SCATTERING THEORY APPROACH TO THERMODYNAMICS OF HADRONS

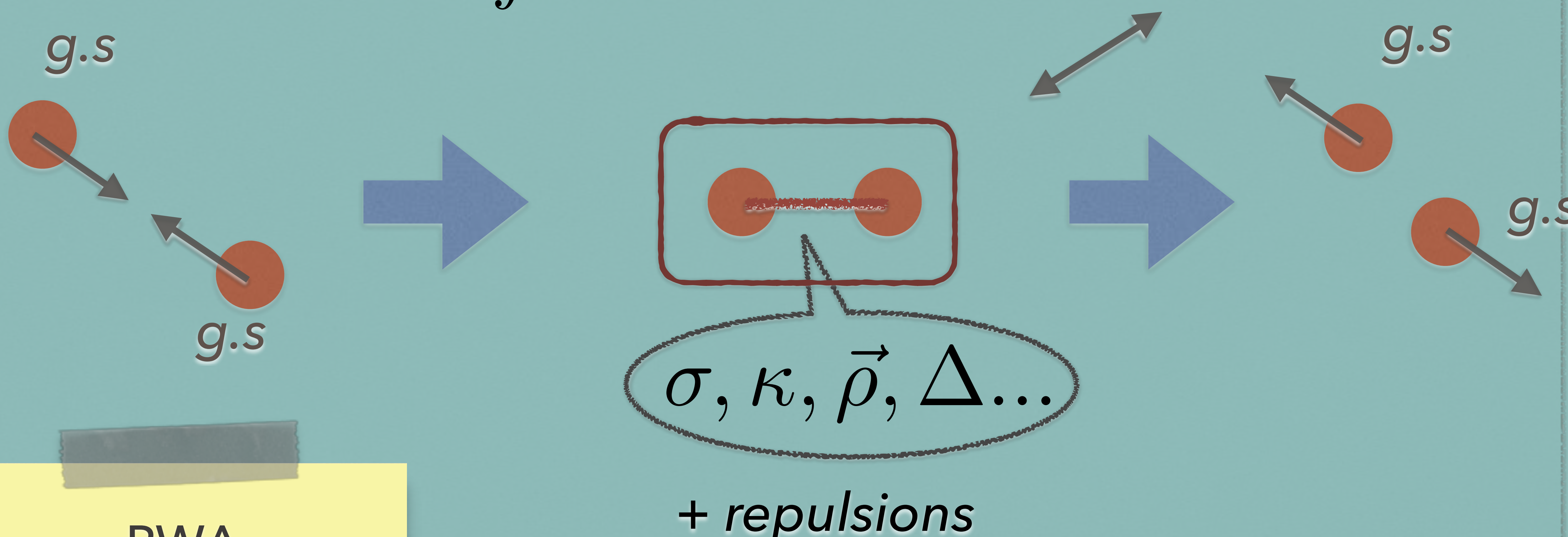
POK MAN LO (盧博文)

University of Wroclaw

HIRSCHEGG 2019

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE \, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \mathrm{tr} \, (\delta_E) \, .$$



$$\delta \longrightarrow \mathcal{Q}(M) \equiv \frac{1}{2} \, \mathrm{Im} \, (\mathrm{tr} \ln \mathrm{S})$$

PWA
X
S-matrix thermo.

IN COLLABORATION WITH

Bengt Friman

Anton Andronic

Peter Braun-Munzinger

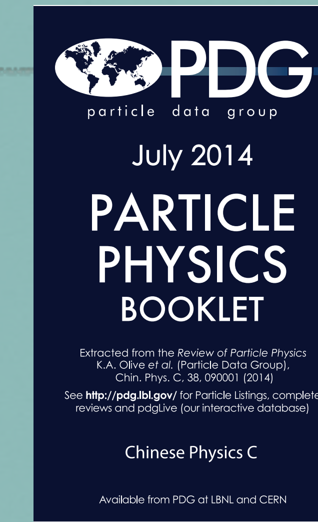
Johanna Stachel

Pasi Huovinen

Chihiro Sasaki

Krzysztof Redlich

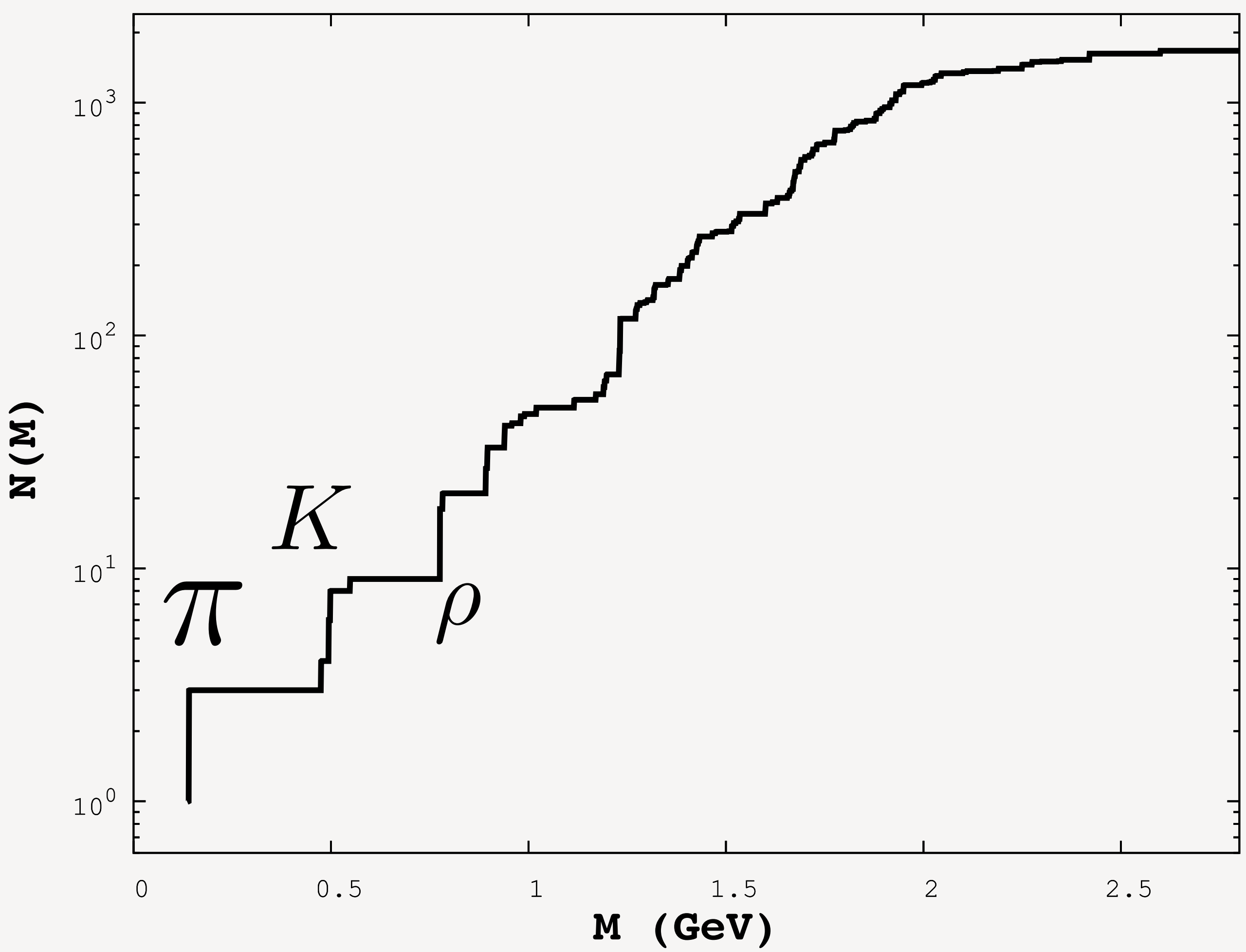
HRG MODEL

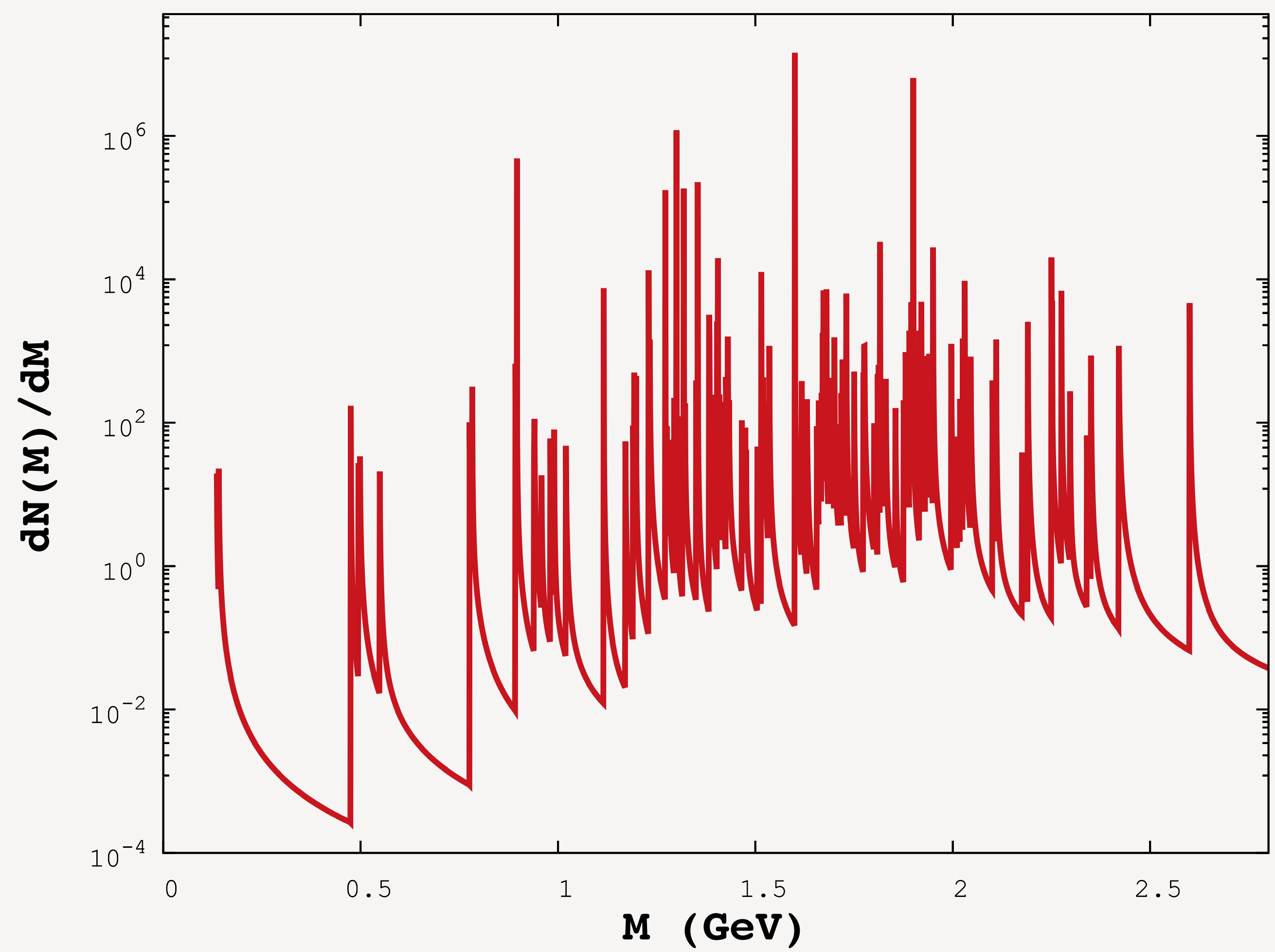


$$\Delta \ln Z = \int dE\, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \mathrm{tr}\,(\delta_E)\,.$$

$$\pi \times \theta(\sqrt{s} - m_{\mathrm{res}})$$

HRG approx.





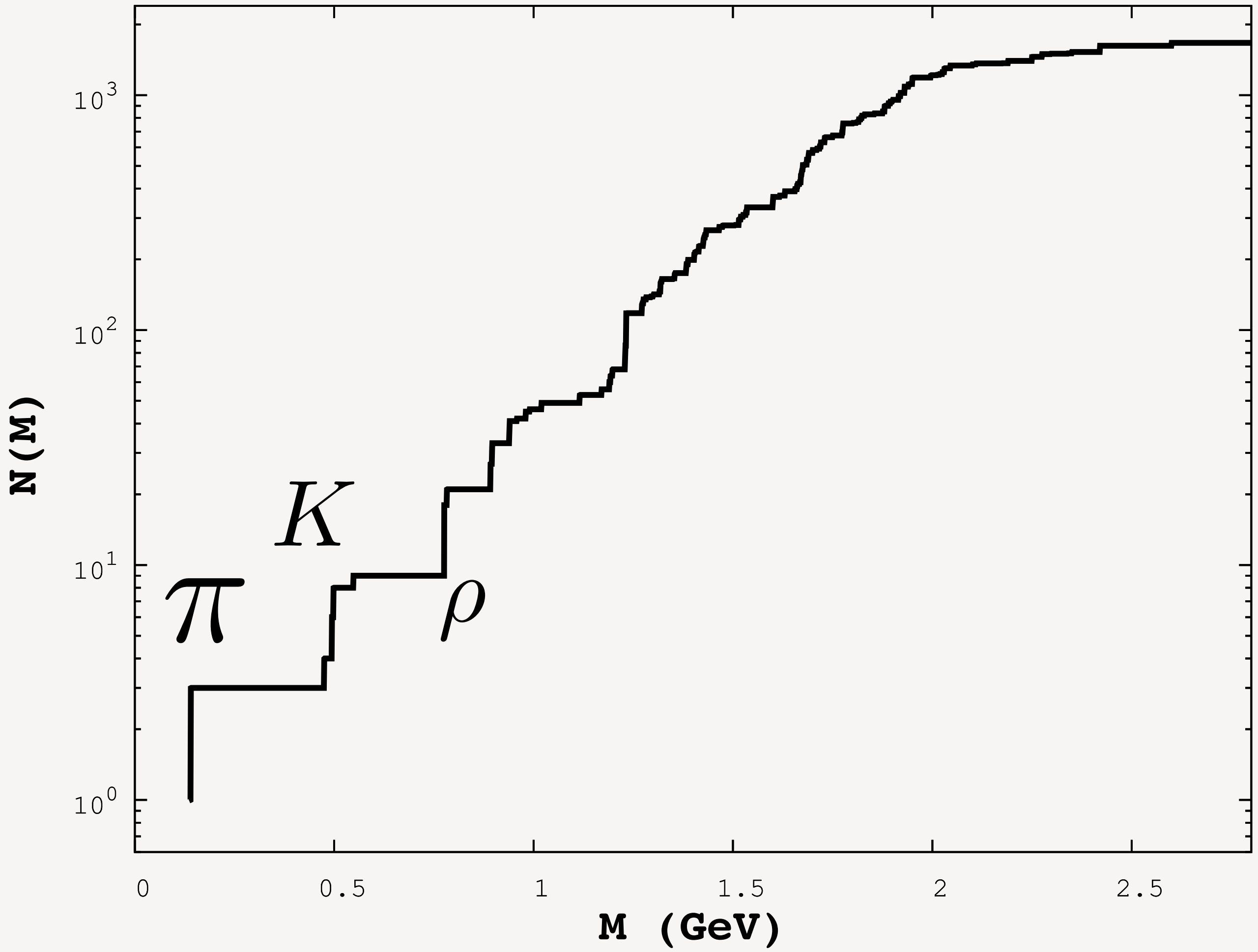
HRG MODEL

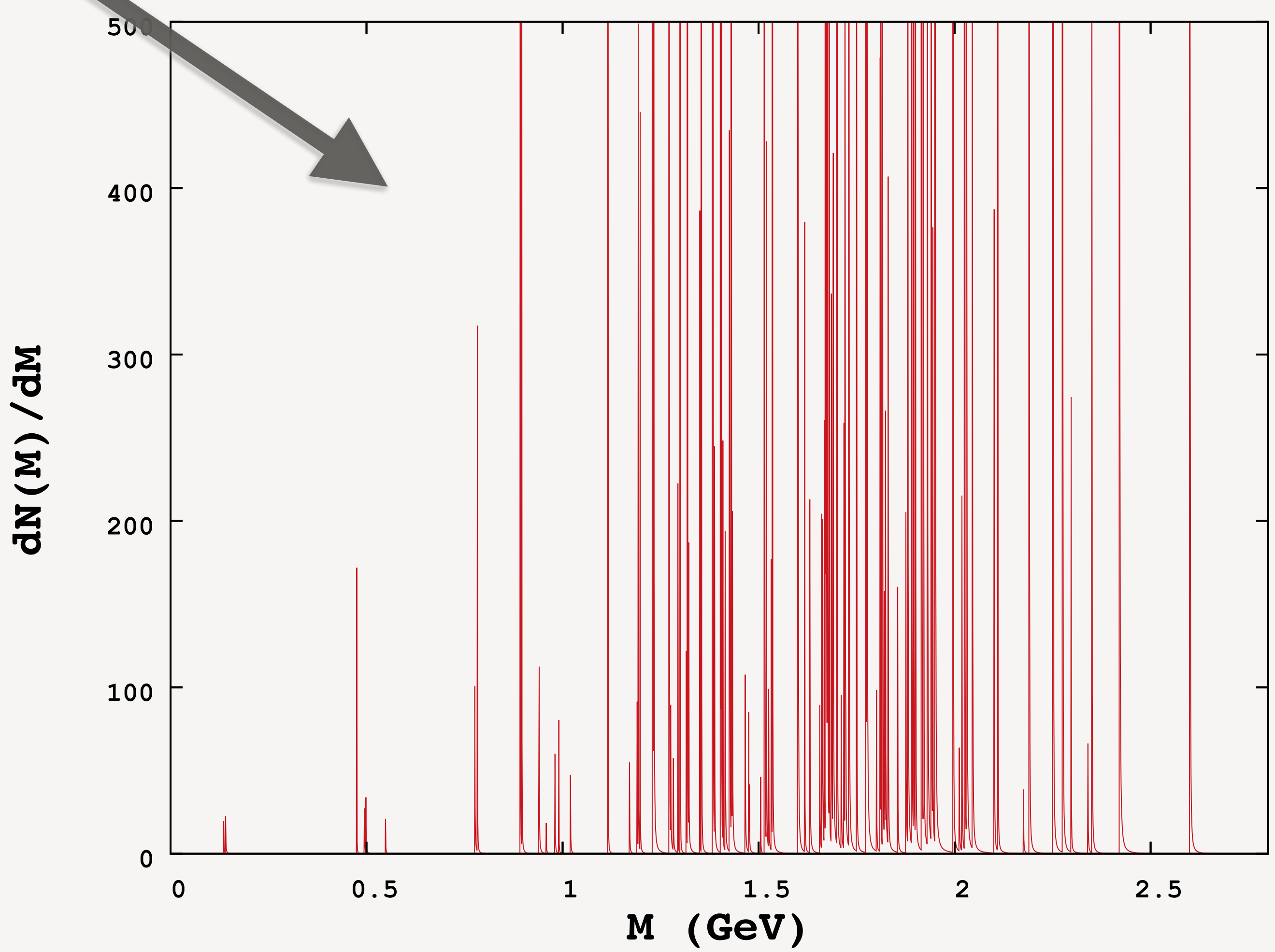
$$\Delta \ln Z = \int dE\, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \mathrm{tr}\, (\delta_E)\,.$$

HRG approx.

$$g(E,\epsilon) = \frac{d_g}{\pi} \sum_n (\tan^{-1}(\frac{E-E_n}{\epsilon}) + \frac{\pi}{2})$$

$$\frac{\partial}{\partial E}$$





WHAT'S IN A NAME? THAT WHICH WE CALL A RESONANCES?

- A resonance is MORE than a MASS and a **WIDTH**

$f_0(500)$ [g] $I^G(J^{PC}) = 0^+(0^{++})$

Mass (T-Matrix Pole $\sqrt{s}$) = (400–550)$-i$(200–350) MeV
Mass (Breit-Wigner) = (400–550) MeV
Full width (Breit-Wigner) = (400–700) MeV

$\rho(770)$ [h] $I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV
$\Gamma_{ee} = 7.04 \pm 0.06$ keV

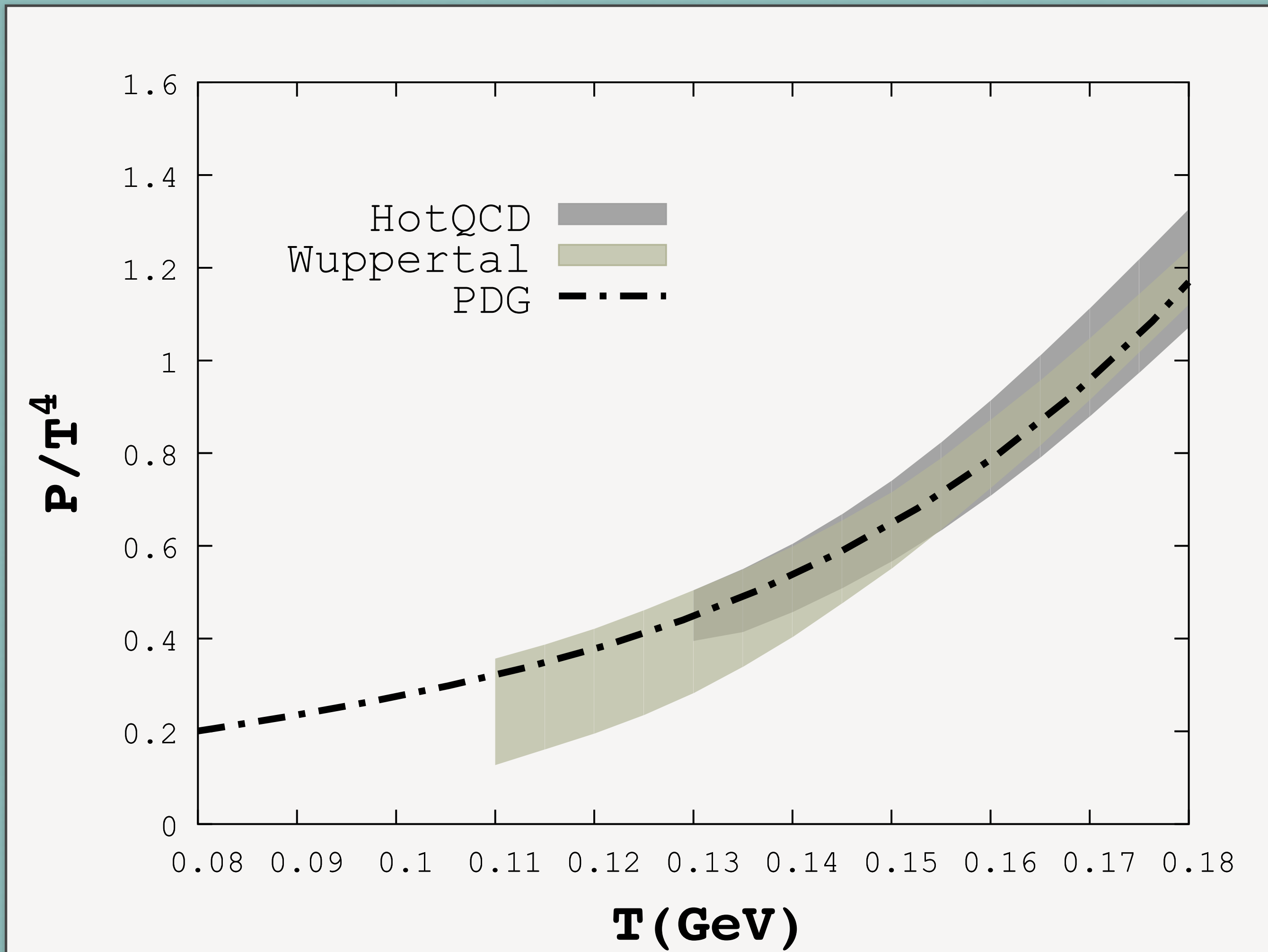
MISSION STATEMENT

- Take the full energy dependence of the S-matrix:

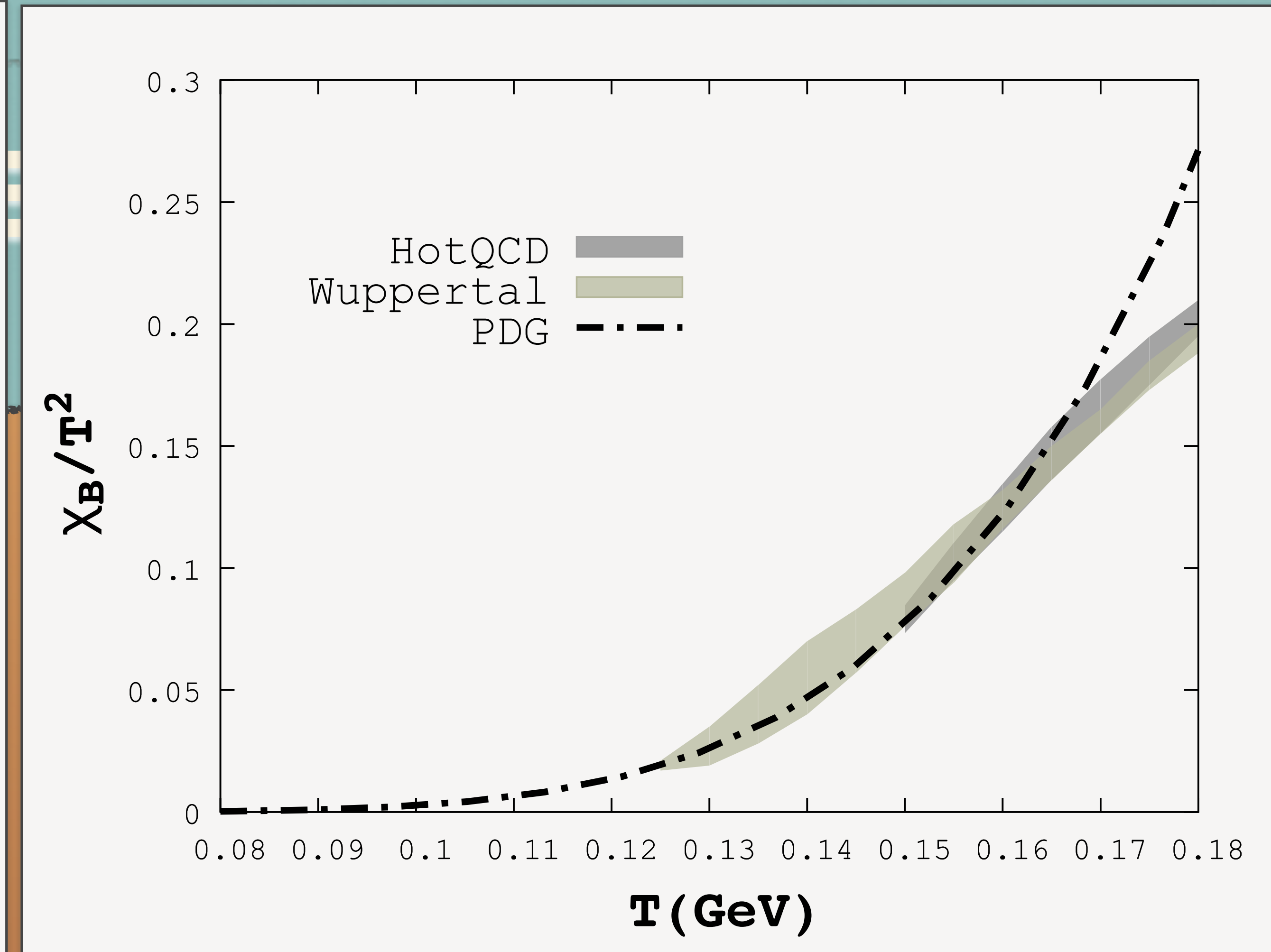
  masses, widths, branching fractions of resonances;
  coupled-channel effects;
  non-resonant interactions;
  unitarity, exchange symmetry, etc.

- incomplete knowledge of QCD spectrum
  e.g. hyperon spectrum

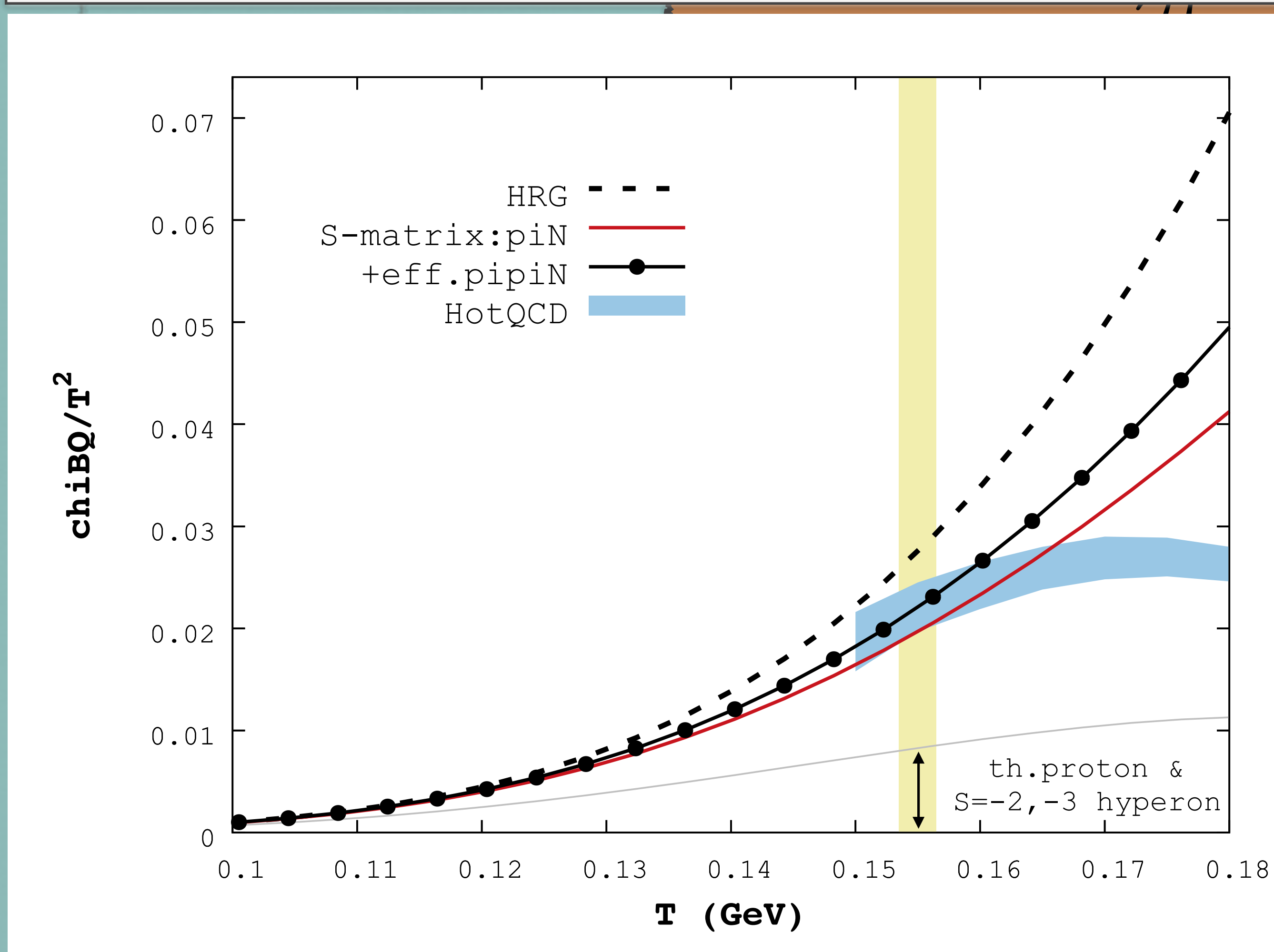
Scattering theory + thermodynamics



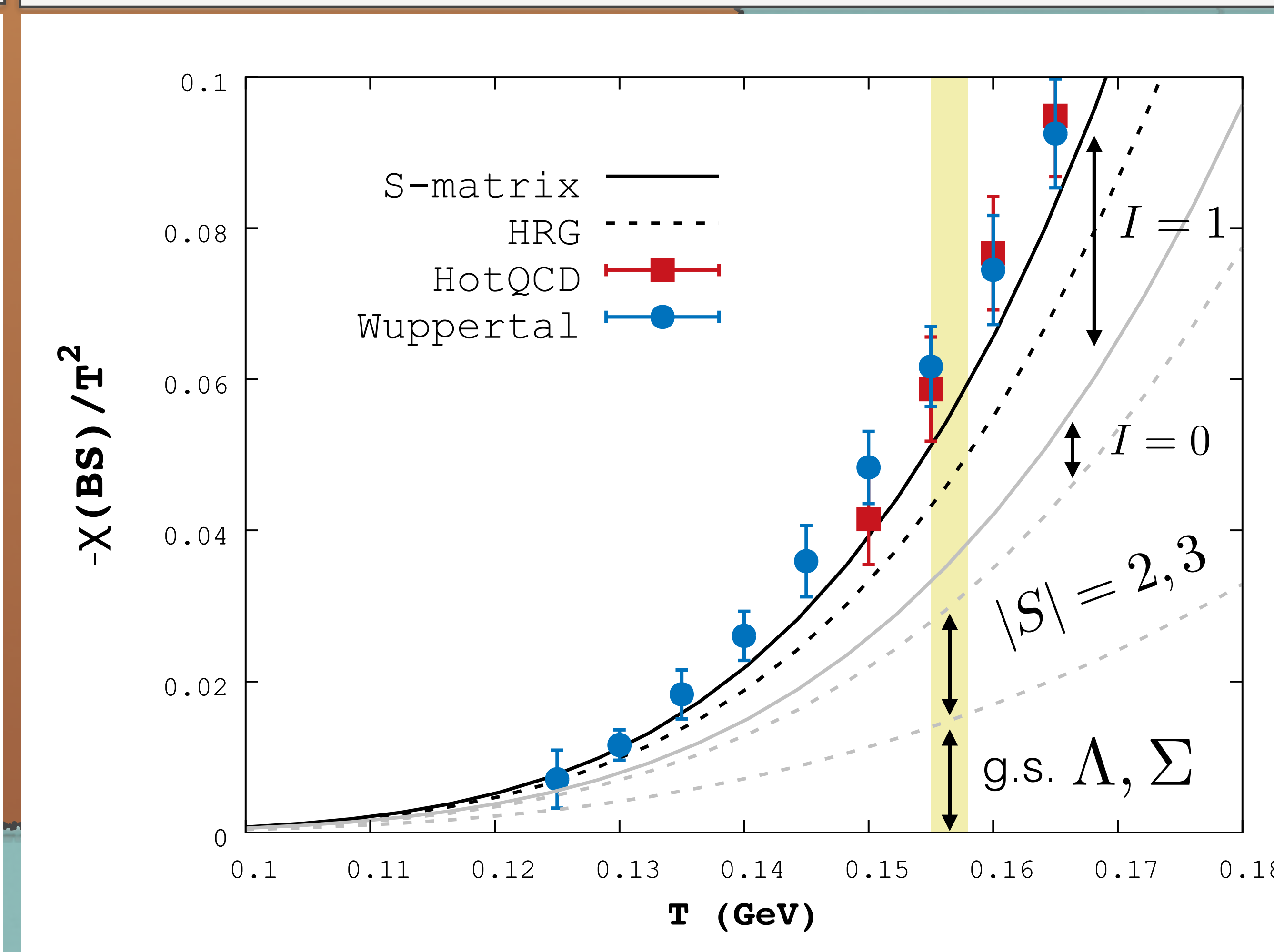
P/T^4
T(GeV)
HotQCD
Wuppertal
PDG



χ_B/T^2
T(GeV)
HotQCD
Wuppertal
PDG



chiBQ/T^2
T (GeV)
HRG
S-matrix:piN
+eff.pipiN
HotQCD
th.proton &
S=-2,-3 hyperon



-χ(BS)/T^2
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ

from S. Ceci
meson 2008

Q&A: 2
Who (else) needs the speed plot?

We Do...

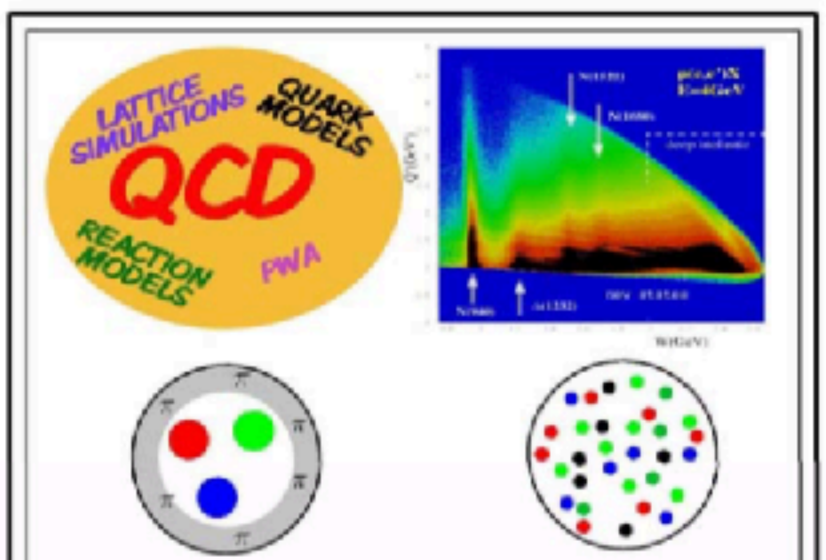
Excited Baryon Program at JLab

A Contribution to the NSAC Long Range Plan

January 11, 2007

Editors: V. Burkert, R. Gothe, T.-S. H. Lee

Contributors:
I. Aznauryan, M. Bellis, C. Bennhold, W. Briscoe, V. Burkert, S. Capstick, P. Cole, V. Crede, R. Gothe, H. Haberzettl, B. Julia-Diaz, F. Klein, S. Krewald, T.-S. H. Lee, C. Meyer, V. Mokeev, V. Pascalutsa, E. Pasyuk, D. Richards, W. Roberts, A. Sandorfi, P. Stoler, L.C. Smith, I. Strakovsky, M. Vanderhaeghen, R. Workman.



IV. EXCITED BARYON ANALYSIS CENTER

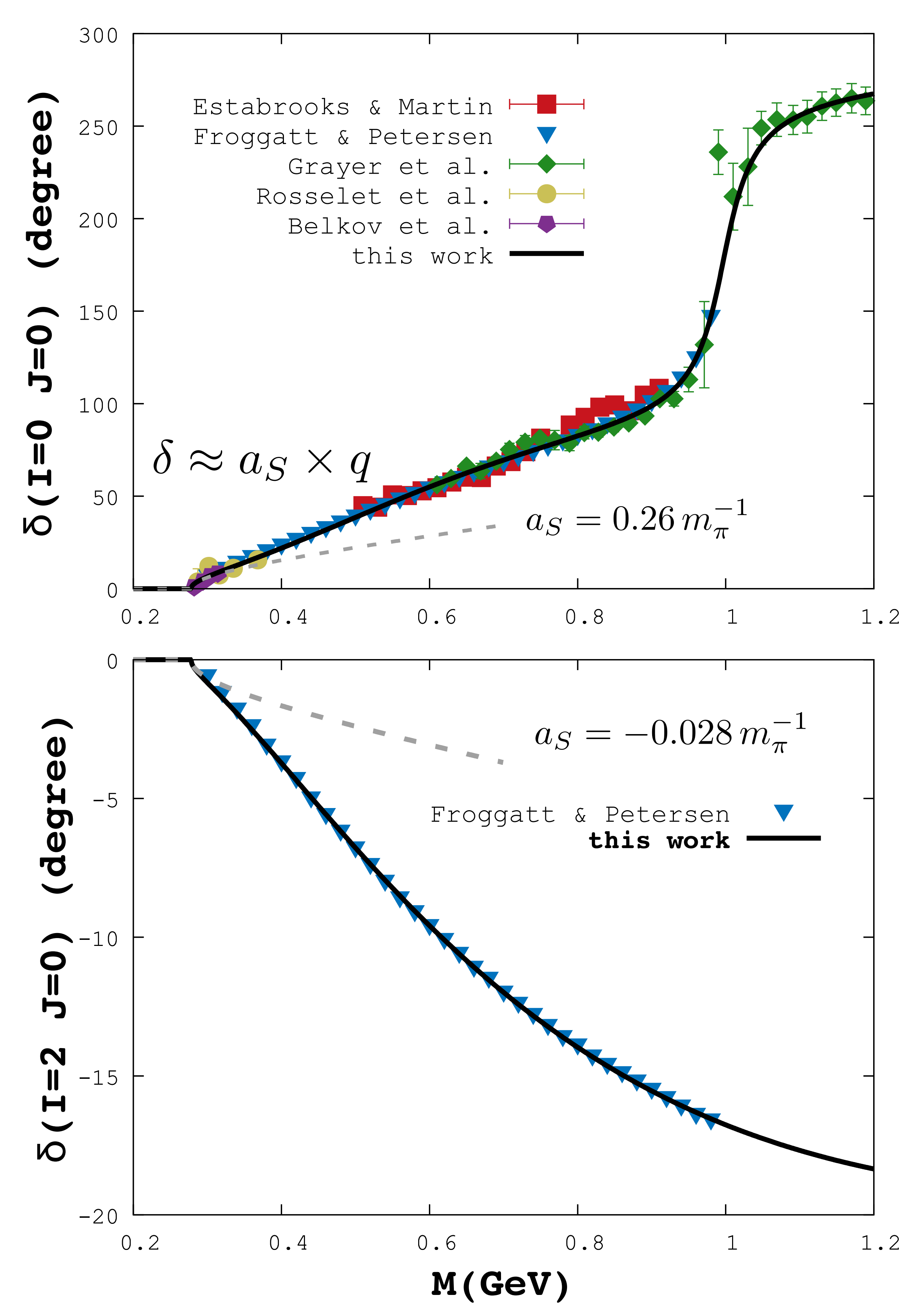
The Excited Baryon Analysis Center (EBAC) was established at JLab in January, 2006 to provide theoretical support to the excited baryon program. EBAC's program has two components. The first one is to identify new baryon states and extract the N^* parameters ...

...

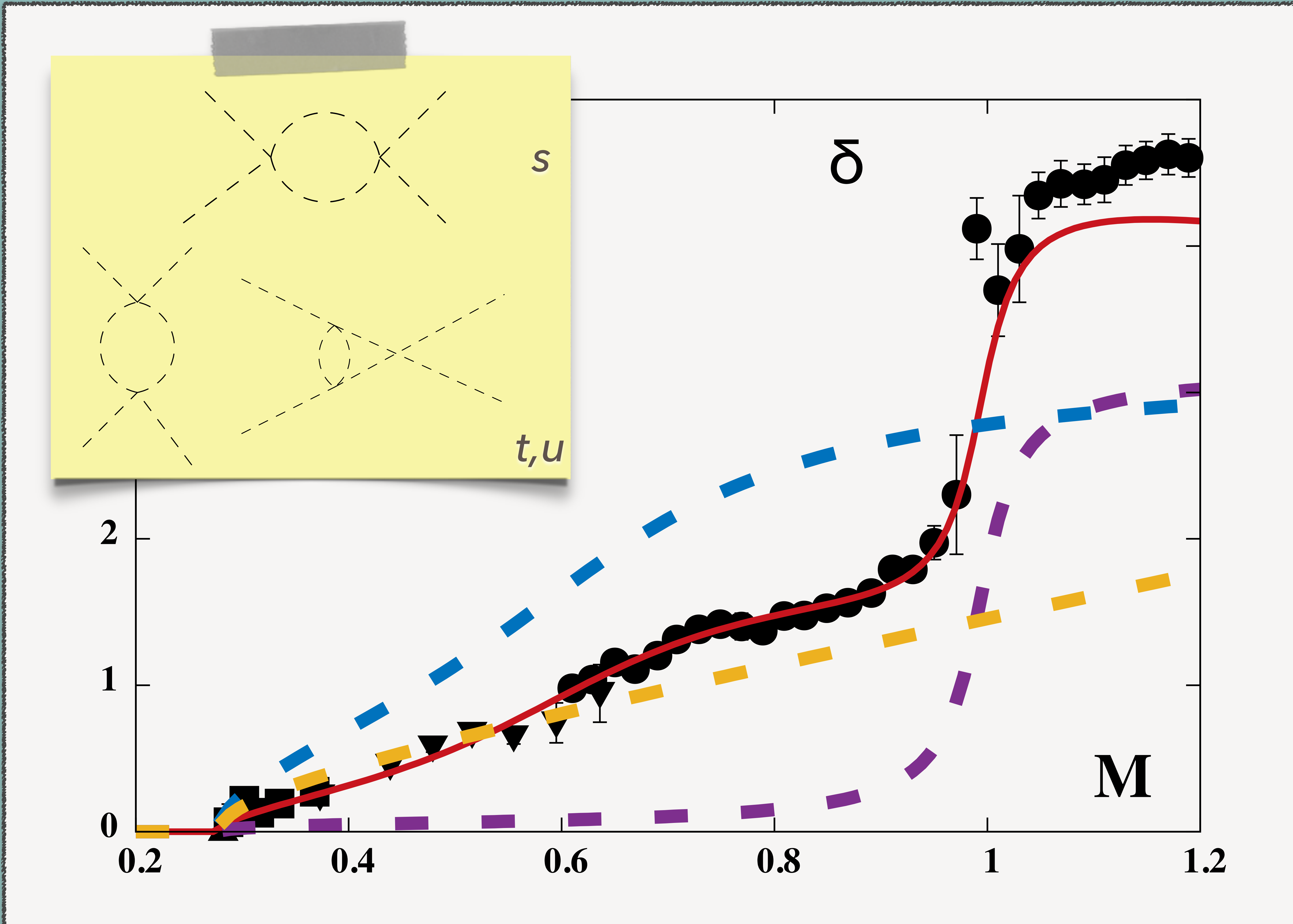
analyses must be examined. The validity of the often used speed-plot or time-delayed plot methods in extracting the N^* parameters from the determined partial-wave amplitudes should also be studied.

phase shifts encode hadronic interactions.

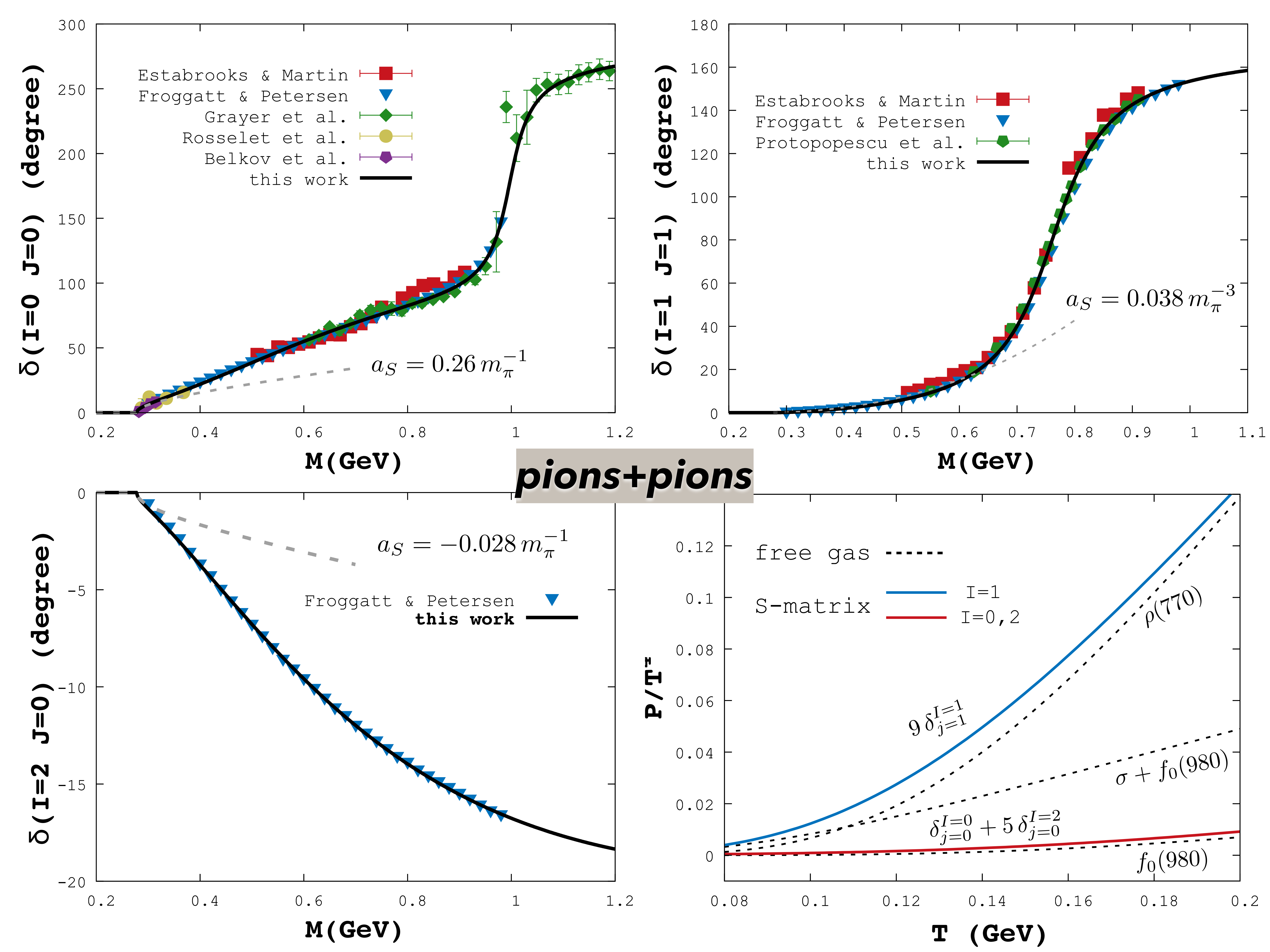
- positive:
  - attractive forces
  - resonances formation
- negative:
  - repulsive forces
  - hard-core
  - channel opening up
  - resonance not as strong



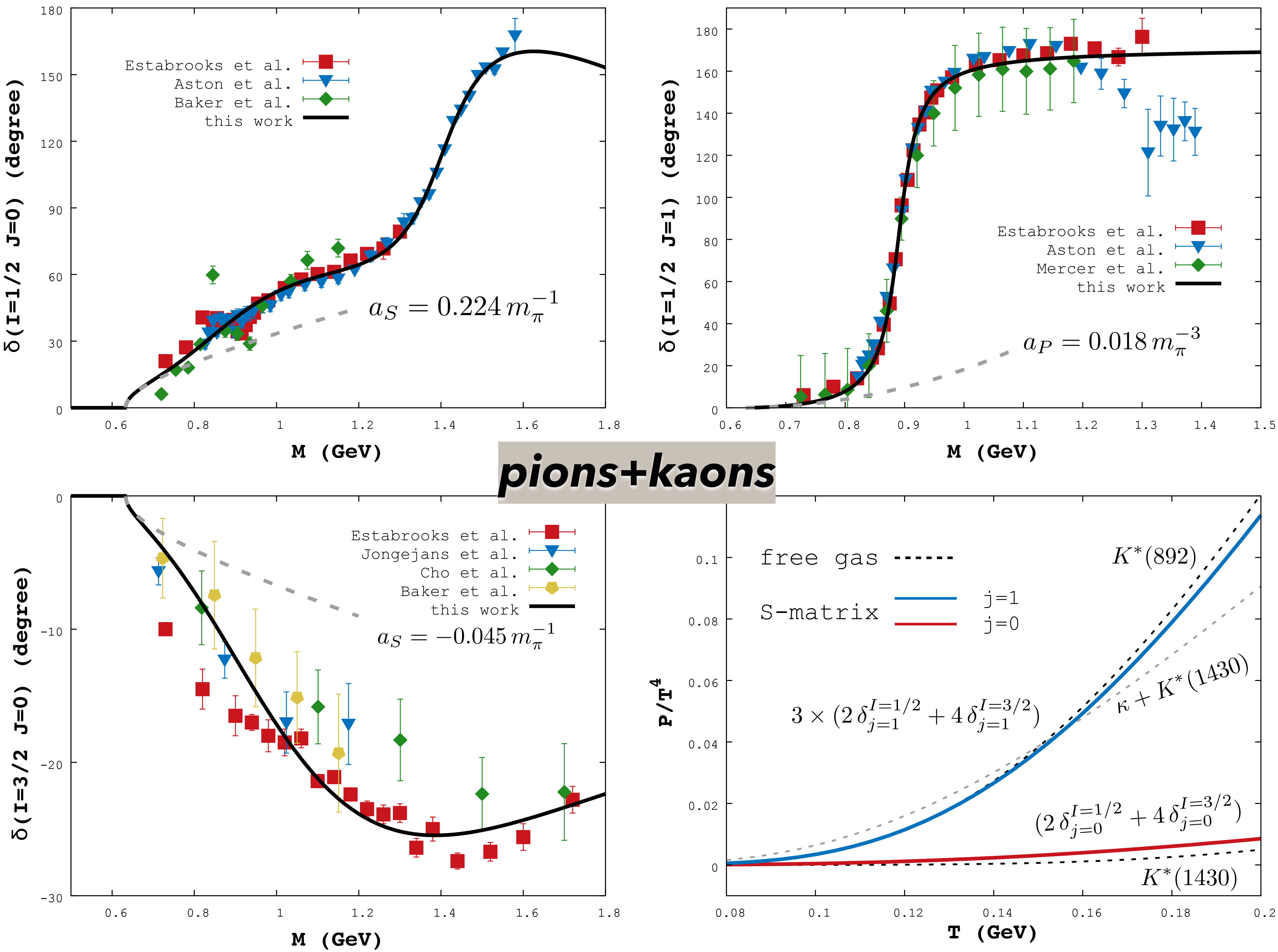
W. Broniowski, F. Giacosa and V. Begun, PRC **92**, 034905 (2015)



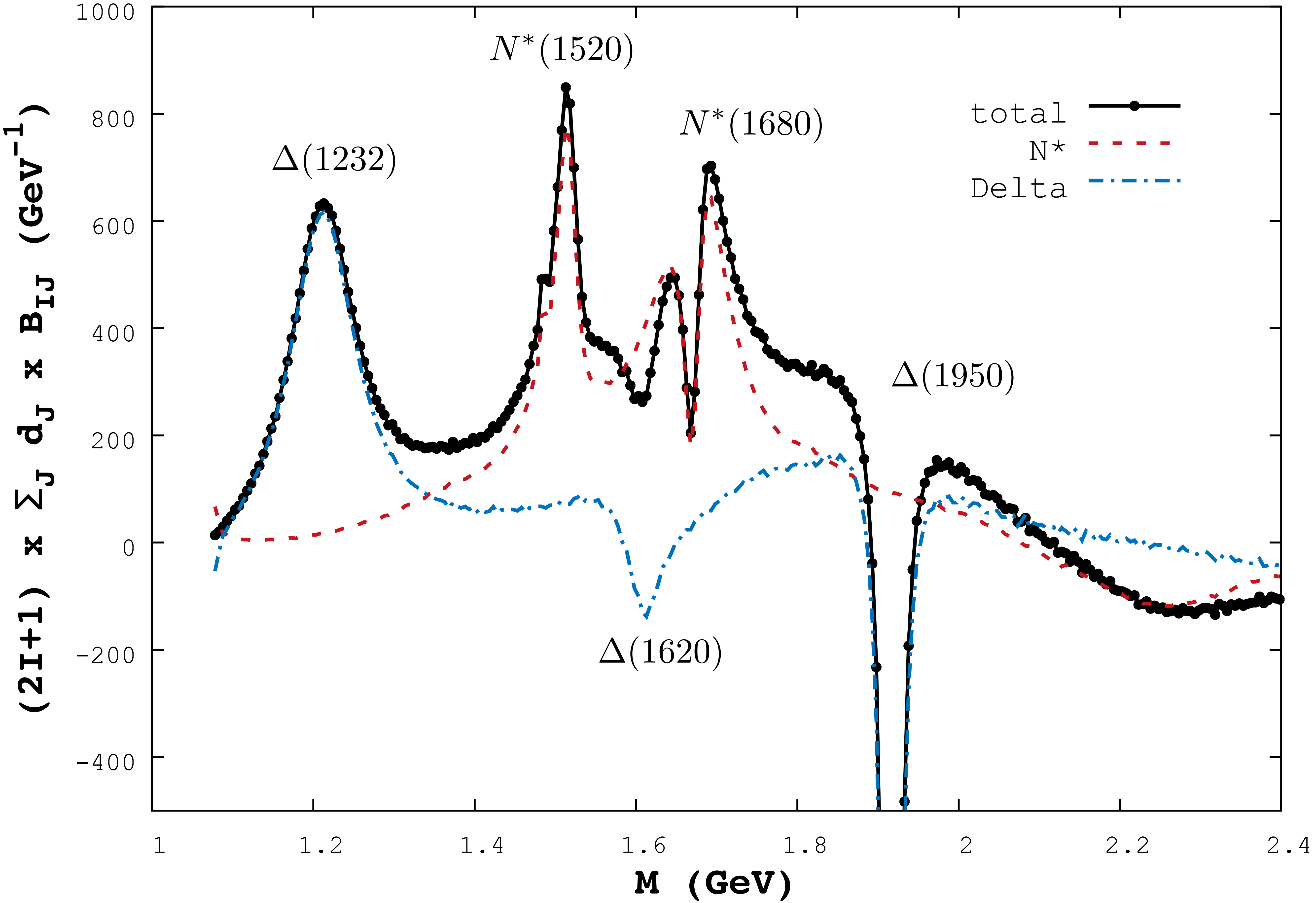
Ishida et al.
Prog. Theor. Phys. **95** (1996)

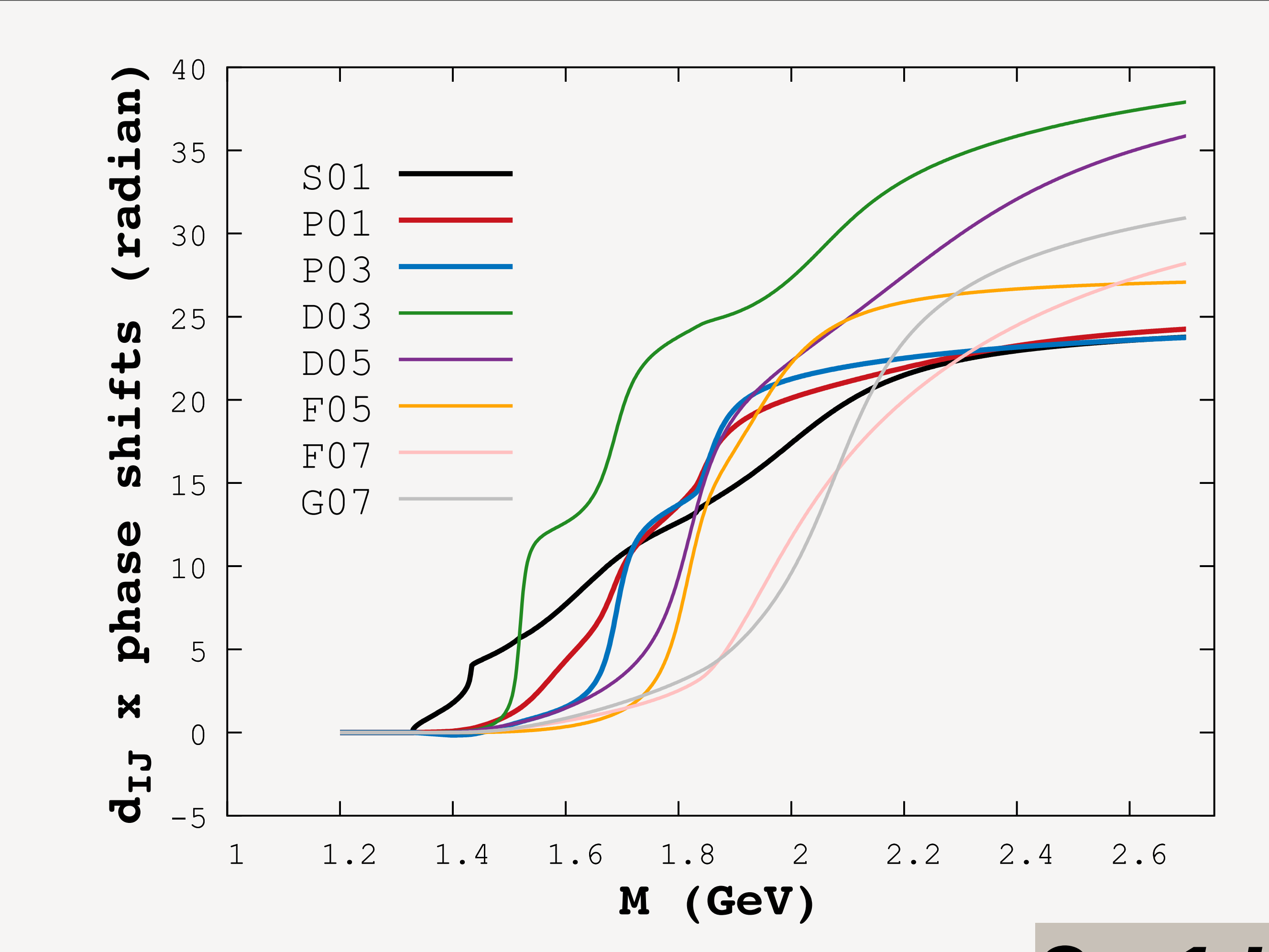


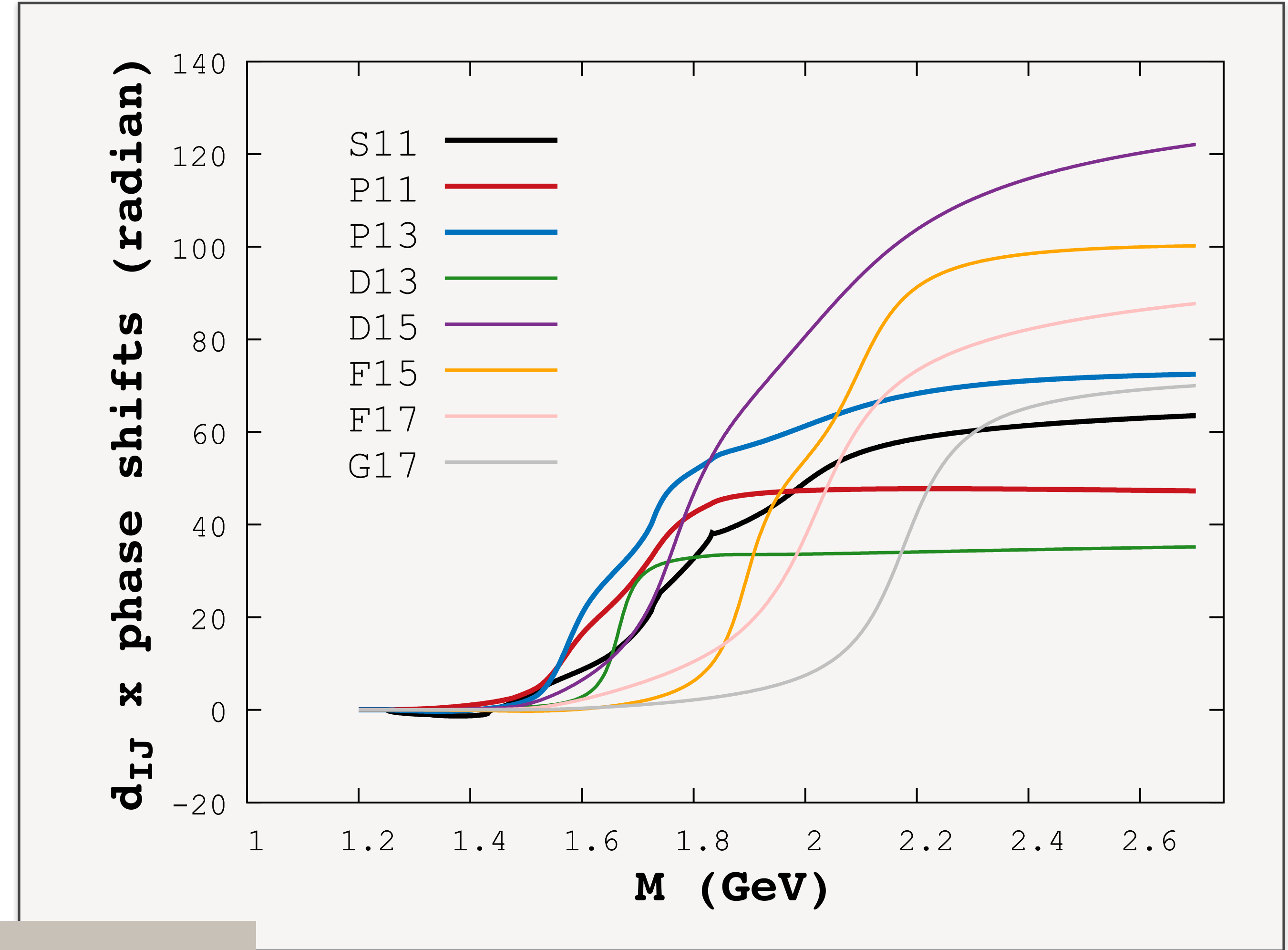
pions+pions
δ(I=0 J=0) (degree)
M(GeV)
Estabrooks & Martin
Froggatt & Petersen
Grayer et al.
Rosselet et al.
Belkov et al.
this work
$a_S = 0.26\, m_\pi^{-1}$
δ(I=1 J=1) (degree)
M(GeV)
Estabrooks & Martin
Froggatt & Petersen
Protopopescu et al.
this work
$a_S = 0.038\, m_\pi^{-3}$
δ(I=2 J=0) (degree)
M(GeV)
$a_S = -0.028\, m_\pi^{-1}$
Froggatt & Petersen
this work
P/T⁴
T (GeV)
free gas
S-matrix
I=1
I=0,2
$\rho(770)$
$9\,\delta_{j=1}^{I=1}$
$\sigma + f_0(980)$
$\delta_{j=0}^{I=0} + 5\,\delta_{j=0}^{I=2}$
$f_0(980)$



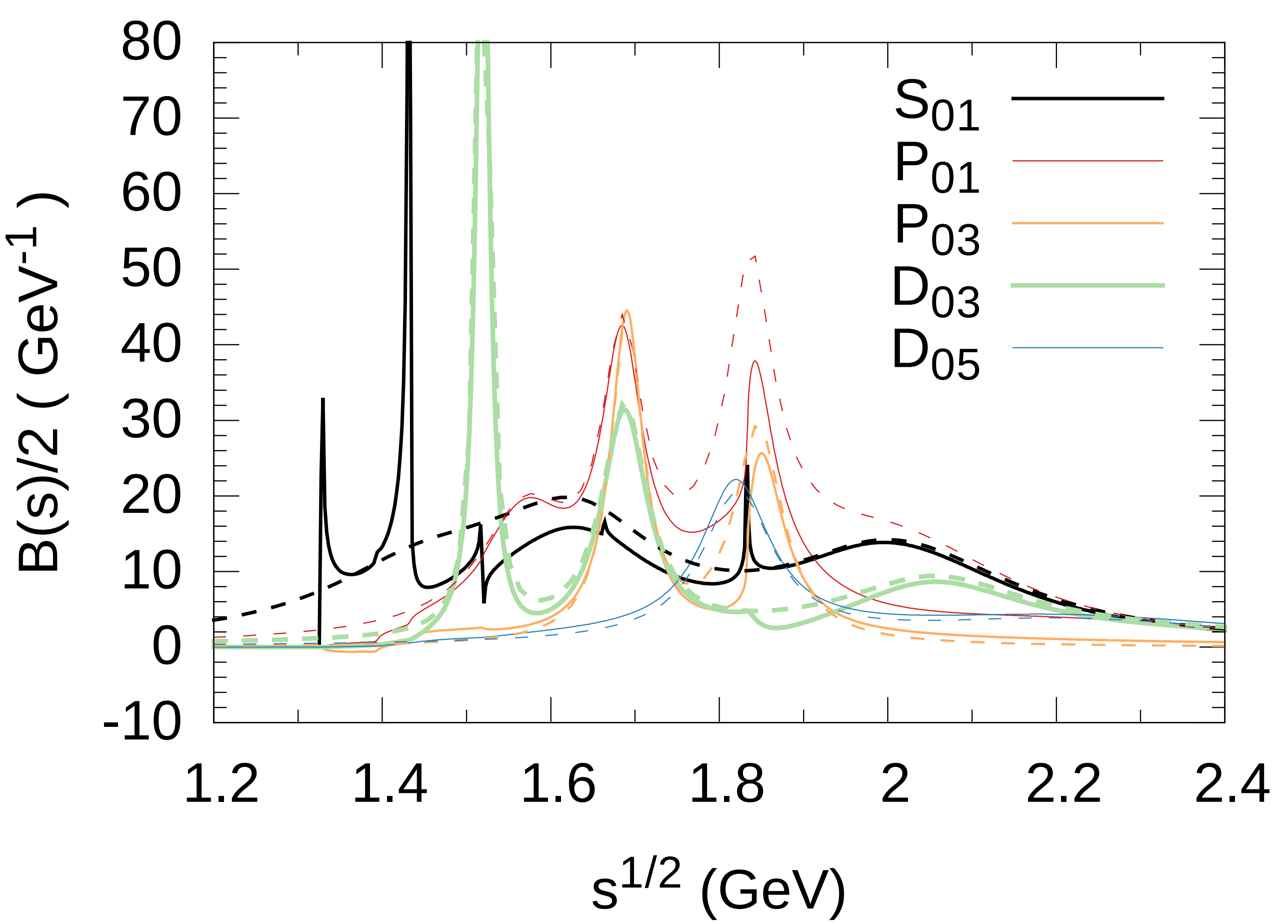
B. Friman, PML, M.Marczenko, K. Redlich, C. Sasaki,
PRD **92** 074003 (2015)

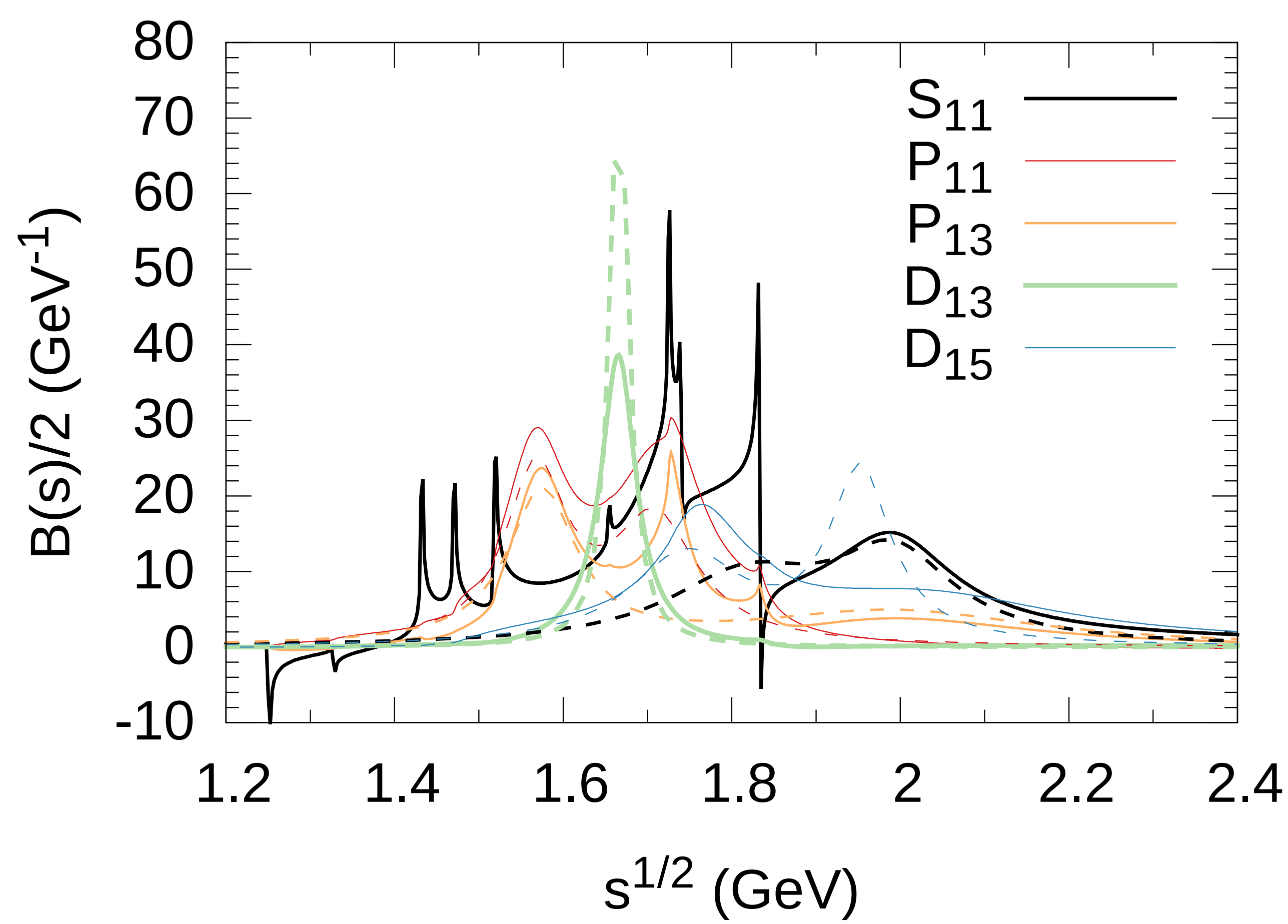

$N^*(1520)$
$N^*(1680)$
$\Delta(1232)$
$\Delta(1950)$
$\Delta(1620)$
total
N*
Delta
1000
800
600
400
200
0
-200
-400
1
1.2
1.4
1.6
1.8
2
2.2
2.4
M (GeV)
$(2I+1) \times \Sigma_J d_J \times B_{IJ}$ (GeV^{-1})


d_IJ x phase shifts (radian)
M (GeV)
S01
P01
P03
D03
D05
F05
F07
G07


d_IJ x phase shifts (radian)
M (GeV)
S11
P11
P13
D13
D15
F15
F17
G17

S=-1 Hyperon


B(s)/2 (GeV-1)
s1/2 (GeV)
S01
P01
P03
D03
D05


B(s)/2 (GeV-1)
s1/2 (GeV)
S11
P11
P13
D13
D15

S-MATRIX APPROACH

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

P. Fre, Fortschritte der Physik, 25 1-12 (1977)

PML, EPJC **77** no.8 533 (2017).

S-MATRIX FORMULATION OF THERMODYNAMICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

TODO'S

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

S-matrix interpretation of medium effects:

Leutwyler and Smilga (1990),
A. Schenk (1991),
Kacir and Zahed (1996)

VS std. QFT

VS imaginary time FTFT

P. Fre (1977)

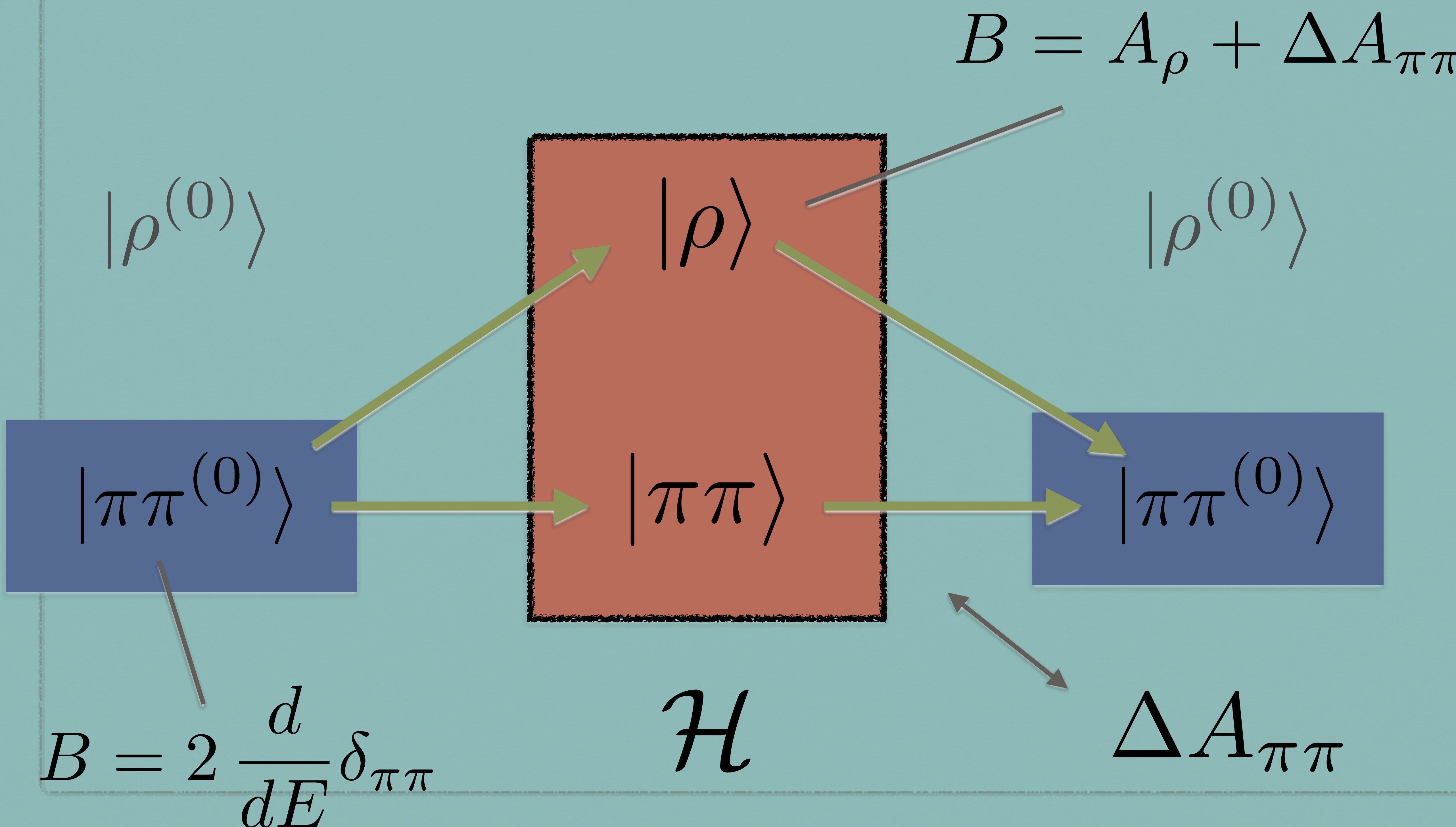
S-matrix Functional

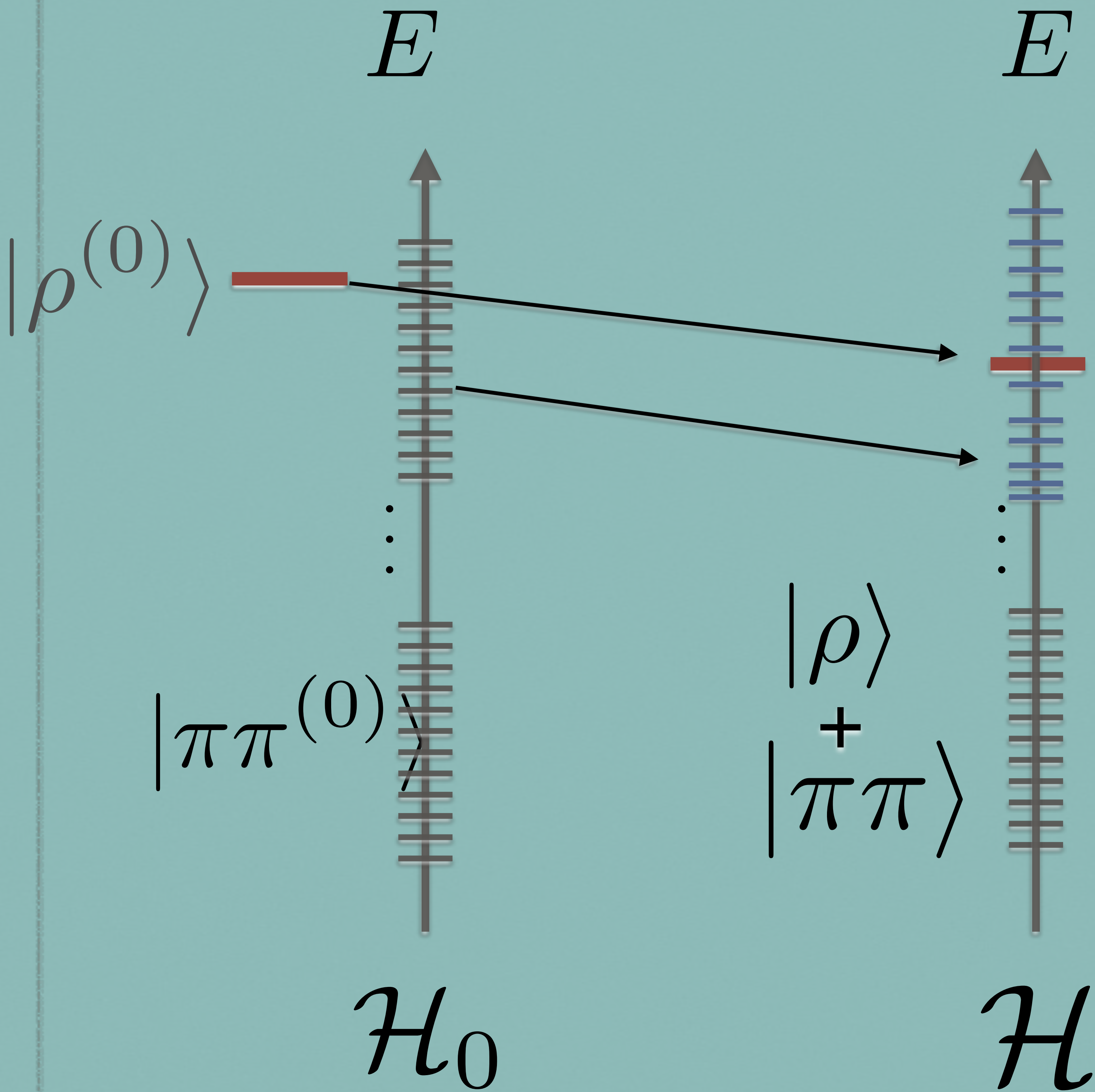
Jevicki and Lee (1987)

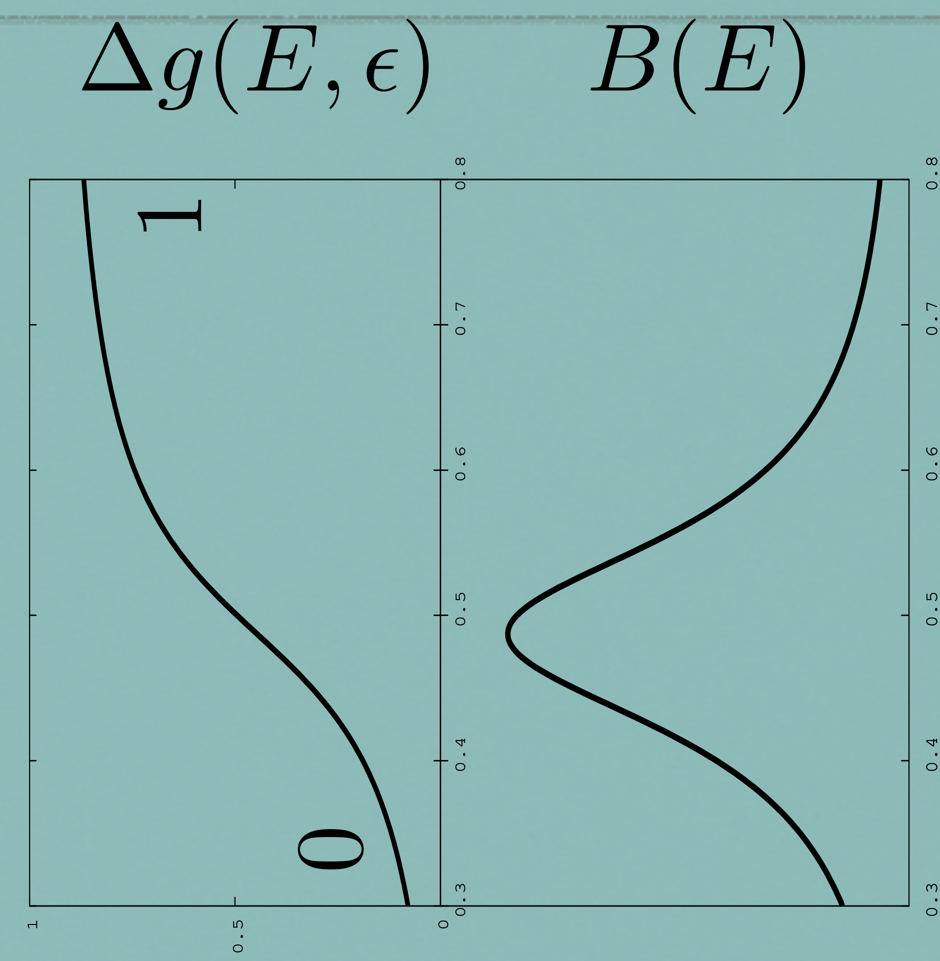
3+-body: e.g. pipiN

coupled-channel system

SCATTERING THEORY VS HAMILTONIAN (LEE MODEL)







$$g(E, \epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E, \epsilon)$$

M. G. Krein, J. Friedel, P. Llyod

A SIMPLE TRICK

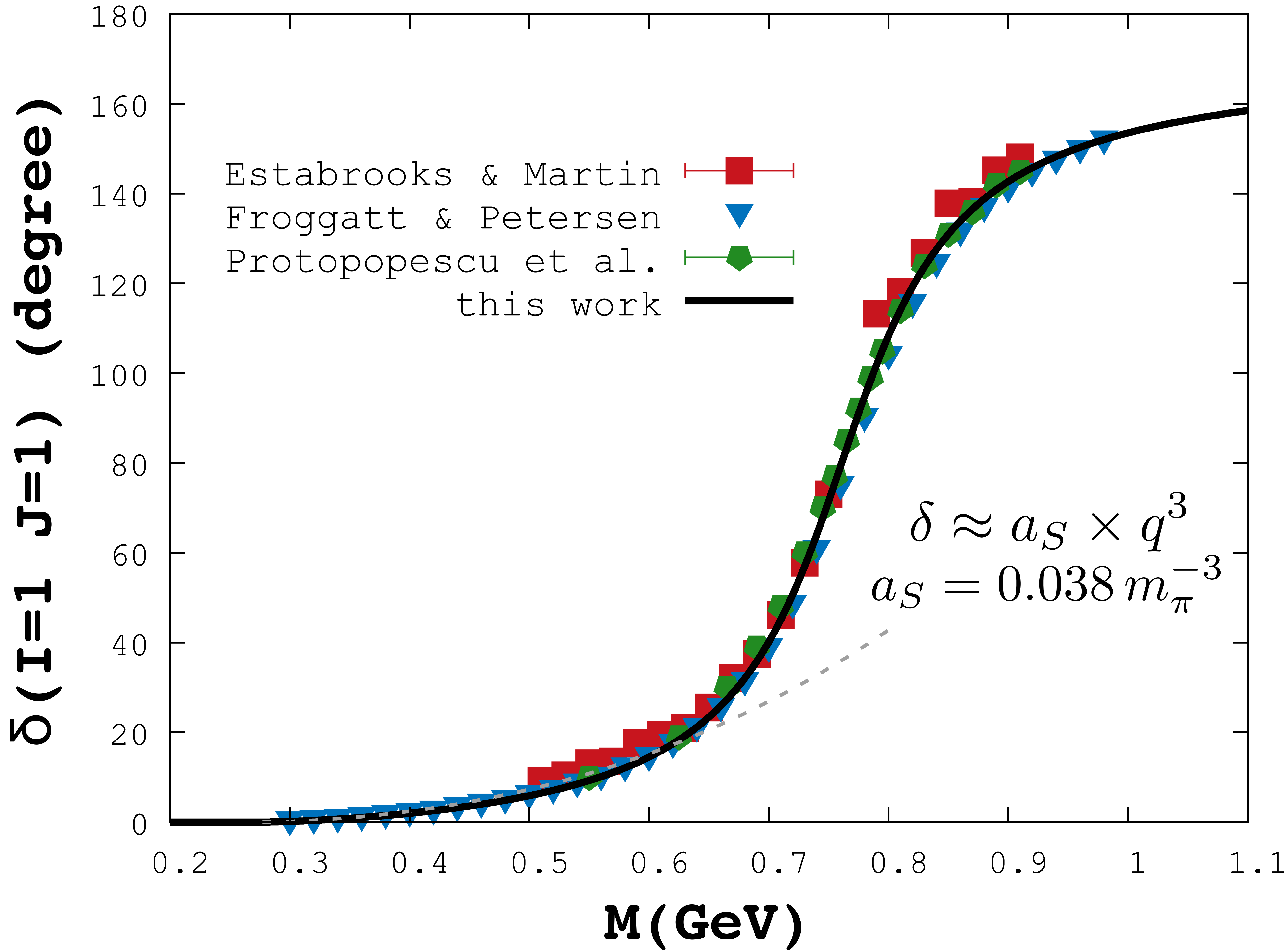
$$S_E = e^{2i\delta_E}$$

$$\frac{1}{4\pi i}\,\mathrm{tr}\left\{S_E^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S_E\right\}_c$$

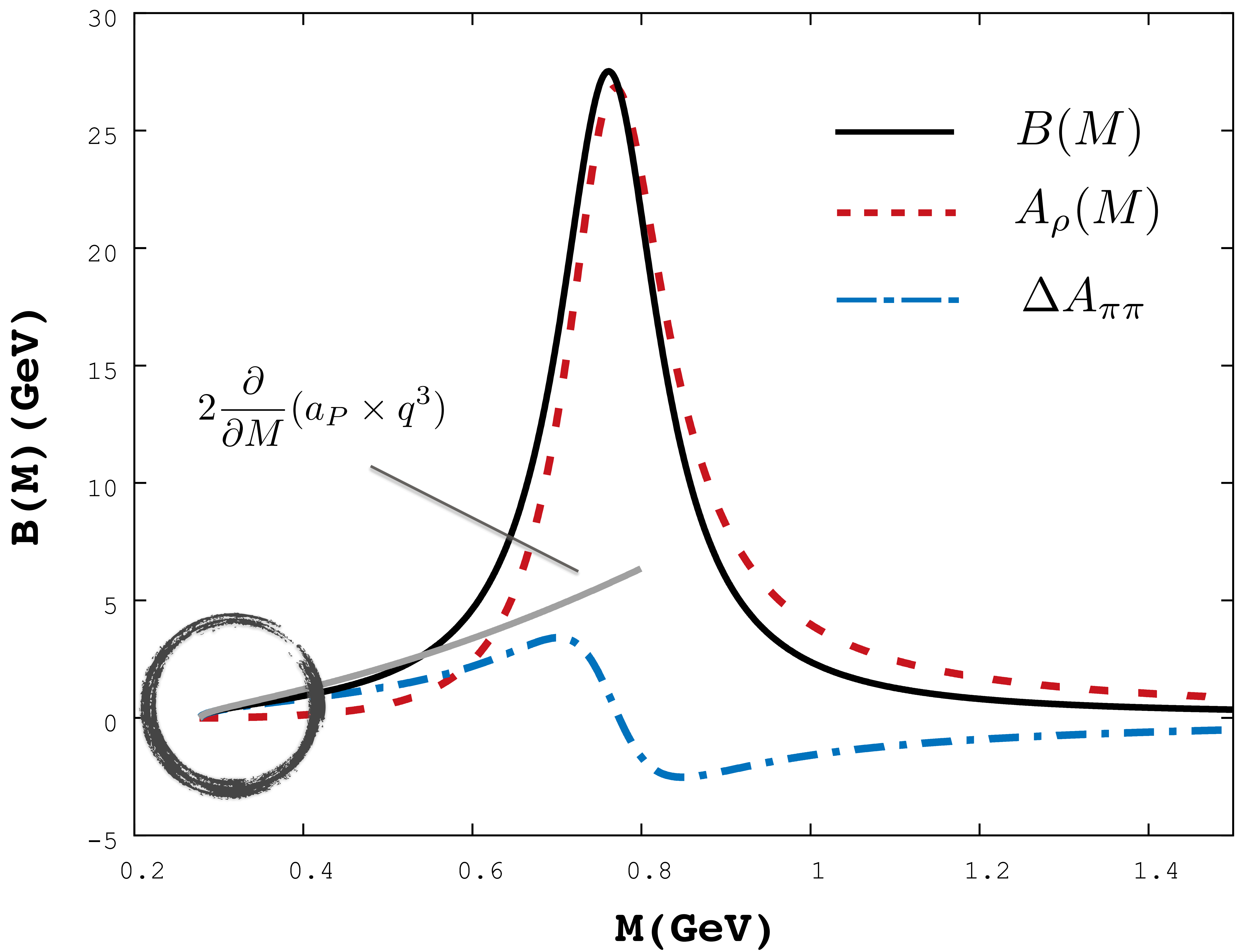
$$= \frac{1}{2\pi}\times 2\,\frac{\partial}{\partial E}\,\frac{1}{2}\,\mathrm{Im}\,\mathrm{tr}\left\{\ln S_E\right\}$$

$$\Delta \ln Z = \int dE\, e^{-\beta E}\times\frac{1}{\pi}\frac{\partial}{\partial E}\,\mathrm{tr}\,(\delta_E)\,.$$

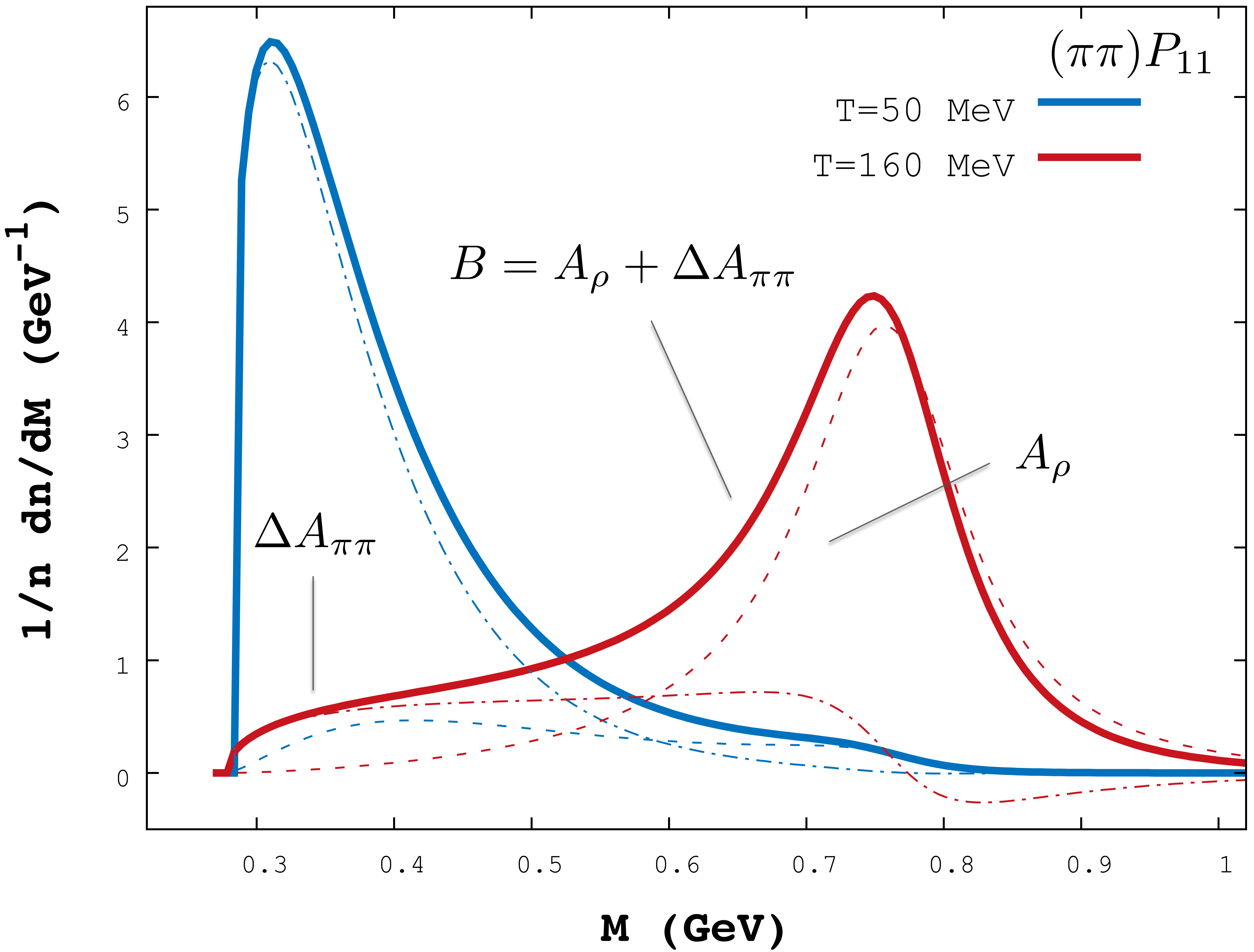
E. Beth and G. Uhlenbeck,
Physica (Amsterdam) 4, 915 (1937).



Estabrooks & Martin
Froggatt & Petersen
Protopopescu et al.
this work
$\delta \approx a_S \times q^3$
$a_S = 0.038\, m_\pi^{-3}$
δ(I=1 J=1) (degree)
M(GeV)
0
20
40
60
80
100
120
140
160
180
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1



B(M) (GeV)
M(GeV)
$B(M)$
$A_\rho(M)$
$\Delta A_{\pi\pi}$
$2\frac{\partial}{\partial M}(a_P \times q^3)$
30
25
20
15
10
5
0
-5
0.2
0.4
0.6
0.8
1
1.2
1.4



$(\pi\pi)P_{11}$
T=50 MeV
T=160 MeV
$B = A_\rho + \Delta A_{\pi\pi}$
A_ρ
$\Delta A_{\pi\pi}$
1/n dn/dM (GeV^{-1})
M (GeV)
0
1
2
3
4
5
6
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

PHYSICS OF B

$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

$$B = 2\,\frac{\partial}{\partial E}\delta$$

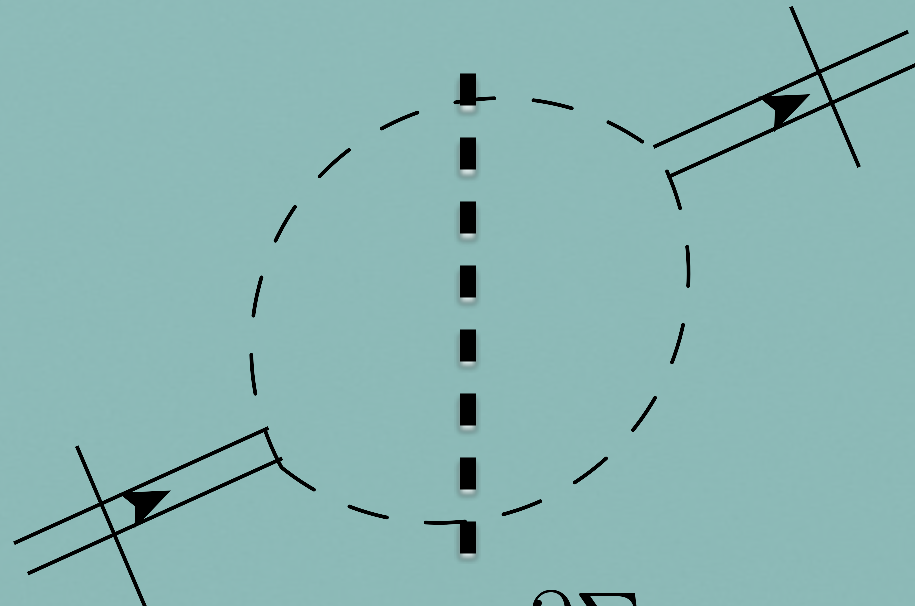
$$= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1}$$

$$= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial \Sigma_\rho}{\partial E} G_\rho]$$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$

physical interpretation:

contribution from correlated pi pi pair

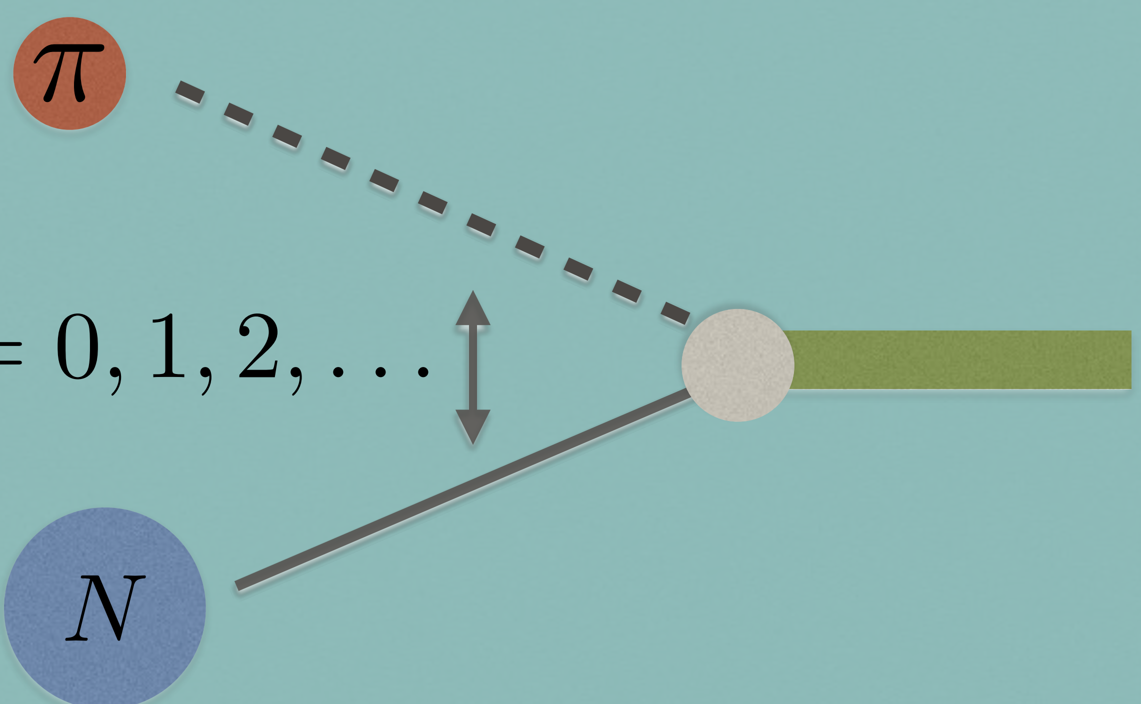


$$\frac{\partial \Sigma_\rho}{\partial E}$$

PROTON PUZZLE

N* AND DELTAS

$$I = 1, j = 0$$



$$l = 0, 1, 2, \ldots$$

$$I = 1/2, j = 1/2$$

N^* $\quad\quad$ Δ

$$I = 1/2, 3/2$$

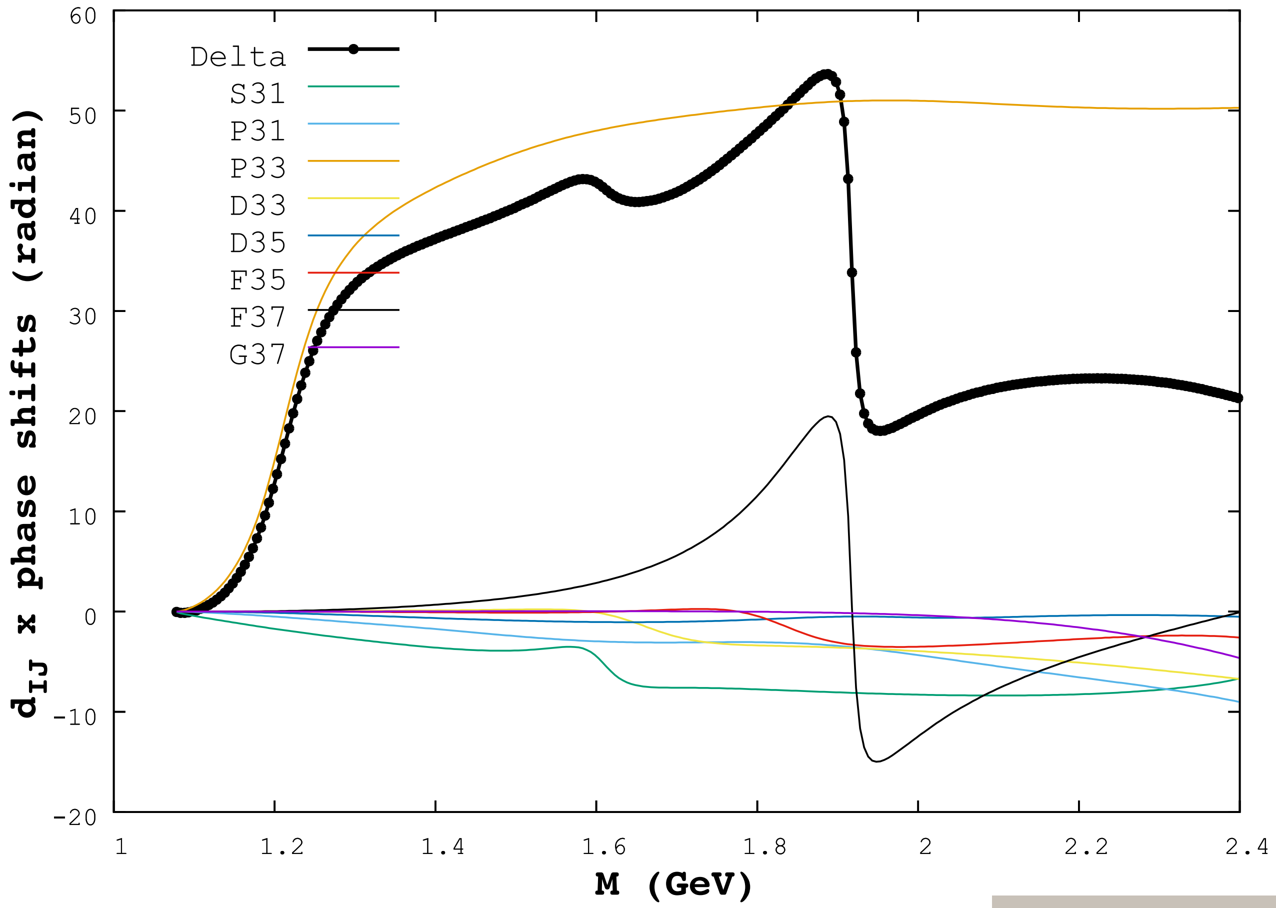
$$j = |\, l \pm 1/2 \,|,$$

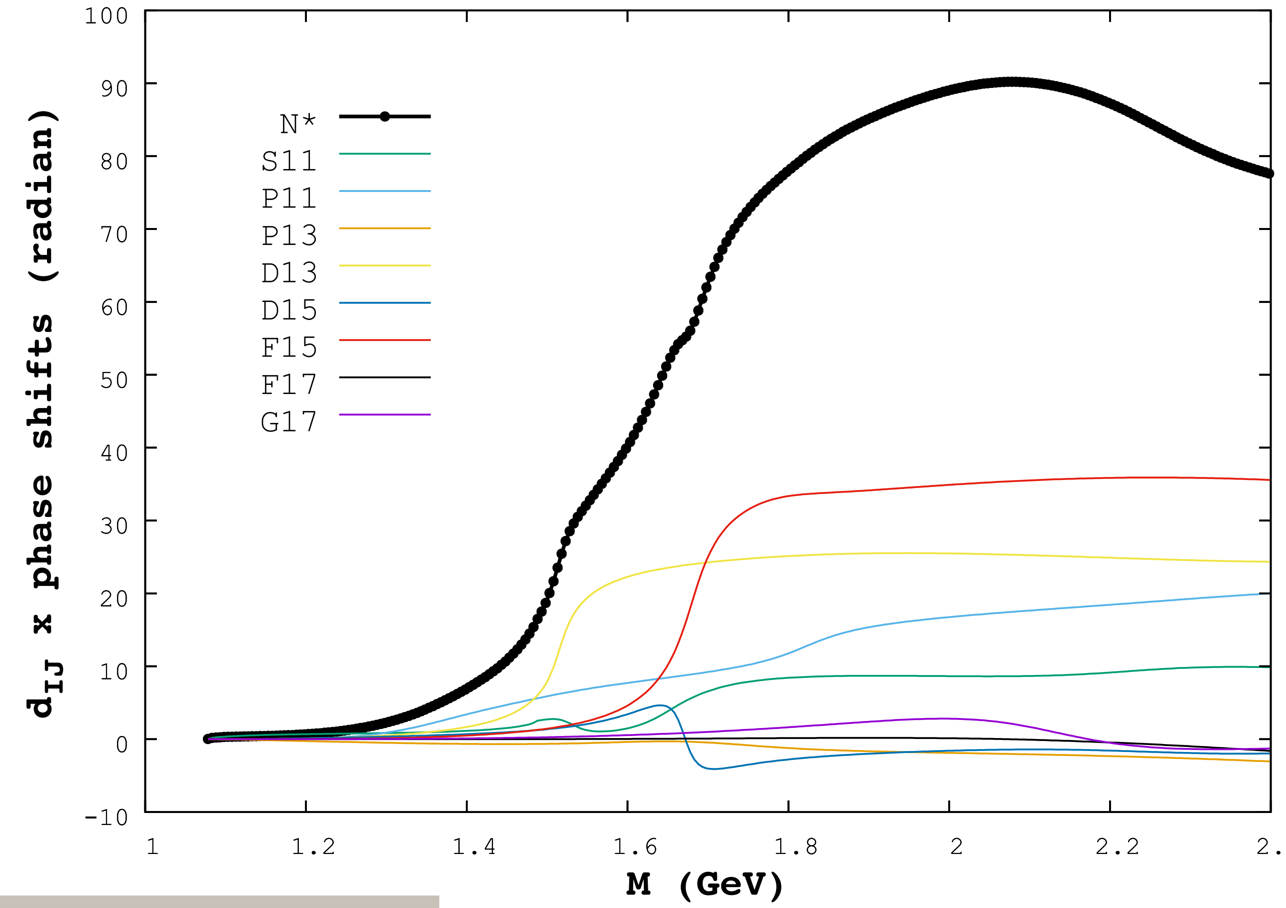
$$P = (-1)^{l+1}$$

N* AND DELTAS

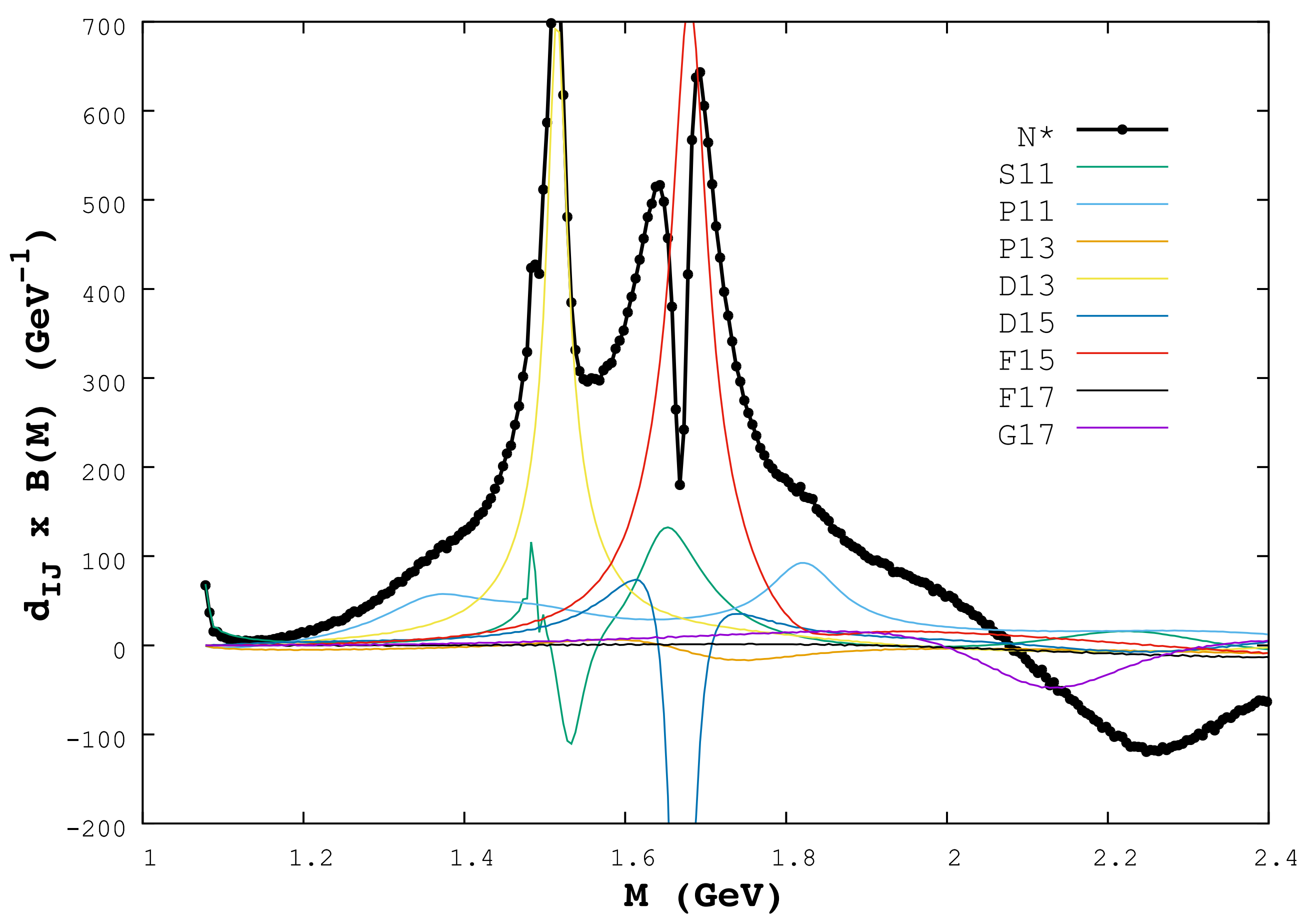
- N*: 1535 (S11), 1440 (P11), 1520 (D13) ...
  Δ : 1232 (P33), 1620 (S31) ...
- Repulsive forces between pions and nucleons
- BQ-correlation: S = -1 hyperons are excluded!

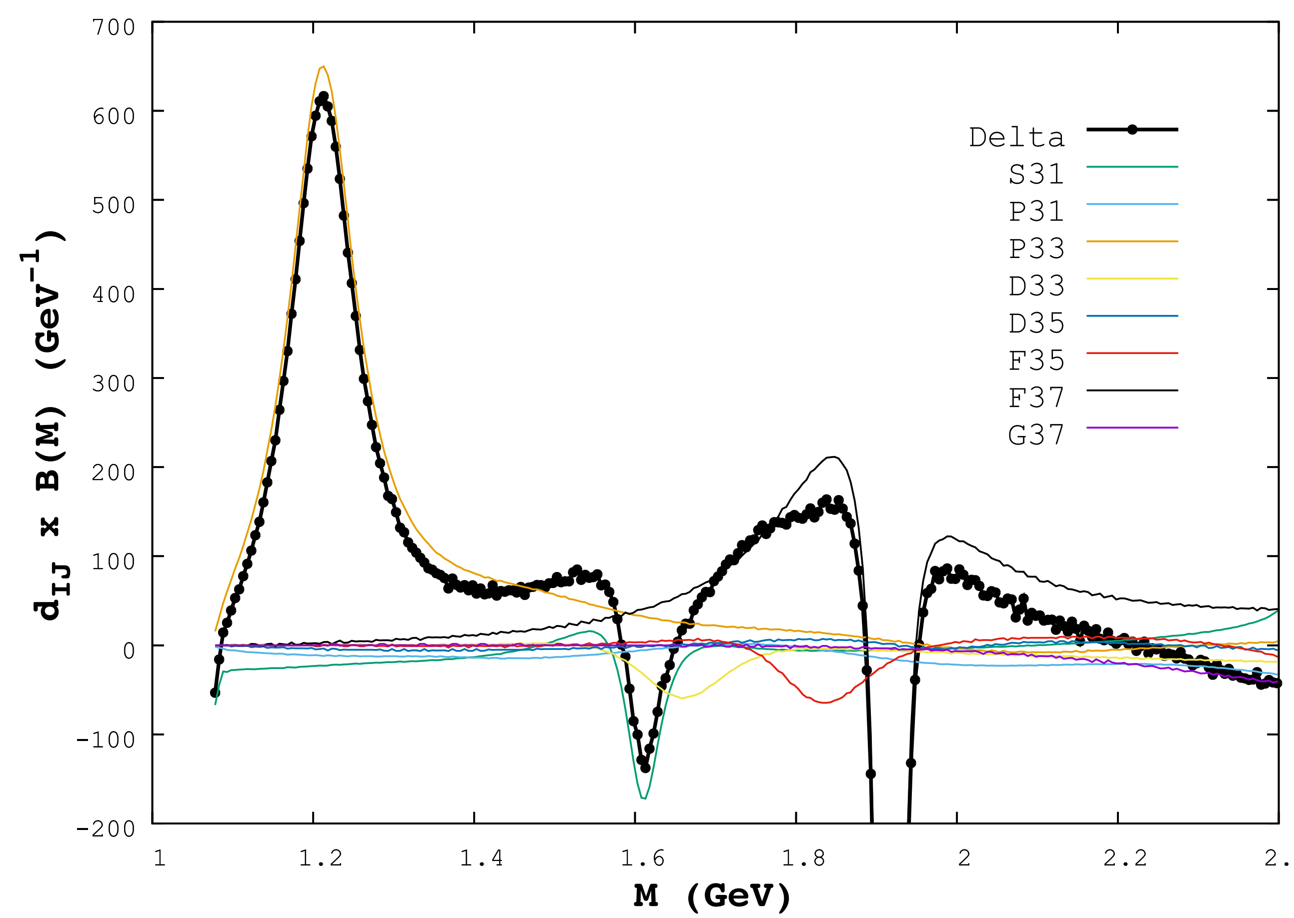
$$Q = I_z + \frac{1}{2}(B + S)$$

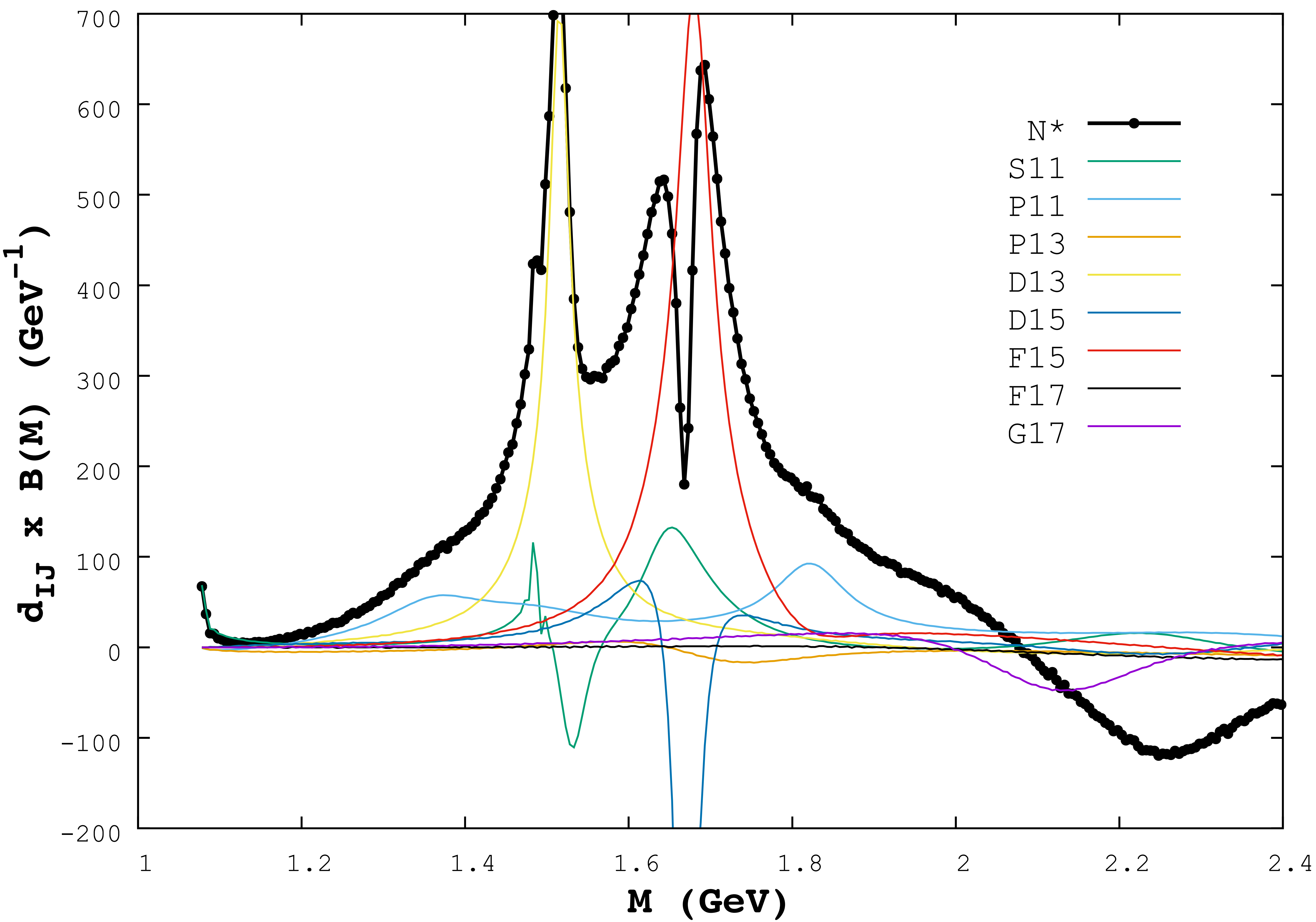




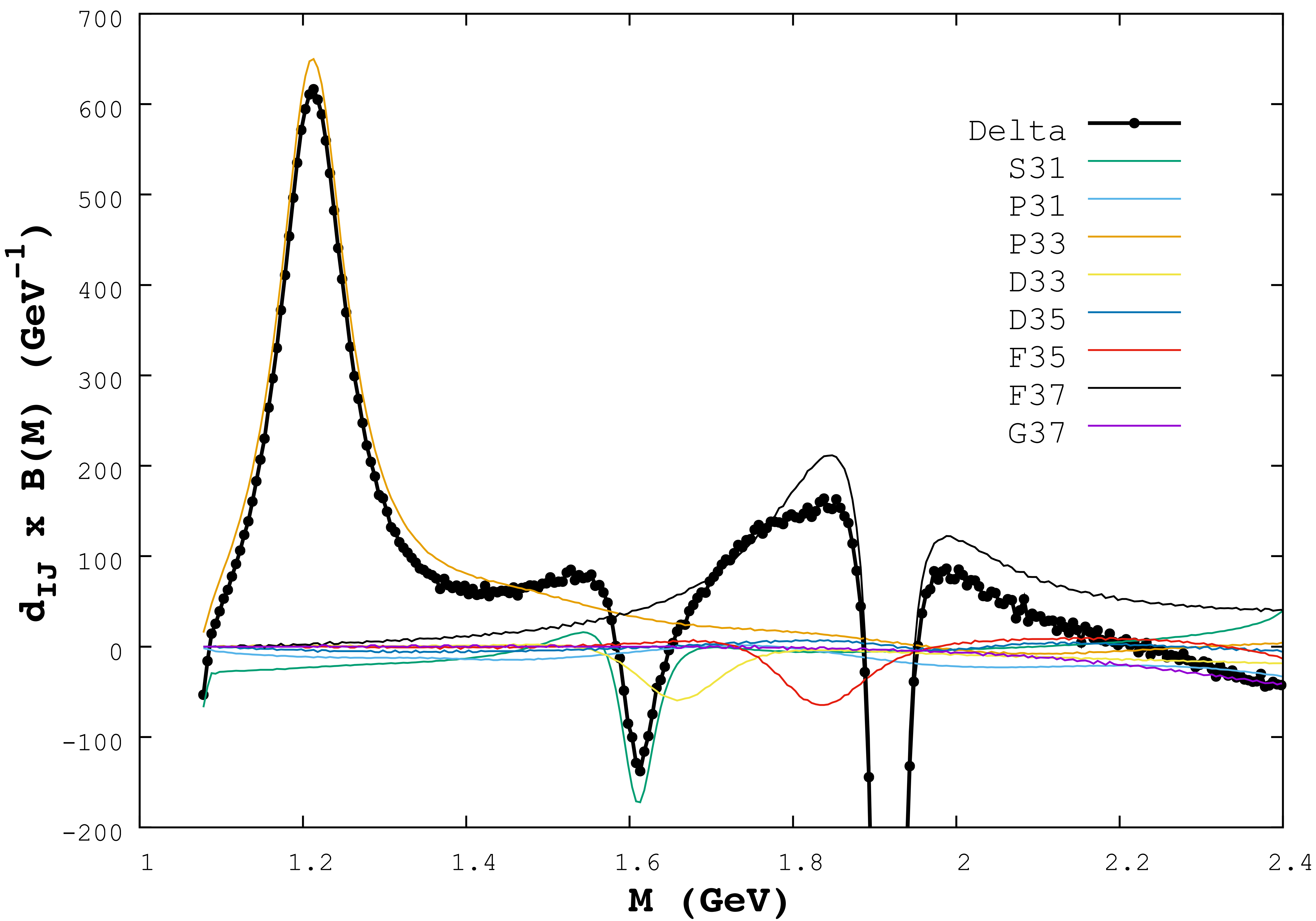
unflavored Baryons



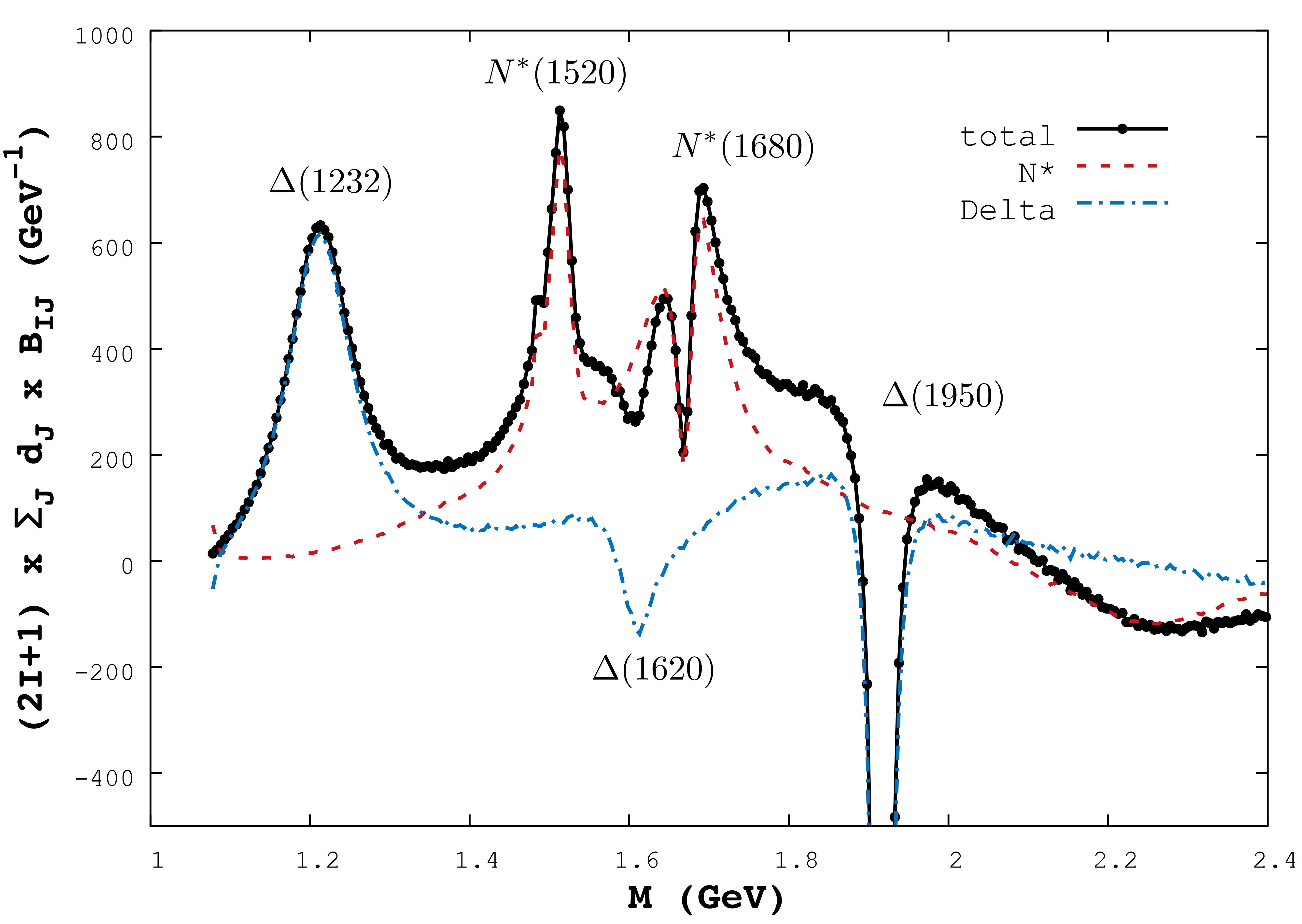




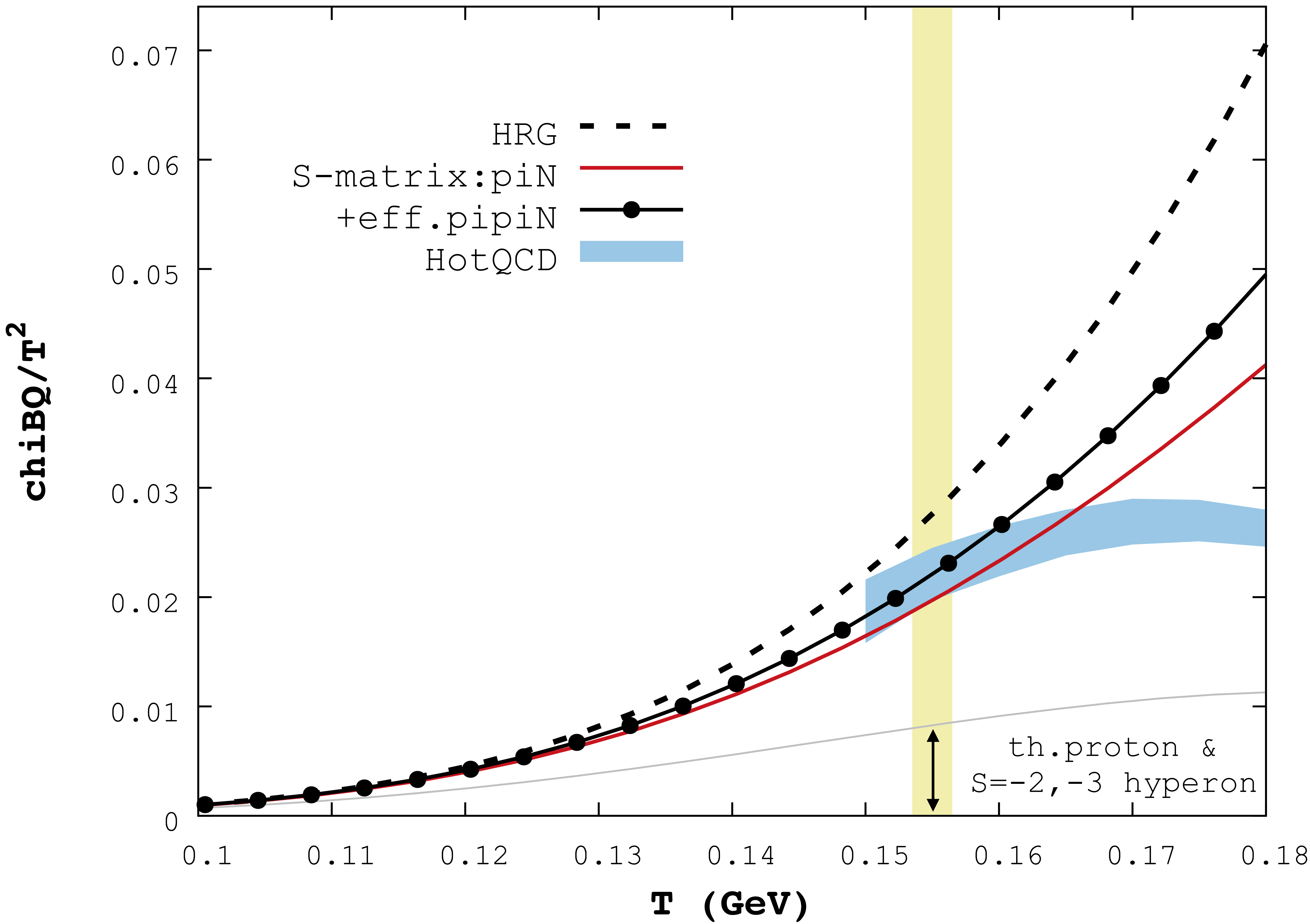
700
600
500
400
300
200
100
0
−100
−200
1
1.2
1.4
1.6
1.8
2
2.2
2.4
M (GeV)
d_IJ x B(M) (GeV^-1)
N*
S11
P11
P13
D13
D15
F15
F17
G17



700
600
500
400
300
200
100
0
-100
-200
1
1.2
1.4
1.6
1.8
2
2.2
2.4
M (GeV)
d_{IJ} x B(M) (GeV^{-1})
Delta
S31
P31
P33
D33
D35
F35
F37
G37



$N^*(1520)$
$N^*(1680)$
$\Delta(1232)$
$\Delta(1950)$
$\Delta(1620)$
total
N*
Delta
1000
800
600
400
200
0
-200
-400
1
1.2
1.4
1.6
1.8
2
2.2
2.4
M (GeV)
(2I+1) x Σ_J d_J x B_{IJ} (GeV^{-1})


HRG
S-matrix:piN
+eff.pipiN
HotQCD
chiBQ/T^2
T (GeV)
th.proton &
S=-2,-3 hyperon
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18

RECIPE

Feynman amplitude

- generalized phase shift

$$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1 + \int d\phi_N\, i\mathcal{M}\right)\right]$$

$$d\phi_N = \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\cdots\frac{d^3p_N}{(2\pi)^3}\frac{1}{2E_N}\times$$
$$(2\pi)^4\,\delta^4(P - \sum_i p_i).$$

phase space approach

PHASE SPACE DOMINANCE

$$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1 + \int d\phi_N\, i\mathcal{M}\right)\right]$$

- structureless scattering

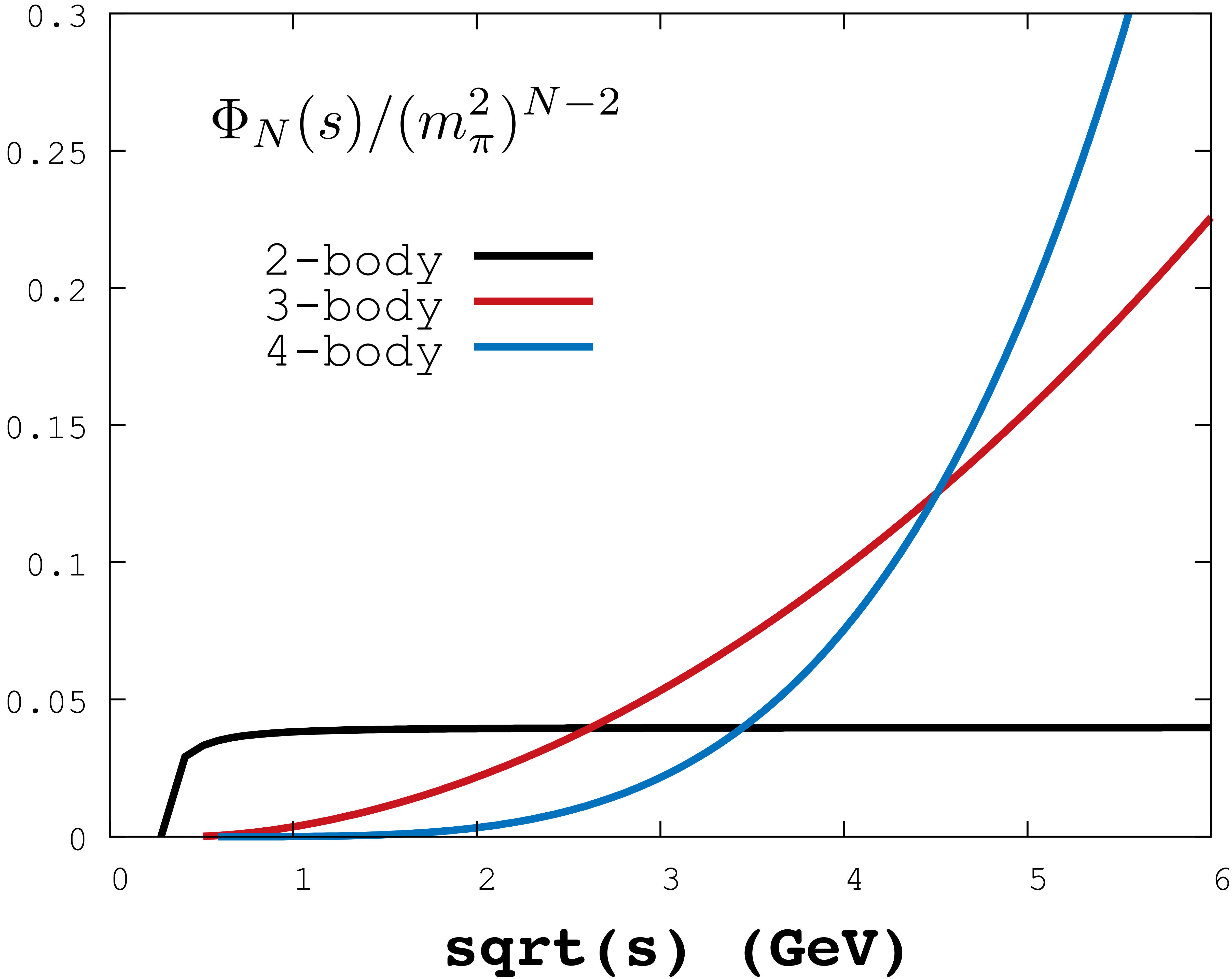
Dimension: $\sim E^{2N-4}$

$$i\mathcal{M} = i\,\lambda_N$$

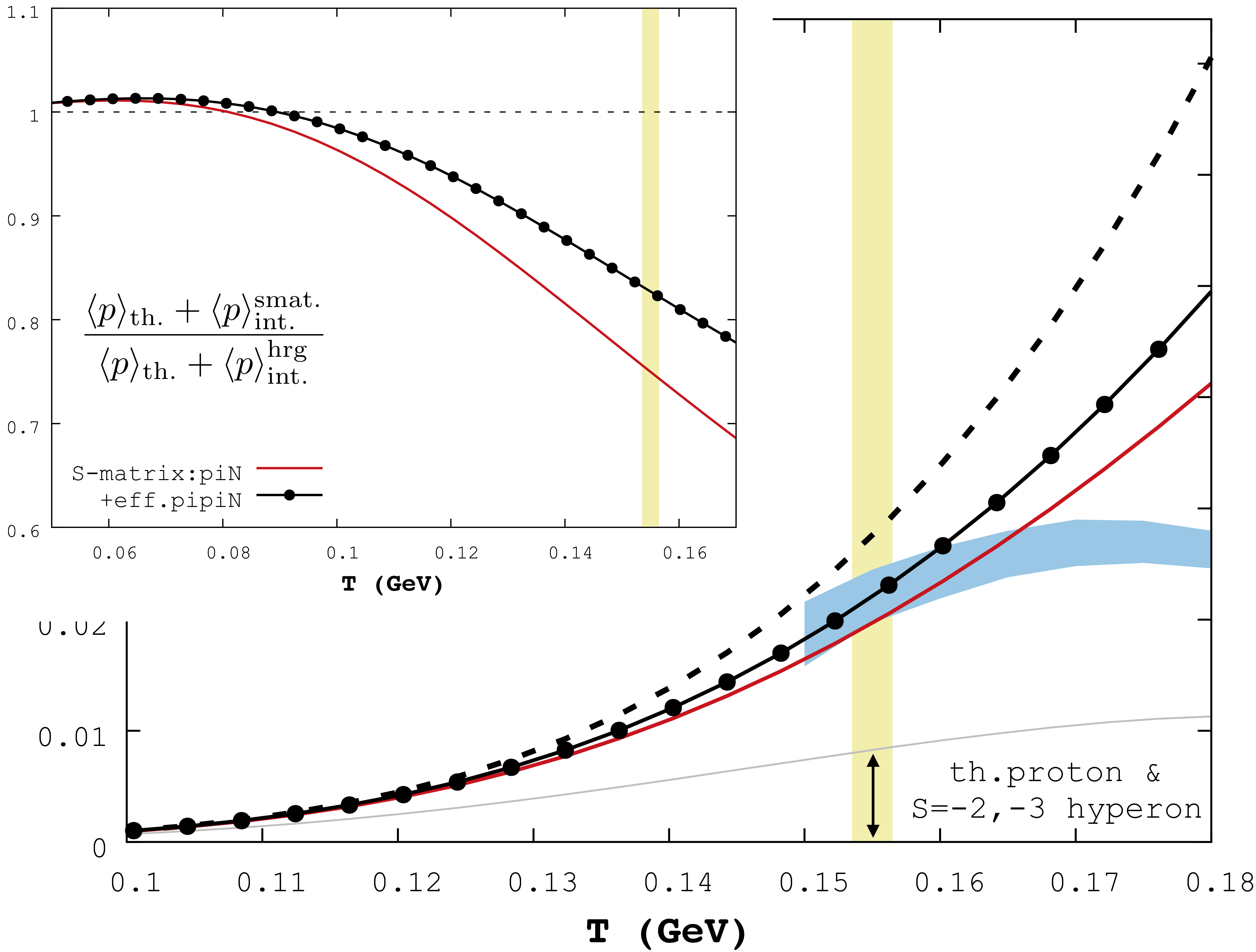
Källén triangle function

$$\phi_N(s) = \frac{1}{16\,\pi^2 s}\int_{s'_-}^{s'_+} ds'\,\sqrt{\lambda(s, s', m_N^2)}\,\times$$

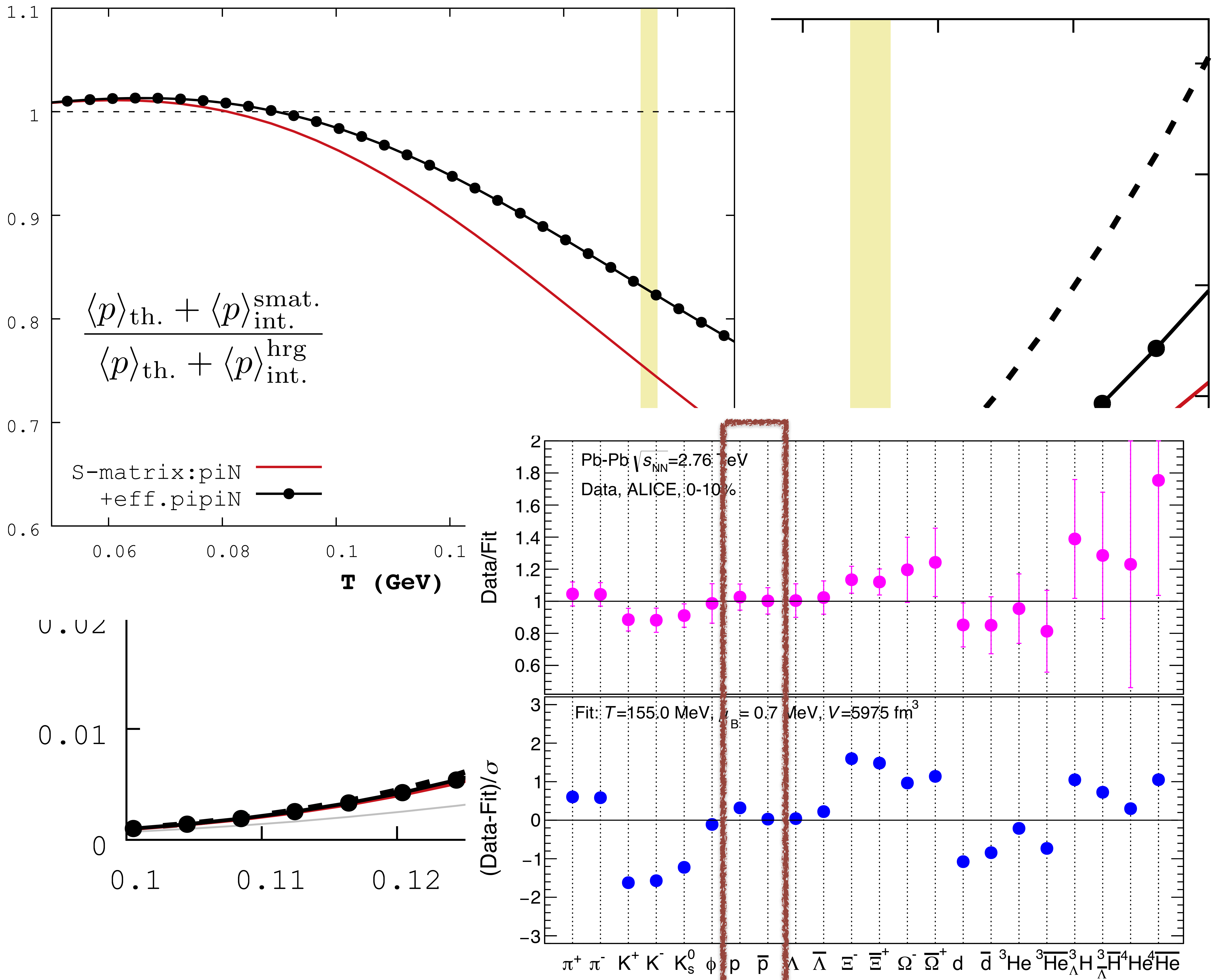
$$\phi_{N-1}(s', m_1^2, m_2^2, ..., m_{N-1}^2)$$



0.3
0.25
0.2
0.15
0.1
0.05
0
$\Phi_N(s)/(m_\pi^2)^{N-2}$
2-body
3-body
4-body
0
1
2
3
4
5
6
sqrt(s) (GeV)

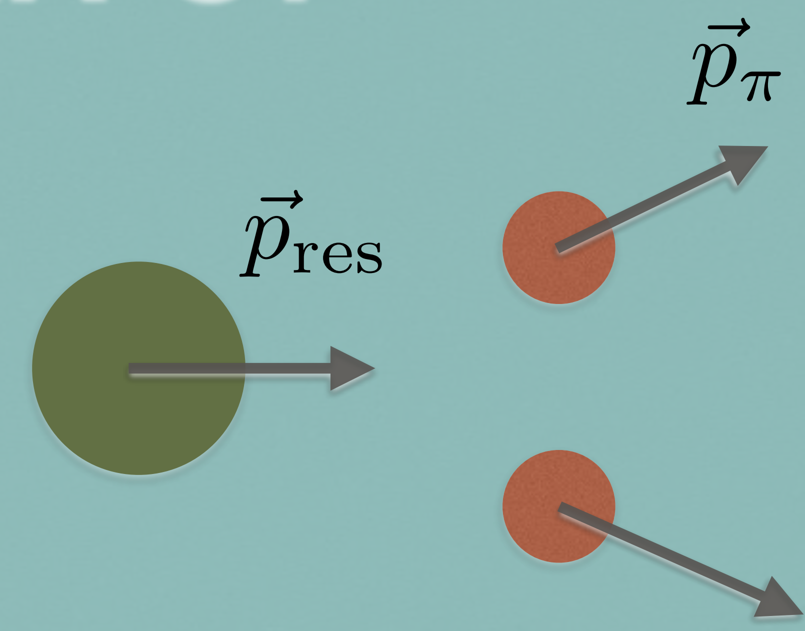


$\frac{\langle p \rangle_{th.} + \langle p \rangle_{int.}^{smat.}}{\langle p \rangle_{th.} + \langle p \rangle_{int.}^{hrg}}$
S-matrix:piN
+eff.pipiN
T (GeV)
th.proton &
S=-2,-3 hyperon
T (GeV)



1.1
1
0.9
0.8
0.7
0.6
$\frac{\langle p \rangle_{th.} + \langle p \rangle_{int.}^{smat.}}{\langle p \rangle_{th.} + \langle p \rangle_{int.}^{hrg}}$
S-matrix:piN
+eff.pipiN
0.06
0.08
0.1
0.1
T (GeV)
0.01
0
0.1
0.11
0.12
Pb-Pb $\sqrt{s_{NN}}$=2.76 TeV
Data, ALICE, 0-10%
Data/Fit
2
1.8
1.6
1.4
1.2
1
0.8
0.6
Fit: T=155.0 MeV, μ_B= 0.7 MeV, V=5975 fm^3
(Data-Fit)/σ
3
2
1
0
−1
−2
−3
π^+ π^- K^+ K^- K^0_s ϕ p $\bar{p}$ Λ $\bar{\Lambda}$ Ξ^- $\bar{\Xi}^+$ Ω^- $\bar{\Omega}^+$ d $\bar{d}$ ^{3}He $^3\overline{He}$ $^3_\Lambda$H $^3_{\bar{\Lambda}}\bar{H}$ ^{4}He $^4\overline{He}$

MOMENTUM SPECTRA OF DECAY PRODUCTS



$$E_\pi \frac{dn_\pi^{\mathrm{dec}}}{d^3p_\pi} = \int \frac{dM}{2\pi} B(M) \times$$

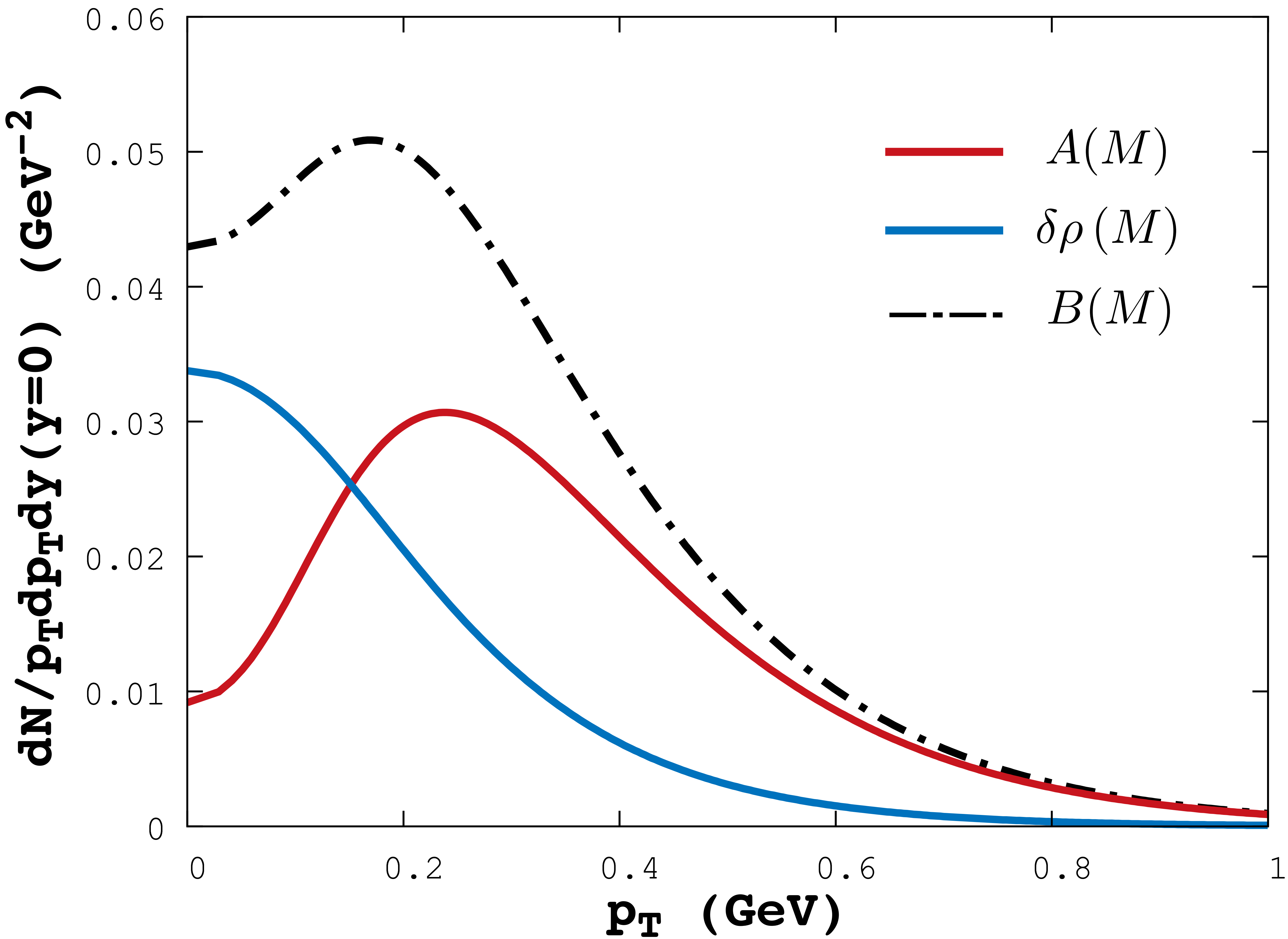
$$\int_{m_{\mathrm{res}} \to M} d^3p_{\mathrm{res}} \frac{dn_{\mathrm{res}}}{d^3p_{\mathrm{res}}} \times E_\pi^\star \times \mathrm{dPS}(\vec{p}_\pi^{\,\star})$$

decay kinematics

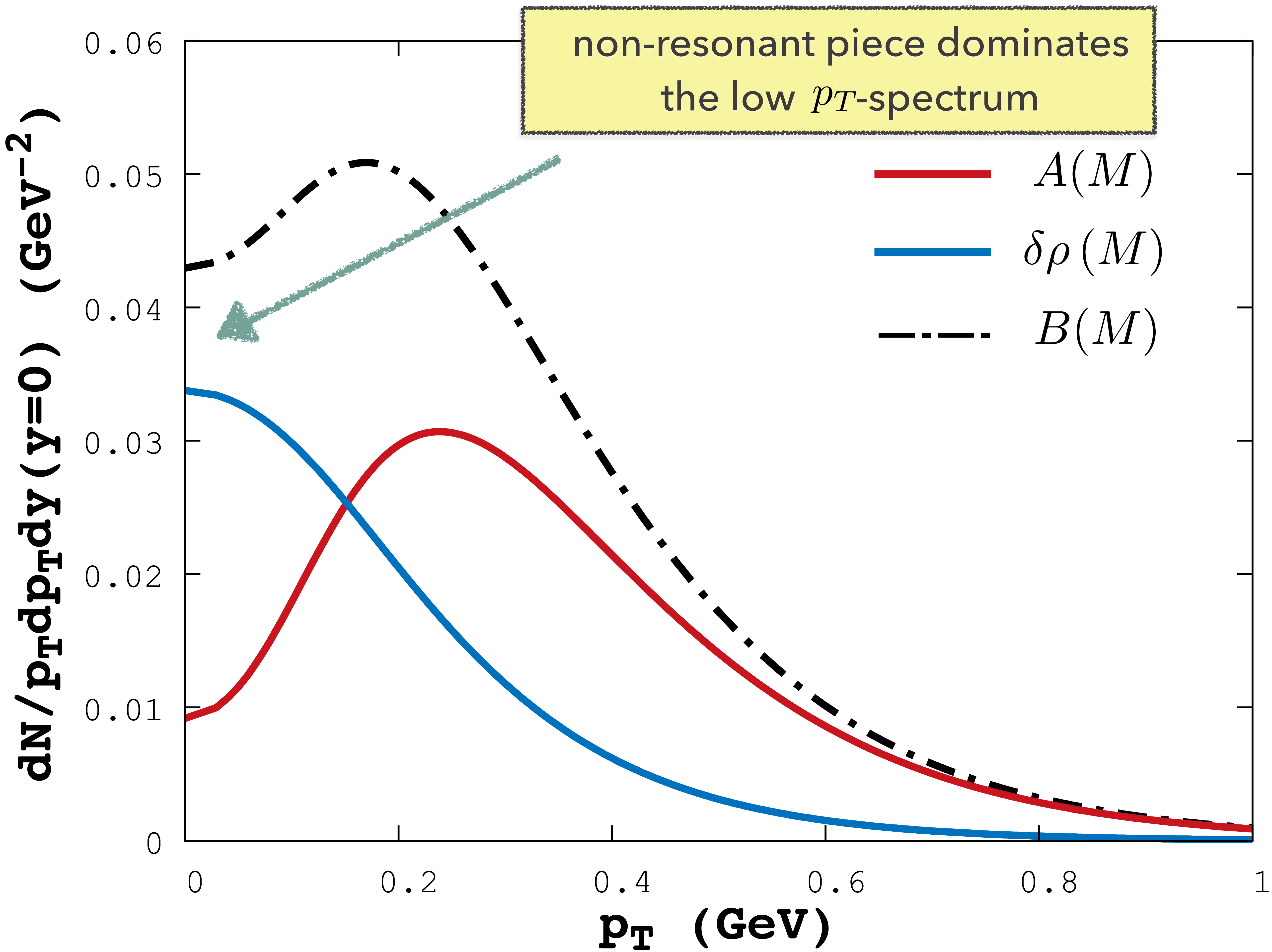
$$\frac{dn_{\mathrm{res}}}{d^3p_{\mathrm{res}}} \to \frac{g_{\mathrm{res}}}{(2\pi)^3} \frac{1}{e^{\beta E_{\mathrm{res}}} - 1}$$

$$\mathcal{F}[p_\pi, M]$$

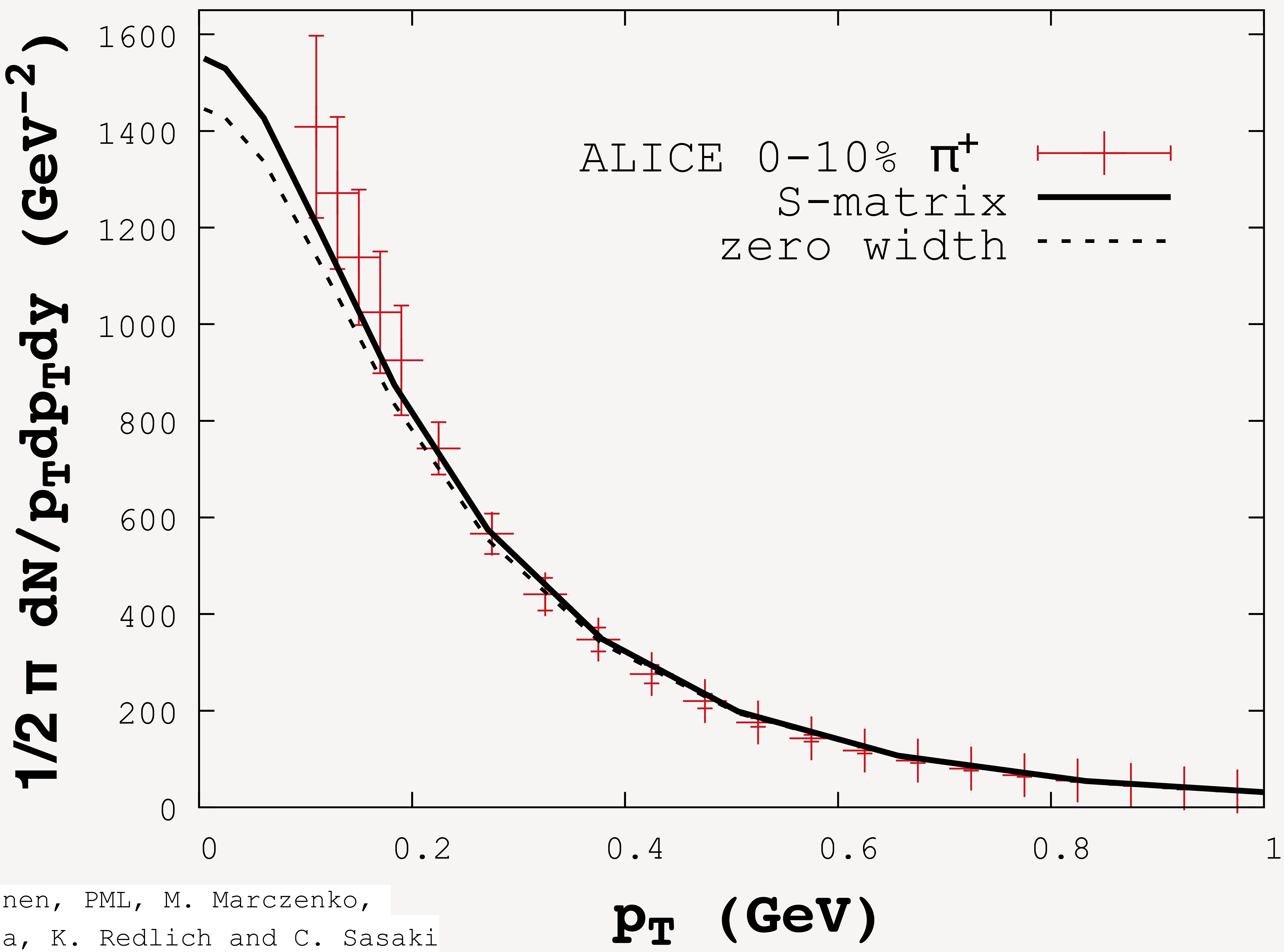
$$\mathrm{dPS} \to \frac{1}{4\pi q^2} \delta(p_\pi^\star - q)$$



0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.2
0.4
0.6
0.8
1
$A(M)$
$\delta\rho(M)$
$B(M)$
$dN/p_T dp_T dy(y=0)$ (GeV^{-2})
p_T (GeV)



non-resonant piece dominates the low p_T-spectrum
$A(M)$
$\delta\rho(M)$
$B(M)$
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.2
0.4
0.6
0.8
1
$dN/p_T dp_T dy(y=0)$ (GeV^{-2})
p_T (GeV)

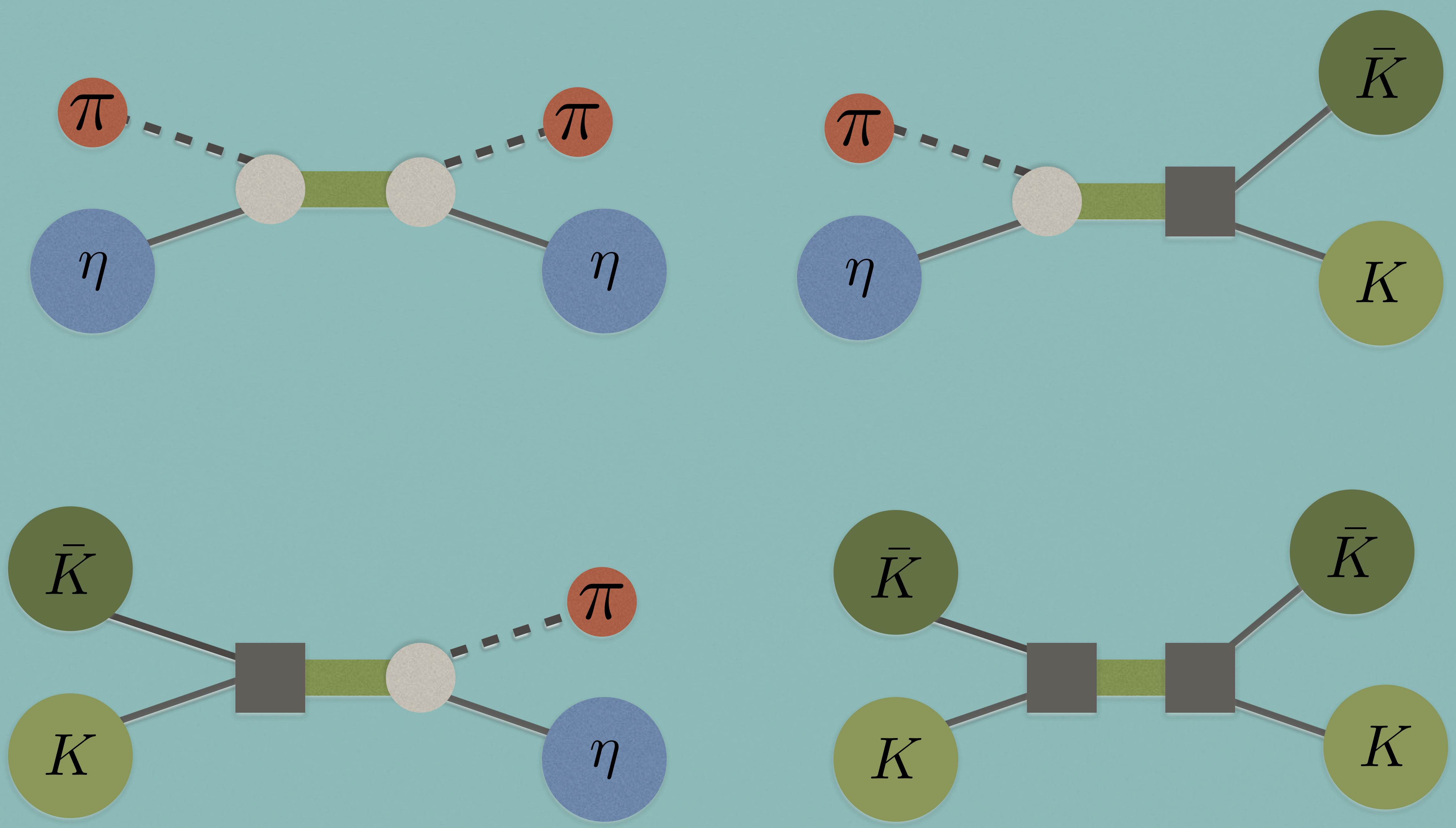


P. Huovinen, PML, M. Marczenko,
K. Morita, K. Redlich and C. Sasaki
Phys.Lett. B769 (2017) 509-512

COUPLED CHANNEL SYSTEM

C. Fernandez-Ramirez, PML, and P. Petreczky,
PRC **98**, 044910 (2018)

1 RES. 2 CHANNELS PROBLEM



COUPLED-CHANNEL PROBLEM

$$\{\gamma_1, \gamma_2, m_{\rm res}\} \longleftrightarrow \{\delta_1, \delta_2, \eta\}$$

$$S = \begin{pmatrix} \eta\, e^{2\,i\,\delta_I} & i\sqrt{1-\eta^2}\, e^{i\,(\delta_I+\delta_{II})} \\ i\sqrt{1-\eta^2}\, e^{i\,(\delta_I+\delta_{II})} & \eta\, e^{2\,i\,\delta_{II}} \end{pmatrix}$$

$$\begin{aligned} \mathcal{Q}(M) &\equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\ \mathrm{S}) \\ &= \frac{1}{2}\,\mathrm{Im}\,(\ln\ \det\,[\mathrm{S}]) \\ &= \delta_I + \delta_{II}. \end{aligned}$$

$a_0(980)$ *system*

$$\pi\eta \to \begin{pmatrix} \pi\eta \\ K\bar{K} \end{pmatrix} \to \pi\eta$$

$$K\bar{K} \to \begin{pmatrix} \pi\eta \\ K\bar{K} \end{pmatrix} \to K\bar{K}$$

WIGNER, EISENBUD, SMITH, …

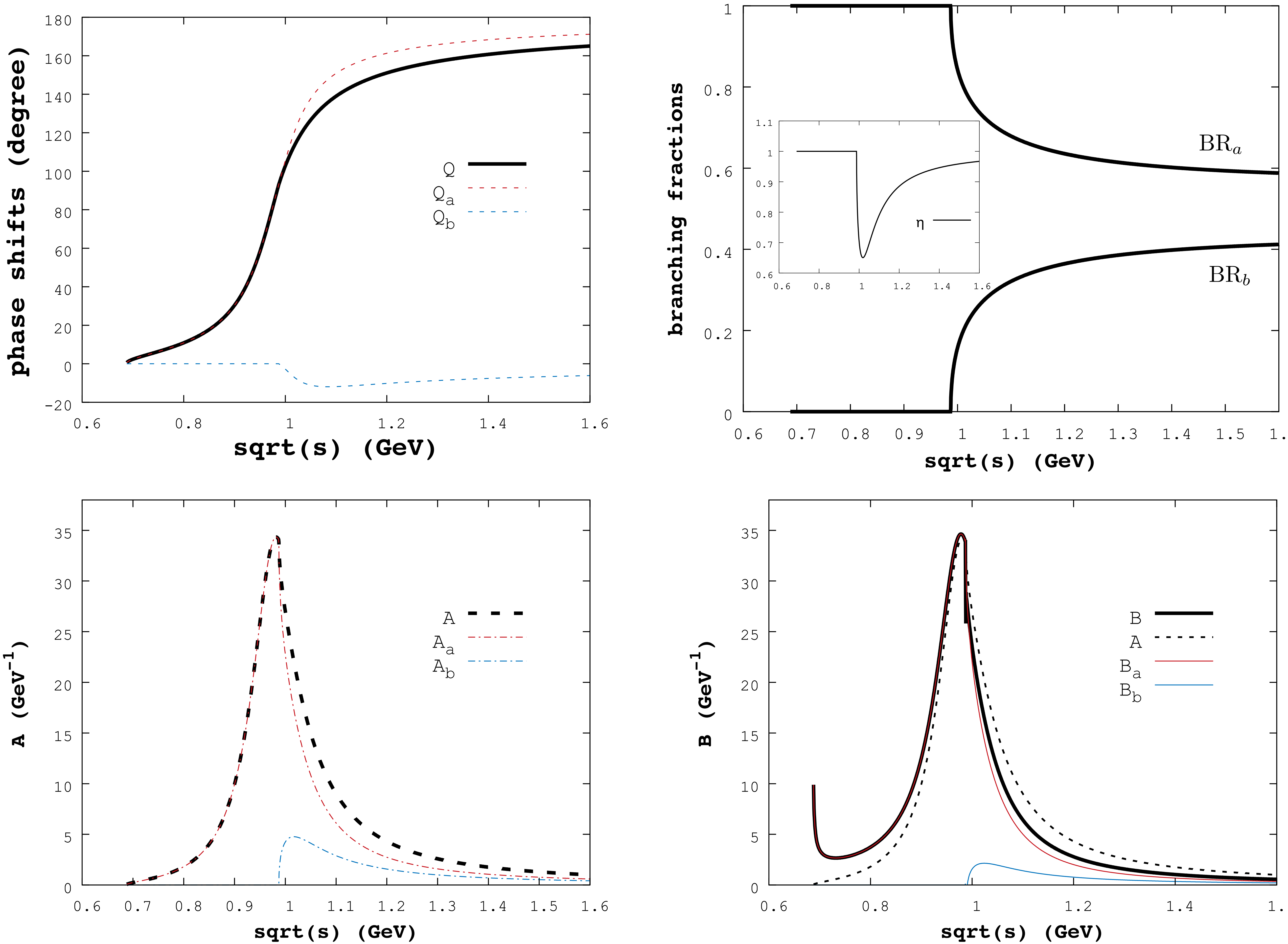
$$S \to U^\dagger S_d U$$

$$S_d = \begin{pmatrix} e^{2i\delta_{\rm res}(s)} & 0 \\ 0 & 1 \end{pmatrix},$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

$$\mathrm{BR}_a = \cos^2\theta = \frac{g_a^2\,\phi_a}{g_a^2\,\phi_a + g_b^2\,\phi_b},$$

$$\mathrm{BR}_b = \sin^2\theta = \frac{g_b^2\,\phi_b}{g_a^2\,\phi_a + g_b^2\,\phi_b}.$$



phase shifts (degree)
sqrt(s) (GeV)
Q
Q_a
Q_b
180
160
140
120
100
80
60
40
20
0
-20
0.6
0.8
1
1.2
1.4
1.6
branching fractions
BR_a
BR_b
η
A (GeV^-1)
A
A_a
A_b
B (GeV^-1)
B
A
B_a
B_b

PHASE SHIFT FROM PWA

```
------------------------------------------------------------------------------------

Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

------------------------------------------------------------------------------------
------------------------------------------------------------------------------------

Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)
Igor V. Danilkin (Jefferson Lab)
Vincent Mathieu (Indiana University)
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosphy. If your
parameters do not make sense, the output will not make sense either.
2 - You can use, share and modify this code under your own responsability.
3 - This code is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
4 - No PhD students or postdocs were severely damaged during the development
of this project.

----------------------------------------------------------------------------
```

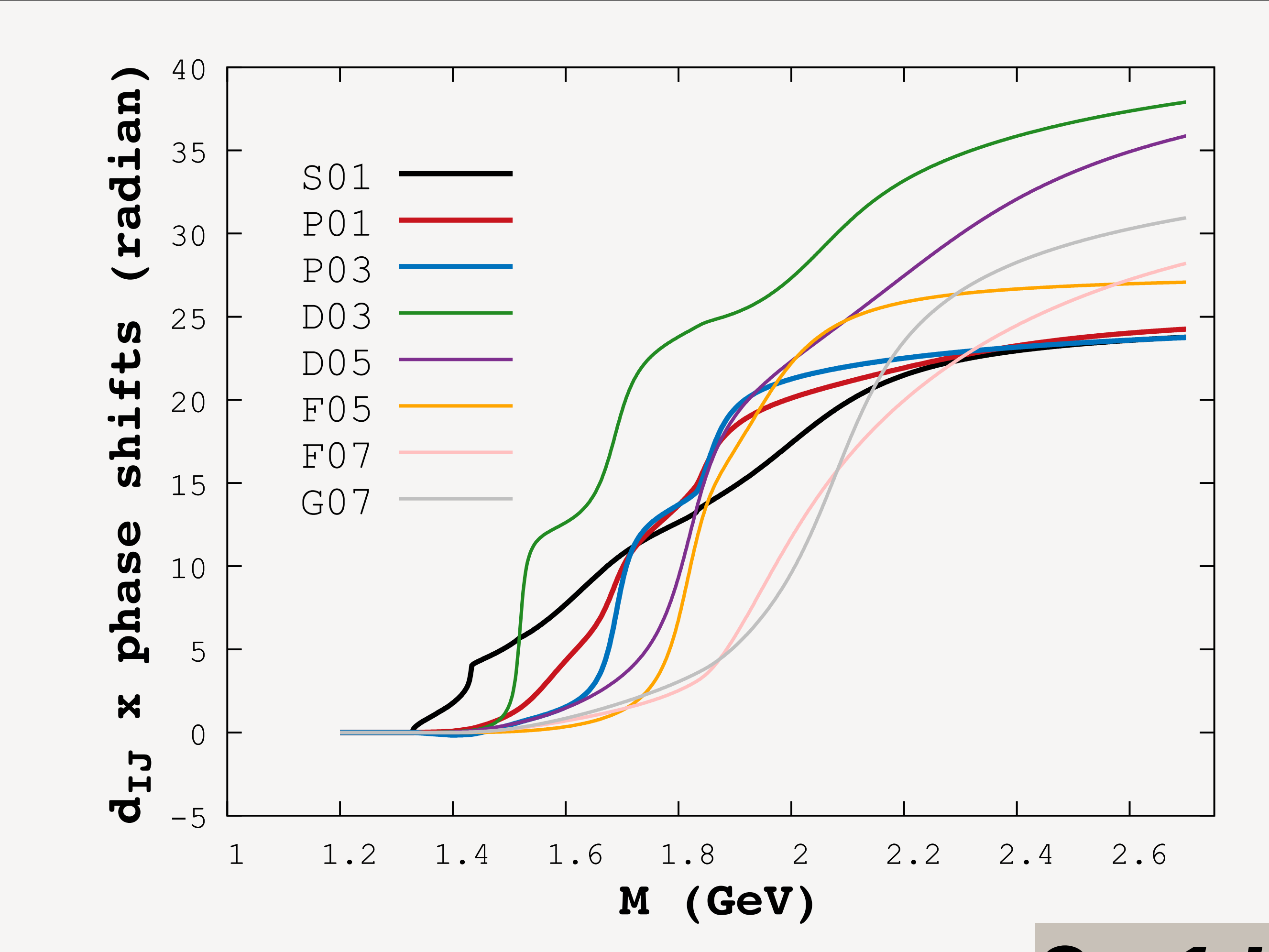
STRANGENESS CONTENT IN A HADRON GAS

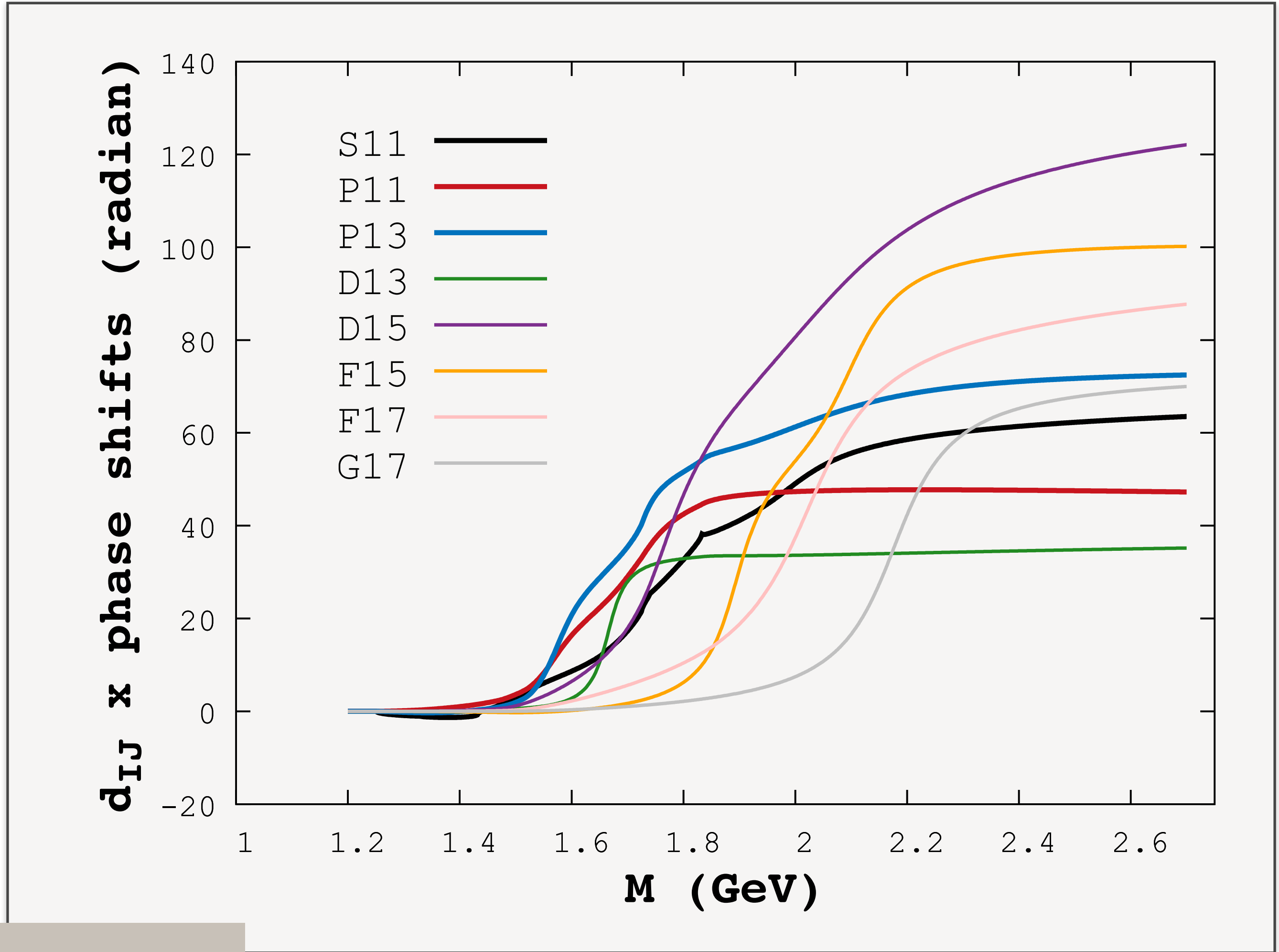
- K-N system requires a coupled channel analysis

$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \ldots$ *16 basis states*

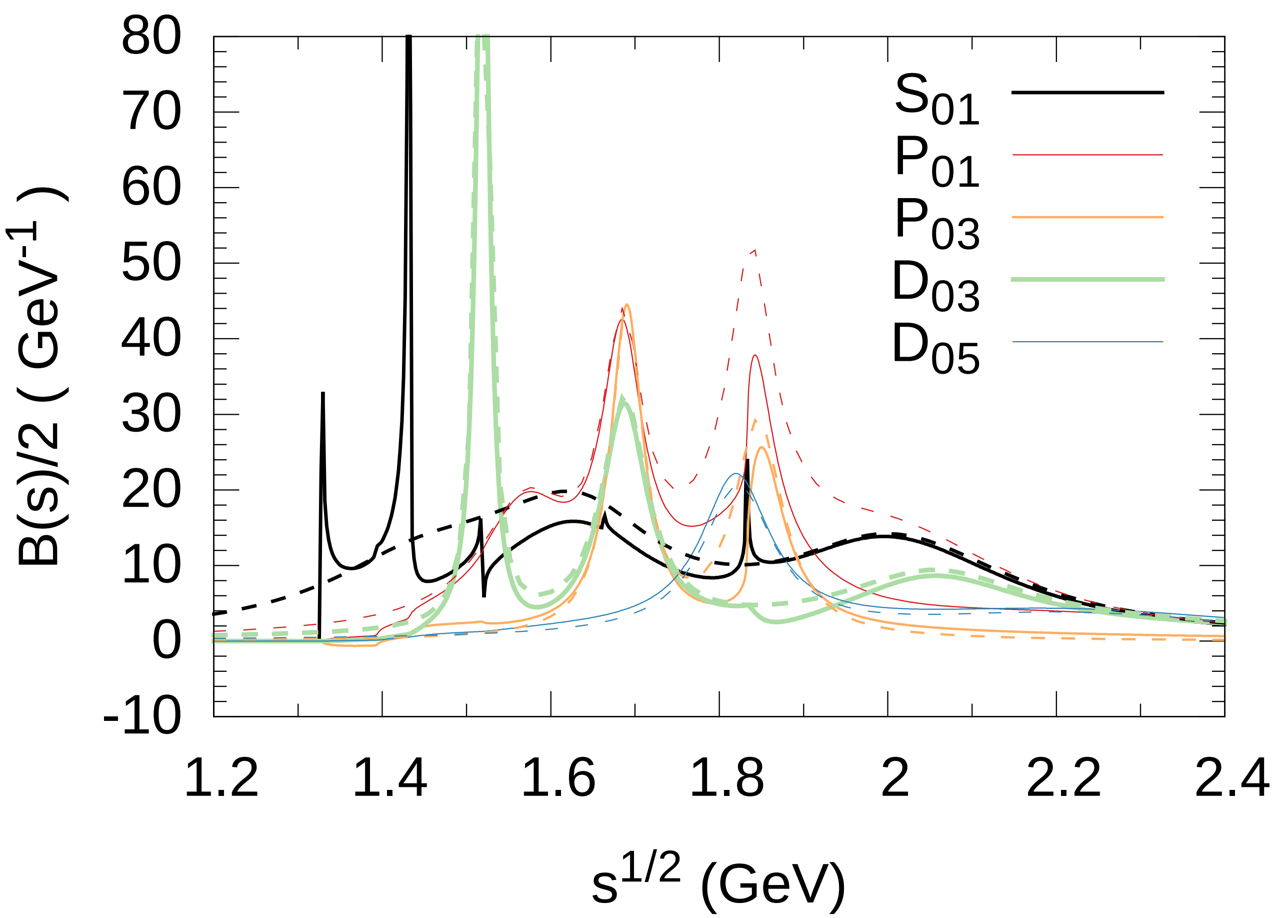
$$\mathcal{Q}(M) \equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\,\mathrm{S}) = \frac{1}{2}\,\mathrm{Im}\,(\ln\,\det\,[\mathrm{S}]) = \delta_{\bar{K}N} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \ldots$$

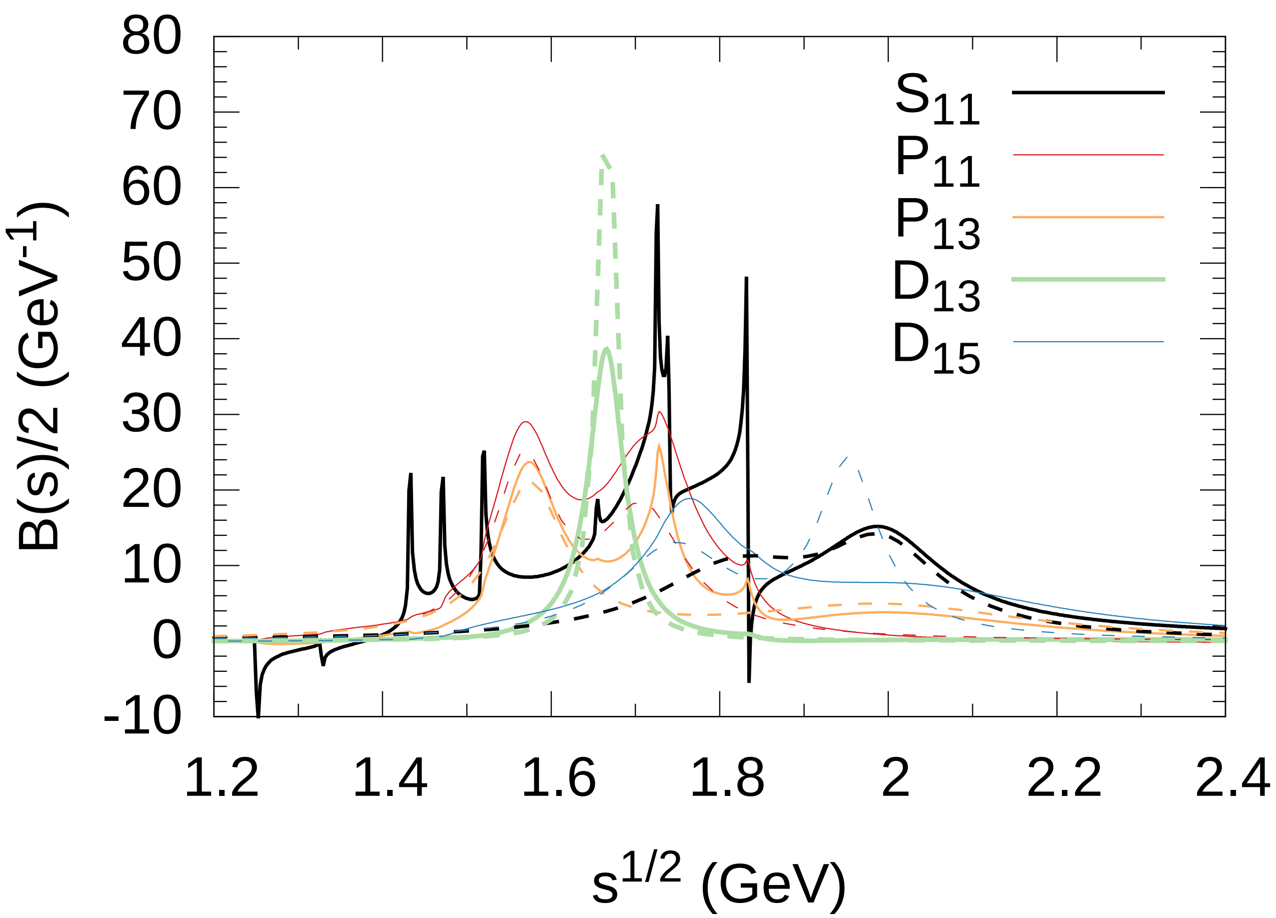
recipe to extract eigenphases from PWA

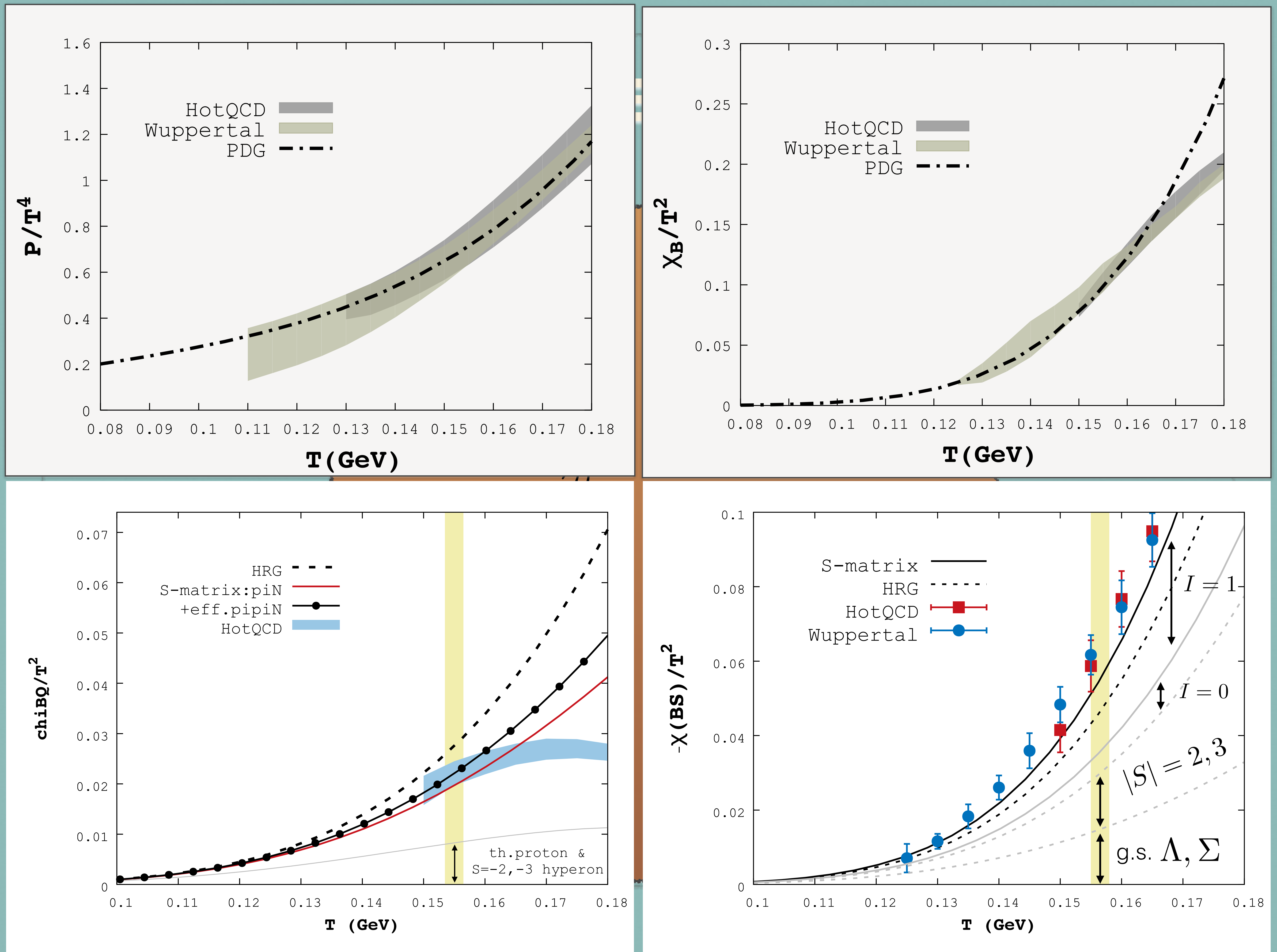

S01
P01
P03
D03
D05
F05
F07
G07
d_{IJ} x phase shifts (radian)
M (GeV)


S11
P11
P13
D13
D15
F15
F17
G17
d_{IJ} x phase shifts (radian)
M (GeV)

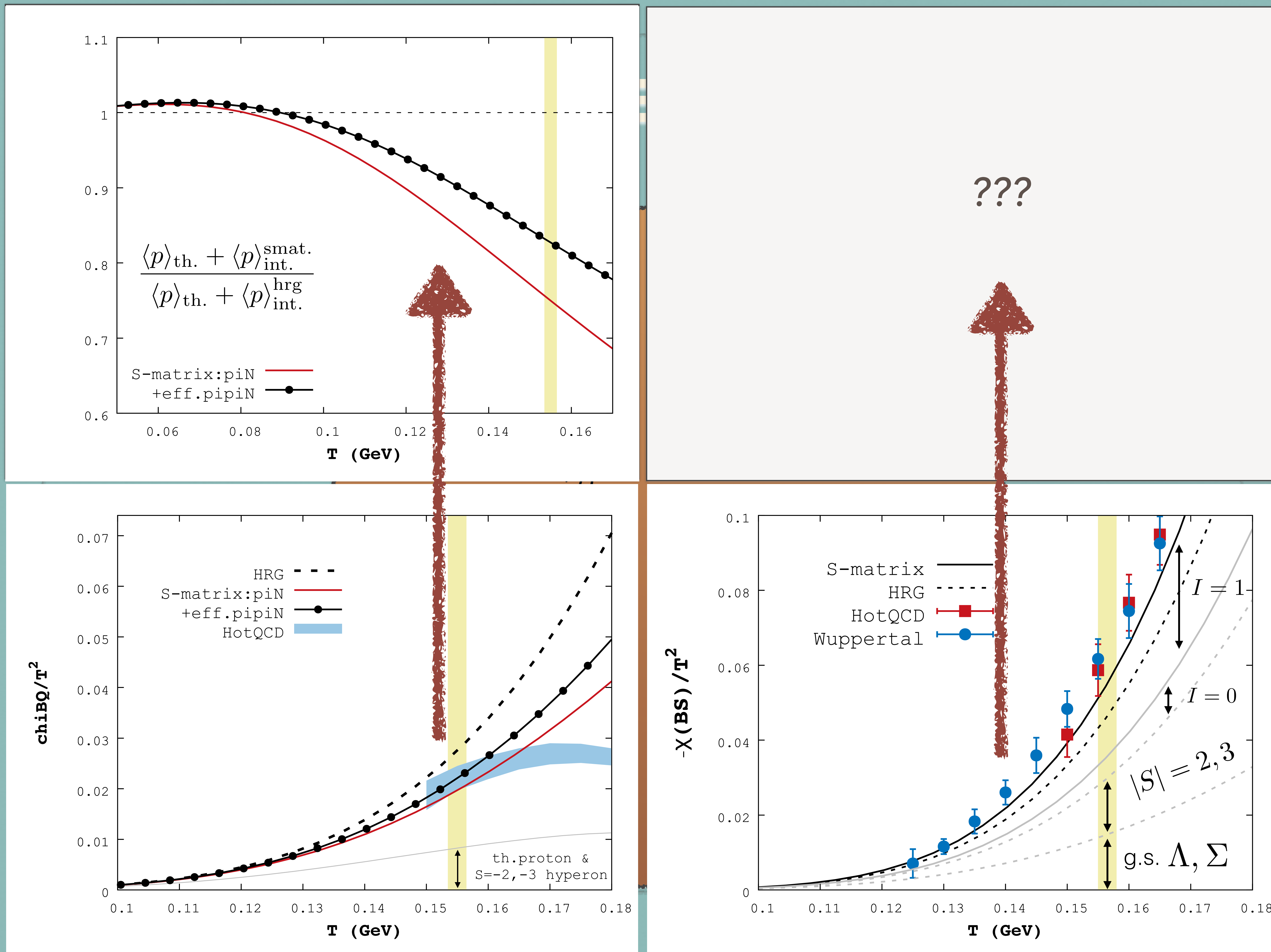
S=-1 Hyperon


S_{01}
P_{01}
P_{03}
D_{03}
D_{05}
B(s)/2 (GeV^{-1})
$s^{1/2}$ (GeV)


S_{11}
P_{11}
P_{13}
D_{13}
D_{15}
B(s)/2 (GeV^{-1})
$s^{1/2}$ (GeV)



HotQCD
Wuppertal
PDG
P/T^4
T(GeV)
χB/T^2
HRG
S-matrix:piN
+eff.pipiN
chiBQ/T^2
T (GeV)
th.proton &
S=-2,-3 hyperon
S-matrix
-χ(BS)/T^2
I = 1
I = 0
|S| = 2,3
g.s. Λ, Σ



$\frac{\langle p\rangle_{th.} + \langle p\rangle_{int.}^{smat.}}{\langle p\rangle_{th.} + \langle p\rangle_{int.}^{hrg}}$
S-matrix:piN
+eff.pipiN
T (GeV)
???
chiBQ/T²
HRG
S-matrix:piN
+eff.pipiN
HotQCD
th.proton &
S=-2,-3 hyperon
T (GeV)
-χ(BS)/T²
S-matrix
HRG
HotQCD
Wuppertal
$I = 1$
$I = 0$
$|S| = 2, 3$
g.s. Λ, Σ
T (GeV)

WHAT'S DONE...

- thermal abundances of channels:
  $\pi\pi, \pi K, \pi N, \text{hyperon}, NN$
- momentum spectra of decay products
- PWA x S-matrix $\mathcal{Q}(M) \equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\,\mathrm{S})$
- 3- and 4-body S-matrix (nuclear matter)

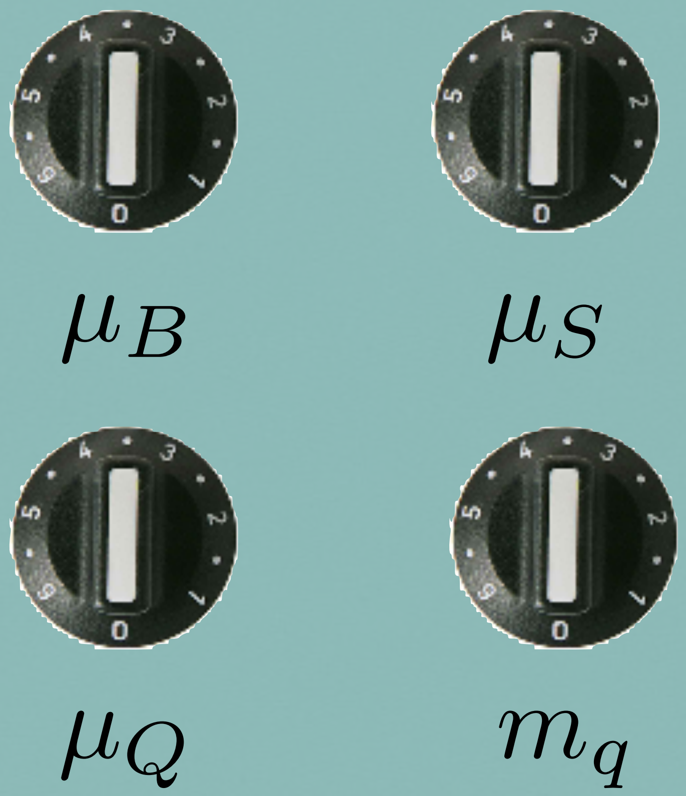
ON-GOING PROJECTS

- impact of pipiN on thermodynamics
- lifetimes of unstable states / exotics
  in a thermal system
- S-matrix interpretation of in-medium effect
- N-body forces

FLUCTUATIONS

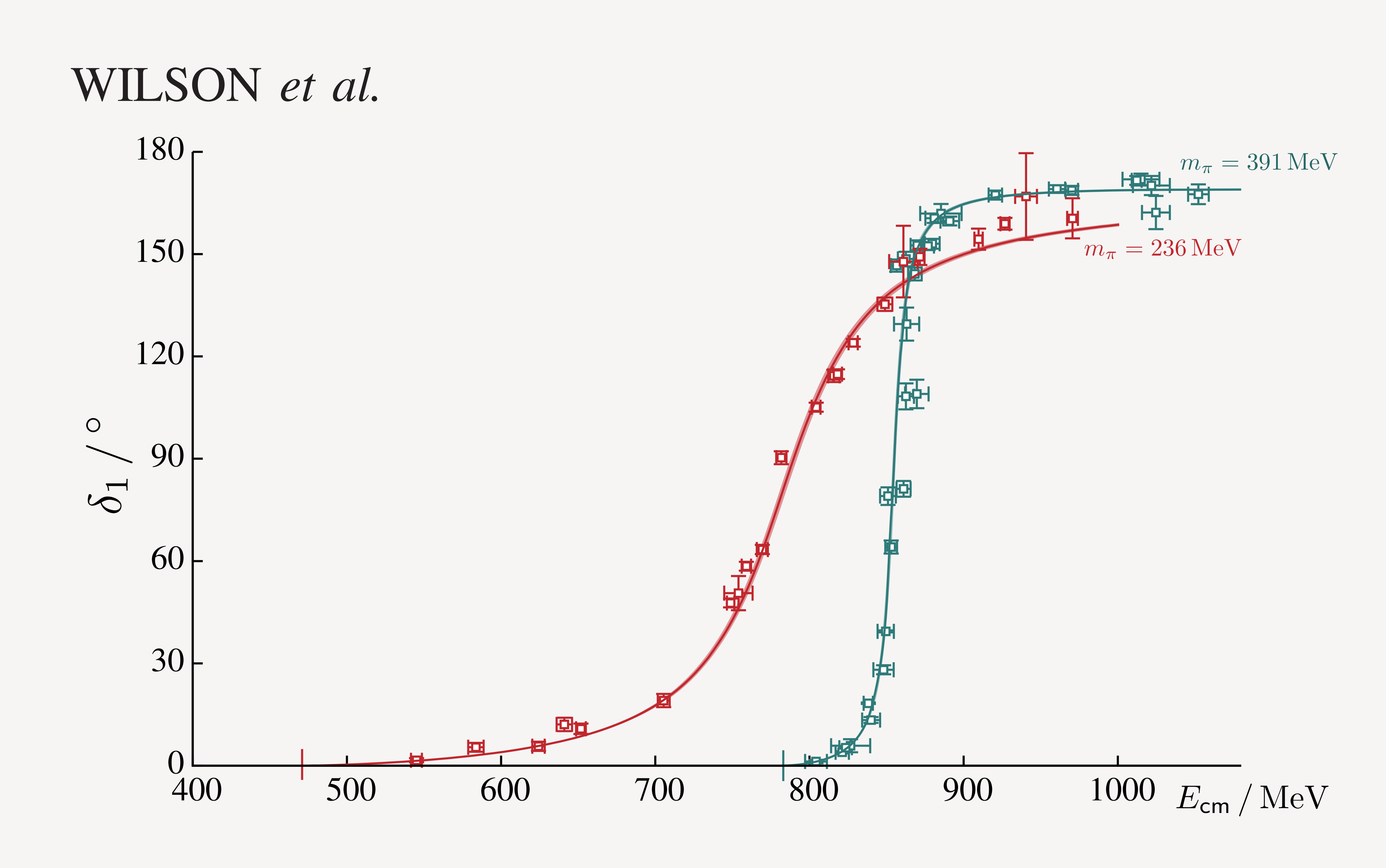
- studying the system by linear response

$$\mu = \mu_B B + \mu_Q Q + \mu_S S$$



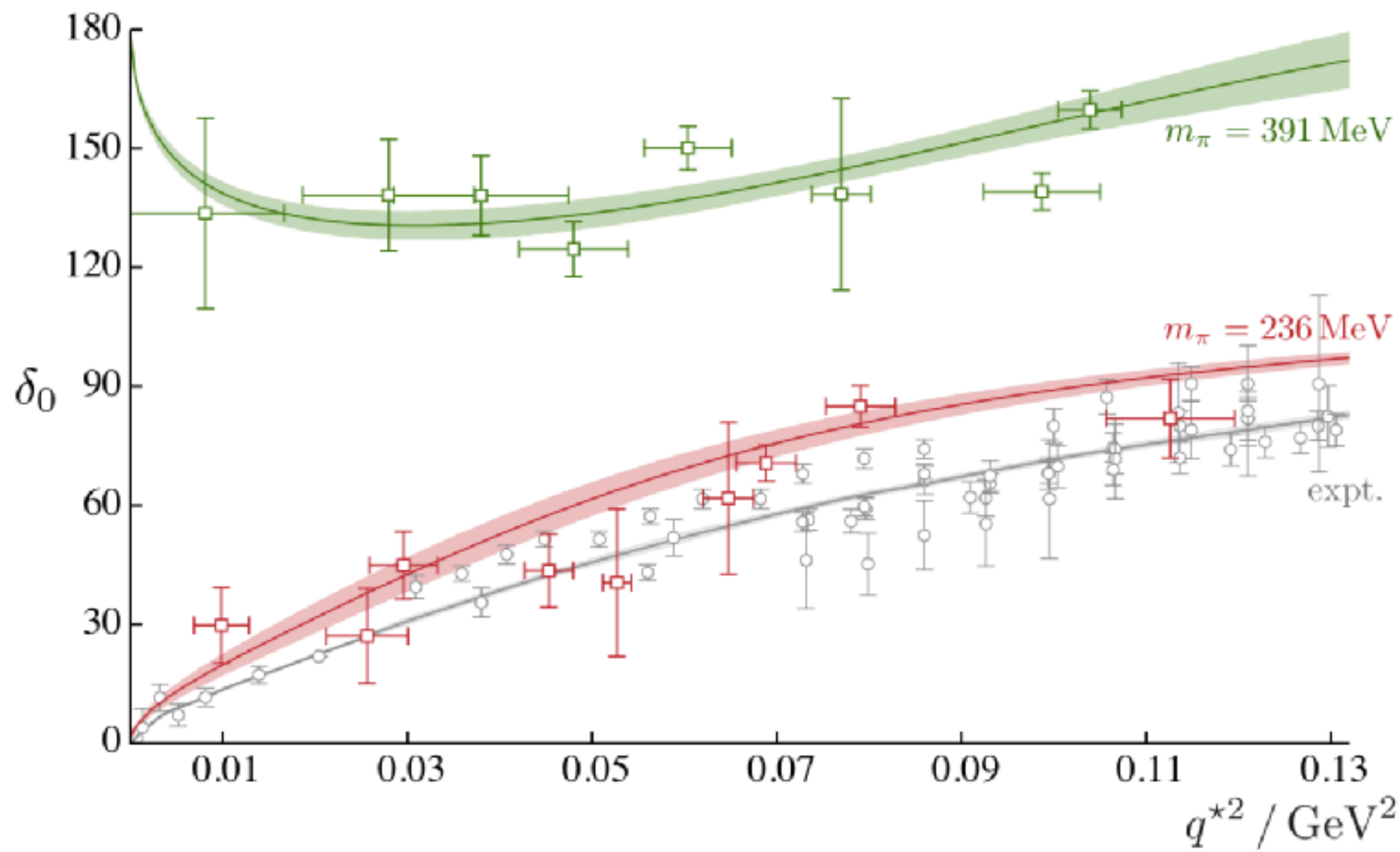
$$\chi_{B,S,\ldots} = \frac{1}{\beta V} \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_S \ldots} \ln Z$$

LATTICE COMPUTATIONS ON PHASE SHIFT



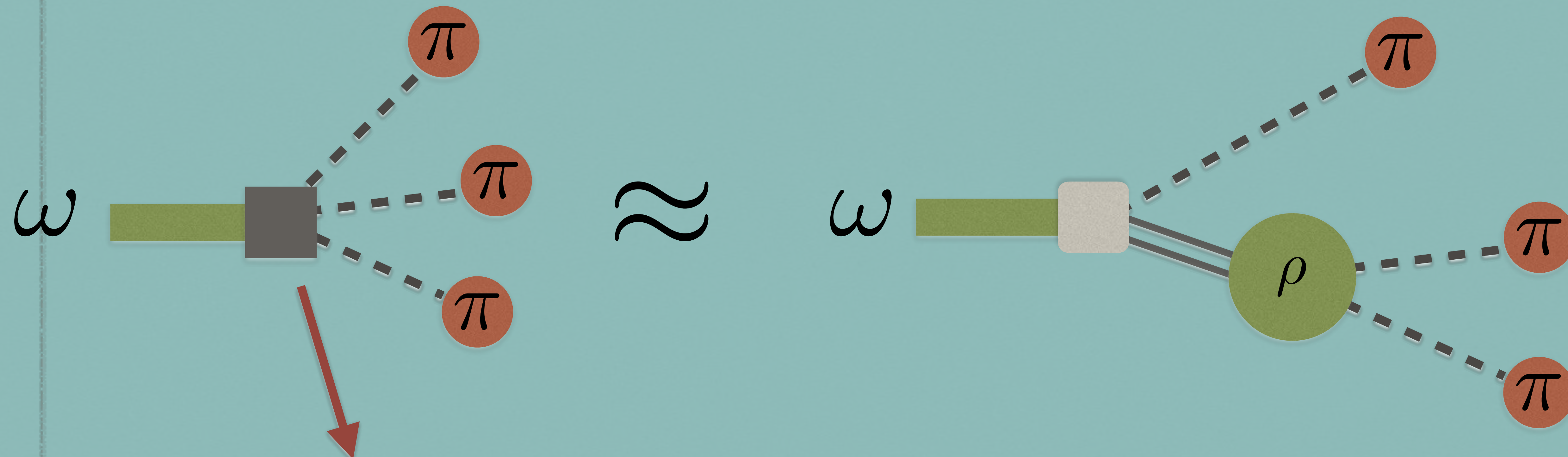
LATTICE COMPUTATIONS ON PHASE SHIFT

deuteron physics?



R. A. Briceno, J. J. Dudek and R. D. Young, arXiv:1706.06223 [hep-lat].

OMEGA TO 3 PI: GSW

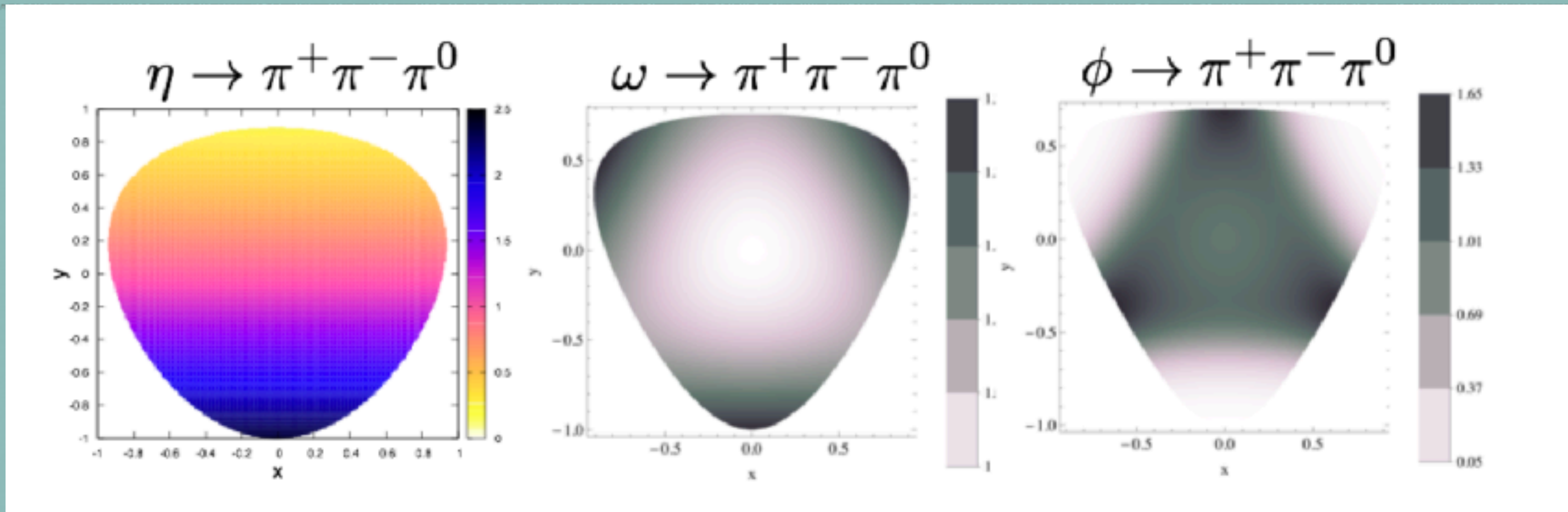


$$|\Gamma_{\omega\to 3\pi}|^2 = \mathcal{P} \times |C_{V\to 123}|^2$$

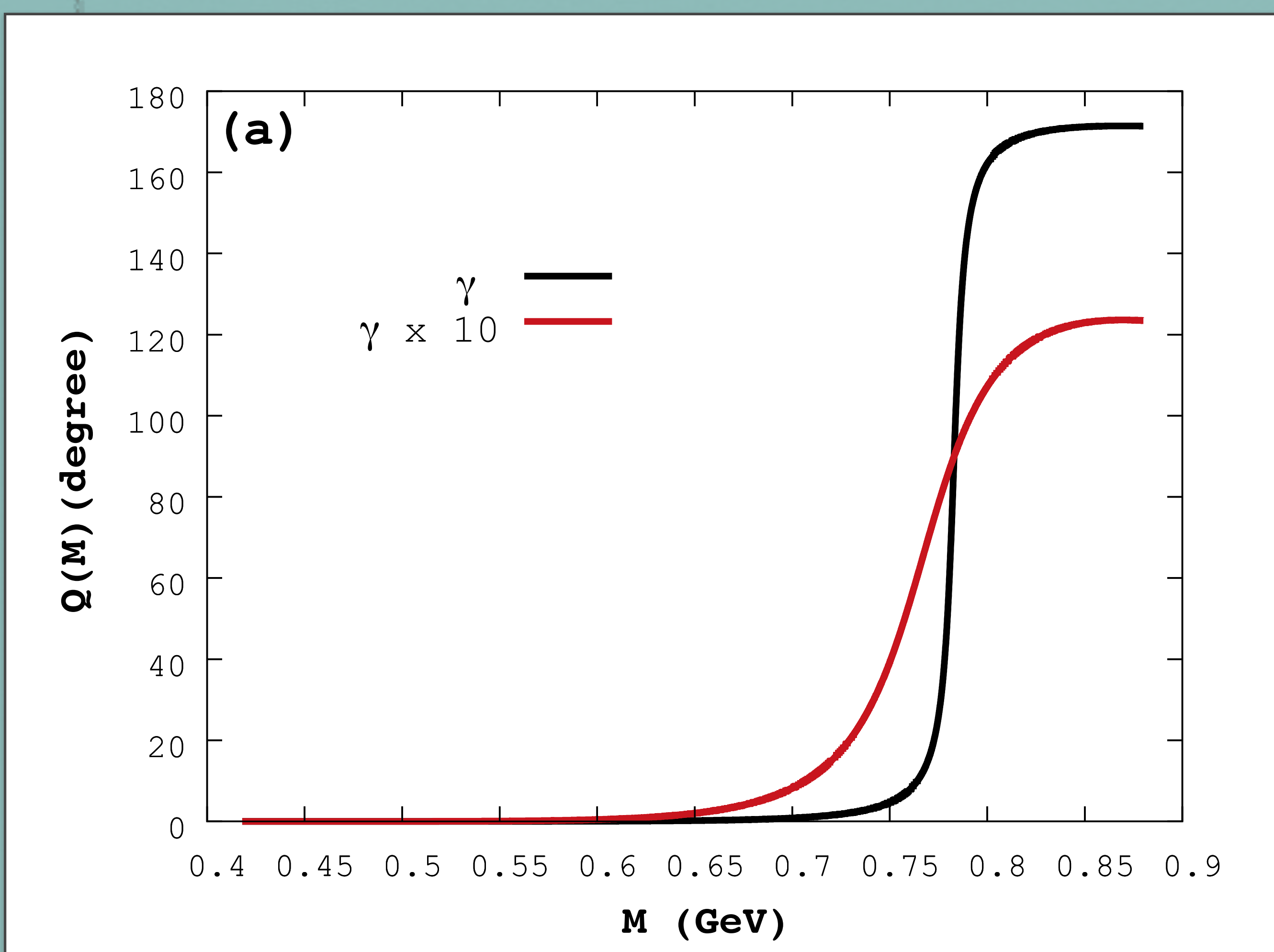
$$\mathcal{P} = -\frac{1}{3}\epsilon_{\mu\nu\alpha\beta}\epsilon_{abcd}P^{\mu}\, p_1^{\nu}\, p_2^{\alpha}\, P^{a}\, p_1^{b}\, p_2^{c}\, g^{\beta d}$$

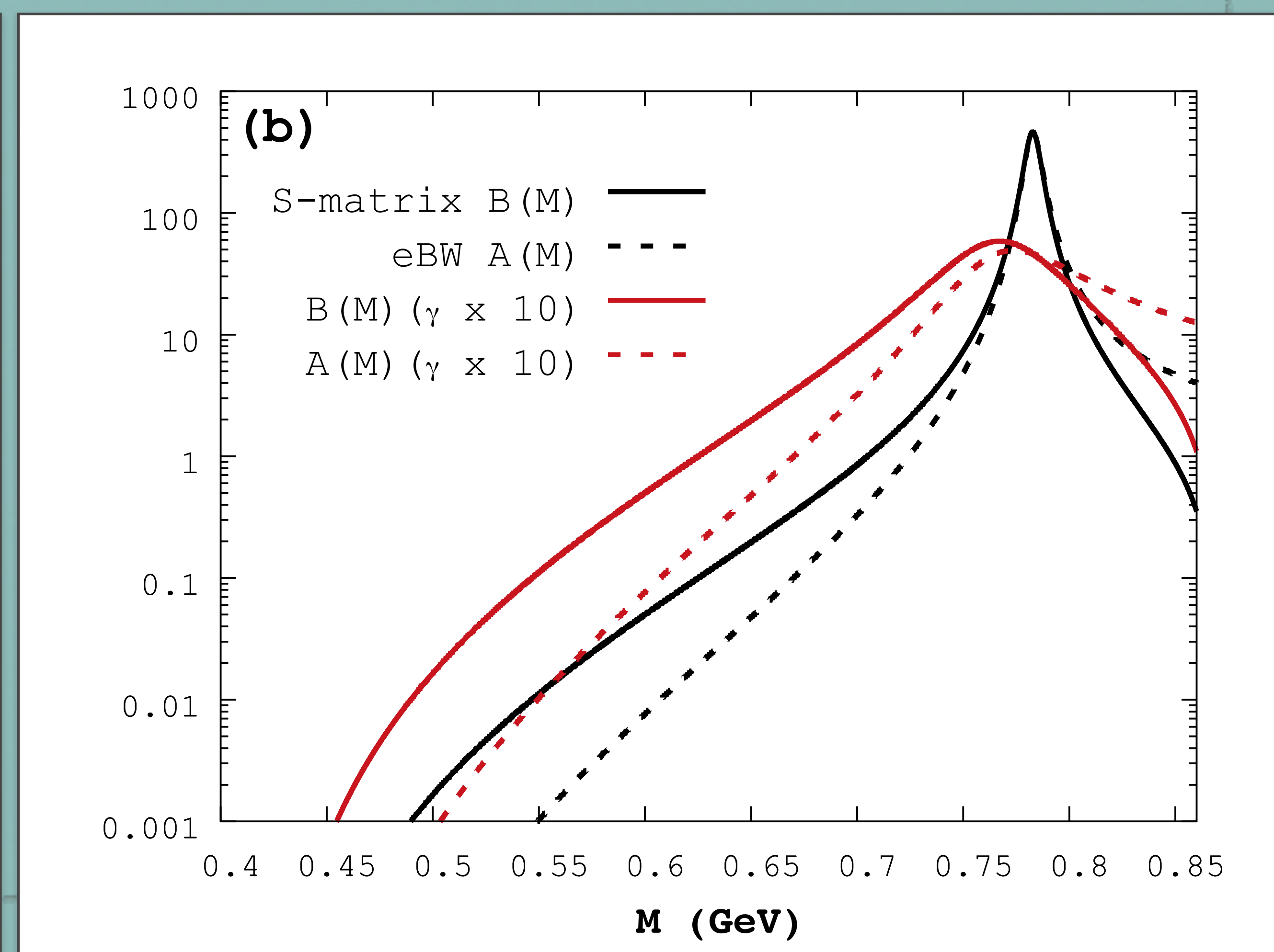
$$= \frac{1}{12} \times \left(s_{12}s_{23}s_{13} - m_\pi^2(m_{\text{res}}^2 - m_\pi^2)^2\right),$$

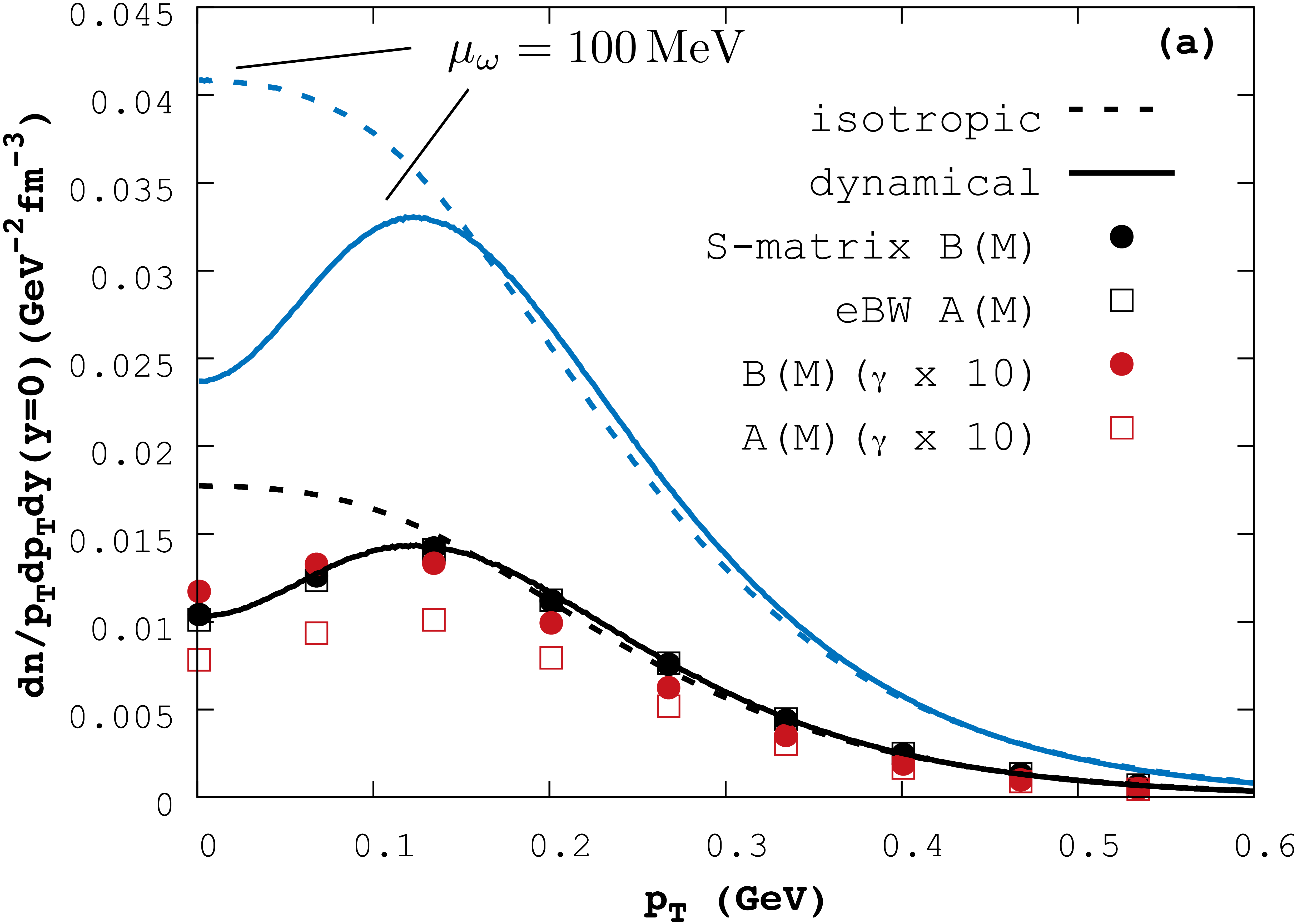
Dalitz info. <=>``3-body phase shift"



JPAC: http://www.indiana.edu/~jpac/



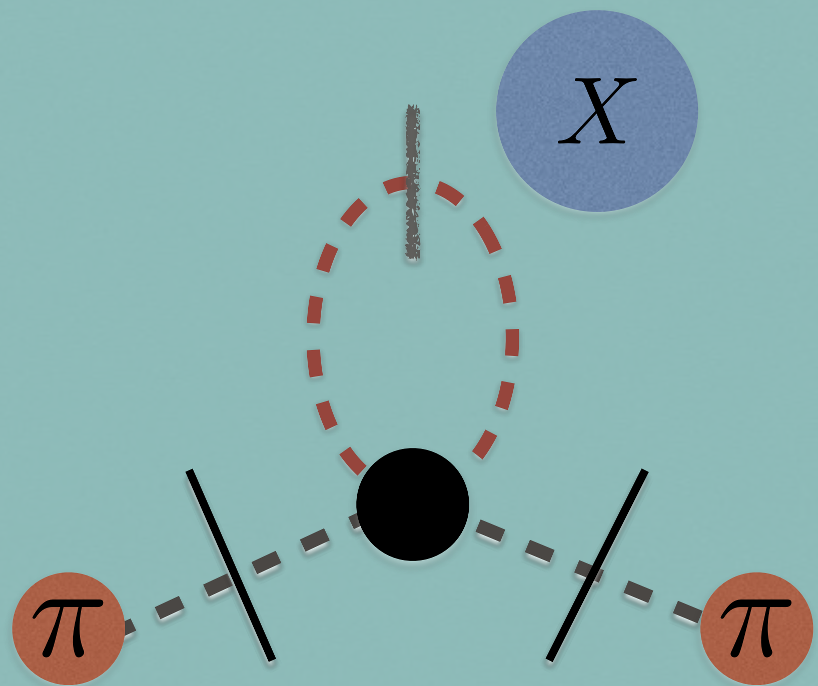




PML, PRC **97**, no. 3, 035210 (2018)

IN-MEDIUM EFFECTS

$$\Sigma_\pi =$$



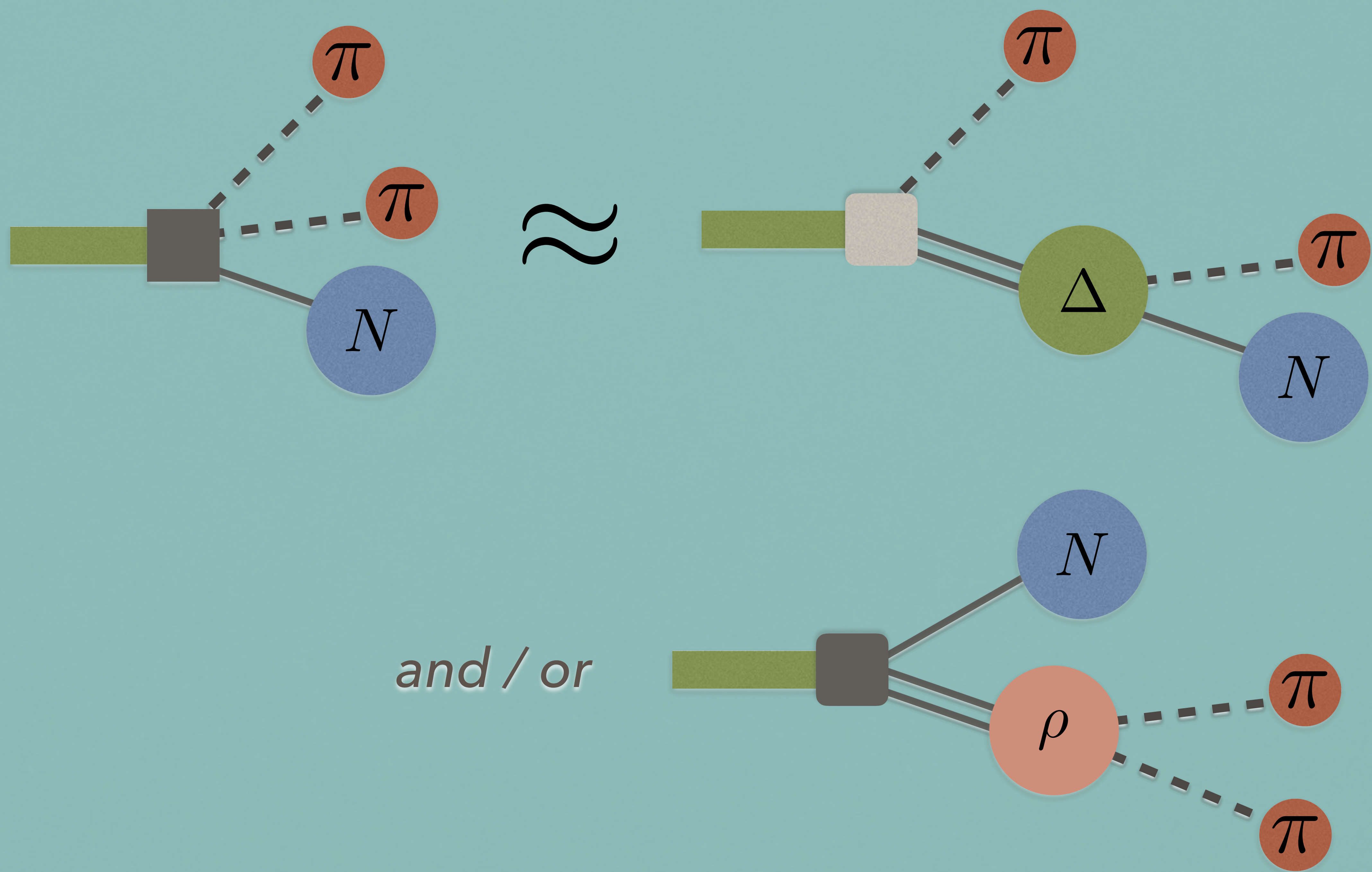
$$\propto \int \frac{d^3 q}{\omega_p \, \omega_q} \, n_X \times T_{\pi X}(s)$$

forward amplitude

A. Schenk NPB 363(1991)

ISOBAR MODEL

sequential decay model

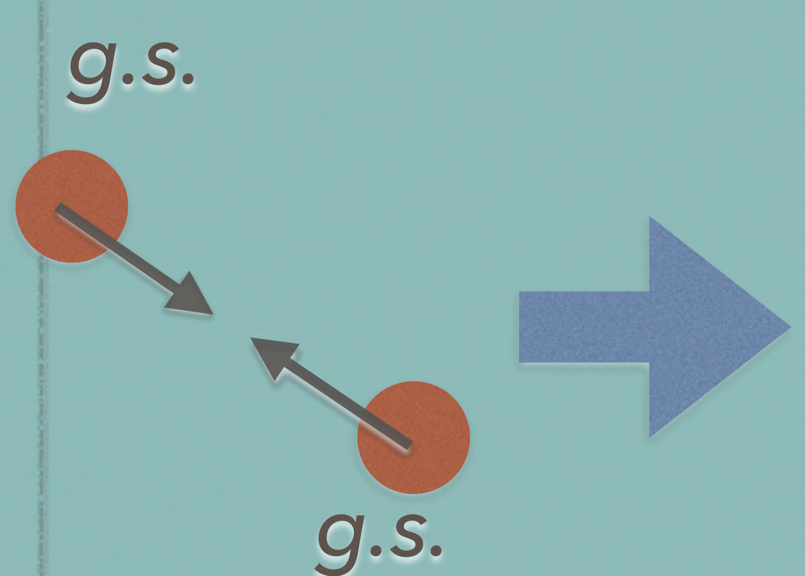


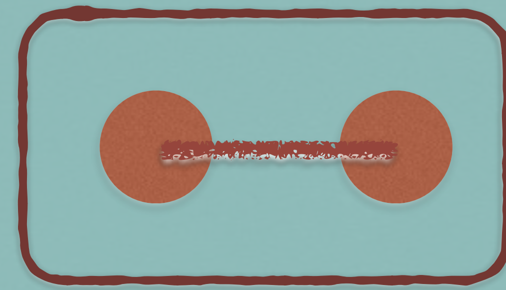
TIME DELAY

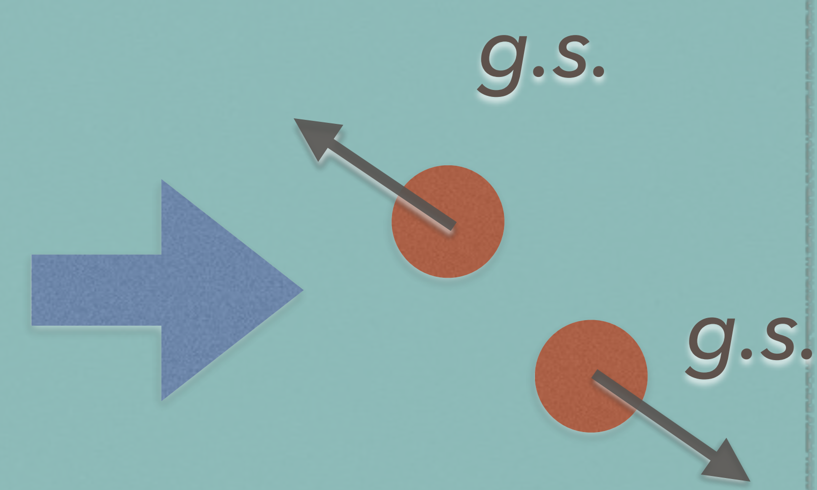
P. Danielewicz and S. Pratt
Phys.Rev. C53 (1996) 249-266

S. Leupold
Nucl.Phys. A695 (2001) 377-394

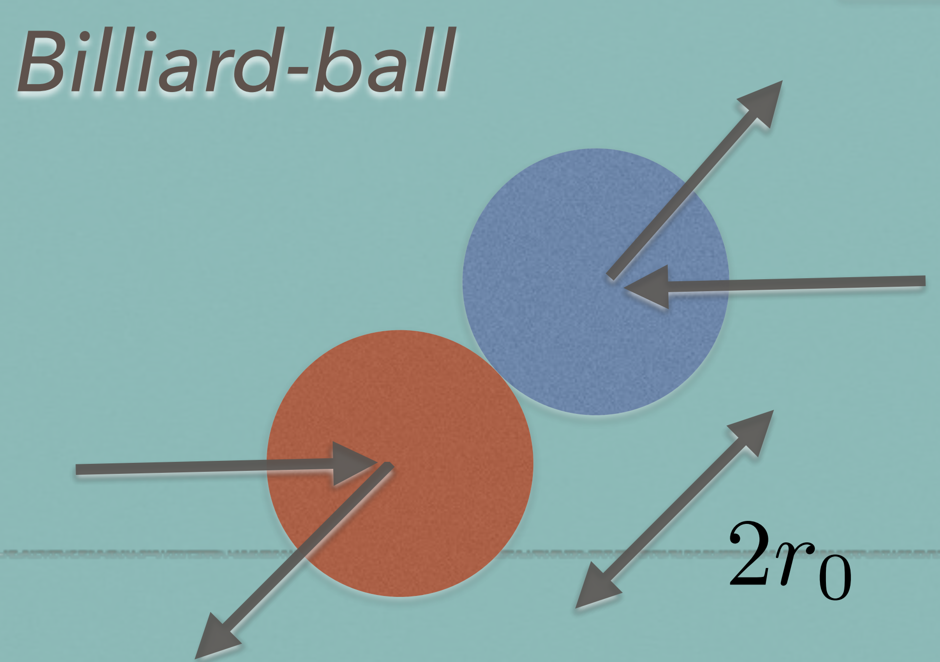
Yu. B. Ivanov et al
Phys.Atom.Nucl.64:652-669,2001

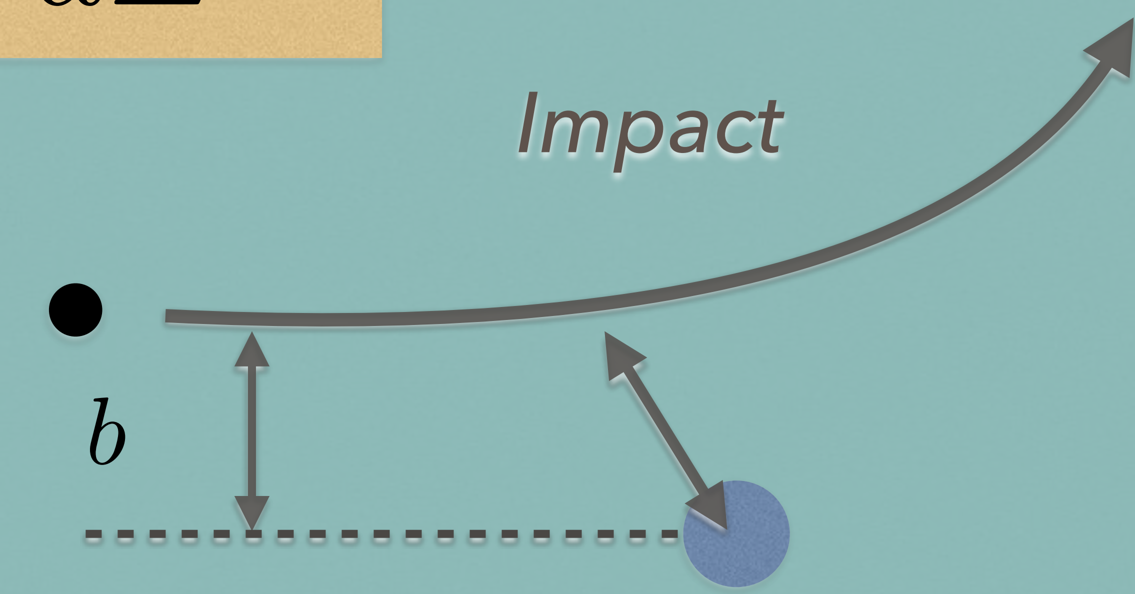






$$\tau \sim 2\frac{d\delta}{dE}$$





THANK YOU

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