Constraining the Radius of Neutron Stars Through the Moment of Inertia

TECHNISCHE UNIVERSITÄT DARMSTADT

Neutron star mergers: From gravitational waves to nucleosynthesis

International Workshop XLV on Gross Properties of Nuclei and Nuclear Excitations

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Observation

Theoretical description of neutron stars

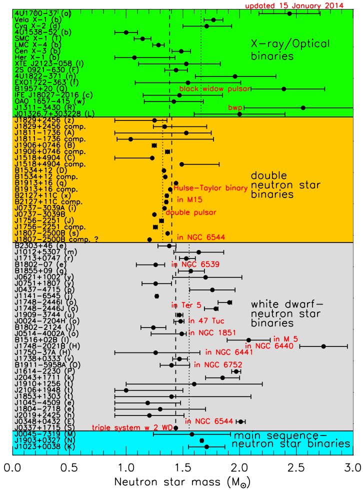
Equation of state and neutron star structure

Constraining the radius of neutron stars

Summary

Observation

- knowledge of EOS is restricted
- relationship between EOS and *MR* relation
- precise mass measurements are possible
- radius determination influenced by systematic uncertainties
- moment of inertia measurement seems feasible in the future
  - PSR J0737–3039A, $M = 1.337(5)\ M_\odot$ Burgay et al., Nature (2003); Lyne et al., Science (2004)
- goal: constraining radius/EOS by moment of inertia



Courtesy of J. M. Lattimer

Observation
Mass measurement

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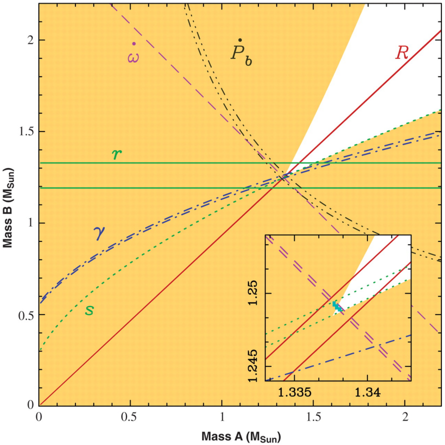

Kramer et al., Science (2006)

- pulsar observation ⟶ determine post–Keplerian parameters Stairs, Living Rev. Relativity (2003)
- post–Keplerian parameters as functions of the masses Damour and Deruelle, Ann. Inst. Henri Poincare (1986)
- intersection area yields masses for binary system

see also talk of Paulo Freire

Observation
Moment of inertia measurement

- relativistic spin-orbit (SO) coupling causes an additional contribution to advance of the periastron
- spin of PSR J0737–3039B is negligible
- contributions to advance of periastron

$$\dot{\omega} = \dot{\omega}_{1pN} + \dot{\omega}_{2pN} + \dot{\omega}_{SO}$$

(pN: post-Newtonian)

Kramer & Wex, Class. Quant. Grav. (2009)

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- candidate for future measurement: PSR J0737–3039A
  - highly relativistic binary

Kramer & Wex, Class. Quantum Grav. (2009)

Theoretical description of neutron stars
Non-rotating neutron stars

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- hydrostatic equilibrium
- Schwarzschild metric

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

- TOV equations

$$\frac{dP}{dr} = -\frac{1}{r^2}(\epsilon + P)\left(m + 4\pi r^3 P\right)\left(1 - \frac{2m}{r}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi\epsilon r^2$$

$$\frac{d\nu}{dr} = \frac{2\left(m + 4\pi r^3 P\right)}{r^2}\left(1 - \frac{2m}{r}\right)^{-1}$$

Theoretical description of neutron stars
Slowly rotating neutron stars

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- hydrostatic equilibrium
- Schwarzschild metric $\rightarrow$ Hartle-Thorne metric Hartle, APJ (1967); Hartle & Thorne, APJ (1968)

$$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \left(d\phi - \left(\omega + O\left(\Omega^3\right)\right) \ dt\right)^2$$

- TOV equations remain

$$\frac{dP}{dr} = -\frac{1}{r^2}(\epsilon + P)\left(m + 4\pi r^3 P\right)\left(1 - \frac{2m}{r}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi\epsilon r^2$$

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Theoretical description of neutron stars
Moment of inertia

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Hartle, APJ (1967), Hartle & Thorne, APJ (1968)

- auxiliary function

$$j = e^{-\frac{1}{2}\nu}\left(1 - \frac{2m}{r}\right)^{\frac{1}{2}}$$

- in addition to TOV eqns. ($\bar{\omega} = \Omega - \omega$)

$$\frac{d}{dr}\left(r^4 j \frac{d\bar{\omega}}{dr}\right) = -4r^3 \frac{dj}{dr}\bar{\omega}$$

- moment of inertia

$$I = \frac{8\pi}{3}\int_0^R dr\; r^4\left(\epsilon + P\right) j \left(1 - \frac{2m}{r}\right)^{-1}\frac{\bar{\omega}}{\Omega}$$

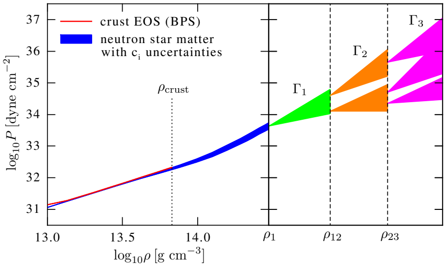
Equation of state and neutron star structure
Piecewise polytropes

Hebeler, Lattimer, Pethick, Schwenk, Phys. Rev. Lett. (2010); Hebeler, Lattimer, Pethick, Schwenk, APJ (2013)

see also talk of Kai Hebeler

- low density regime: knowledge of nuclear physics
  - BPS crust up to $\rho_{\text{sat}}/2$
  - chiral EFT expansion up to $\sim \rho_{\text{sat}}$
- high density regime: requirement of causality and constraints from 2.01 $M_\odot$ neutron stars
  - polytropic expansion Read, Lackey, Owen, Friedman, Phys. Rev. D (2009)

$$P(\rho) = K\rho^{\Gamma}$$



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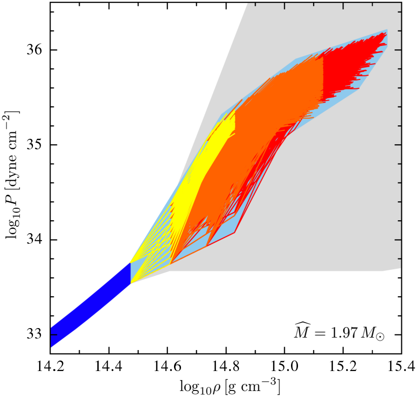

Hebeler, Lattimer, Pethick, Schwenk, APJ (2013)

Equation of state and neutron star structure
MR relation

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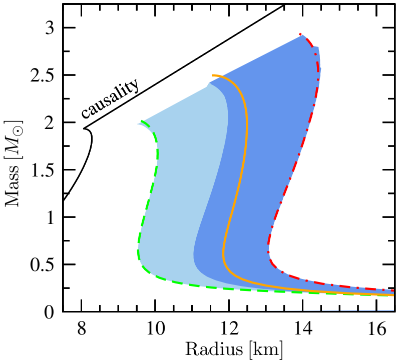
$$P(\rho) = K\rho^{\Gamma}$$

- radius prediction for PSR J0737–3039A: $R \approx (9.9 - 13.6)$ km



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Constraining the radius of neutron stars
Universal relation

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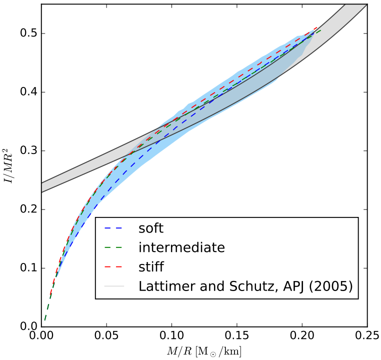

grey band holds for

$0.07\ M_\odot\ \mathrm{km}^{-1} \lesssim MR^{-1}$

- several studies have investigated $I\left(MR^{-1}\right)$ Bejger & Haensel , A&A (2002); Lattimer & Schutz, APJ (2005); Breu & Rezzolla, MNRAS (2016); ...

  $\rightarrow$ dimensionless moment of inertia is not sensitive to EOS
- find fit and determine radius constraints

Constraining the radius of neutron stars
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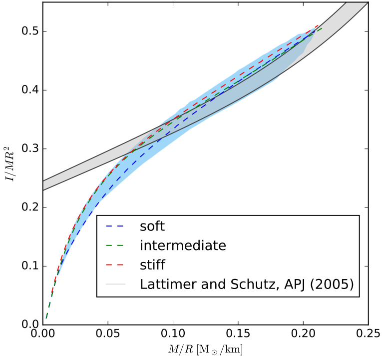

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- find fit and determine radius constraints
- our approach: use the whole EOS uncertainty band created by polytropic expansion

Constraining the radius of neutron stars

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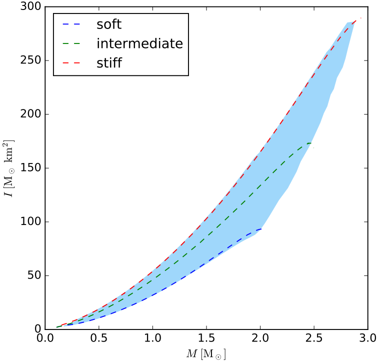
- use all EOS from uncertainty band
- consider different masses
- use $I(M)$ band in order to find reasonable moment of inertia values



Constraining the radius of neutron stars

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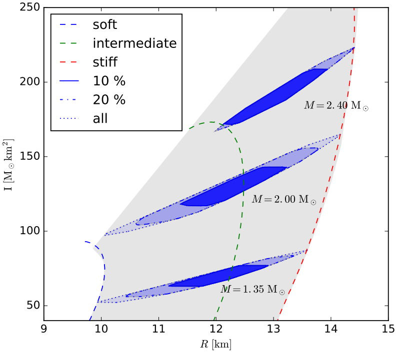
- combine mass, moment of inertia and ΔI



Constraining the radius of neutron stars

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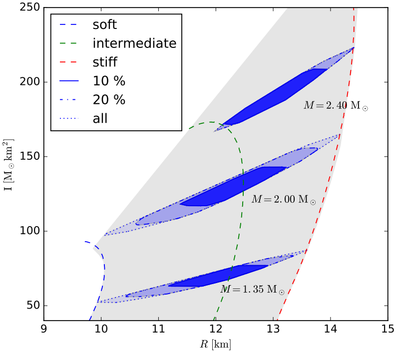
- assumption for PSR J0737–3039A: $I = (70 \pm 7)\ M_\odot \mathrm{km}^2$

$$R \approx (9.9 - 13.6)\ \mathrm{km}$$



Constraining the radius of neutron stars

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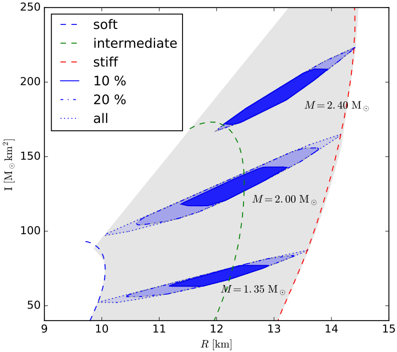
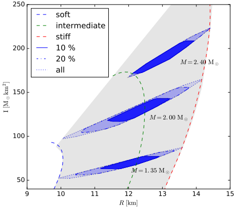
$$\Downarrow$$

$$R \approx (11.2 - 12.9)\ \text{km}$$



Summary

- derived constraints for the EOS over wide range of densities and resulting NS radii using
  - BPS crust EOS up to $\rho_{\text{sat}}/2$
  - results based on chiral EFT interactions up to $\sim \rho_{\text{sat}}$
  - causality at all densities
  - $M_{\text{max}} > 2.01\ \text{M}_\odot$ + assumed fixed measured values for moment of inertia (including uncertainties)
- developed a framework to perform an extensive large scale sampling of all possible high-density extensions for EOS compatible with constraints
- find a reduction of radius uncertainty from moment of inertia measurements by about 50 % ($\Delta I = \pm 10$ %)



In collaboration with K. Hebeler and A. Schwenk.

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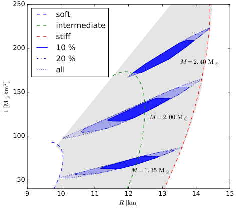

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Thanks for your attention!