Hadronization and freeze-out from lattice QCD

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QCD matter under extreme conditions

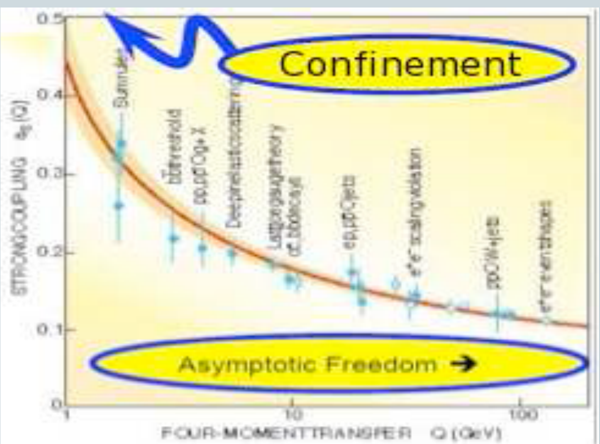
To address this topic, we need fundamental theory and experiment

Theory: Quantum Chromodynamics

- QCD is the fundamental theory of strong interactions
- It describes interactions among quarks and gluons

$$L_{QCD} = \sum_{i=1}^{n_f} \overline{\psi}_i \gamma_\mu \left(i\partial^\mu - g A_a^\mu \frac{\lambda_a}{2} \right) \psi_i - m_i \overline{\psi}_i \psi_i - \frac{1}{4} \sum_a F_a^{\mu\nu} F_a^{\mu\nu}$$

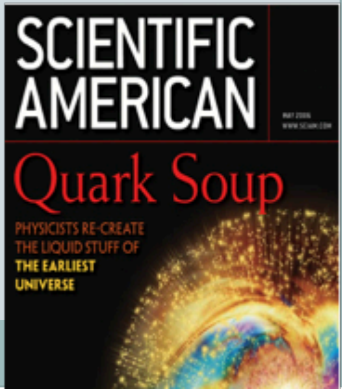
$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + i f_{abc} A_b^\mu A_c^\mu$$



Experiment: heavy-ion collisions

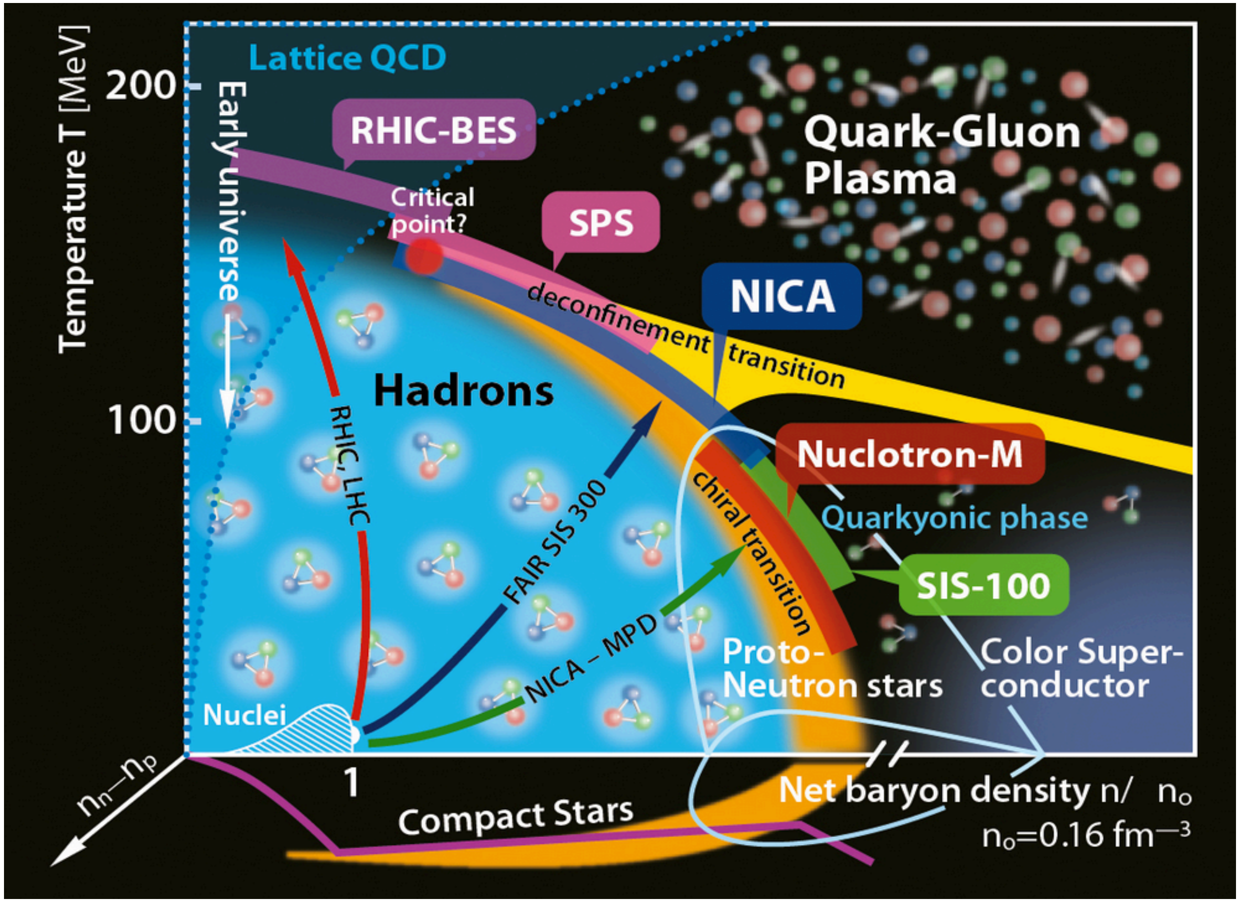
- Quark-Gluon Plasma (QGP) discovery at RHIC and LHC:
- SURPRISE!!! QGP is a PERFECT FLUID
- Changes our idea of QGP

  (no weak coupling)
- Microscopic origin still unknown



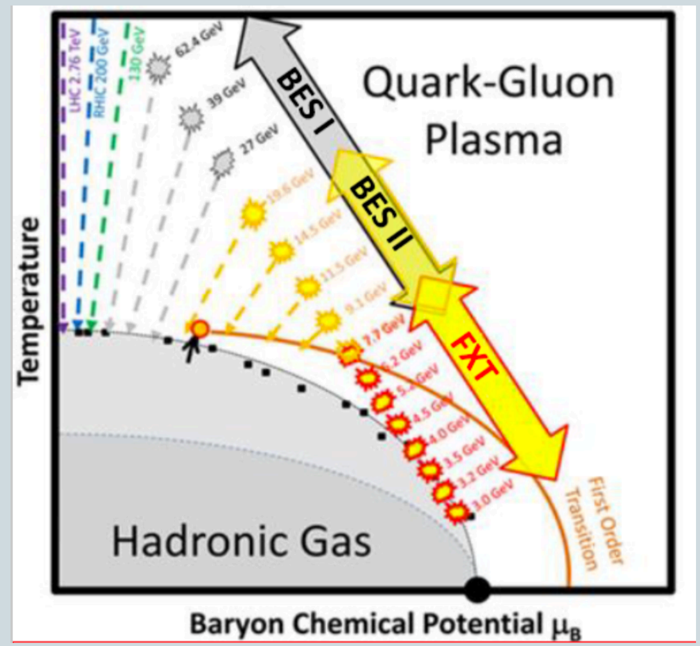
Open Questions

- Is there a critical point in the QCD phase diagram?
- What are the degrees of freedom in the vicinity of the phase transition?
- Where is the transition line at high density?
- What are the phases of QCD at high density?
- Are we creating a thermal medium in experiments?



Second Beam Energy Scan (BESII) at RHIC

- Planned for 2019-2020
- 24 weeks of runs each year
- Beam Energies have been chosen to keep the μ_B step ~50 MeV
- Chemical potentials of interest: μ_B/T~1.5…4



√s (GeV)	19.6	14.5	11.5	9.1	7.7	6.2	5.2	4.5
μ_B (MeV)	205	260	315	370	420	487	541	589
# Events	400M	300M	230M	160M	100M	100M	100M	100M

Collider — Fixed Target

Comparison of the facilities

Compilation by D. Cebra

Facilty	RHIC BESII	SPS	NICA	SIS-100 SIS-300	J-PARC HI
Exp.:	STAR +FXT	NA61	MPD + BM@N	CBM	JHITS
Start:	2019-20 2018	2009	2020 2017	2022	2025
Energy: $\sqrt{s_{NN}}$ (GeV)	7.7– 19.6 2.5-7.7	4.9-17.3	2.7 - 11 2.0-3.5	2.7-8.2	2.0-6.2
Rate: At 8 GeV	100 HZ 2000 Hz	100 HZ	<10 kHz	<10 MHZ	100 MHZ
Physics:	CP&OD	CP&OD	OD&DHM	OD&DHM	OD&DHM
	Collider Fixed target	Fixed target Lighter ion collisions	Collider Fixed target	Fixed target	Fixed target

CP=Critical Point OD= Onset of Deconfinement DHM=Dense Hadronic Matter

How can lattice QCD support the experiments?

- Equation of state
  - Needed for hydrodynamic description of the QGP

- QCD phase diagram
  - Transition line at finite density
  - Constraints on the location of the critical point

- Fluctuations of conserved charges
  - Can be simulated on the lattice and measured in experiments
  - Can give information on the evolution of heavy-ion collisions
  - Can give information on the critical point

Taylor expansion of EoS

- Taylor expansion of the pressure:

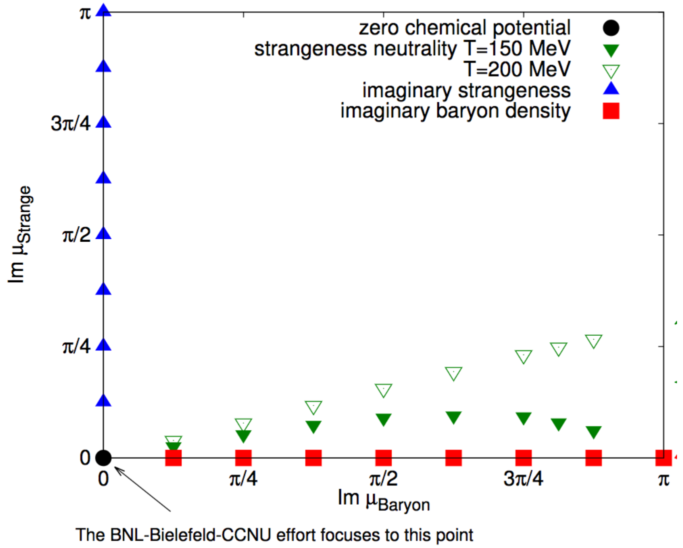
$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \frac{\mathrm{d}^{2\mathrm{n}}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}}\Bigg|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

- Two ways of extracting the Taylor expansion coefficients:
  - Direct simulation
  - Simulations at imaginary μ_B
- Two physics choices:
  - $\mu_B \neq 0$, $\mu_S = \mu_Q = 0$
  - μ_S and μ_Q are functions of T and μ_B to match the experimental constraints:

$\langle n_S \rangle = 0$ $\quad\quad$ $\langle n_Q \rangle = 0.4 \langle n_B \rangle$

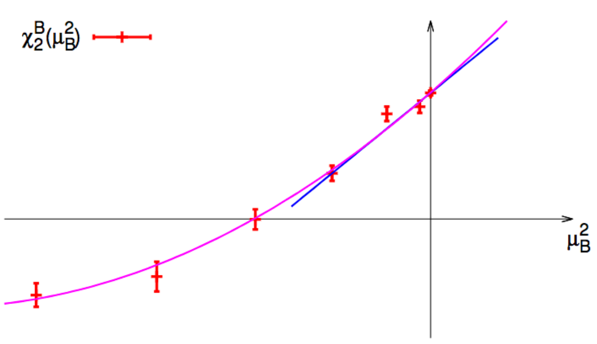
Pressure coefficients: simulations at imaginary μ_B

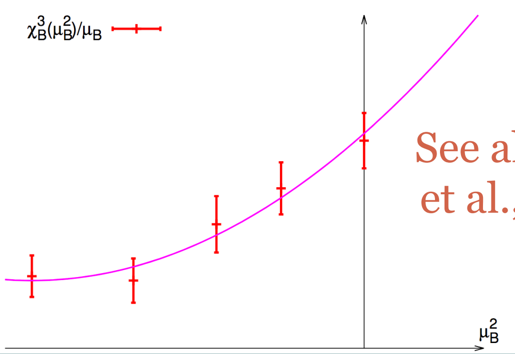
Simulations at imaginary μ_B:



Common technique: [de Forcrand, Philipsen (2002)], [D'Elia and Lombardo, (2002)], [Bonati et al., (2015), (2018)], [Cea et al., (2015)]

Strategy: simulate lower-order fluctuations and use them in a combined, correlated fit





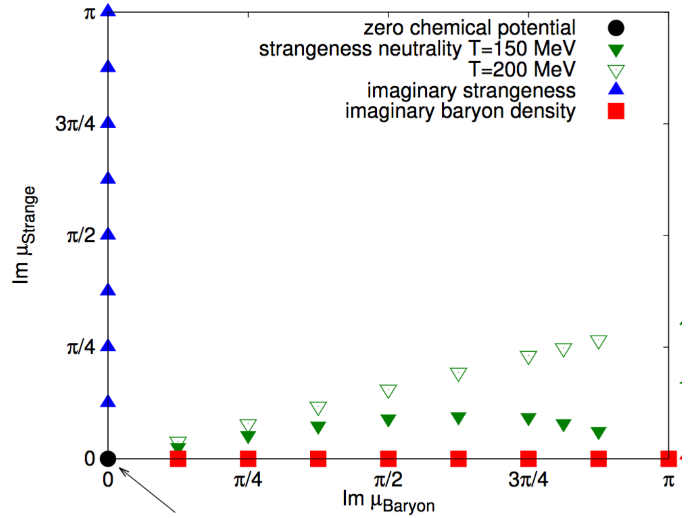
See also M. D'Elia et al., PRD (2017)

$$\chi_2^B(\mu_B^2) \approx \chi_2^B(0) + \frac{1}{2}\mu_B^2\chi_4^B(0) + \frac{1}{24}\mu_B^4\chi_6^B(0) + \ldots$$

$$\frac{\chi_3^B(\mu_B^2)}{\mu_B} \approx \chi_4^B(0) + \frac{1}{6}\mu_B^2\chi_6^B(0) + \frac{1}{120}\mu_B^4\chi_8^B(0)$$

Pressure coefficients: simulations at imaginary μ_B

Simulations at imaginary μ_B:



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Strategy: simulate lower-order fluctuations and use them in a combined, correlated fit

$$\chi_1^B(\hat{\mu}_B) = 2c_2\hat{\mu}_B + 4c_4\hat{\mu}_B^3 + 6c_6\hat{\mu}_B^5 + \frac{4!}{7!}c_4\epsilon_1\hat{\mu}_B^7 + \frac{4!}{9!}c_4\epsilon_2\hat{\mu}_B^9$$

$$\chi_2^B(\hat{\mu}_B) = 2c_2 + 12c_4\hat{\mu}_B^2 + 30c_6\hat{\mu}_B^4 + \frac{4!}{6!}c_4\epsilon_1\hat{\mu}_B^6 + \frac{4!}{8!}c_4\epsilon_2\hat{\mu}_B^8$$

$$\chi_3^B(\hat{\mu}_B) = 24c_4\hat{\mu}_B + 120c_6\hat{\mu}_B^3 + \frac{4!}{5!}c_4\epsilon_1\hat{\mu}_B^5 + \frac{4!}{7!}c_4\epsilon_2\hat{\mu}_B^7$$

$$\chi_4^B(\hat{\mu}_B) = 24c_4 + 360c_6\hat{\mu}_B^2 + c_4\epsilon_1\hat{\mu}_B^4 + \frac{4!}{6!}c_4\epsilon_2\hat{\mu}_B^6.$$

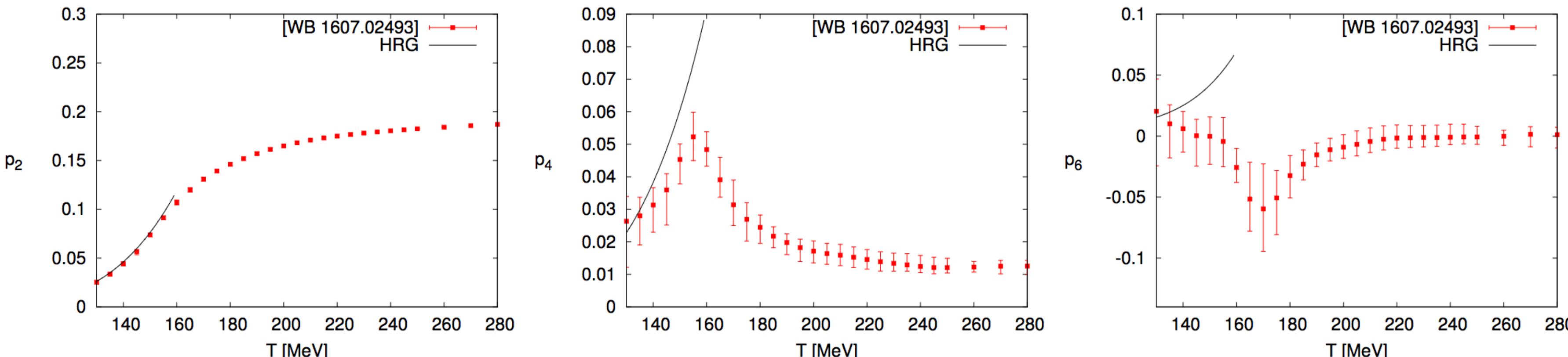
See also M. D'Elia et al., PRD (2017)

Pressure coefficients

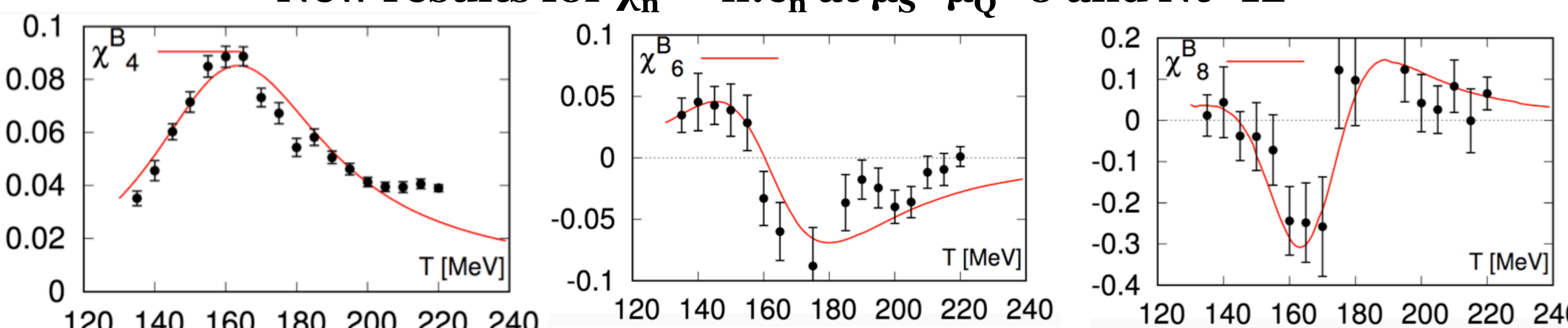
<u>Simulations at imaginary μ_B</u>:

Continuum, O(10^4) configurations, errors include systematics (WB: NPA (2017))

Strangeness neutrality



New results for $\chi_n^B = n!c_n$ at $\mu_S=\mu_Q=0$ and Nt=12



WB, JHEP (2018)

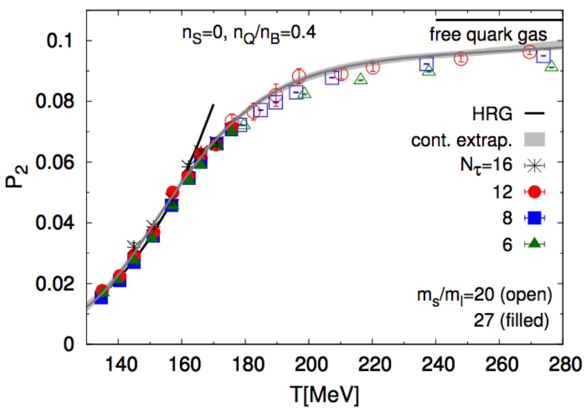
Red curves are obtained by shifting χ_1^B/μ_B to finite μ_B: consistent with no-critical point

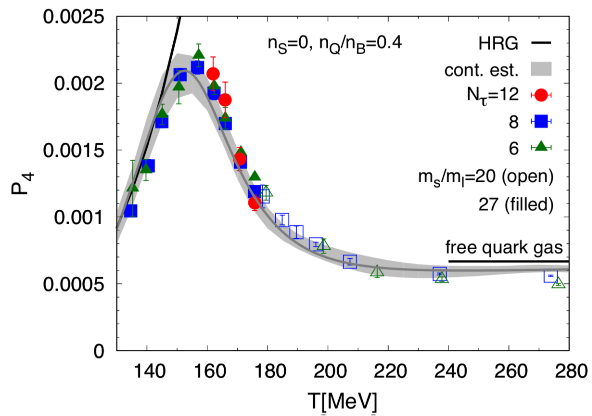
Pressure coefficients

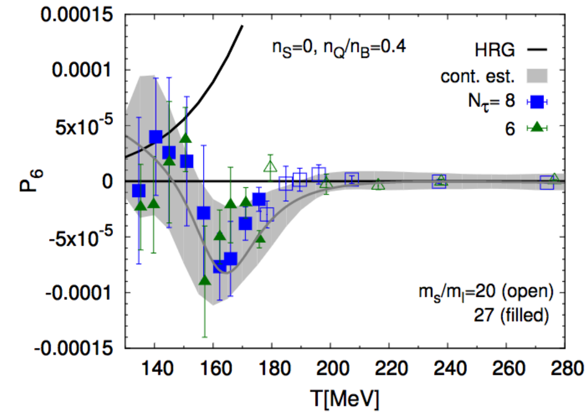
Direct simulation:

O(10^5) configurations (hotQCD: PRD (2017) and update 06/2018)

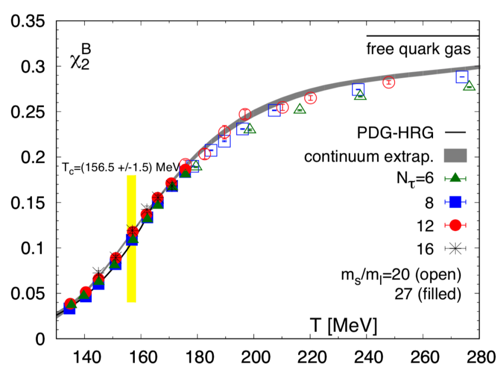
Strangeness neutrality

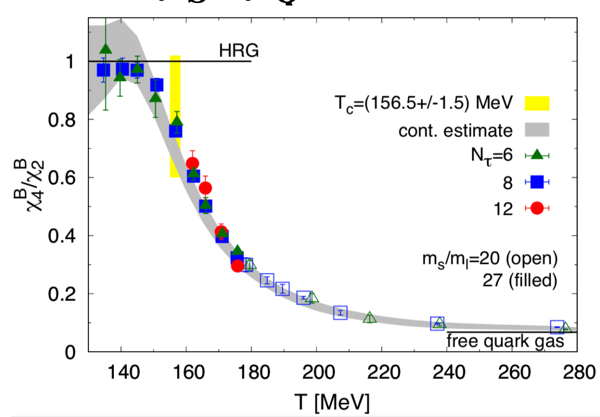


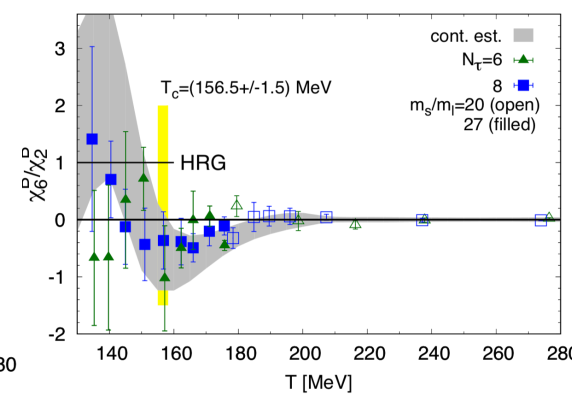




$\mu_S=\mu_Q=0$



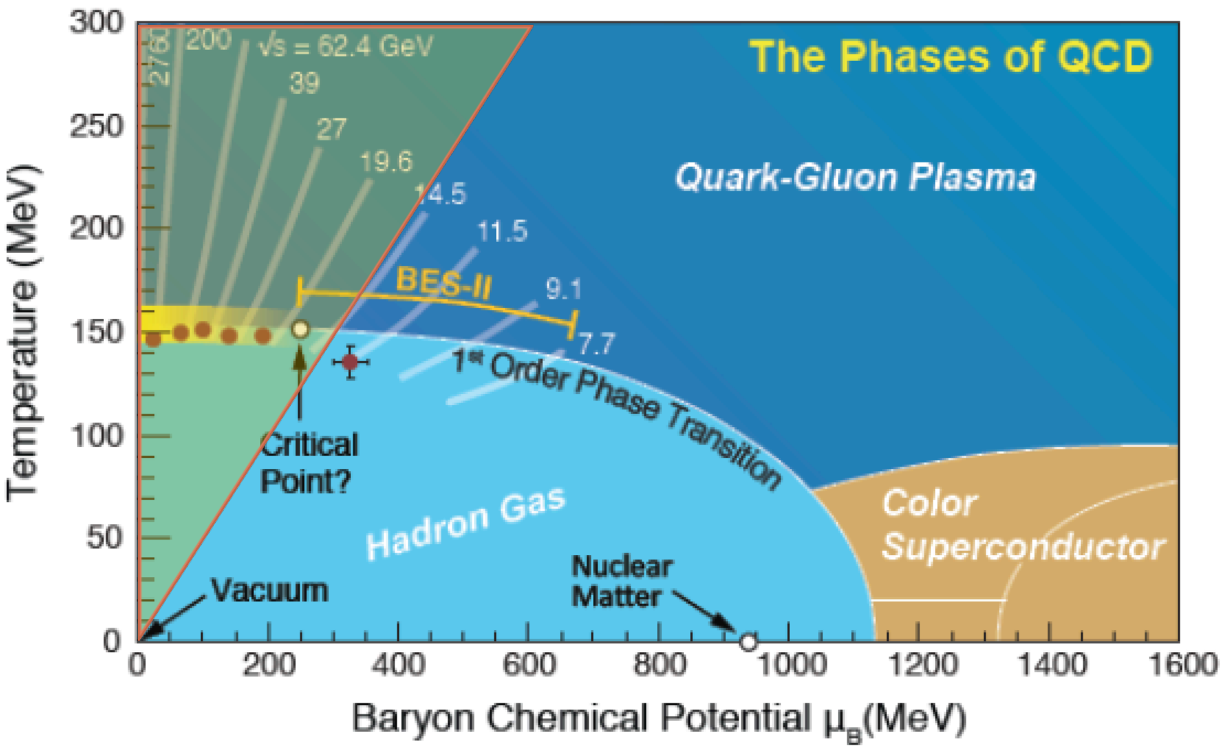




Range of validity of equation of state

- We now have the equation of state for $\mu_B/T \leq 2$ or in terms of the RHIC energy scan:

$$\sqrt{s} = 200,\ 62.4,\ 39,\ 27,\ 19.6,\ 14.5\text{GeV}$$



QCD EoS with critical point

P. Parotto, C.R. et al., 1805.05249 (2018)

- Currently, first principle EoS is given from Lattice QCD as Taylor expansion around $\mu_B = 0$

$$P_{QCD} = T^4 \sum_n c^n(T) \left(\frac{\mu_B}{T}\right)^n \quad , \qquad c^n(T) = \left.\frac{\partial (P/T^4)}{\partial (\mu_B/T)}\right|_{\mu_B=0}$$

- QCD is in the **3D Ising static universality class**
- **Idea:** use 3D Ising EoS mapped onto QCD to estimate critical contribution to $c^n(T)$:

$$c^n(T) = c^n_{\text{reg}}(T) + c^n_{\text{crit}}(T)$$

- Expand over the whole phase diagram

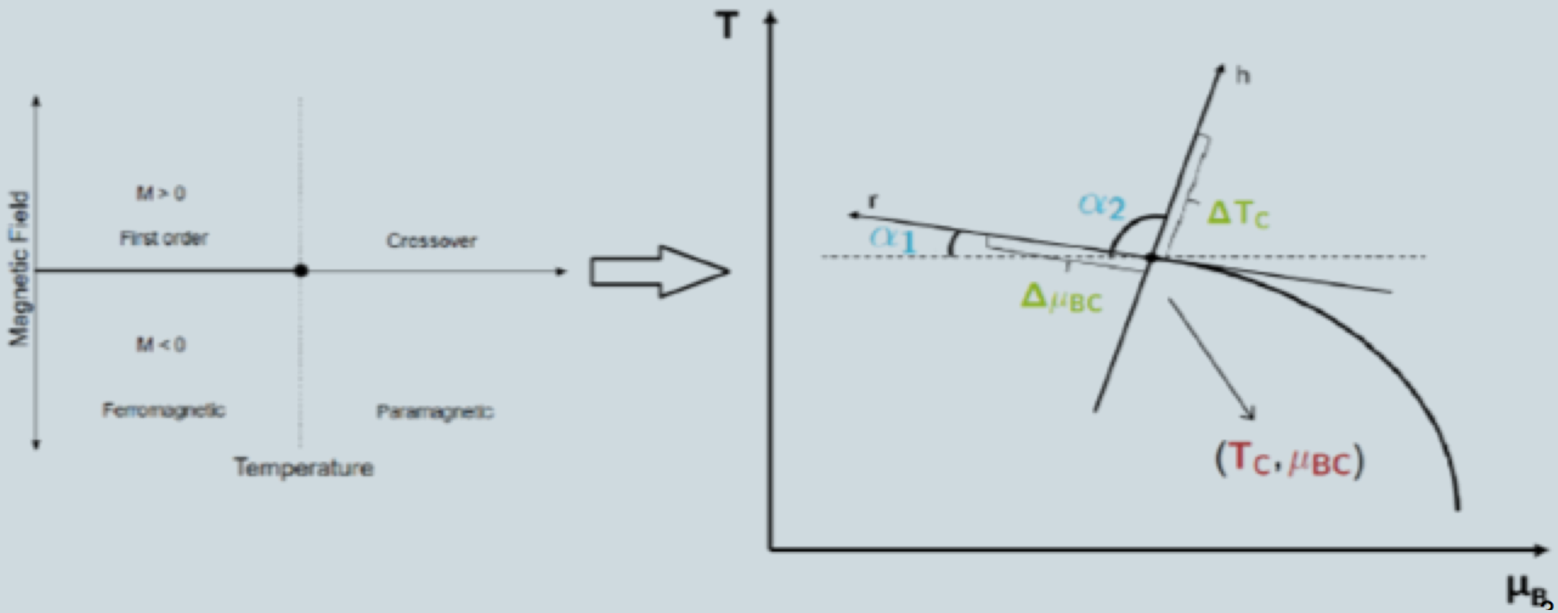
$$P(T, \mu_B \neq 0) = T^4 \sum c^n_{\text{reg}}(T) \left(\frac{\mu_B}{T}\right)^n + P_{\text{crit}}(T, \mu_B)$$

QCD EoS with critical point

We need 6 parameters to map Ising model's phase diagram onto the QCD one:

$$(r, h) \longmapsto (T, \mu_B): \qquad T = T_C + r \sin \alpha_1 \Delta\mu_{BC} + h \sin \alpha_2 \Delta T_C$$

$$\mu_B = \mu_{BC} - r \cos \alpha_1 \Delta\mu_{BC} - h \cos \alpha_2 \Delta T_C$$



QCD EoS with critical point

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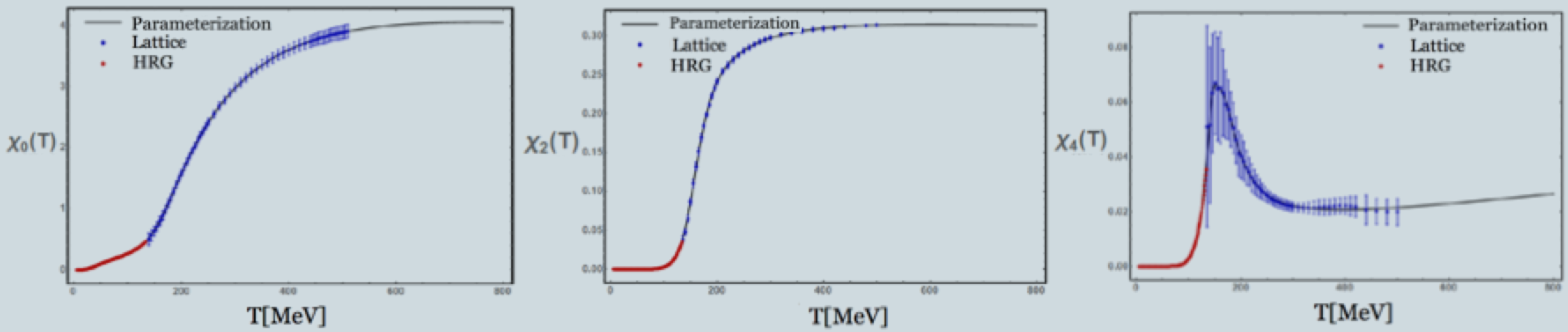
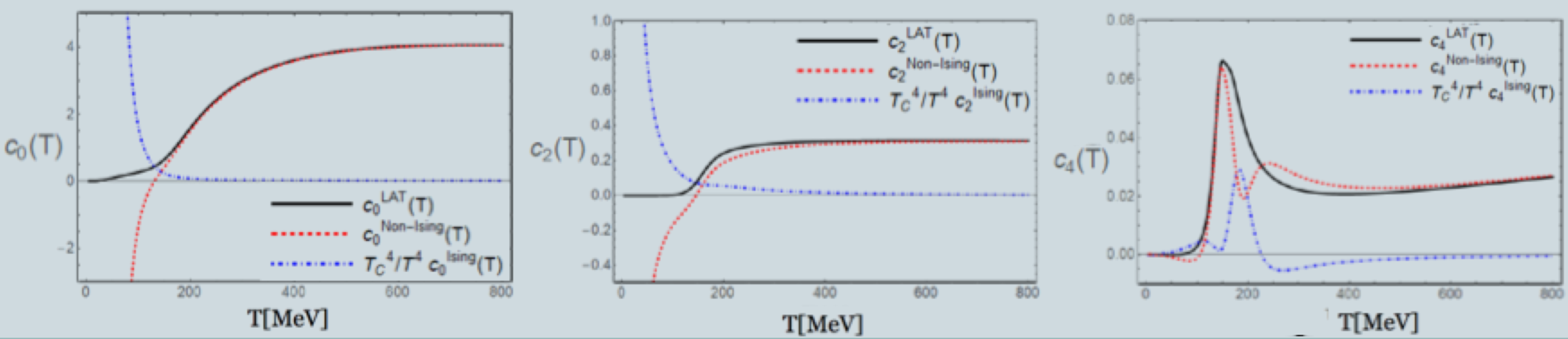


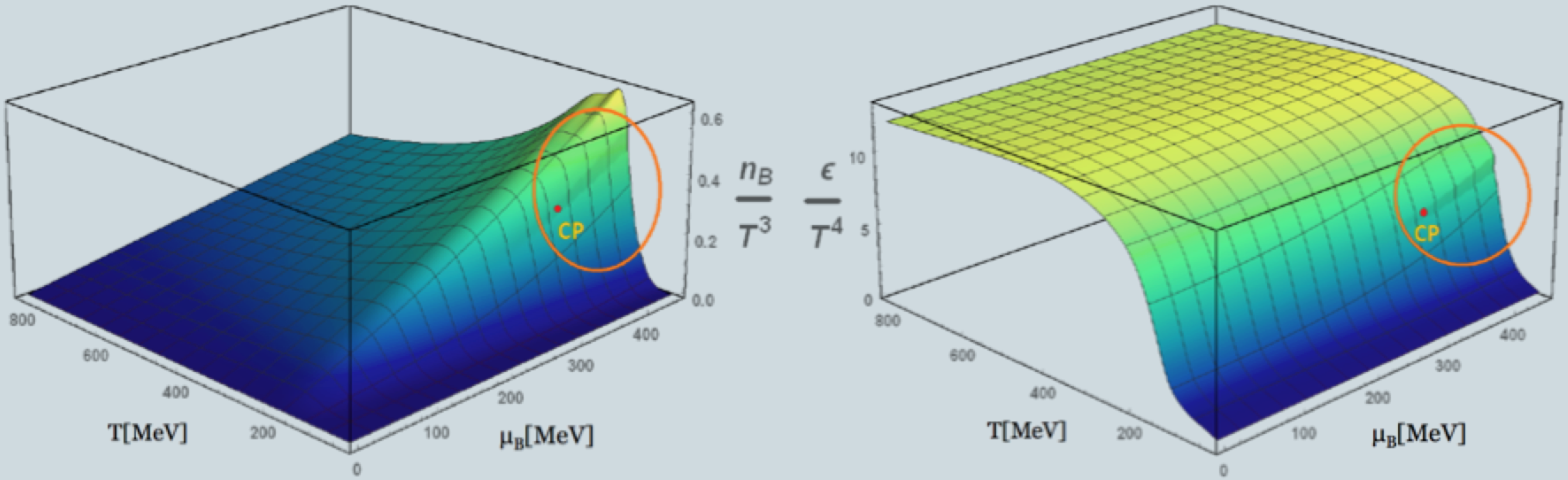
Extract the "regular" contribution as the difference between the lattice and Ising ones

$$T^4 c_n^{\rm LAT}(T) = T^4 c_n^{\rm Non\text{-}Ising}(T) + T_C^4 c_n^{\rm Ising}(T)$$



QCD EoS with critical point

P. Parotto, C.R. et al., 1805.05249 (2018)



- Thermodynamic consistency constrains the parameter space
- EoS will be used as input in hydro simulations
- Comparison with experimental data will constrain the parameter space too, including the critical point location

QCD phase diagram

TRANSITION TEMPERATURE

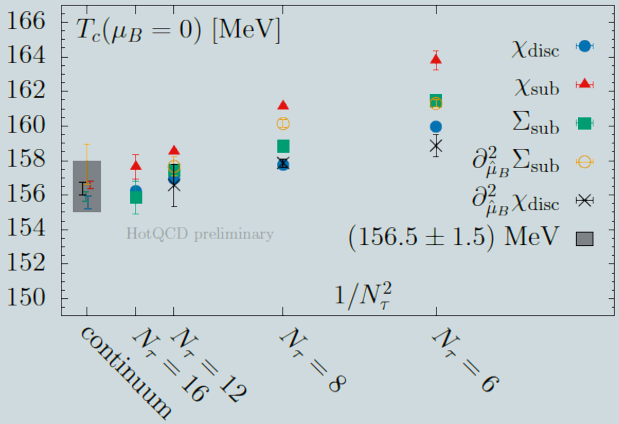
CURVATURE

RADIUS OF CONVERGENCE
OF TAYLOR SERIES

QCD transition temperature and curvature

$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$$

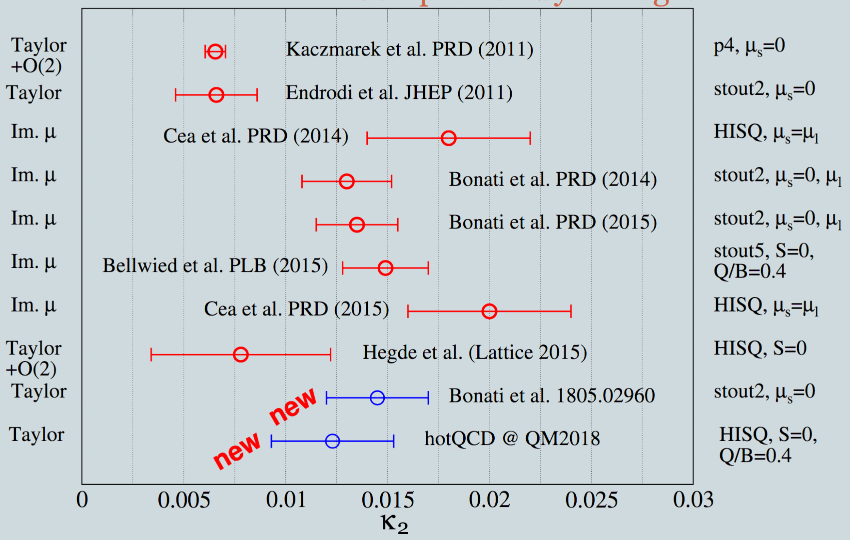
- QCD transition at μ_B=0 is a crossover
  Aoki et al., Nature (2006)
- Latest results on T_0 from HotQCD based on subtracted chiral condensate and chiral susceptibility



P. Steinbrecher for HotQCD, 1807.05607

T_0=156.5±1.5 MeV

Compilation by F. Negro

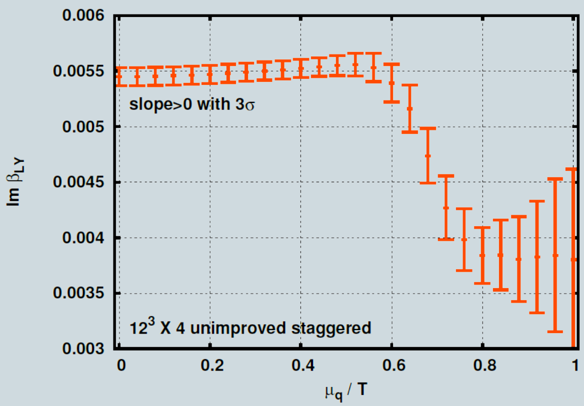


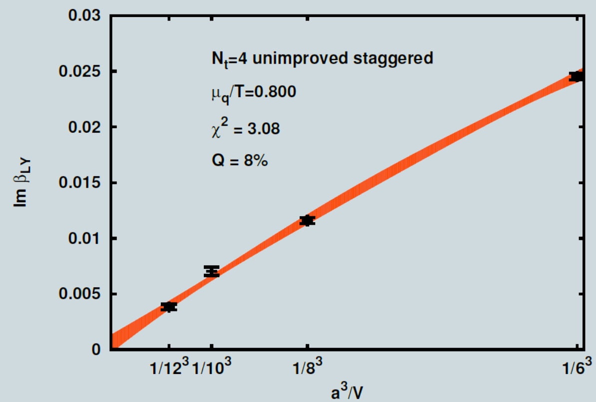
- Curvature very small at μ_B=0
- New results from HotQCD and from Bonati et al. agree with previous findings

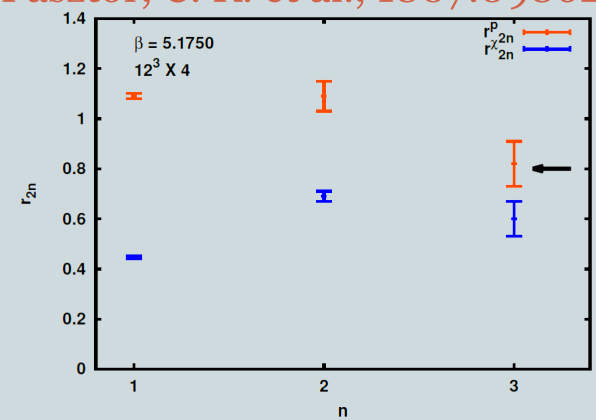
Radius of convergence of Taylor series

- For a genuine phase transition, we expect the ∞-volume limit of the Lee-Yang zero to be real

A. Pasztor, C. R. et al., 1807.09862

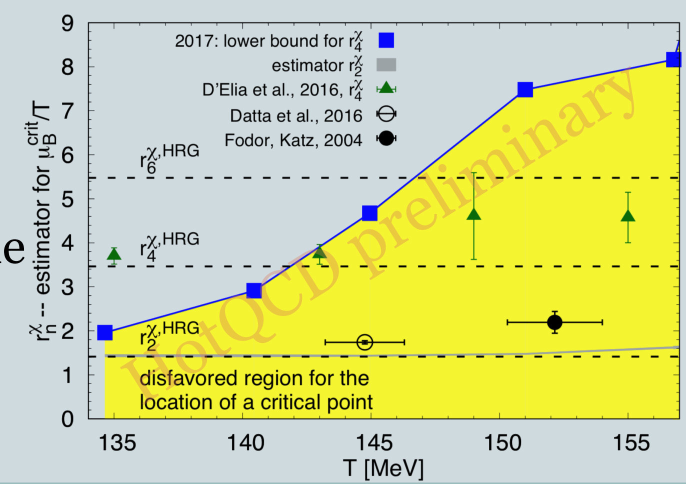


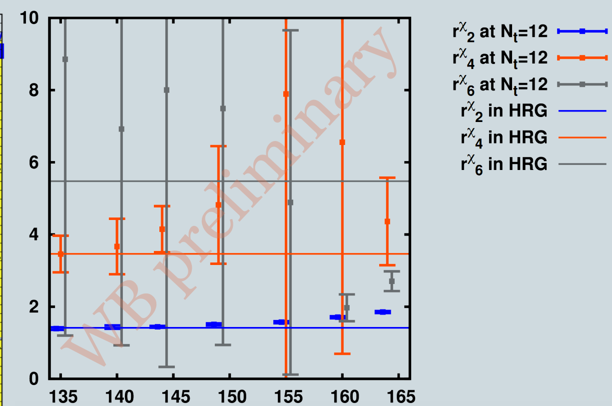




$$r^\chi_{2n} = \left| \frac{2n(2n-1)\chi^B_{2n}}{\chi^B_{2n+2}} \right|^{1/2}$$

- It grows as ~n in the HRG model



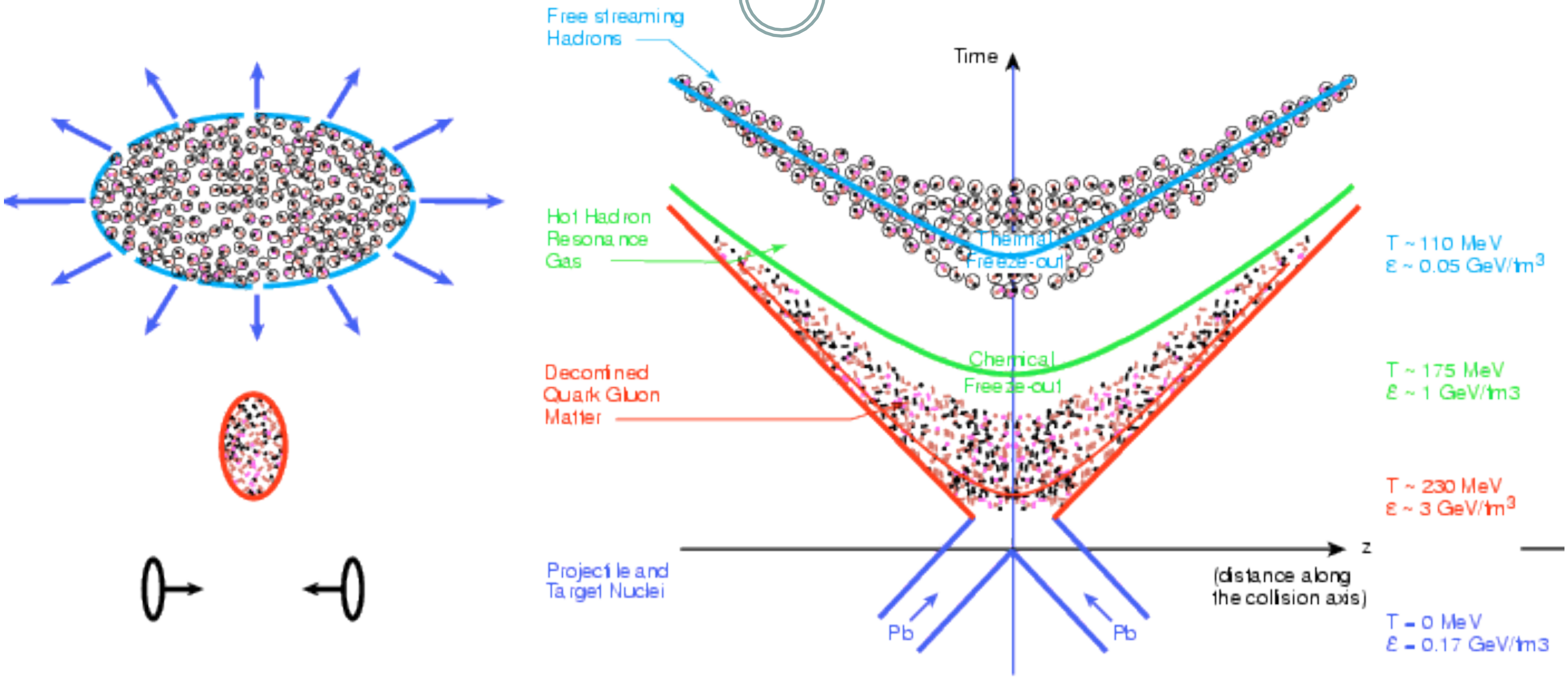


Fluctuations of conserved charges

COMPARISON TO EXPERIMENT: CHEMICAL FREEZE-OUT PARAMETERS

ADS/CFT-BASED APPROACH: SEARCH FOR THE CRITICAL POINT

Evolution of a heavy-ion collision



- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

Fluctuations on the lattice

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

- Definition: $$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- They can be calculated on the lattice and compared to experiment

- variance: $\sigma^2 = \chi_2$ Skewness: $S = \chi_3/(\chi_2)^{3/2}$ Kurtosis: $\kappa = \chi_4/(\chi_2)^2$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

V. Skokov et al., PRC (2013), P. Braun-Munzinger et al., NPA (2017), V. Begun and M. Mackowiak-Pawlowska (2017)

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution

A.Bzdak,V.Koch, PRC (2012)

- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance

V. Koch, S. Jeon, PRL (2000)

- Baryon number conservation
  - Experimental data need to be corrected for this effect

P. Braun-Munzinger et al., NPA (2017)

- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

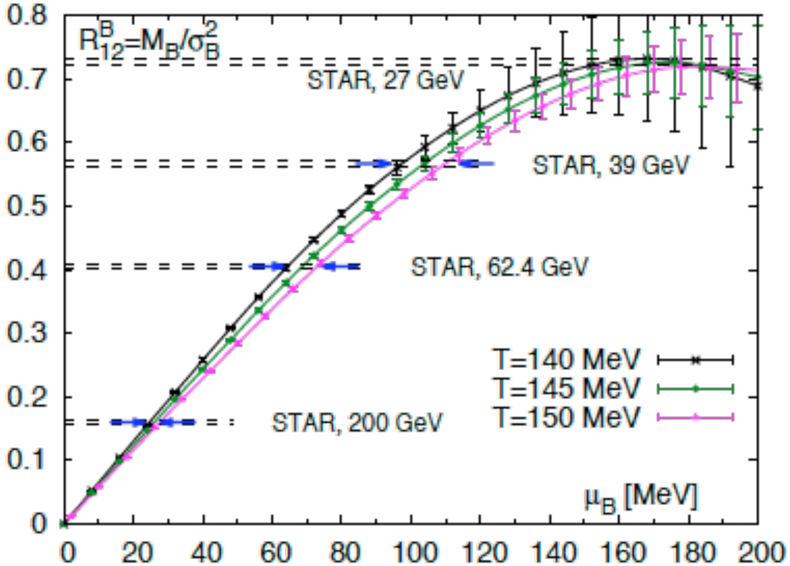
M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

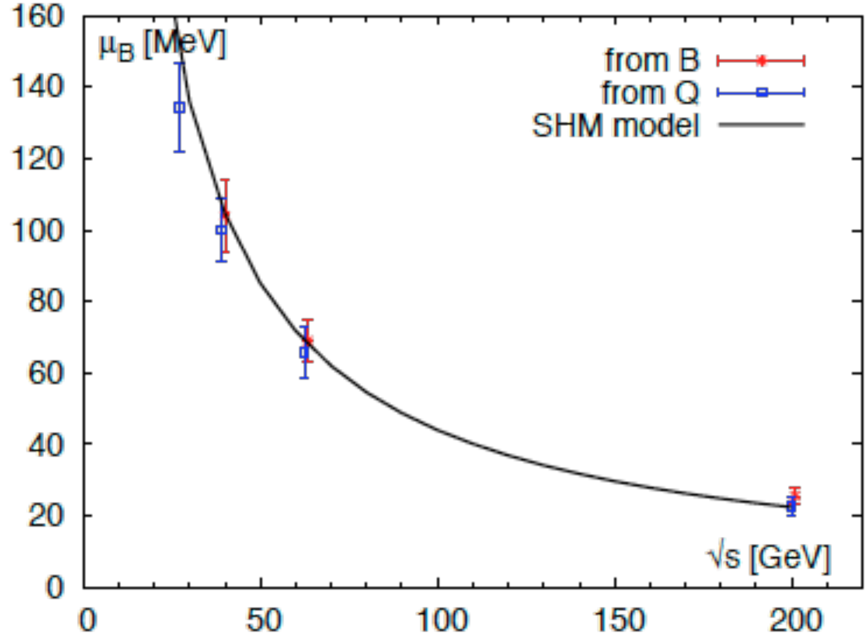
- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

J.Steinheimer et al., PRL (2013)

Consistency between freeze-out of B and Q

- Independent fit of of R_{12}^Q and R_{12}^B: consistency between freeze-out chemical potentials





$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

WB: PRL (2014)
STAR collaboration, PRL (2014)

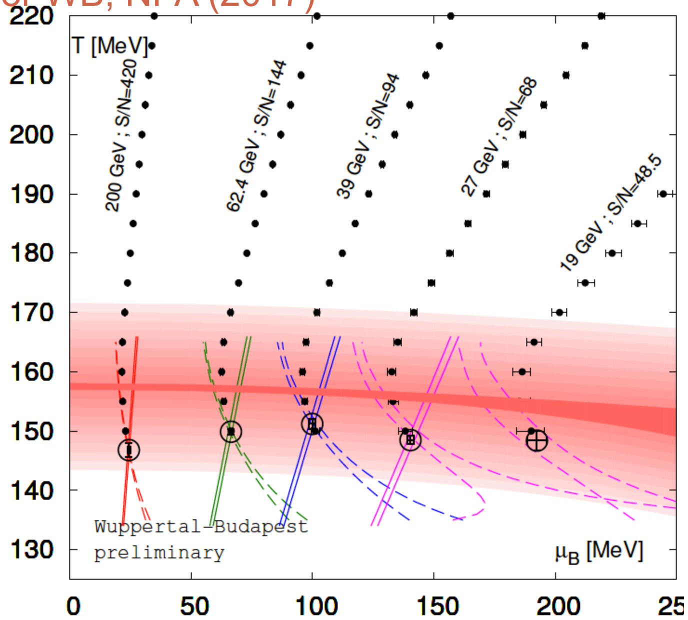
Freeze-out line from first principles

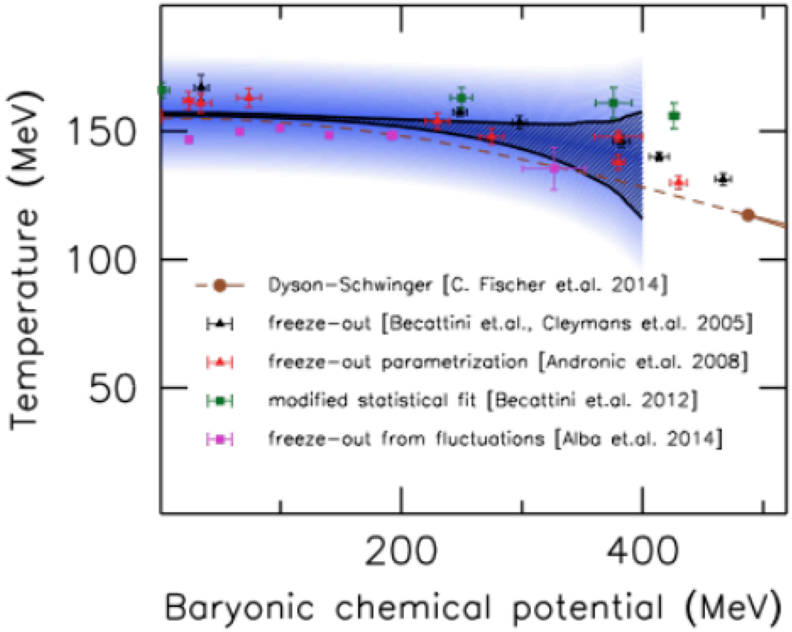
- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^{Q}(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

$$R_{12}^{B}(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

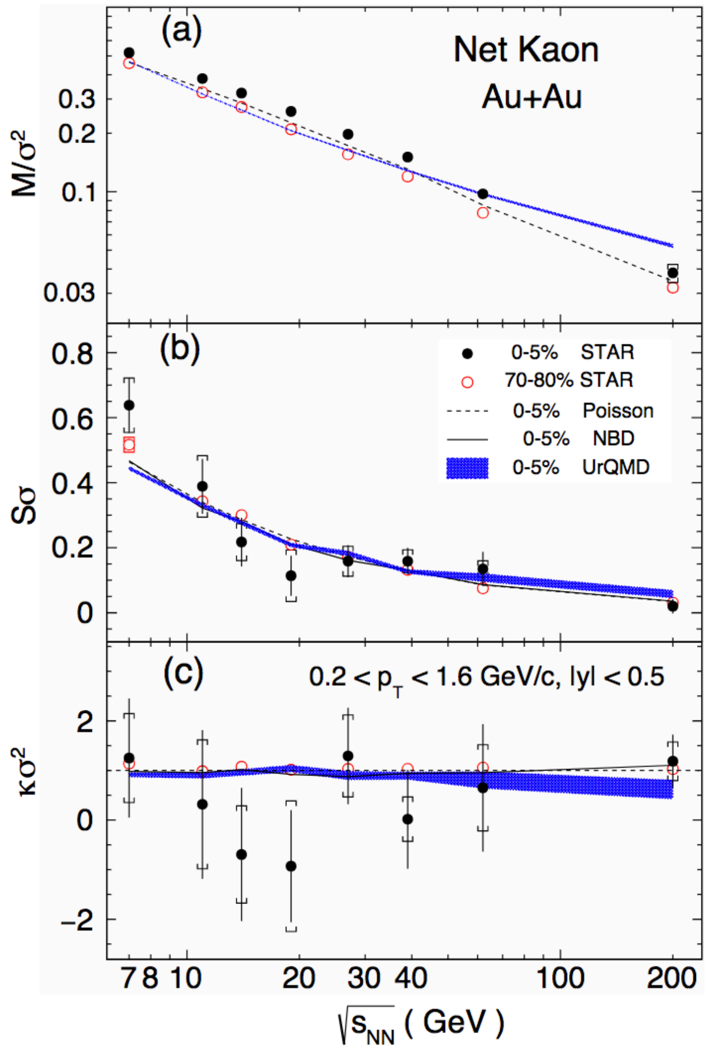
C. Ratti for WB, NPA (2017)





Freeze-out of kaons in the HRG model

STAR Collaboration 1709.00773

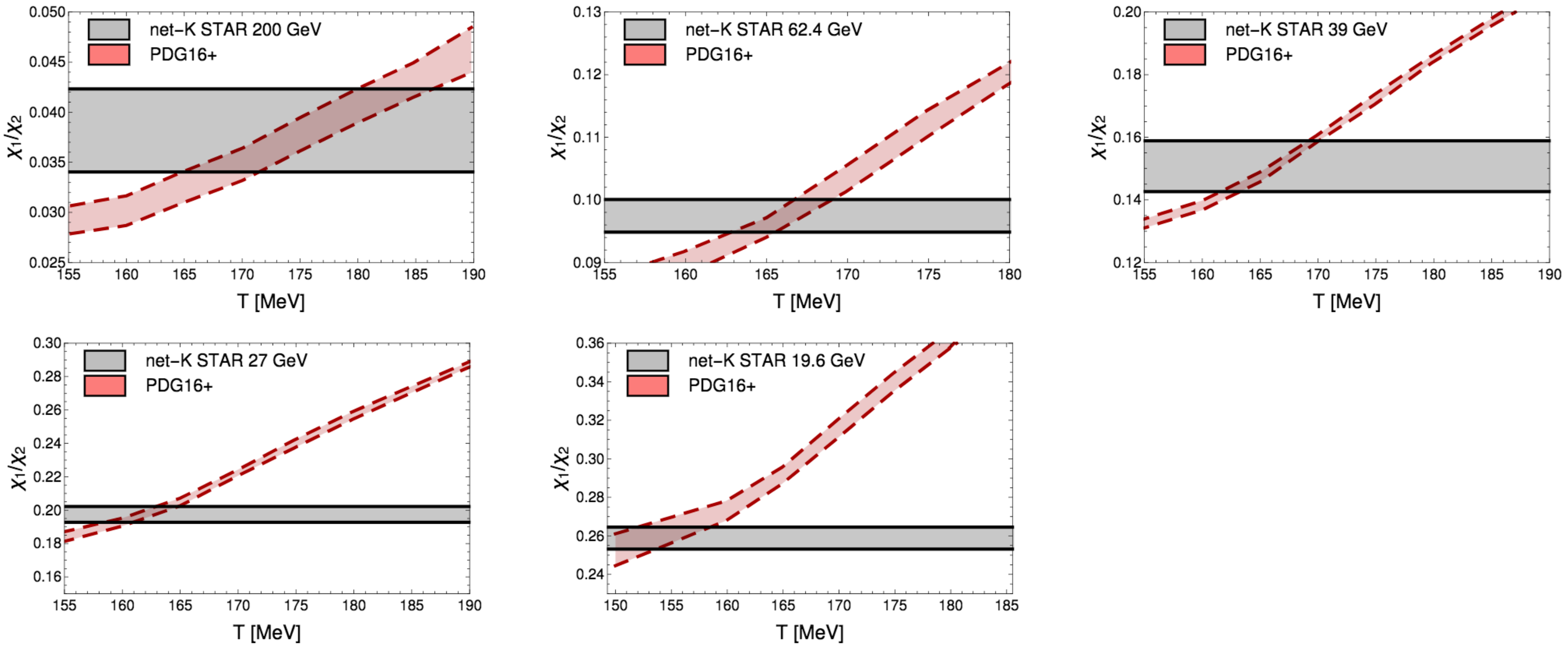


- Calculate χ_1/χ_2 for kaons in the HRG model, including resonance decays and acceptance cuts
- Calculate it along the isentropes
- Fit χ_1/χ_2 and extract T_{fo}
- Obtain μ_{Bfo} from the isentropes

R. Bellwied, C. R. et al., 1805.00088

Freeze-out of kaons in the HRG model

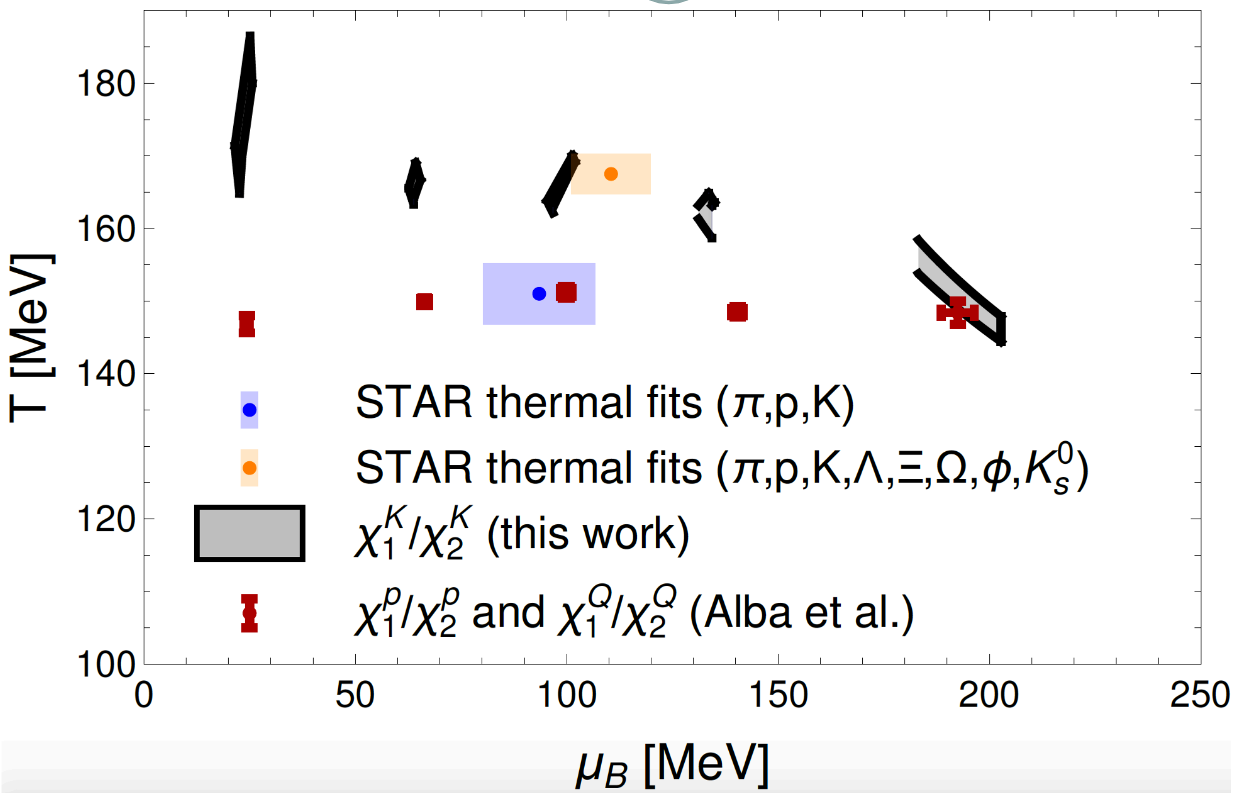
R. Bellwied, C. R. et al., 1805.00088



- χ_1/χ_2 for kaons needs a higher freeze-out temperature than net-p/net-Q

Freeze-out of kaons in the HRG model

R. Bellwied, C. R. et al., 1805.00088



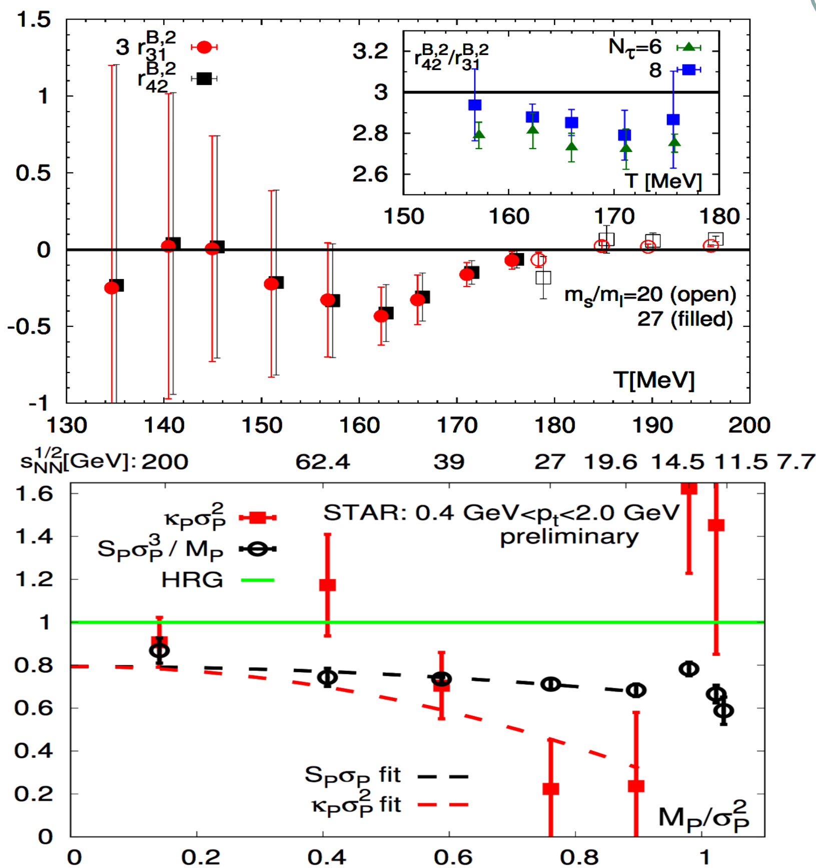
- χ_1/χ_2 for kaons needs a higher freeze-out temperature than net-p/net-Q
- The f.o. parameters agree with the STAR fit of yields (including strange particles)

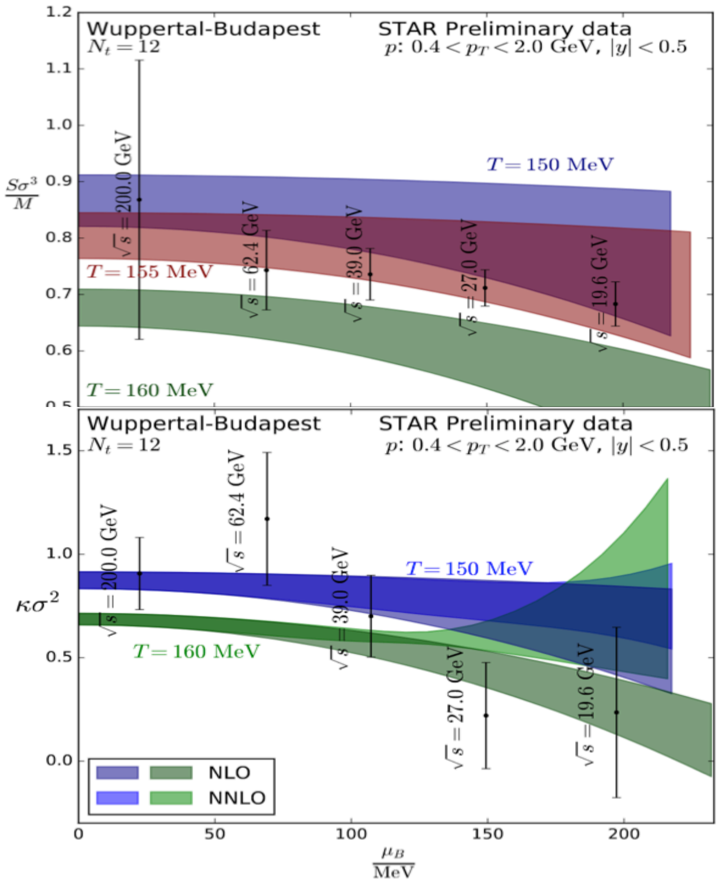
STAR Collaboration, PRC (2017)

Higher order fluctuations

HotQCD, PRD (2017)

WB, JHEP (2018)





$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = \frac{\chi_4^B + s_1\chi_{31}^{BS} + q_1\chi_{31}^{BQ}}{\chi_2^B + s_1\chi_{11}^{BS} + q_1\chi_{11}^{BQ}} + \mathcal{O}(\mu_B^2) \equiv r_{31}^{B,0} + r_{31}^{B,2}\hat{\mu}_B^2 + \mathcal{O}(\mu_B^4)$$

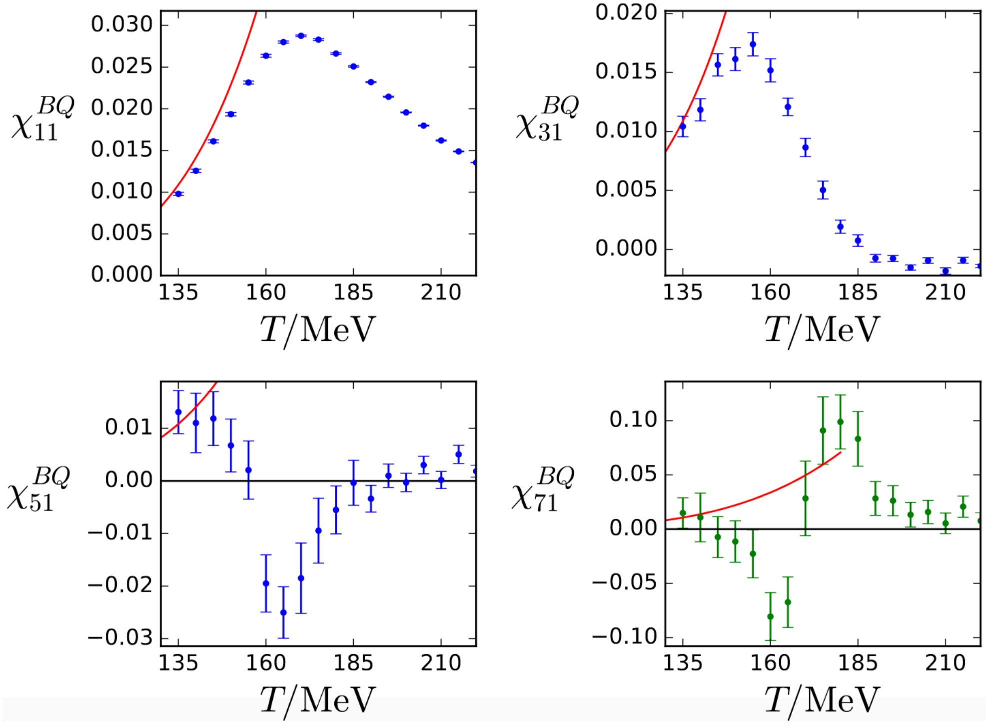
$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^2) \equiv r_{42}^{B,0} + r_{42}^{B,2}\hat{\mu}_B^2 + \mathcal{O}(\mu_B^4)\,,$$

Off-diagonal correlators

- Simulation of the lower order correlators at imaginary μ_B
- Fit to extract higher order terms
- Results exist also for BS, QS and BQS correlators

WB, JHEP (2018)

Nt=12



$$\chi_{11}^{BS}(\hat\mu_B) = \chi_{11}^{BS} + \frac{1}{2!}\chi_{31}^{BS}\hat\mu_B^2 + \frac{1}{4!}\chi_{51}^{BS}\hat\mu_B^4 + \frac{1}{6!}\chi_{71}^{BS}\hat\mu_B^6 + \frac{1}{8!}\chi_{91}^{BS}\hat\mu_B^8$$

$$\chi_{21}^{BS}(\hat\mu_B) = \chi_{31}^{BS}\hat\mu_B + \frac{1}{3!}\chi_{51}^{BS}\hat\mu_B^3 + \frac{1}{5!}\chi_{71}^{BS}\hat\mu_B^5 + \frac{1}{7!}\chi_{91}^{BS}\hat\mu_B^7$$

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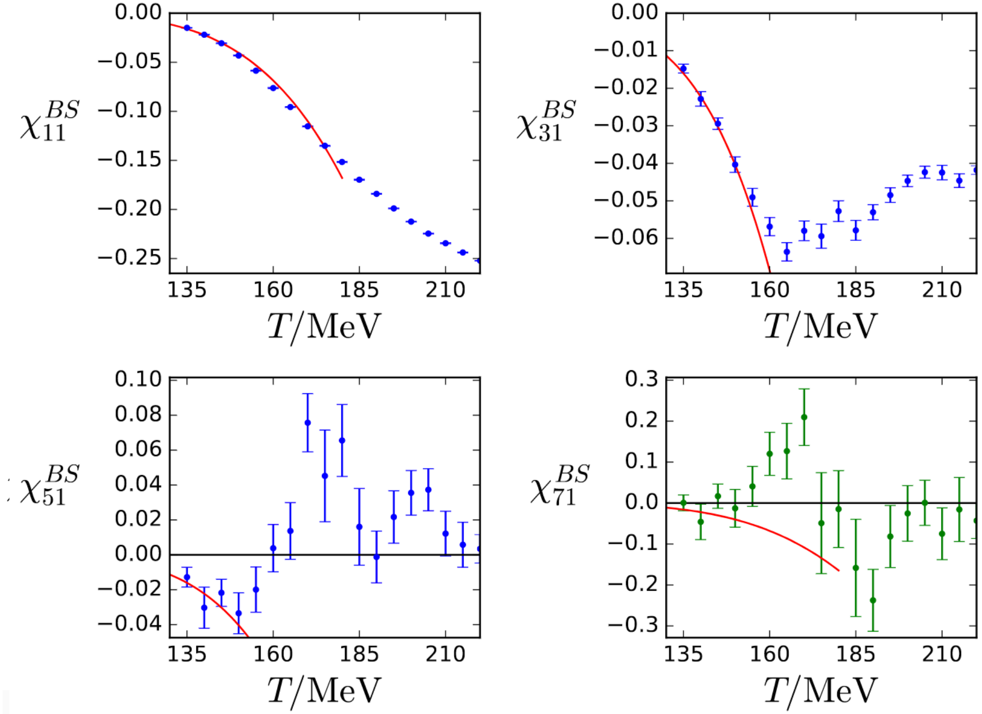

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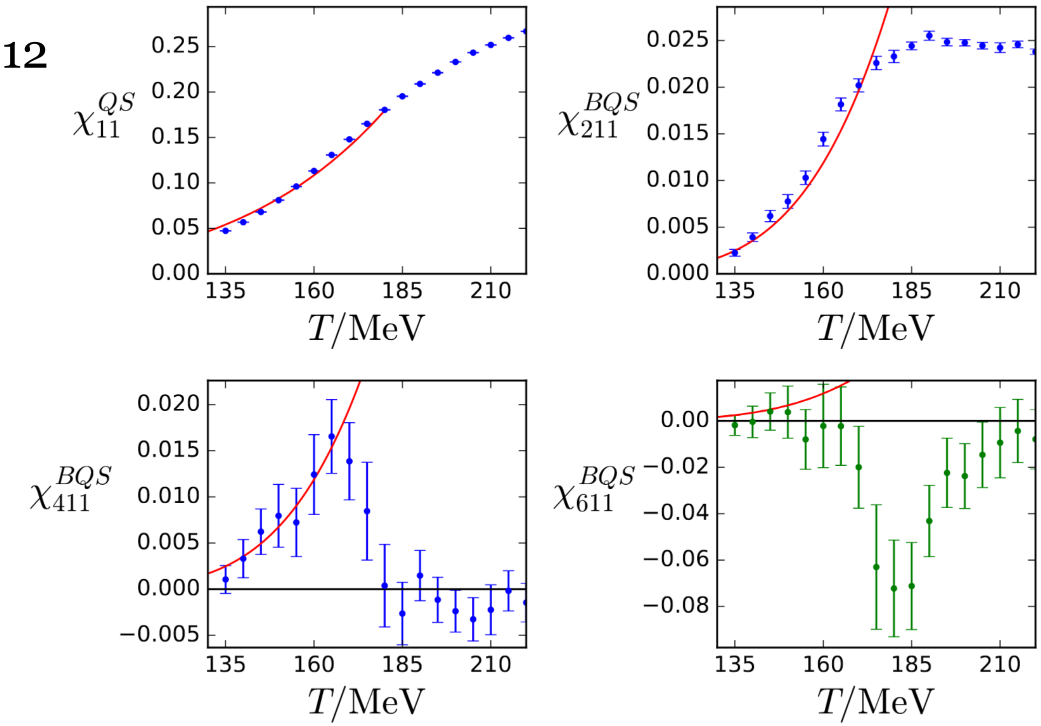

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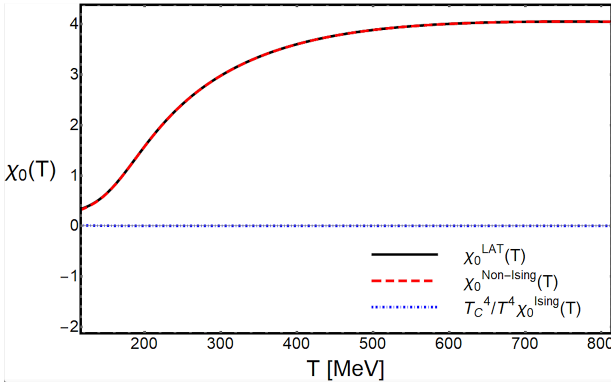
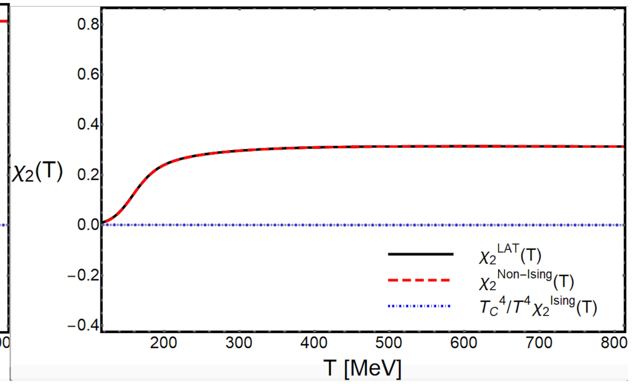
Conclusions

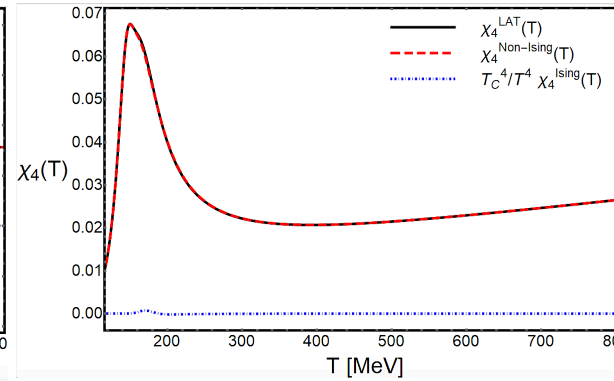
- Need for quantitative results at finite-density to support the experimental programs
  - Equation of state
  - Phase transition line
  - Fluctuations of conserved charges
- Current lattice results for thermodynamics up to $\mu_B/T \leq 2$
- Extensions to higher densities by means of lattice-based models
- No indication of Critical Point from lattice QCD in the explored μ_B range

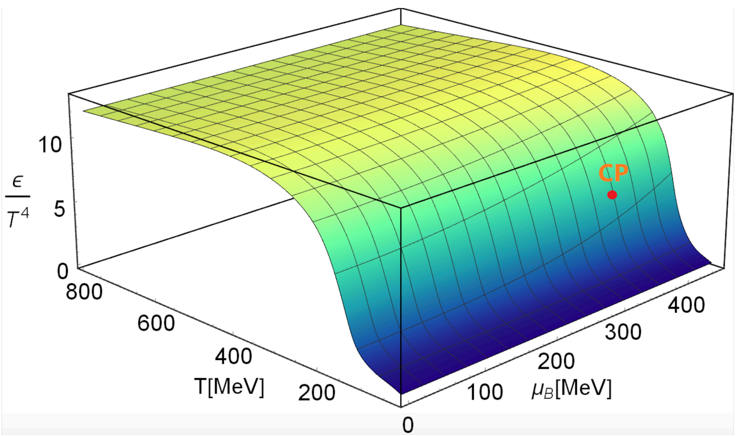
Backup slides

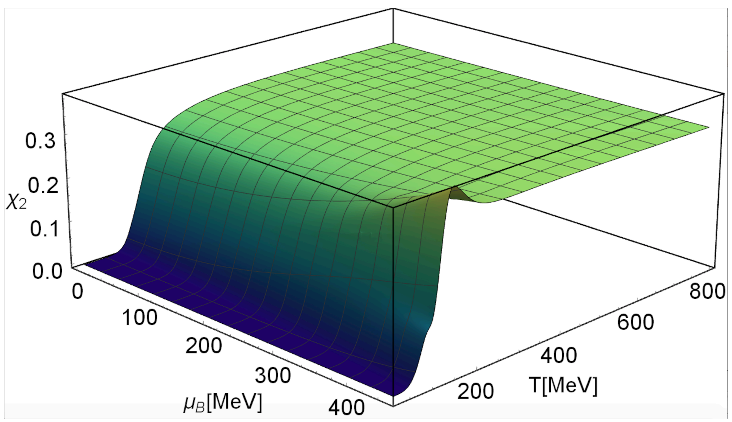
Choosing e.g. w=20, we can minimize the effect due to the critical point





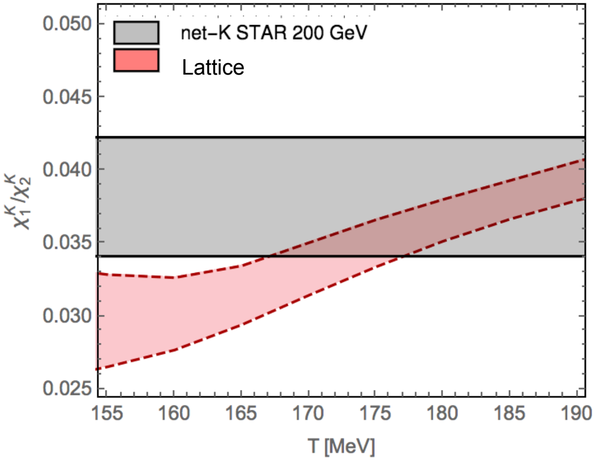


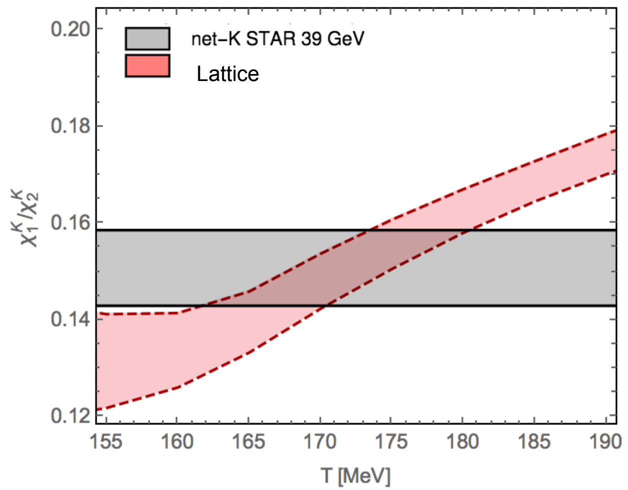


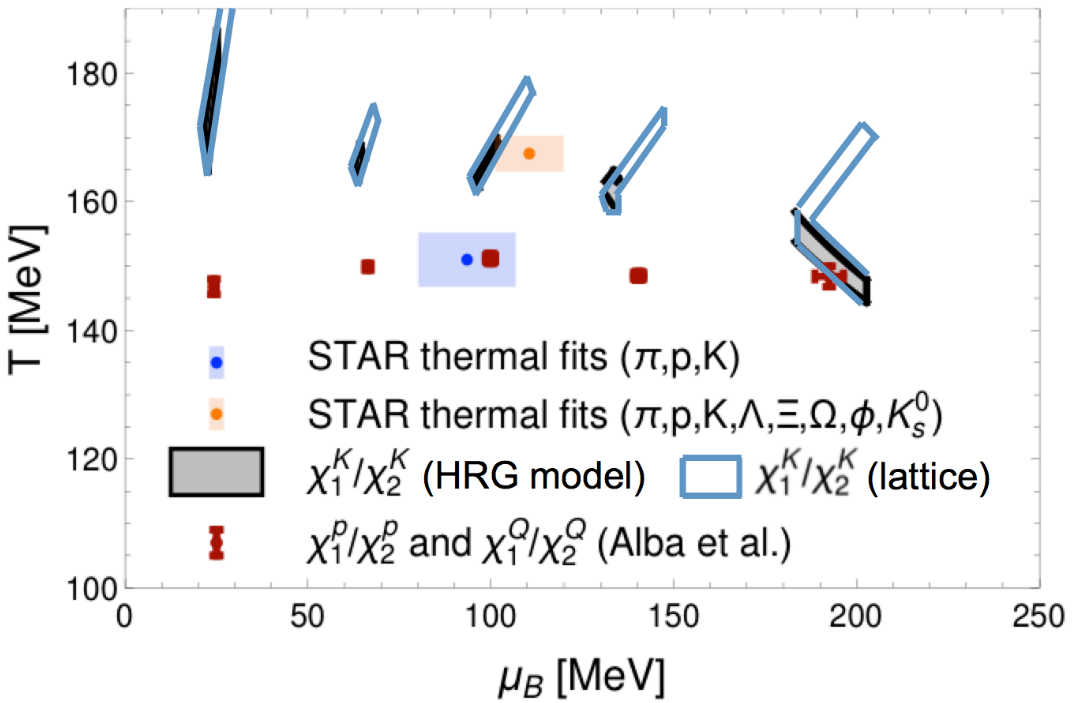


Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al. forthcoming





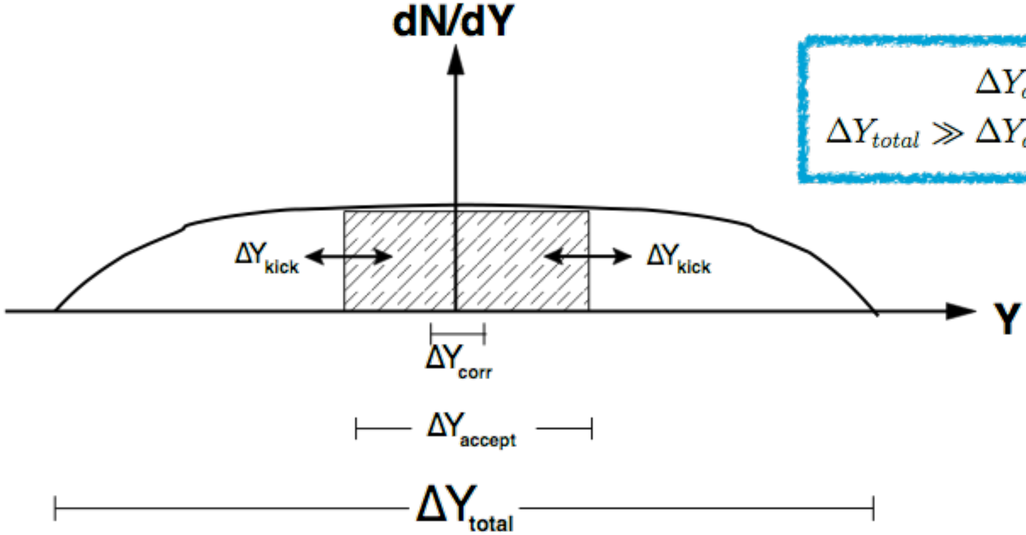


- Lattice QCD temperatures have a large uncertainty but they are above the light flavor ones

Fluctuations of conserved charges?

- If we look at the entire system, none of the conserved charges will fluctuate

- By studying a sufficiently small subsystem, the fluctuations of conserved quantities become meaningful



$$\Delta Y_{accept} \gg \Delta Y_{corr}$$
$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

- ❑ ΔYtotal: range for total charge multiplicity distribution
- ❑ ΔYaccept: interval for the accepted charged particles
- ❑ ΔYkick: rapidity shift that charges receive during and after hadronization

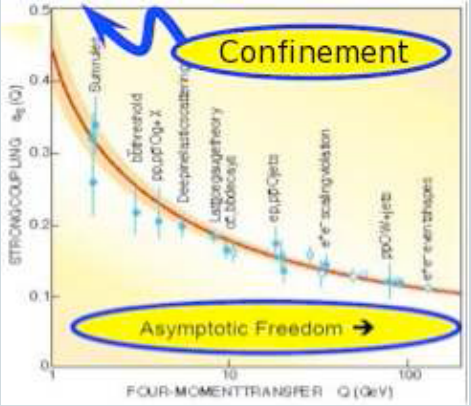
QCD matter under extreme conditions

To address these questions we need fundamental theory and experiment

Theory: Quantum Chromodynamics

- QCD is the fundamental theory of strong interactions
- It describes interactions among quarks and gluons

$$L_{QCD} = \sum_{i=1}^{n_f} \overline{\psi}_i \gamma_\mu \left(i\partial^\mu - g A_a^\mu \frac{\lambda_a}{2} \right) \psi_i - m_i \overline{\psi}_i \psi_i - \frac{1}{4} \sum_a F_a^{\mu\nu} F_a^{\mu\nu}$$



Experiment: heavy-ion collisions

- Quark-gluon plasma (QGP) discovery at RHIC and the LHC
- QGP is a strongly interacting (almost) perfect fluid

Cumulants of multiplicity distribution

- Deviation of N_Q from its mean in a single event: $\delta N_Q = N_Q - \langle N_Q \rangle$
- The cumulants of the event-by-event distribution of NQ are:

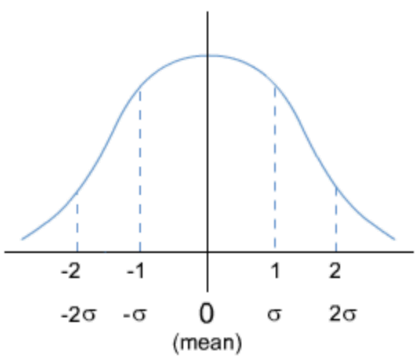
$$\chi_2 = \langle(\delta NQ)^2\rangle \qquad \chi_3 = \langle(\delta NQ)^3\rangle \qquad \chi_4 = \langle(\delta NQ)^4\rangle - 3\langle(\delta NQ)^2\rangle^2$$

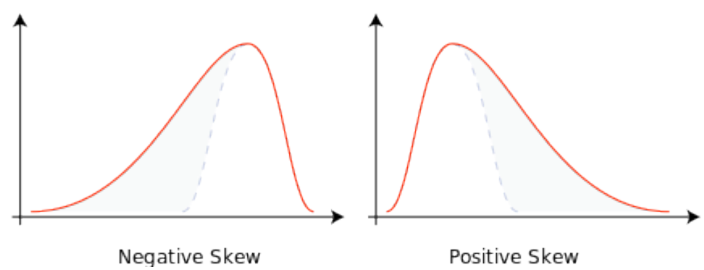
- The cumulants are related to the central moments of the distribution by:

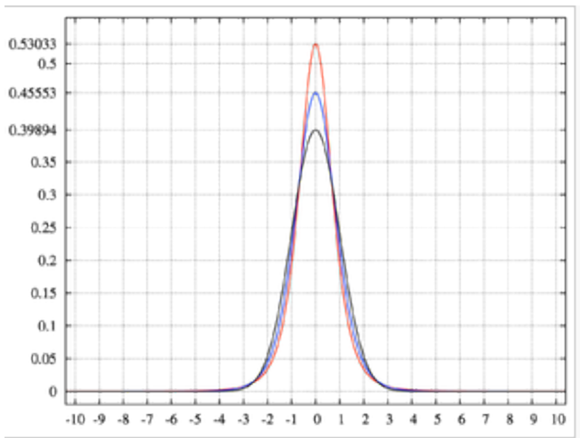
variance: $\sigma^2 = \chi_2$

Skewness: $S = \chi_3/(\chi_2)^{3/2}$

Kurtosis: $\kappa = \chi_4/(\chi_2)^2$





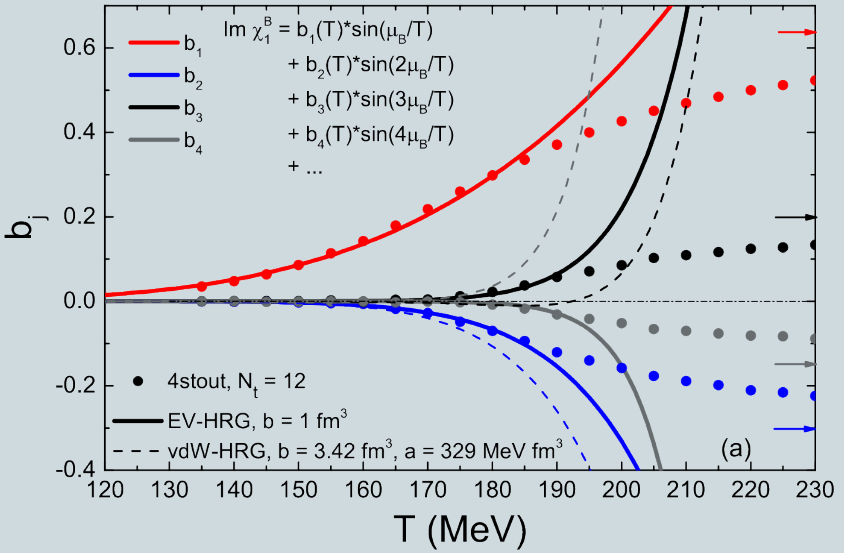


Fluctuations and hadrochemistry

$$\chi_1^B(T,\mu_B) = \frac{\rho_B(T,\mu_B)}{T^3} = \sum_{k=1}^{\infty} b_k(T)\sinh(k\,\mu_B/T)$$

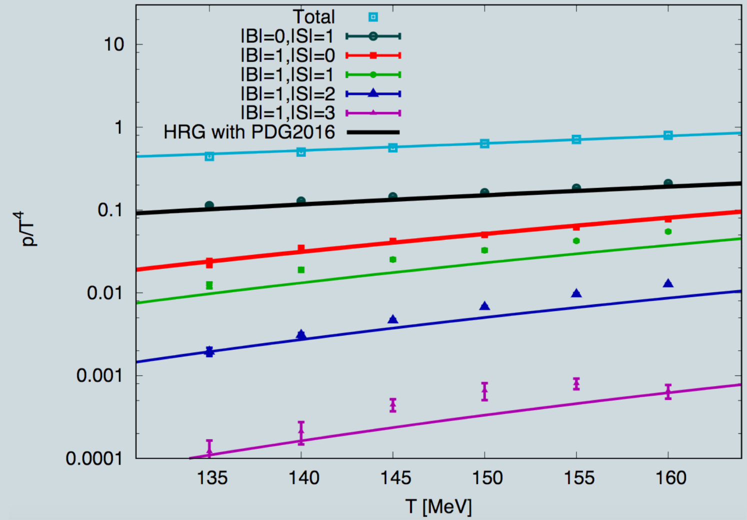
$$P(\hat\mu_B,\hat\mu_S) = P_{00}^{BS} + P_{10}^{BS}\cosh(\hat\mu_B) + P_{01}^{BS}\cosh(\hat\mu_S) + P_{11}^{BS}\cosh(\hat\mu_B-\hat\mu_S) + P_{12}^{BS}\cosh(\hat\mu_B-2\hat\mu_S) + P_{13}^{BS}\cosh(\hat\mu_B-3\hat\mu_S)$$

V. Vovchenko et al., PLB (2017)



- Consistent with HRG at low temperatures
- Consistent with approach to ideal gas limit
- b_2 departs from zero at T~160 MeV
- Deviation from ideal HRG

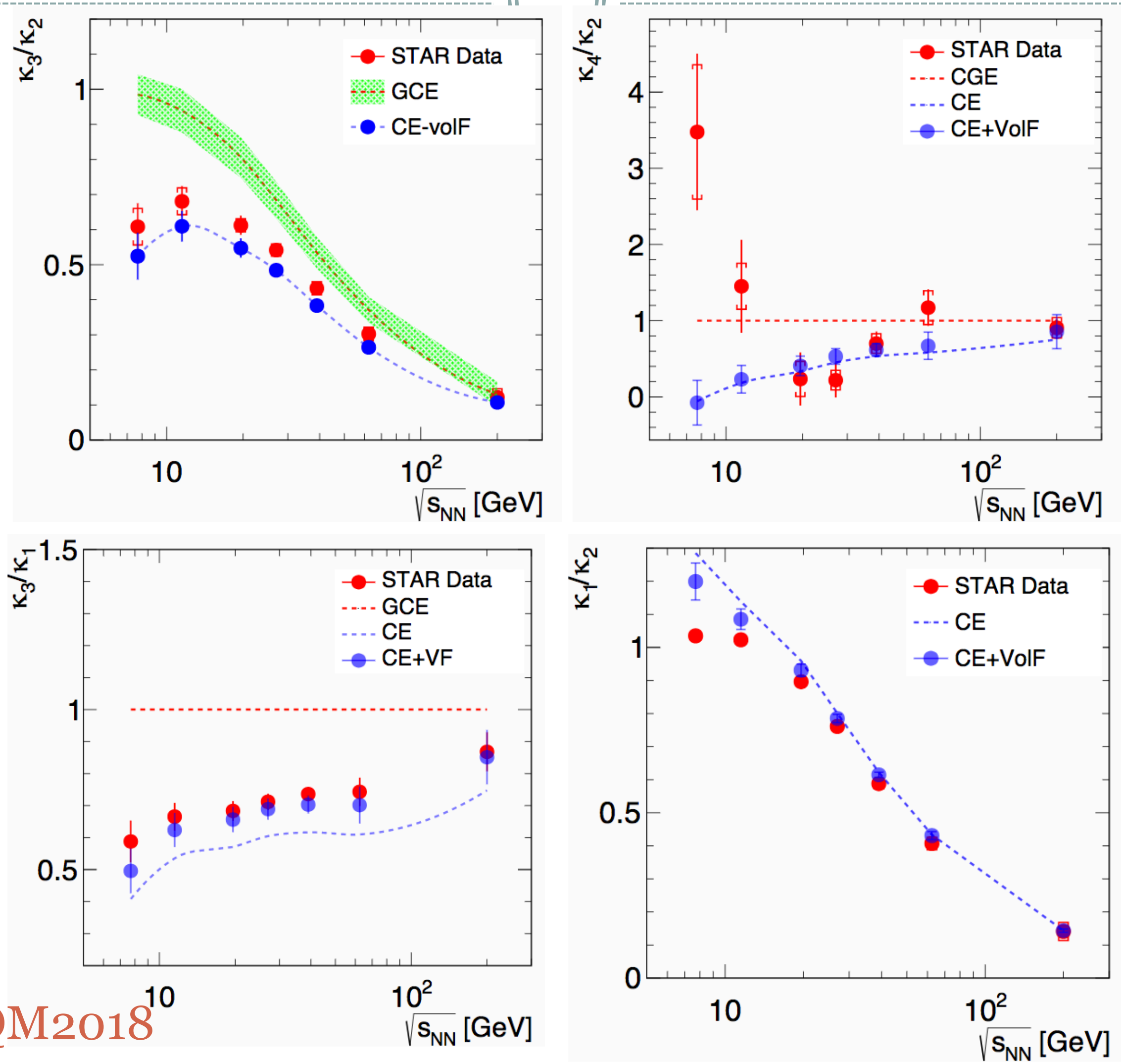
P. Alba et al., PRD (2017)



- Need of additional strange hadrons, predicted by the Quark Model but not yet detected
- First pointed out in Bazavov et al., PRL(2014)

(see talk by J. Glesaaen on Friday)

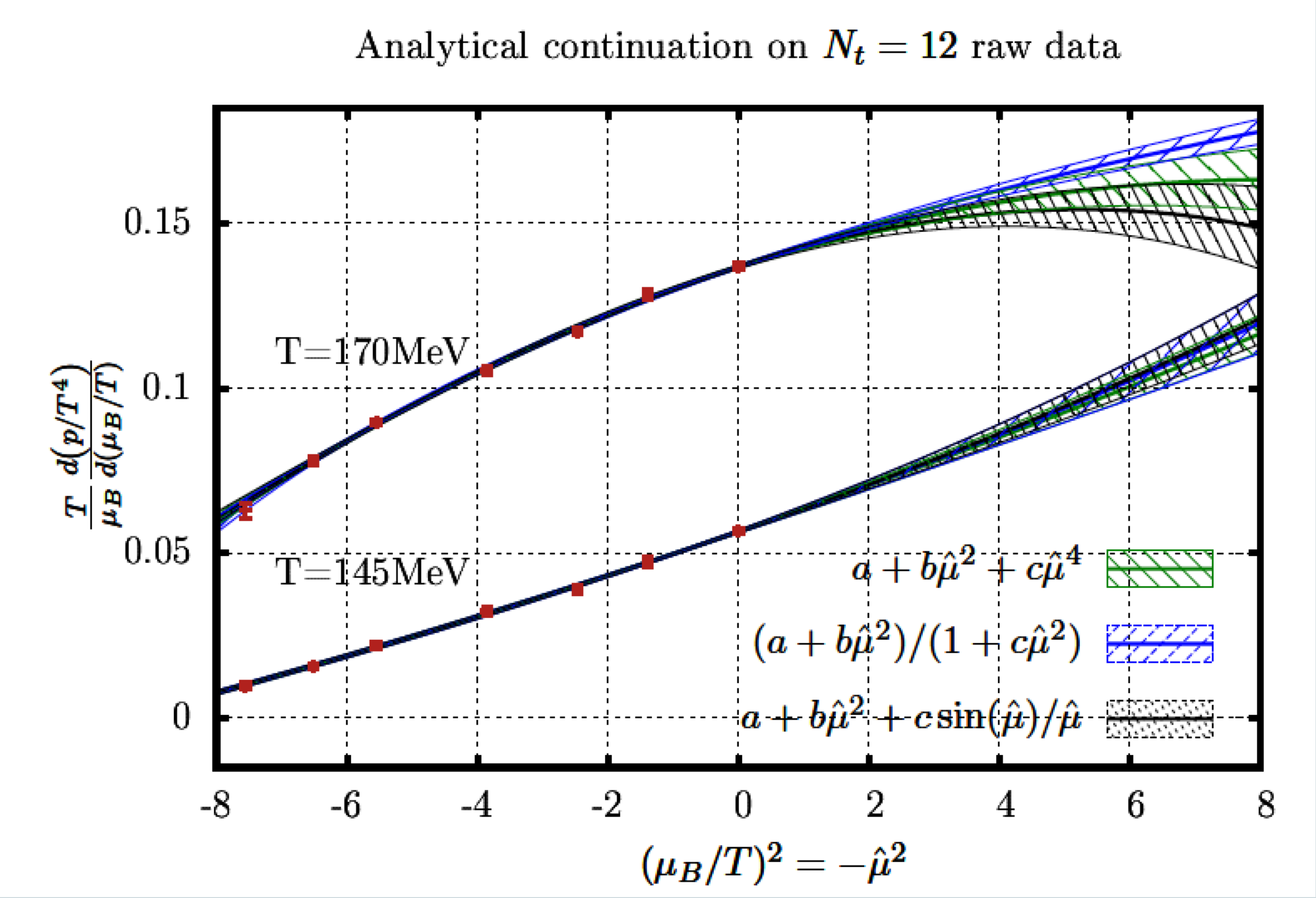
Canonical suppression



A. Rustamov @QM2018

above 11.5 GeV CE suppression accounts for measured deviations from GCE

Analytical continuation – illustration of systematics



Analytical continuation – illustration of systematics

Condition: $\chi_8 \lesssim \chi_4 \longrightarrow f(\hat{\mu}_B) = a + b\hat{\mu}_B^2 + c\hat{\mu}_B^4 + \frac{b\epsilon}{840}\hat{\mu}_B^6$