Chemical Freeze-out parameters from Hadron Resonanse Gas

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Hirschegg 2019

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(manuscript under preparation)

January 14, 2019

Heavy Ion Collisions : :

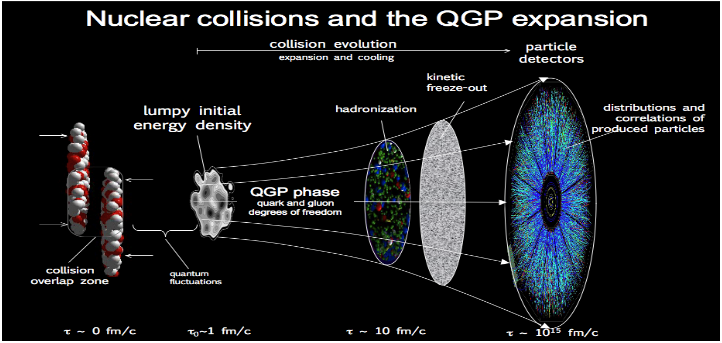

Figure: Evolution of fireball [1]

[1] www.quora.com ;

Equilibrium Physics : :

Chemical equilibrium means the equilibration of conserved charges. For strong interactions Quantum Chromodynamics (QCD) ensures the conservation of baryon number (B), electric charge (Q), and strangeness (S).

Thus, the equilibrium thermodynamic state of QCD matter is completely determined by temperature (T) and the three chemical potentials μ_B, μ_Q, μ_S and corresponding to B ,Q and S, respectively.

Hadron Resonance Gas Model : :

The grand canonical partition function ($\mathcal{Z}$) for ideal Hadron Resonance Gas (HRG) model can be written as,

$$\boxed{ln\mathcal{Z}^{id} = \sum_i ln\mathcal{Z}_i^{id}}$$

Where, the sum is over all hadrons and their resonances.

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$$\boxed{ln\mathcal{Z}^{id} = \pm \sum_i \frac{Vg_i}{2\pi^2} \int_0^\infty p^2 \; dp \; ln\Big[1 \pm exp\Big\{-\frac{(\mathcal{E}_i - \mu_i)}{T}\Big\}\Big]}$$

Thermodynmic observables : :

From partition fuction, we can derive different thermodynamical quantities.

Number density can be calculated according to :

$$n^{id} = \frac{T}{V}\sum_i \Big(\frac{\partial ln\mathcal{Z}_i}{\partial\mu_i}\Big)_{V,T} = \sum_i \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2\ dp}{[exp\{(\mathcal{E}_i-\mu_i)/T\}\pm 1]}$$

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Entropy density can be written as:

$$s^{id} = \pm\sum_i \frac{g_i}{2\pi^2}\int_0^\infty p^2\ dp\ \Big[ln\Big\{1\pm exp\Big(-\frac{(\mathcal{E}_i-\mu_i)}{T}\Big)\Big\}\pm\frac{(\mathcal{E}_i-\mu_i)}{T(exp(\frac{(\mathcal{E}_i-\mu_i)}{T})\pm 1)}\Big]$$

Thermodynamic observables : :

At vanishing chemical potential, various thermodynamic observables like pressure, energy density, number density and even different susceptibilities of conserved charges are in good agreement with lattice results in low temperature phase.

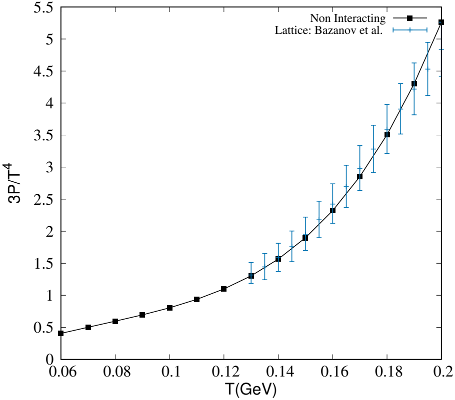

Figure: Pressure with temperature

Freeze-out : :

• **Chemical freeze-out (CFO)** surface is determined by analysing the measured hadron yields [2-3].

• **Kinetic freeze-out (KFO)** surface can be determined by studying the data of transverse momentum (p_T) distribution of produced particles [4].

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[2] Physics Letters B, vol. 365, pp. 1-6, 1996 ; [3] Physical Review C, vol. 48, pp. 2462-2475, 1993;
[4] Physics Letters B, vol. 503, no. 1-2, pp. 58-64, 2001;

Freeze-out parameters : :

Systematics of Chemical Freeze-out are thermodynamic parameters : T, μ_B, μ_Q, μ_S and normalization factor volume V.

Hadron rapidity density can be written as :

$$\boxed{\langle \frac{dN_j}{dy} \rangle = \frac{dV}{dy} n_j(T, \mu_Q, \mu_B, \mu_S)}$$

To extract the parameters of the model one perform fits of experimental data with model calculations :

$$\boxed{\frac{n_j(T, \mu_B, \mu_Q, \mu_S)}{n_k(T, \mu_B, \mu_Q, \mu_S)} = \frac{\frac{dN_j}{dY}}{\frac{dN_k}{dY}}}$$

Traditional approach to χ^2 Method : :

The best fit is obtained by minimizing the distribution of χ^2.

$$\chi^2 = \sum_i \frac{(R_i^{exp} - R_i^{therm})^2}{(\sigma_i^{exp})^2}$$

Out of the three chemical potentials it is a common approach to fix μ_Q and μ_S from the following constraints :

$$\frac{\sum_i n_i B_i}{\sum_i n_i Q_i} = constant \qquad \text{and} \qquad \sum_i n_i S_i = 0$$

With this two constraint equations the problem is reduced to a two dimensional problem.

Our approach to χ^2 Method : :

Four equations we have used from the definition of χ^2 ,

$$\frac{d\chi^2}{dT} = 0,$$

$$\frac{d\chi^2}{d\mu_k} = 0, \quad \text{where}, k = B, Q, S$$

We did not use any constraint relations to reproduce the ratios.

Few important informations : :

- We have used the mid-rapidity data of most central collision of Au-Au nuclei for $\sqrt{S}$ of AGS, SPS, RHIC, LHC.

- σ is the uncertainity and to derive σ we have quadratically add statistical and systematic errors of measured yields.

- Error of ratios have been calculated from error propagation method.

$$\sigma_i^{exp} = \frac{\frac{dN_j}{dY}}{\frac{dN_k}{dY}} \sqrt{\left(\frac{\sigma_j^{exp}}{\frac{dN_j}{dY}}\right)^2 + \left(\frac{\sigma_k^{exp}}{\frac{dN_k}{dY}}\right)^2}$$

Variation of Freeze-out Parameters : :

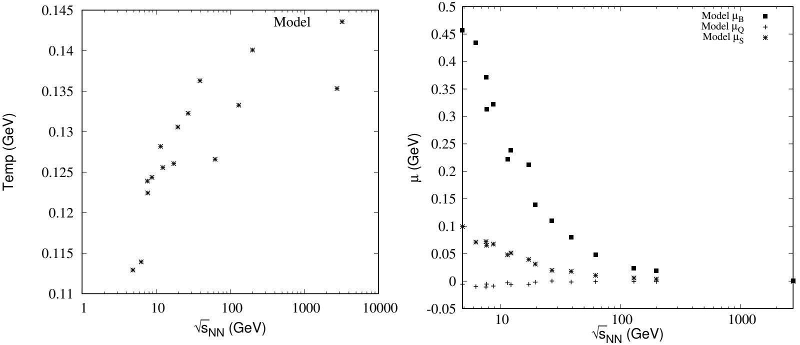

Figure: Variation of freeze-out parameters using π^+, π^-, k^+, k^-, p, $\bar{p}$.

Particle Yield Ratios with four parameters : :

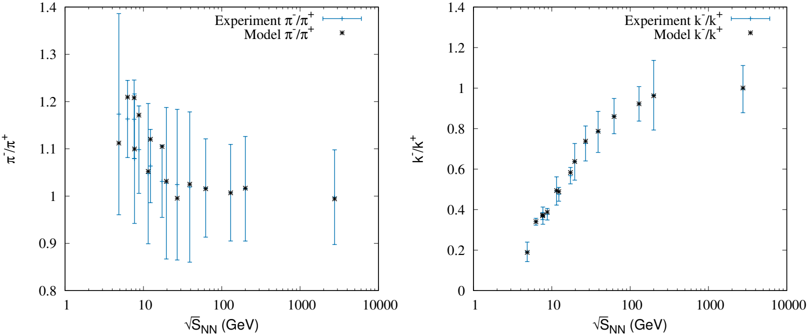

Figure: π^-/π^+ , k^-/k^+ with $\sqrt{S}$.

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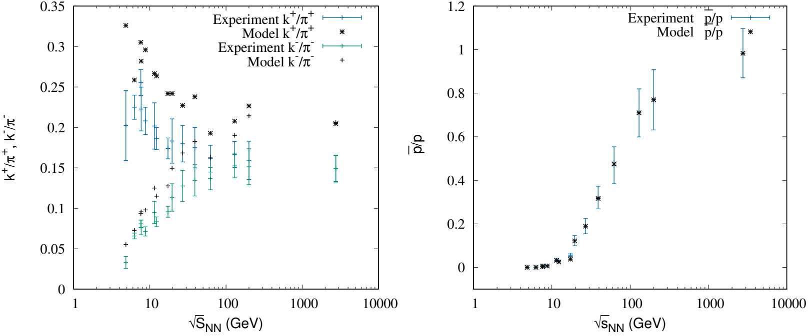

Figure: k^+/π^+, k^-/π^- and $\bar{p}/p$ with $\sqrt{S}$.

Our approach to χ^2 Method : :

However, an additional fifth parameter, γ_s, called the "*strangeness suppression factor*", often used in thermal model to fit the model predicted value with experimental data.

Thus, the fifth equation we have used,

$$\frac{d\chi^2}{d\gamma_s} = 0,$$

Thus the modified equation of Number density become,

$$\boxed{n^{id} = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 \ dp}{[\gamma_s^{S_i} exp\{(\mathcal{E}_i - \mu_i)/T\} \pm 1]}}$$

Our approach to χ^2 Method : :

Using these five equations with the central value of hadron yields, we can obtain freeze-out parameters for two sets of data of identified particles.

Set I	Set II
$\pi^+, \pi^-, k^+, k^-, p, \bar{p}$	$\pi^+, \pi^-, k^+, k^-, p, \bar{p}, \Lambda, \bar{\Lambda}, \Xi^-, \Xi^+$

Variation of Freeze-out Parameters : :

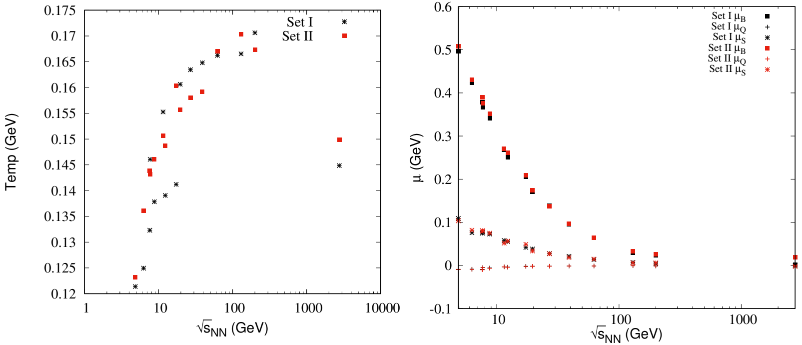

Figure: Variation of freeze-out parameters.

γ_s and χ^2 with collisional energy : :

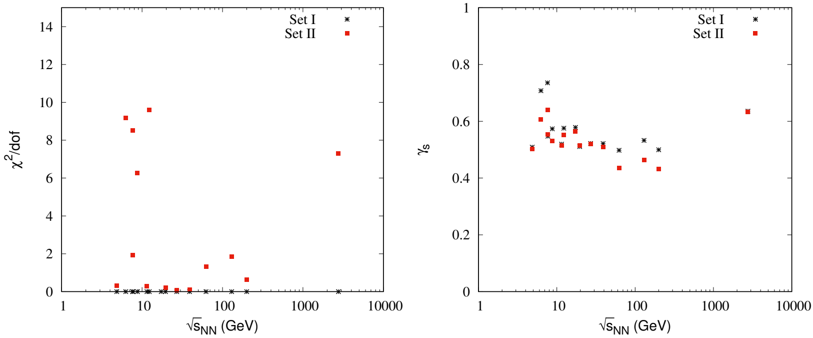

Figure: Variation of γ_s and χ^2.

Particle Yield Ratios with five parameters : :

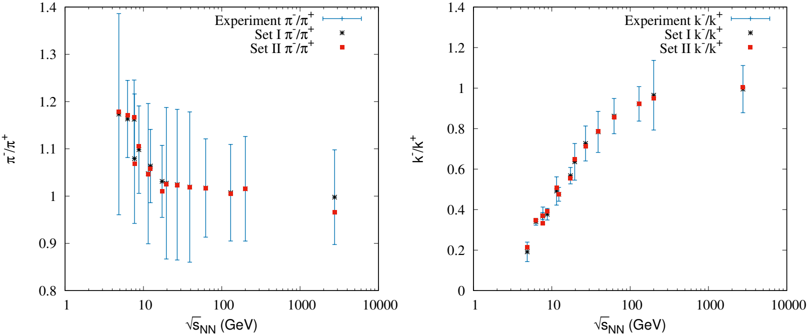

Figure: π^-/π^+ , k^-/k^+ with $\sqrt{S}$.

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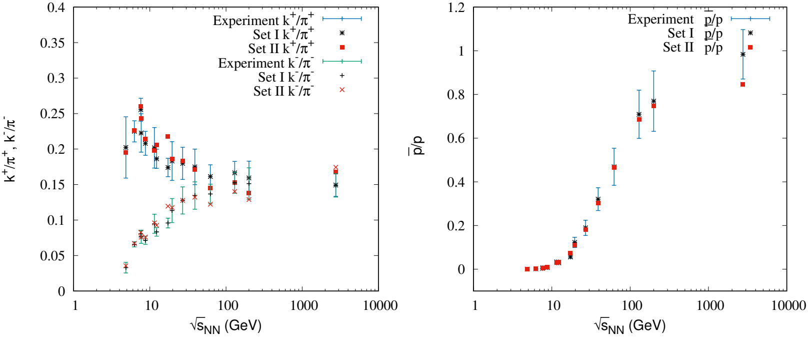

Figure: k^+/π^+, k^-/π^- and $\bar{p}/p$ with $\sqrt{S}$.

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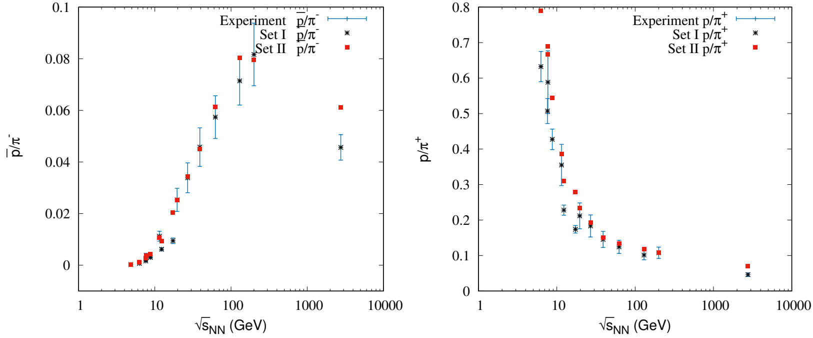

Figure: $\bar{p}/\pi^-$ and p/π^+ with $\sqrt{S}$.

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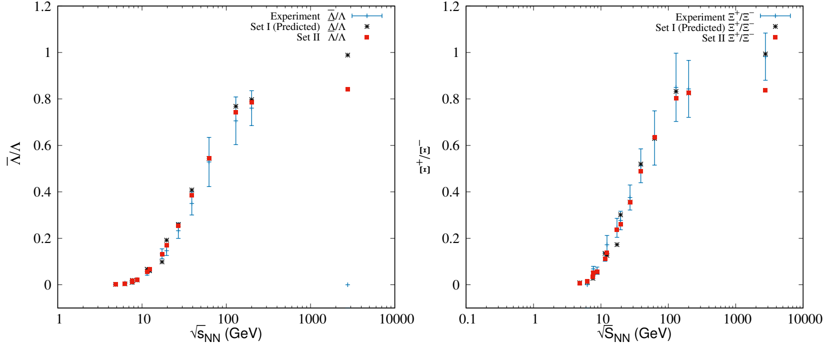

Figure: $\bar{\Lambda}/\Lambda$, Ξ^+/Ξ^- with $\sqrt{S}$.

Particle Yield Ratios with five parameters : :

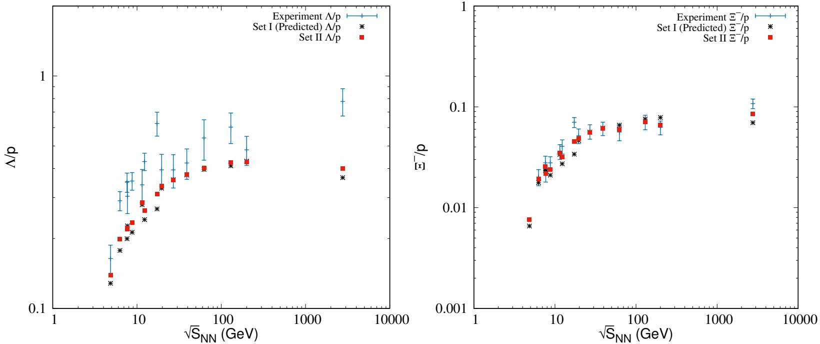

Figure: Λ/p, Ξ^-/p with $\sqrt{S}$.

Discussion : :

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- We have solved a multidimensional problem without using any constraint relations.

- The predicted ratios of higher mass particle show deviation for cross ratios whereas they are in good agreement with experimental data for particle-antiparticle ratios.

Discussion : :

• In order to have a more realistic approach one needs to include all the decay chains and production channels in HRG model and detector efficiencies specific to each particle yield.

• The repulsive interactions included in hadron gas via van der Waals excluded volume procedure corresponds to a substitution of the system volume V by the available volume V_{avl}, which can be included in our recent study also.

• Calculations have been done with a grand-canonical treatment of the conserved charges B, S and Q. The same is possible within other statistical ensembles like a fully canonical treatment of the conserved charges, and a mixedcanonical ensemble combining a canonical treatment of strangeness with a grand canonical treatment of baryon number and electric charge.


Thank You!