# How to verify the Quark-Gluon Plasma with Gravitational-Wave Detectors

Talk at the Hirschegg Workshop "Neutron Star Mergers: From gravitational waves to nucleosythesis"Hirschegg (Austria), 16.01.2017

<u>Matthias Hanauske</u>, Kentaro Takami, Luke Bovard, Luciano Rezzolla, Filippo Galeazzi, José A. Font, and Horst Stöcker

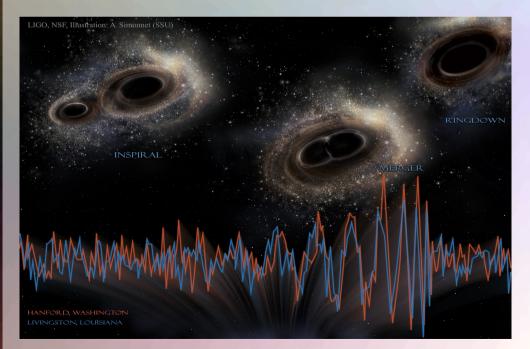
> Frankfurt Institute for Advanced Studies Johann Wolfgang Goethe-University Institute for Theoretical Physics Department of Relativistic Astrophysics Frankfurt am Main, Germany

How to verify the Quark-Gluon Plasma with Gravitational-Wave Detectors

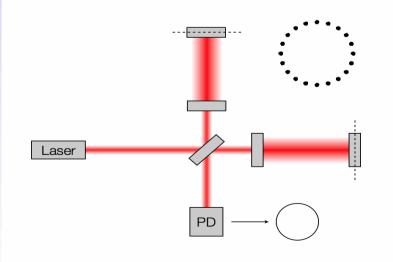
- 1. Introduction
- 2. Numerical General Relativity
- 3. Equation of State and the Hadron-Quark Phase Transition
- 4. Internal Properties of Hypermassive Neutron and Hybrid Stars in the Post-Merger Phase
- 5. Outlook and Summary

# First observation gravitational waves from binary black hole merger by LIGO

<u>Facts about GW150914</u> Merger of two black holes of around 36 and 29 solar masses Distance: 410 Mpc (1340 million Ly)





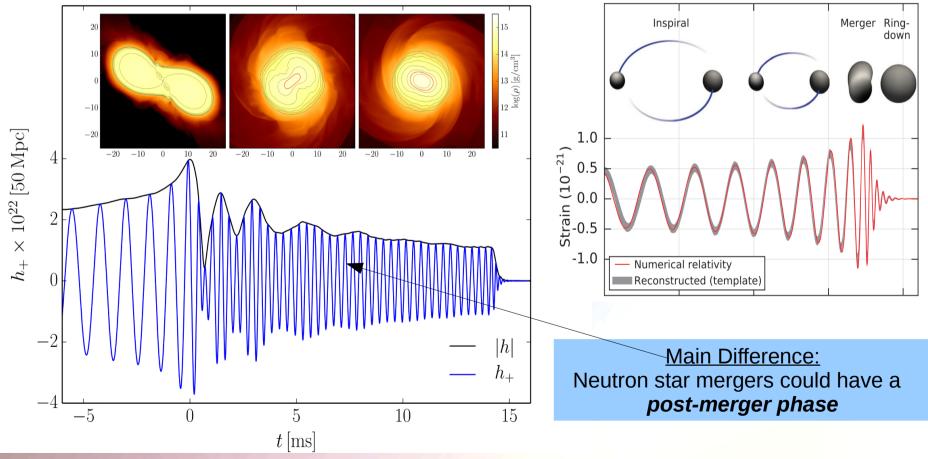


Credit: Les Wade from Kenyon College.

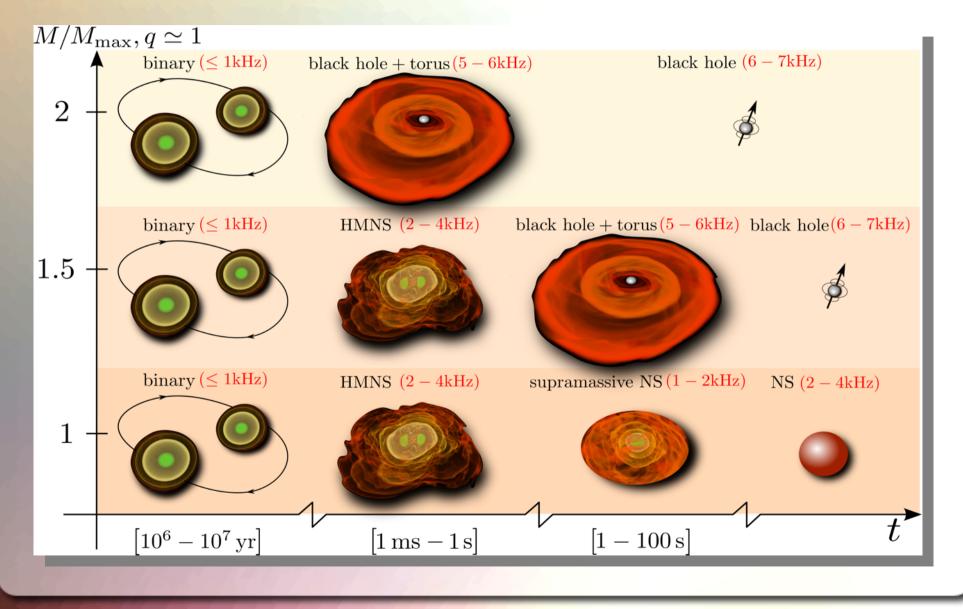
# Gravitational Waves from Binary Neutron Star Mergers

**Neutron star merger (Simulation)** 

Merger of two Black Holes

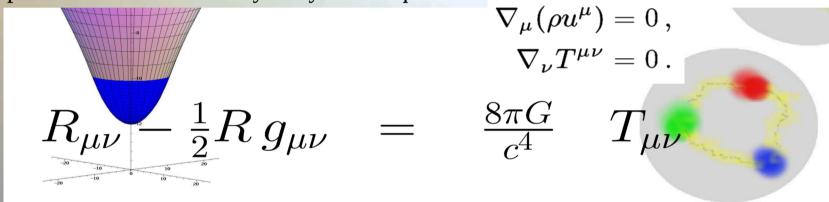


### The Neutron Star Merger Product



# Relativistic Hydrodynamics and Numerical General Relativity

The time evolution of a merger scenario of a binary neutron star system requires the (3+1)-Split of the Einstein- and hydrodynamic equations.



$$g_{\mu\nu} = \begin{pmatrix} -\alpha^{2} + \beta_{i}\beta^{i} & \beta_{i} \\ \beta_{i} & \gamma_{ij} \end{pmatrix} \xrightarrow{(3+1)} decomposition of spacetime} d\tau^{2} = \alpha^{2}(t, x^{j})dt^{2} \qquad x_{t+dt}^{i} = x_{t}^{i} - \beta^{i}(t, x^{j})dt \qquad \Sigma_{t}$$

All figures and equations from: Luciano Rezzolla, Olindo Zanotti: Relativistic Hydrodynamics, Oxford Univ. Press, Oxford (2013)

### The ADM equations

The ADM (Arnowitt, Deser, Misner) equations come from a reformulation of the Einstein equation using the (3+1) decomposition of spacetime.

$$\partial_{t}\gamma_{ij} = -2\alpha K_{ij} + \mathscr{L}_{\beta}\gamma_{ij}$$

$$= -2\alpha K_{ij} + D_{i}\beta_{j} + D_{j}\beta_{i}$$

$$\partial_{t}K_{ij} = -D_{i}D_{j}\alpha + \beta^{k}\partial_{k}K_{ij} + K_{kj}\partial_{j}\beta^{k} + K_{kj}\partial_{i}\beta^{k}$$

$$+ \alpha \left( {}^{(3)}R_{ij} + KK_{ij} - 2K_{ik}K^{k}_{j} \right) + 4\pi\alpha \left[ \gamma_{ij} \left( S - E \right) - 2S_{ij} \right]$$
Time evolving part of ADM
$$D_{j}(K^{ij} - \gamma^{ij}K) = 8\pi S^{i}$$

$$(^{(3)}R + K^{2} - K_{ij}K^{ij} = 16\pi E$$
Constraints on each hypersurface

Three dimensional covariant derivative

 $D_
u \coloneqq \gamma^\mu_{\,\,
u} 
abla_\mu = (\delta^\mu_
u + n_
u n^\mu) 
abla_\mu$ 

Three dimensional Riemann tensor

 ${}^{(3)}R^{\mu}_{\phantom{\mu}\nu\kappa\sigma} = \partial^{(3)}_{\kappa}\Gamma^{\mu}_{\nu\sigma} - \partial^{(3)}_{\sigma}\Gamma^{\mu}_{\nu\kappa} + {}^{(3)}\Gamma^{\mu}_{\lambda\kappa}{}^{(3)}\Gamma^{\lambda}_{\nu\sigma} - {}^{(3)}\Gamma^{\mu}_{\lambda\sigma}{}^{(3)}\Gamma^{\lambda}_{\nu\kappa}$ 

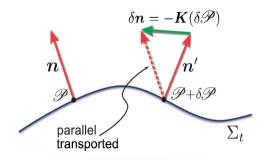
 $^{(3)}\Gamma^{lpha}_{eta\gamma}=rac{1}{2}\gamma^{lpha\delta}\left(\partial_{eta}\gamma_{\gamma\delta}+\partial_{\gamma}\gamma_{\deltaeta}-\partial_{\delta}\gamma_{eta\gamma}
ight)$ 

Spatial and normal projections of the energy-momentum tensor:

$$egin{aligned} S_{\mu
u} \coloneqq \gamma^{lpha}_{\ \ \mu} \gamma^{eta}_{\ \ 
\nu} T_{lphaeta}\,, \ S_{\mu} \coloneqq -\gamma^{lpha}_{\ \ \mu} n^{eta} T_{lphaeta}\,, \ S \coloneqq S^{\mu}_{\ \ \mu}\,, \ E \coloneqq n^{lpha} n^{eta} T_{lphaeta}\,. \end{aligned}$$

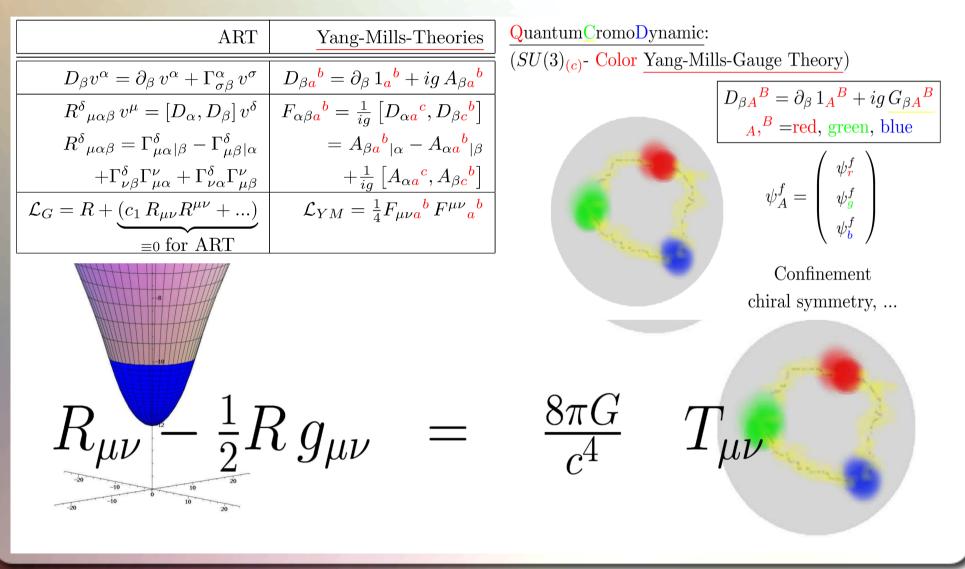


$$K_{\mu
u} \coloneqq -\gamma^{\lambda}_{\ \mu} 
abla_{\lambda} n_{
u}$$

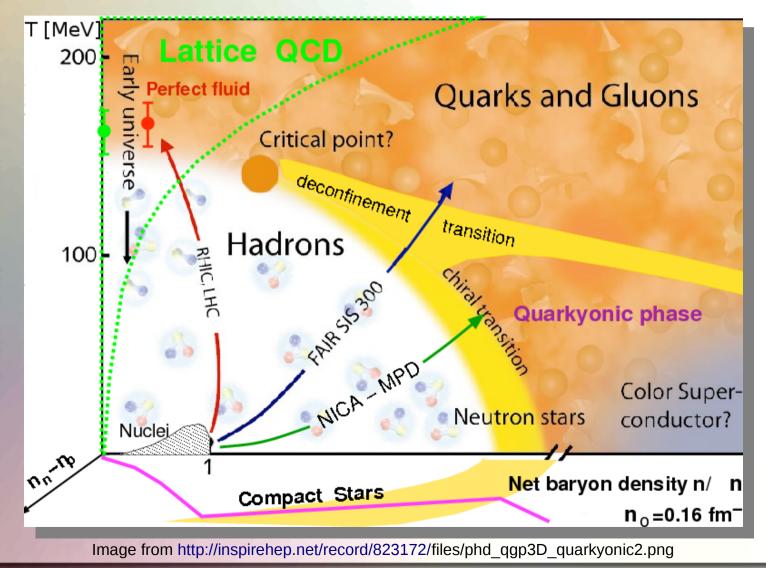


All figures and equations from: Luciano Rezzolla, Olindo Zanotti: Relativistic Hydrodynamics, Oxford Univ. Press, Oxford (2013)

# General Relativity and Quantum Cromodynamics

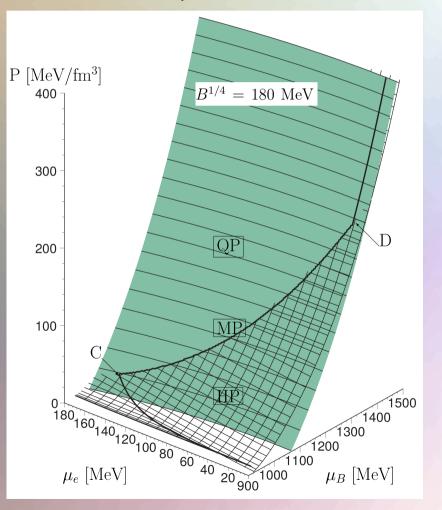


# The Equation of State and the QCD Phase Diagram



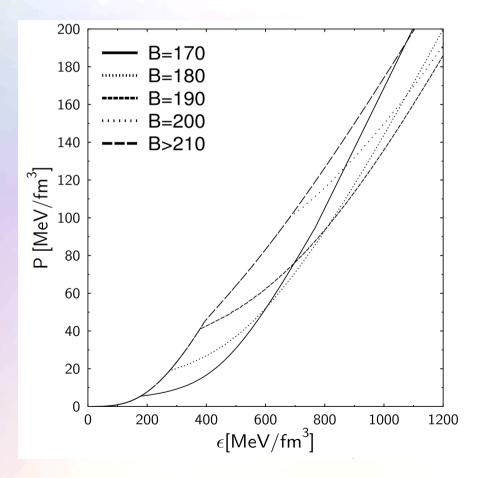
### The Gibbs Construction

Hadronic and quark matter surface:

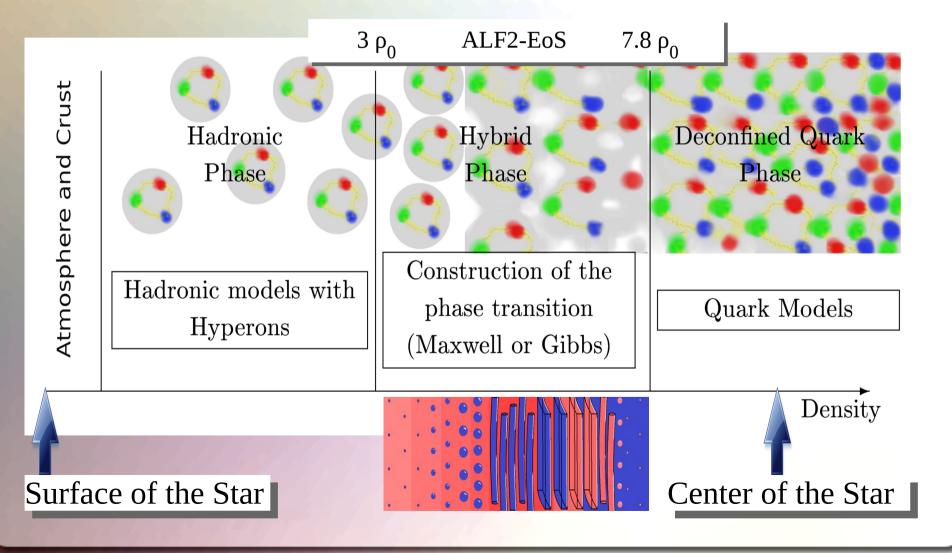


Charge neutrality condition is only globally realized

$$\rho_e := (1 - \chi) \rho_e^H(\mu_B, \mu_e) + \chi \rho_e^Q(\mu_B, \mu_e) = 0.$$

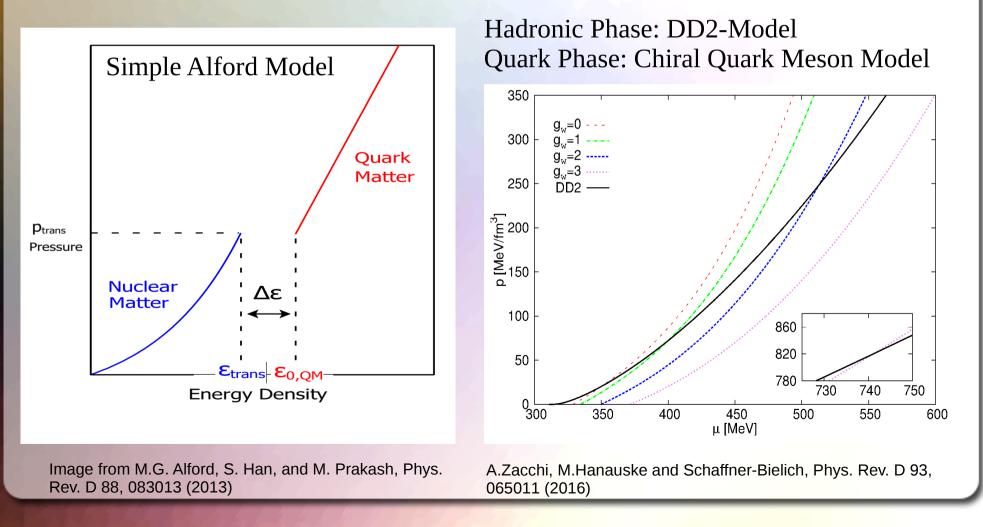


# The QCD – Phase Transition and the Interior of a Hybrid Star



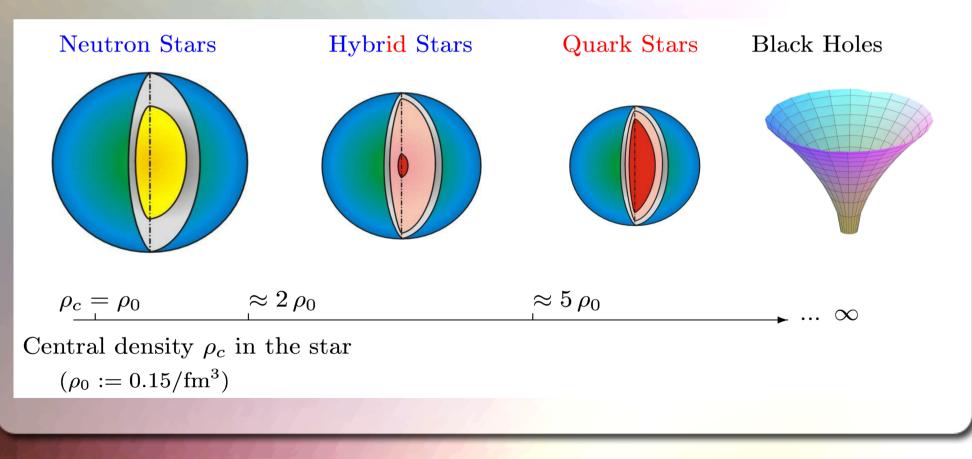
#### The Maxwell Construction No Mixed Phase Region

Pressure and baryon chemical potential stays constant, while the density and the charge chemical potential jump discontinuously during the phase transition.



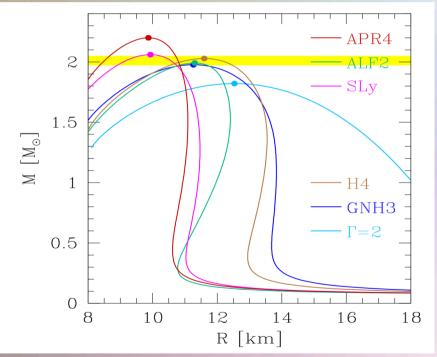
### The Compact Star Zoo

Depending on the model used, the compact star zoo consists of different inhabitants: e.g. neutron stars with and without hyperons, quark stars and strange quark stars, hybrid stars with color superconducting quark matter, hybrid stars with Bose-Einstein condensates of antikaons.



# Numerical Setup

Several different EOSs : ALF2, APR4, GNH3, H4 and Sly, approximated by piecewise polytopes. Thermal ideal fluid component ( $\Gamma$ =2) added to the nuclearphysics EOSs.



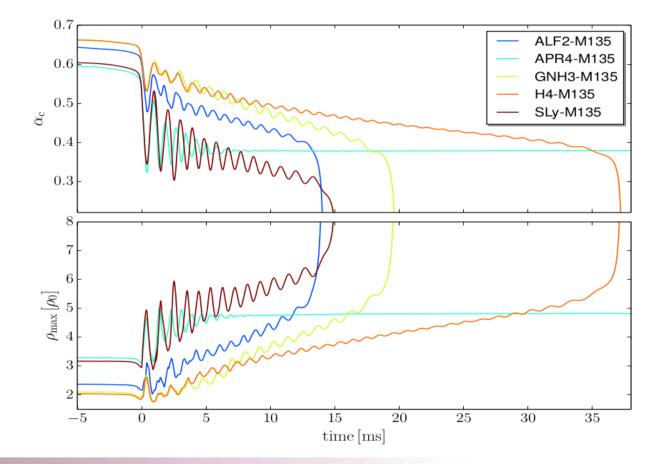
LS220: Temperature dependent EOS-table (Lattimer-Swesty) BSSNOK conformal traceless formulation of the ADM equations. 3+1 Valencia formulation and high resolution shock capturing methods for the hydrodynamic evolution. Full general relativity using the **Einstein-Toolkit** and the **WHISKY/WhiskyTHC code** for the general-relativistic hydrodynamic equations.

#### **Grid Structure:**

Adaptive mesh refinement (six ref. levels) Grid resolution: (from 221 m to 7.1 km) Outer Boundary: 759 km Initial separation of stellar cores: 45 km

# **HMNS Evolution** for different EoSs

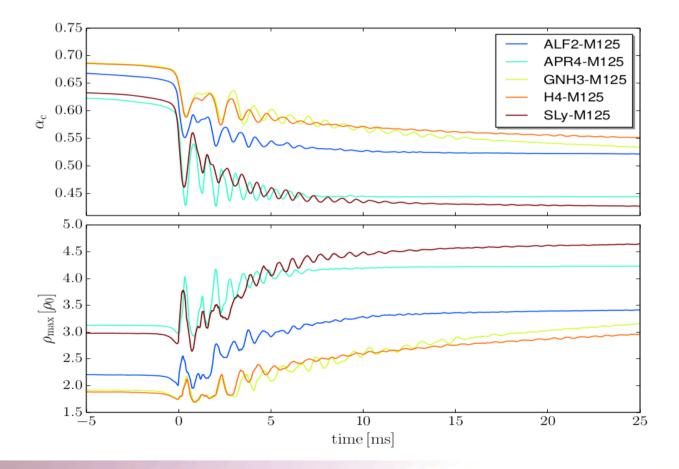
High mass simulations (M=1.35)



Central value of the lapse function  $\alpha_c$  (upper panel) and maximum of the rest mass density  $\rho_{max}$  in units of  $\rho_0$  (lower panel) versus time for the high mass simulations.

### **HMNS Evolution** for different EoSs

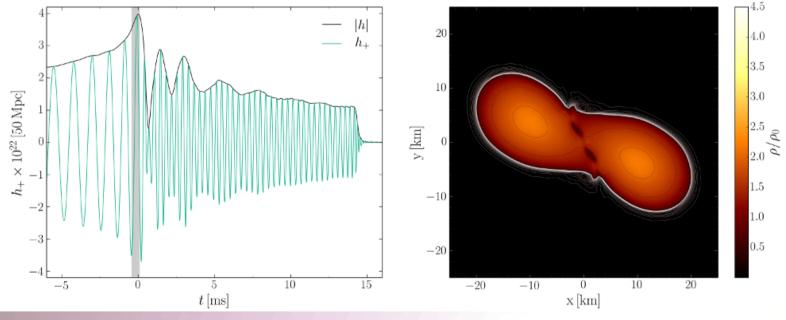
Low mass simulations (M=1.25)



Central value of the lapse function  $\alpha_c$  (upper panel) and maximum of the rest mass density  $\rho_{max}$  in units of  $\rho_0$  (lower panel) versus time for the low mass simulations .

# Evolution of the rest-mass density distribution

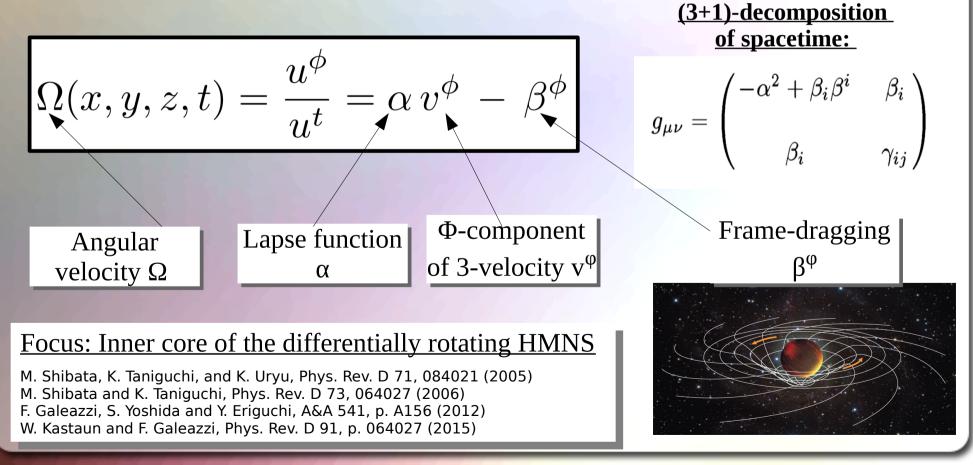
ALF2, High mass model: Mixed phase region starts at  $3\rho_0$ Initial NS mass: 1.35 M<sub>solar</sub>



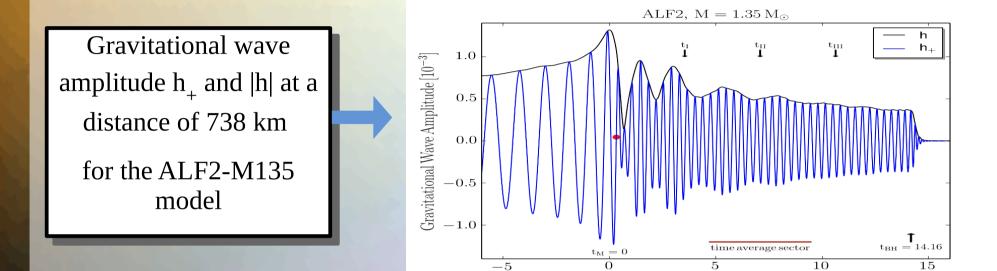
Gravitational wave amplitude at a distance of 50 Mpc Rest mass density distribution  $\rho(x,y)$ in the equatorial plane in units of the nuclear matter density  $\rho_0$ 

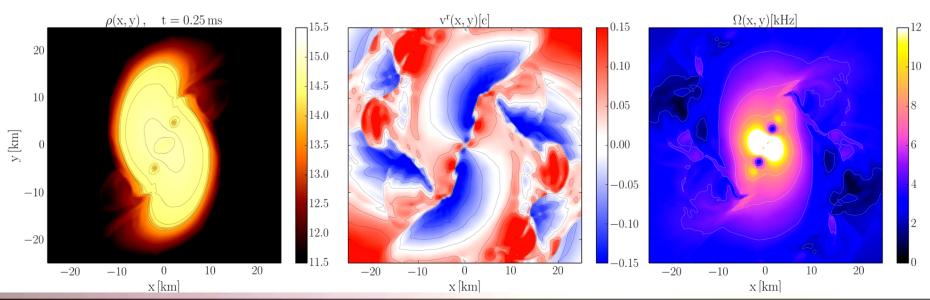
# The Angular Velocity in the (3+1)-Split

The angular velocity  $\Omega$  in the (3+1)-Split is a combination of the lapse function  $\alpha$ , the  $\varphi$ -component of the shift vector  $\beta^{\varphi}$  and the 3-velocity  $v^{\varphi}$  of the fluid (spatial projection of the 4-velocity **u**):



## EoS: ALF2, M=1.35 Post-Merger Phase





Time [ms]

### Rotation Profiles (ALF2-1.35 Model)

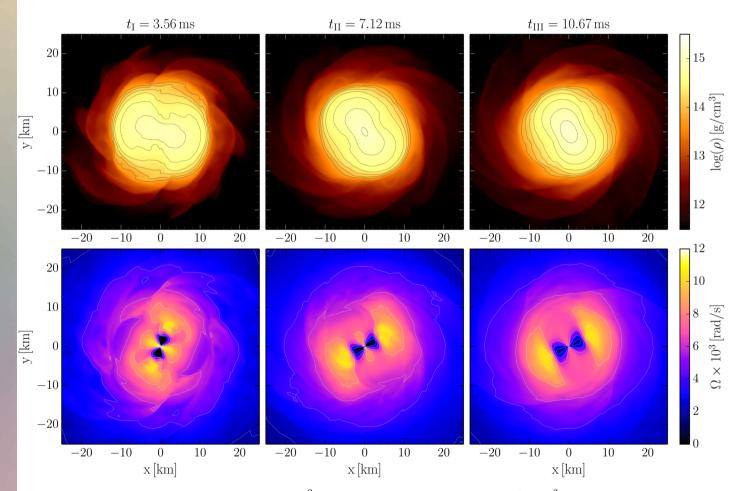


FIG. 3. Logarithm of the rest mass density  $Log(\rho)$  [g/cm<sup>3</sup>] (upper row) and fluid angular velocity  $\Omega \times 10^3$  [rad/s] (lower row) in the xy-plane for the ALF2-M135 model at three different post-merger times. The iso-contour curves have been drawn at 13.8 + 0.2n (upper row) and 2n (lower row),  $n \in \mathbb{N}$ .

### Rotation Profiles (ALF2-1.25 Model)

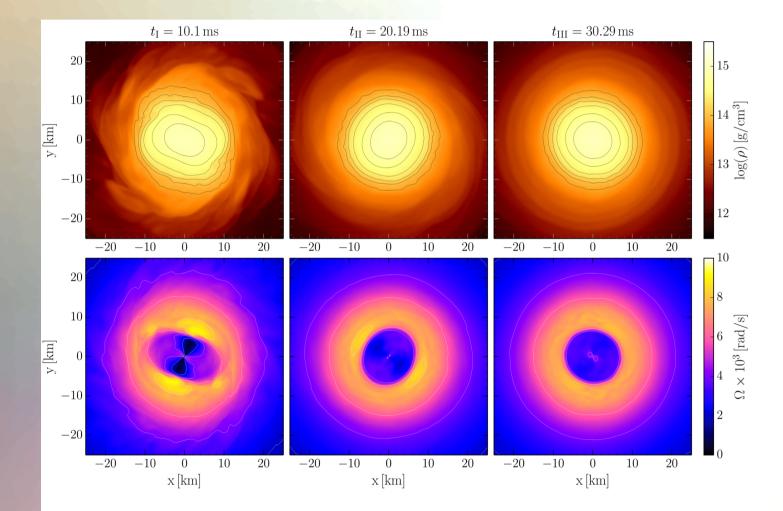
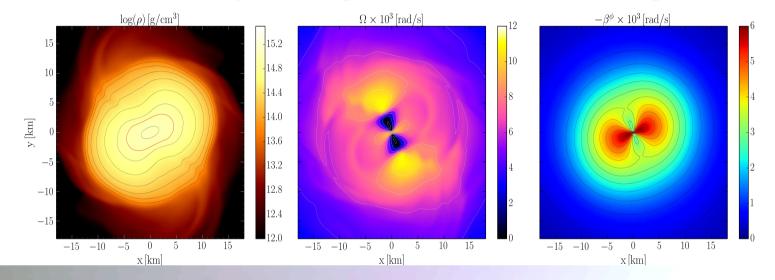
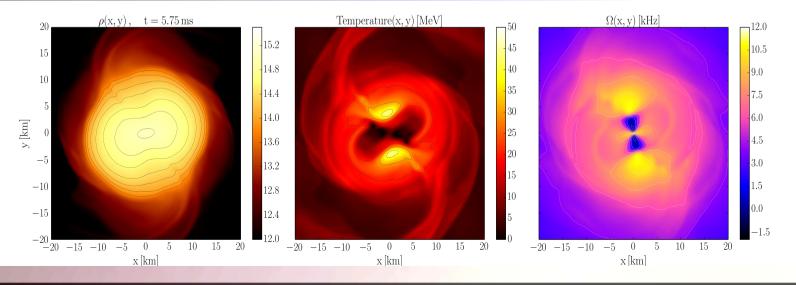


FIG. 11. Logarithm of the rest mass density  $Log(\rho)$  [g/cm<sup>3</sup>] (upper row) and fluid angular velocity  $\Omega \times 10^3$ [rad/s] (lower row) in the xy-plane for the ALF2-M125 model at three different post-merger times. The iso-contour curves have been drawn at 13.8 + 0.2n (upper row) and 2n (lower row),  $n \in \mathbb{N}$ 

#### Rest mass density, rotationprofile, Shift vector and Temperature





#### **Co-rotating** frame and fluid tracers

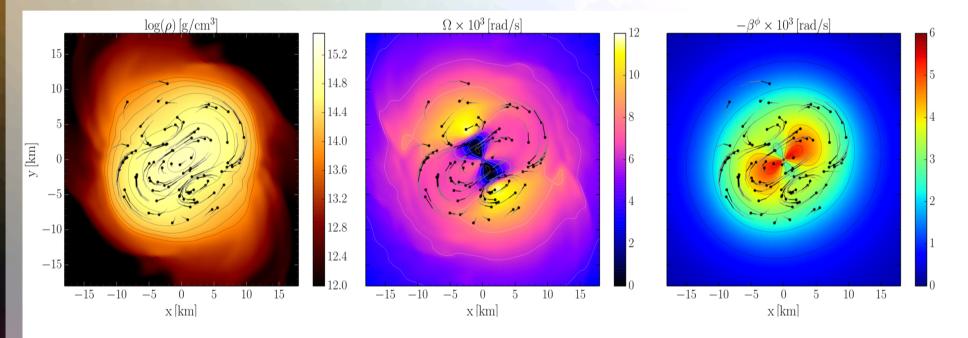
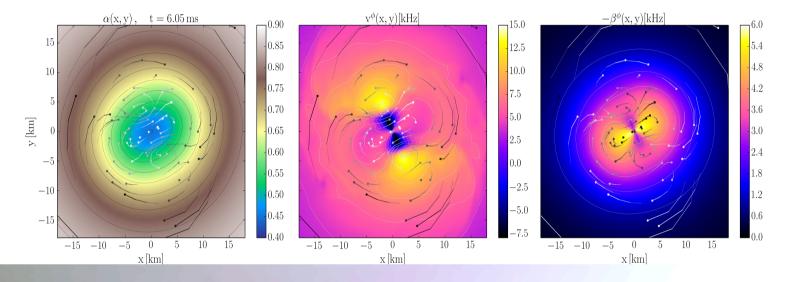
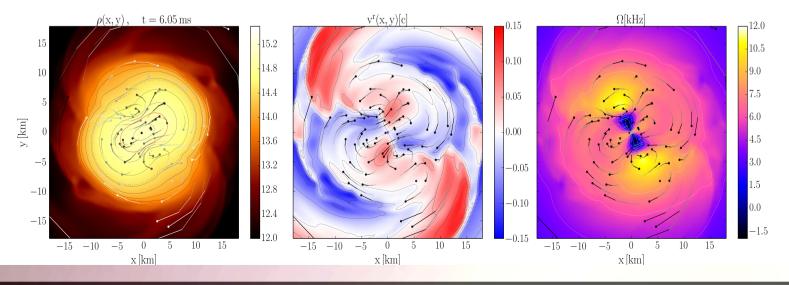


FIG. 7. Logarithm of the rest mass density  $\text{Log}(\rho)$  [g/cm<sup>3</sup>] (left picture), the fluid angular velocity  $\Omega \times 10^3$  [rad/s] (middle) and  $-\beta^{\phi} \times 10^3$  [rad/s] (right picture) in the xy-plane for the SLY-M132 model at t = 6.71 ms. The trajectories of several tracer-cells are additionally mapped for two previous times (separation  $\Delta t_{Tr} = 0.095$  ms).

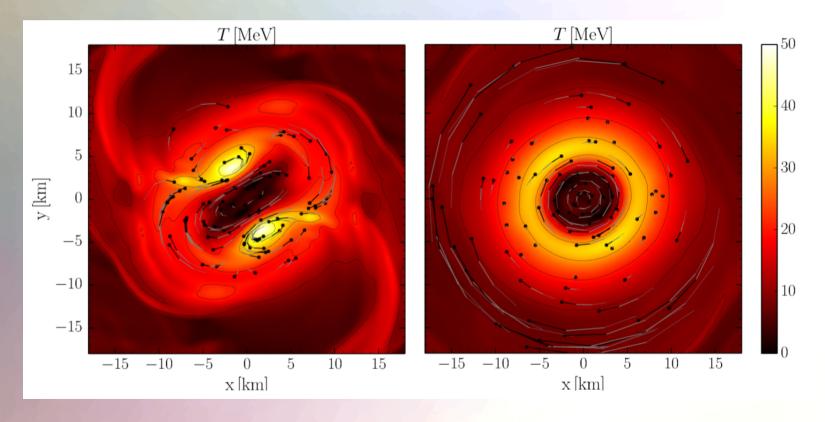
#### **Co-rotating frame and fluid tracers**





## **Temperature profiles** in the (x,y)-plane

LS220-M132 binary at t = 6.7 ms (left panel) and t = 23.8 ms (right panel). The trajectories of several tracer-cells are additionally visualized.



#### **Co-rotating frame and fluid tracers**

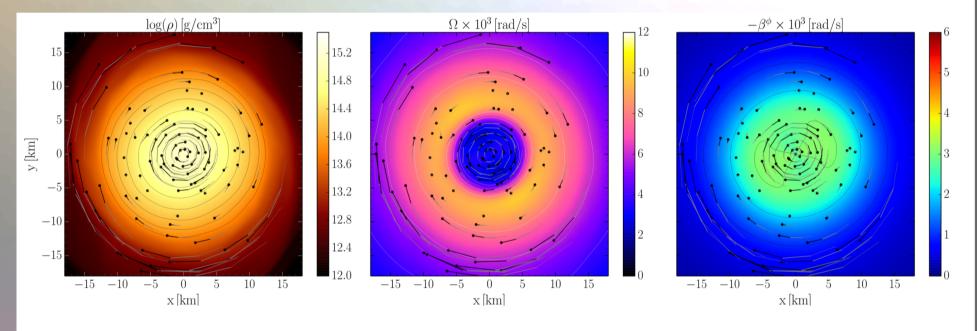
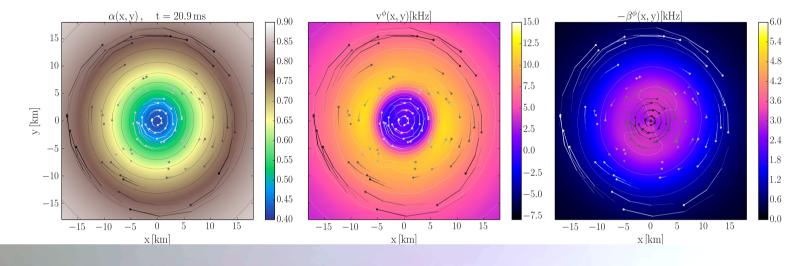
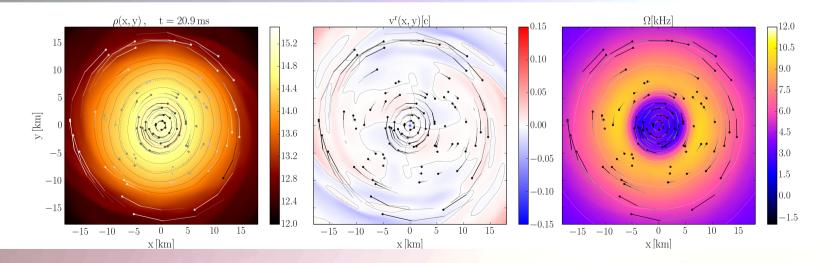


FIG. 8. Same as in Fig. 7, but at t = 23.83 ms.

#### Co-rotating frame and fluid tracers





## Averaging Procedure for $\Omega$

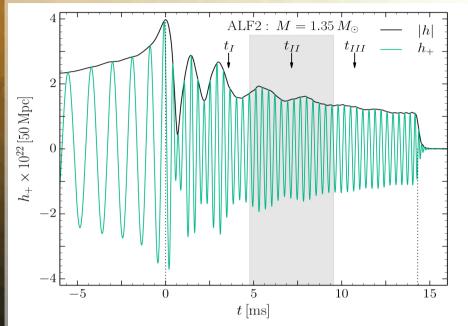


FIG. 2. Gravitational wave amplitude |h| and  $h_+$  at a distance of 50 Mpc for the ALF2-M135 model.

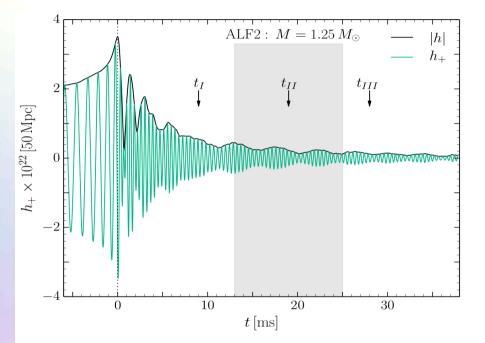
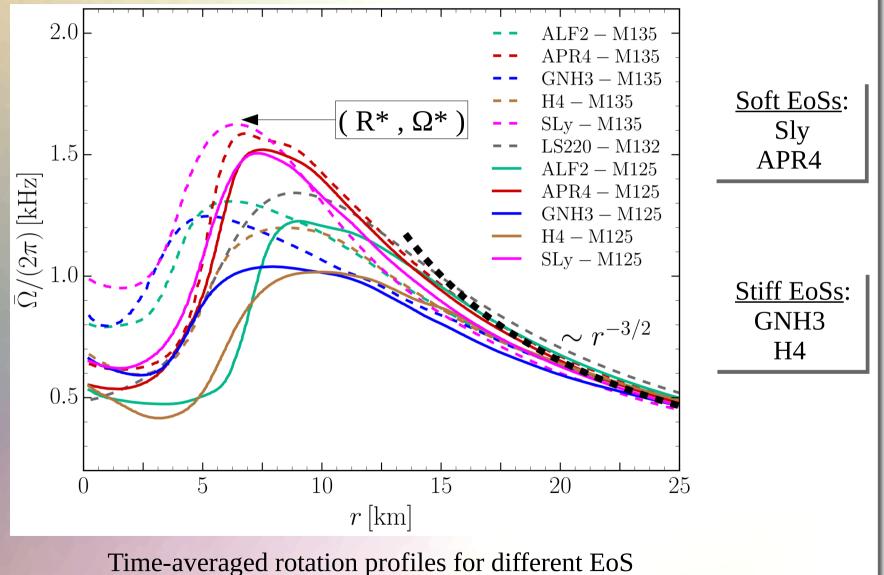


FIG. 10. Gravitational wave amplitude  $h_+$  and |h| at a distance of 50 Mpc for the ALF2-M125 model.

In order to compare the structure of the rotation profiles between the different EOSs, a certain time averaging procedure has been used:

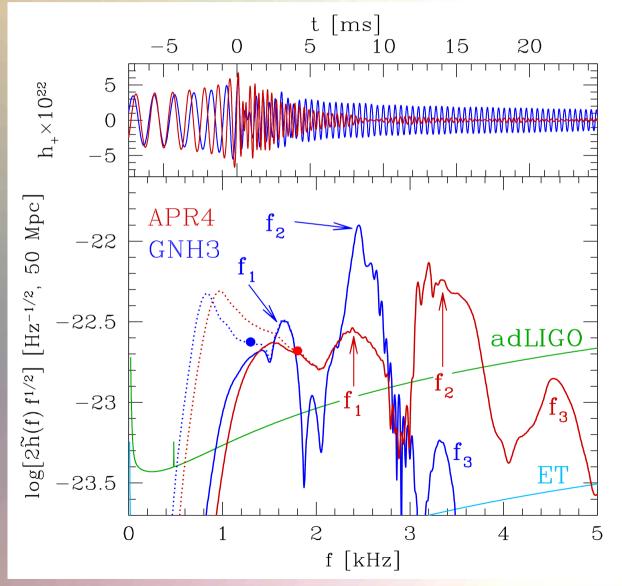
$$\bar{\Omega}(r,t_c) = \int_{t_c - \Delta t/2}^{t_c + \Delta t/2} \int_{-\pi}^{\pi} \Omega(r,\phi,t') \, d\phi \, dt$$

## **Time-averaged Rotation Profiles**



Low mass runs (solid curves), high mass runs (dashed curves).

### **GW-Spectrum** for different EoSs



See:

Kentaro Takami, Luciano Rezzolla, and Luca Baiotti, Physical Review D 91, 064001 (2015)

Hotokezaka, K., Kiuchi, K., Kyutoku, K., Muranushi, T., Sekiguchi, Y. I., Shibata, M., & Taniguchi, K. (2013). Physical Review D, 88(4), 044026.

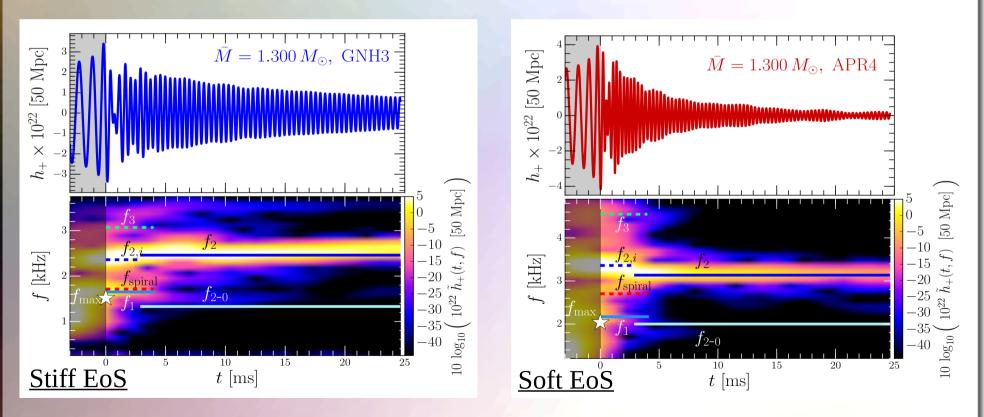
Bauswein, A., & Janka, H. T. (2012). Physical review letters, 108(1), 011101.

Clark, J. A., Bauswein, A., Stergioulas, N., & Shoemaker, D. (2015). arXiv:1509.08522.

Bernuzzi, S., Dietrich, T., & Nagar, A. (2015). Physical review letters, 115(9), 091101.

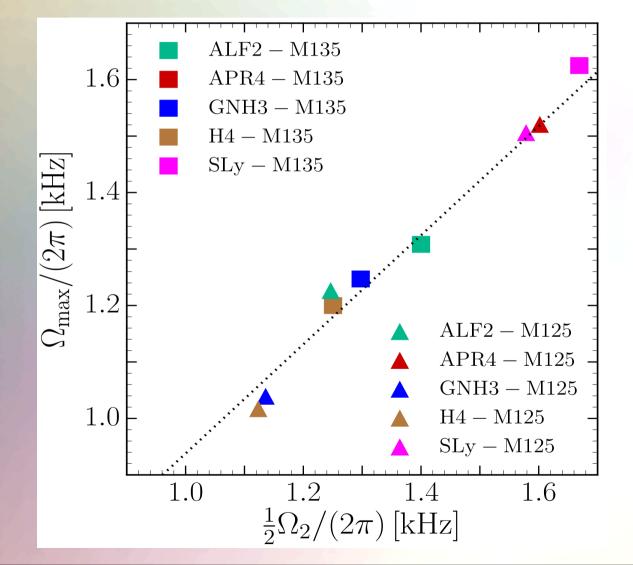
## Time Evolution of the GW-Spectrum

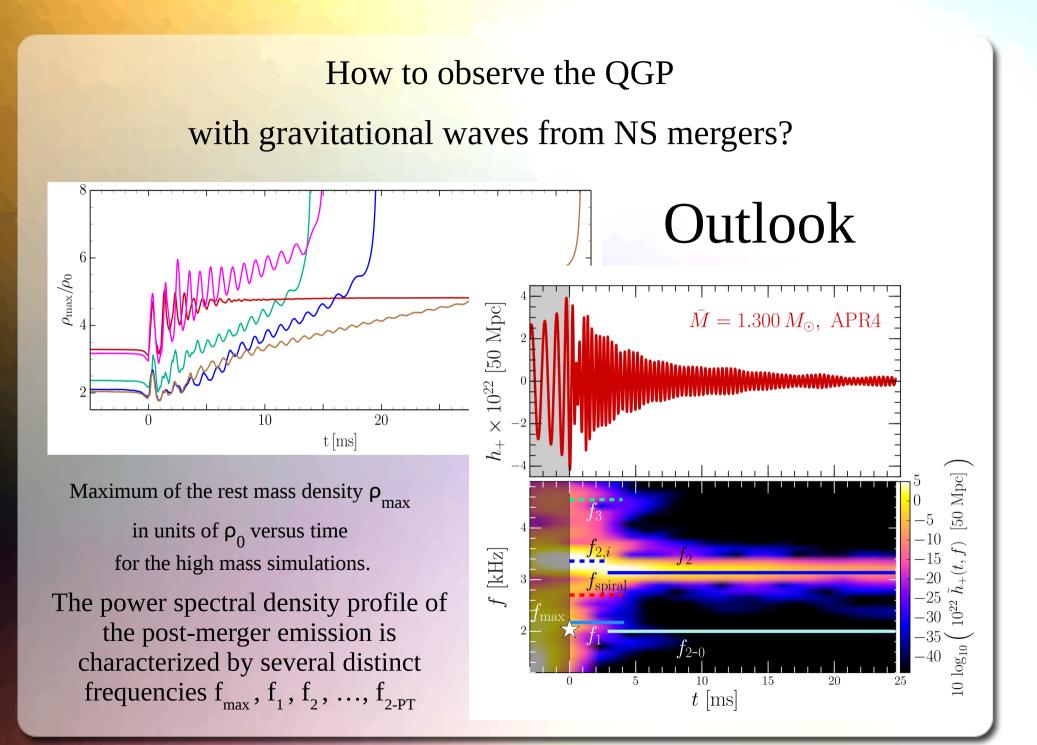
The power spectral density profile of the post-merger emission is characterized by several distinct frequencies  $f_{max}$ ,  $f_1$ ,  $f_2$ ,  $f_3$  and  $f_{2-0}$ . After approximately 5 ms after merger, the only remaining dominant frequency is the  $f_2$ -frequency. See L.Rezzolla and K.Takami, arXiv:1604.00246



Evolution of the frequency spectrum of the emitted gravitational waves for the stiff GNH3 (left) and soft APR4 (right) EOS.

# Gravitational Waves and the maximum of the Rotation Curve





#### Summary

- 1. With the first observation of gravitational waves from binary black hole merger by LIGO, the whole branch of observational astronomy will enter a new era the so called gravitational-wave astronomy.
- 2. GWs emitted from merging neutron star binaries are on the verge of their first detection.
- 3. The spectrum of the emitted GWs (within the merger and postmerger phase) depend strongly on the high density regime of the EOS.
- 4. With the knowledge of the  $f_1$  and  $f_2$ -frequency peak and the total mass the system, the GW signal can set tight constraints on the EOS.
- 5. The phasetransition to the Quark-Gluon-Plasma might be observable with Gravitational Wave Detektors ( $f_{2-PT}$  peak)

#### **Rotational properties of hypermassive neutron stars from binary mergers**

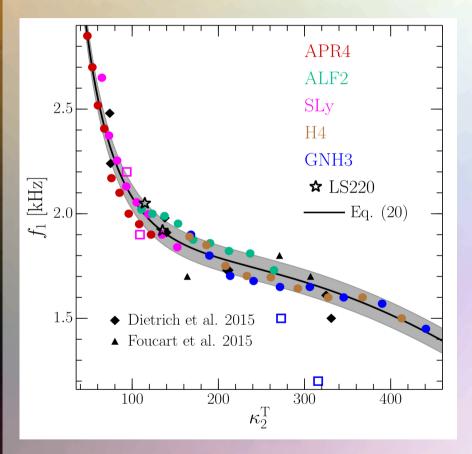
Matthias Hanauske,<sup>1,2</sup> Kentaro Takami,<sup>3,1</sup> Luke Bovard,<sup>1</sup> Luciano Rezzolla,<sup>1,2</sup> José A. Font,<sup>4,5</sup> Filippo Galeazzi,<sup>1</sup> and Horst Stöcker<sup>1,2,6</sup>

<sup>1</sup>Institut für Theoretische Physik, Max-von-Laue-Straße 1, 60438 Frankfurt, Germany <sup>2</sup>Frankfurt Institute for Advanced Studies, Ruth-Moufang-Straße 1, 60438 Frankfurt, Germany <sup>3</sup>Kobe City College of Technology, 651-2194 Kobe, Japan

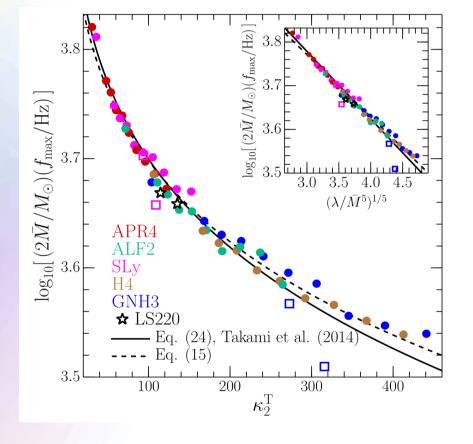
<sup>4</sup>Departamento de Astronomía y Astrofísica, Universitat de València, Dr. Moliner 50, 46100, Burjassot (València), Spain
 <sup>5</sup>Observatori Astronòmic, Universitat de València, C/ Catedrático José Beltrán 2, 46980, Paterna (València), Spain
 <sup>6</sup>GSI Helmholtzzentrum für Schwerionenforschung GmbH, D-64291 Darmstadt, Germany

Determining the differential-rotation law of compact stellar objects produced in binary neutron stars mergers or core-collapse supernovae is an old problem in relativistic astrophysics. Addressing this problem is important because it impacts directly on the maximum mass these objects can attain and hence on the threshold to blackhole formation under realistic conditions. Using the results from a large number of numerical simulations in full general relativity of binary neutron star mergers described with various equations of state and masses, we study the rotational properties of the resulting hypermassive neutron stars. We find that the angular-velocity distribution shows only a modest dependence on the equation of state, thus exhibiting the traits of "quasiuniversality" found in other aspects of compact stars, both isolated and in binary systems. The distributions are characterized by an almost uniformly rotating core and a quasi-Keplerian "disk". Such a configuration is significantly different from the j – constant differential-rotation law that is commonly adopted in equilibrium models of differentially rotating stars. Furthermore, the rest-mass contained in such a disk can be quite large, ranging from  $\simeq 0.03 M_{\odot}$  in the case of high-mass binaries with stiff equations of state, up to  $\simeq 0.2 M_{\odot}$  for low-mass binaries with soft equations of state. We comment on the astrophysical implications of our findings and on the long-term evolutionary scenarios that can be conjectured on the basis of our simulations.

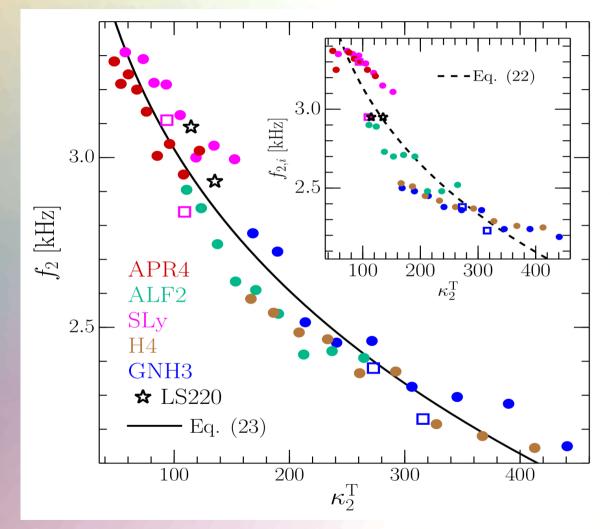
# **Universal Behavior of** f<sub>1</sub> and f<sub>max</sub>



Values of the low-frequency peaks  $f_1$ shown as a function of the tidal deformability parameter  $\kappa_2^T$ . Mass-weighted frequencies at amplitude maximum  $f_{max}$  shown as a function of the tidal deformability parameter  $\kappa_{2}^{T}$ .



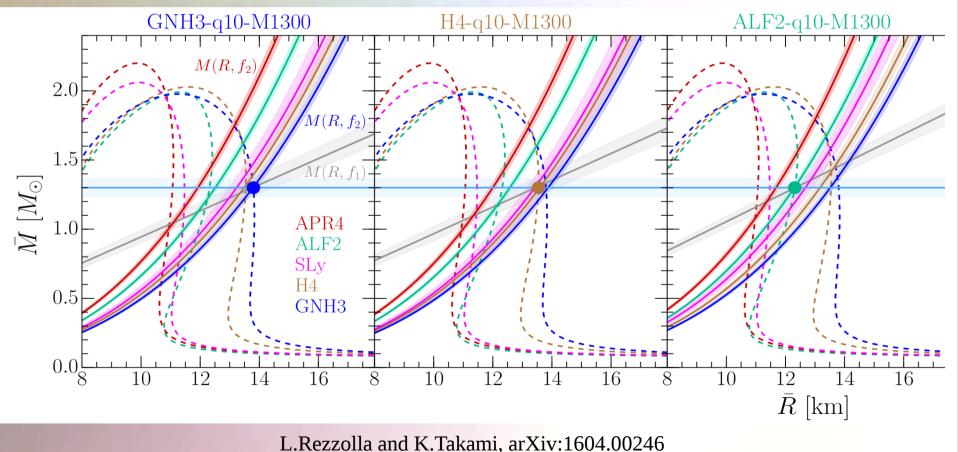
## **Universal behavior of the f<sub>2</sub>-peak**



Values of the low-frequency peaks  $f_2$  shown as a function of the tidal deformability parameter  $\kappa_2^T$ .

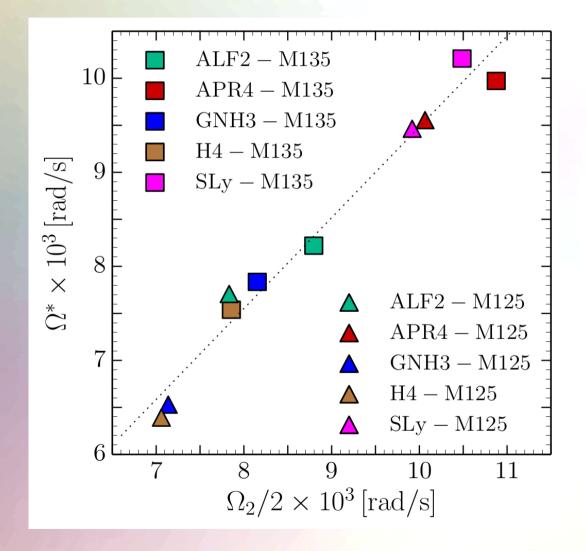
# 

The detection of GWs from merging neutron star binaries can be used to determine the high density regime of the EOS. With the knowledge of  $f_1$ ,  $f_2$  and the total mass the system, the GW signal can set tight constraints on the EOS.

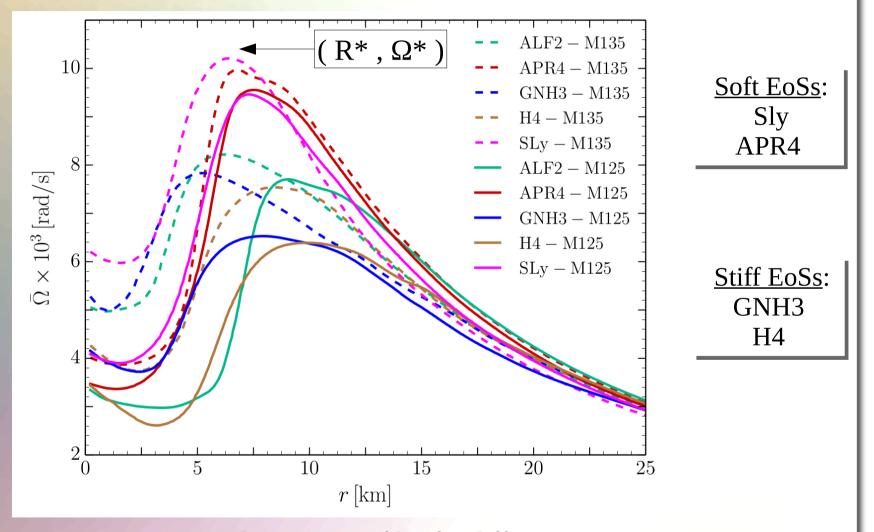


K.Takami, L.Rezzolla, and L.Baiotti, Physical Review D 91, 064001 (also PRL 113, 091104)

## Gravitational Waves and the maximum of the Rotation Curve



#### **Time-averaged** Rotation Profiles



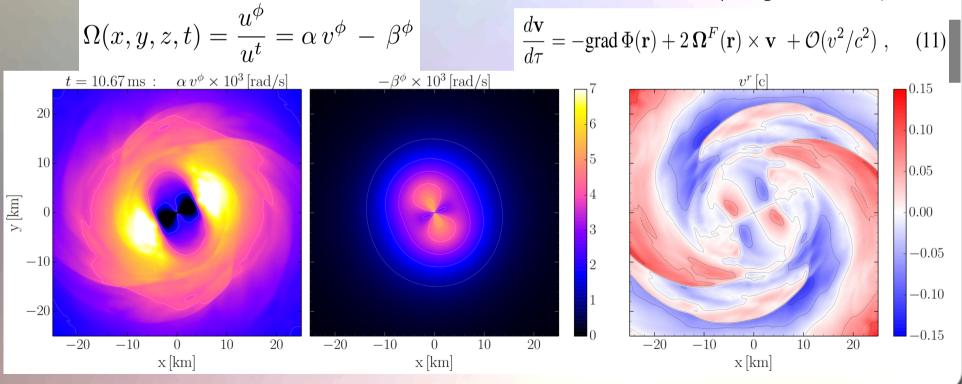
Time-averaged rotation profiles for different EoS Low mass runs (solid curves), high mass runs (dashed curves).

## Frame-Dragging and Gravitomagnetism

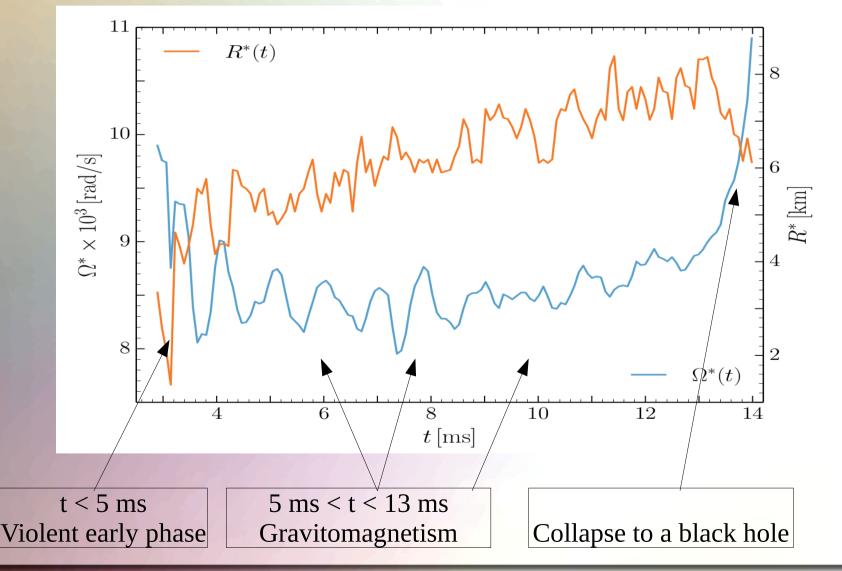
The dragging of local inertial frames is quite large in the interior of the HMNS and therefore an additional gravitomagnetic force is present. Like the Lorentz-force in electromagnetism, it acts orthogonal to the fluids velocity and the frame dragging.

Ingredients of the angular velocity:

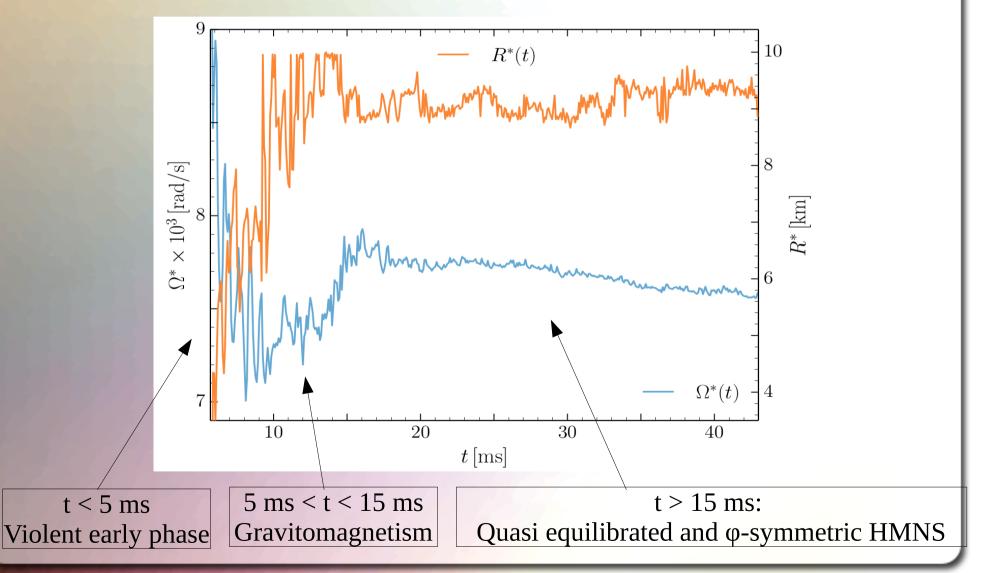
(strong time and φ-dependence !)



# Maximum Value of $\Omega$ and its Radial Position (high mass run)



# Maximum Value of $\Omega$ and its Radial Position (low mass run)

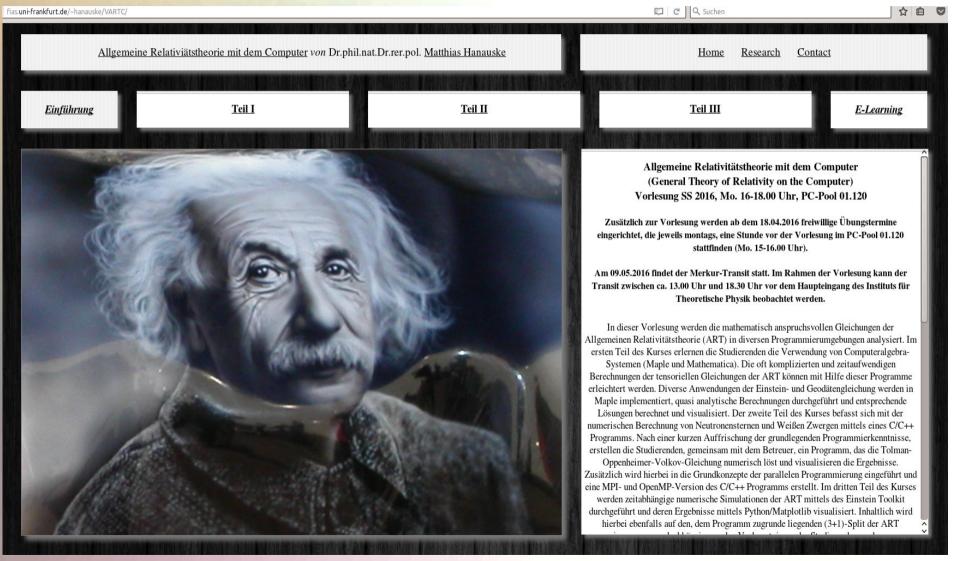


# Einstein inside Ausstellung in Frankfurt am Main



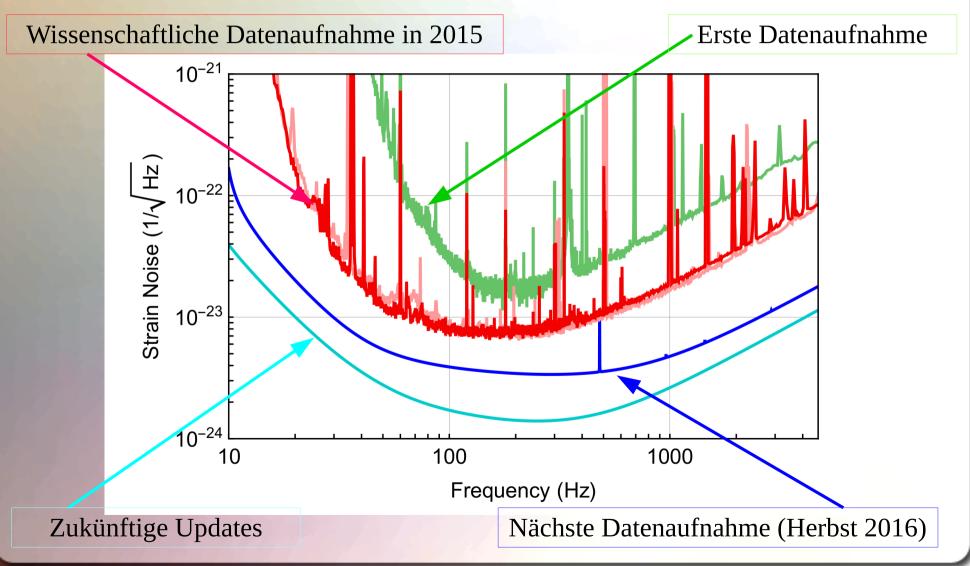
Die multimediale Mitmachausstellung

# Vorlesung: ART mit dem Computer



www.fias.uni-frankfurt/~hanauske/VARTC/

# Warum (noch) keine Gravitationswellen von kollidierenden Neutronensternen?



#### The ballroom dance of NS-mergers

