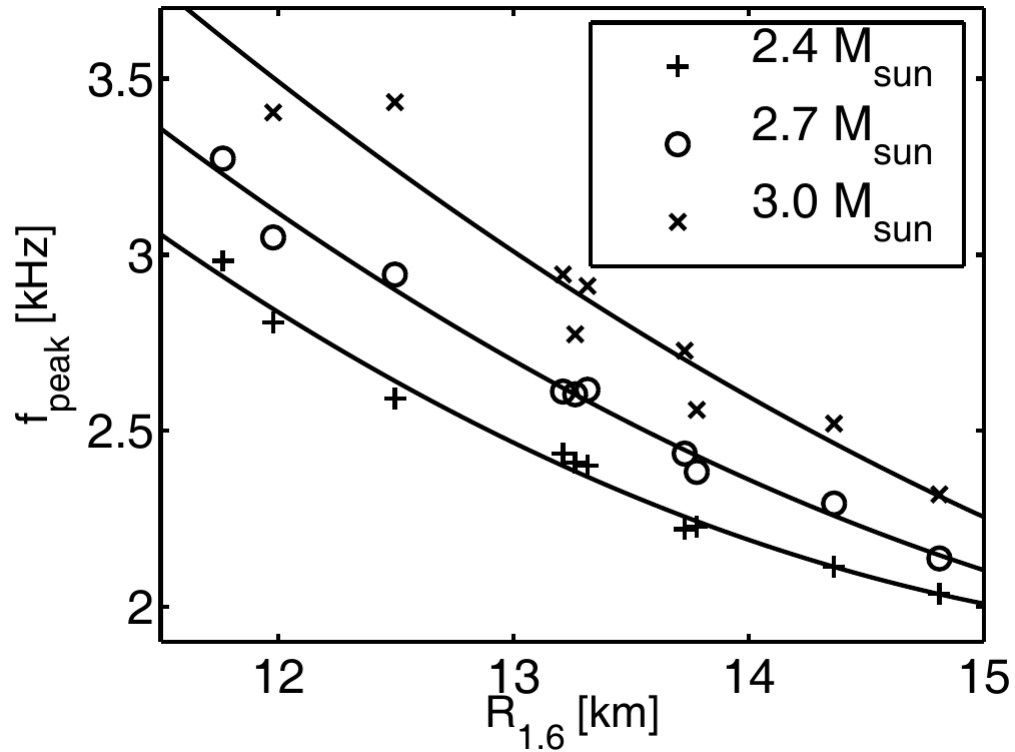


→ No impact on f_{peak}

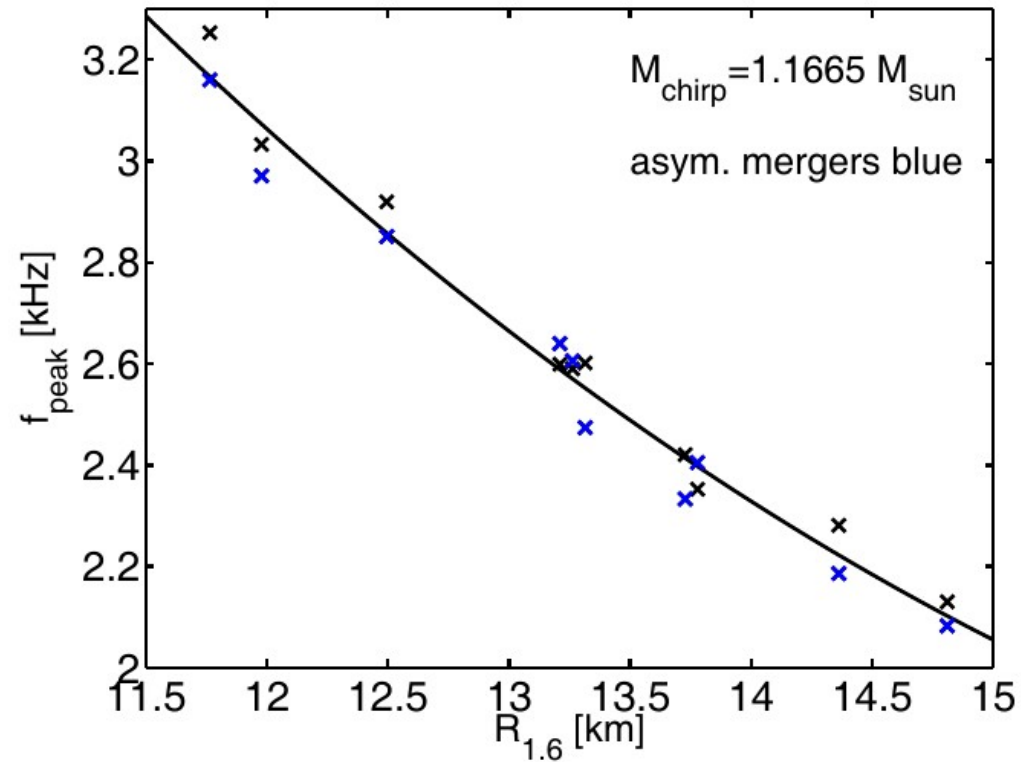
Strategy

- ▶ Measure binary masses from inspiral
- ▶ Construct $f_{\text{peak}} - R$ relation for fixed binary mass
- ▶ Measure f_{peak} from postmerger GW signal
- ▶ Obtain radius by inverting $f_{\text{peak}} - R$ relation

Binary mass variations



Different total binary masses
(symmetric)

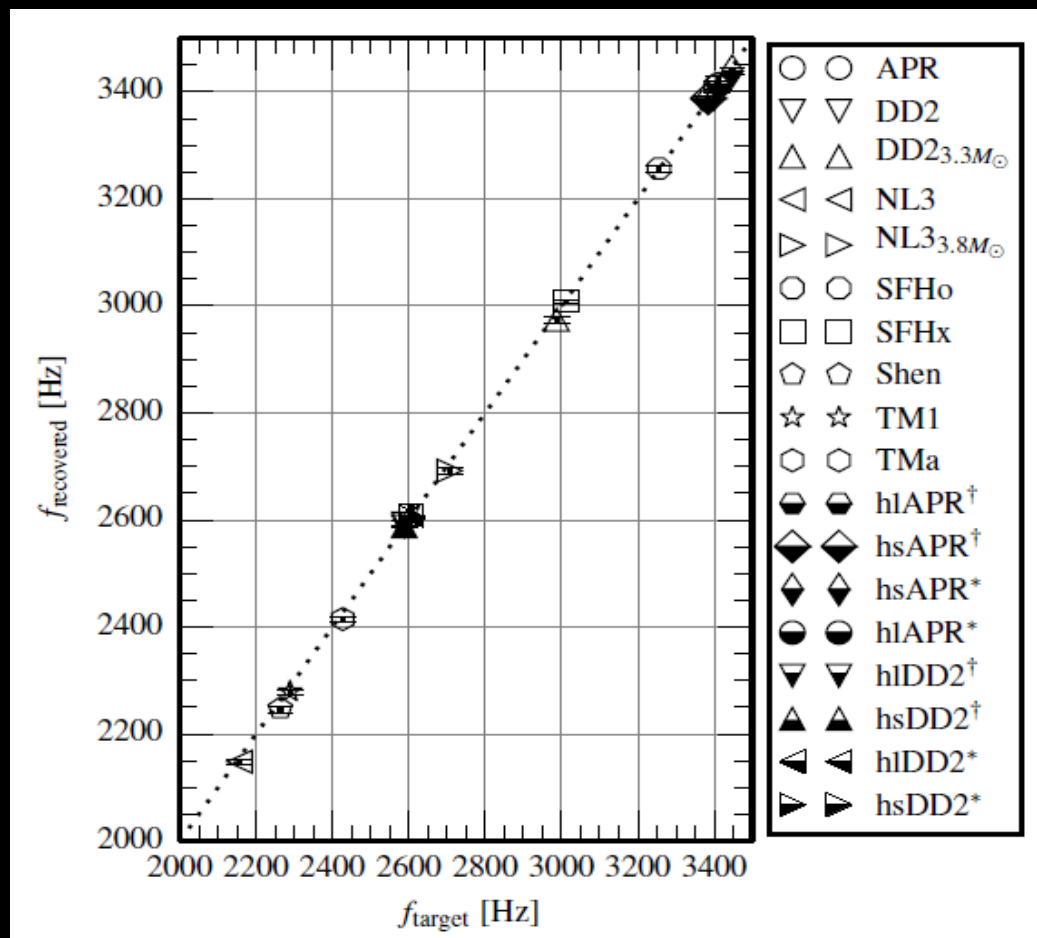


Fixed chirp mass (incl.
Asymmetric binaries)

Bauswein et al. 2016

Detectability

Unmodeled searches: Measuring the dominant GW frequency



Model waveforms hidden in rescaled LIGO noise

Peak frequency recovered with burst search analysis

Error ~ 10 Hz

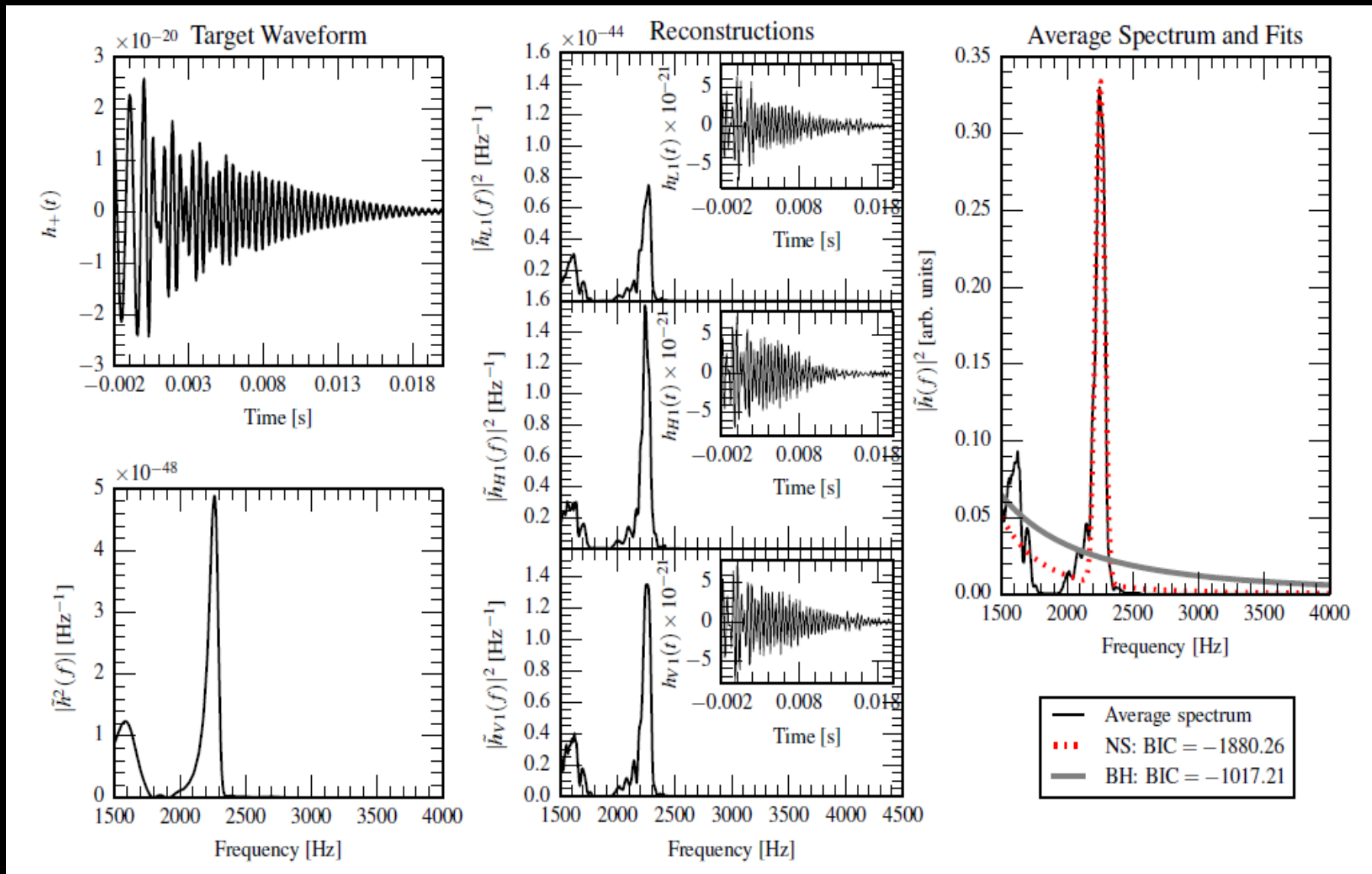
For signals within ~ 10 -25 Mpc

=> for near-by event radius measurable with high precision (~ 0.01 -1/yr)

Proof-of-principle study
→ improvements likely

Clark et al. 2014

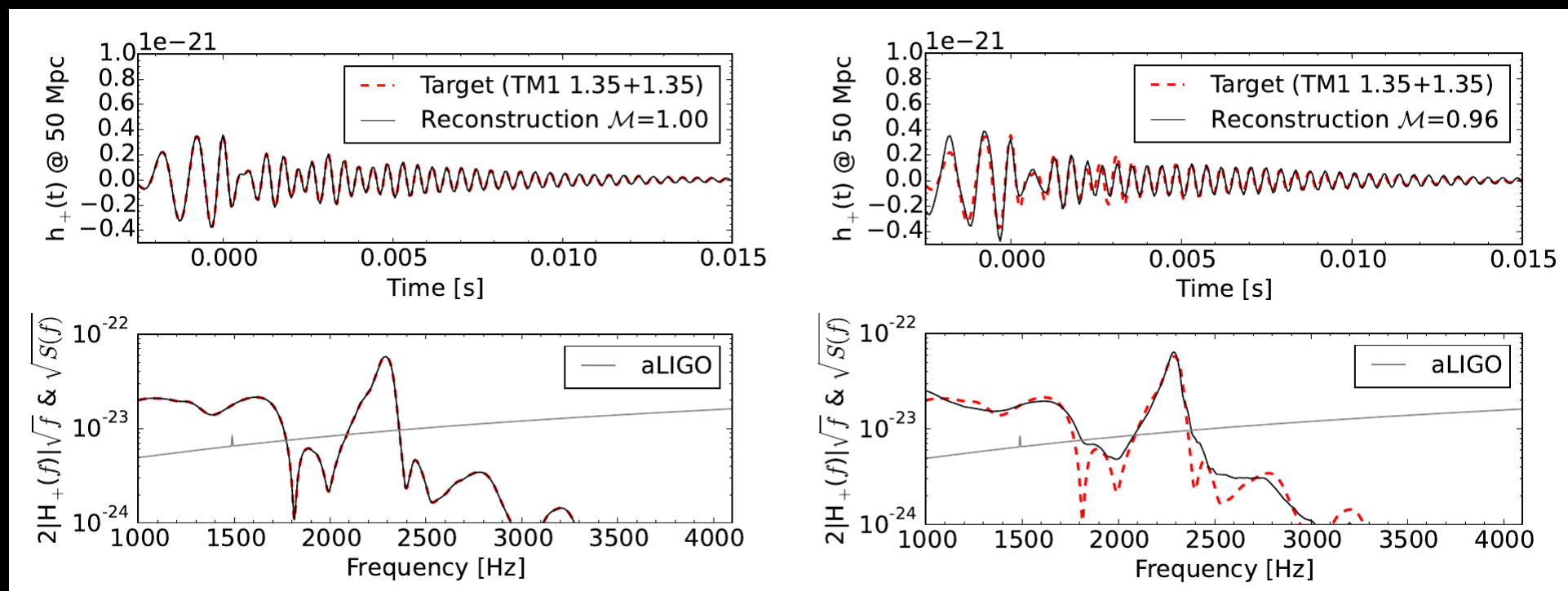
Unmodeled search – less sensitive



Discrimination between prompt collapse and delay/no collapse possible

Shen 1.35-1.35; Clark et al. 2014

Principal Component Analysis



Only first component

Excluding the reconstructed waveform from catalogue

Clark et al 2016

PCA - prospects

Instrument	SNR _{full}	SNR _{post}	D_{hor} [Mpc]	$\dot{\mathcal{N}}_{\text{det}}$ [year ⁻¹]
aLIGO	2.99 ^{3.86} _{2.37}	1.48 ^{1.86} _{1.13}	29.89 ^{38.57} _{23.76}	0.01 ^{0.03} _{0.01}
A+	7.89 ^{10.16} _{6.25}	4.19 ^{5.35} _{3.26}	78.89 ^{101.67} _{62.52}	0.13 ^{0.20} _{0.10}
LV	14.06 ^{18.13} _{11.16}	7.28 ^{9.30} _{5.64}	140.56 ^{181.29} _{111.60}	0.41 ^{0.88} _{0.21}
ET-D	26.65 ^{34.28} _{20.81}	12.16 ^{15.31} _{9.34}	266.52 ^{342.80} _{208.06}	2.81 ^{5.98} _{1.33}
CE	41.50 ^{53.52} _{32.99}	20.52 ^{25.83} _{15.72}	414.62 ^{535.221} _{329.88}	10.59 ^{22.78} _{5.33}

$$\delta R_{1.6} = \sqrt{(\delta R_{1.6}^{\text{stat}})^2 + (\delta R_{1.6}^{\text{sys}})^2}.$$

Instrument	\mathcal{M}	δf_{peak} [Hz]	$\delta R_{1.6}^{\text{stat}}$ [m]	$\delta R_{1.6}$ [m]
aLIGO	0.93 ^{0.96} _{0.91}	51.8 ^{139.4} _{23.1}	145.7 ^{228.9} _{78.3}	227.7 ^{288.1} _{191.7}
A+	0.93 ^{0.96} _{0.89}	44.4 ^{116.6} _{20.5}	125.5 ^{195.0} _{69.2}	215.3 ^{262.0} _{188.2}
LV	0.93 ^{0.96} _{0.90}	46.5 ^{123.1} _{21.3}	131.4 ^{204.8} _{71.9}	218.8 ^{269.4} _{189.2}
CE	0.91 ^{0.96} _{0.93}	51.6 ^{138.9} _{23.1}	78.1 ^{228.2} _{145.2}	191.6 ^{287.6} _{227.4}
ET-D	0.94 ^{0.97} _{0.92}	59.9 ^{163.3} _{26.0}	168.0 ^{267.5} _{88.4}	242.6 ^{319.7} _{196.1}

Clark et al. 2016

Note:

- rate unclear, could be much higher
- Conservative estimate – numerical damping and finite duration → underestimation of signal strength

Systematics error:

- Intrinsic scatter in relation
- Uncertainties in simulations

Better error for closer systems

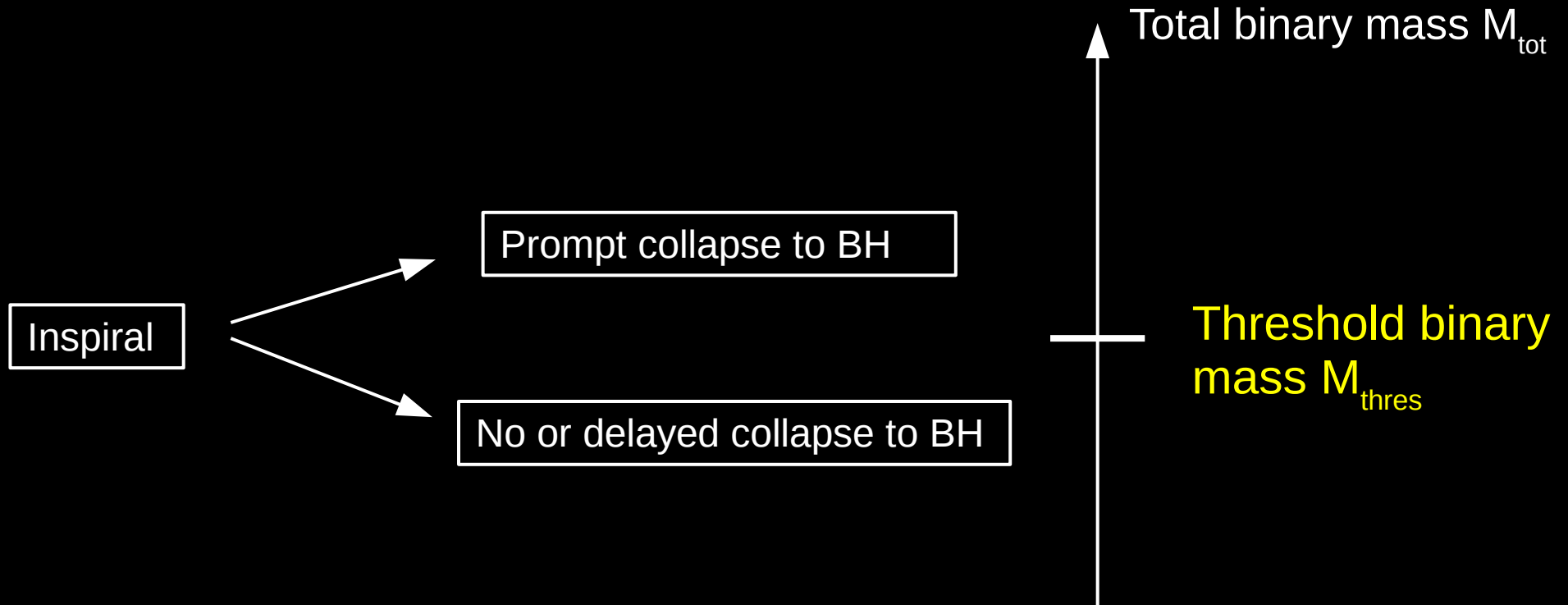
→ improvements possible

Collapse behavior and the maximum mass of (non-rotating) neutron stars

Key quantity: Threshold binary mass for prompt collapse

Note: Collapse behavior important for nucleosynthesis,
kilonovae and short gamma-ray bursts !!!

Collapse behavior



EoS dependent !!! - somehow M_{max} should play a role

Simulations

EoS	M_{\max} (M_{\odot})	R_{\max} (km)	C_{\max}	$R_{1.6}$ (km)	M_{thres} (M_{\odot})	ρ_c/ρ_0	$f_{\text{peak}}^{\text{stab}}$ (kHz)
NL3 [37,38]	2.79	13.43	0.307	14.81	3.85	5.6	2.78
GS1 [39]	2.75	13.27	0.306	14.79	3.85	5.7	2.81
LS375 [40]	2.71	12.34	0.325	13.71	3.65	6.5	3.05
DD2 [38,41]	2.42	11.90	0.300	13.26	3.35	7.2	3.06
Shen [42]	2.22	13.12	0.250	14.46	3.45	6.7	2.85
TM1 [43,44]	2.21	12.57	0.260	14.36	3.45	6.7	2.91
SFHX [45]	2.13	10.76	0.292	11.98	3.05	8.9	3.52
GS2 [46]	2.09	11.78	0.262	13.31	3.25	7.6	3.19
SFHO [45]	2.06	10.32	0.294	11.76	2.95	9.8	3.67
LS220 [40]	2.04	10.62	0.284	12.43	3.05	9.4	3.52
TMA [44,47]	2.02	12.09	0.247	13.73	3.25	7.2	2.96
IUF [38,48]	1.95	11.31	0.255	12.57	3.05	8.1	3.31



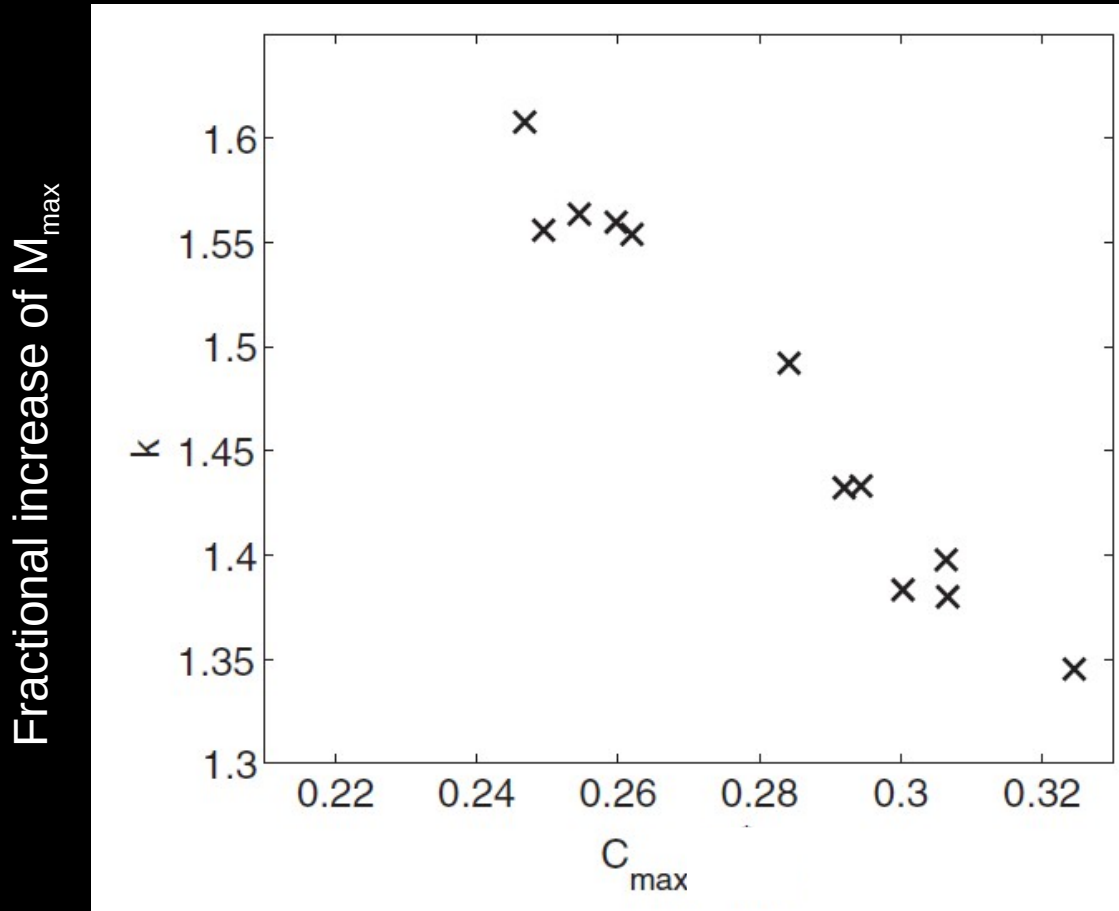
Simulations with many different binary masses

$$k = \frac{M_{\text{thres}}}{M_{\text{max}}}$$

← from simulations

← TOV property of employed EoS

Key quantity: **Threshold binary mass M_{thres}** for prompt BH collapse



$$M_{\text{thres}} = k * M_{\text{max}}$$

with $k = k(C_{\text{max}})$

$$C_{\text{max}} = G M_{\text{max}} / (c^2 R_{\text{max}})$$

(compactness of TOV maximum-mass configuration)

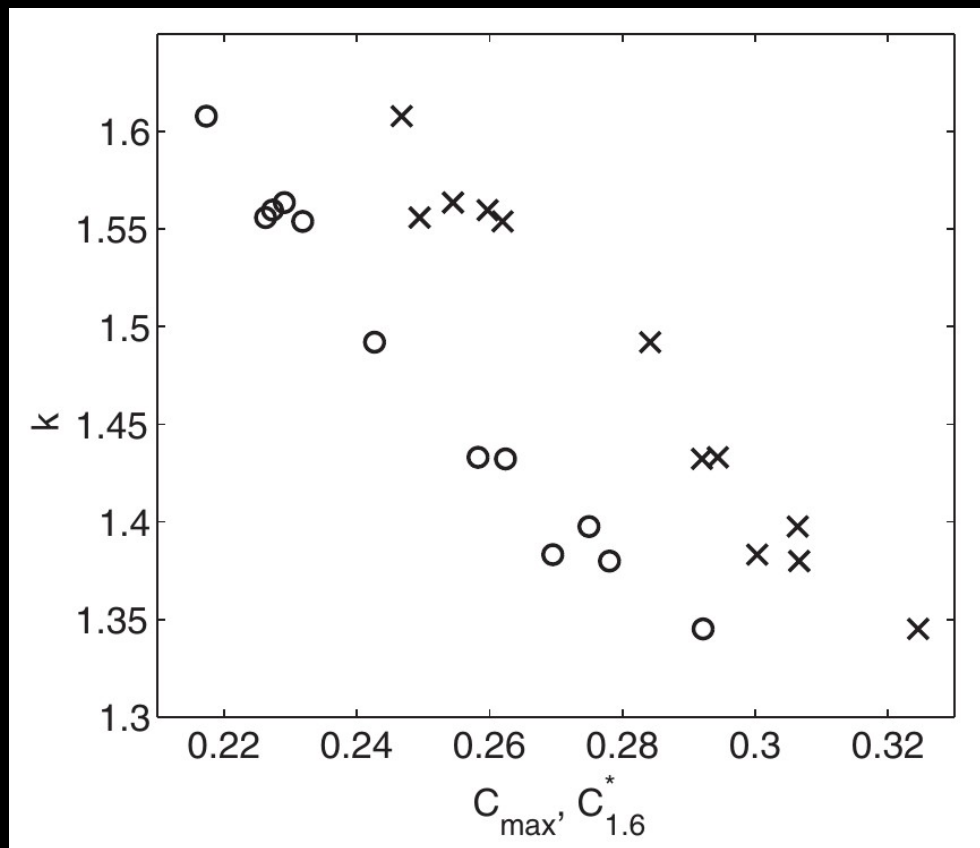
$$\Rightarrow M_{\text{thres}} = M_{\text{thres}}(M_{\text{max}}, R_{\text{max}})$$

Bauswein et al. 2013

$$k = \frac{M_{\text{thres}}}{M_{\text{max}}}$$

← From simulations with different M_{tot}

← TOV property of employed EoS



Fit:
$$k = \frac{M_{thres}}{M_{max}} = -3.6 \frac{GM_{max}}{c^2 R_{1.6}} + 2.38$$

← e.g. from f_{peak} measurement

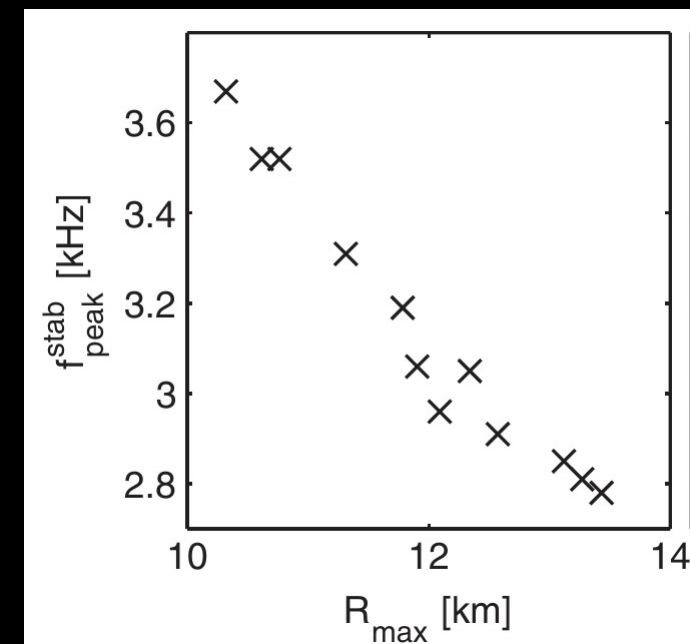
M_{\max} - Method 1

- ▶ Measure several NS mergers – check if postmerger GW emission present
→ M_{thres} estimate
- ▶ (We ~ know radii at that point)
- ▶ Invert fit

$$M_{\text{thres}} = \left(-3.6 \frac{GM_{\max}}{c^2 R_{1.6}} + 2.38 \right) M_{\max}$$

→ M_{\max}

- ▶ Note: already a single/few measurement could provide interesting constraints !!!

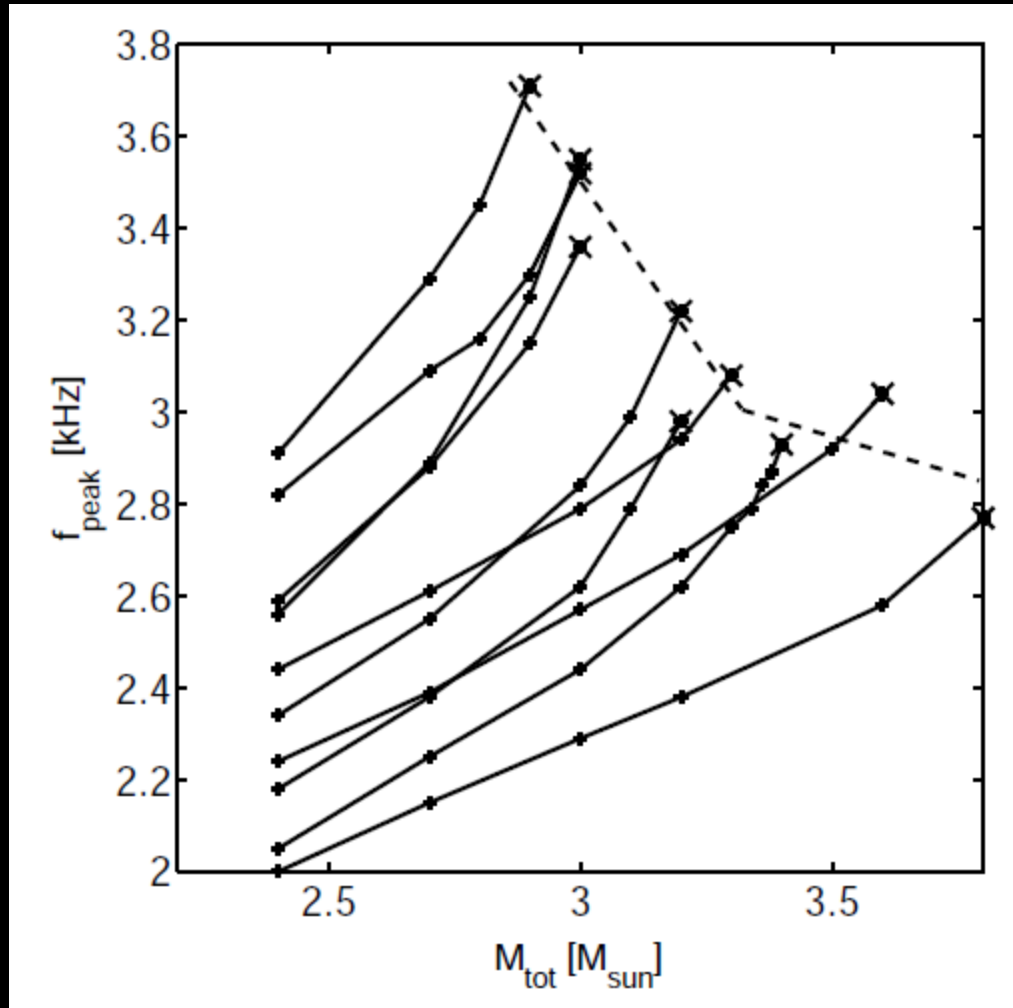


M_{\max} – Method 2

- ▶ Possibly there are not many binaries with $M_{\text{tot}} \sim M_{\text{thres}}$
- ▶ May be hard to measure M_{thres}

- ▶ Method 2: extrapolate from observations as “moderate” binary masses
- ▶ e.g. two events: one with 1.25-1.25 M_{sun} and one with 1.4-1.4 M_{sun}

f_{peak} dependence on total binary mass



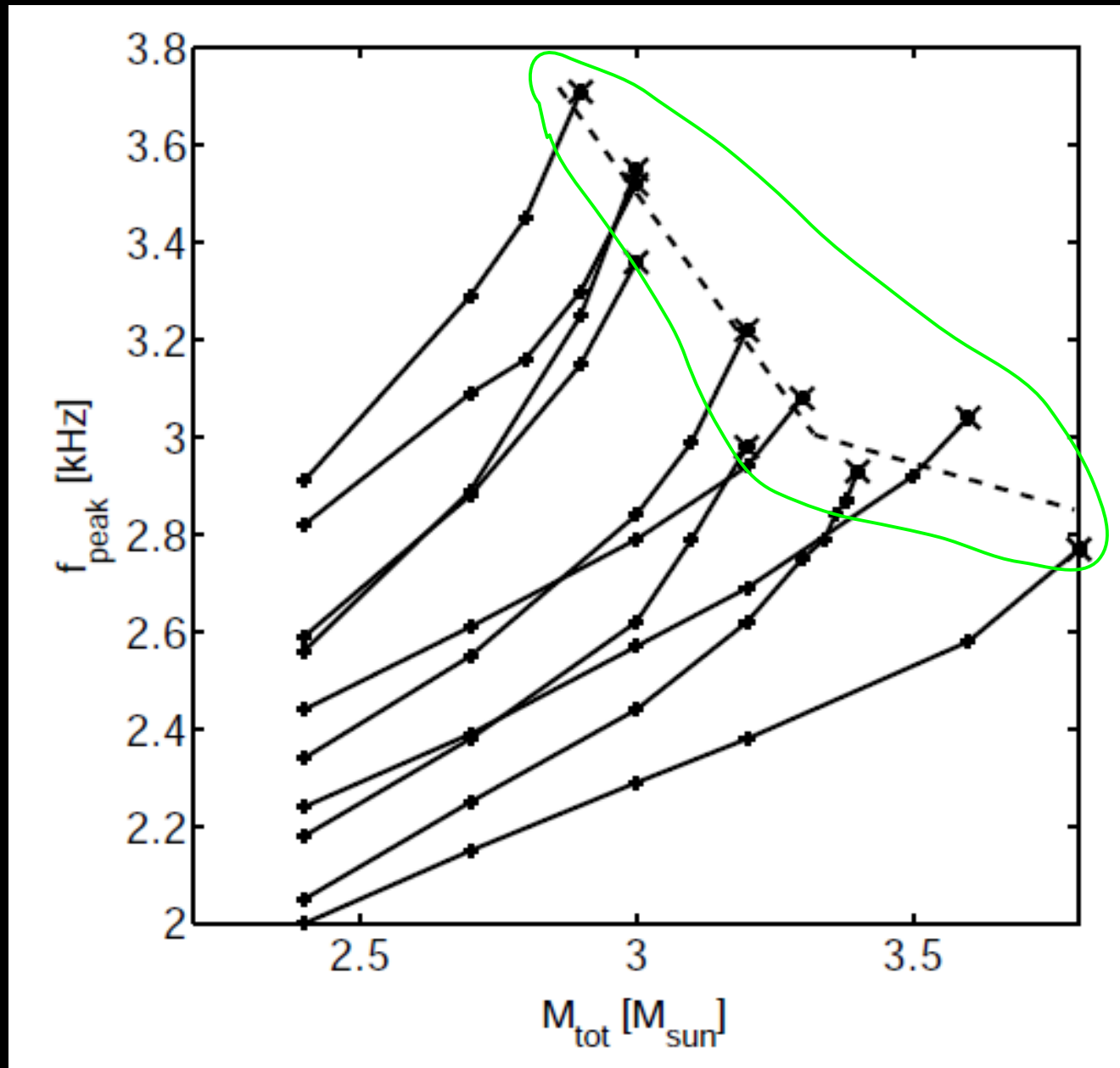
(every single line corresponds to a specific EoS
→ only one line can be the true EoS)

$$f_{\text{peak}} \sim \sqrt{\frac{M}{R^3}}$$

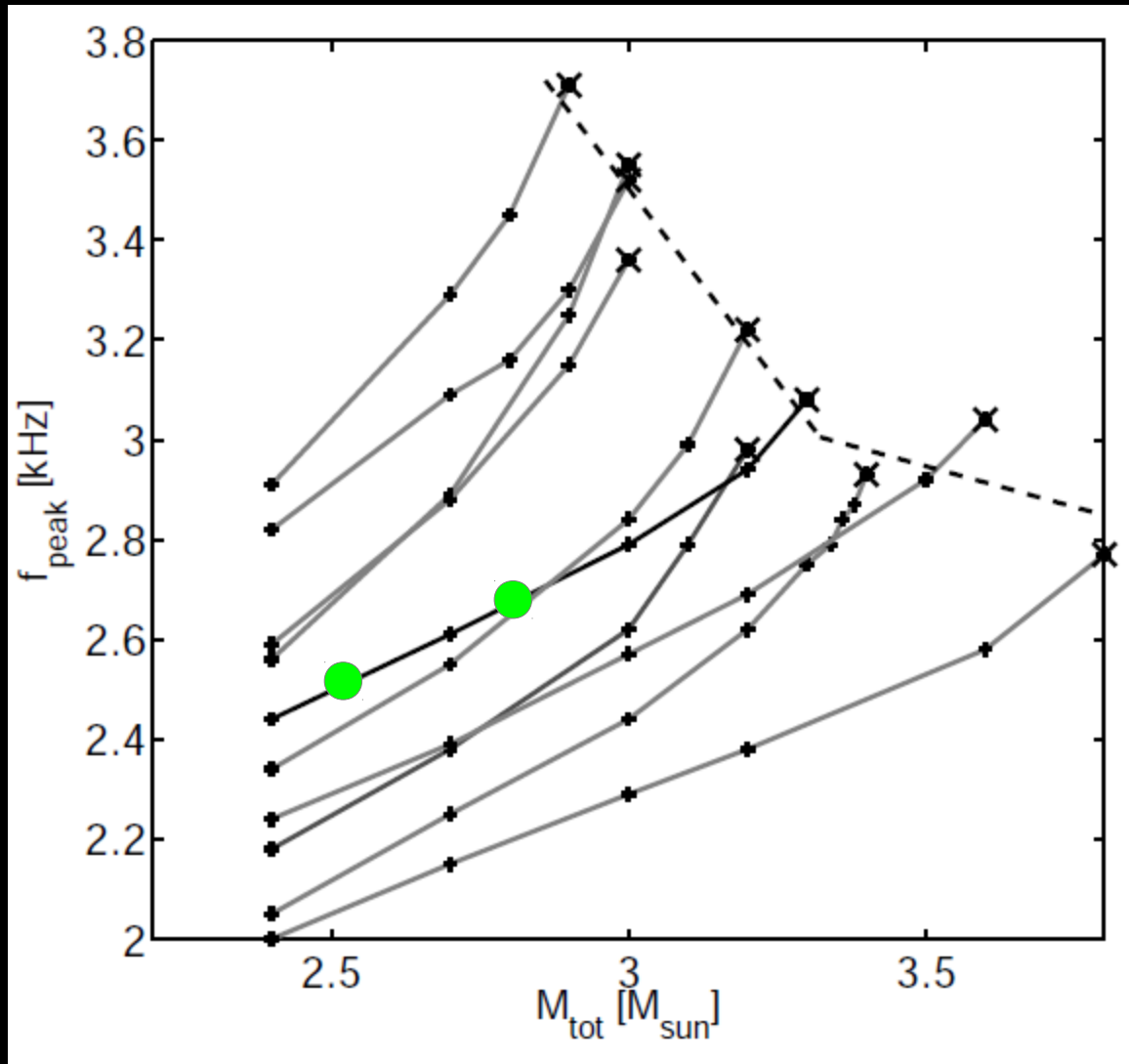
Dominant GW frequency monotone function of M_{tot}

Threshold to prompt BH collapse shows a clear dependence on M_{tot}
(dashed line)

Threshold to prompt BH collapse



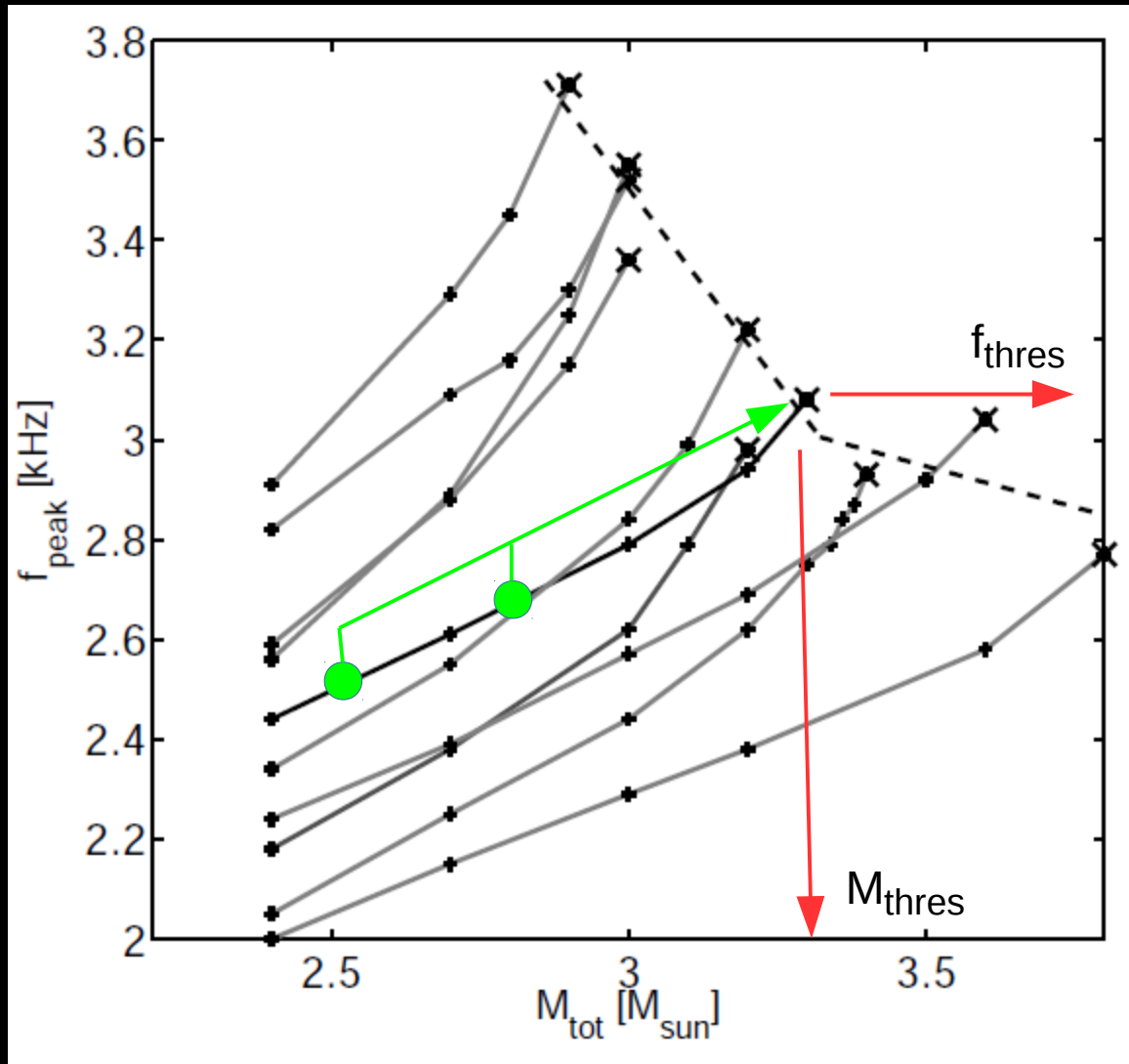
Extrapolation procedure



Details in Bauswein
et al. 2014

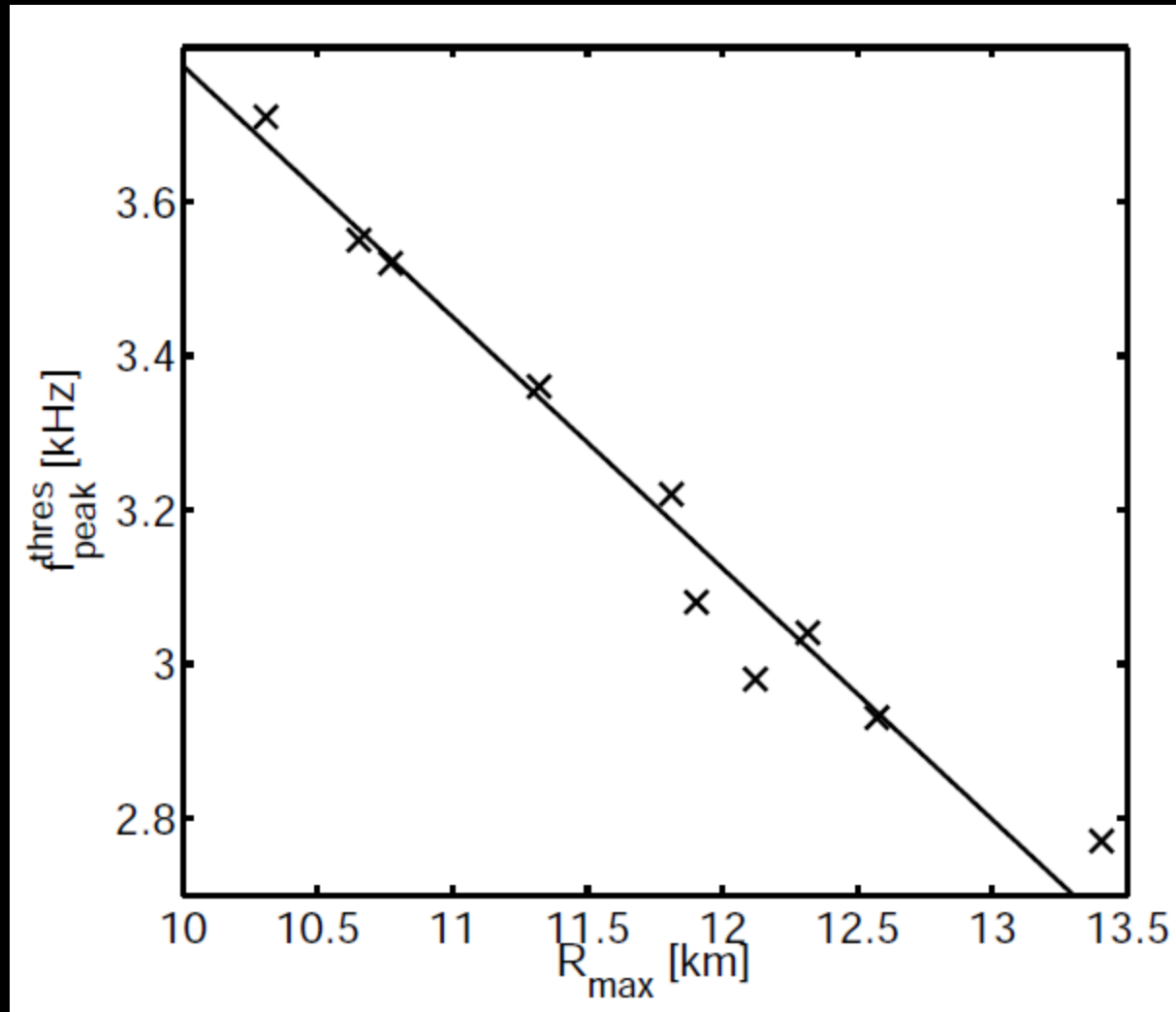
Two f_{peak} measurements at different M_{tot} yield threshold mass
and “threshold frequency” !!!

Extrapolation procedure



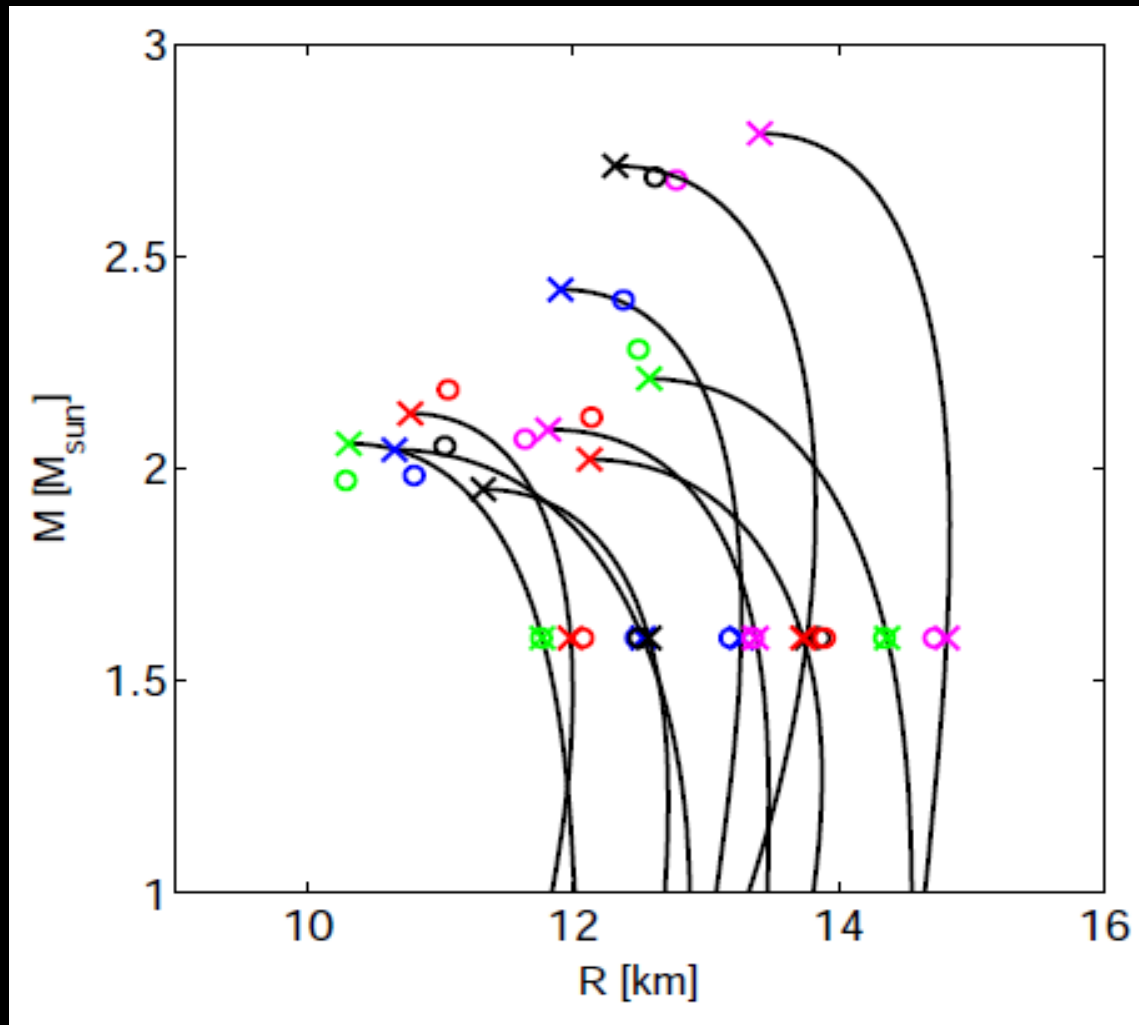
Two f_{peak} measurements at different M_{tot} yield threshold mass and “threshold frequency” !!!

R_{max} determination via extrapolation



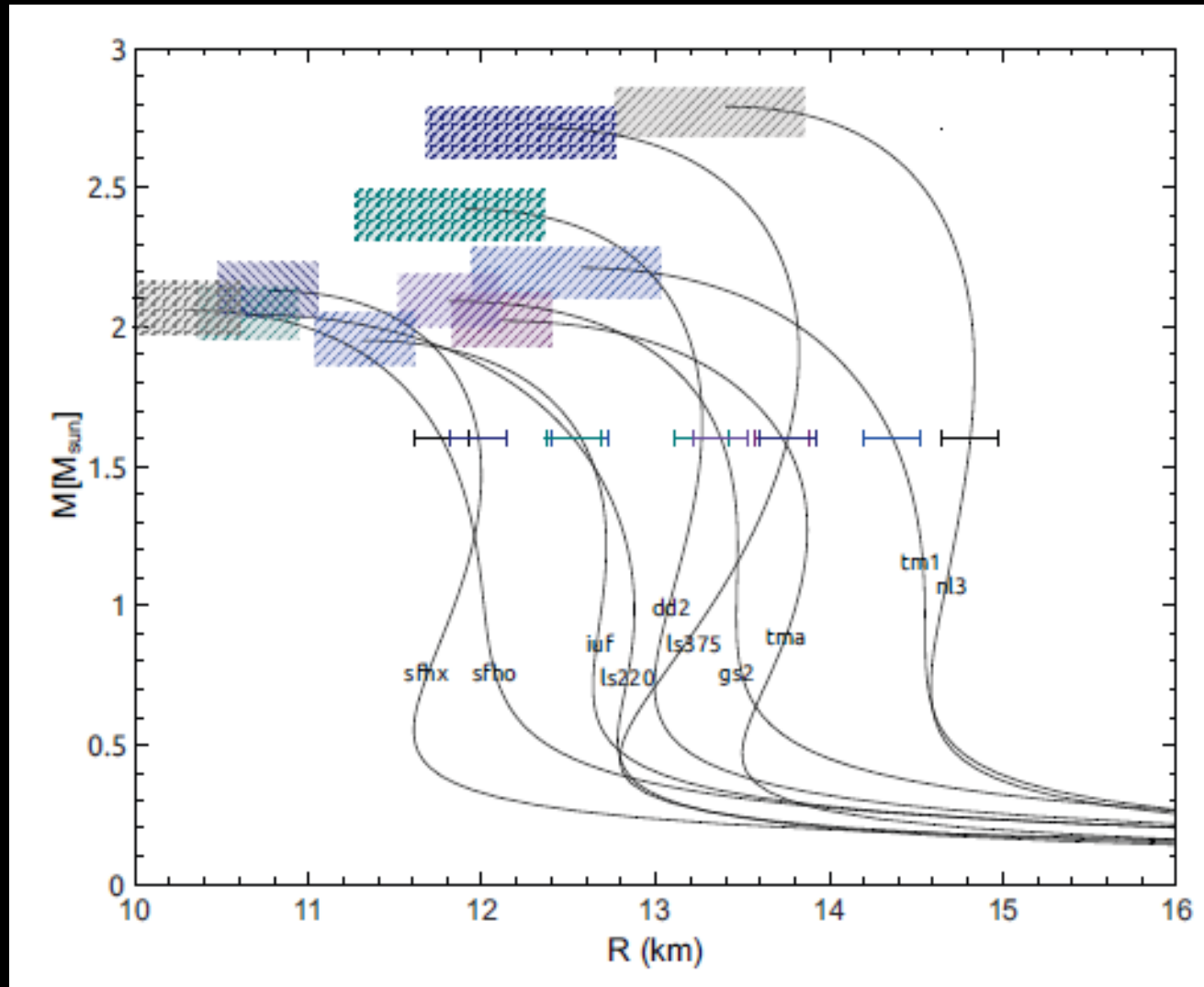
Threshold frequency f_{thres} yields a good estimate of the **radius of the TOV maximum mass configuration** (a few 100 meters)

Maximum mass via extrapolation



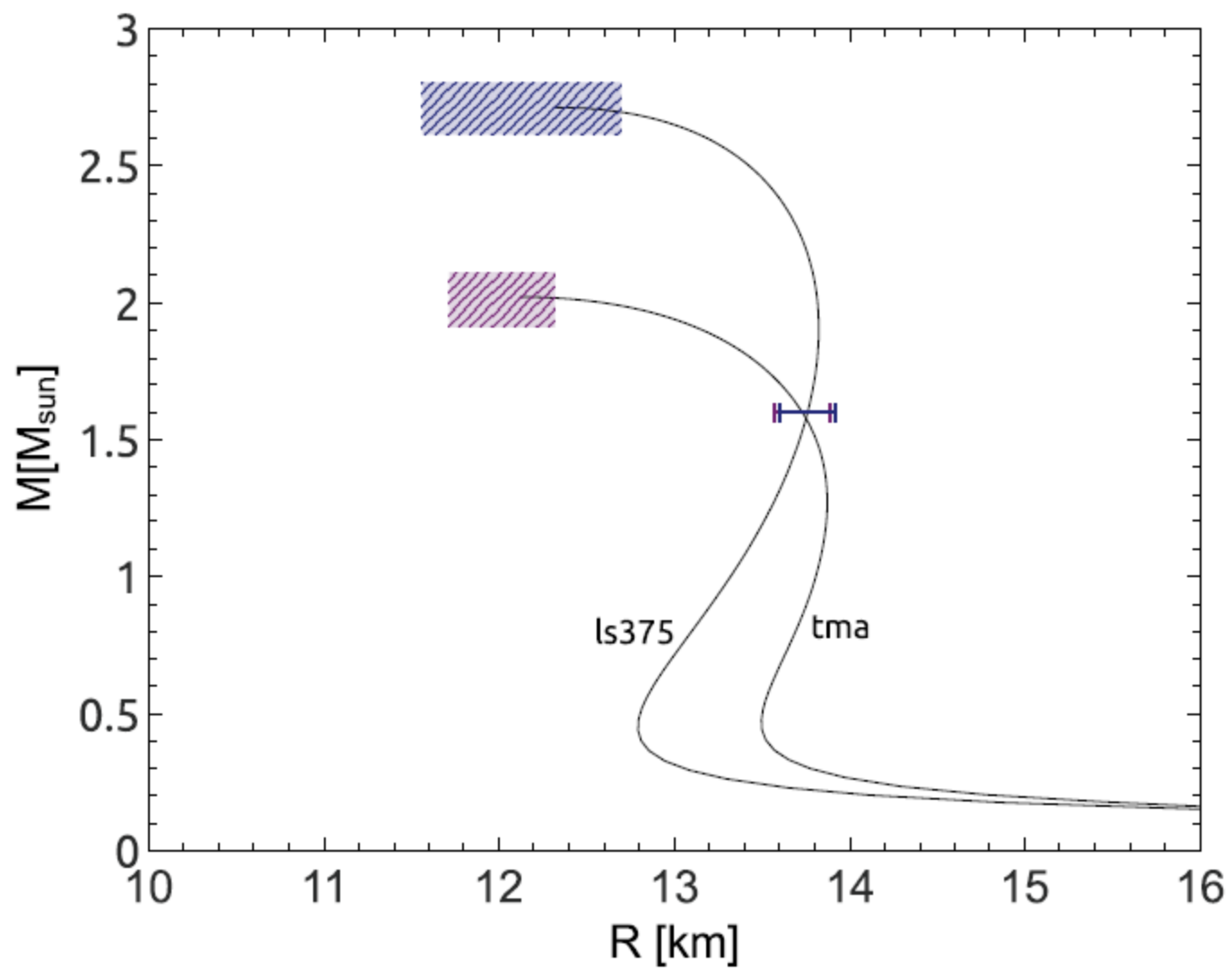
M_{max} within $0.1 M_{\text{sun}}$, R_{max} within a few 100 m
(from f_{peak} detections at common M_{tot})

from two measurements of f_{peak} at moderate M_{tot}

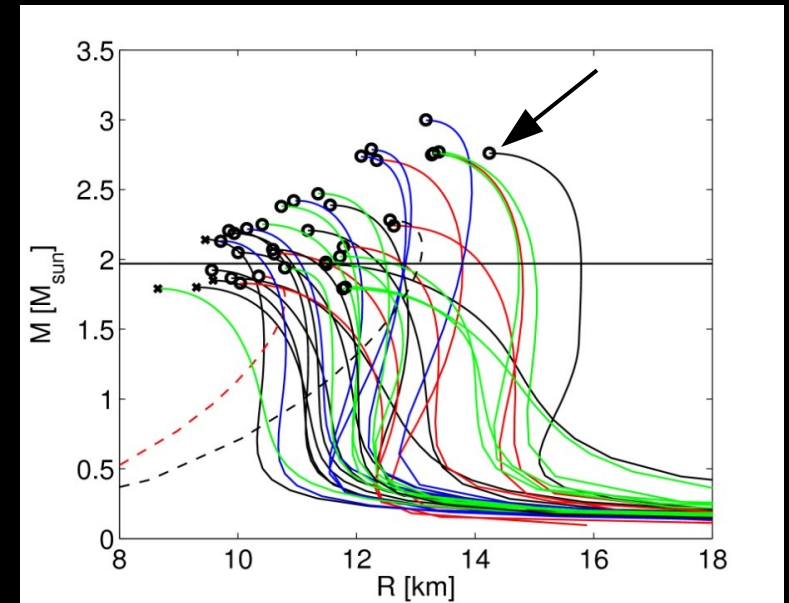
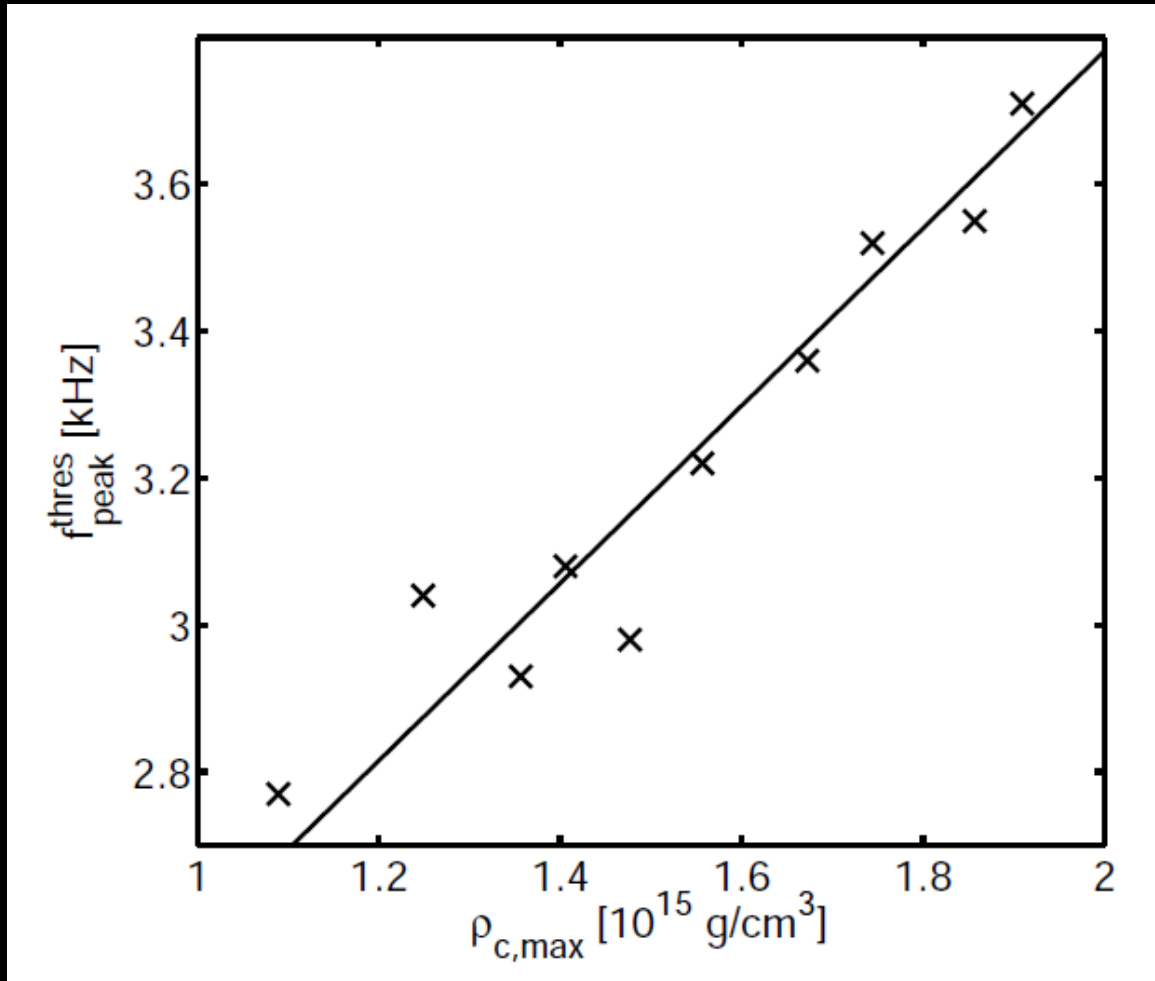


(final error will depend on EoS and exact systems measured)

Note: M_{thres} may also be constrained from prompt collapse directly



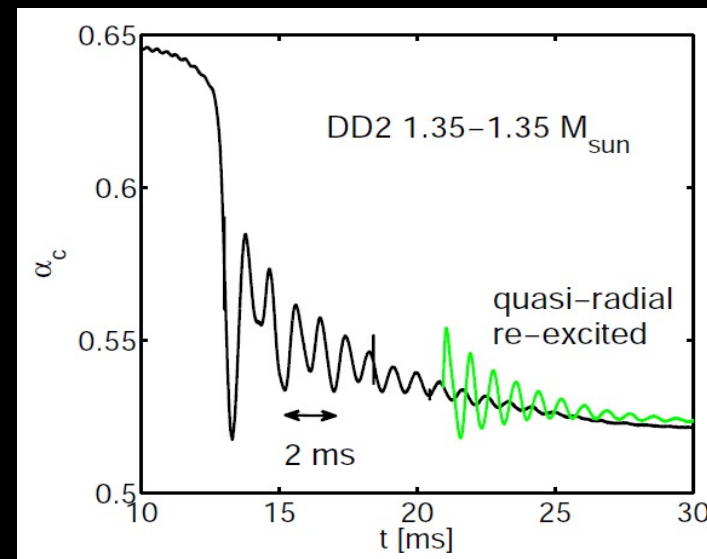
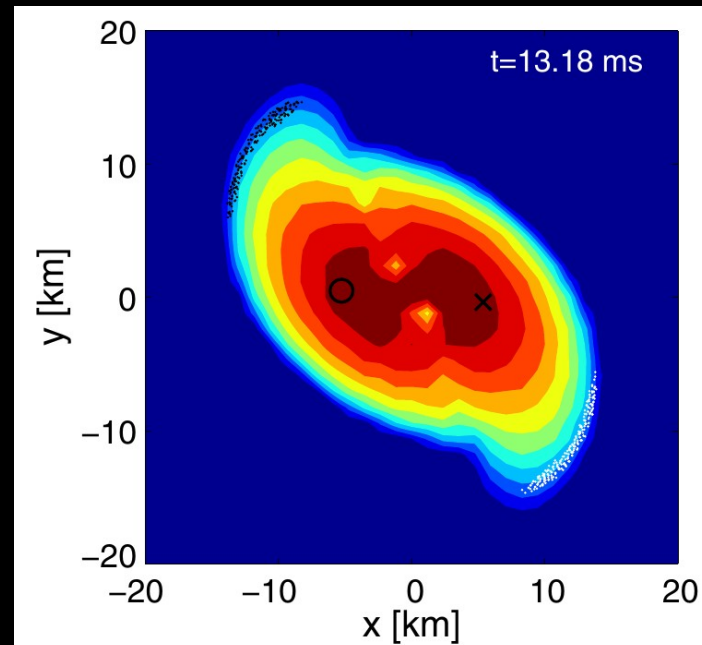
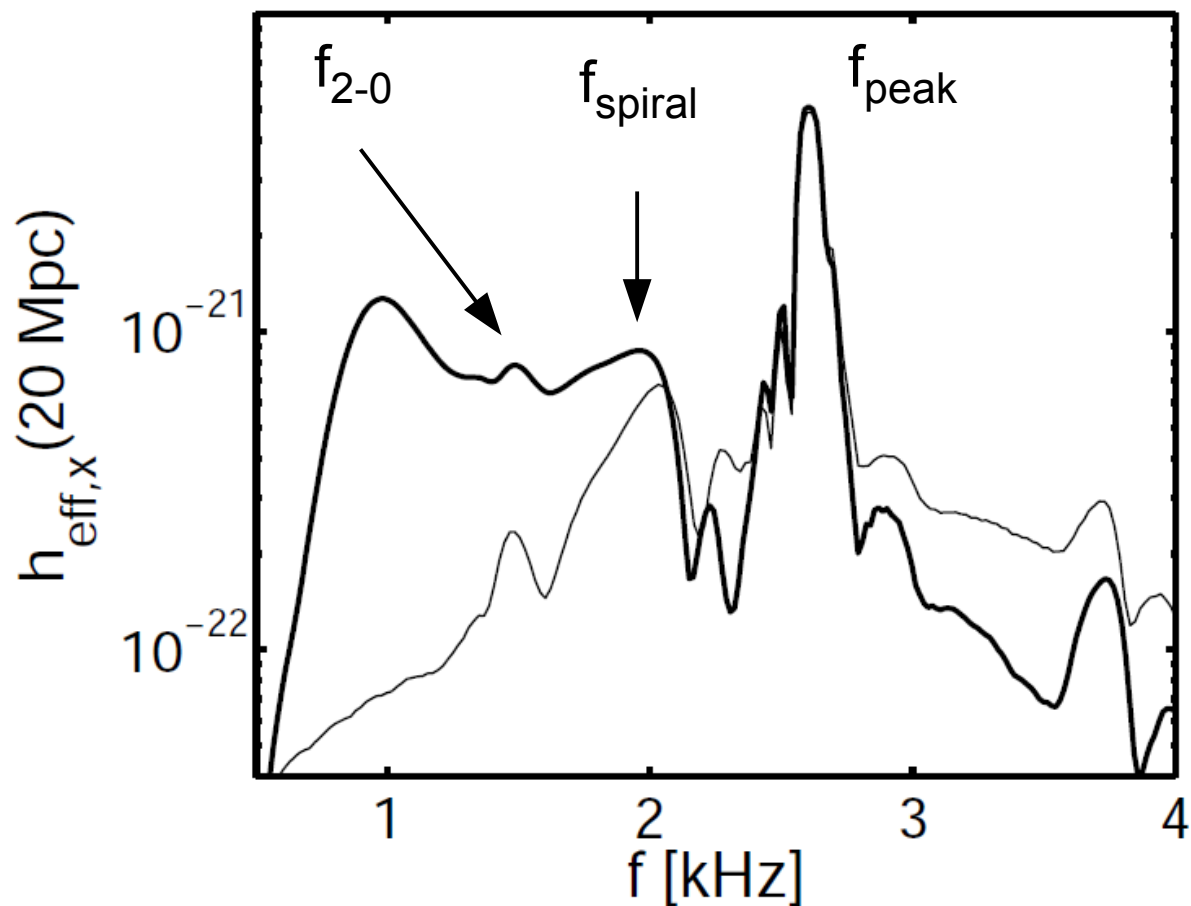
Maximum density via extrapolation



Maximum density of non-rotating NS within 10 per cent

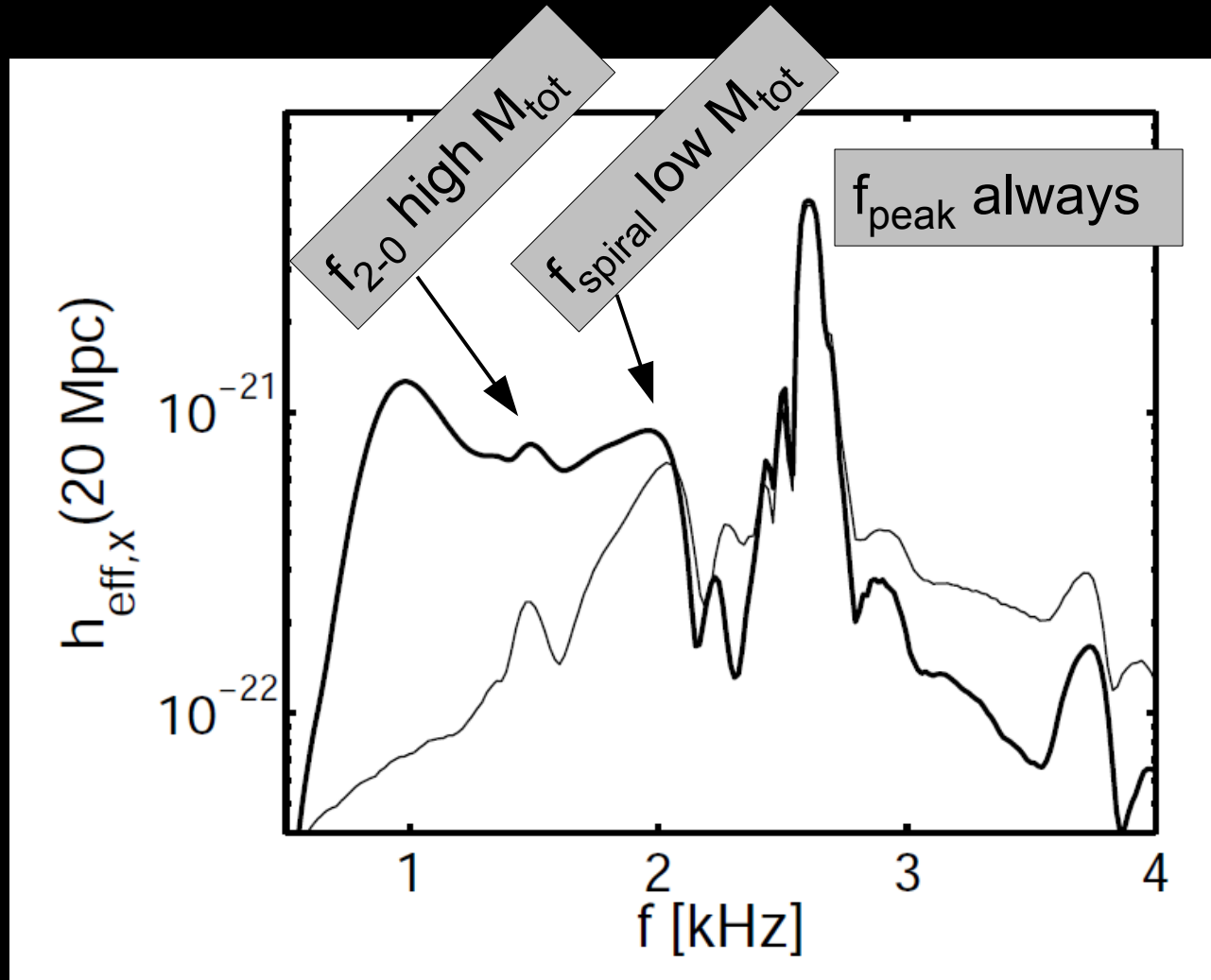
Secondary GW peaks

Generic GW spectrum



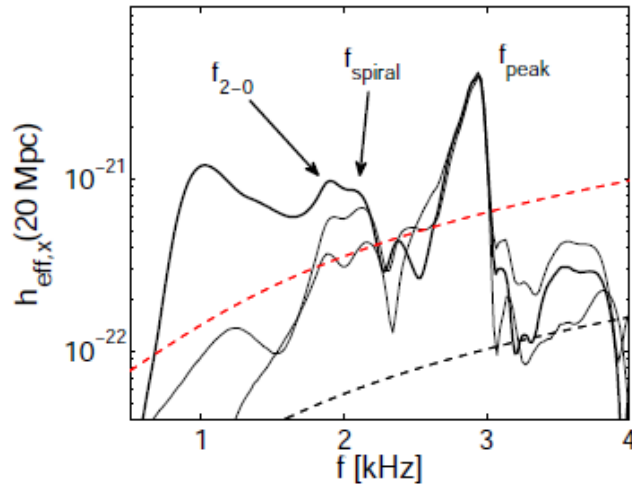
- Orbital motion of antipodal bulges generate peak at f_{spiral}
- Coupling between quasi-radial and quadrupolar mode

Survey of GW spectra

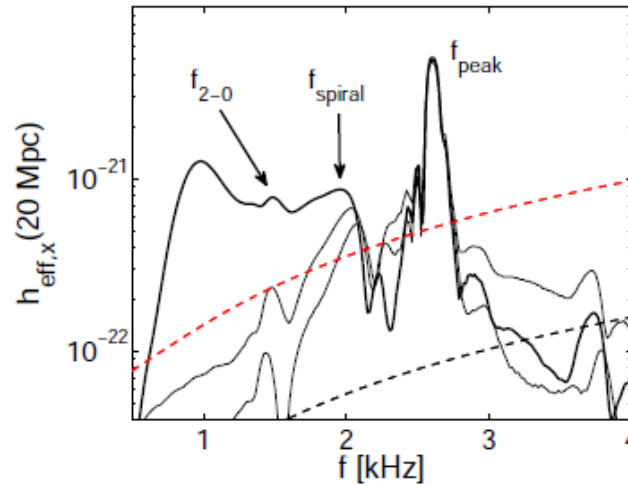


- Considering different models (EoS, M_{tot}): 3 types of spectra depending on presence of secondary features (dominant f_{peak} is always present)
- Secondary frequencies depend on EoS (similar as f_{peak})

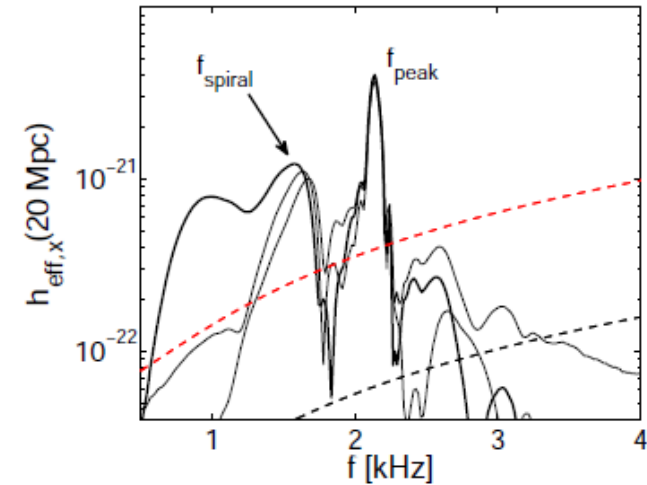
Survey of GW spectra



Type I



Type II



Type III

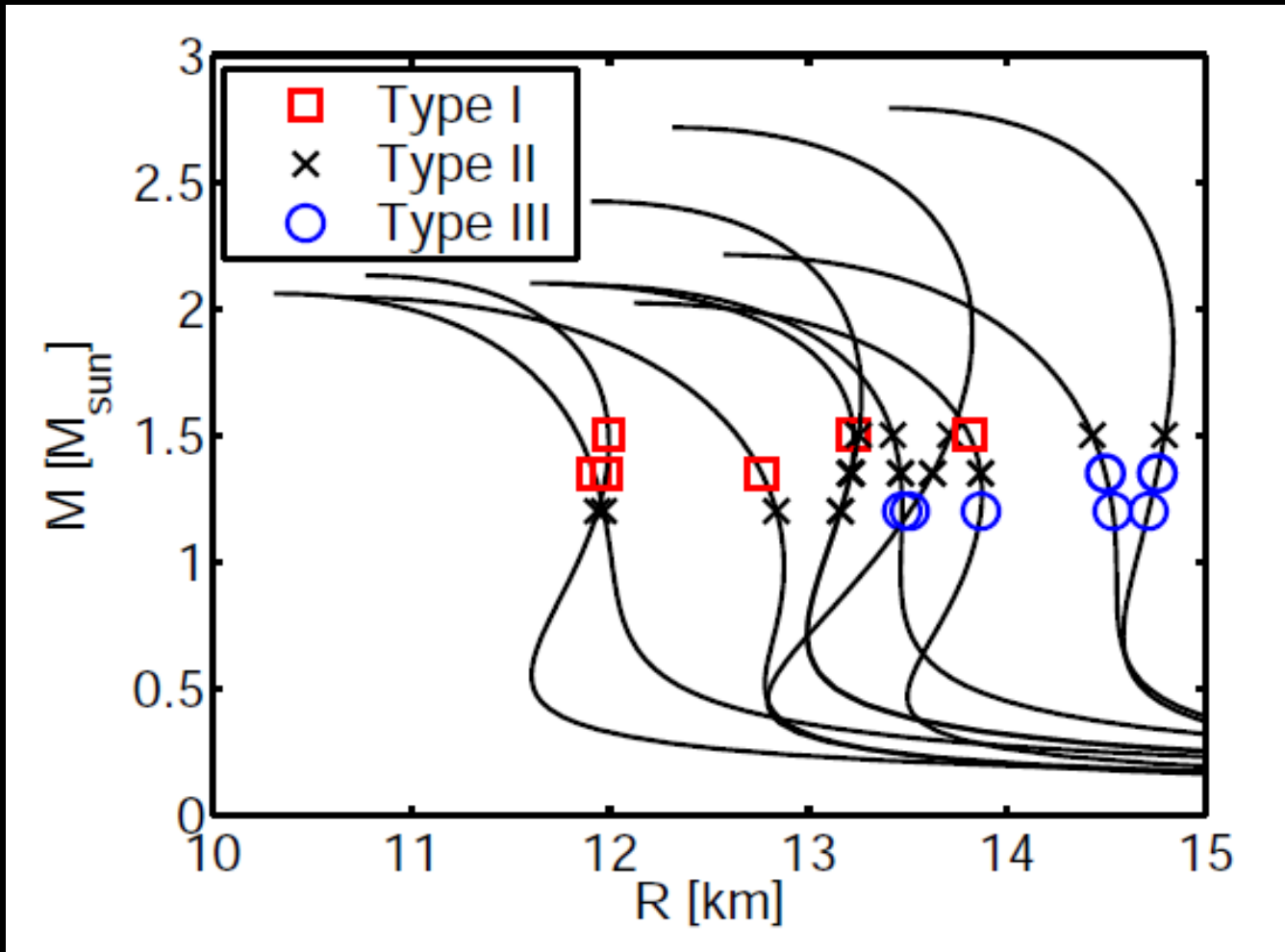
LS220, DD2, NL3 EoS all with $M_{\text{tot}} = 2.7 M_{\text{sun}}$ \rightarrow consider M_{tot} relative M_{thres}

\Rightarrow Depending on binary model (EoS, $M1/2$) **either one or the other or both features** are present / dominant (if you measure a secondary peak you should always think whether it is f_{2-0} or f_{spiral})

Classification scheme

- **Type I:** 2-0 feature dominates, f_{spiral} hardly visible, radial mode strongly excited, observed for soft EoS, relatively high M_{tot}
 - **Type II:** both secondary features have comparable strength, clearly distinguishable, moderate binary masses
 - **Type III:** f_{spiral} dominates, f_{2-0} hardly visible, found for stiff EoS, relatively low binary masses, (central lapse, GW amplitude, ρ_{max} show low-frequency modulation in addition to radial oscillation)
-
- Different types show also different dynamical behavior, e.g. in central lapse, maximum density, GW amplitude,
 - High mass / low mass relative to threshold binary mass for prompt BH collapse (\rightarrow EoS dependent)
 - Continuous transition between different types: a given EoS shows all types depending on M_{tot} : Type III for low M_{tot} \rightarrow Type I towards M_{thres}

Classification scheme



Type of M_1 - M_2 merger indicate at $M_{\text{tot}}/2 = M_1$

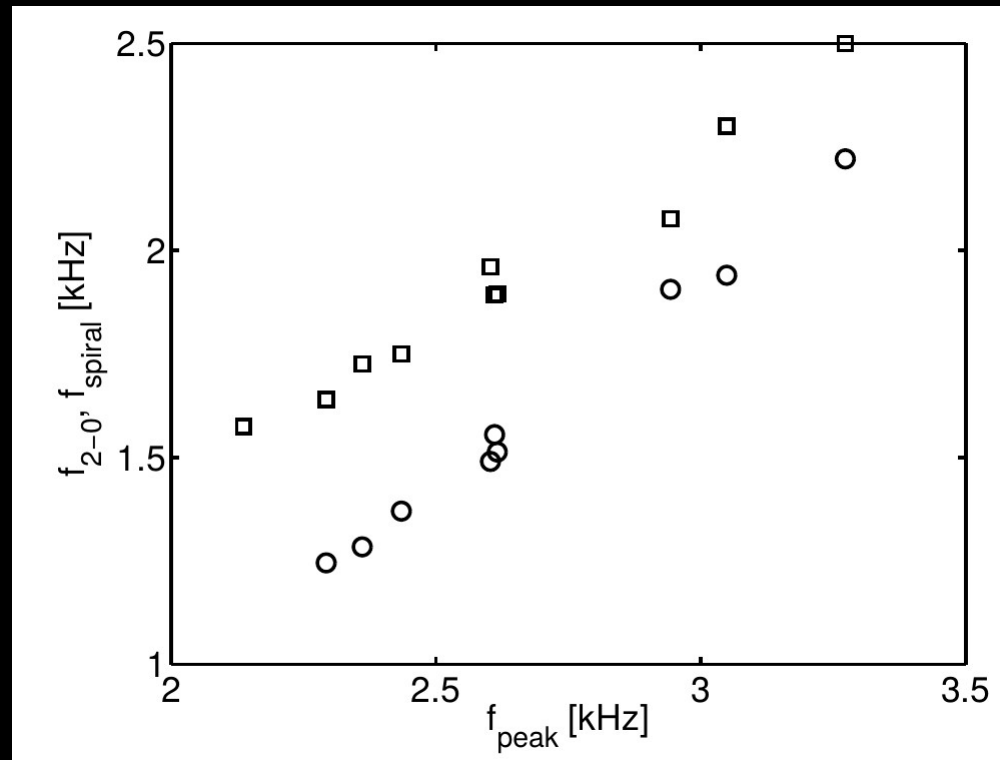
Bauswein et al. 2015

(Continuous transition between types \rightarrow tentative association)

For $M_{\text{tot}} = 2.7 M_{\text{sun}}$ all Types are possible depending on EoS

Classification intuitive: merger dynamics affected by compactness

Dependencies of secondary frequencies



1.35-1.35 M_{sun}

Clark et al. 2016

→ secondary frequencies are essentially given by dominant frequency

Conclusions

- ▶ NS radii scale tightly with dominant postmerger GW frequency
- ▶ GW data analysis → radii measurable with precision of a few 100 m
- ▶ a single measurement of only the dominant frequency is sufficient
- ▶ Collapse behavior determined by EoS → maximum mass M_{\max} of non-rotating NSs
 - directly determine M_{thres} by several merger events
 - extrapolation method: 2 events with “moderate” binary masses
- ▶ Radius R_{\max} of maximum-mass TOV star
- ▶ Maximum density in NSs
- ▶ Secondary peaks generated by distinct mechanisms, EoS dependent, classification scheme based on presence of secondary features