Hirschegg 2019 workshop “From QCD matter to hadrons"

Axial Charge Fluctuation and Chiral Magnetic Effect from Stochastic Hydrodynamics

in Bjorken flow

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17/01/2019

Motivation

Current phenomenological studies assume large axial charge chemical potential μ_5 produced in Glasma phase and assume the conservation of axial charge throughout the evolution, which is valid in the long relaxation time limit.

We want to phenomenologically quantify the CME purely due to fluctuations and make comparisons with experiments.

arXiv.org > nucl-th > arXiv:1802.04941

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1.Stochastic hydrodynamics for axial charge

(1) Hydrodynamics equations and Fluctuations

Based on holographic D4/D8 model model proposed by

Iatrakis, Lin, and Yin (arXiv: 1506.01384),

the hydrodynamic framework incorporating both axial charge generation and dissipation effect can be written as,

$$\begin{cases} \partial_t n_5 + \nabla \cdot \mathbf{j}_5 = -2q, \\ \mathbf{j}_5 = -D\nabla n_5 + \xi, \\ q = \frac{n_5}{2\tau_{\mathrm{CS}}} + \xi_q, \end{cases}$$

thermal fluctuation

topological fluctuation

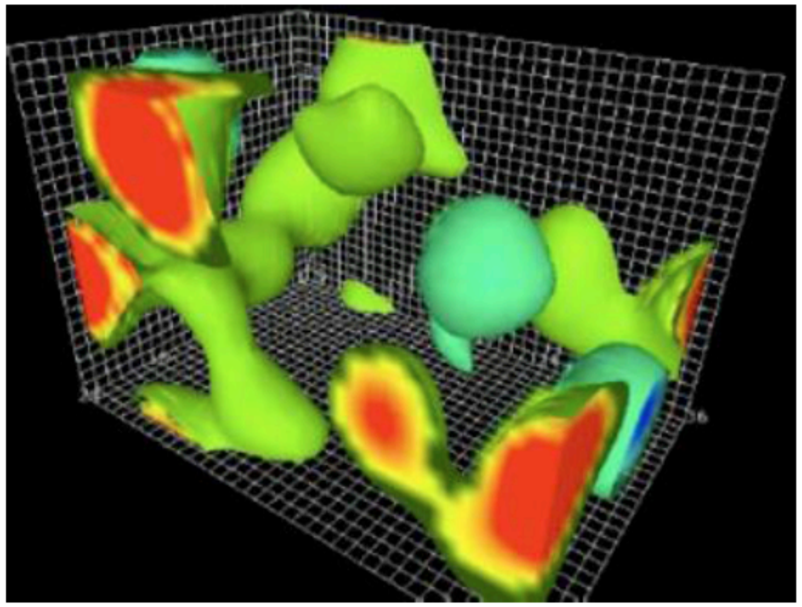
dissipation

$$\tau_{\mathrm{CS}} = 2\chi T/\Gamma_{\mathrm{CS}}$$

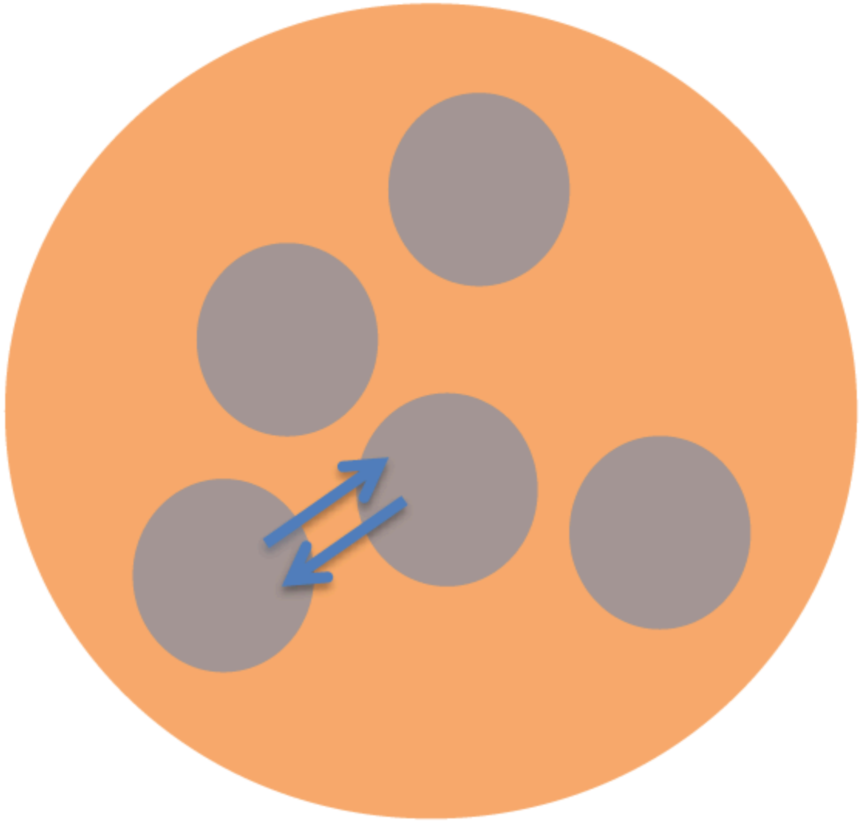
where χ is the charge susceptibility and Γ_{CS} is the Chern-Simon (CS) diffusion rate.

Topological noise vs Thermal noise

Topological noise
within fluid cell



Thermal noise
between fluid cells



The fluctuation relation is given by

$$\langle \boldsymbol{\xi}(t,\boldsymbol{x})\rangle = 0\,, \qquad \langle \xi_i(t,\boldsymbol{x})\xi_j(t,\boldsymbol{x}')\rangle = 2\sigma T\delta_{ij}\delta(t-t')\delta^3(\boldsymbol{x}-\boldsymbol{x}')$$

$$\langle \xi_q(t,\boldsymbol{x})\rangle = 0\,, \qquad \langle \xi_q(t,\boldsymbol{x})\xi_q(t,\boldsymbol{x}')\rangle = \Gamma_{\mathrm{CS}}\delta(t-t')\delta^3(\boldsymbol{x}-\boldsymbol{x}')$$

$$\langle \xi_q(t,\boldsymbol{x})\xi_i(t,\boldsymbol{x}')\rangle = 0$$

Assumptions:

1. Correlations of the mixture of these two types of noise vanish.

2. Axial Charge is small so that its dynamics decouples from the evolution of energy density and flow.

Correlation Function

$$C_{nn}(t,\boldsymbol{x}) = \langle [n_A(t,\boldsymbol{x}) - n_A(0,\boldsymbol{x})]\,[n_A(t,\boldsymbol{x}) - n_A(0,\boldsymbol{x})]\rangle$$

(2) Equations in Covariant form and in Bjorken flow

$$\begin{cases} \nabla_\mu J_5^\mu = -2q, \\ J_5^\mu = n_5 u^\mu - \sigma T P^{\mu\nu} \nabla_\nu \left(\frac{\mu_5}{T}\right) + P^{\mu\nu}\xi_\nu, \\ q = \frac{n_5}{2\tau_{\rm CS}} + \xi_q, \end{cases}$$

$$\langle P^{\mu\alpha}\xi_\alpha(x) P^{\nu\beta}\xi_\beta(x')\rangle = P^{\mu\alpha}P^{\nu\beta} g_{\alpha\beta} 2\sigma T \frac{\delta^4(x-x')}{\sqrt{-g}},$$

$$\langle \xi_q(x)\xi_q(x')\rangle = \Gamma_{\rm CS}\frac{\delta^4(x-x')}{\sqrt{-g}},$$

$$\langle P^{\mu\alpha}\xi_\alpha(x)\xi_q(x')\rangle = 0.$$

$$\nabla_\mu (n_5 u^\mu) - \nabla_\mu \left(D\chi T P^{\mu\nu}\nabla_\nu \left(\frac{n_5}{\chi T}\right)\right) + \frac{n_5}{\tau_{\rm CS}} = -\nabla_\mu (P^{\mu\nu}\xi_\nu) - 2\xi_q.$$

Einstein relations

$$\sigma = \chi D, \quad \tau_{\rm CS} = \frac{\chi T}{2\Gamma_{\rm CS}}$$

Apply to Bjorken flow

$$\partial_\tau n_5 + \frac{n_5}{\tau} + \frac{n_5}{\tau_{\rm CS}} - D\left(\partial_\perp^2 + \frac{\partial_\eta^2}{\tau^2}\right) n_5 = s$$

$$s = -\nabla_\mu \xi^\mu - 2\xi_q$$

Master Equation

(3) Choice of Parameters in Bjorken Flow

In Bjorken flow, we have $T \sim \tau^{-1/3}$. We adopt the following parametrization:

$$T = T_0 \left(\frac{\tau}{\tau_0}\right)^{-1/3},$$

The parameter τ_0 is an arbitrary time scale, which we take to be the initial time of hydrodynamics $\tau_0 = 0.6$ fm. Parameters with index 0 correspond to their values at $\tau = \tau_0$, or equivalently $T = T_0$.

Dimensional arguments lead to $\tau_{\rm CS} \sim \frac{1}{T}$ and $\Gamma_{\rm CS} \sim T^4$.

$$\tau_{\rm CS} = \left(\frac{\tau}{\tau_0}\right)^{1/3} \tau_{\rm CS0}, \quad \Gamma_{\rm CS} = \Gamma_0 \left(\frac{\tau}{\tau_0}\right)^{-4/3}$$

Using similar dimensional analysis for the scaling of diffusion constant

$\sigma \sim T \sim \tau^{-1/3}$ and $D \sim 1/T \sim \tau^{1/3}$, we adopt

$$\sigma = \sigma_0 \left(\frac{\tau}{\tau_0}\right)^{-1/3}, \quad D = D_0 \left(\frac{\tau}{\tau_0}\right)^{1/3}.$$

2.Axial Charge Evolution with Vanishing Initial Value

(1) Total correlator and Late Time Limit

The total axial charge N_5 is defined as an integration of n_5 on hypersurface with constant τ.

$$N_5 = \int d\eta d^2 x_\perp \tau n_5$$

Equal-time correlator:

$$\langle (N_5(\tau_2)^2) \rangle = \chi_0 T_0 \tau_0 \int d\eta d^2 x_\perp \left(1 - e^{3\left(1-\left(\frac{\tau_2}{\tau_0}\right)^{2/3}\right)\left(\frac{\tau_0}{\tau_{CS0}}\right)}\right)$$

$\tau_2' \to \tau_2$

Non-equal-time total correlator:

$$\langle N_5(\tau_2)\ N_5(\tau_2') \rangle = \chi_0 T_0 \tau_0 \int d^2 x_\perp \int d\eta \int d\eta' \left(e^{-C'}\big|_{\tau=\tau 2} - e^{-C'}\big|_{\tau=\tau 1=\tau 0} \right) \qquad (\tau_2 < \tau_2')$$

$$C' = \frac{3}{2\tau_0^{\frac{2}{3}}} \left(\tau_2^{\frac{2}{3}} + \tau_2'^{\frac{2}{3}} - 2\tau^{\frac{2}{3}} \right) \left(\frac{\tau_0}{\tau_{CS0}} \right)$$

At late time $\tau_2, \tau_2' \gg \tau_0, \tau_{CS0}$

$$\left\{ \begin{array}{c} \langle (N_5(\tau_2)^2) \rangle \\ \langle N_5(\tau_2)\ N_5(\tau_2') \rangle \end{array} \right. = \int \tau_0 d\eta d^2 x_\perp \chi_0 T_0 \sim \chi T V$$

Equilibrium Limit

(2) Rapidity-dependent correlators and late time limit

Equal-time correlator:

$$\int d^2x_\perp \langle \tau_2 n_5(\tau_2,\eta,x_\perp)\tau_2 n_5(\tau_2,0)\rangle = \chi_0 T_0 \tau_0 \int \frac{dk_\eta}{2\pi} e^{-ik_\eta \eta}\left(1 - e^{-2(c+bk_\eta^2)}|_{\tau=\tau_1}\right)$$

$$= \chi_0 T_0 \tau_0 \left(\delta(\eta) - e^{-2c}\frac{e^{-\eta^2/(8b)}}{2\sqrt{2\pi b}}|_{\tau=\tau_1}\right).$$

localized in rapidity

from topological fluctuation

from diffusion

$$b = \frac{3D_0}{2\tau_0^{1/3}}\left(\frac{1}{\tau^{2/3}} - \frac{1}{\tau_2^{2/3}}\right),$$

$$c = \frac{3}{2\tau_0^{2/3}}\left(-\tau^{2/3} + \tau_2^{2/3}\right)\left(\frac{\tau_0}{\tau_{\rm CS0}}\right).$$

Note that the first term is a positive delta function, meaning that it is localized in one fluid cell.

The second term is negative, corresponding to anti-correlation over rapidity.

Some arguments on the delta function

It was argued that the delta function term corresponds to self-correlation of particles, thus should be excluded in the calculation of balance function.

But in case of CME, we think it should be kept.

The reasoning is clearest in the late time limit when the second term can be ignored.

$$\langle n_5(\tau_2, \eta, x_\perp) n_5(\tau_2, 0)\rangle \simeq \frac{\chi_0 T_0 \tau_0}{\tau_2} \frac{\delta(\eta)}{\tau_2 \int d^2 x_\perp}.$$

It is easy to show the first factor is nothing but $\chi(\tau_2)T(\tau_2)$, while the second factor is volume of fluid cell at τ_2, with $\delta(\eta)$ setting size of the cell in rapidity direction. Therefore it contains more than just self-correlation.

The first term should be interpreted as a measure of amount of charge in fluid cells.

Equal-time correlator:

$$\int d^2x_\perp \langle \tau_2 n_5(\tau_2,\eta,x_\perp)\tau_2 n_5(\tau_2,0)\rangle = \chi_0 T_0 \tau_0 \left(\delta(\eta) - e^{-2c_1}\frac{e^{-\eta^2/(8b_1)}}{2\sqrt{2\pi b_1}}\right)$$

when $\tau_2' \to \tau_2$

Non-equal-time local correlator:

$$\int_{\tau_2 < \tau_2'} d^2x_\perp \langle \tau_2 n_5(\tau_2,\eta,x_\perp)\ \tau_2' n_5(\tau_2',0)\rangle = \chi_0 T_0 \tau_0 \left(e^{-C_2'}\frac{e^{-\frac{\eta^2}{4B_2'}}}{2\sqrt{\pi B_2'}} - e^{-C_1'}\frac{e^{-\frac{\eta^2}{4B_1'}}}{2\sqrt{\pi B_1'}}\right)$$

$$B' = \frac{3D_0}{2\tau_0^{\frac{1}{3}}}\left(\frac{2}{\tau_2^{\frac{2}{3}}} - \frac{1}{\tau_2'^{\frac{2}{3}}} - \frac{1}{\tau^{\frac{2}{3}}}\right)$$

$$C' = \frac{3}{2\tau_0^{\frac{2}{3}}}\left(\tau_2^{\frac{2}{3}} + \tau_2'^{\frac{2}{3}} - 2\tau^{\frac{2}{3}}\right)\left(\frac{\tau_0}{\tau_{CS0}}\right)$$

Late Time Limits $(\tau_2 >> \tau_0, \quad \tau_2' >> \tau_0)$

$$\chi_0 T_0 \tau_0 \delta(\eta)$$

3.Phenomenology

(1) Conversion to electric charge correlators via CME

The CME current in unit of e :

$$\vec{j} = C_e \mu_5 e \vec{B}$$

$$\left(C_e = \sum_f q_f^2 \frac{N_c}{2\pi^2} = \frac{1}{\pi^2},\ B = B_0 e^{-\frac{\tau}{\tau_B}},\ eB_0 = 10 m_\pi^2,\ \tau_B = 3 fm \right)$$

Charge separation
with respect to reaction plane:

$$Q = \int_{\tau_0}^{\tau_f} d\tau \tau d\eta 2Rj$$

$$\left(\tau_0 = 0.6\ fm,\ \tau_f = 7\ fm \right)$$

The electric chemical potential
at freeze-out time:

$$\mu_e(\tau_f) = \frac{Q}{V\chi_Q}$$

$$\left(V = \frac{1}{2}\pi R^2 \tau_f \int d\eta,\ \chi_Q = \sum_f q_f^2 T^2 = \frac{2}{3} T^2 \right)$$

$$\langle \mu_e(\tau_f) \mu_e(\tau_f) \rangle$$

$$= \left(\frac{4}{R\tau_f \pi^3 \chi_Q(\tau_f) \Delta\eta} \right)^2 \int_{\tau_0}^{\tau_f} d\tau \int_{\tau_0}^{\tau_f} d\tau' B(\tau) B(\tau') \int \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta d\eta' \frac{\langle \tau n_5(\tau_2, \eta, x_\perp) \tau n_5(\tau_2', \eta', x_\perp') \rangle}{\chi(\tau_2)\chi(\tau_2')}$$

Bring our correlators here

$$\left(\langle \mu_e(\tau_f) \rangle = 0,\ \langle \mu_e(\tau_f) \mu_e(\tau_f) \rangle \approx \mu_e(\tau_f)^2 \right)$$

If we assume axial charge fluctuation reaches equilibrium at the beginning of the evolution time , we can roughly estimate an effective electric chemical potential versus centrality:

$$\int d^2x_\perp \langle \tau_2 n_5(\tau_2,\eta,x_\perp)\tau_2 n_5(\tau_2,0)\rangle = \chi_0 T_0 \tau_0\,\delta(\eta)$$

TABLE III. The centrality dependence of $e\mu_e(\tau_f)$.

Centrality	60-70%	50-60%	40-50%	30-40%	20-30%	10-20%	5-10%	0-5%
$e\mu_e(\tau_f)$(MeV)	4.33	3.03	2.26	1.74	1.37	1.07	0.88	0.76

If we don't assume fluctuation equilibrium, we can have a similar table, but the values gets smaller.

$$\int d^2x_\perp \langle \tau_2 n_5(\tau_2,\eta,x_\perp)\;\tau_2' n_5(\tau_2',0)\rangle \;=\chi_0 T_0 \tau_0 \left(e^{-C_2'}\frac{e^{-\frac{\eta^2}{4B_2'}}}{2\sqrt{\pi B_2'}} - e^{-C_1'}\frac{e^{-\frac{\eta^2}{4B_1'}}}{2\sqrt{\pi B_1'}} \right)$$

TABLE I. The centrality dependence of $e\mu_e(\tau_f)$.

Centrality	60-70%	50-60%	40-50%	30-40%	20-30%	10-20%	5-10%	0-5%
$e\mu_e(\tau_f)$(MeV)	2.80	1.96	1.46	1.13	0.88	0.69	0.57	0.49

Charge fluctuation is more significant in peripheral collisions than in central regions.

This is a reflection of the fact that fluctuation is suppressed by volume factor.

(2) Cooper-Frye Freezeout Procedure

The spectrum of Cooper-Frye freeze-out:

$$\frac{dN_Q^i}{d\phi} = \frac{g_i}{(2\pi)^3}\int dy p_\perp dp_\perp \int d\sigma_\mu p^\mu f_i(x,p)$$

$$f_i = e^{p_\mu u^\mu/T_f + Q\mu_e/T_f + \mu_i/T_f}$$

In case of Bjorken flow:

$$\frac{dN_Q^i}{d\phi} = \frac{g_i}{(2\pi)^3}\int dy dm_\perp m_\perp^2 \int \tau_f d\eta d^2x_\perp \cosh(\eta - y) f_i(x,p)$$

$$m_\perp = \sqrt{p_\perp^2 + m^2}$$

To the lowest order in μ_e :

$$\delta\frac{dN_Q^i}{d\phi} = \frac{g_i}{(2\pi)^3}\int dy m_\perp^2 dm_\perp \int \tau_f d\eta d^2x_\perp \cosh(\eta - y) f_i(\mu_e = 0)\mu_e(\tau_f, \eta)$$

Asymmetric charged particle distribution due to CME

$$\frac{dN^i_Q}{d\phi} \quad \longrightarrow \quad \delta\frac{dN^i_Q}{d\phi}$$

$$N_Q = \sum_{i \in Q} \int d\phi \, \frac{dN^i_Q}{d\phi} \quad \longrightarrow \quad \Delta_Q = \sum_{i \in Q} \int d\phi \, \delta\frac{dN^i_Q}{d\phi}$$

(Charged particle multiplicity)

(Induced charged particle due to CME)

$$a_{QQ} \equiv \boxed{\frac{\pi^2}{16}\frac{\langle \Delta_Q \Delta_Q \rangle}{\langle N_Q \rangle \langle N_Q \rangle}} = \frac{\pi^2}{16 T_f^2} \frac{\int dy_1 d\eta_1 dy_2 d\eta_2 \; H(y_1 - \eta_1) H(y_2 - \eta_2) \; \langle \mu_e(\tau_f, \eta_1) \mu_e(\tau_f, \eta_2) \rangle}{\left[\int H(y-\eta) dy d\eta\right]^2}$$

$$\begin{cases} H(y-\eta) = \sum_{i=\pi,K} h_i(y-\eta) \\ h_i(y-\eta) = e^{\frac{-m\cosh(y-\eta)+\mu_i}{T_f}} \left(m^2 + 2m\cosh^{-1}(y-\eta) T_f + 2\cosh(y-\eta)^{-2} T_f^2\right) \end{cases}$$

(3) Centrality&Rapidity-Dependent Correlators

We assume the following ansatz for the generated charged single-particle spectrum,

$$\frac{dN_\pm}{d\phi} = \frac{\langle N_\pm \rangle}{2\pi}\left[1 + 2\sum_{n=1} v_n \cos n(\phi - \Psi_n)\right] + \frac{1}{4}\Delta_\pm \sin(\phi - \Psi_{RP}).$$

If we take the correlation to be a simple product of single-particle distribution, as

$$\rho(\phi_1, \phi_2) = \left\langle \frac{dN^\alpha}{d\phi_1^\alpha}\frac{dN^\beta}{d\phi_2^\beta} \right\rangle$$

Consider only the parity-even v1, we get

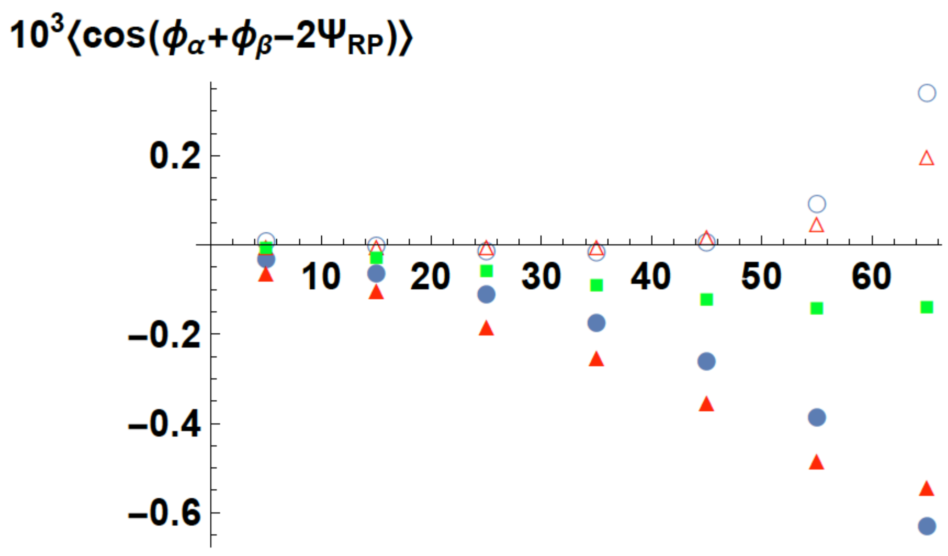
$$\gamma_{\alpha\beta} = \langle \cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\mathrm{RP}})\rangle = \frac{\int \rho(\phi_1, \phi_2)\cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\mathrm{RP}})\,\mathrm{d}\phi_1^\alpha\,\mathrm{d}\phi_2^\beta}{\int \rho(\phi_1, \phi_2)\,\mathrm{d}\phi_1^\alpha\,\mathrm{d}\phi_2^\beta}$$

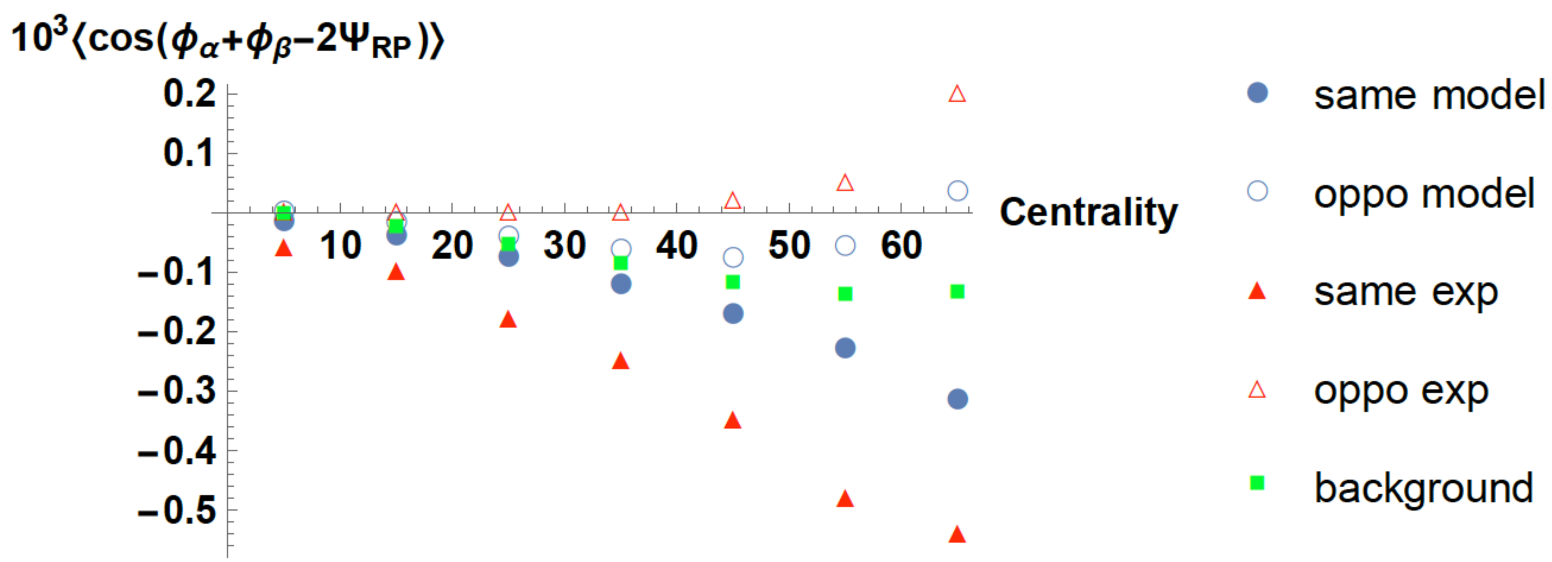
$$= \langle v_1^2 \cos 2(\Psi_1 - \Psi_{RP})\rangle - \boxed{\frac{\pi^2}{16}\frac{\langle \Delta_\alpha \Delta_\beta\rangle}{\langle N_\alpha\rangle\langle N_\beta\rangle}}$$

Background

asymmetry due to CME

Estimated **Non-equal-time integrated**





Centrality dependence of the calculated Gamma correlator (circle dots), in comparison to that in Au-Au 200GeV (triangle dots).

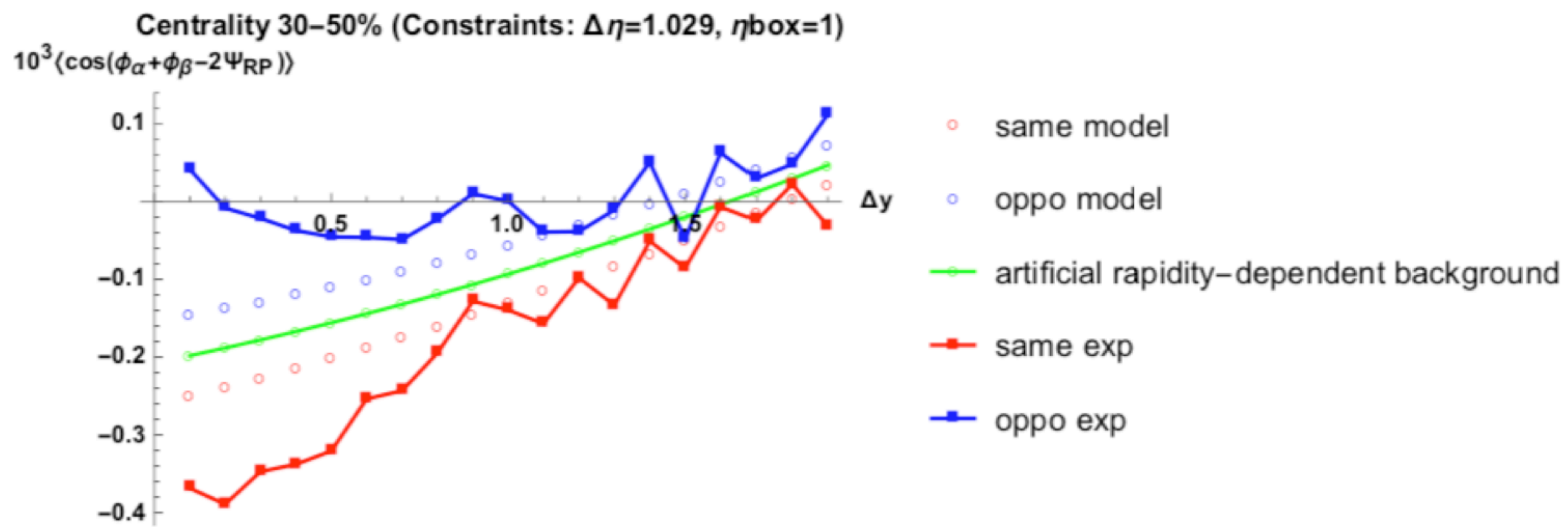
Combined effects from CME and a charge-independent background shows the reasonably-shaped centrality-dependence: the correlations get stronger as in more peripheral collisions.

Reference for experimental datas:

B. I. Abelev et al. Observation of charge-dependent azimuthal correlations and possible local strong parity violation in heavy ion collisions. Phys. Rev., C81:054908, 2010.

Rapidity-dependent correlators:

$$a_{QQ}(\Delta y) = \frac{\pi^2}{16T_f^2} \frac{\int_0^{1-\frac{\Delta y}{2}} dy \int d\eta_1 d\eta_2 \; H(y+\frac{\Delta y}{2}-\eta_1)H(y-\frac{\Delta y}{2}-\eta_2)\langle \mu_e(\tau_f,\eta_1)\mu_e(\tau_f,\eta_2)\rangle}{\int_0^{1-\frac{\Delta y}{2}} dy \int d\eta_1 d\eta_2 \; H(y+\frac{\Delta y}{2}-\eta_1)H(y-\frac{\Delta y}{2}-\eta_2)}$$



Rapidity dependence of the Gamma correlator.

I temporally make artificial background contribution for comparison.

Extensions

- We will include elliptic flow in our background, re-calculate our correlation considering the “underlying correlation”.

- We will try to plot the Gamma correlator versus average transverse momentum and transverse momentum difference.

- We will generalize our method to include non-trivial rapidity-dependent temperature profile, and compare our correlator with cases in lower beam energy collisions.

Constraints of our current model

- Background from other sources not included.
- An uncertainty in the magnetic field.
- Moment conservation and charge conservation not considered.
- Back-reaction to energy density and flow from axial charge dynamics ignored.
- ….

Thank you!