Exploring the QCD phase diagram with Fluctuations and correlations

A. Bzdak, VK, N. Strodthoff: arXiv:1607.07375

A. Bzdak, VK, V. Skokov: arXiv:1612.05128

A. Bzdak, VK: arXiv:1707.02640

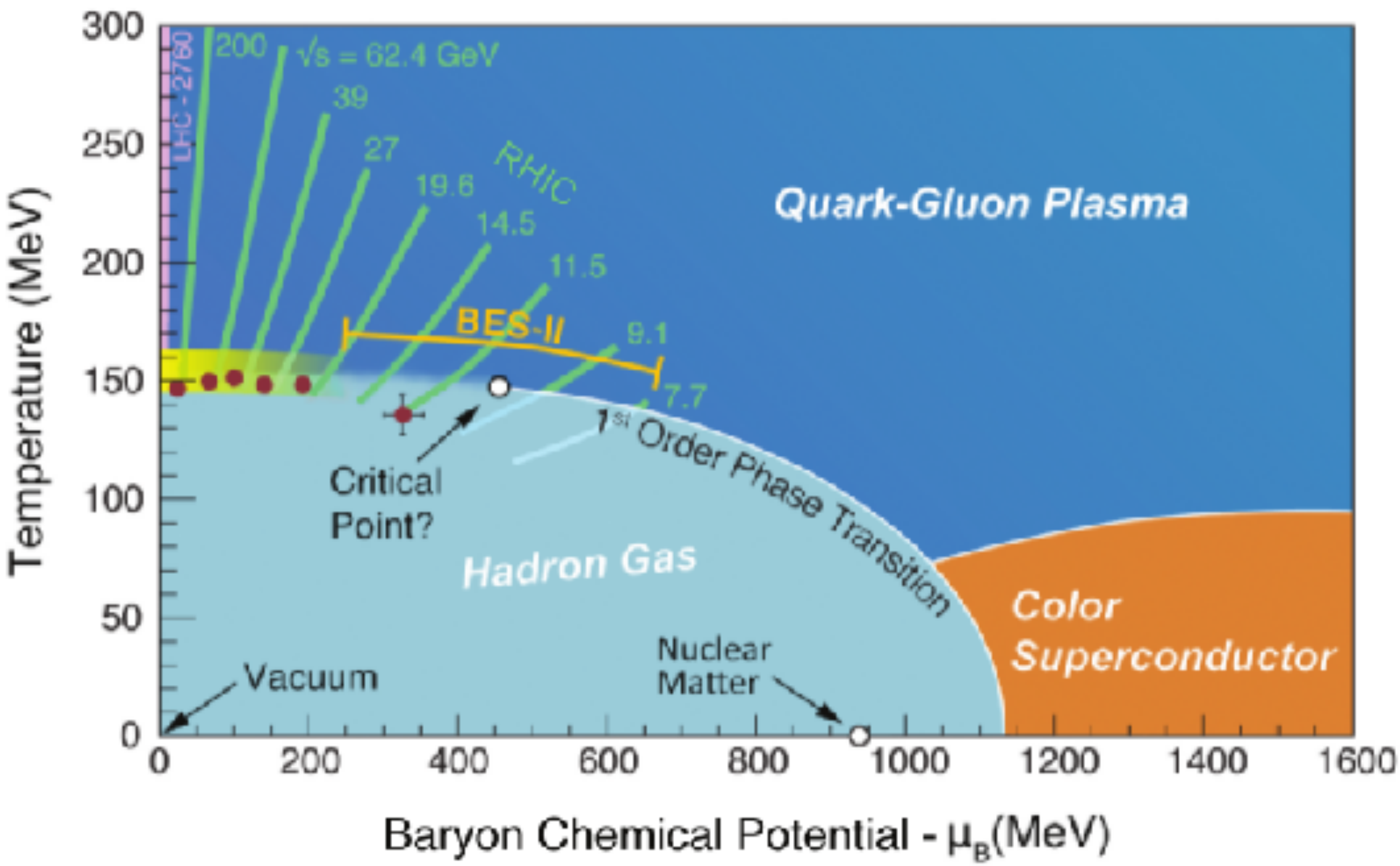
A. Bzdak, D. Oliinychenko, J. Steinheimer, VK: arXiv:1804.04463

A. Bzdak, VK: arXiv:1810.01913

A. Bzdak and V.K: arXiv:1811.04456

BEST COLLABORATION

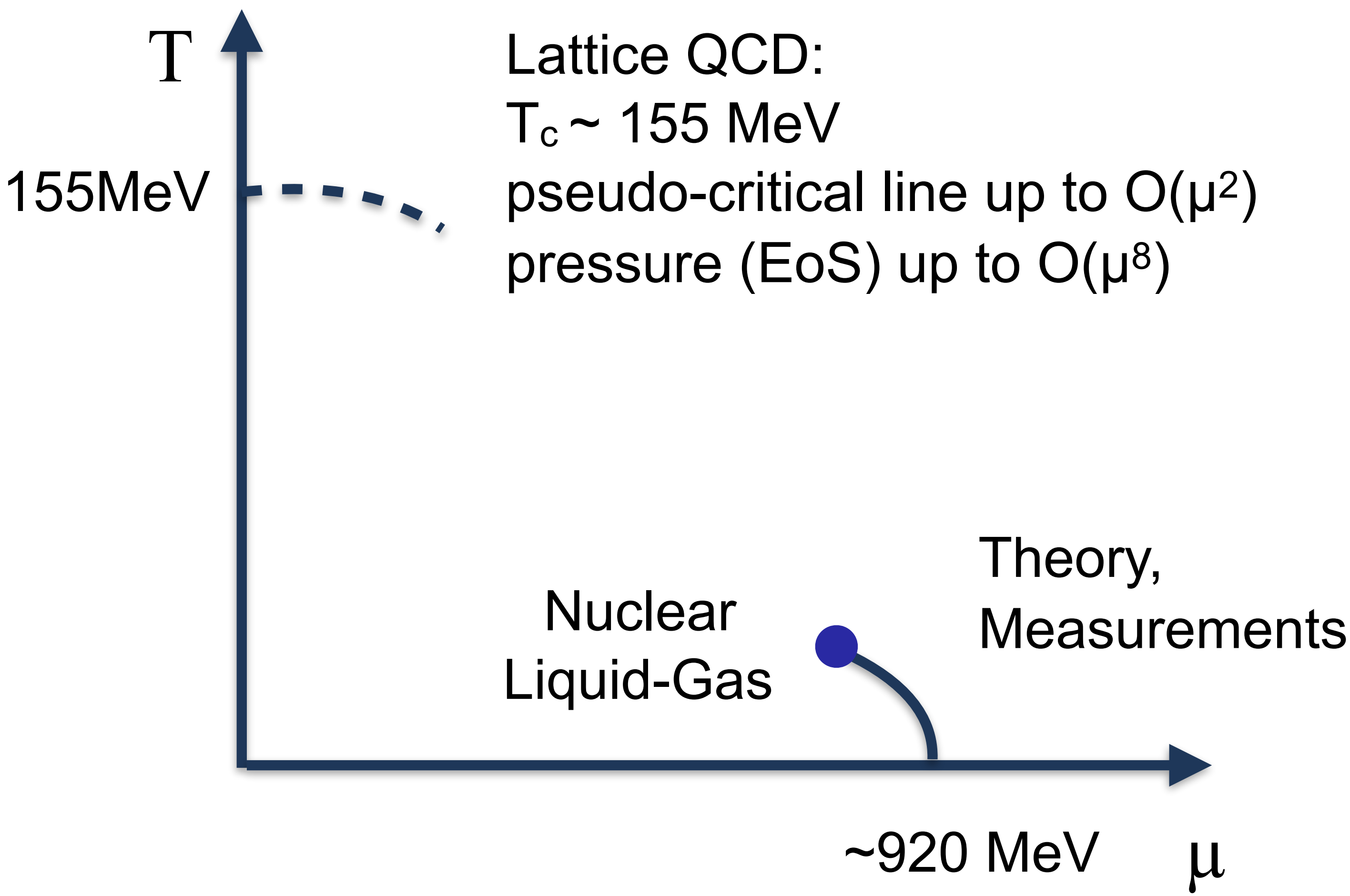
The phase diagram



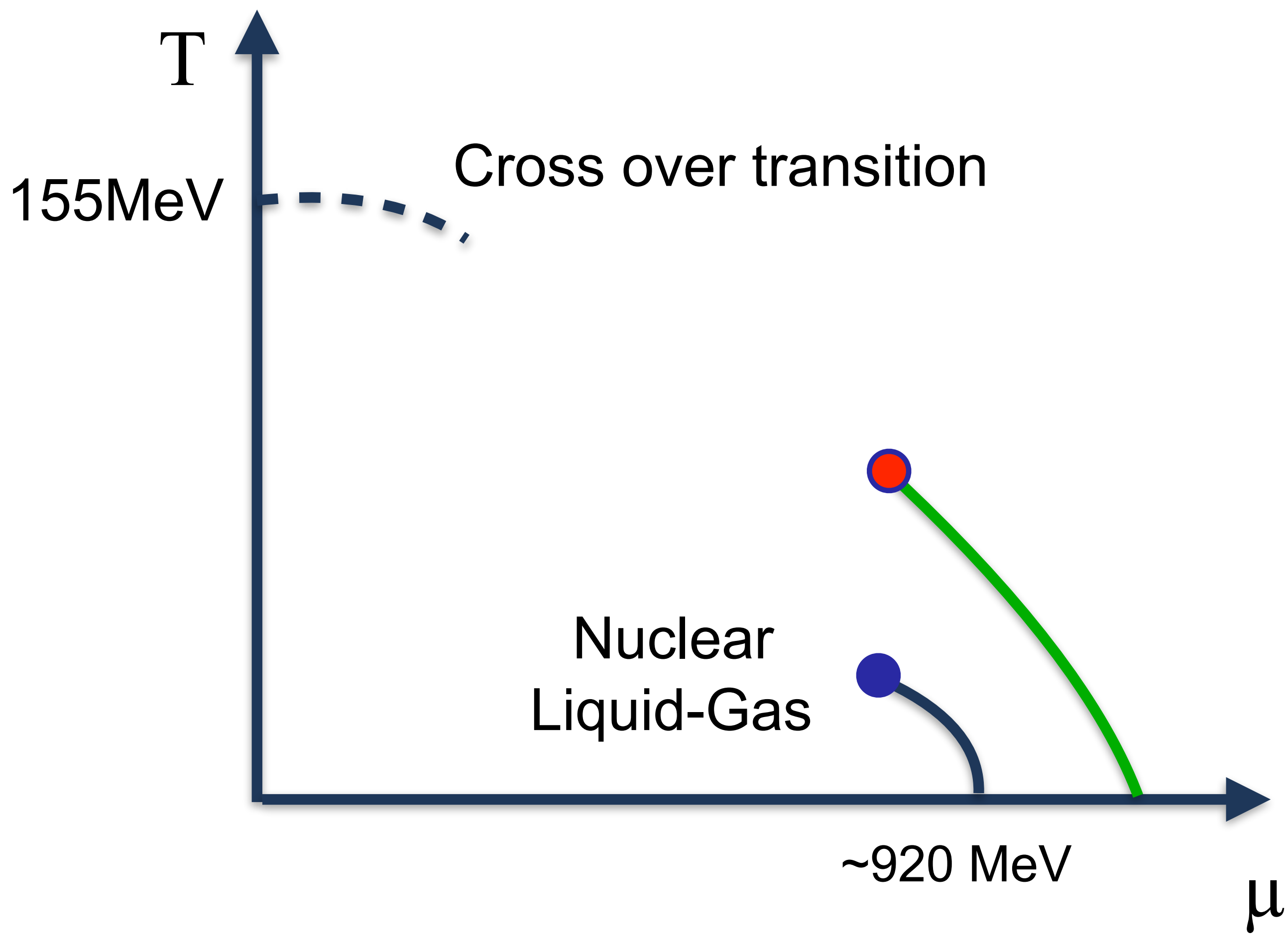
Increase chemical potential by lowering the beam energy

In reality, we add baryons (nucleons) from target and projectile to mid-rapidity

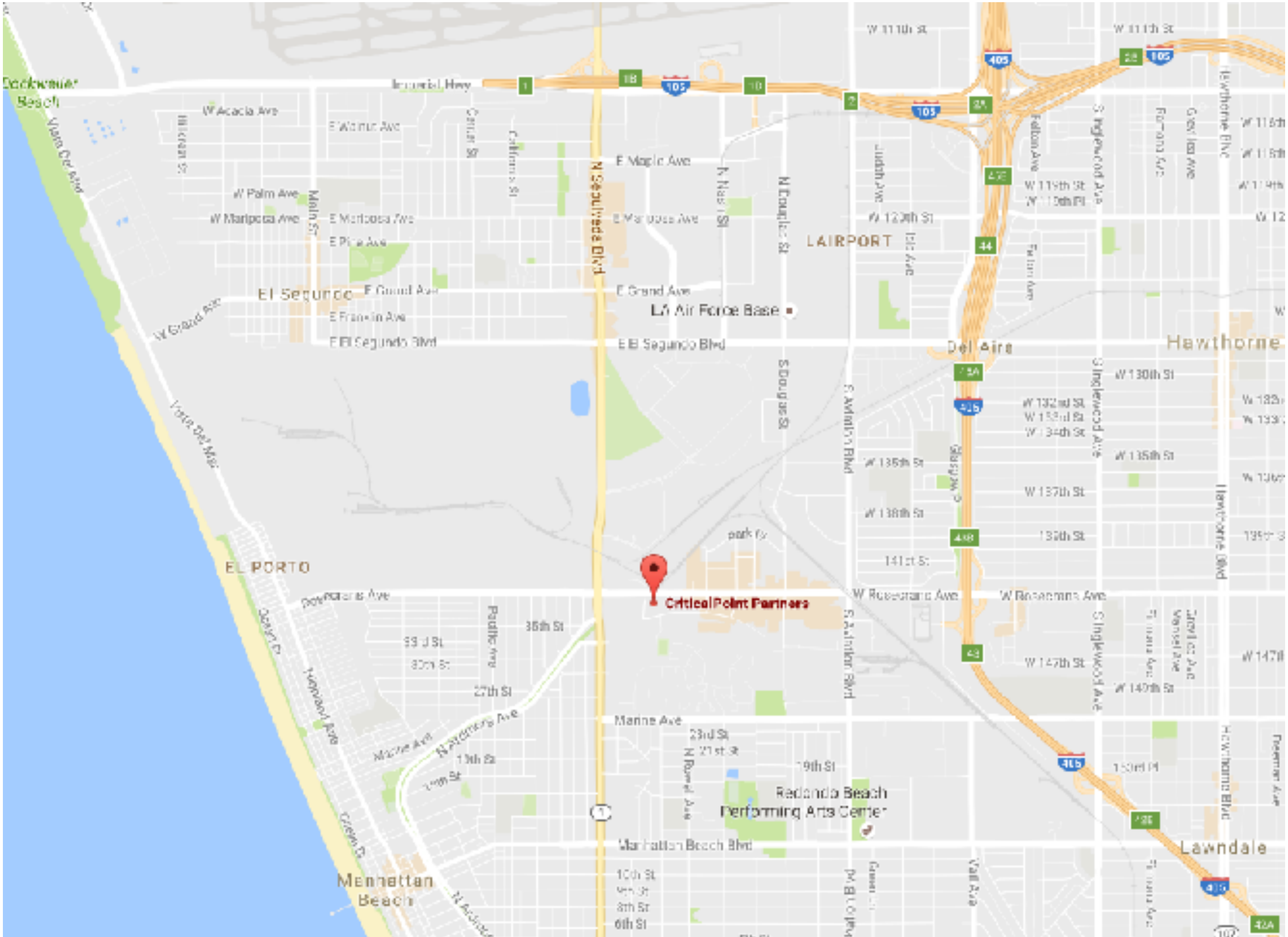
What we know about the Phase Diagram



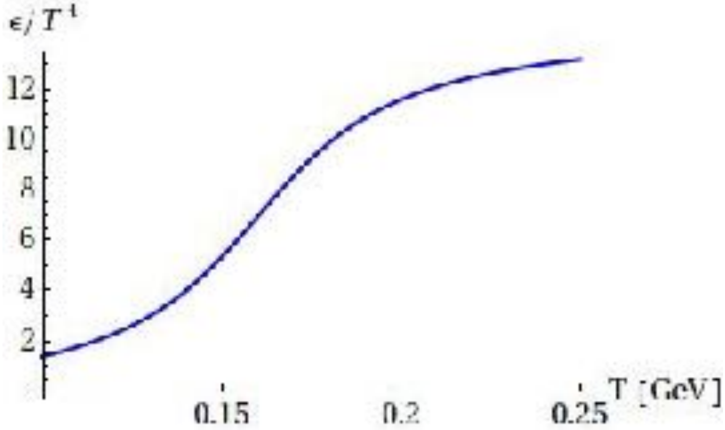
What we "hope" for

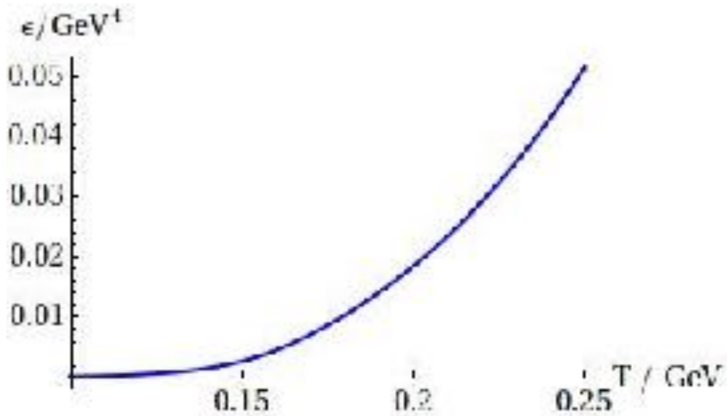


Is there a critical point?



Cumulants and phase structure



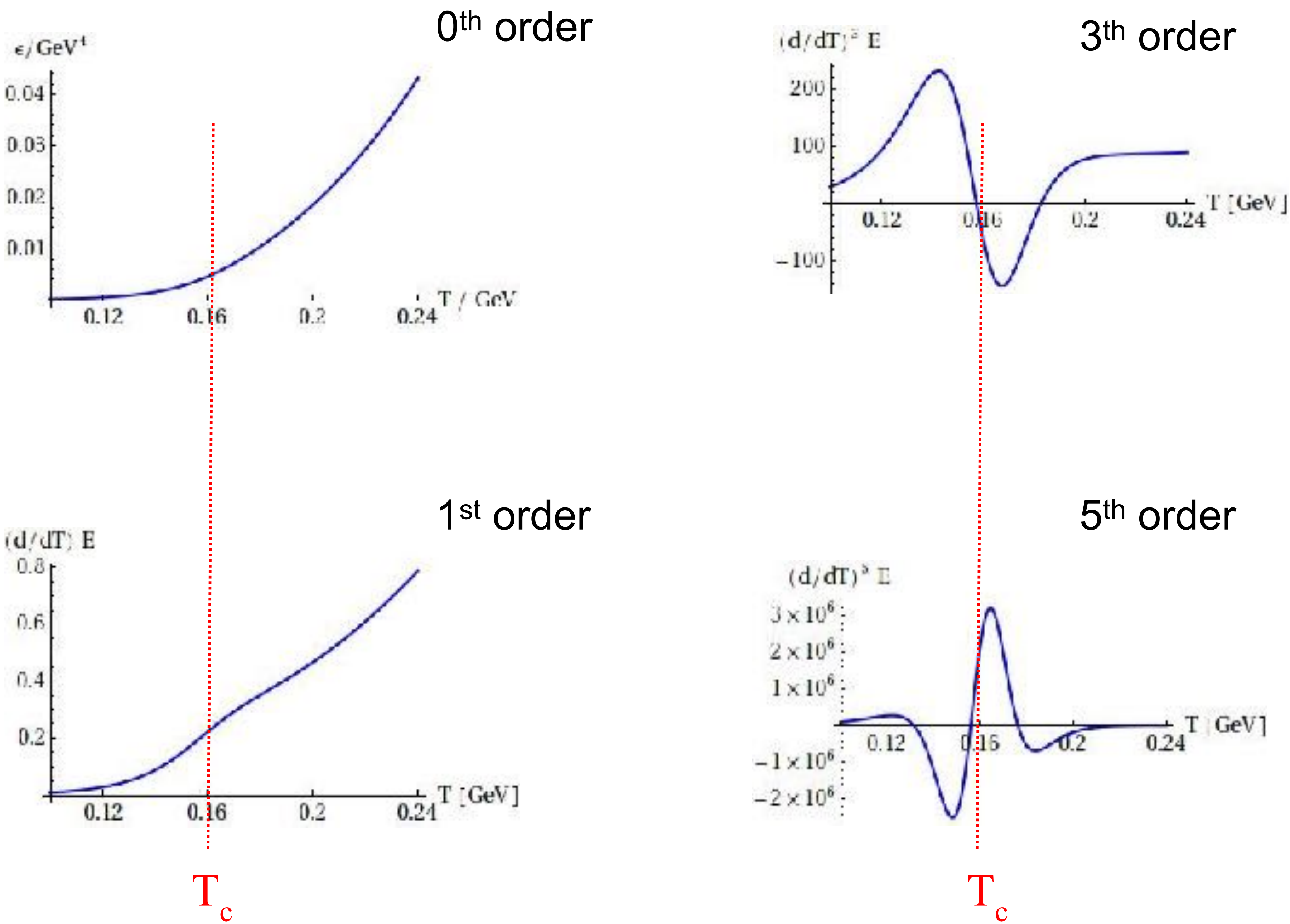


What we always see....

What it really means....

"T_c" ~ 160 MeV

Derivatives



How to measure derivatives

At μ = 0:

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z}\, tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

Cumulants of (Baryon) Number

$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle\,, \quad K_2 = \langle N - \langle N \rangle \rangle^2\,, \quad K_3 = \langle N - \langle N \rangle \rangle^3$$

Cumulants scale with volume (extensive): $K_n \sim V$

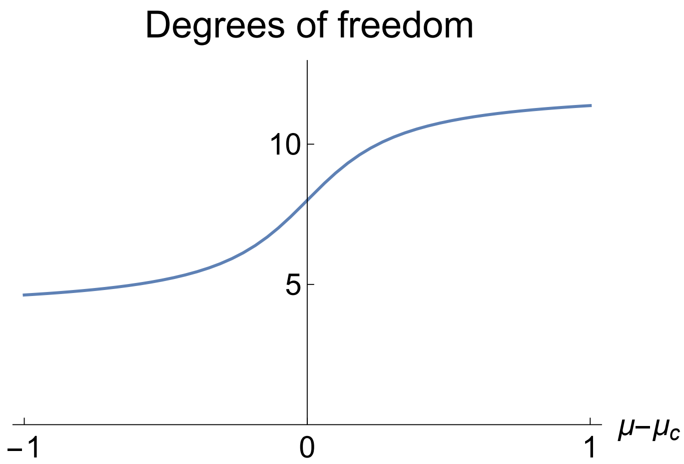
Volume not well controlled in heavy ion collisions

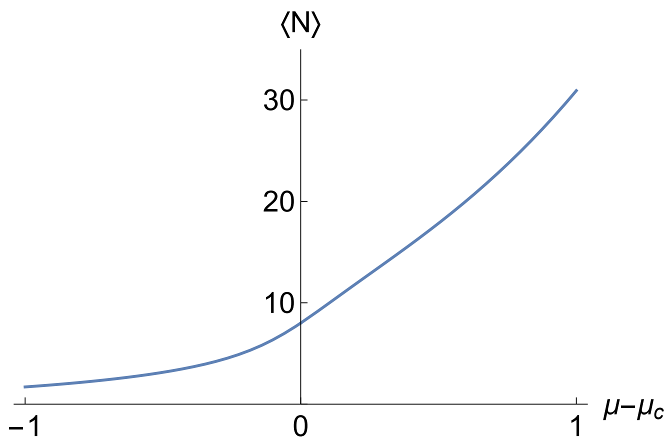
Cumulant Ratios: $\frac{K_2}{\langle N \rangle}, \quad \frac{K_3}{K_2}, \quad \frac{K_4}{K_2}$

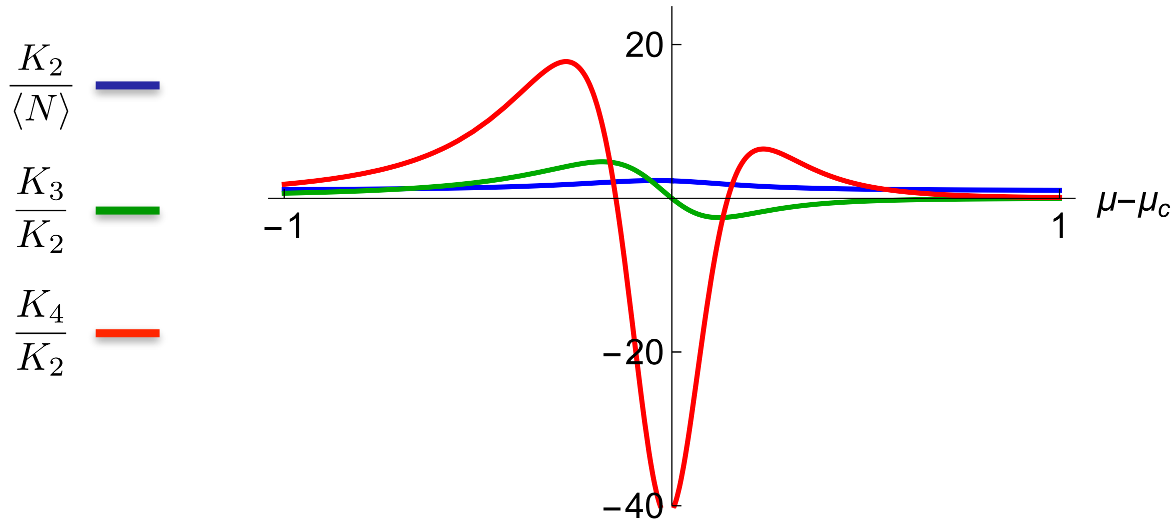
Simple model

Change degrees of freedom at phase transition

$$\langle N \rangle = dof(\mu)\, e^{\mu/T} \int d^3p e^{-E/T}$$



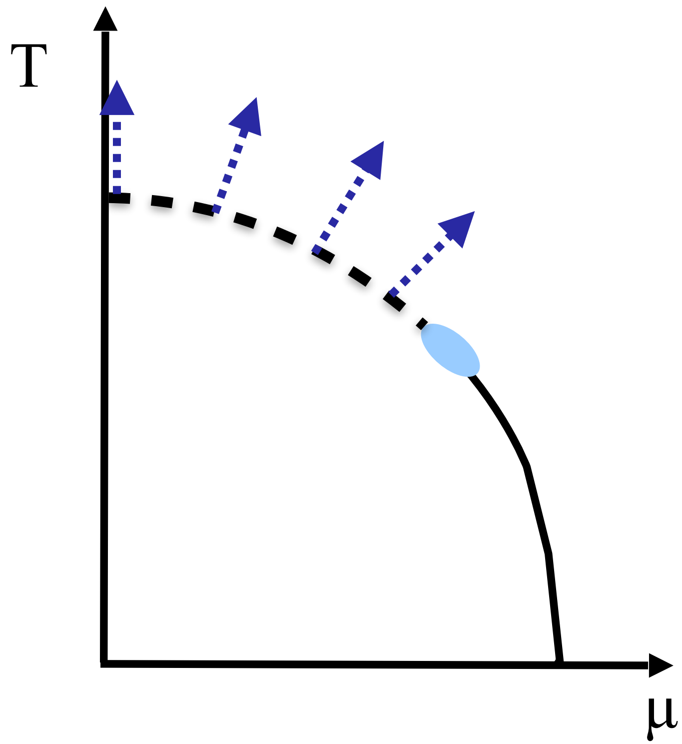




Close to μ=0

$$F = F(r), \quad r = \sqrt{T^2 + a\mu^2}$$

a ~ curvature of critical line

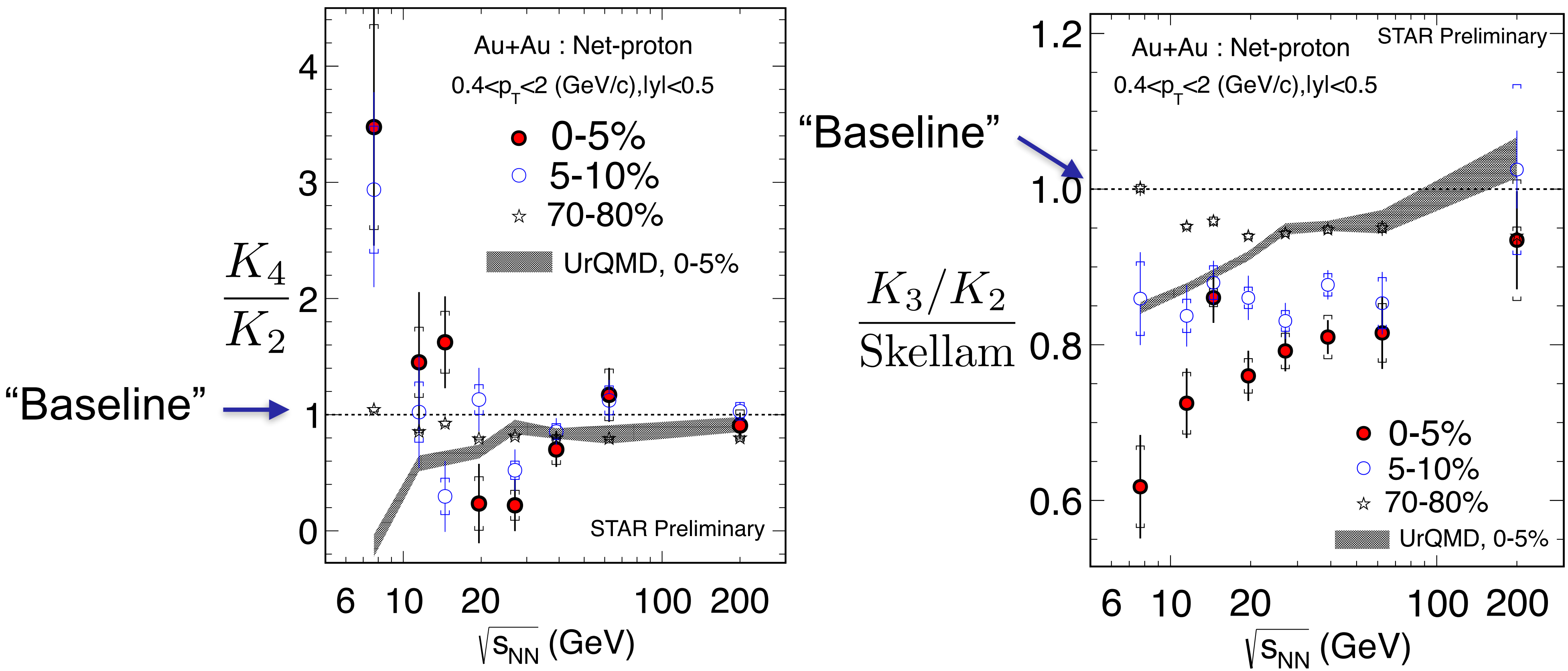


$$\frac{\partial^2}{\partial \mu^2} F(T,\mu)|_{\mu=0} = \frac{a}{T}\frac{\partial}{\partial T} F(T,\mu=0) \sim \langle E \rangle$$

Needs higher order cumulants (derivatives) at μ ~ 0

Latest STAR result on net-proton cumulants

X. Luo, NPA 956 (2016) 75



K_4/K_2 follows expectation for CP , K_3/K_2 no so much…..
URQMD totally fails to get trend for K_4/K_2 !

Let's take the preliminary STAR data at face value

Further insights: Correlations

Cumulants $K_n = \frac{\partial^n}{\partial \hat{\mu}^n} P/T^4$

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \langle (\delta N)^2 \rangle$$

$$\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2),$$

C_2: Correlation Function

$$K_3 = \langle (\delta N)^3 \rangle$$

$$\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)\underline{C_2(p_2, p_3)} + \rho_1(p_2)\underline{C_2(p_1, p_3)} + \rho_1(p_3)\underline{C_2(p_1, p_2)} + \underline{C_3(p_1, p_2, p_3)}$$

From Cumulants to Correlations
(no anti-protons)

Defining integrated correlations function a.k.a factorial cumulants

$$C_n = \int dp_1 \dots dp_n C_n(p_1, \dots, p_n)$$

Simple Algebra leads to relation between correlations C_n and K_n

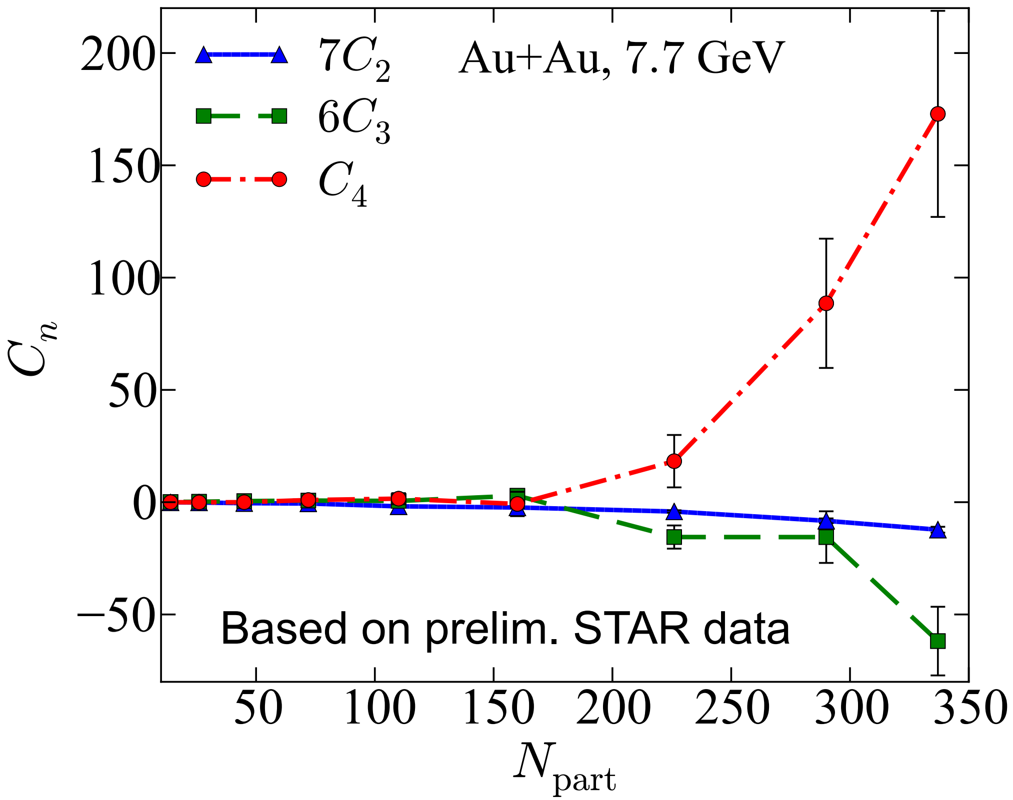
$$C_2 = -K_1 + K_2,$$
$$C_3 = 2K_1 - 3K_2 + K_3,$$
$$C_4 = -6K_1 + 11K_2 - 6K_3 + K_4, .$$

or vice versa

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Preliminary Star Data

(X. Luo, PoS Cpod 2014 (019))

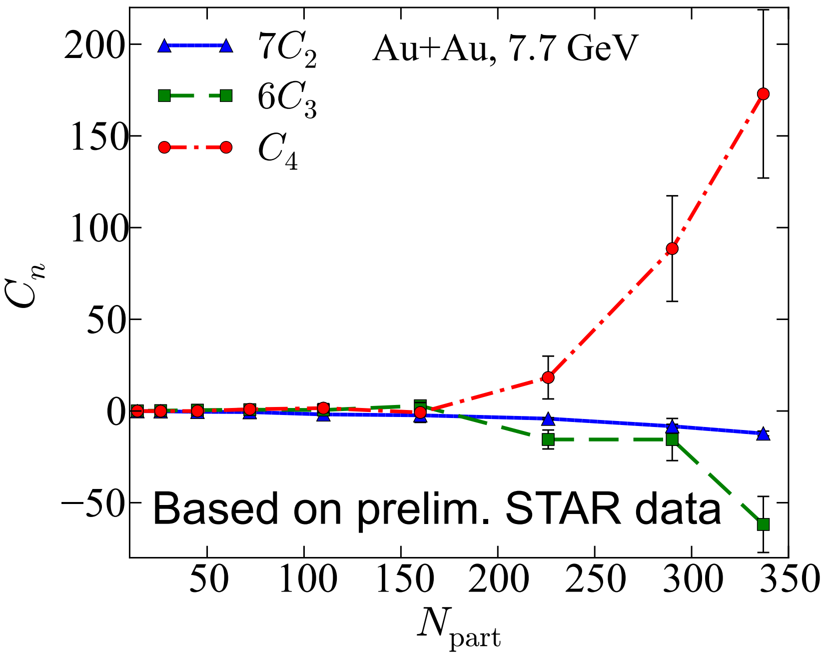


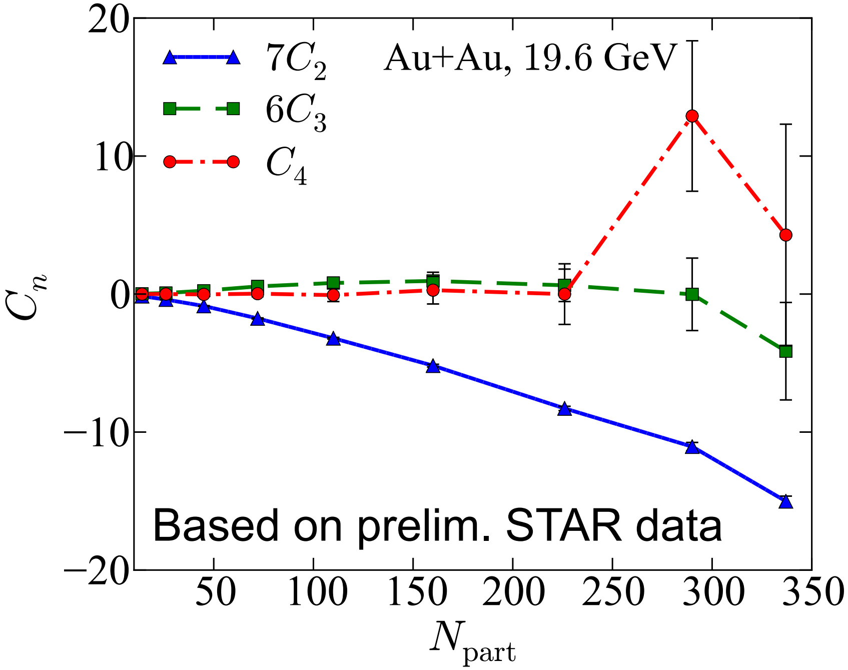
Significant four particle correlations!

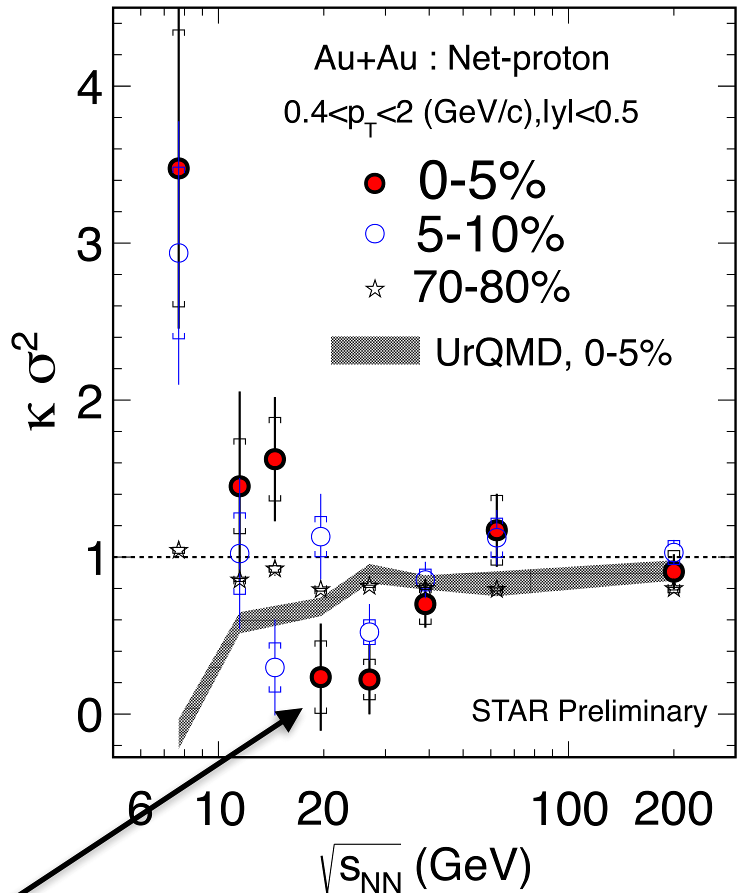
Four particle correlation dominate K_4 for central collisions at 7.7 GeV

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Correlations







Dip at 19.6 GeV from NEGATIVE C_2 !

Rapidity dependence

$$C_k(\Delta Y) = \int_{\Delta Y} dy_1 \ldots dy_k \rho_1(y_1) \ldots \rho_1(y_k) c_k(y_1, \ldots, y_k)$$

Assume: $\rho_1(y) \simeq const.$

short range correlations:

$$c_k(y_1, \ldots, y_k) \sim \delta(y_1 - y_2) \ldots \delta(y_{k-1} - y_k)$$

$$C_k(\Delta Y) \sim \Delta Y \to K_k \sim \Delta Y$$

Long range correlations:

$$c_k(y_1, \ldots, y_k) = const.$$

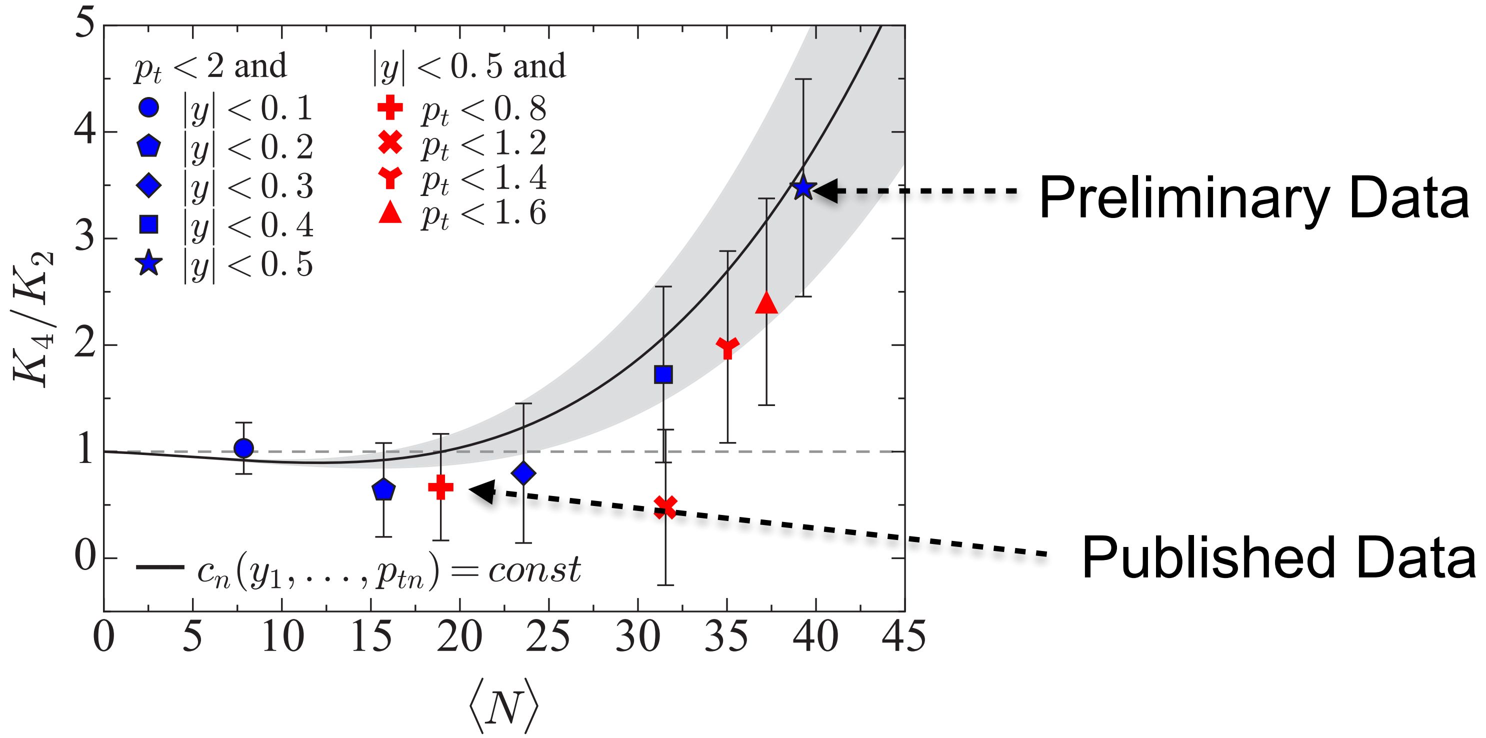
$$C_k(\Delta Y) \sim (\Delta Y)^k \sim \langle N \rangle^k$$

$$\Rightarrow K_n = K_n(\langle N \rangle)$$

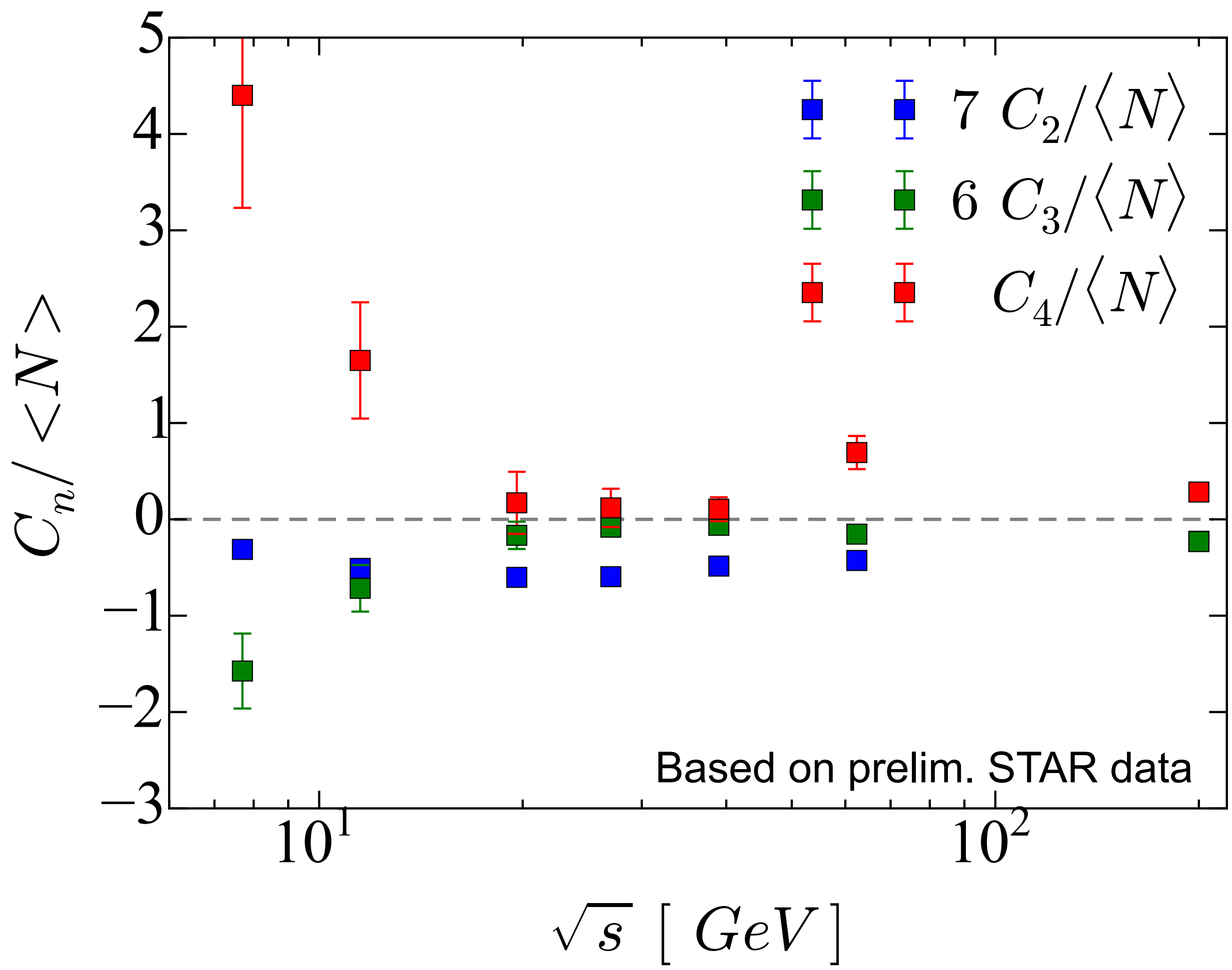
Long range correlations

$$C_k = \langle N \rangle^k c_k$$

$$c_k = const. \Rightarrow K_n = K_n(\langle N \rangle)$$



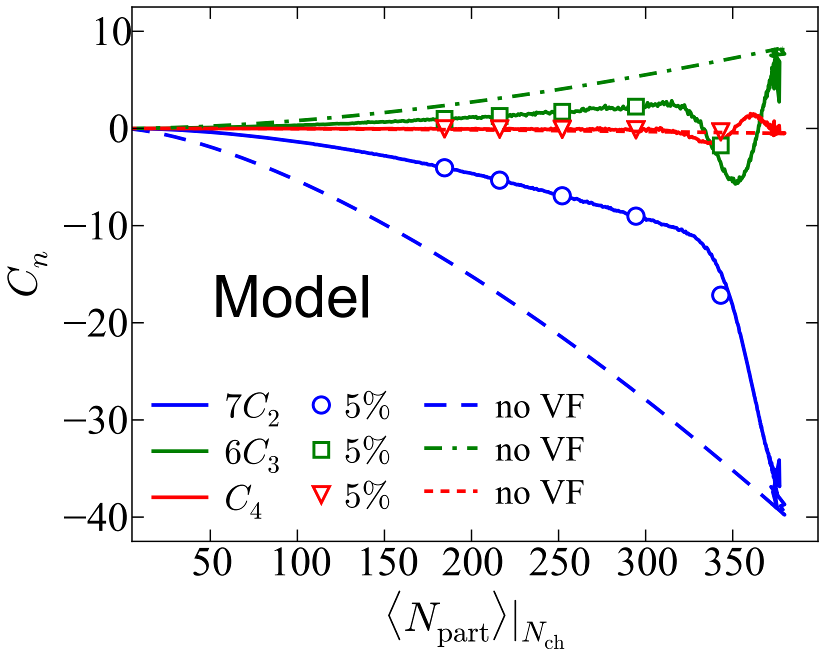
Energy dependence

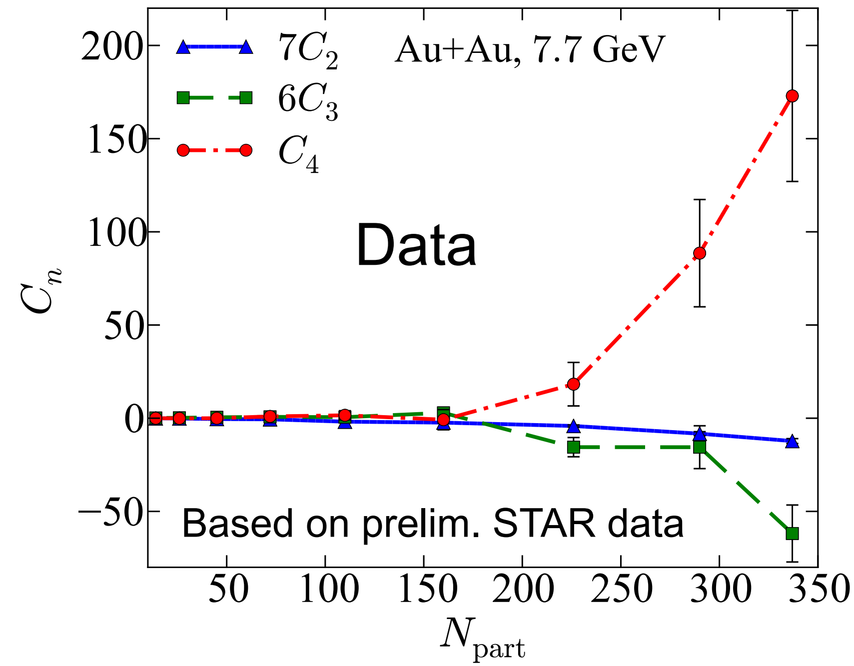


Note: anti-protons are non- negligible above 19.6 GeV
Data are protons only

Can we understand these correlations?

- Two particle correlations can be understood by simple Glauber model + Baryon number conservation



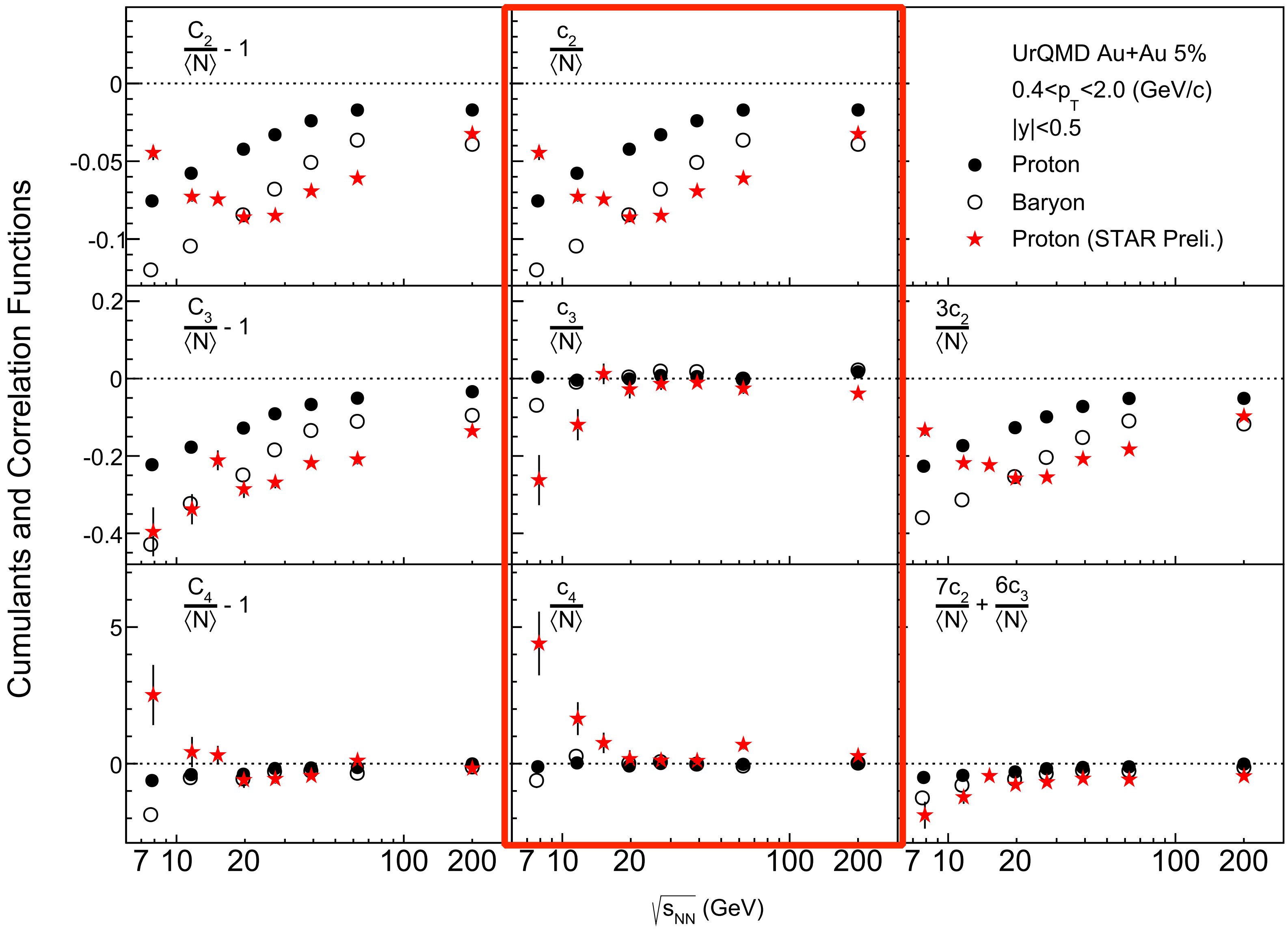


Four particle correlations are orders of magnitudes larger in the data

Also seen in URQMD calculations by He et al. PLB774 (2017) 623

Need to assume the ~40% of protons come from 8-nucleon cluster in order to get magnitude right!

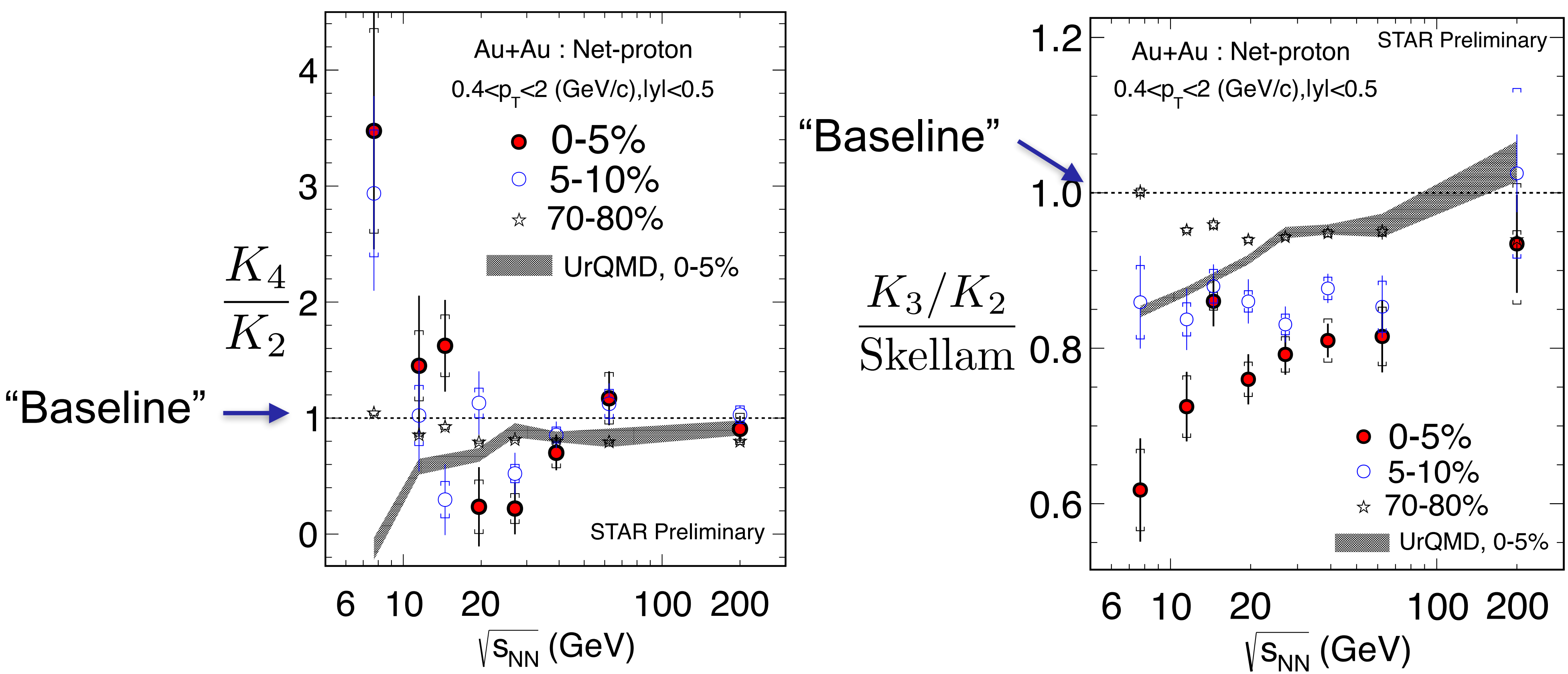
URQMD



He, Luo PLB774 (2017) 623

Latest STAR result on net-proton cumulants

X. Luo, NPA 956 (2016) 75

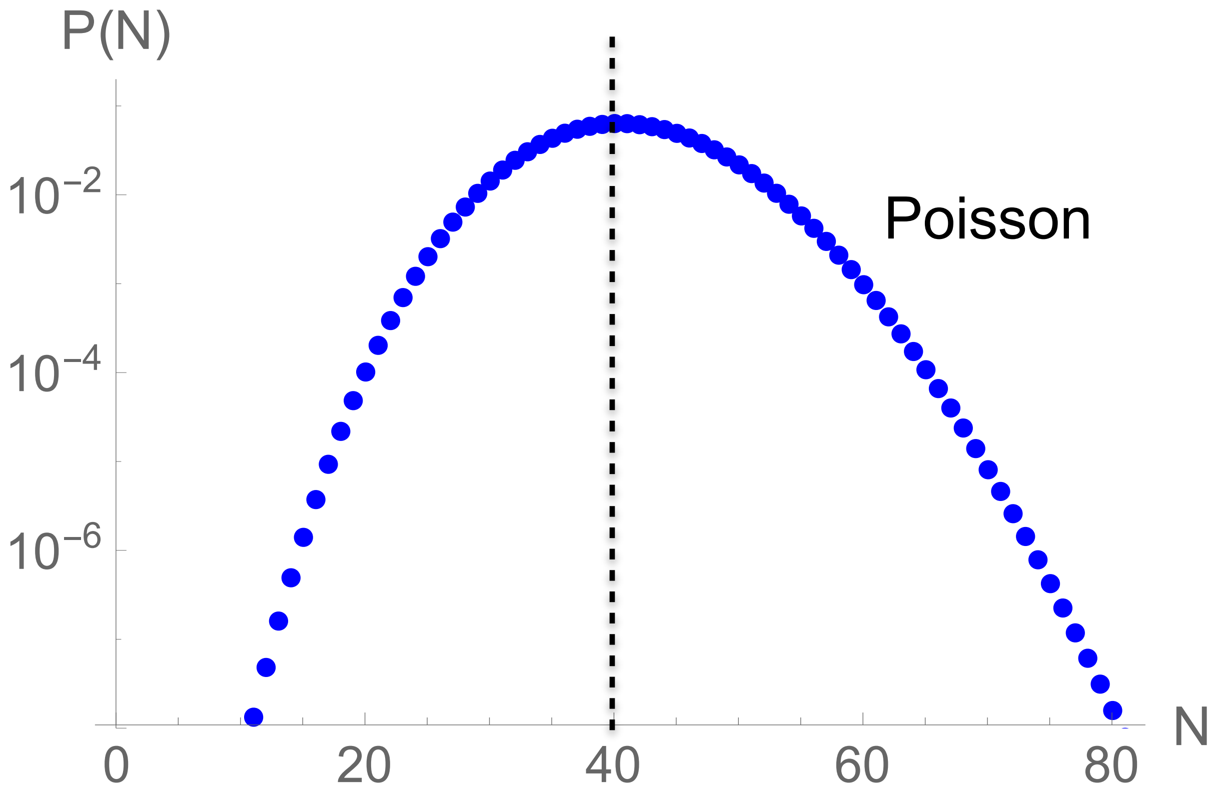


K_4/K_2 above baseline K_3/K_2 below baseline

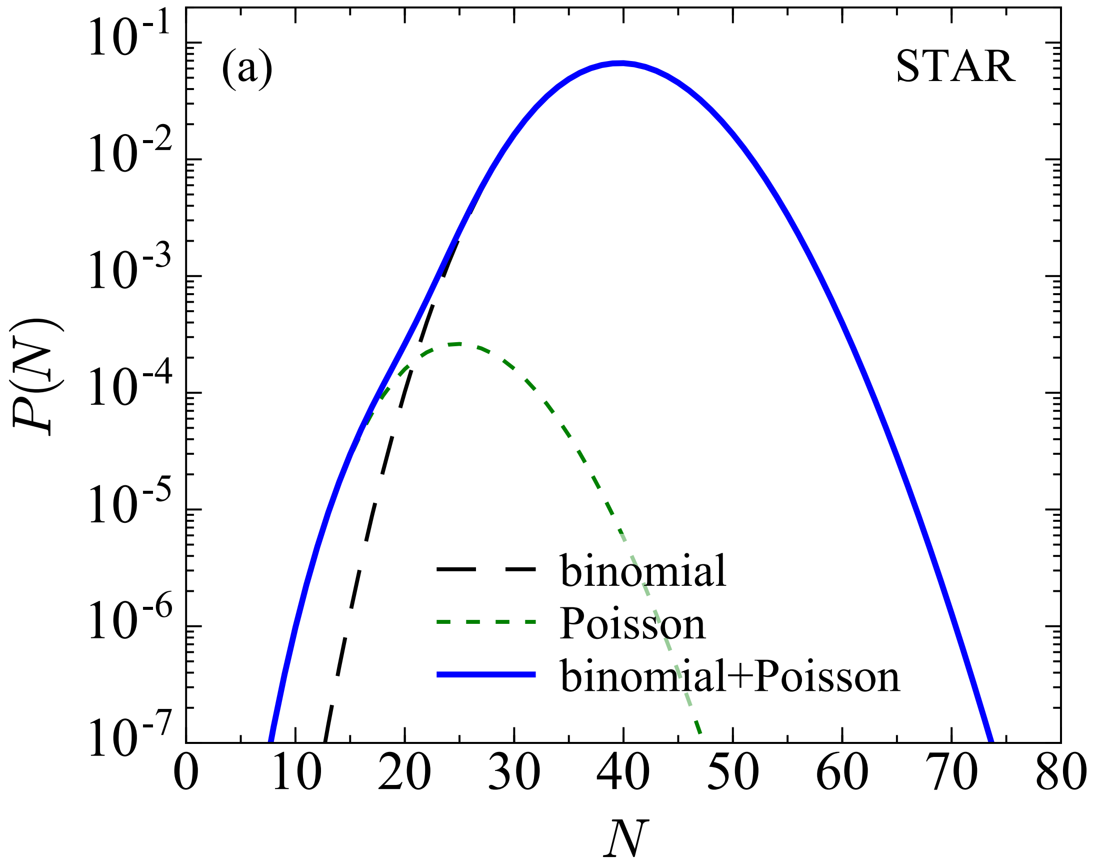
Shape of probability distribution

$$K_3 < \langle N \rangle \qquad K_3 = \langle N - \langle N \rangle \rangle^3$$

$$K_4 > \langle N \rangle \qquad K_4 = \langle N - \langle N \rangle \rangle^4 - 3 \langle N - \langle N \rangle \rangle^2$$



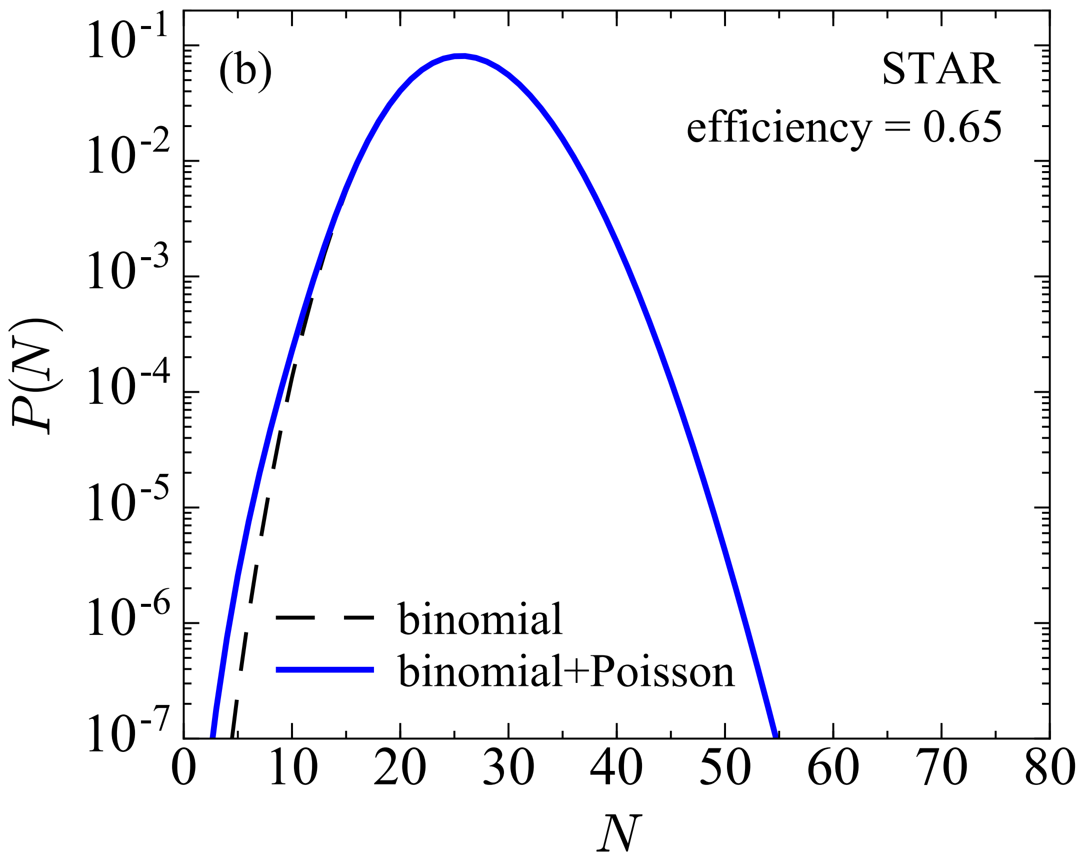
Simple two component model



Weight of small component: ~0.3%

Simple two component model

Difficult to see in the real data with efficiency ε=0.65



Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\overline{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle$$

$$C_2 = C_2^{(a)} - \alpha \left\{ \overline{C}_2 - (1-\alpha)\overline{N}^2 \right\}$$

$$C_3 = C_3^{(a)} - \alpha \left\{ \overline{C}_3 + (1-\alpha) \left[(1-2\alpha)\overline{N}^3 - 3\overline{N}\,\overline{C}_2 \right] \right\}$$

$$C_4 = C_4^{(a)} - \alpha \left\{ \overline{C}_4 - (1-\alpha) \left[\left(1 - 6\alpha + 6\alpha^2\right) \overline{N}^4 - 6(1-2\alpha)\overline{N}^2\overline{C}_2 + 4\overline{N}\,\overline{C}_3 + 3(\overline{C}_2)^2 \right] \right\}$$

$$\overline{C}_n = C_n^{(a)} - C_n^{(b)},$$

For Poisson, $C_{(a)}$, $C_{(b)}=0$

Fit to STAR data: $\langle N_{(a)} \rangle \simeq 40, \ \langle N_{(b)} \rangle \simeq 25, \ \alpha \simeq 0.003$

Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\bar{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle > 0$$

For $P_{(a)}$, $P_{(b)}$ Poisson, or (to good approximation) Binomial

$$C_n = (-1)^n K_n^B \bar{N}^n \quad n \geq 2$$

$$K_n^B : \text{ Cumulant of Bernoulli distribution}$$

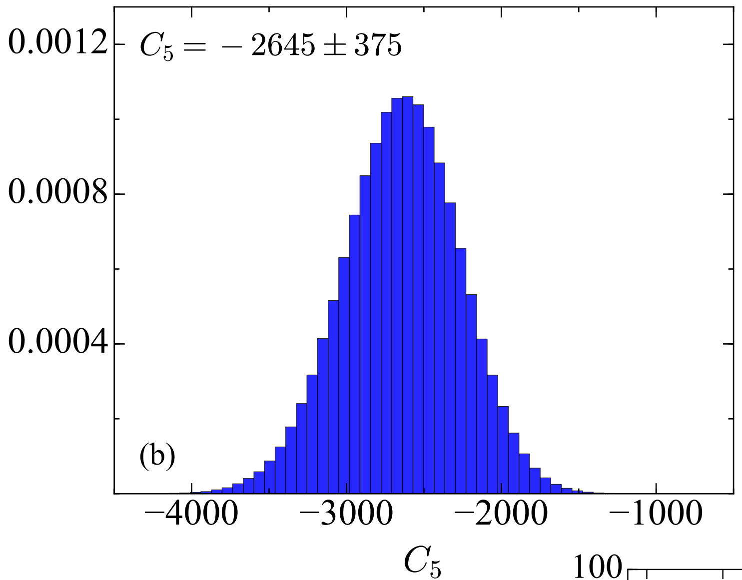
$$\alpha \ll 1, K_n^B = \alpha \;\Rightarrow\; C_n \simeq \alpha(-1)^n \bar{N}^n$$

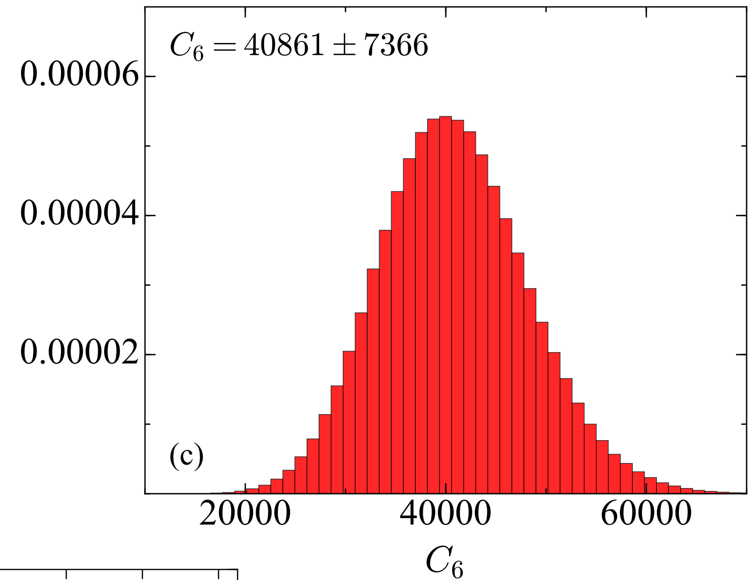
$\Rightarrow |C_n| \sim \langle N \rangle^n$ as seen by STAR (i.e. "infinite" correlation length)

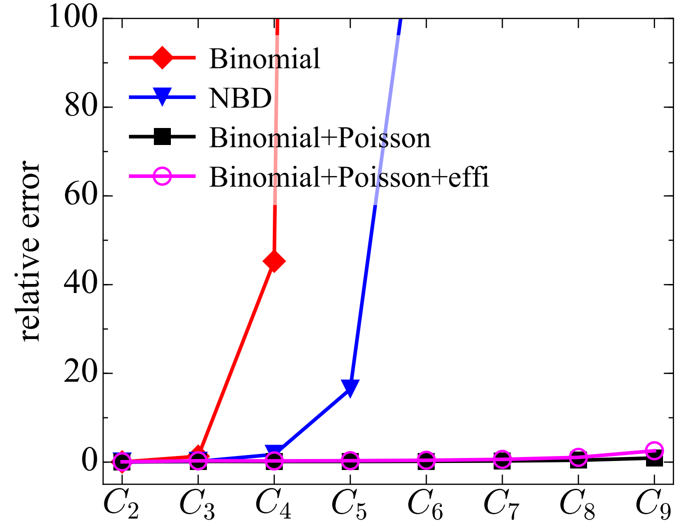
predict: $$\frac{C_4}{C_3} = \frac{C_5}{C_4} = \frac{C_{n+1}}{C_n} = -\bar{N}$$ $\bar{N} \simeq 15$

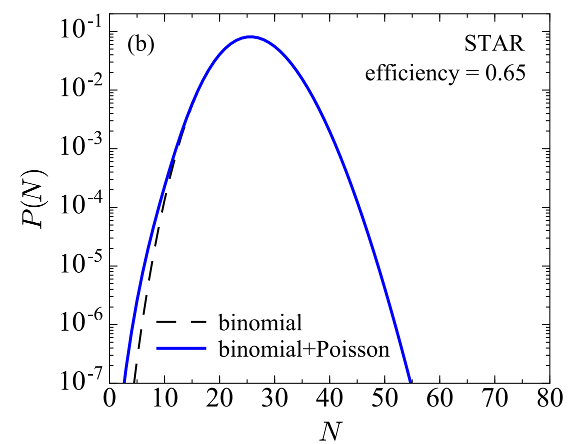
Clear and falsifiable prediction: $C_5 \approx -2650 \quad C_6 \approx 41000$

Two component model is Statistics “friendly”









Based on 144393 events (same as STAR 0-5% at 7.7 GEV)

This model can be tested RIGHT NOW!

Model prediction:

$$C_5 = -2645\,(1 \pm 0.14), \quad C_6 = 40900\,(1 \pm 0.18),$$
$$C_7 = -615135\,(1 \pm 0.26), \quad C_8 = 8520220\,(1 \pm 0.42)$$

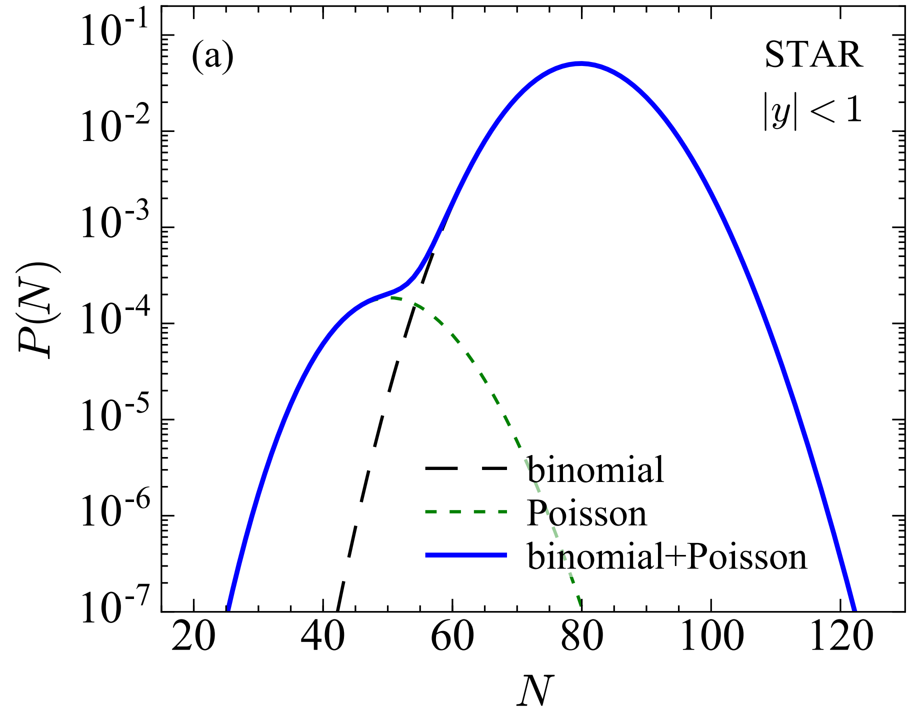
Efficiency corrected

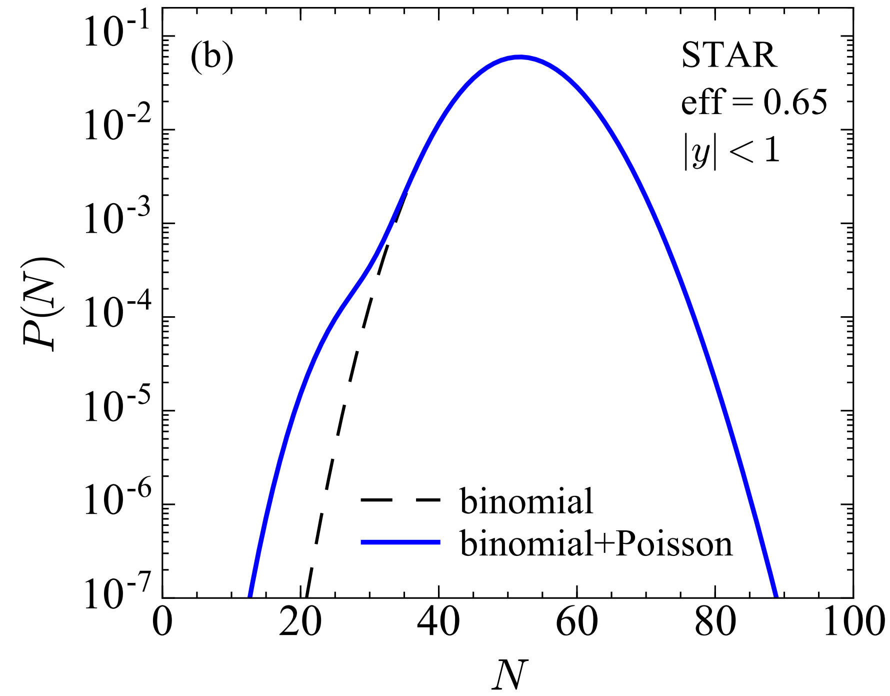
$$C_5 = -307\,(1 \pm 0.31), \quad C_6 = 3085\,(1 \pm 0.41),$$
$$C_7 = -30155\,(1 \pm 0.61), \quad C_8 = 271492\,(1 \pm 1.06),$$

Efficiency UN-corrected

Based on 144393 events (same as STAR 0-5% at 7.7 GEV)

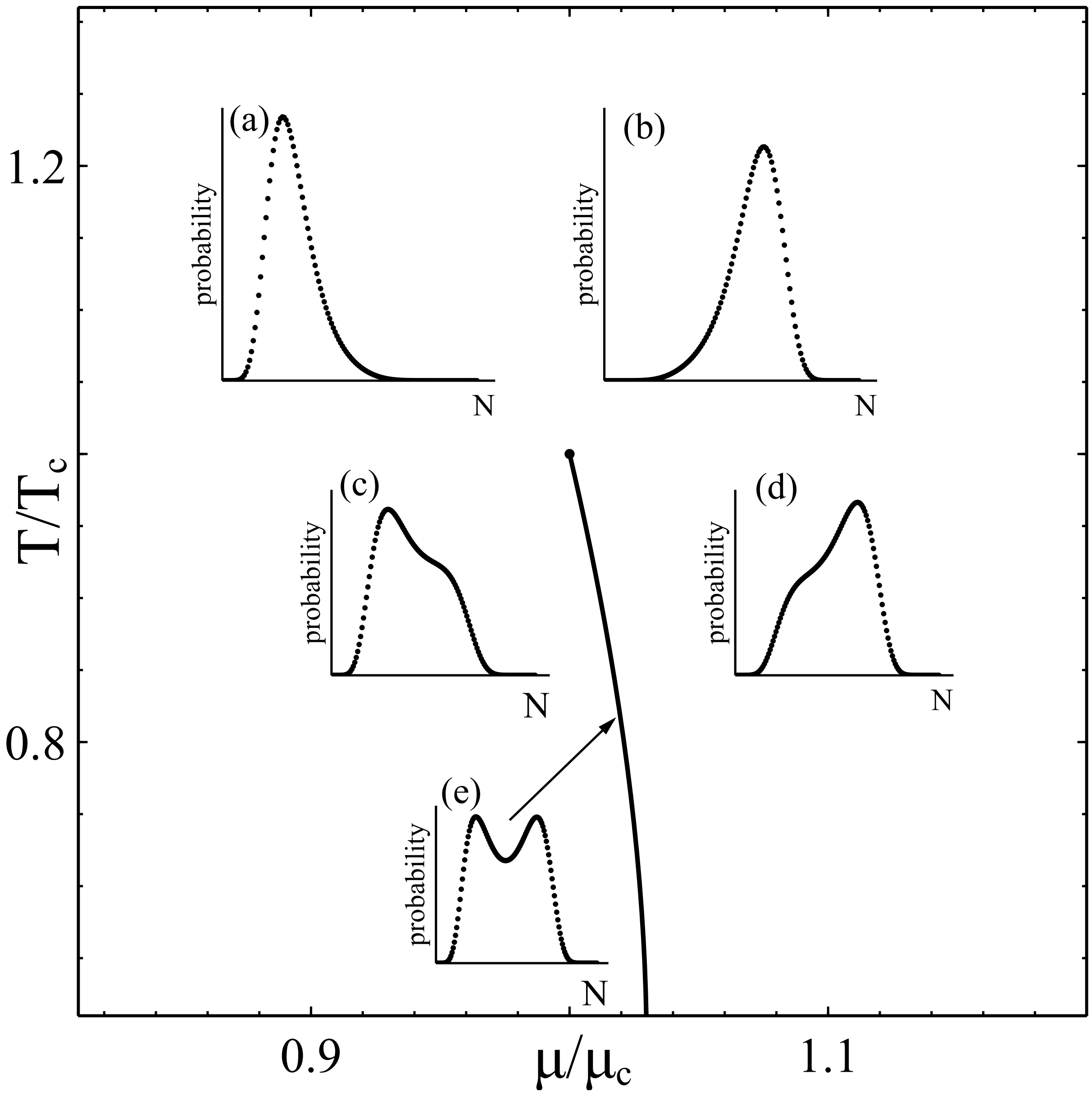
Double the acceptance

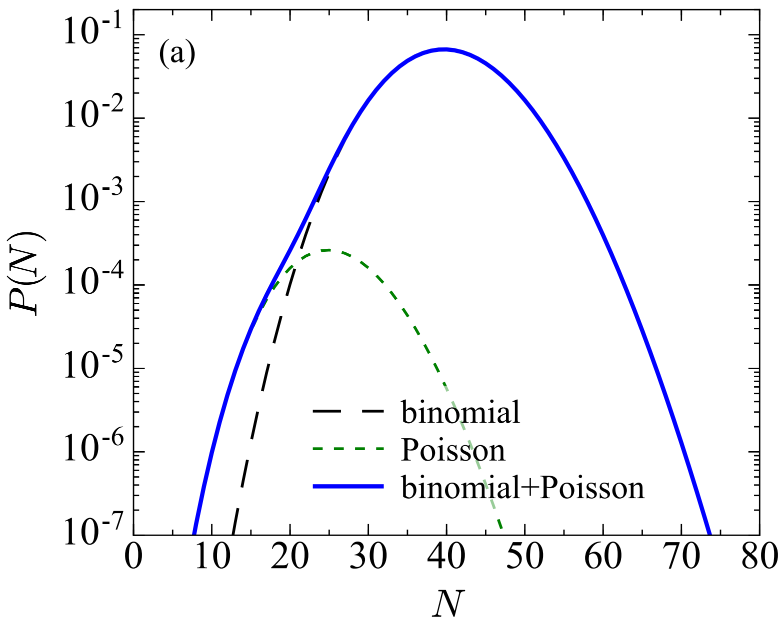




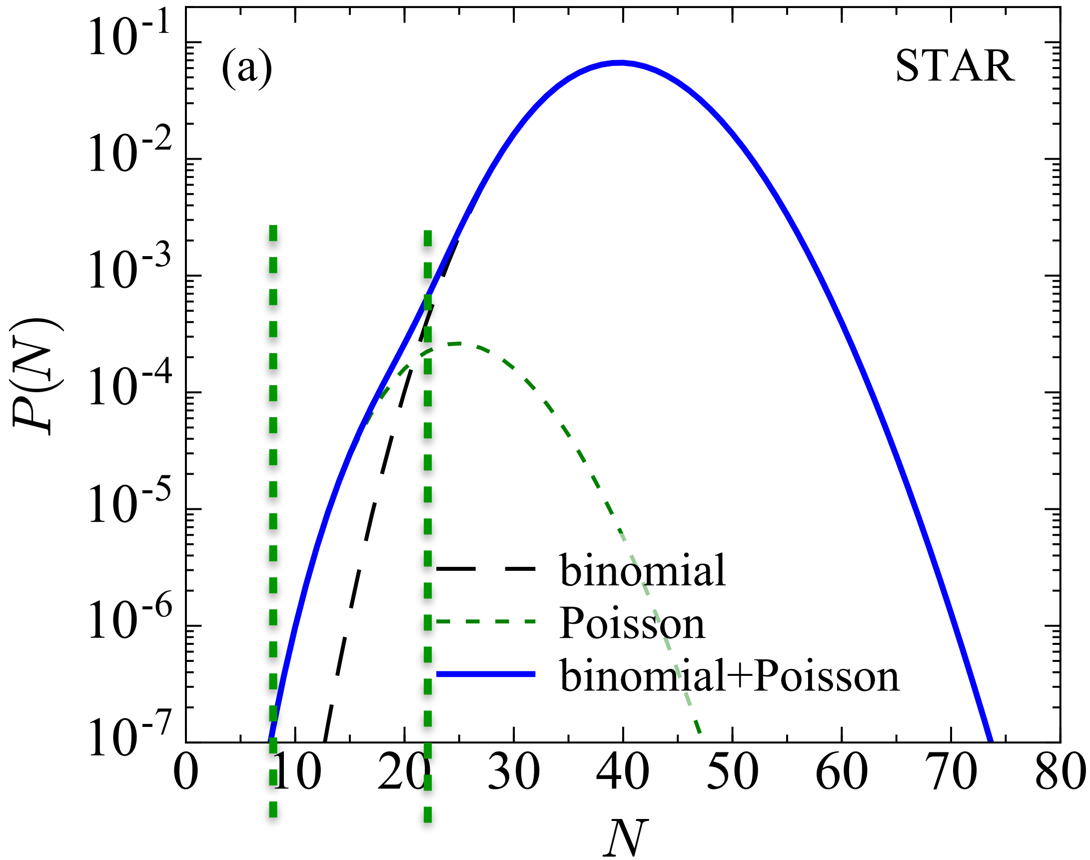
Should be visible in raw (unfolded) data

Speculation





Simple two component model

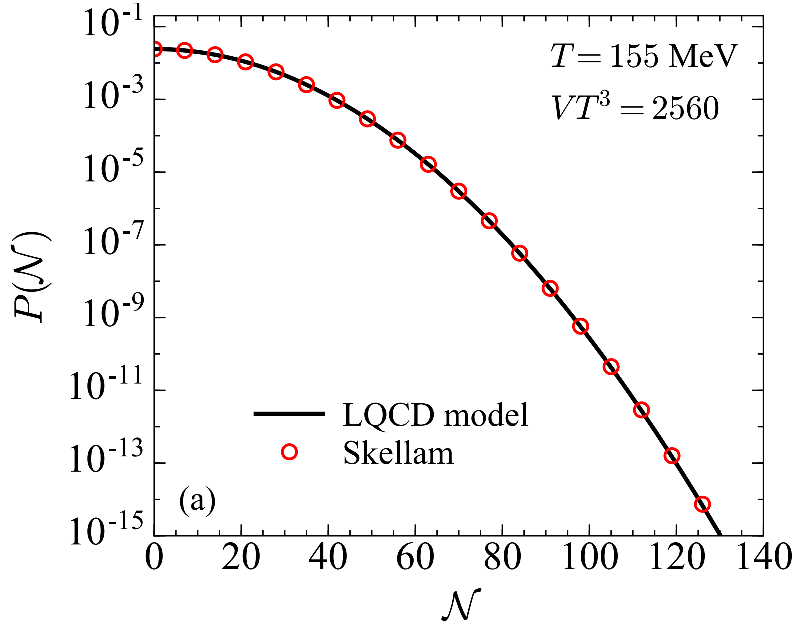


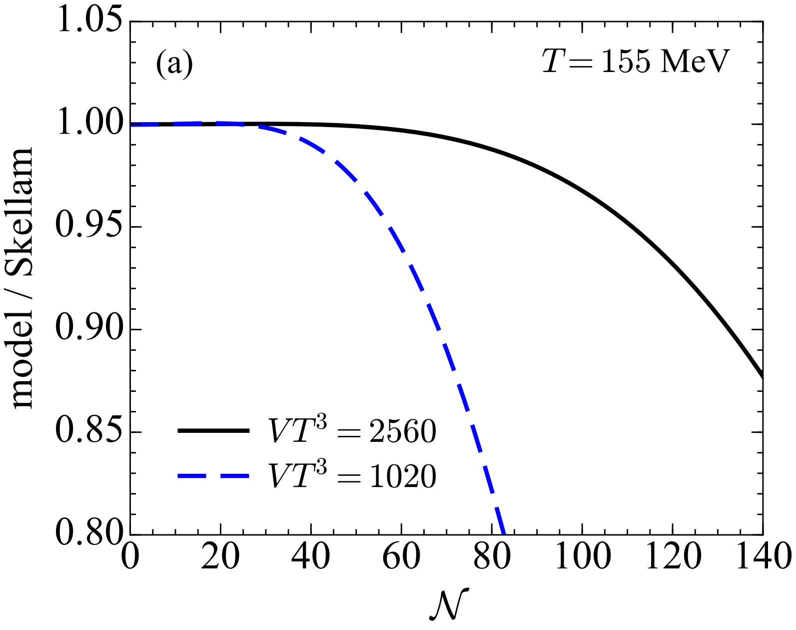
Analyse data for $N_p < 20$

- Is flow etc different?
- “Inspect by eye (<1% of all events)

The return of the camel?

Net-baryon distribution consisted with Lattice QCD





For details, see arXiv:1810.01913

Summary

- Fluctuations sensitive to phase structure:
  - measure “derivatives” of EOS
- Cumulants contain information about correlations
- Preliminary STAR data:
  - Significant four particle correlations at 7.7 and 11.5 GeV
- Fluctuations of system size (N_{part}) and stopping
  - May explain 2-particle correlations
  - Fail to reproduce the magnitude of 3- and 4- particle correlations
- 3 and 4 particle correlations are HUGE!
- “Bi-Modal” distribution works
  - Can be tested RIGHT NOW by STAR.
- Net-baryon number distribution consistent with lattice
  - Deviation from Skellam is very small!

Thank You

Net-baryon multiplicity distribution

Utilize of cluster expansion model of Vovchenko et al arXiv:1711.01261

Virial expansion: $$\frac{P}{T^4} = \frac{1}{VT^3}\ln(Z) = \sum_{k=0}^{\infty} p_k(T)\cosh\left(k\hat{\mu}_B\right)$$

Cluster model: $$p_k = f(p_1, p_2);\ k > 2$$

Lattice QCD: p_1, p_2 Vovchenko et al, arXiv:1708.02852

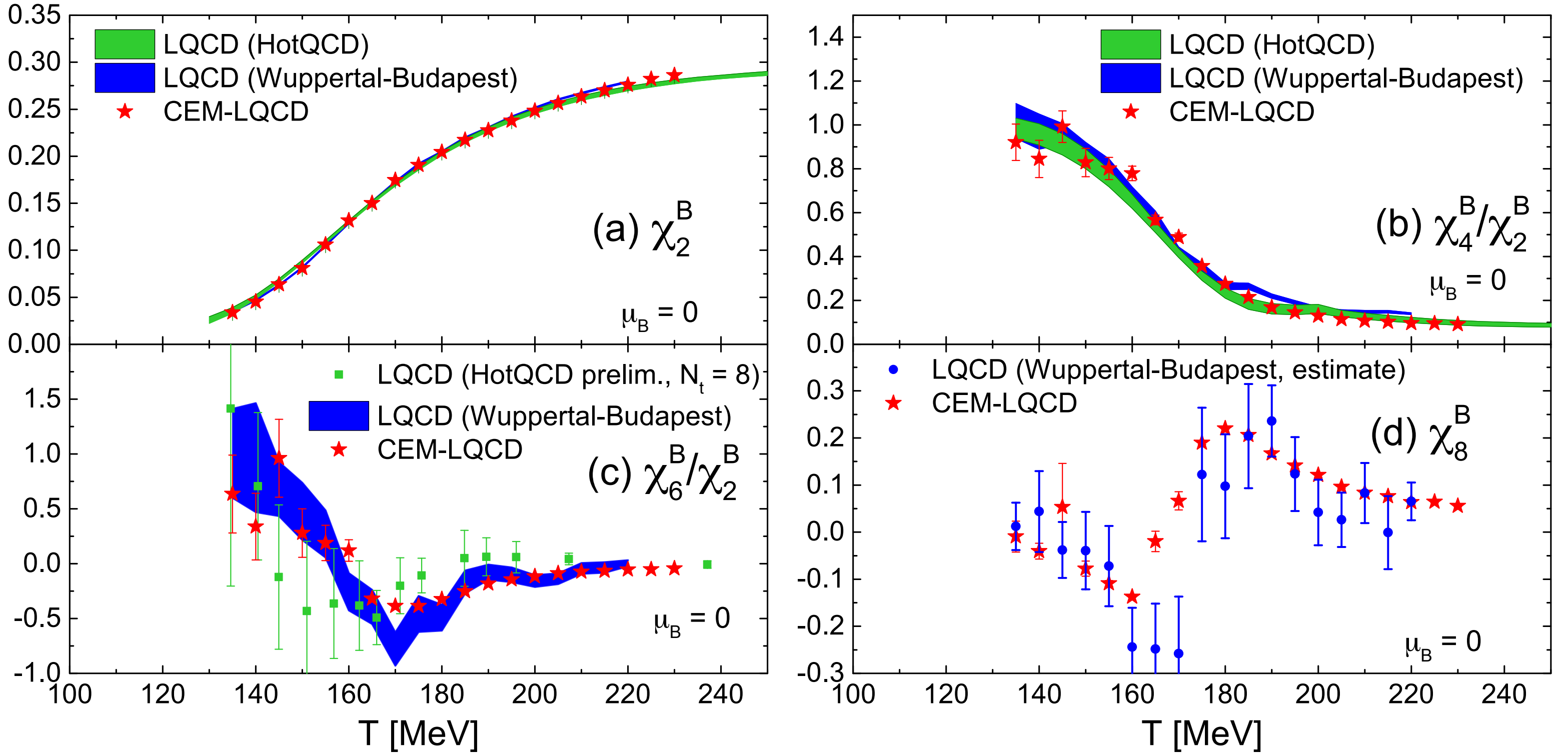


Figure from: arXiv:1807.06472

Net-baryon multiplicity distribution

Virial expansion: $$\frac{P}{T^4} = \frac{1}{VT^3}\ln(Z) = \sum_{k=0}^{\infty} p_k(T)\cosh\left(k\hat{\mu}_B\right)$$

$$Z = \exp\left[VT^3 \sum_{k=0}^{\infty} p_k(T)\cosh(k\hat{\mu}_B)\right] = z_0 + 2\sum_{\mathcal{N}=1}^{\infty} z_{\mathcal{N}}\cosh(\mathcal{N}\hat{\mu}_B)$$

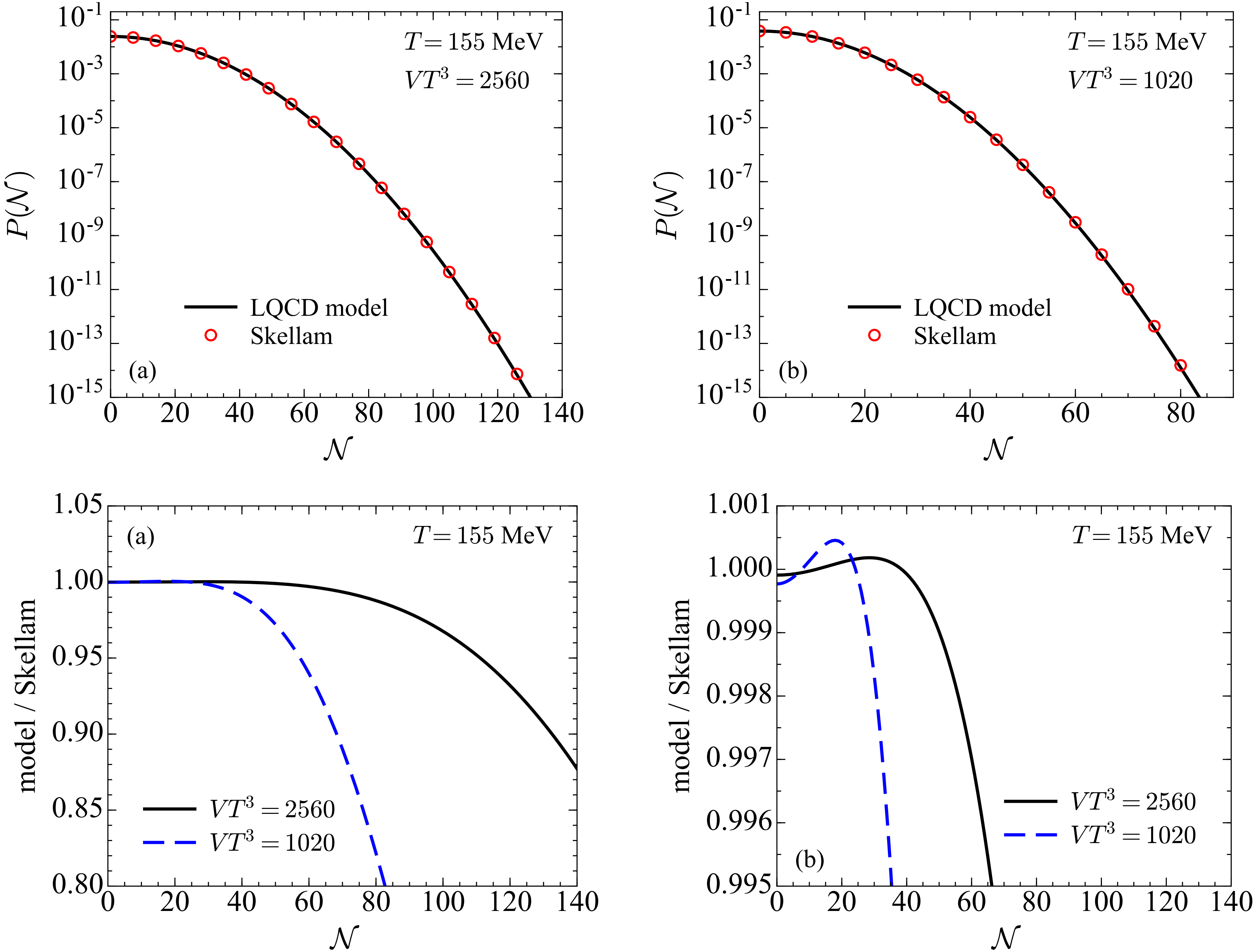
Multiplicity distribution: $$P(\mathcal{N}) = \frac{z_{\mathcal{N}} e^{\hat{\mu}_B \mathcal{N}}}{Z}$$

$$\hat{\mu}_B \to i\bar{\mu}_B$$

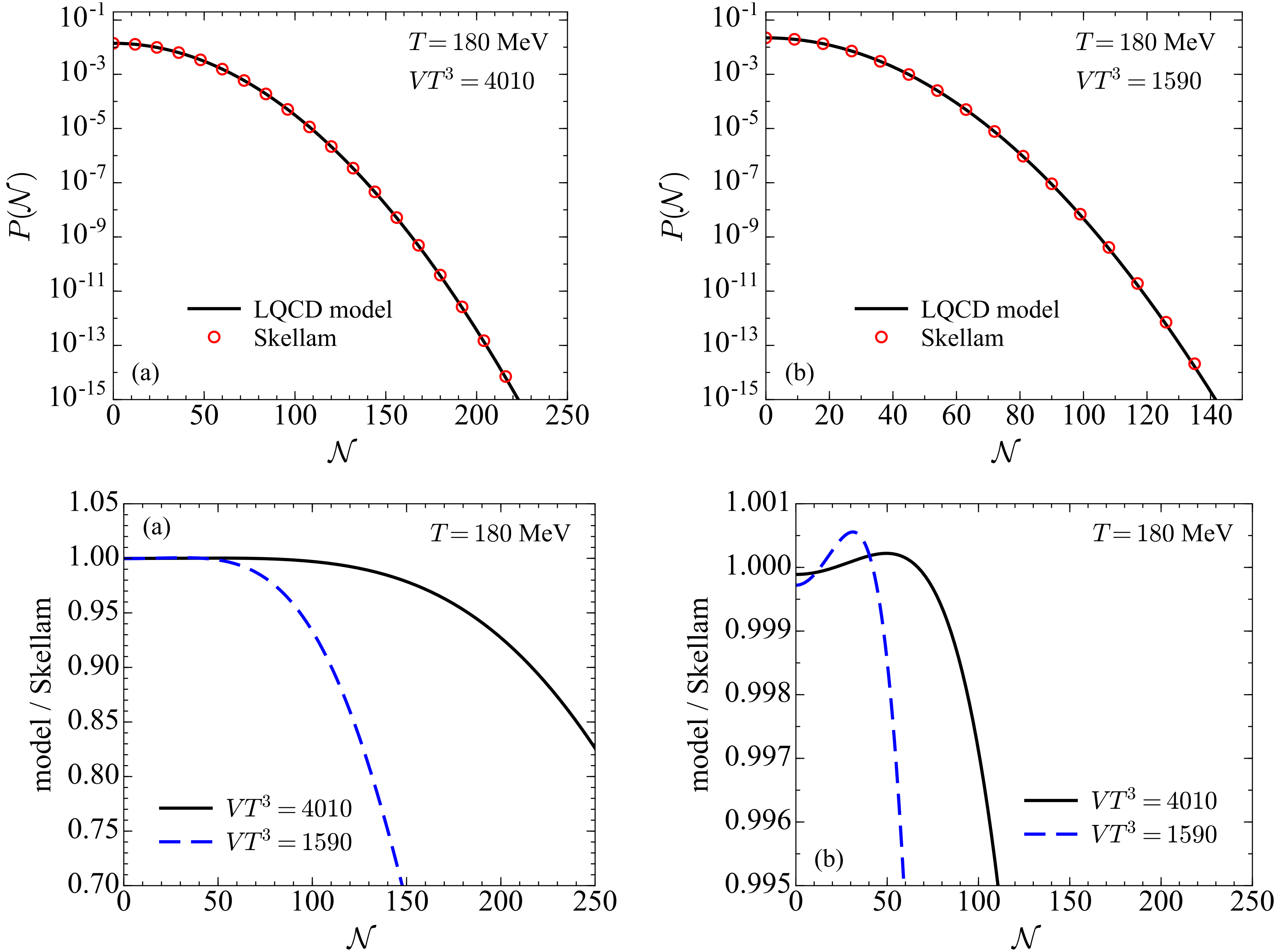
$$P(\mathcal{N}) = \frac{1}{\pi}\int_0^{\pi} d\bar{\mu}_B \, \cos(\mathcal{N}\bar{\mu}_B)\, \frac{\exp\left[VT^3 \sum_{k=1}^{\infty} p_k(T)\cos\left(k\bar{\mu}_B\right)\right]}{\exp\left[VT^3 \sum_{k=1}^{\infty} p_k(T)\right]}$$

$$\hat{\mu}_B = 0$$

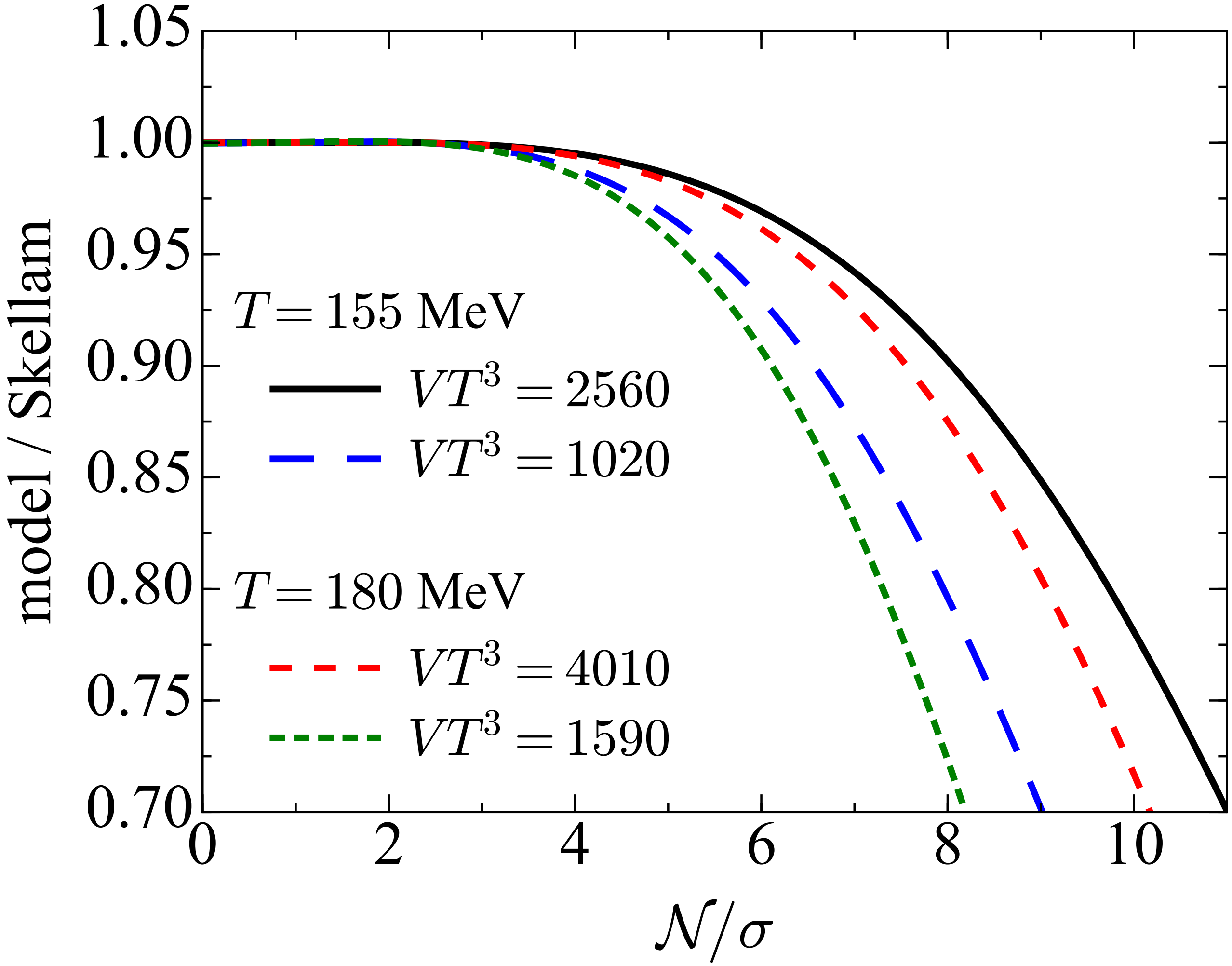
LHC   Top RHIC



$P(\mathcal{N})$
$T = 155$ MeV
$VT^3 = 2560$
LQCD model
Skellam
(a)
$\mathcal{N}$
$T = 155$ MeV
$VT^3 = 1020$
LQCD model
Skellam
(b)
$\mathcal{N}$
model / Skellam
(a)
$T = 155$ MeV
$VT^3 = 2560$
$VT^3 = 1020$
$\mathcal{N}$
model / Skellam
$T = 155$ MeV
$VT^3 = 2560$
$VT^3 = 1020$
(b)
$\mathcal{N}$

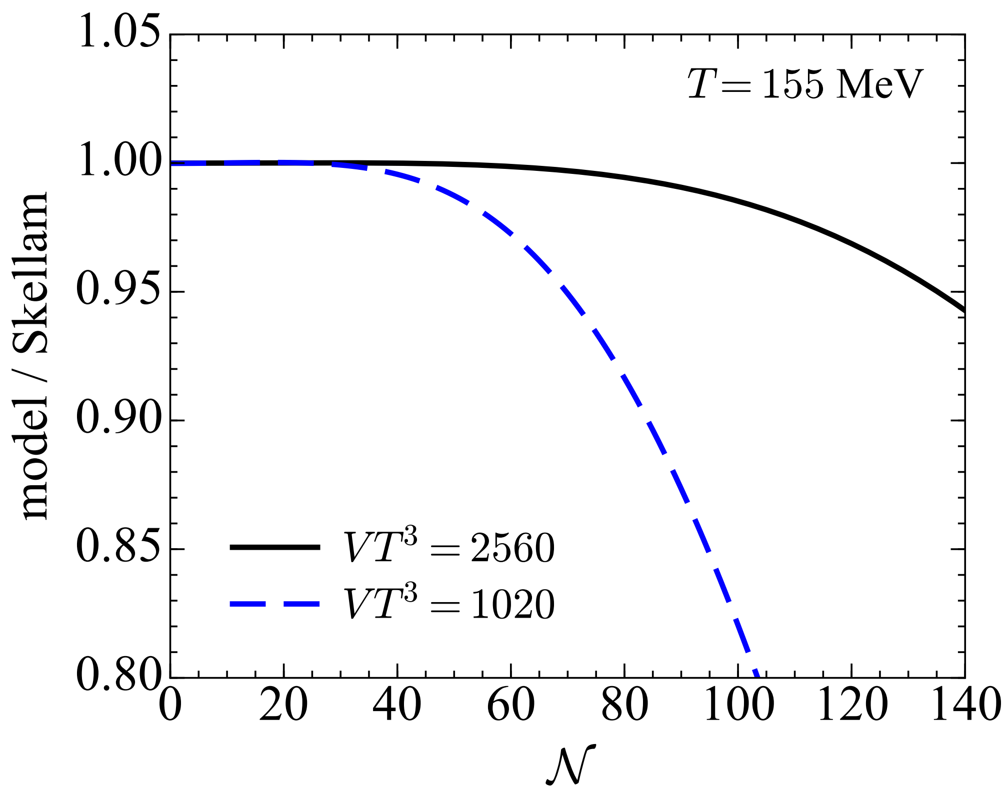


$T = 180$ MeV
$VT^3 = 4010$
LQCD model
Skellam
(a)
$P(\mathcal{N})$
$\mathcal{N}$
$T = 180$ MeV
$VT^3 = 1590$
LQCD model
Skellam
(b)
$P(\mathcal{N})$
$\mathcal{N}$
(a)
$T = 180$ MeV
model / Skellam
$VT^3 = 4010$
$VT^3 = 1590$
$\mathcal{N}$
$T = 180$ MeV
model / Skellam
$VT^3 = 4010$
$VT^3 = 1590$
(b)
$\mathcal{N}$

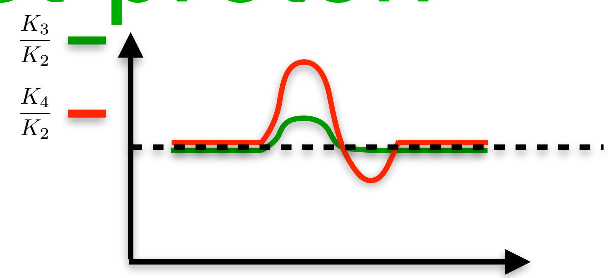


1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
model / Skellam
0
2
4
6
8
10
$\mathcal{N}/\sigma$
$T = 155$ MeV
$VT^3 = 2560$
$VT^3 = 1020$
$T = 180$ MeV
$VT^3 = 4010$
$VT^3 = 1590$

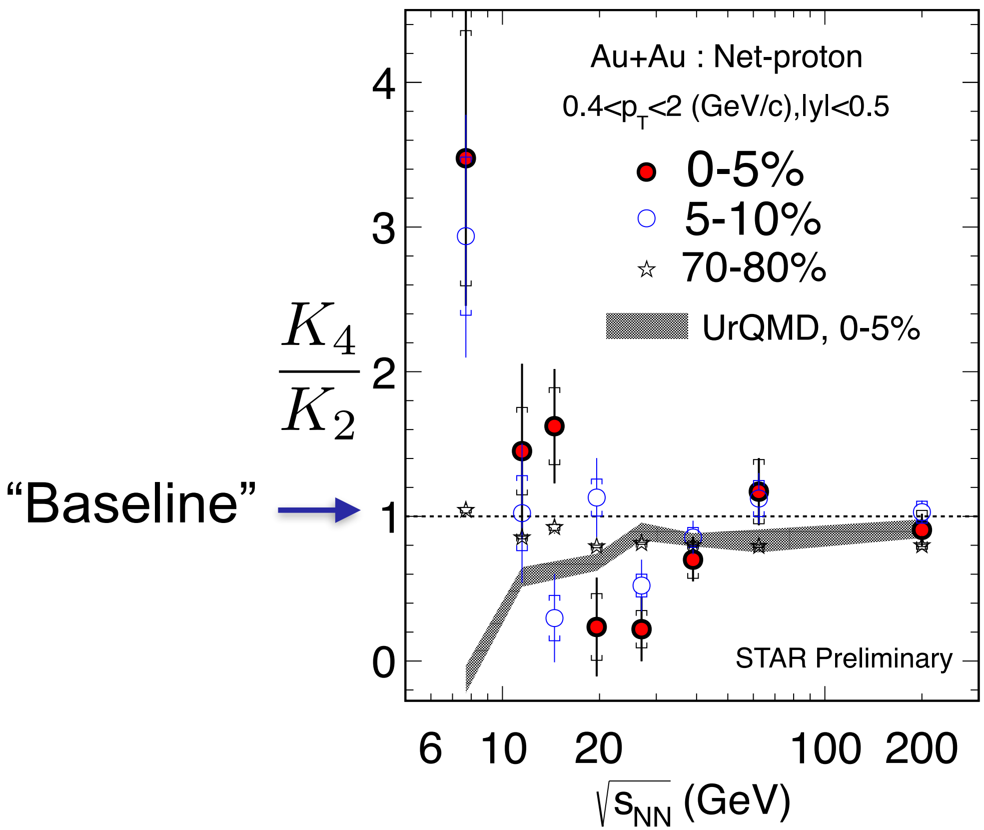
Almasi at al Model

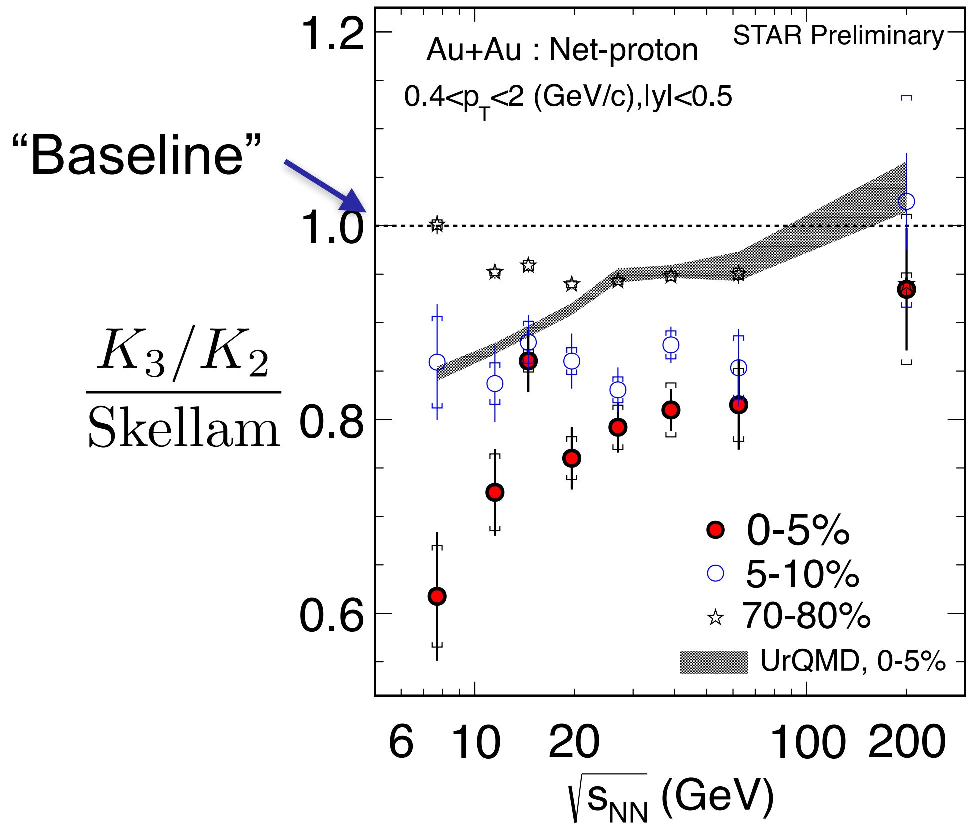


Latest STAR result on net-proton cumulants



X. Luo, NPA 956 (2016) 75





K_4/K_2 follows expectation, K_3/K_2 no so much…..
URQMD totally fails to get trend for K_4/K_2 !

Correlations near the critical point

M. Stephanov, 0809.3450, PRL 102

Scaling of Cumulants K_n with correlation length ξ

$$K_2 \sim \xi^2, \;\; K_3 \sim \xi^{4.5}, \;\; K_4 \sim \xi^7$$

Cumulants from Correlations

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Consequently:

$$C_2 \sim \xi^2, \;\; C_3 \sim \xi^{4.5}, \;\; C_4 \sim \xi^7$$

Correlations C_n pick up the most divergent pieces of cumulants K_n!

Reduced correlation function

Reduced correlation function

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)\, c_k(y_1,...,y_k)\, dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)\, dy_1\cdots dy_k}$$

$$C_k = \langle N \rangle^k c_k$$

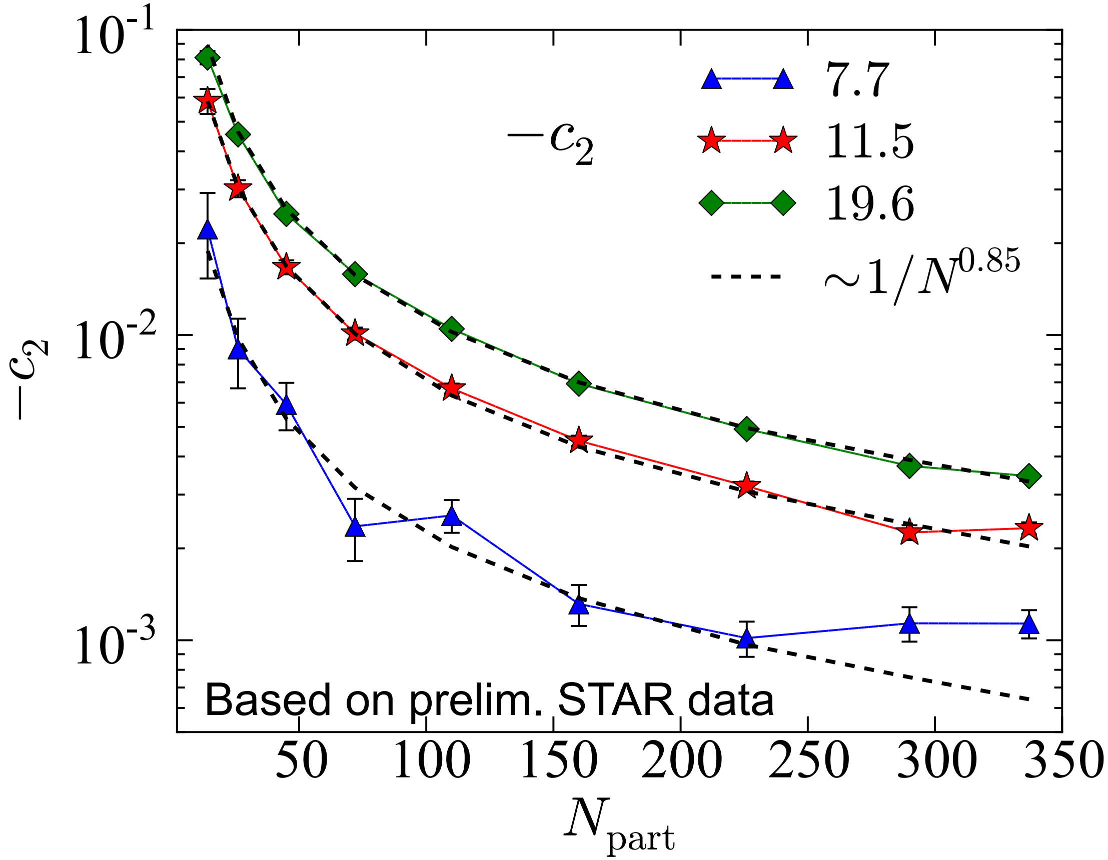
Independent sources such as resonances, cluster, p+p:

$$c_k \sim \frac{\langle N_s \rangle}{\langle N \rangle^k} \sim \frac{1}{\langle N \rangle^{k-1}}$$

For example two particle correlations:

$$c_2 \sim \frac{\text{Number of sources}}{\text{Number of all pairs}} = \frac{\text{Number of correlated pairs}}{\text{Number of all pairs}} = \frac{1}{\langle N \rangle}$$

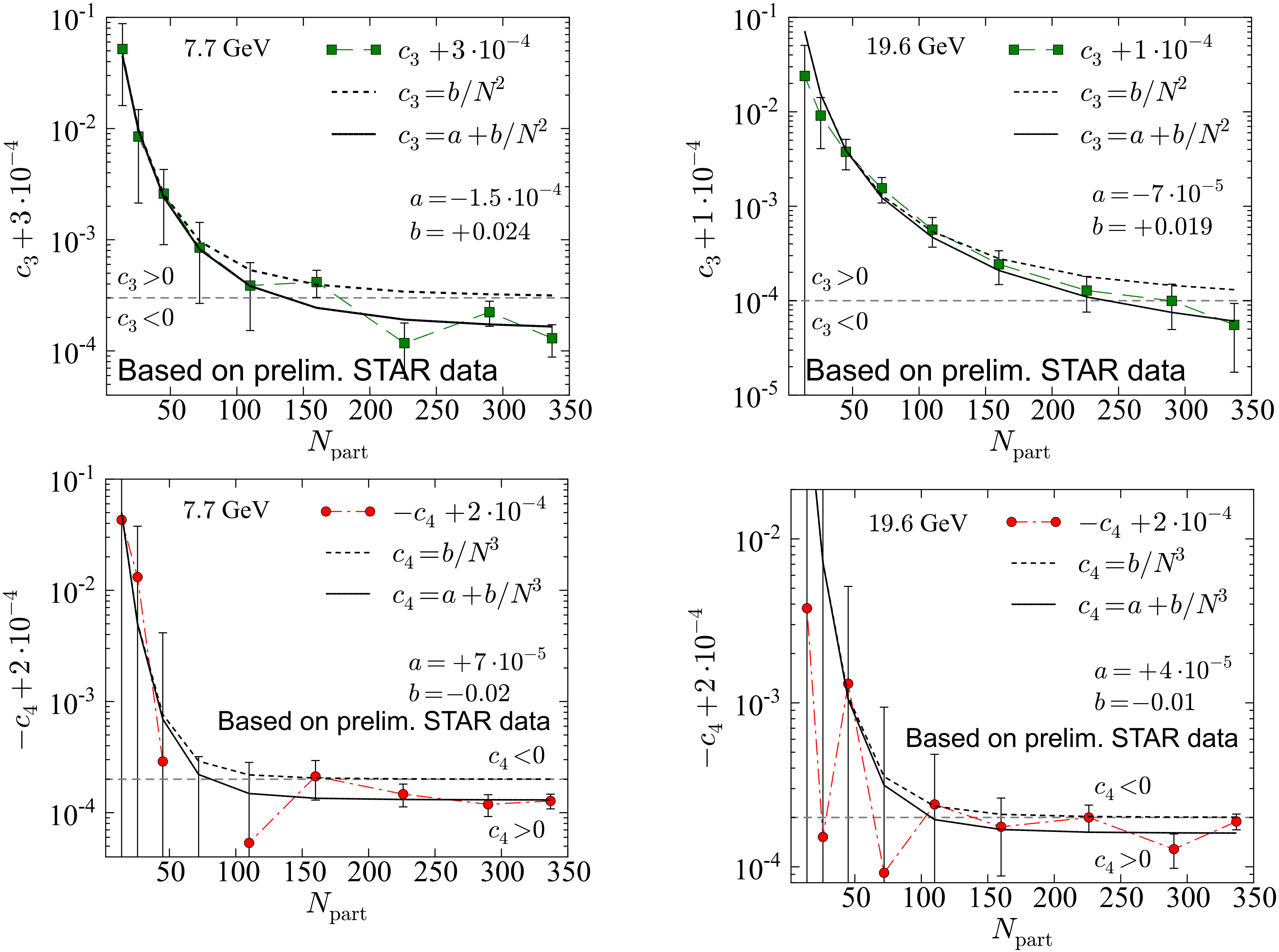
Centrality dependence



Centrality dependence

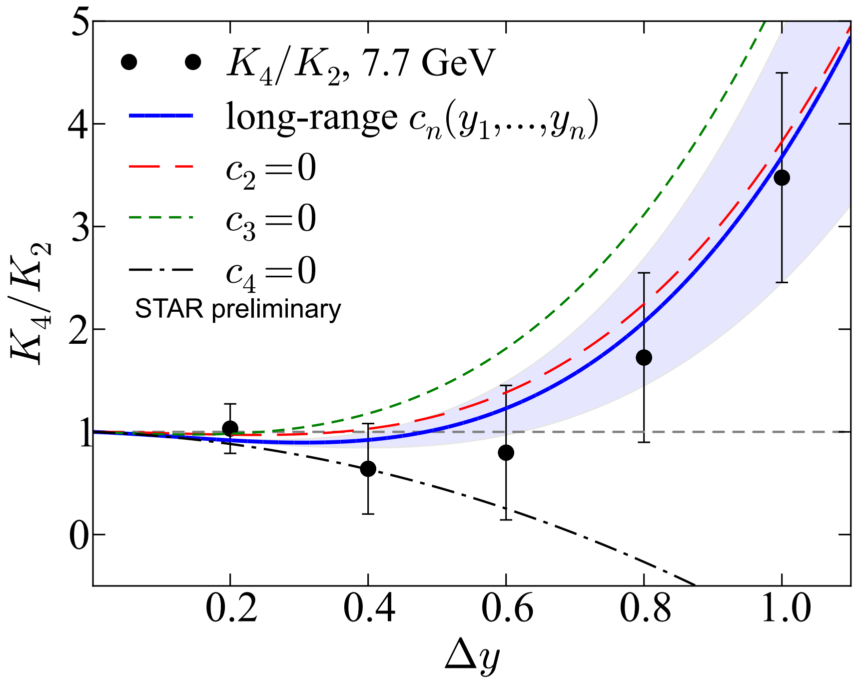
C3

C4

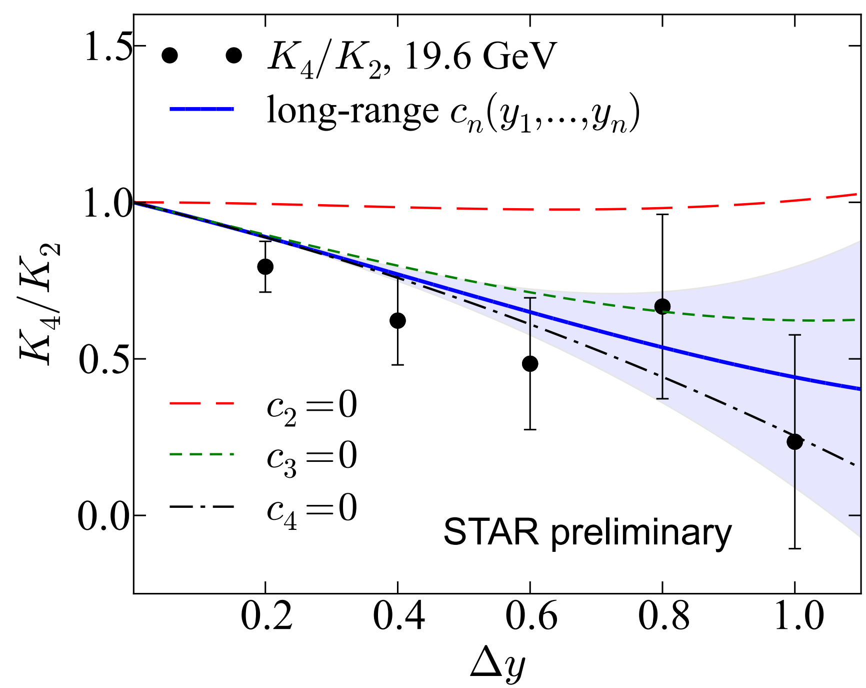


7.7 GeV   19.6 GeV

Preliminary Star data are consistent with long range correlations



7.7 GeV
central



19.6 GeV
central