FastReso – fast resonance decays in nuclear collisions

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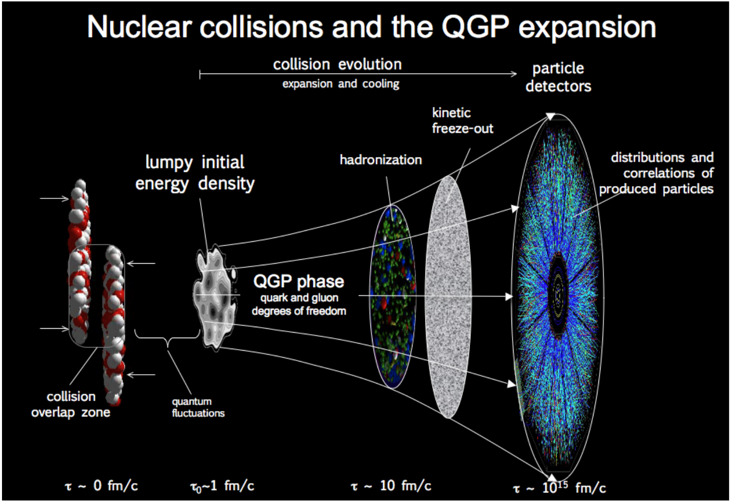
AM, Flörchinger, Grossi, and Teaney, arXiv:1809.11049

https://github.com/amazeliauskas/FastReso

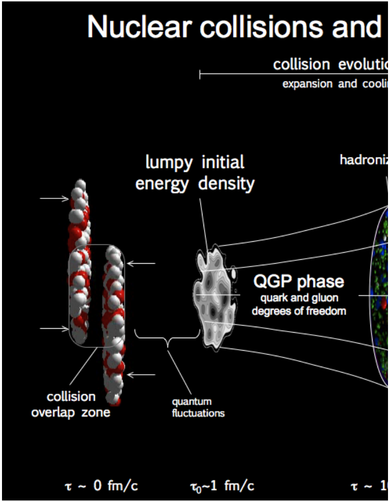
Isolated quantum systems and universality in extreme conditions

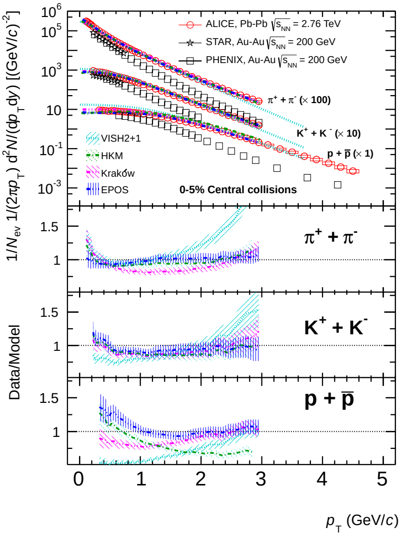
QCD matter to hadrons



Hadronization: QGP ⇒ hadrons ⇒ experimental observables.

QCD matter to hadrons





Phys. Rev. C 88, 044910 (2013)

Hadronization: QGP $\Rightarrow$ hadrons $\Rightarrow$ experimental observables.

Motivation

Resonance decays – most computationally intensive part of HI modelling.

- PDG lists ~ 300 hadron resonances with $m \leq 2{-}3\,\mathrm{GeV}$
- Many are short lived $\tau_{1/2} \sim 10\,\mathrm{fm}/c$ – decay before detected!
- Experimentally measured: π, K, p and strange $\Lambda, \Xi, \Omega, \ldots$
- Decays only depend on particle properties and initial populations

$$\underbrace{E_p \frac{dN_b}{d^3p}}_{\text{decay products}} = \int \frac{d^3q}{(2\pi)^3 2E_q} \underbrace{D_{a\to b}(p,q)}_{\text{decay map}} \underbrace{E_q \frac{dN_a}{d^3q}}_{\text{primary resonances}} .$$

- $b = \pi, K, p, \Lambda, \Sigma, \Omega,$ a – all resonances decaying to b.
- $D_{a\to b}(p,q)$ connects 1-body particle spectra*

If primary resonances are functions of fluid properties $T, u^\mu, \ldots$

$\Rightarrow$ then final spectra are too through $D_{a\to b}(p,q)$

* *neglecting hadronic rescatterings and non-flow n-particle correlations from decays.*

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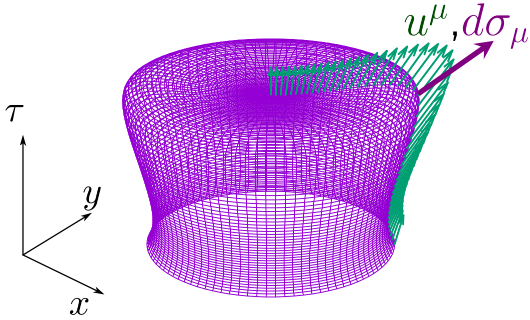
Cooper-Frye freeze-out procedure

Primary hadrons given by the Cooper-Frye fluid-particle conversion

$$\underbrace{E_p \frac{dN_a}{d^3p}}_{\text{primary spectrum}} = \frac{\nu_a}{(2\pi)^3} \int_\sigma f^a(p^\mu; \underbrace{T, u^\mu, \pi^{\mu\nu}, \ldots}_{\text{fluid properties}}) p^\mu d\sigma_\mu$$

Particle distribution f is expanded around equilibrium distribution

$$f(p^\mu; u^\mu, T, \ldots) = \underbrace{f_{\text{eq}}(\bar{E}_p = -p_\mu u^\mu; T, \mu)}_{\text{Bose-Einstein or Fermi-Dirac}} + \underbrace{\delta f_{\text{shear}} + \delta f_{\text{bulk}} + \ldots}_{\text{various ansatzes; } \eta,\, \zeta \text{ dependent}}$$



Iso-thermal $T = T_{\text{fo}}$ freeze-out surface σ for a central PbPb

The invariant decay spectrum

- Usual Cooper-Frye hadronization: freeze-out, then decays

$$\underbrace{E_p \frac{dN_b}{d^3p}}_{\text{decay products}} = \underbrace{\int_{\mathbf{q}} D_{a\to b}(p,q)}_{\text{decay map}} \; \underbrace{\frac{\nu_a}{(2\pi)^3}\int_\sigma}_{\text{freeze-out}} f^a(q^\mu; u^\mu, T, \ldots) q^\mu d\sigma_\mu .$$

- Reverse the order of the freeze-out integration and the decay map

$$E_p \frac{dN_b}{d^3p} = \frac{\nu_b}{(2\pi)^3}\int_\sigma \underbrace{\int_{\mathbf{q}} \frac{\nu_a}{\nu_b} D_{a\to b}(p,q)\; f^a(q^\mu; u^\mu, T, \ldots) q^\mu}_{g_b^\mu(p^\mu; T, u^\mu, , \ldots)} d\sigma_\mu$$

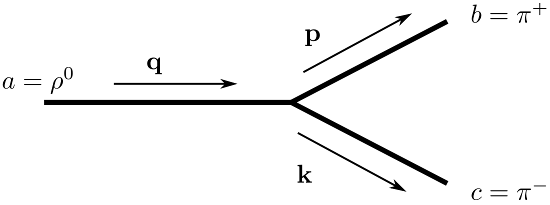
- Define *vector distribution function* g_b^μ of decay particles b

$$E_p \frac{dN_b}{d^3p} = \frac{\nu_b}{(2\pi)^3}\int_\sigma g_b^\mu(p^\mu; T, u^\mu, \ldots) d\sigma_\mu$$

- Only few distributions are needed: $b = \pi, K, p, \Lambda, \Xi, \Omega$

$g_b^\mu(p^\mu; T, u^\mu, \ldots)$ — direct conversion of fluid properties $T, u^\mu, \pi^{\mu\nu}, \ldots$ on freeze-out surface to the final hadron spectra!

Isotropic decay maps



A 2-body decay phase-space integral with $\mathcal{M}^a_{b|c} = \text{const.}$

$$E_p \frac{d^3 N_b}{d^3 p} = \int_{\mathbf{q}} \underbrace{\int_{\boldsymbol{k}} |\mathcal{M}^a_{b|c}|^2 (2\pi)^4 \delta^{(4)}(q^\mu - p^\mu - k^\mu)}_{D^a_{b|c}(p,q)} E_q \frac{d^3 N_a}{d^3 q}$$

For an isotropic decay $D^a_{b|c}(p,q) = D^a_{b|c}(p^\mu q_\mu)$ and

$$D^a_{b|c}(p^\mu q_\mu) = B \frac{4\pi^2 m_a}{p^a_{b|c}} \delta(q^\mu p_\mu + m_a E^a_{b|c}),$$

B – branching ratio, $p^a_{b|c}(m_a, m_b, m_c)$ – decay momentum in c.m.f.

Computing irreducible components

- p^μ and u^μ – the only Lorentz vectors in equilibrium

$$g^\mu = f_1^{\text{eq}}(\bar{E}_p, T, \mu) \underbrace{(p^\mu - \bar{E}_p u^\mu)}_{\text{vector } (0,\vec{p})} + f_2^{\text{eq}}(\bar{E}_p, T, \mu) \underbrace{\bar{E}_p u^\mu}_{\text{scalar } (\bar{E}_p,\vec{0})}$$

- Initially $g_a^\mu(q^\mu; T, u^\mu, \mu) = f_{\text{eq}}^a(\bar{E}_q, T, \mu) q^\mu$. After decays

$$g_b^\mu(p^\mu; T, u^\mu, \mu) = \frac{\nu_a}{\nu_b} \int \frac{d^3q}{(2\pi)^3 2E_q} D_{b|c}^a(p^\nu q_\nu) g_a^\mu(q^\mu; T, u^\mu, \mu).$$

- Irreducible SO(3) representations do not mix during decays

$$f_{i,b}^{\text{eq}}(\bar{E}_p, T, \mu) = B \frac{\nu_a}{\nu_b} \frac{m_a^2}{m_b^2} \frac{1}{2} \int_{-1}^{1} dw A_i(\bar{E}_p, w) f_{i,a}^{\text{eq}} \left(\mathcal{E}(\bar{E}_p, w), T, \mu\right)$$

  $A_i(\bar{E}_p, w)$, $\mathcal{E}(\bar{E}_p, w)$ simple functions of $\bar{E}_p$ and particle masses.
  3-body decay $\Rightarrow$ 2-body decay with variable $m_{\tilde{c}}^2 = -(p_c + p_d)^2$

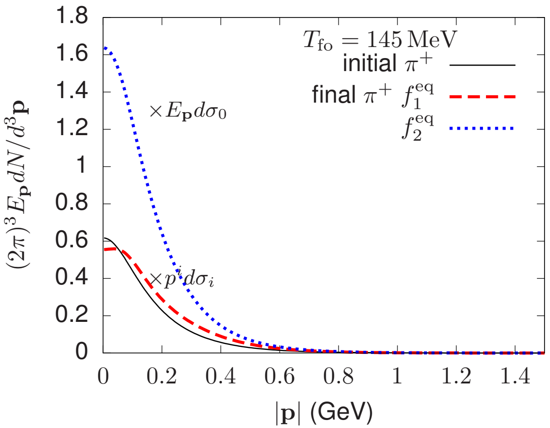
- *Iterate over decay list to get final $g_b^\mu(q^\mu; T, u^\mu, \mu)$*

`FastReso` – public implementation for arbitrary decay lists
`https://github.com/amazeliauskas/FastReso`

Final decay distributions for π^+

Pion π^+ spectrum after resonance decays in fluid rest-frame

$$(2\pi)^3 E_p \frac{dN}{d^3p} = \int_\sigma [f_1^{\text{eq}}(E_p,T,\mu)p^i d\sigma_i + f_2^{\text{eq}}(E_p,T,\mu)E_p d\sigma_0]\Big|_{u^\mu=(1,\vec{0})}$$



$f_1^{eq}(\bar{E}_p,T,\mu)$, $f_2^{eq}(\bar{E}_p,T,\mu)$ computed only once – valid in any frame.

Viscous corrections to the spectrum

Linearized shear and bulk perturbations to initial particle spectrum given by

$$E_p \frac{dN^{\text{visc.}}}{d^3p} = \frac{\nu_a}{(2\pi)^3} \int_\sigma \left[\delta f^{\text{bulk}}(\bar{E}_p, \Pi) + \delta f^{\text{shear}}(\bar{E}_p, \pi^{\mu\nu} p_\mu p_\nu) \right] p^\mu d\sigma_\mu$$

where usual ansatzes are

$$\delta f^{\text{bulk}}(\bar{E}_p, \Pi) = f_{\text{eq}}(1 \pm f_{\text{eq}}) \left[\frac{\bar{E}_p}{T} \left(\frac{1}{3} - c_s^2 \right) - \frac{1}{3} \frac{m^2}{T \bar{E}_p} \right] \frac{\tau_\Pi \Pi}{\zeta}.$$

$$\delta f^{\text{shear}}(\bar{E}_p, \pi^{\mu\nu} p_\mu p_\nu) = f_{\text{eq}}(1 \pm f_{\text{eq}}) \frac{\pi_{\rho\nu} p^\rho p^\nu}{2sT^3}$$

Then the decay particle spectrum is given by

$$E_p \frac{dN_b}{d^3p} = \frac{\nu_b}{(2\pi)^3} \int_\sigma (g^\mu_\Pi(p^\mu, u^\mu, T, \mu) \frac{-\tau_\pi \Pi}{\zeta} + g^{\mu\nu\rho}_\pi(p^\mu, u^\mu, T, \mu) \frac{\pi_{\nu\rho}}{2sT}) d\sigma_\mu$$

g^μ_Π and $g^{\mu\nu\rho}_\pi$ — independent of bulk Π pressure and shear $\pi^{\mu\nu}$ tensor.

Also works for baryon diffusion $\delta f^{\text{diffusion}} \propto j^\mu_D p_\mu$

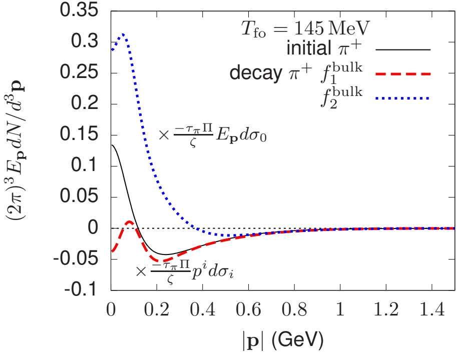
Final decay distributions for π^+ from viscous components

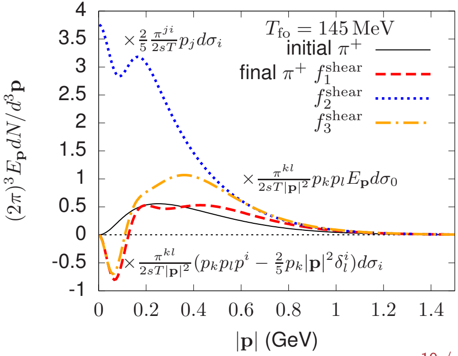
Irreducible SO(3) decomposition of g^μ_Π and $g^{\mu\nu\rho}_\pi$ in terms of p^μ and u^μ.

$$g^\mu_\Pi = \left(p^\mu - \bar{E}_p u^\mu\right) f_1^{\text{bulk}}(\bar{E}_p) + \bar{E}_p u^\mu f_2^{\text{bulk}}(\bar{E}_p),$$

$$g^{\mu\nu\rho}_\pi = [(p^\mu - \bar{E}_p u^\mu) p^\nu p^\rho - \frac{2}{5}|\vec{\bar{p}}|^2 \eta^{\mu\nu} p^\rho] f_1^{\text{shear}}(\bar{E}_p)$$
$$+ \frac{2}{5}|\vec{\bar{p}}|^2 \eta^{\mu\nu} p^\rho f_2^{\text{shear}}(\bar{E}_p) + \bar{E}_p u^\mu p^\nu p^\rho f_3^{\text{shear}}(\bar{E}_p).$$

Viscous π^+ spectrum corrections





Fast freeze-out with irreducible decay spectrum components

Freeze-out procedure for the direct decays

$$E_p \frac{dN_b}{d^3p} = \frac{\nu_b}{(2\pi)^3} \int_\sigma d\sigma_\mu \Big\{ F p^\mu + G u^\mu + H p^\nu \pi_\nu{}^\mu \Big\},$$

where explicitly these terms are

$$\begin{aligned} F =& f_1^{\text{eq}} + f_1^{\text{shear}} \pi_{\rho\sigma} p^\rho p^\sigma + f_1^{\text{bulk}} \Pi, \\ G =& f_2^{\text{eq}} - f_1^{\text{eq}} + \left(f_2^{\text{bulk}} - f_1^{\text{bulk}} \right) \Pi + \left(f_3^{\text{shear}} - f_1^{\text{shear}} \right) \pi_{\rho\sigma} p^\rho p^\sigma \bar{E}_p, \\ H =& \left(f_2^{\text{shear}} - f_1^{\text{shear}} \right) \frac{2}{5} |\bar{\mathbf{p}}|^2. \end{aligned}$$

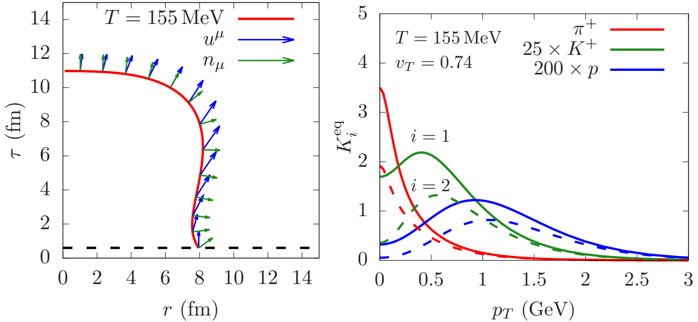
Only 7 Lorentz invariant scalar functions for each $b = \pi, K, p, \Lambda, \Xi, \Omega$ or just $b = N_{ch}$

Huge computational efficiency compared with 3 functions $(f^{\text{eq}}, \delta f^{\text{shear}}, \delta f^{\text{bulk}}) \times \sim 300$ primary resonances + decay integrals for usual Cooper-Frye freeze-out proceedure

Irreducible components f_i easily computed with `FastReso`
`https://github.com/amazeliauskas/FastReso`

Boost-invariant and azimuthally symmetric freeze-out surface

$$\frac{dN_b}{2\pi p_T dp_T dy} = \frac{\nu_b}{(2\pi)^3}\int_0^1 d\alpha\, \tau(\alpha) r(\alpha) \left\{ \frac{\partial r}{\partial \alpha} K_1^{\mathsf{eq}}(p_T, u^r) - \frac{\partial \tau}{\partial \alpha} K_2^{\mathsf{eq}}(p_T, u^r) \right\},$$



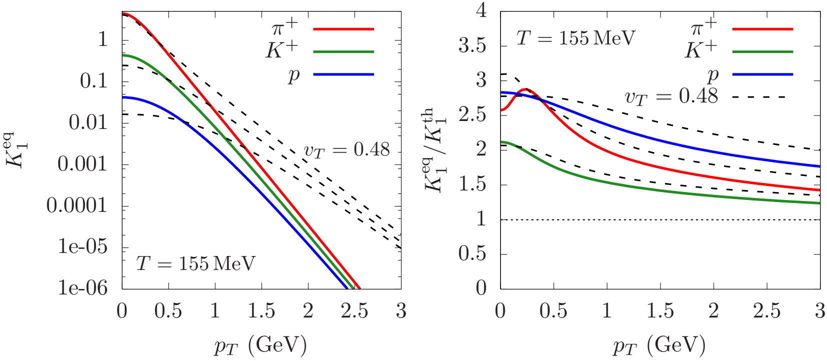
Kernels pre-computed from components $f_i(\bar{E}_p; T, \mu)$ for different T and μ

$$K_1^{\mathsf{eq}}(p_T, u^r) = \int d\phi d\eta \left\{ f_1^{\mathsf{eq}}(\bar{E}_p) m_T \cosh(\eta) + \left(f_2^{\mathsf{eq}}(\bar{E}_p) - f_1^{\mathsf{eq}}(\bar{E}_p) \right) \bar{E}_p u^\tau \right\},$$

$$K_2^{\mathsf{eq}}(p_T, u^r) = \int d\phi d\eta \left\{ f_1^{\mathsf{eq}}(\bar{E}_p) p_T \cos(\phi) + \left(f_2^{\mathsf{eq}}(\bar{E}_p) - f_1^{\mathsf{eq}}(\bar{E}_p) \right) \bar{E}_p u^r \right\}.$$

where $\bar{E}_p = m_T u^\tau - p_T u^r \cos\phi$

Comparison with thermal feed-down



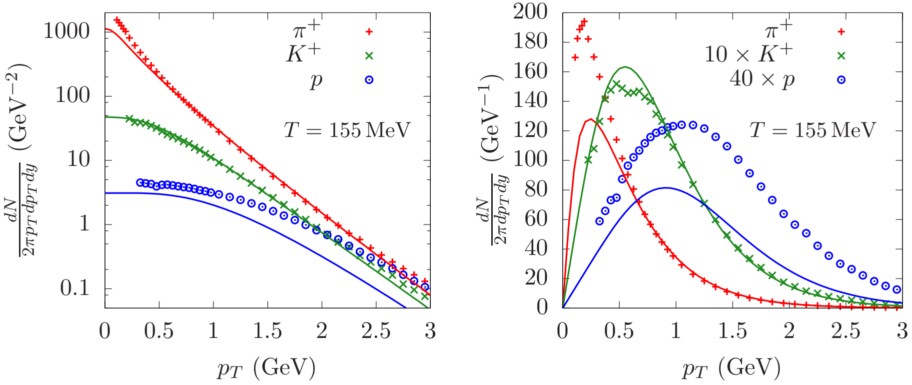
Compare with blast-wave fit – no decays included.

$$\frac{dN_b}{2\pi p_T dp_T dy} = \frac{\nu_b}{(2\pi)^3}\int_0^1 \alpha d\alpha\ \tau R \times \underbrace{C_b}_{\text{arb. norm.}} \times \underbrace{\mathcal{I}_0\left(\frac{p_T u^r_{\mathsf{kin}}(\alpha)}{T_{\mathsf{kin}}}\right)\mathcal{K}_1\left(\frac{m_T u^\tau_{\mathsf{kin}}(\alpha)}{T_{\mathsf{kin}}}\right)}_{K_1^{\text{blast-wave}}}$$

$C_b, T_{\mathsf{kin}}, u^r_{\mathsf{kin}}$ – unphysical fit parameters.

Comparison with experimental data

0-5% Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV data from Phys. Rev. C 88, 044910 (2013)



This is an illustration of fast-freezeout procedure — not best fit.

Single-shot freeze-out surface produced by `Fluid`u`M` – viscous hydrodynamic evolution with new equation of state, $\eta/s = 0.16$ and $\zeta/s(T)$ parametrization

Floerchinger, Grossi and Lion, 1811.01870

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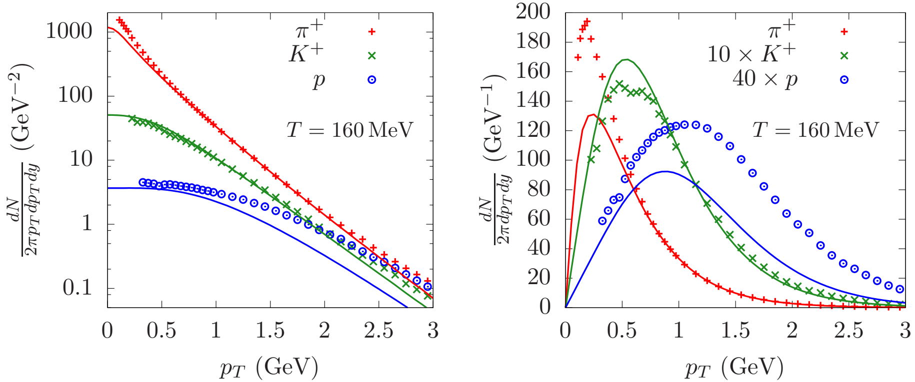

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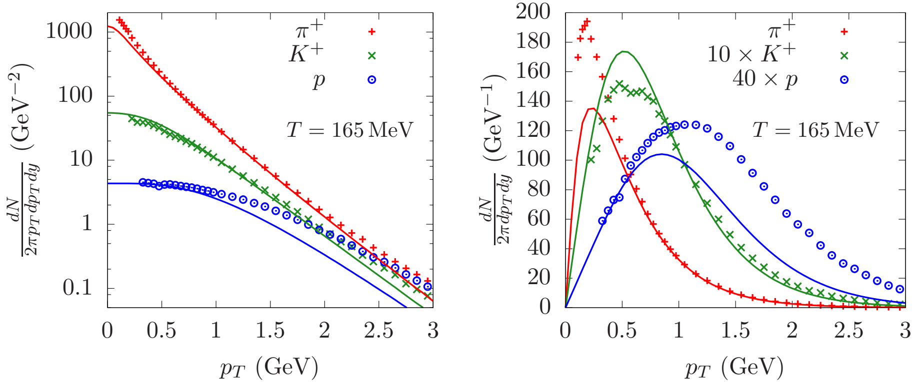

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Summary

Fast resonance decays – a map from properties of QCD matter to hadrons.

- Semi-analytic understanding of viscous corrections to hadron spectra.
  $\Rightarrow$ *easier to study δf corrections, also works for diffusion.*
- Affordable computation of the major hadronization effect.
  $\Rightarrow$ *statistics hungry studies, Byasian analysis of freeze-out*
- Decay kernels can be used for blast-wave fits.
  $\Rightarrow$ *physically meaning extraction of freeze-out parameters*

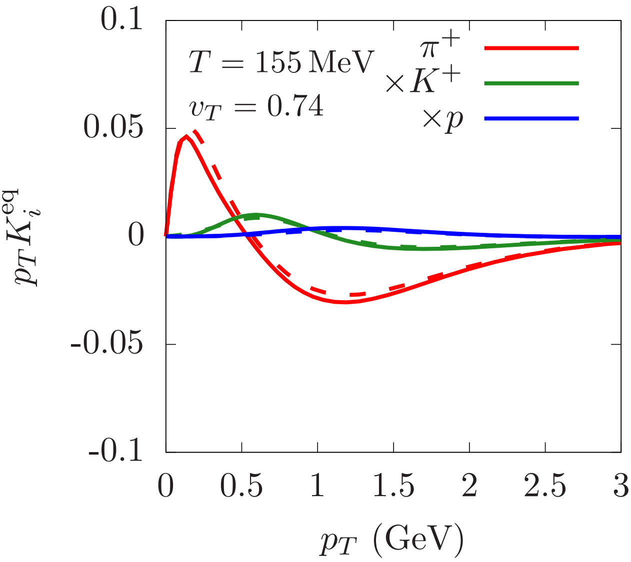
Current limitations:

- isotropic 2 and 3 body decays.
- finite resonance widths. *work in progress*
- massless decays. *workaround $m_\gamma = 1$ MeV*
- computes 1-particle spectra *2-point function in progress*
- no resonance rescatterings.

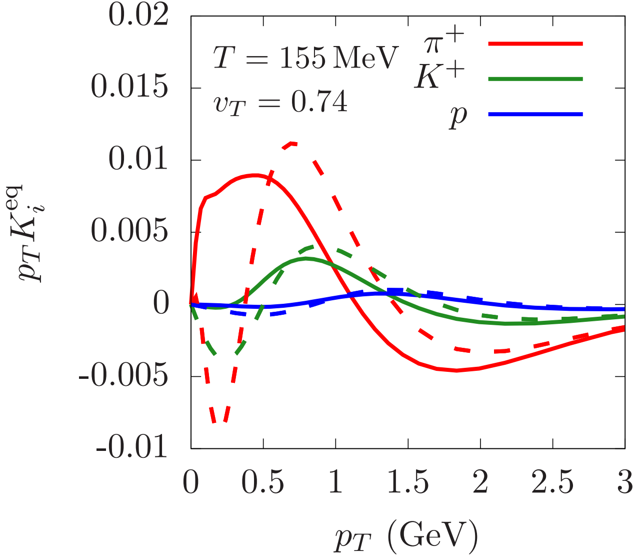
Analytical understanding of the map between initial spectra and decays $\Rightarrow$ more transparent heavy ion modelling.

Backup

Bulk kernels

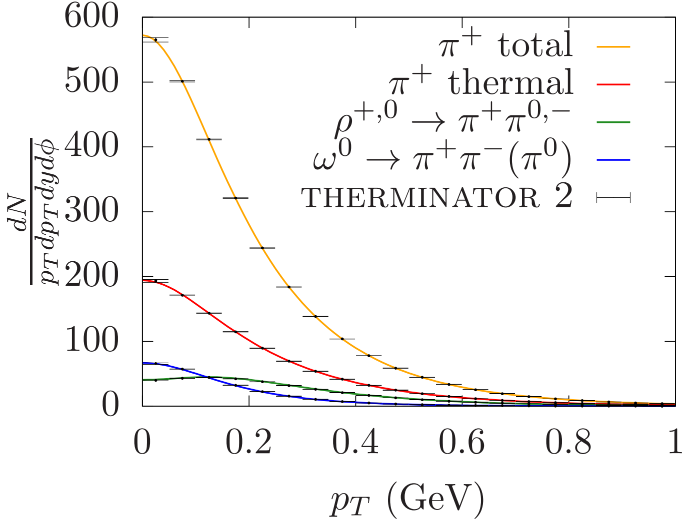


Shear kernels



Final pion spectrum for a blast-wave surface

Blast wave surface with $T_{\mathsf{fo}} = 145\,\mathsf{MeV}$ $v_T = 0.341$ $(\mu_B = 0)$



Direct computation of resonance decay spectra from freeze-out surface!

Particle data and decay data

`particle.data`

```
# Name       Mass      Gamma       Spin       Isospin  I3        Nq
jp3096zer   3.0968700    0.0000000        1.0       0.
Dc1800plu   1.8693000    0.0000000        0.0       0.5
Dc1800min   1.8693000    0.0000000        0.0       0.5        -
Dc1800zer   1.8693000    0.0000000        0.0       0.5
Dc1800zrb   1.8693000    0.0000000        0.0       0.5        -
...
eta547zer   0.5473000    1.2900e-6        0.0       0.
Ka0492zer   0.4976720    7.335e-16        0.0       0.5        -
Ka0492zrb   0.4976720    7.335e-16        0.0       0.5
Ka0492plu   0.4936770    0.0000000        0.0       0.5
Ka0492min   0.4936770    0.0000000        0.0       0.5        -
pi0139plu   0.1395699    0.0000000        0.0       1.
pi0139min   0.1395699    0.0000000        0.0       1.         -
pi0135zer   0.1349764    0.0000000        0.0       1.
gam000zer   0.0010000    0.0000000        1.0       0.
```

Particle data and decay data

decays.data

```
#Father Child1 Child2 [Child3] BranchingRatio ClebschGordanCoe
Dl2420plp   pr0938plu   pi0139plu   0.1    0
Dl2420plp   Dl1232plu   pi0139plu   0.2    1
Dl2420plp   Dl1232plp   pi0135zer   0.2    1
Dl2420plp   pr0938plu   rho770plu   0.7    0
Dl2420plu   Dl1232plp   pi0139min   0.2    1
...
rho770plu   pi0139plu   pi0135zer   1.      1
rho770min   pi0139min   pi0135zer   1.      1
f00600zer   pi0135zer   pi0135zer   1.0     1
f00600zer   pi0139plu   pi0139min   1.0     1
eta547zer   gam000zer   gam000zer   0.3943  0
eta547zer   pi0135zer   pi0135zer   pi0135zer   0.3251   0
eta547zer   pi0139plu   pi0139min   pi0135zer   0.226    0
eta547zer   pi0139plu   pi0139min   gam000zer   0.0468   0
```

Three body decays and finite resonance width

3-body decay $a \to b + c + d$ as 2-body decay with variable mass $a \to b + \tilde{c}$, $m_{\tilde{c}} = \sqrt{-(p_c + p_d)^2}$

$$D^a_{b|cd}(p^\mu q_\mu) = \frac{\int_{m_c+m_d}^{m_a-m_b} dm_{\tilde{c}}\ p^a_{b|\tilde{c}} p^{\tilde{c}}_{c|d} D^a_{b|\tilde{c}}(p^\mu q_\mu)}{\int_{m_c+m_d}^{m_a-m_b} dm_{\tilde{c}}\ p^a_{b|\tilde{c}} p^{\tilde{c}}_{c|d}}$$

Finite resonance width

$$D^a_{b|c}(p^\mu q_\mu) = \int_{(m_b+m_c)^2}^{\infty} ds\, \delta(s - m_a^2)\ D^a_{b|c}(p^\mu q_\mu)\Big|_{q^2=-s}$$

Breit-Wigner distribution

$$\rho(s) = \frac{\xi}{\pi} \frac{m_a \Gamma(s)}{(s + m_a^2) + m_a^2 \Gamma^2(s)}, \quad \int_{(m_b+m_c)^2}^{\infty} ds \rho(s) = 1$$

or, better, see talks of Pok Man Lo and Pasi Huovinen.

Only one additional mass integral.

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