Universität Bielefeld

CRC-TR 211
Strong-interaction matter under extreme conditions

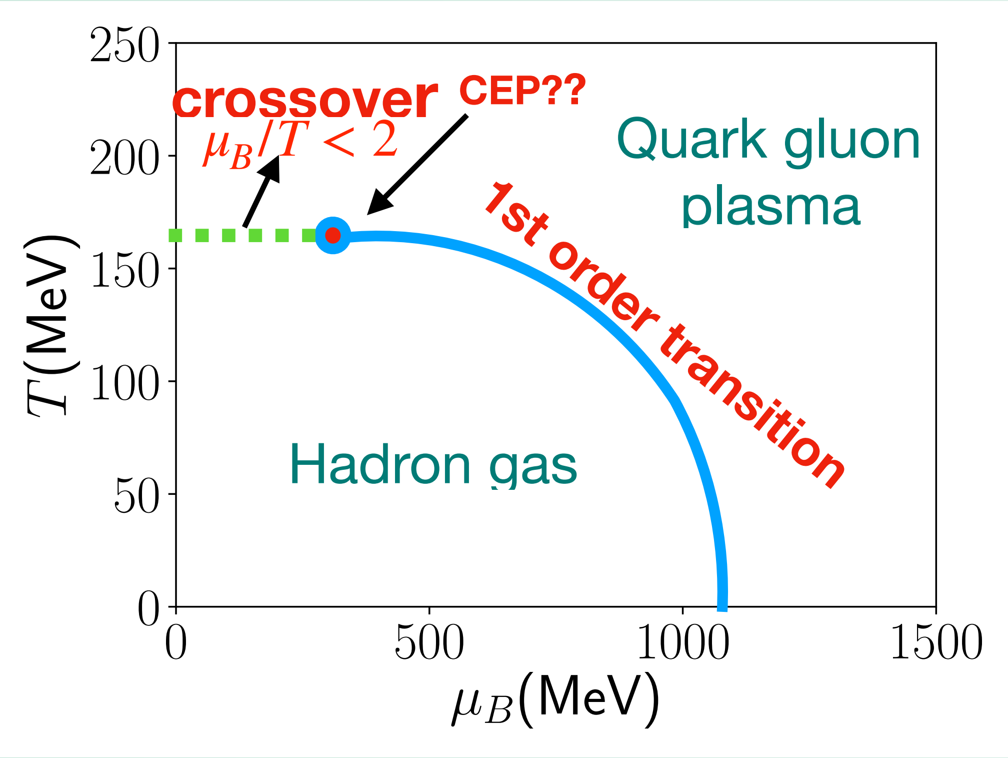
Critical end points in (2+1)-flavor QCD with imaginary chemical potential

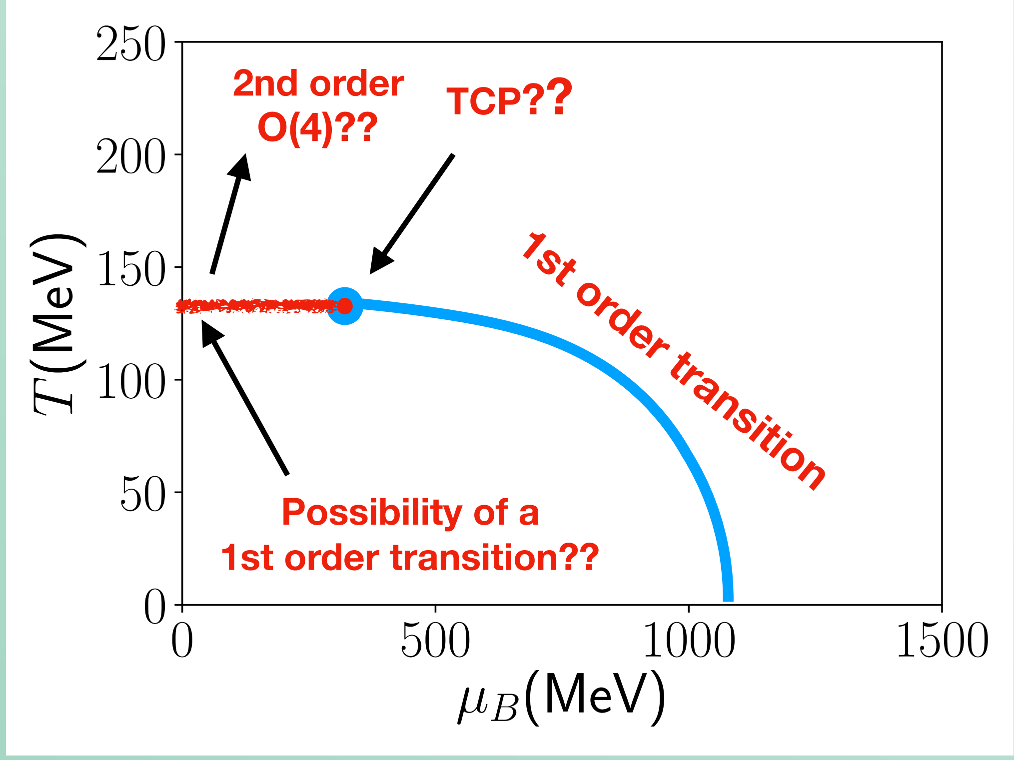
In collaboration with: Frithjof Karsch, Christian Schmidt, and Anirban Lahiri

Jishnu Goswami

From QCD matter to hadrons
"Hirschegg 2019"

QCD phase diagram





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Pisarski, Wilczek, PRD 29 (1984)

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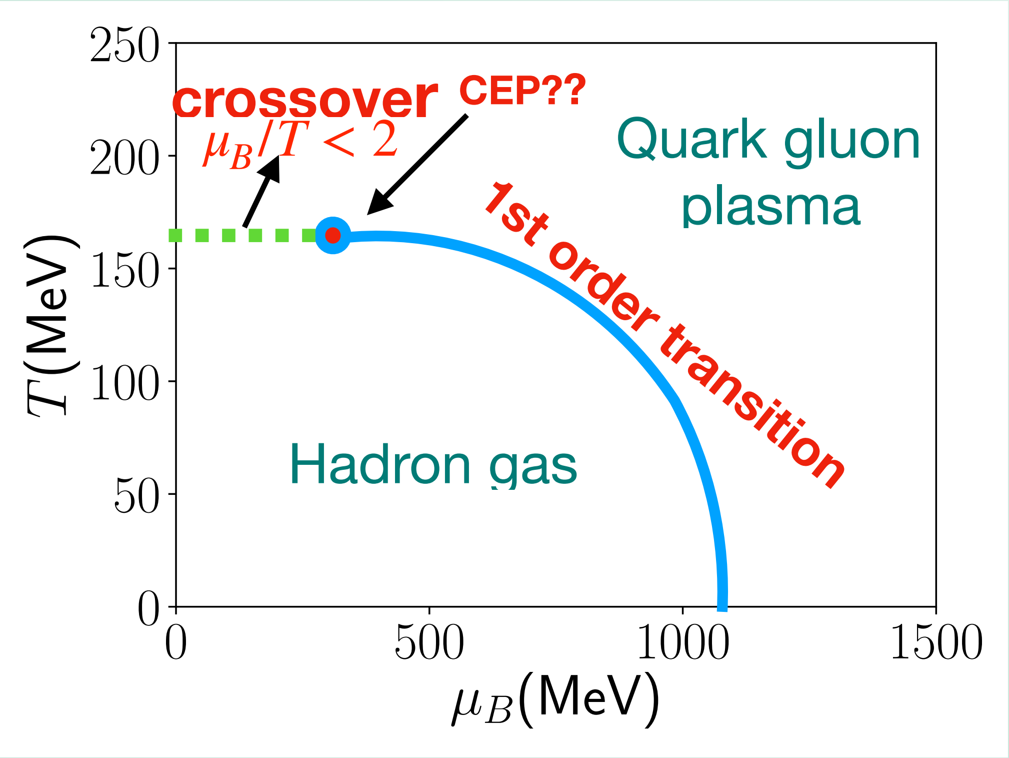

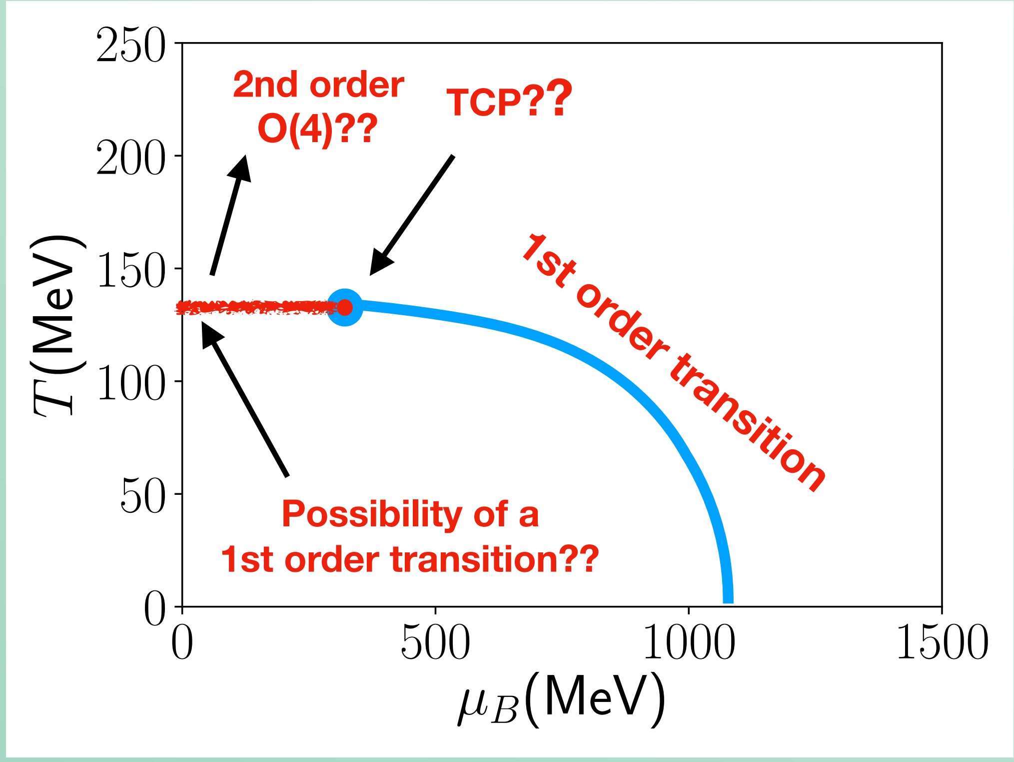

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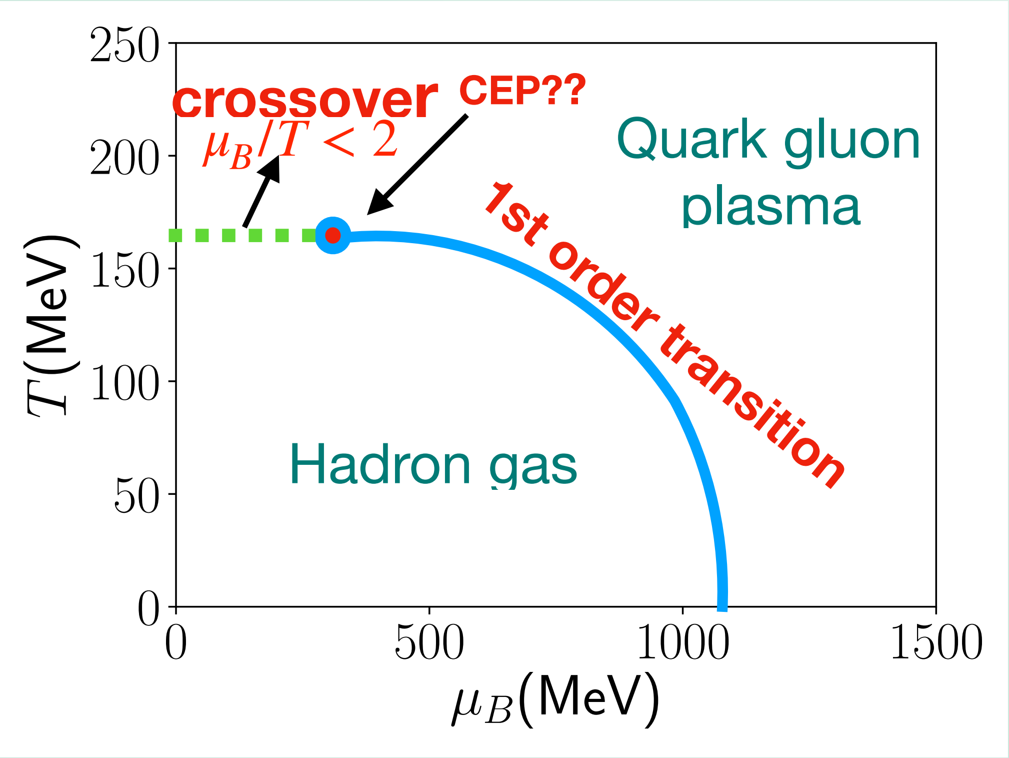

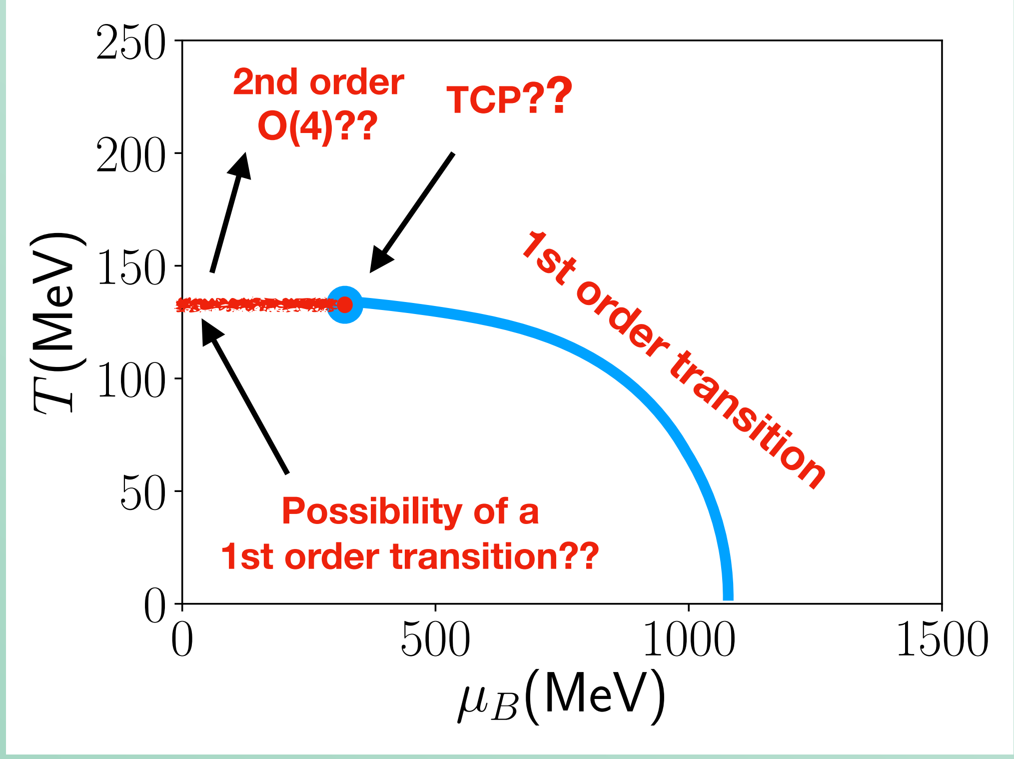

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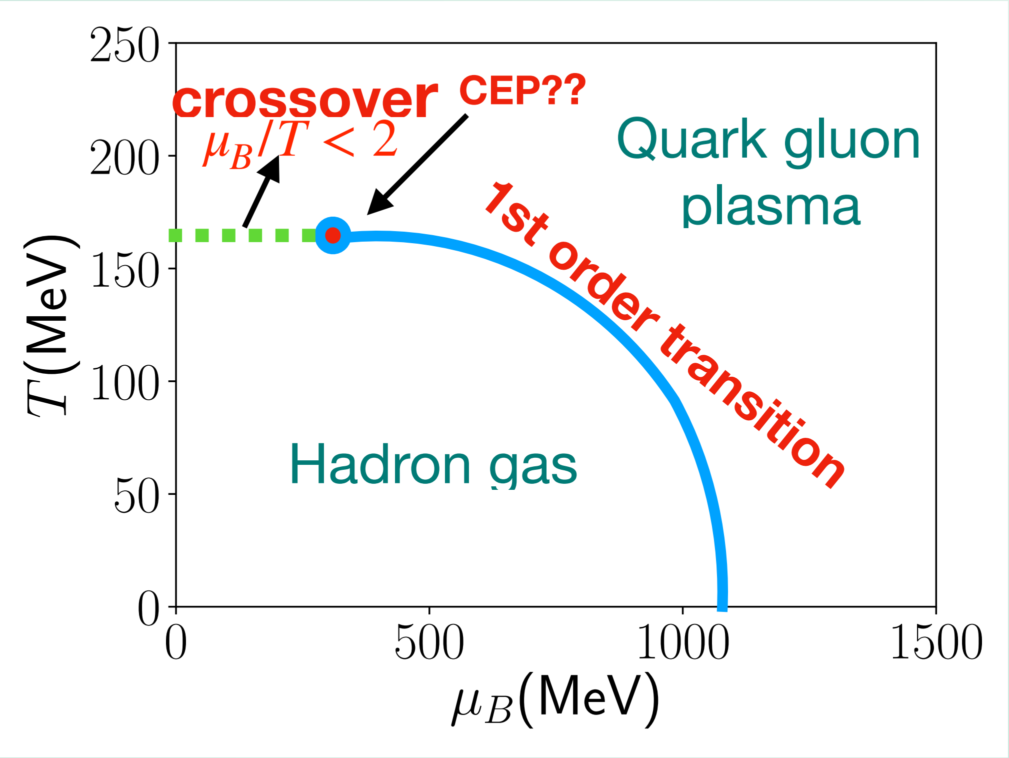

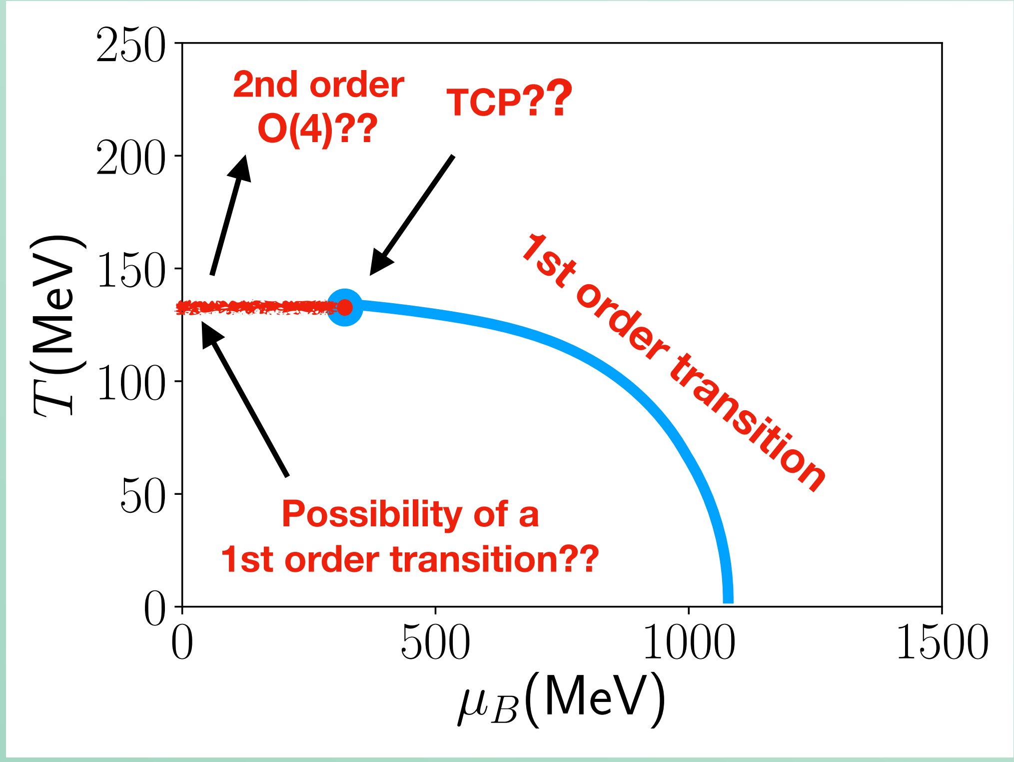

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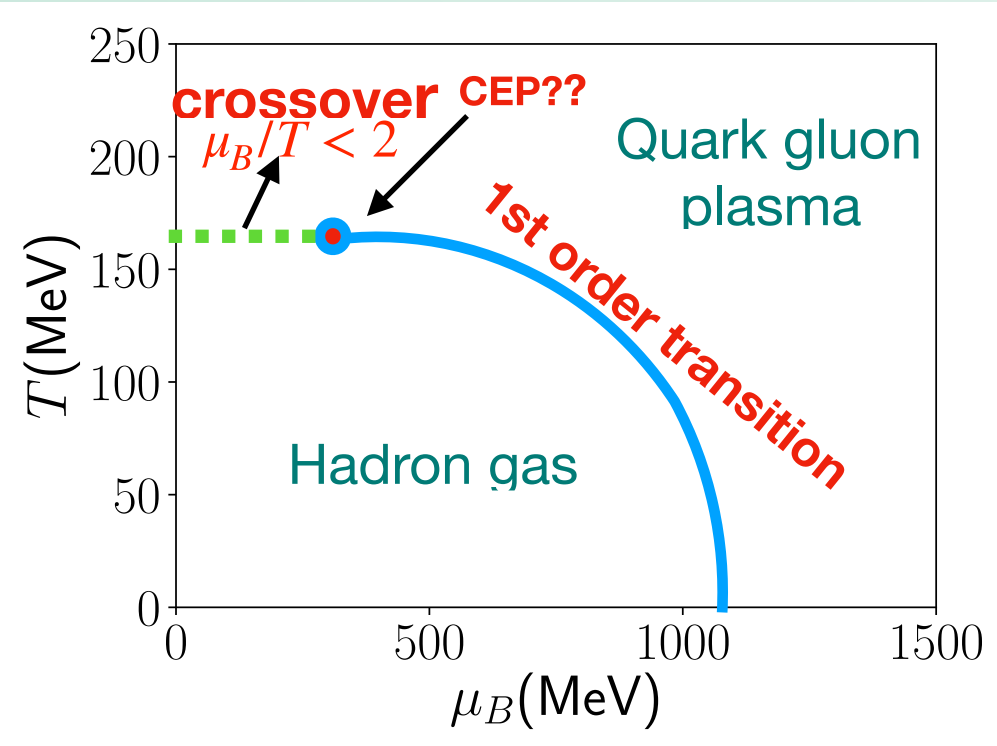

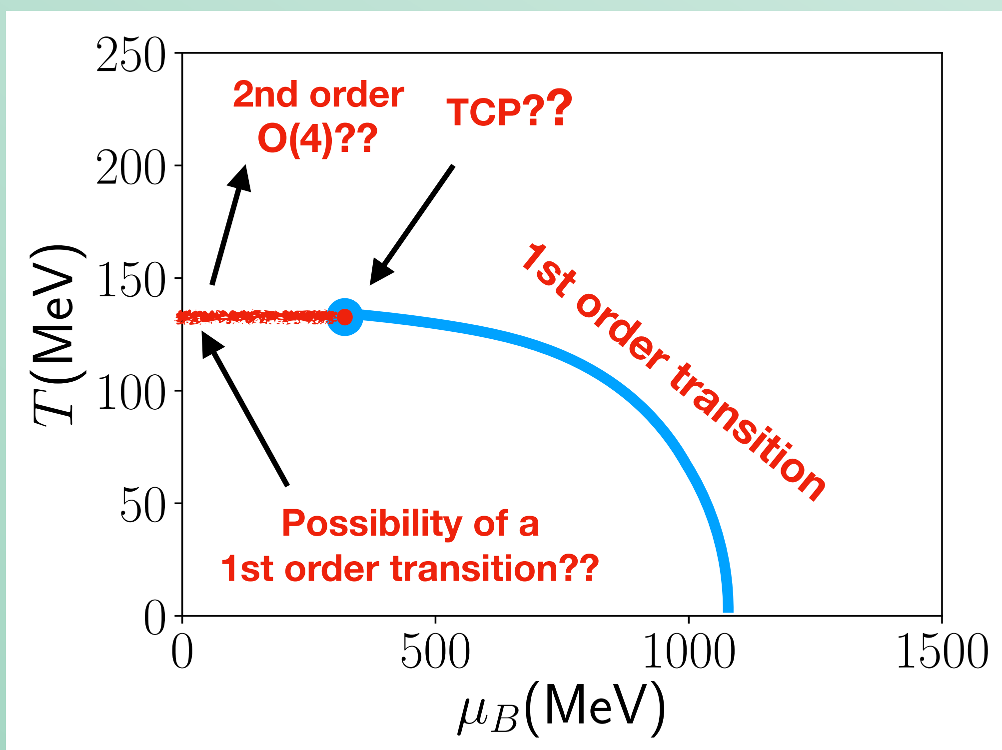

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- Although, if $U(1)_A$ restoration happens at T_c then there is a possibility of a fluctuation induced 1st order transition in the chiral limit.[See talk by Lukas]
- Order of the phase transition in the chiral limit is also of relevance for studies of fluctuations at the LHC.

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Chiral transition for zero chemical potential with HISQ

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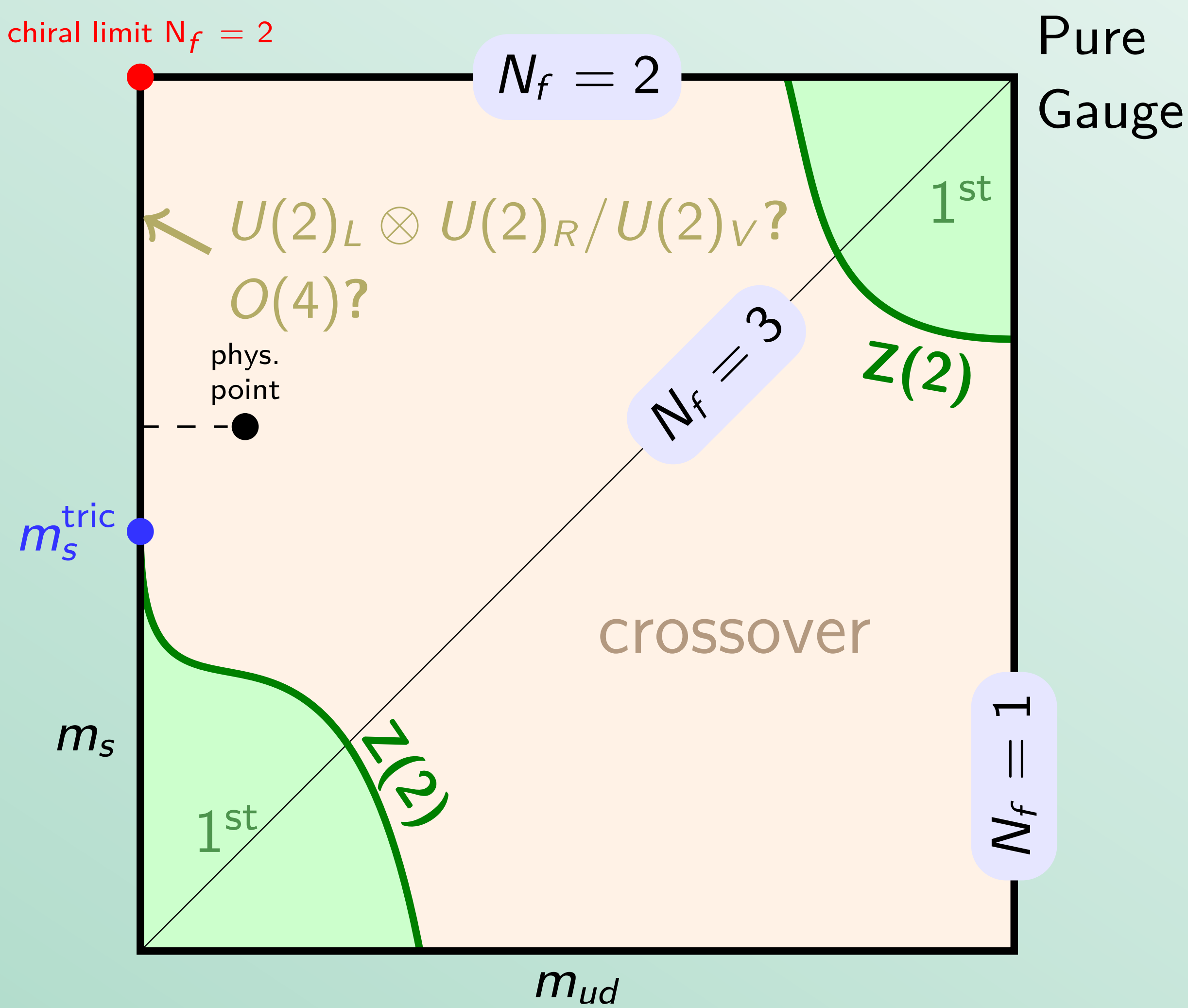

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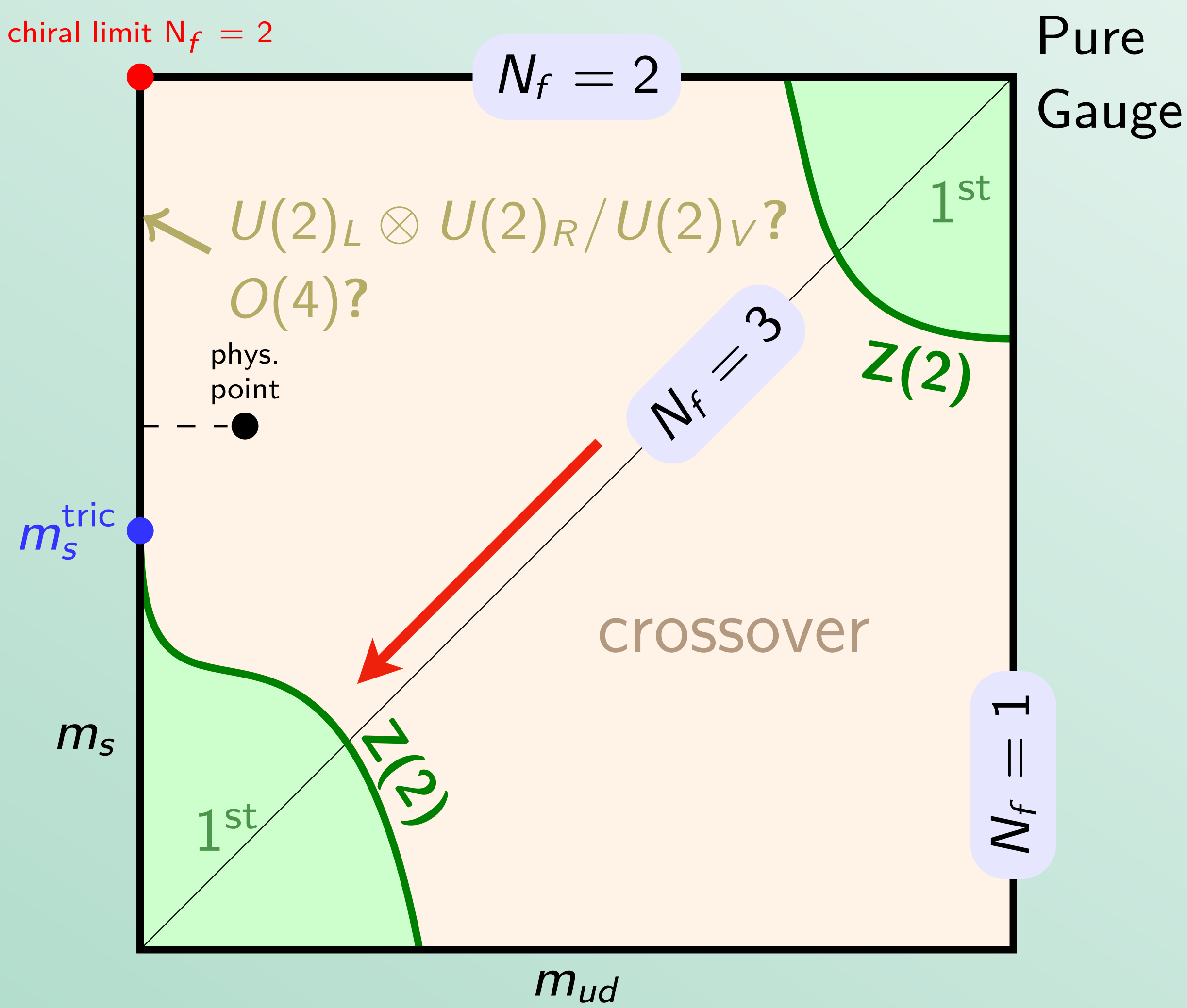

N_f=3 : 1st order phase transition ruled out for 230 MeV > m_π >80 MeV. Bound on critical pion mass is given as, $m_\pi^{cr} \lesssim 50$ MeV from the scaling analysis.

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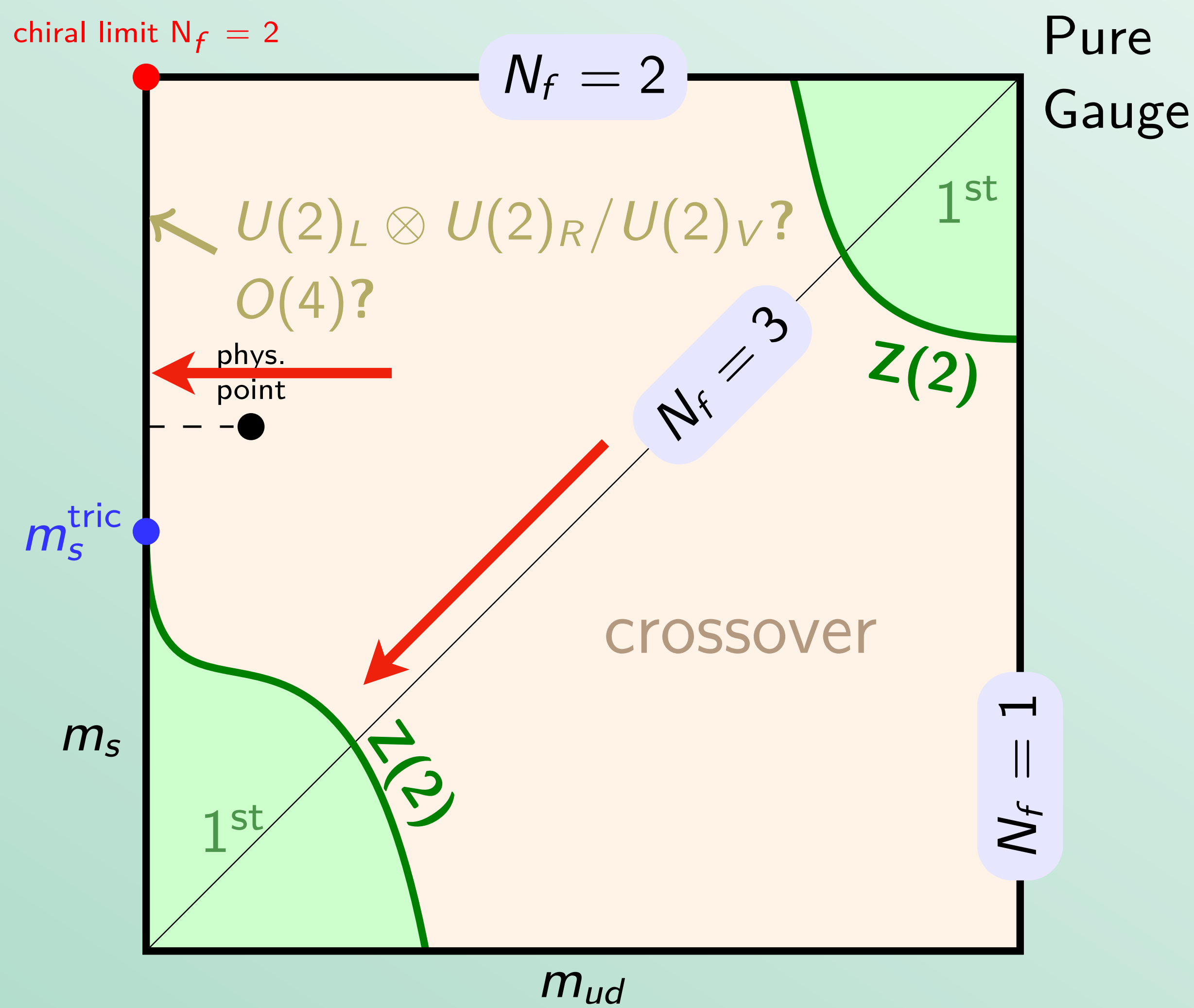

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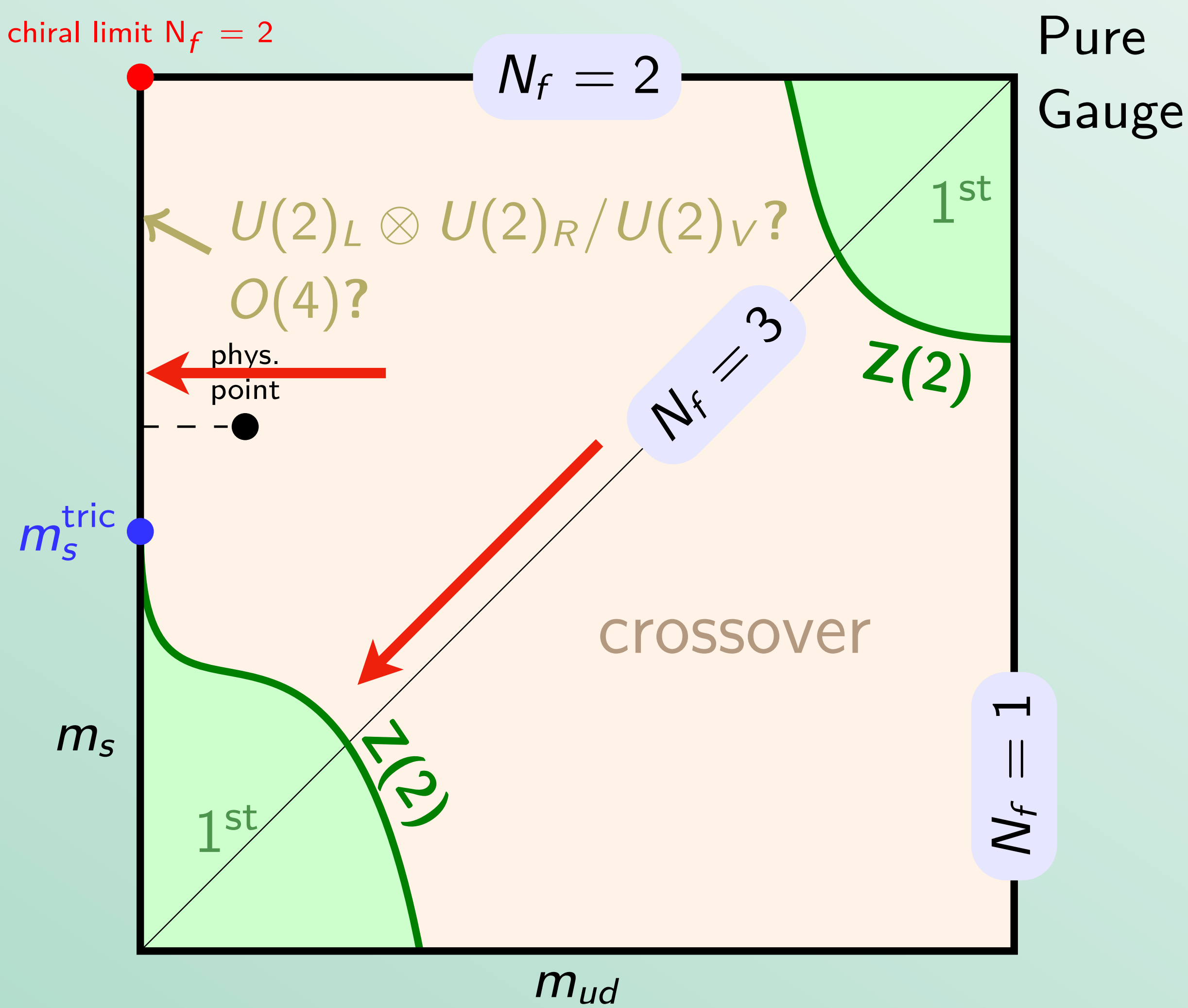

See talk by Frithjof Karsch

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Bazavov et. al. PRD 95, 074505 (2017)

N_f=2+1 : No hint of 1st order phase transition for m_π >55 MeV. Chiral transition is most likely 2nd order O(4) rather than 1st order.

A. Lahiri et. al. , QM 2018, arXiv:1807.05727

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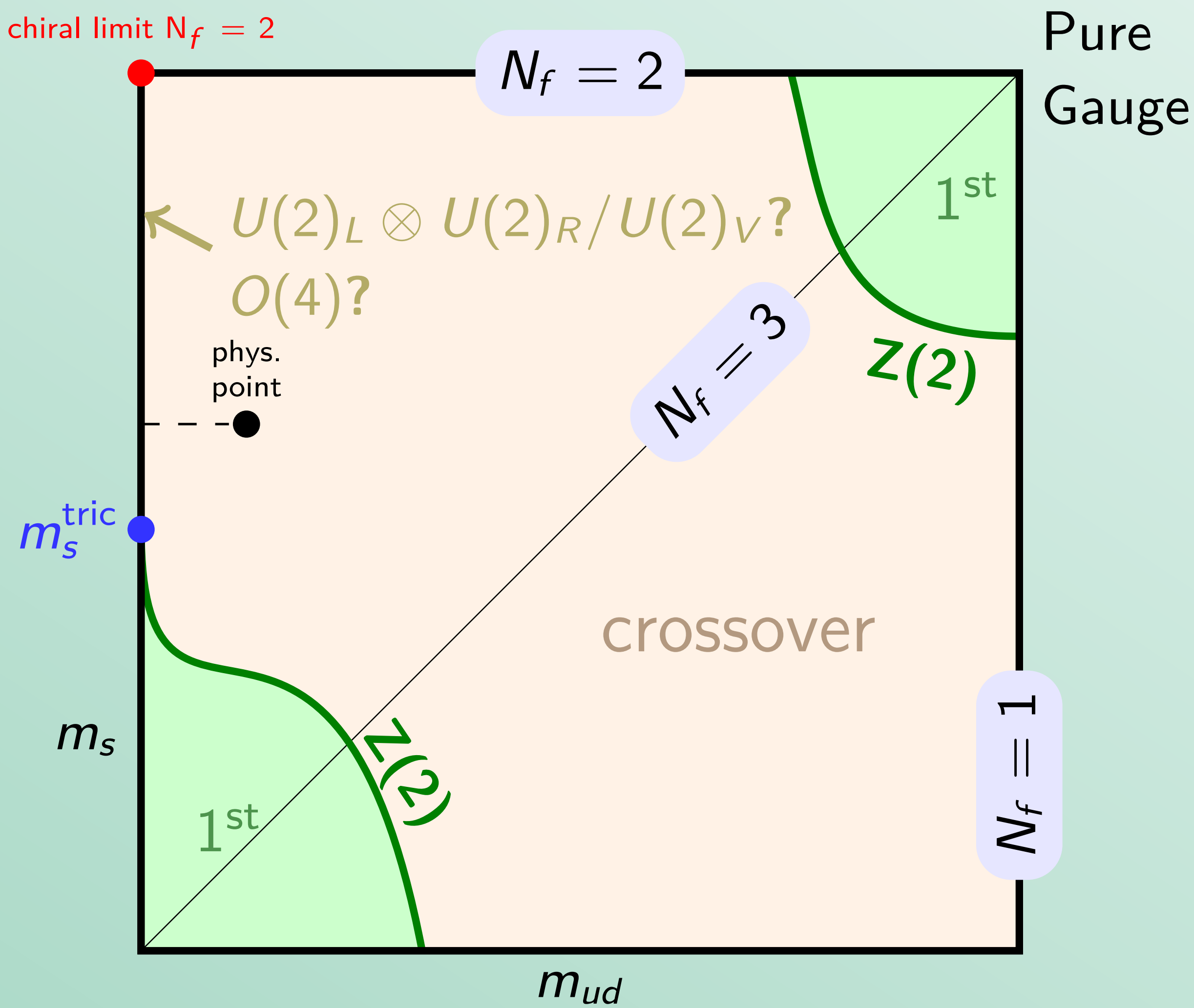

HotQCD: Based on studies with HISQ action

Chiral transition for zero chemical potential with HISQ

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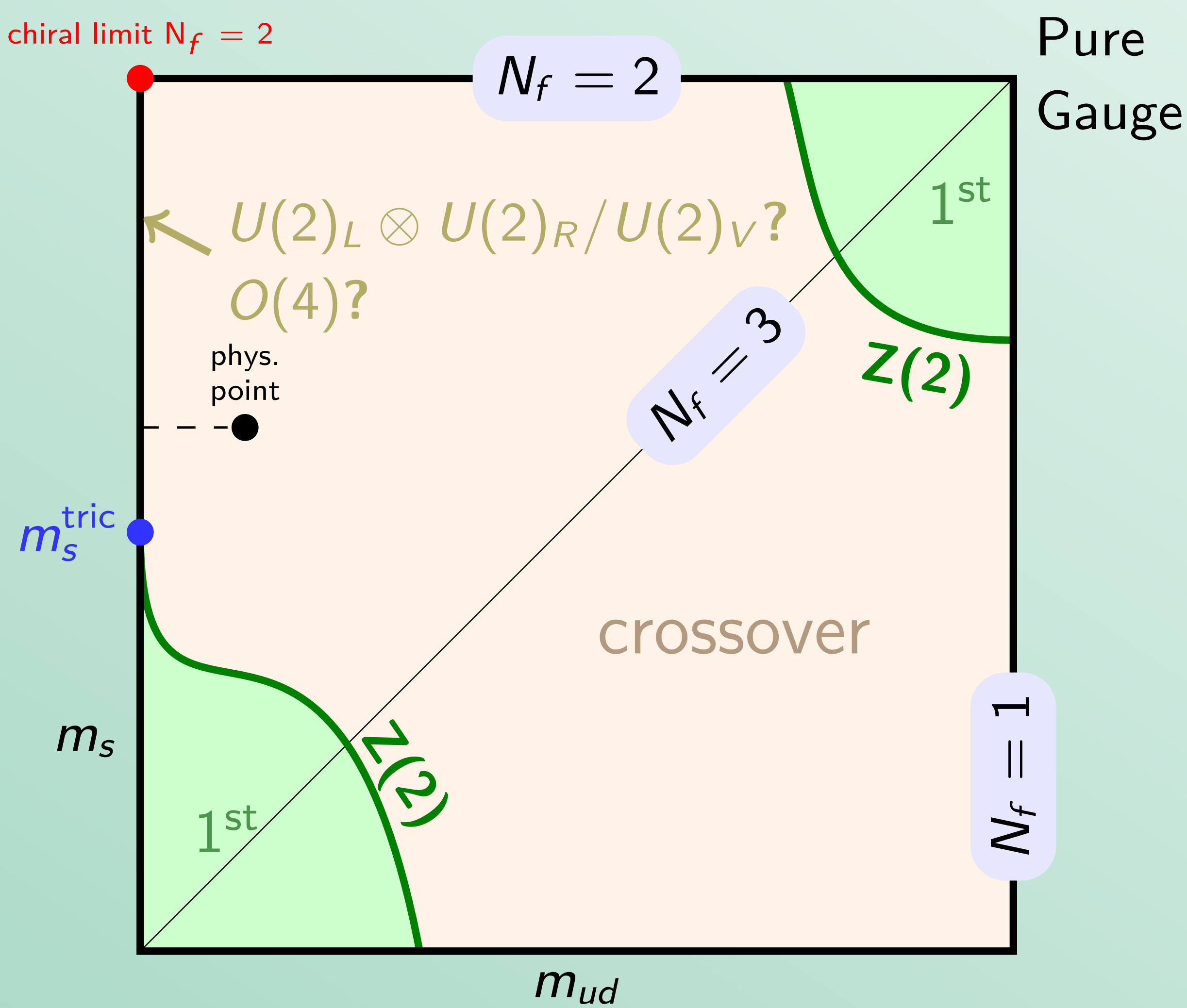

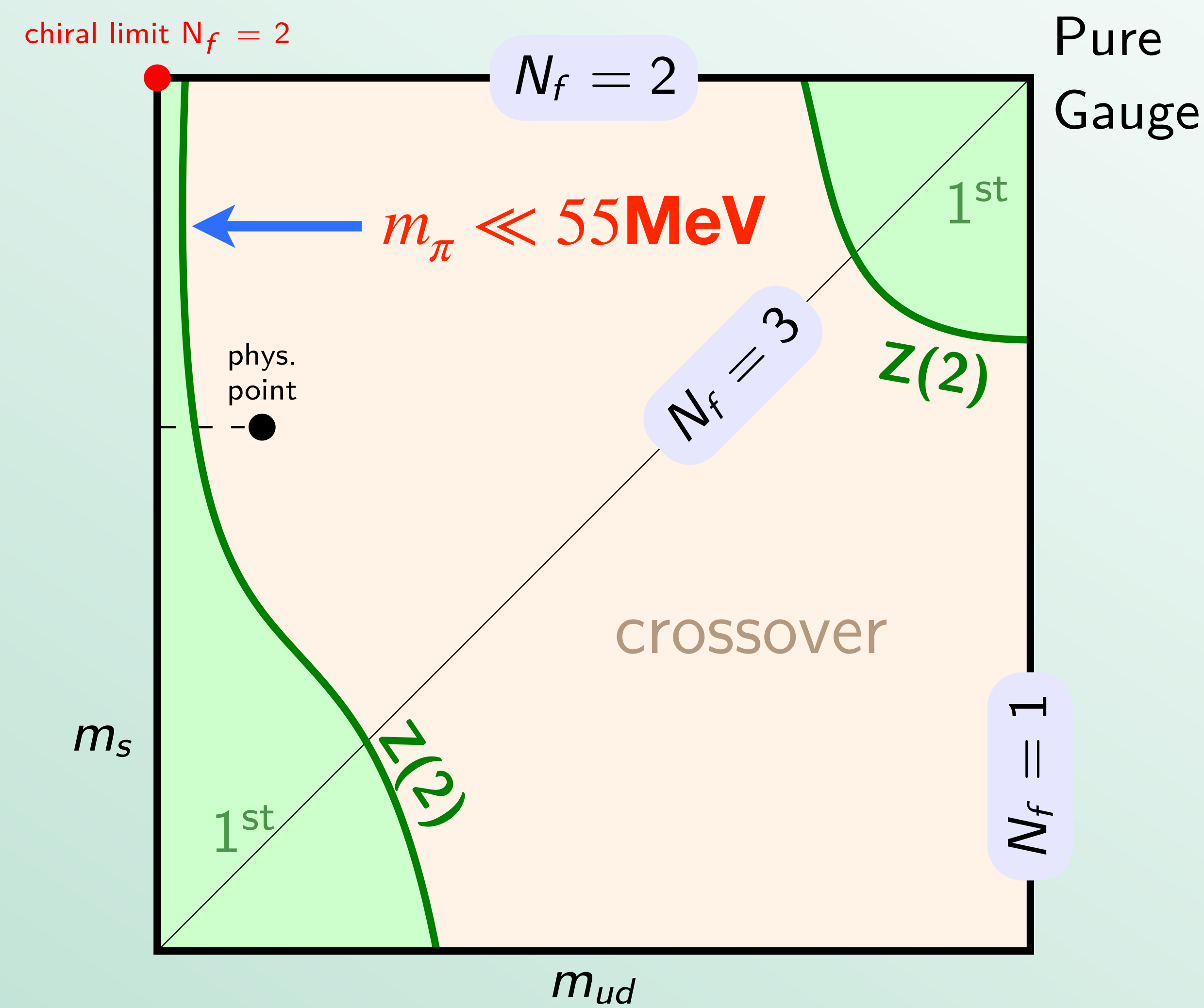

HotQCD: Based on studies with HISQ action

However, still one may argue that 1st order transition can be possible for, $m_\pi \ll 55$ **MeV**

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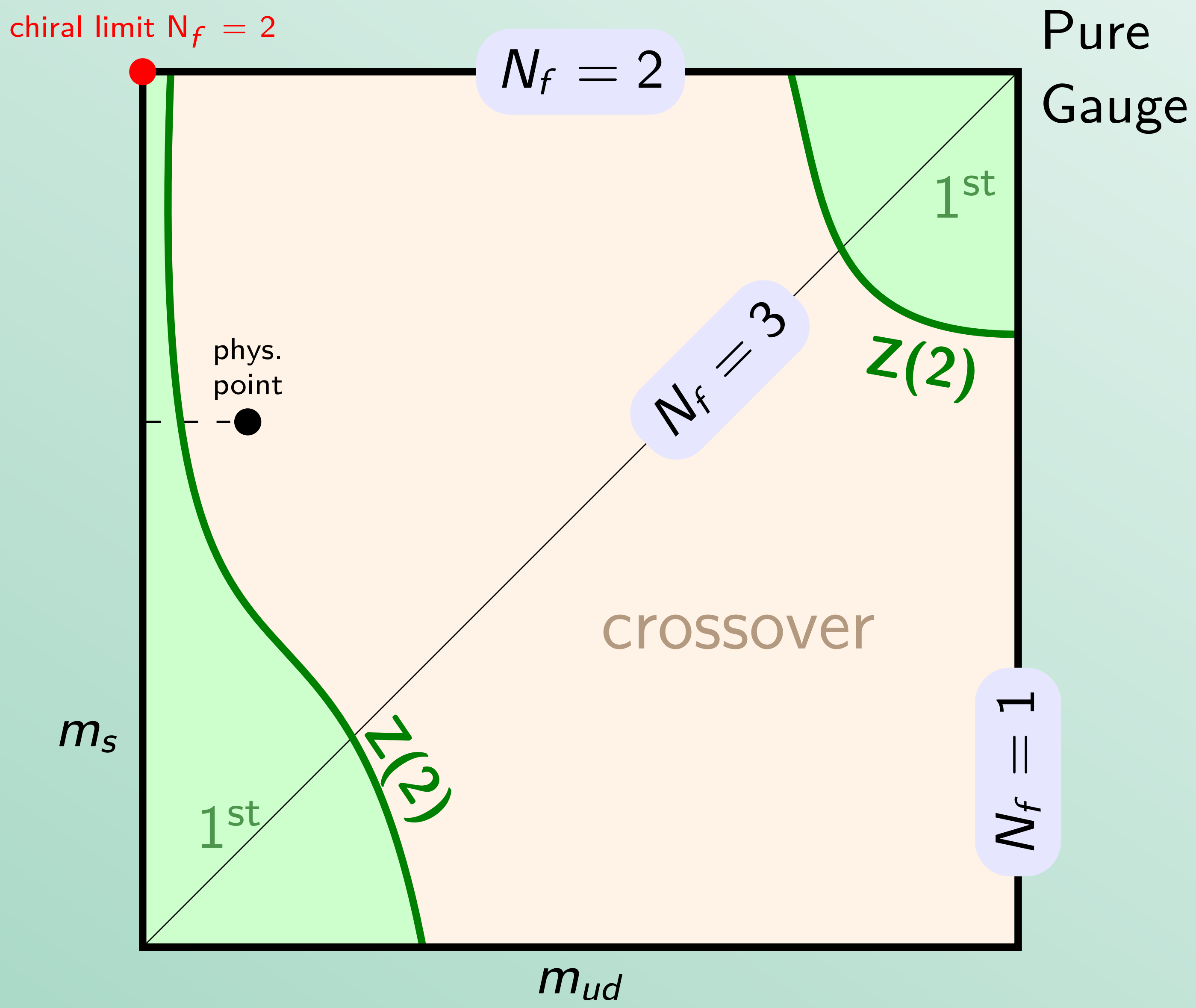
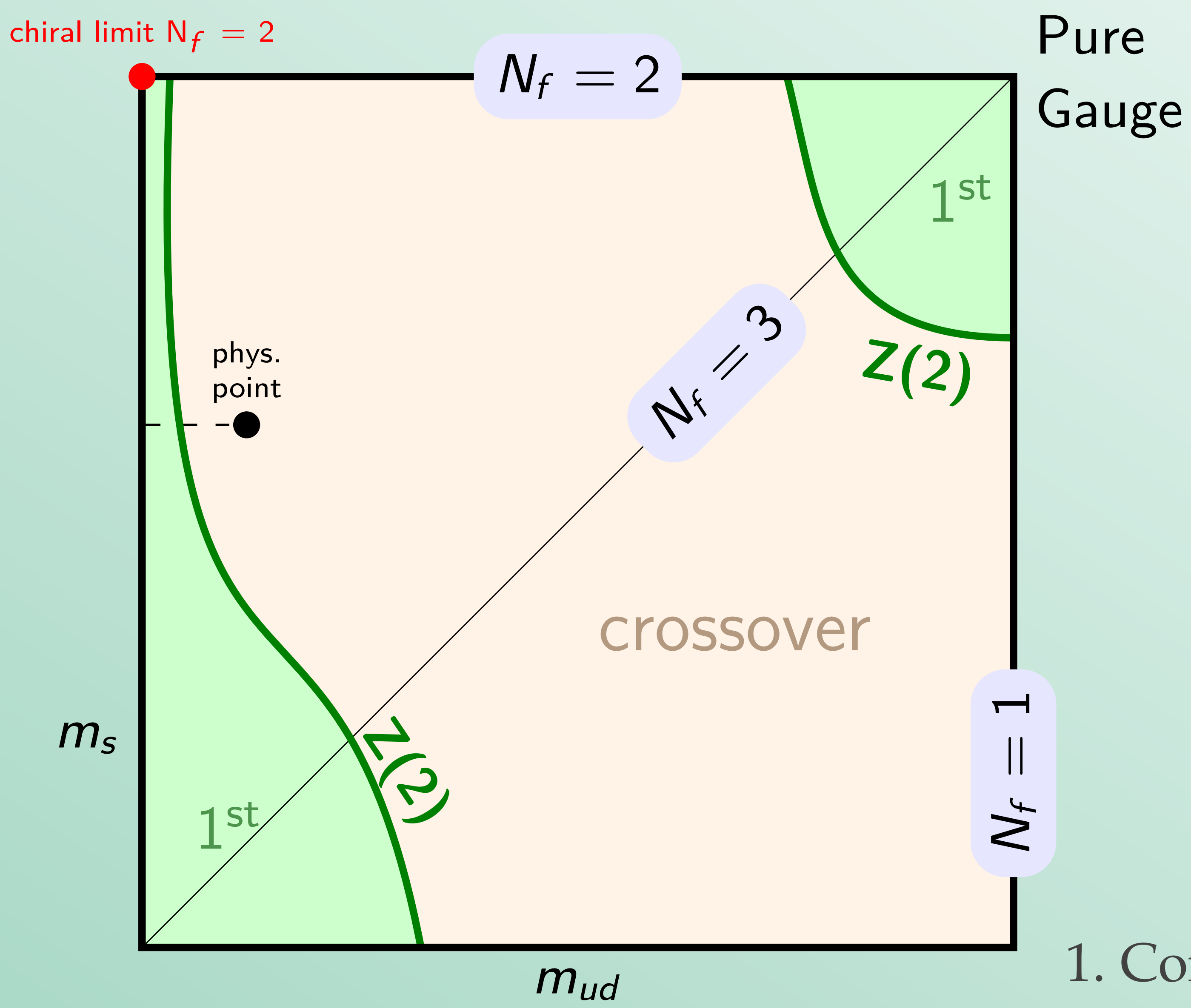


Philipsen, Pinke, PRD 93 (2016)

❖ A few Lattice actions show a first order chiral transition but.....

1. These results are done on fixed N_τ. Not continuum extrapolated.
2. They use unimproved actions which have large cut-off effects, means the calculations are done away from the continuum.
3. The order of the transition can change if calculations are done close to continuum.

Chiral transition for zero chemical potential



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Possible Solutions:

1. Continuum extrapolate the results (requires decent effort).
2. Verify with actions which allows us to do the calculations close to the continuum i.e. with improved actions

Nature of the chiral symmetry restoring transition at μ=0 at the chiral limit??

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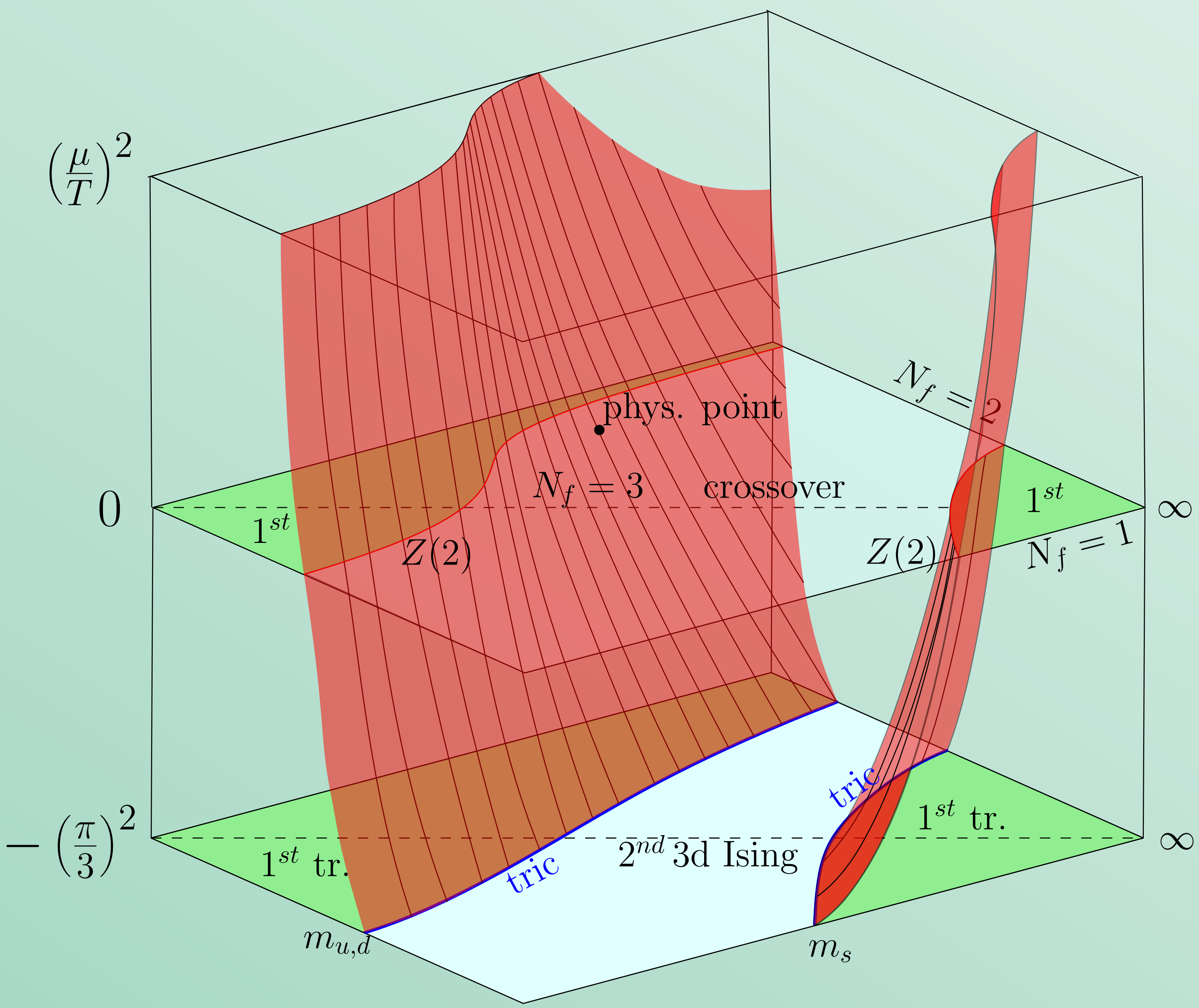

Possible scenario of extended 3d Columbia plot

Another possible way to examine the nature of chiral transition(1st order??) at μ=0 is to study the phase diagram with imaginary chemical potential. It is expected that the first order region, if exists, increases in the imaginary chemical potential direction.

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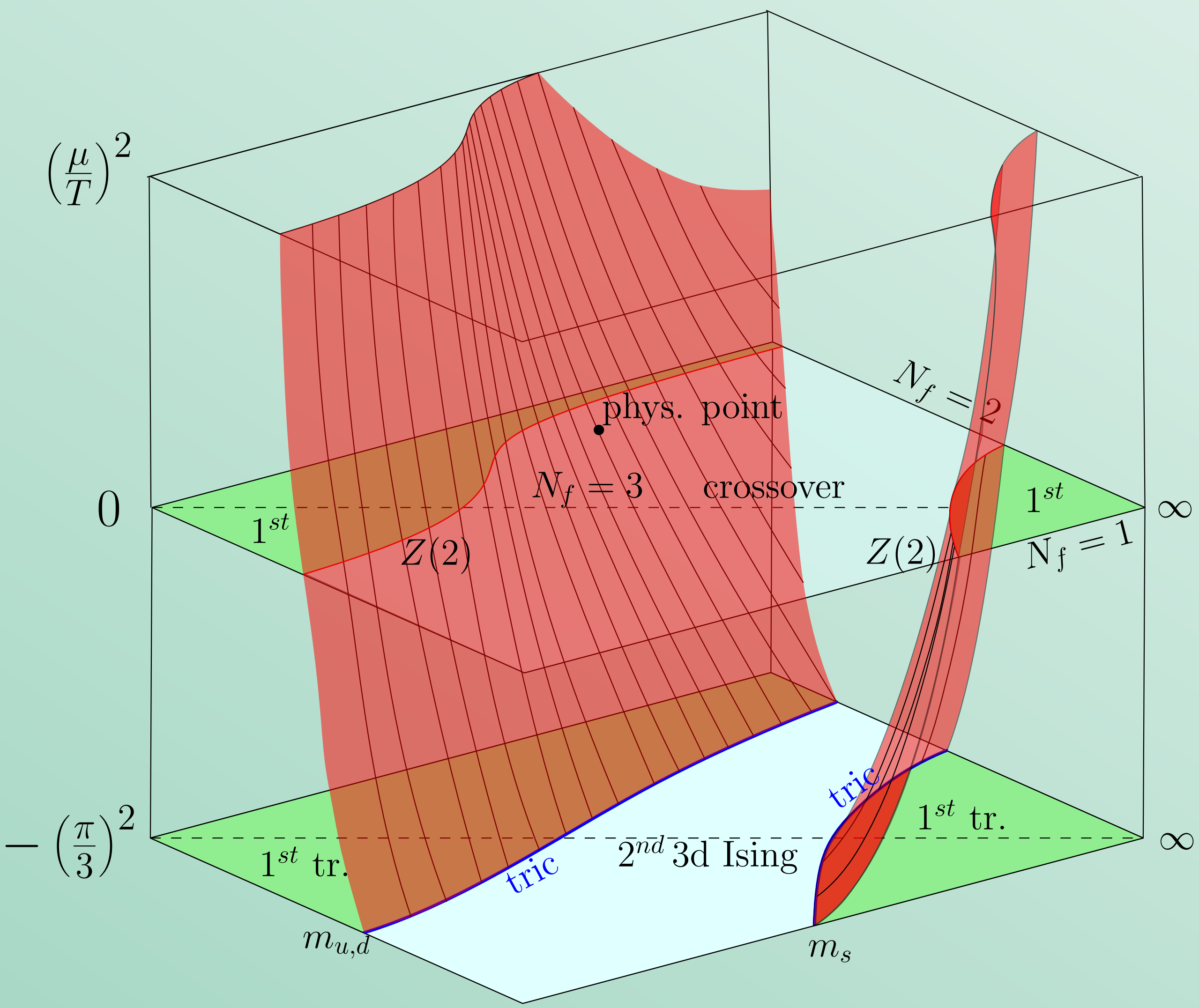

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Largest 1st order region:
Roberge-Weiss (RW) plane, $i\mu/T = i\pi/3$

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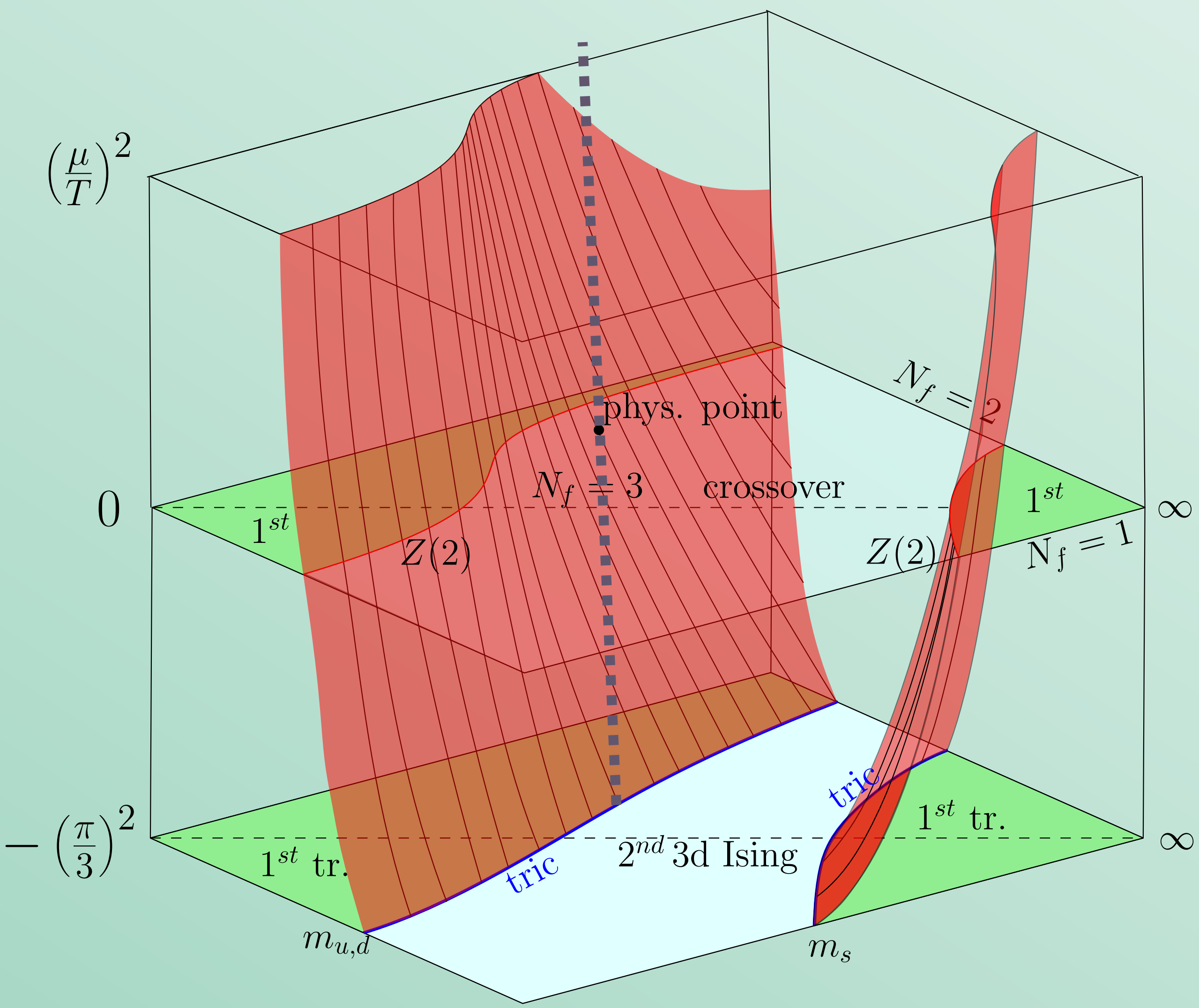

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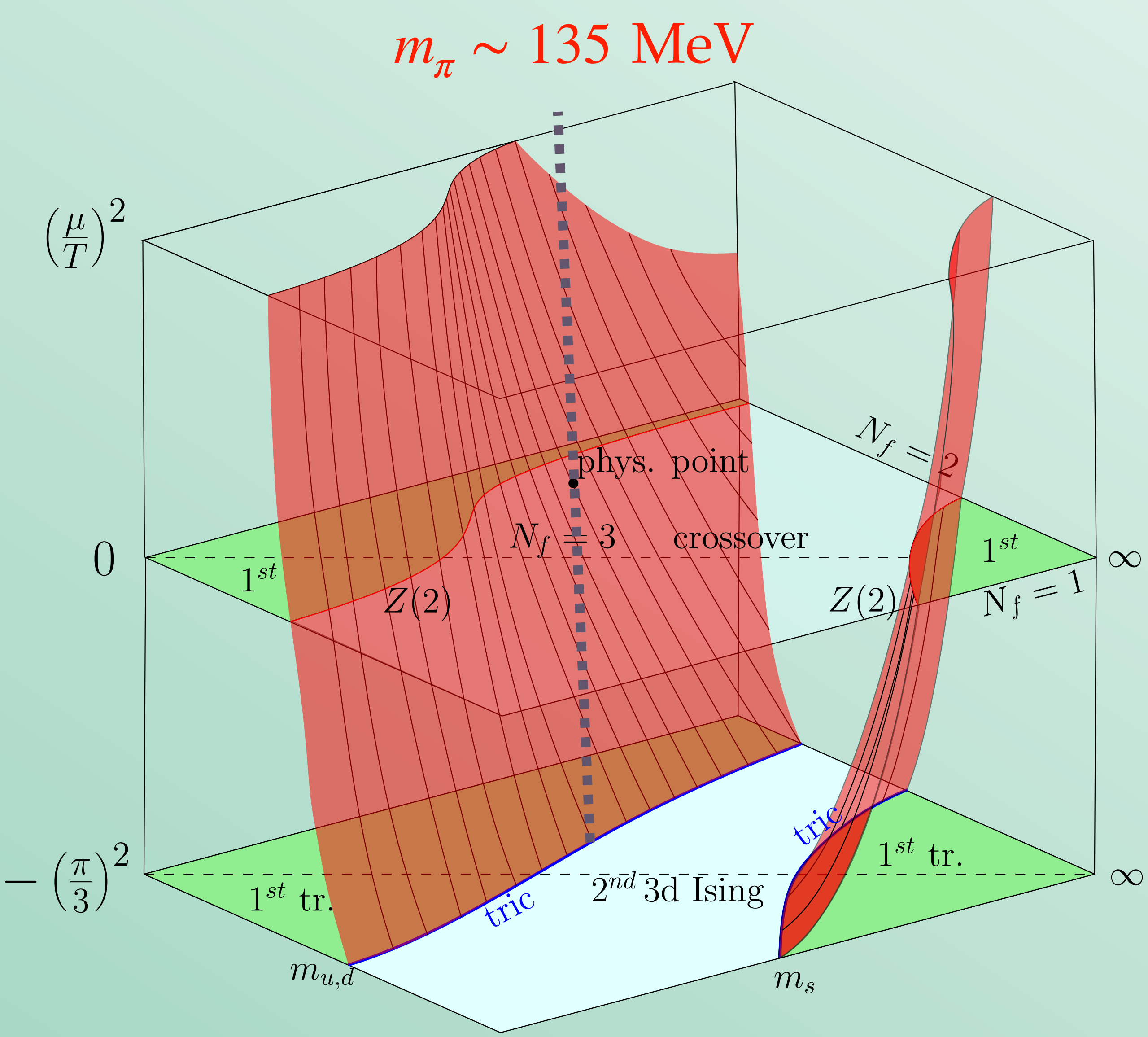

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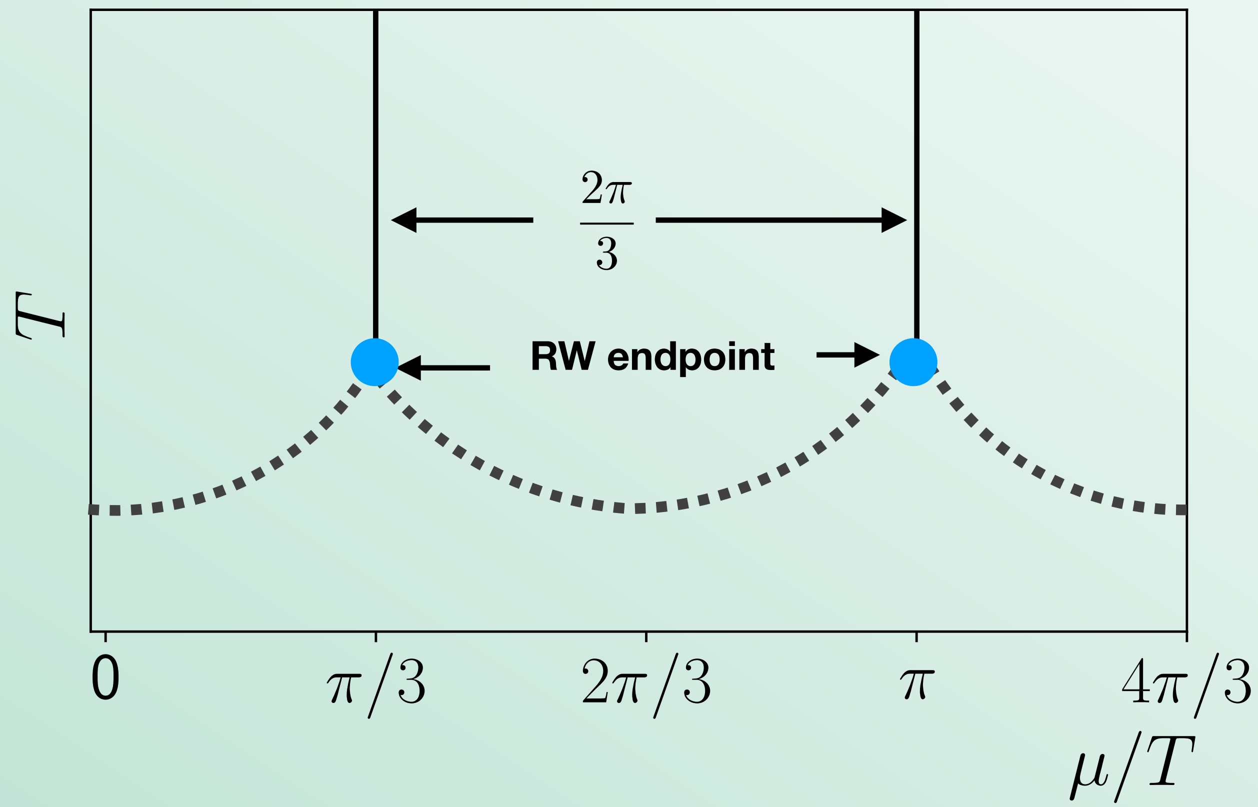
General strategy:Locate the physical point for N_f=2+1, approach the chiral limit while keeping the strange quark mass fix to its physical value. Extrapolate the result to μ=0.

Details about the RW plane



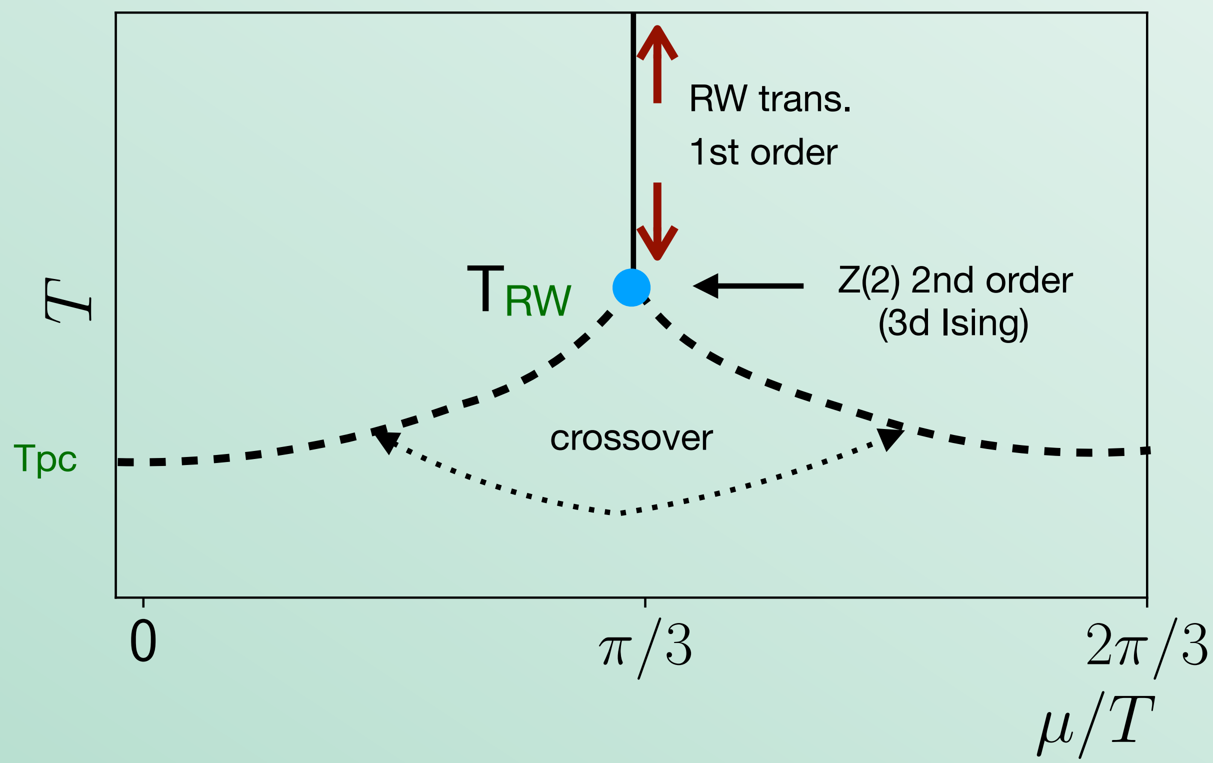
- If $(i\mu)$ is purely imaginary then we can compare the results with the real chemical potential as,

$$\langle O \rangle = \sum_n c_n \left(\frac{i\mu}{T}\right)^n, n = 2, 4, 6.....$$

- The partition function is periodic in μ/T as, $Z(\mu/T)=Z(\mu/T+2k\pi/3)$, known as , Roberge-Weiss(RW) periodicity. And $\mu/T=(2k+1)\pi/3$, is known as RW plane.

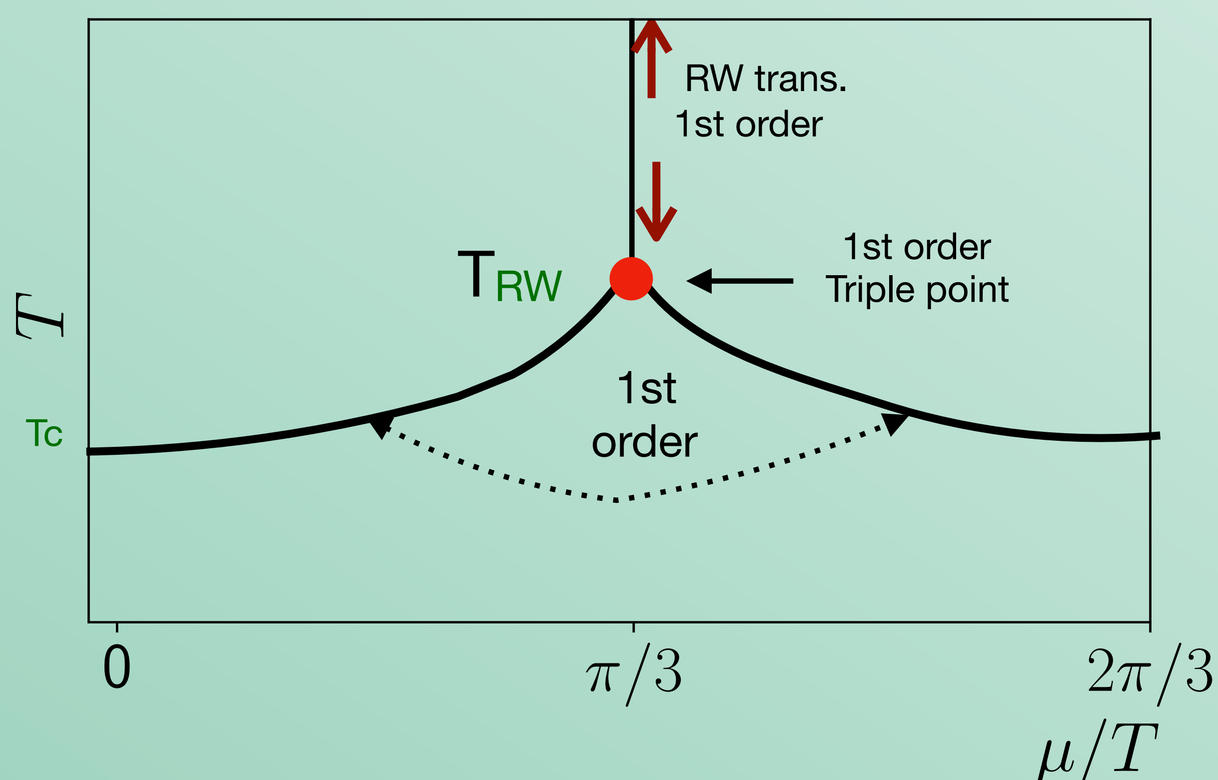
Possible fate of the RW end point

Intermediate quark mass



For, intermediate pion mass RW end point is of Z(2) 2nd order.

Approaching Chiral limit



For, smaller pion mass RW end point may become a 1st order triple point.

This could indicate that the chiral transition at μ=0 is 1st order.

Studies in the RW plane

Mostly with unimproved actions

$N_f = 2$: 1st order triple point (at the end of the line of 1st order RW transitions) exist for $\mu/T = \pi/3$ and $m_{cri} > m_{phy.}$

Standard staggered action:

$m_\pi \sim 400$ MeV(N_τ=4)

Standard Wilson action:

$m_\pi \sim 930$ MeV(N_τ=4)

$m_\pi \sim 680$ MeV (N_τ=6)

The results are strongly fermion discretization scheme and cut-off(N_τ) dependent.

P. de Forcrand et. al, PRL 105, 152001(2010), Owe Philipsen et. al, PRD 89, 094504(2014), Christopher Czaban et al, PRD 93, 054507 (2016)

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Very recent studies with improved actions,

Stout improved staggered fermions($N_f=2+1$): For physical quark mass the RW endpoint is belongs to the Z(2) universality class.

C. Bonati et. al,PRD 93, 074504 (2016)

No, 1st order triple point exist at least for $m_\pi >50$ MeV.

C. Bonati et. al,arXiv:1807.02106 [hep-lat]

HISQ($N_f=2$): Order of the phase transition at physical point is not clear(large cut-off effects) .

L.K.Wu, et al. PRD 97,114514(2018)

Studies with HISQ in the RW plane

Action, $Z(T,\mu) = \int [\mathscr{D}U] det[M_{ud}(\mu_f)]^{1/2} det[M_s(\mu_f)]^{1/4} \exp[-S_G]$

$M_q = D_{HISQ}(\mu_f) + m_q$, μ_f is purely imaginary

Simulation details, $N_f = 2+1, \frac{\mu_f}{T} = \frac{\pi}{3}$

N_σ	N_τ	$\frac{m_l}{m_s}$	m_π (MeV)
8	4	1/27	135
12	4	1/27	135
16	4	1/27,1/40, 1/60	135, 110, 90
24	4	1/27,1/40, 1/160,1/320	135, 110,55,40

T is varied in the range, 176-215 MeV, corresponds to, $T \sim T_c \pm 0.1 T_c$

Generally we generated 20k trajectory per T value away from T_c and 80k trajectory near T_c

Ising endpoint of a first order line

$$H_{eff}(t,\xi) = t\mathscr{E} + h\mathscr{M}$$

→ Effective Ising Hamiltonian which defines the universal critical behaviour of the system

temperature like field

energy like operator

magnetic field like

magnetization like operator (order parameter)

under Z(2) transformation,

$$\mathscr{E} \rightarrow \mathscr{E}$$

$$\mathscr{M} \rightarrow -\mathscr{M}$$

Corresponding critical behaviour of QCD in 2nd RW plane [Z(2) transformation],

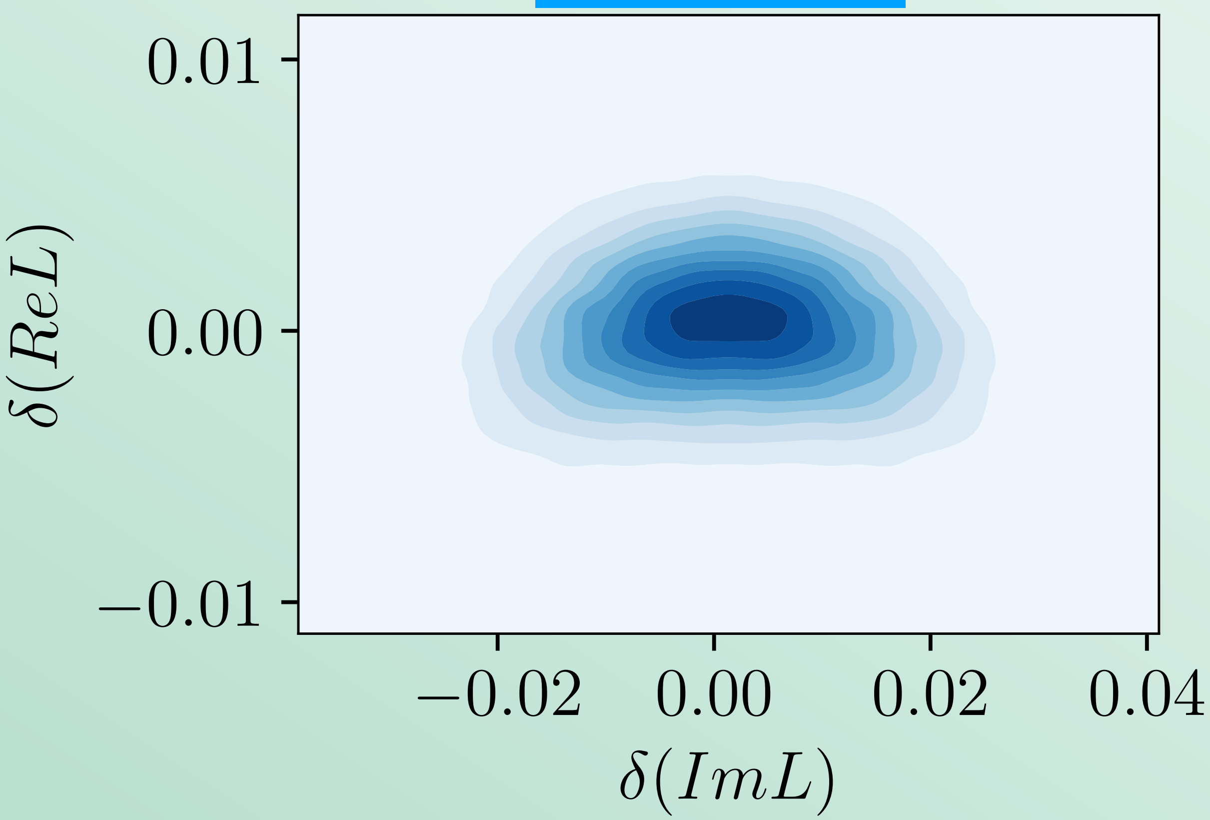
$$Im\ L \rightarrow -Im\ L$$ → order parameter

$$Re\ L \rightarrow Re\ L$$ → energy like

i.e. at $\mu = \mu_{RW}$

$$\lim_{h\to 0}\lim_{V\to\infty}\langle Im\ L\rangle \equiv \lim_{V\to\infty}\langle|Im\ L|\rangle_{h=0} = \begin{cases} 0, & \text{if } \beta < \beta_c \\ \text{non-zero}, & \text{if } \beta > \beta_c \end{cases}$$

$T<T_c$



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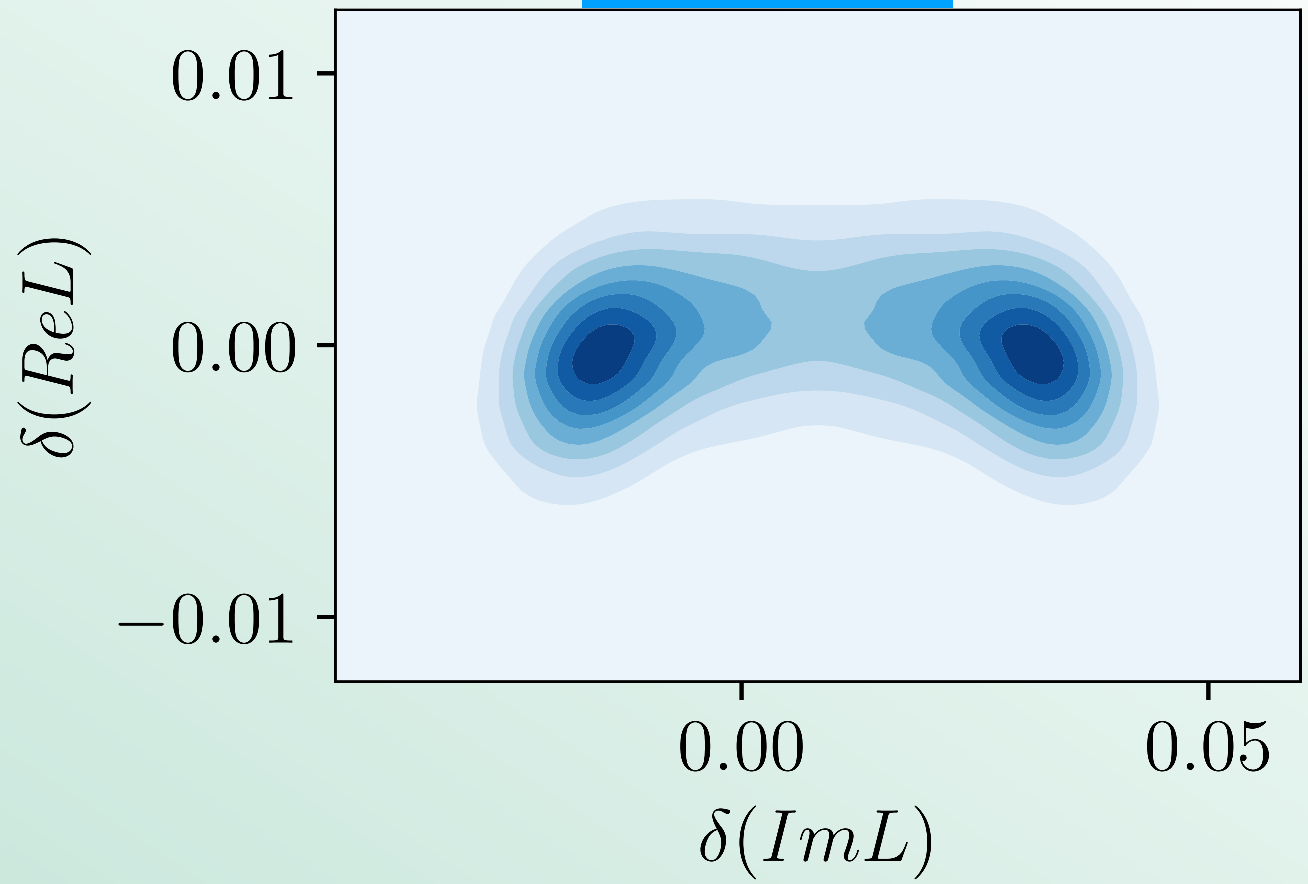

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Finite size scaling and Z(2) universality class

Free energy and the universal functions for second order transition near the critical point can be written as,[t=(T-T$_c$)/T$_c$]

$$f = b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma) + f_{ns} \;, \quad u_t \sim c_t t, u_t \sim c_h h$$

$$\langle |Im\ L| \rangle = \left.\frac{\partial f}{\partial h}\right|_{h\to 0} \sim N_\sigma^{-\beta/\nu} f_h(z_0 t N_\sigma^{1/\nu})$$

universal scaling function of order parameter

$$\chi_h = \left.\frac{\partial^2 f}{\partial h^2}\right|_{h\to 0} \sim N_\sigma^{\gamma/\nu}\ f_\chi(z_0 t N_\sigma^{1/\nu})$$

universal scaling function of order parameter susceptibility

$$B_4 - (1/N_\sigma^d) B_{ns} \sim f_B(z_0 t N_\sigma^{1/\nu})$$

universal scaling function of Binder cumulant

$$\chi_t = \left.\frac{\partial^2 f}{\partial t^2}\right|_{h\to 0} \sim N_\sigma^{\alpha/\nu}\ f_c(z_0 t N_\sigma^{1/\nu})$$

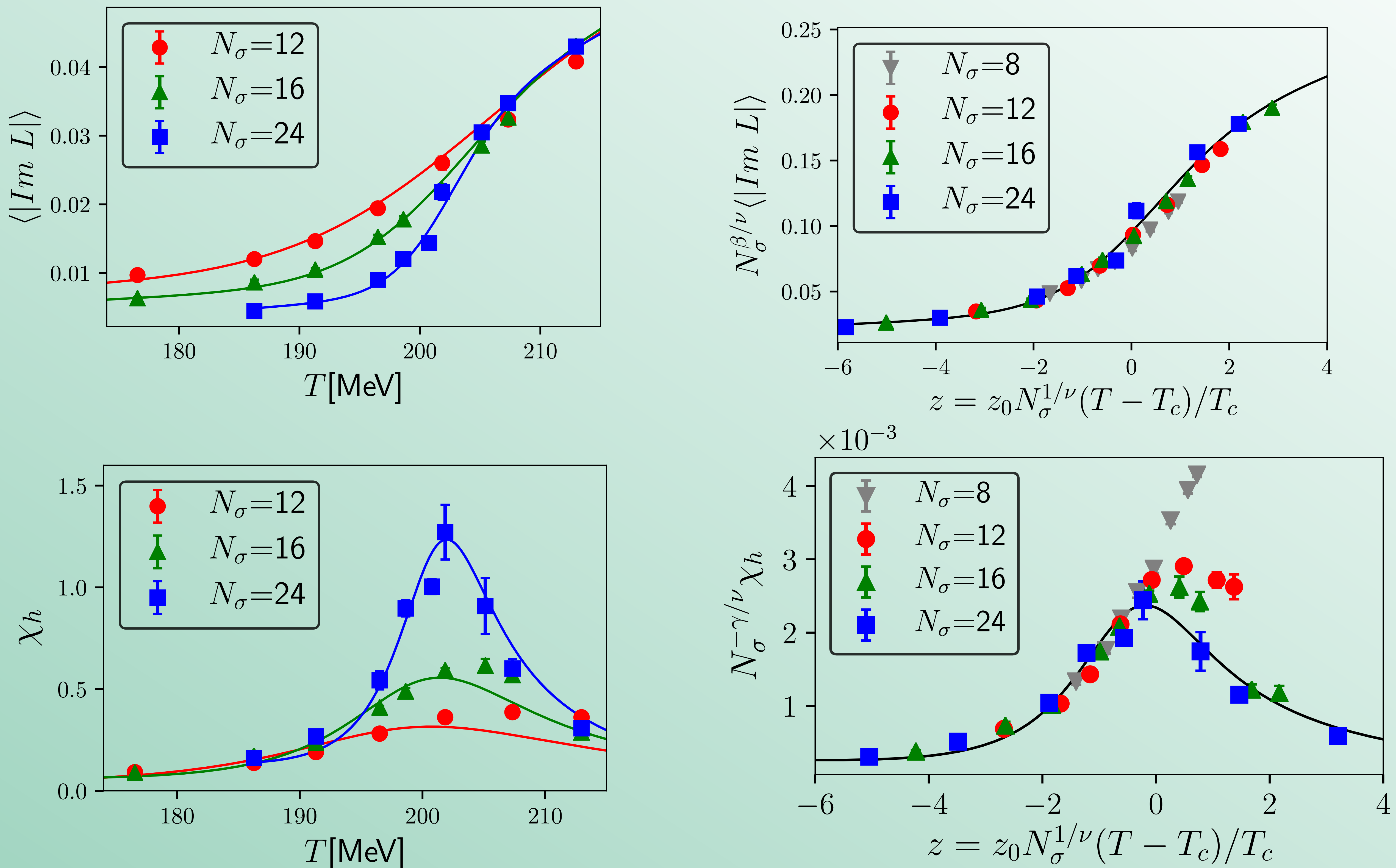
universal scaling function of specific heat

In our case β,ν,α and γ *are Z(2) critical exponents*

Finite size scaling of order parameter and its susceptibility

$m_\pi \sim 135$ MeV

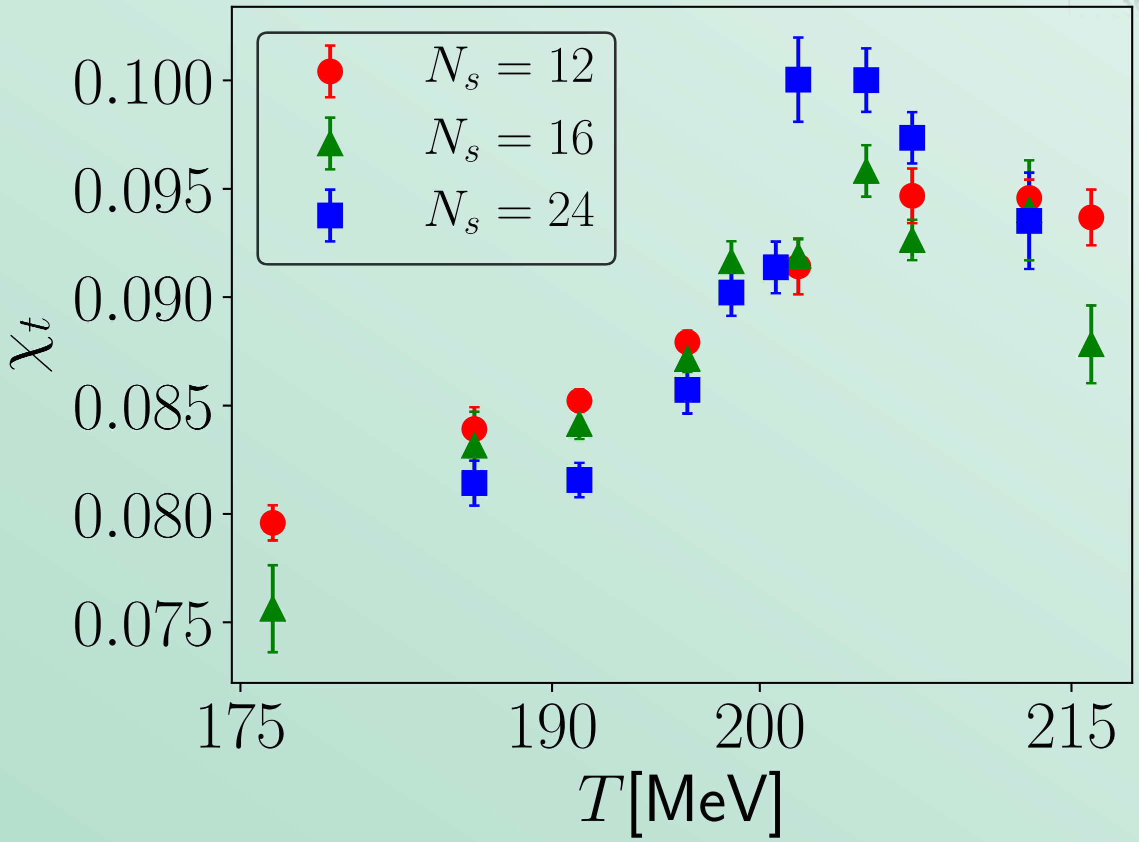
$\beta = 0.32 \quad \gamma = 1.24 \quad \nu = 0.62$

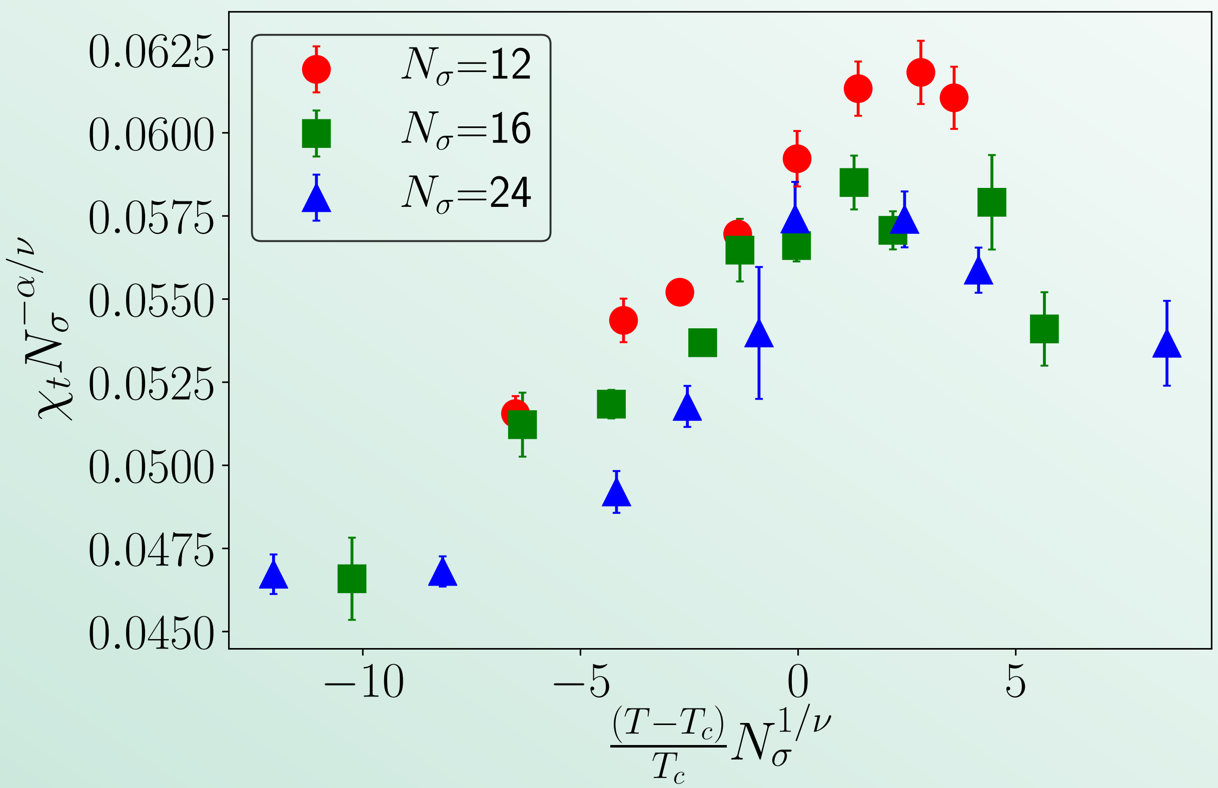


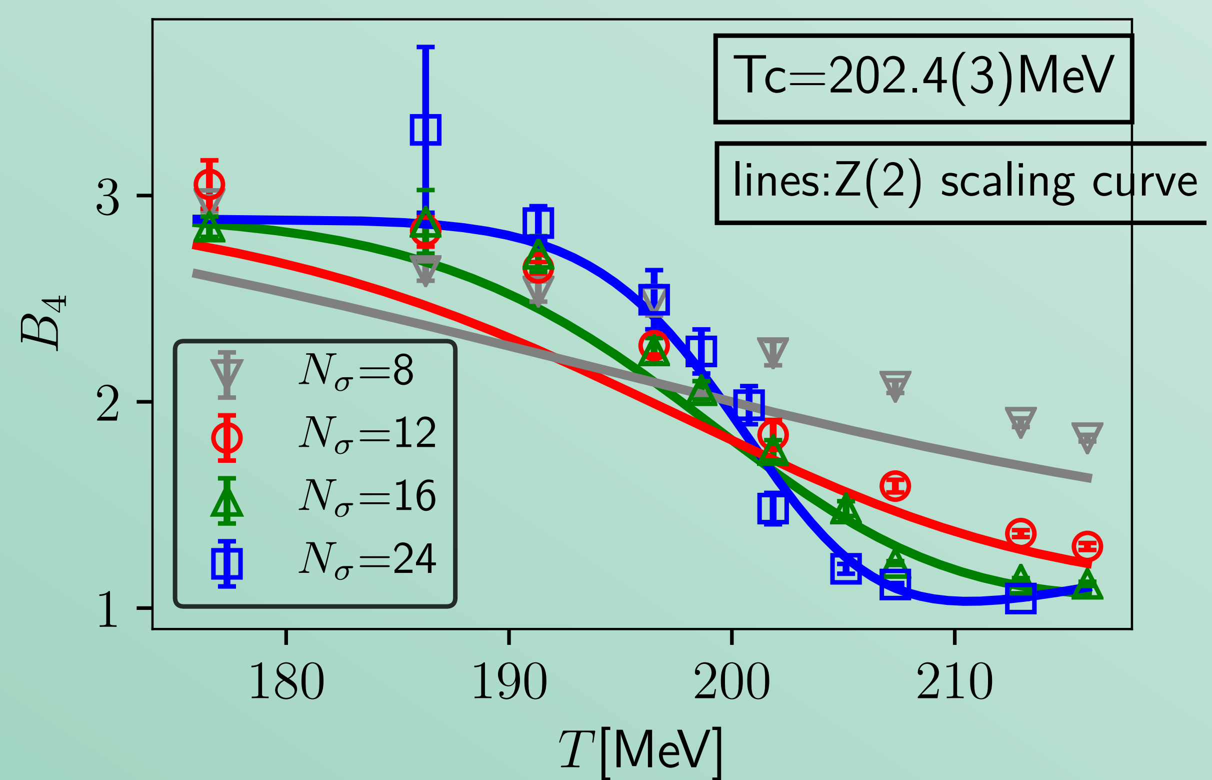
Finite size scaling of specific heat and Binder cumulant

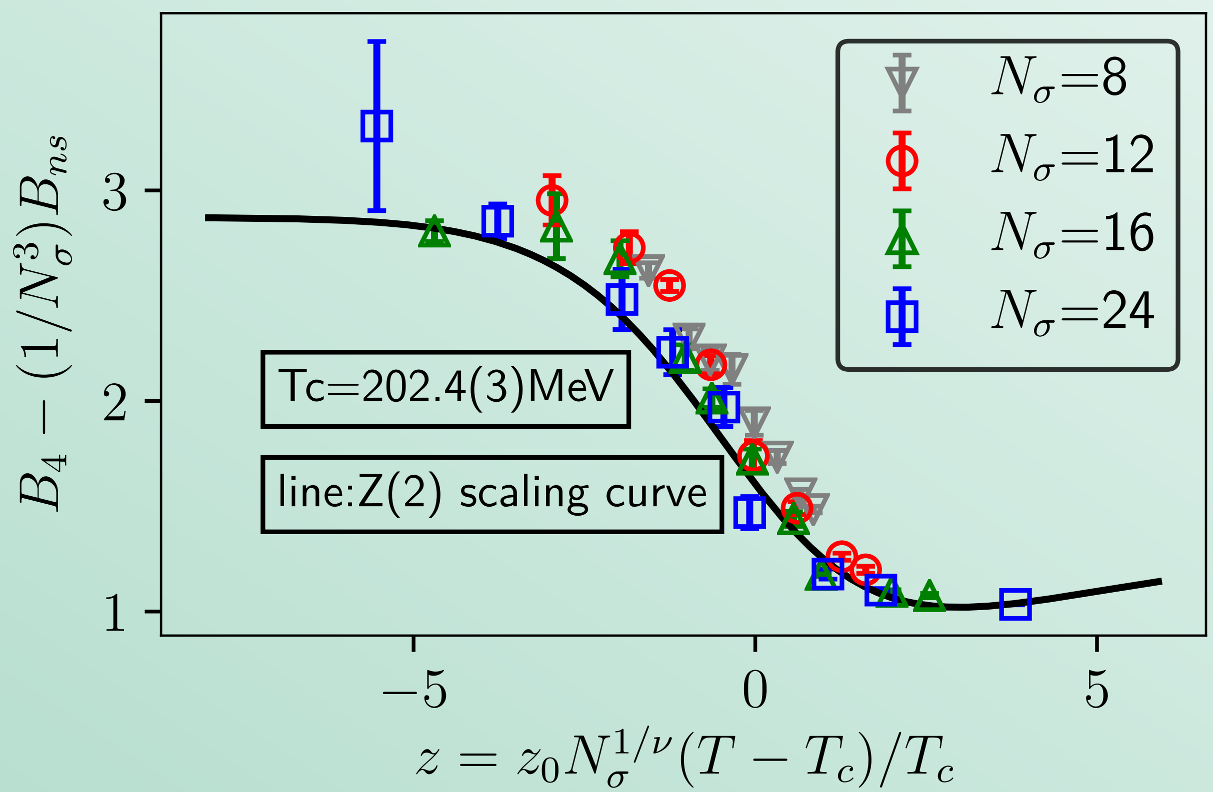
$m_\pi \sim 135$ MeV

$\alpha = 0.11$

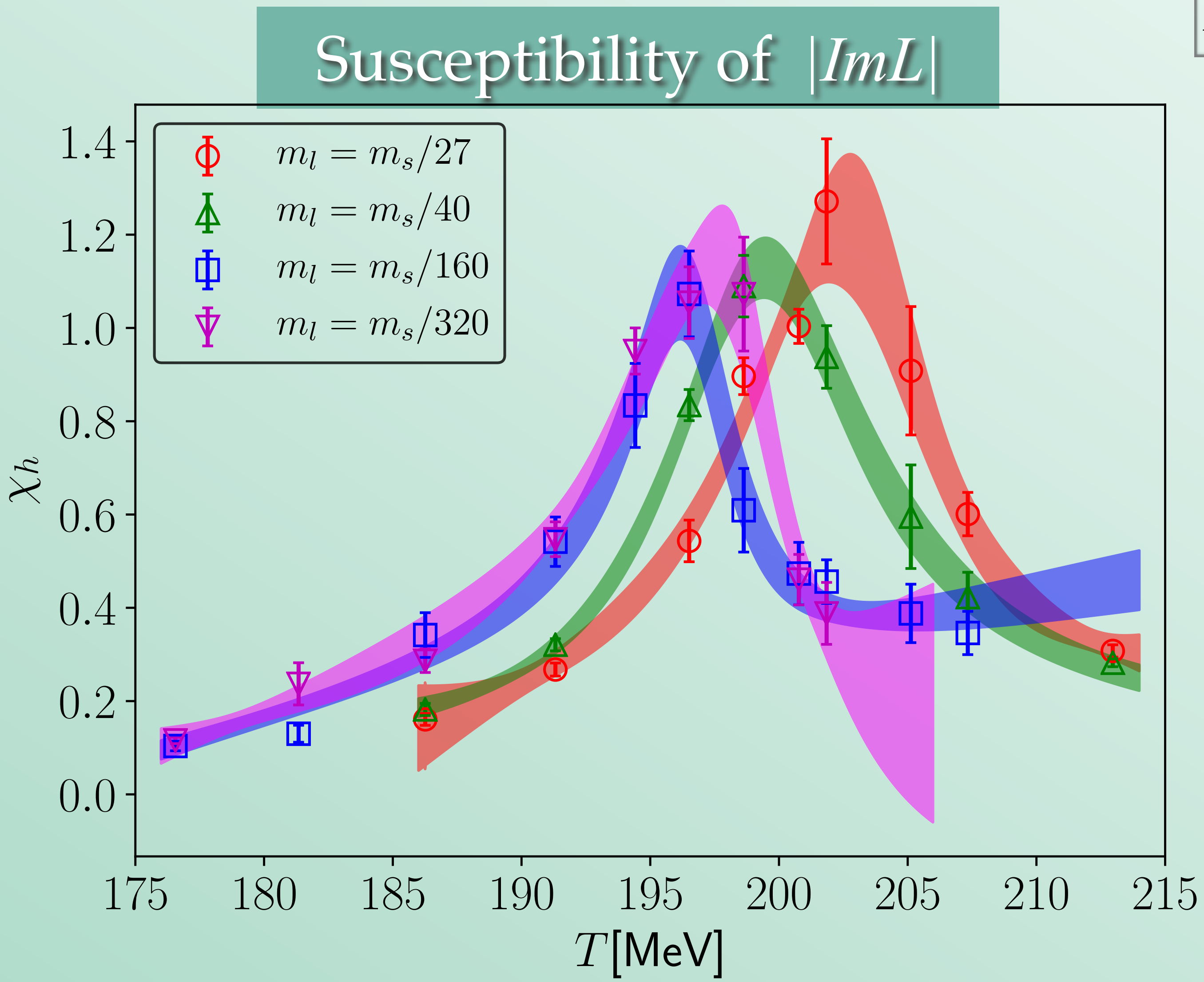








Quark mass dependence of RW transition



$N_\sigma = 24$

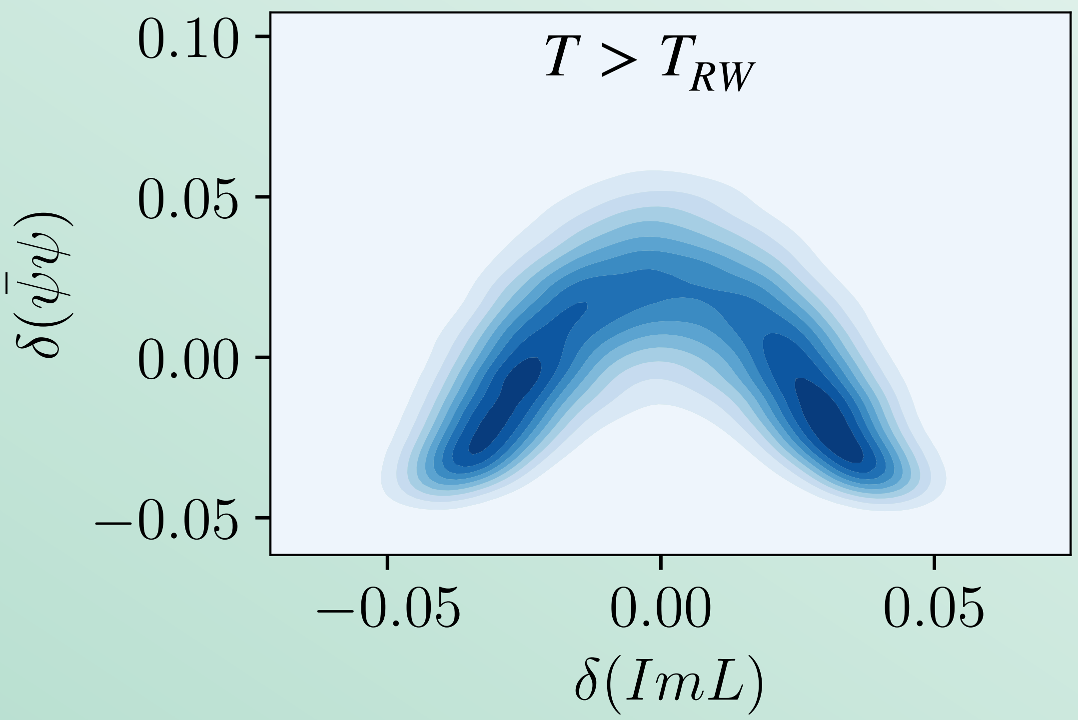
m_l	m_π(MeV)
$m_s/27$	135
$m_s/40$	110
$m_s/160$	55
$m_s/320$	40

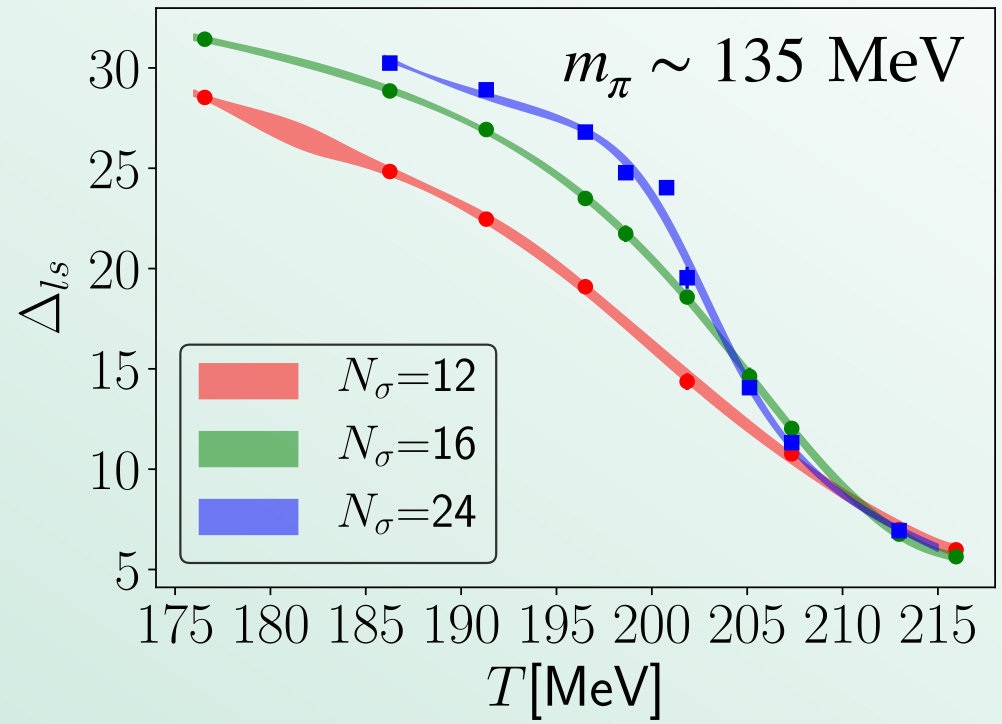
No significant rise in the peak of the susceptibility of the order parameter with respect to pion mass down to, $m_\pi \sim 40$ MeV

Order of the RW transition seems to be unchanged ??

Preliminary finite size scaling study with $m_\pi \sim 55$ MeV is consistent with Z(2) second order transition.

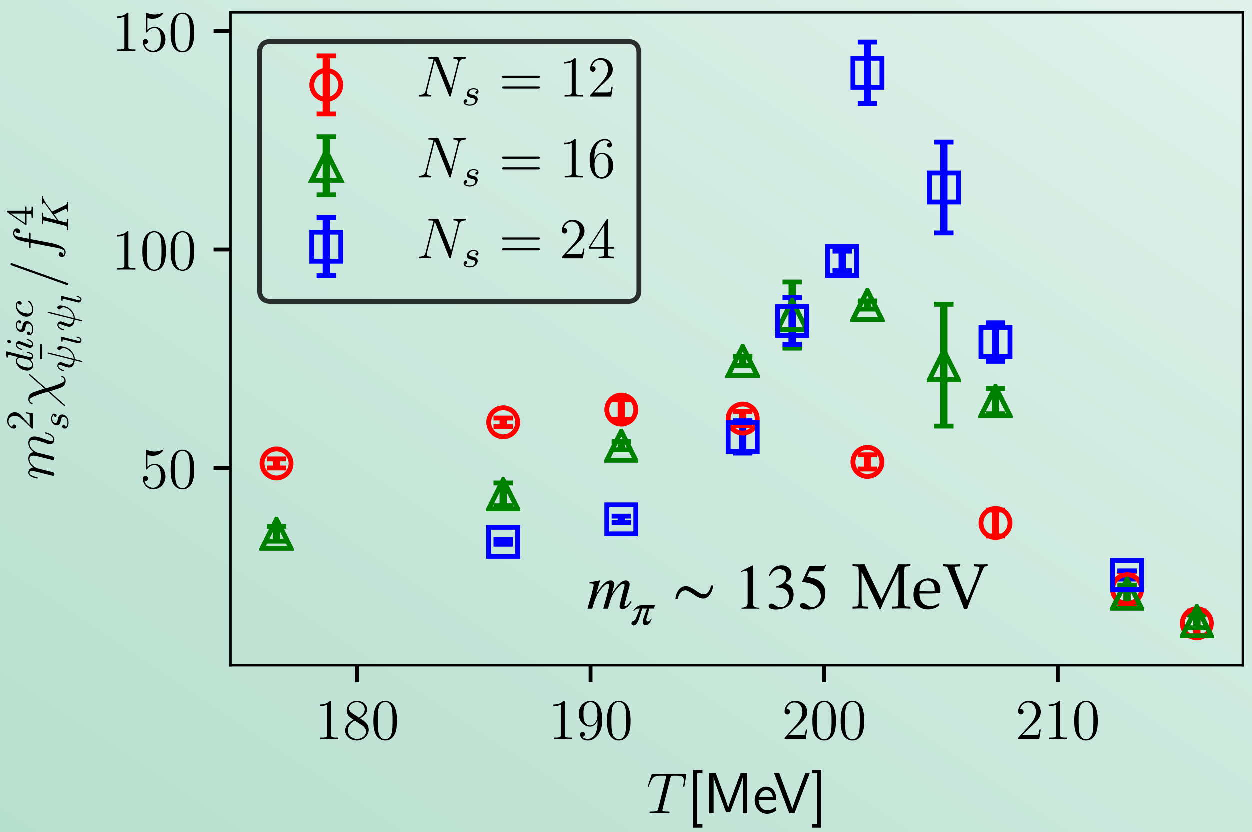
Finite size effects on chiral observables



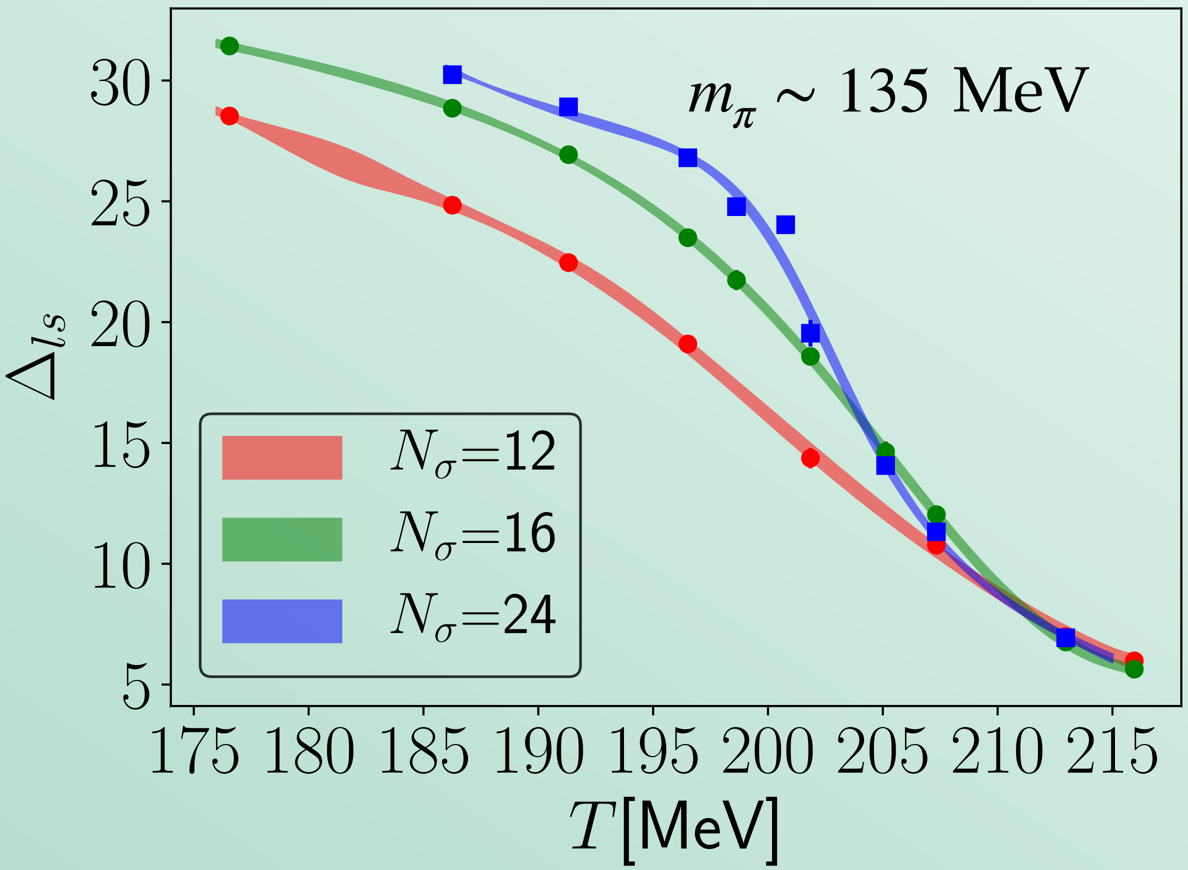


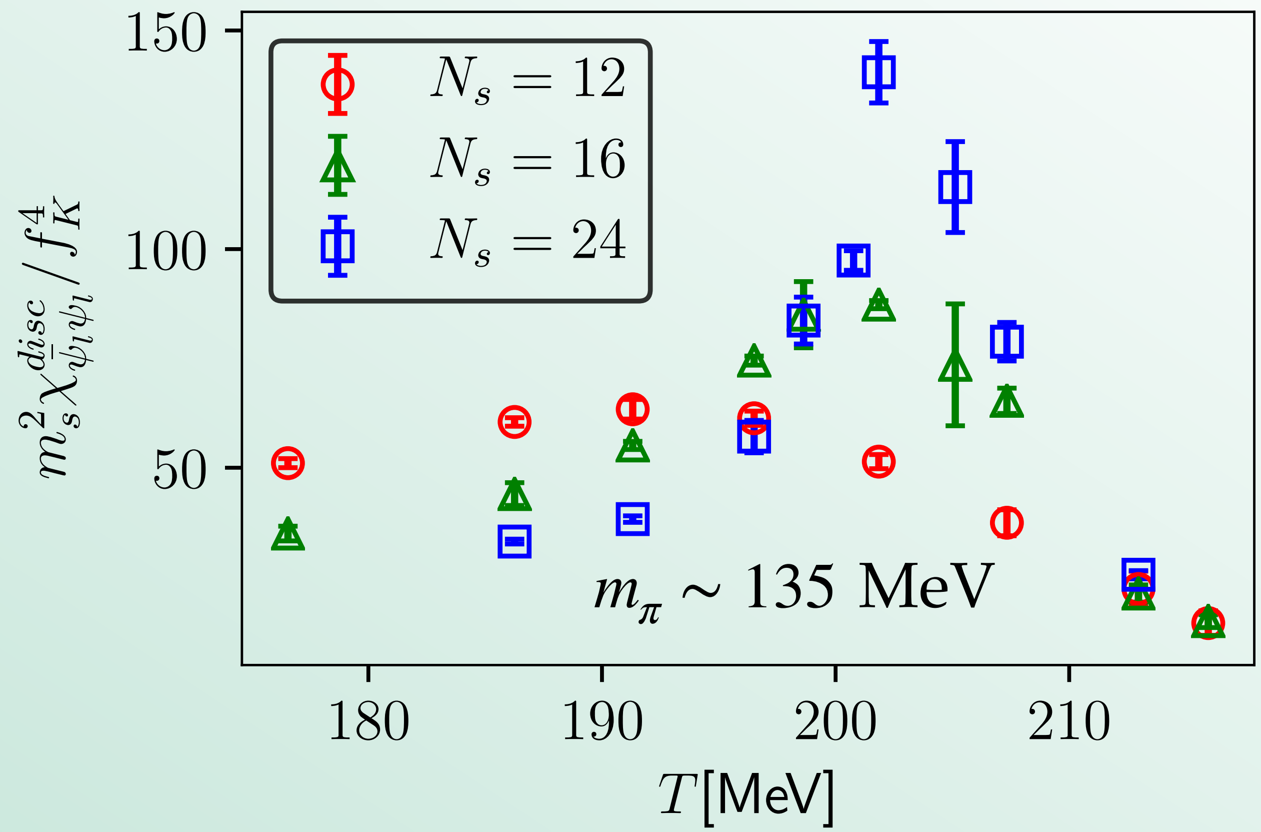
$$\Delta_{ls} = (m_s/f_k^4)\left(\langle \bar{l}l \rangle - (m_l/m_s)\langle \bar{s}s \rangle\right)$$

Strong volume dependence of "subtracted chiral condensate" at fixed m_l/m_s below T_{RW}



Finite size effects on chiral observables





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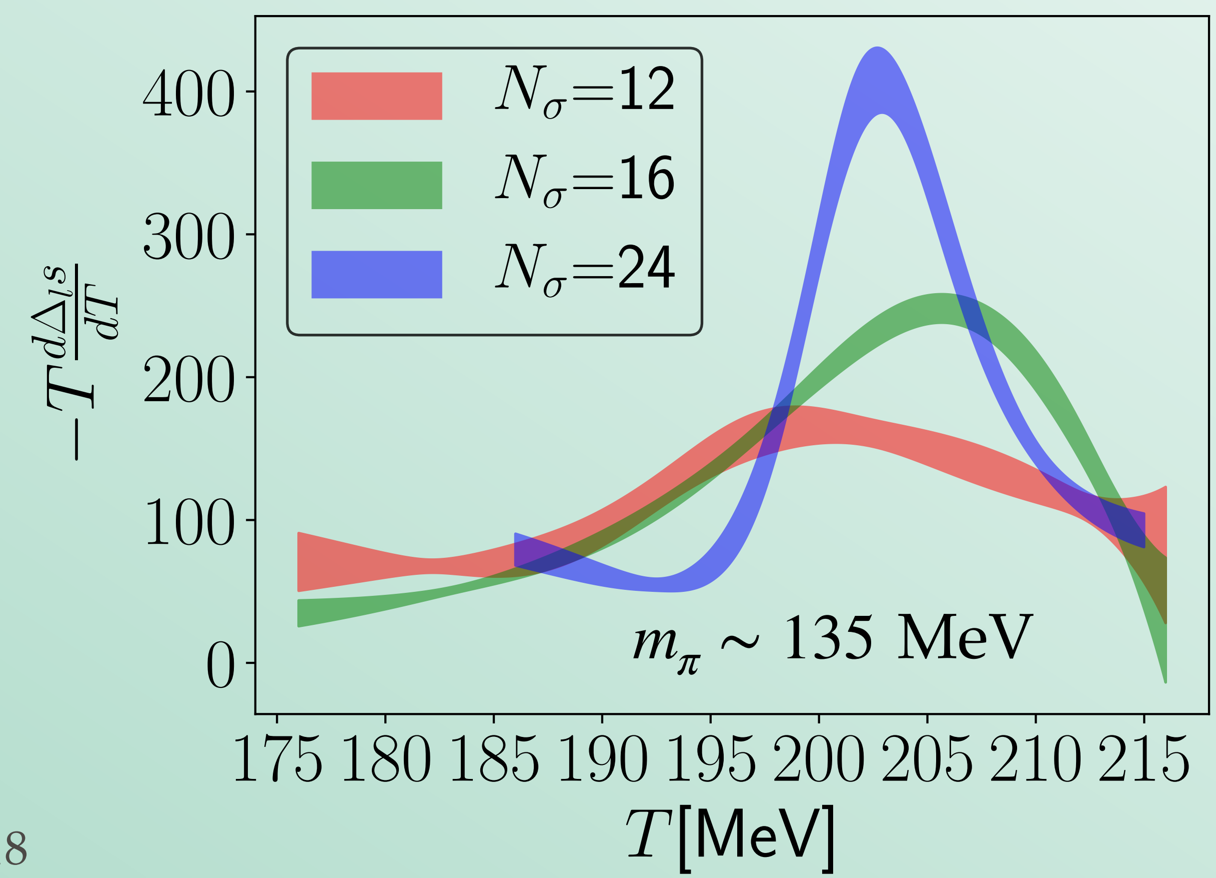
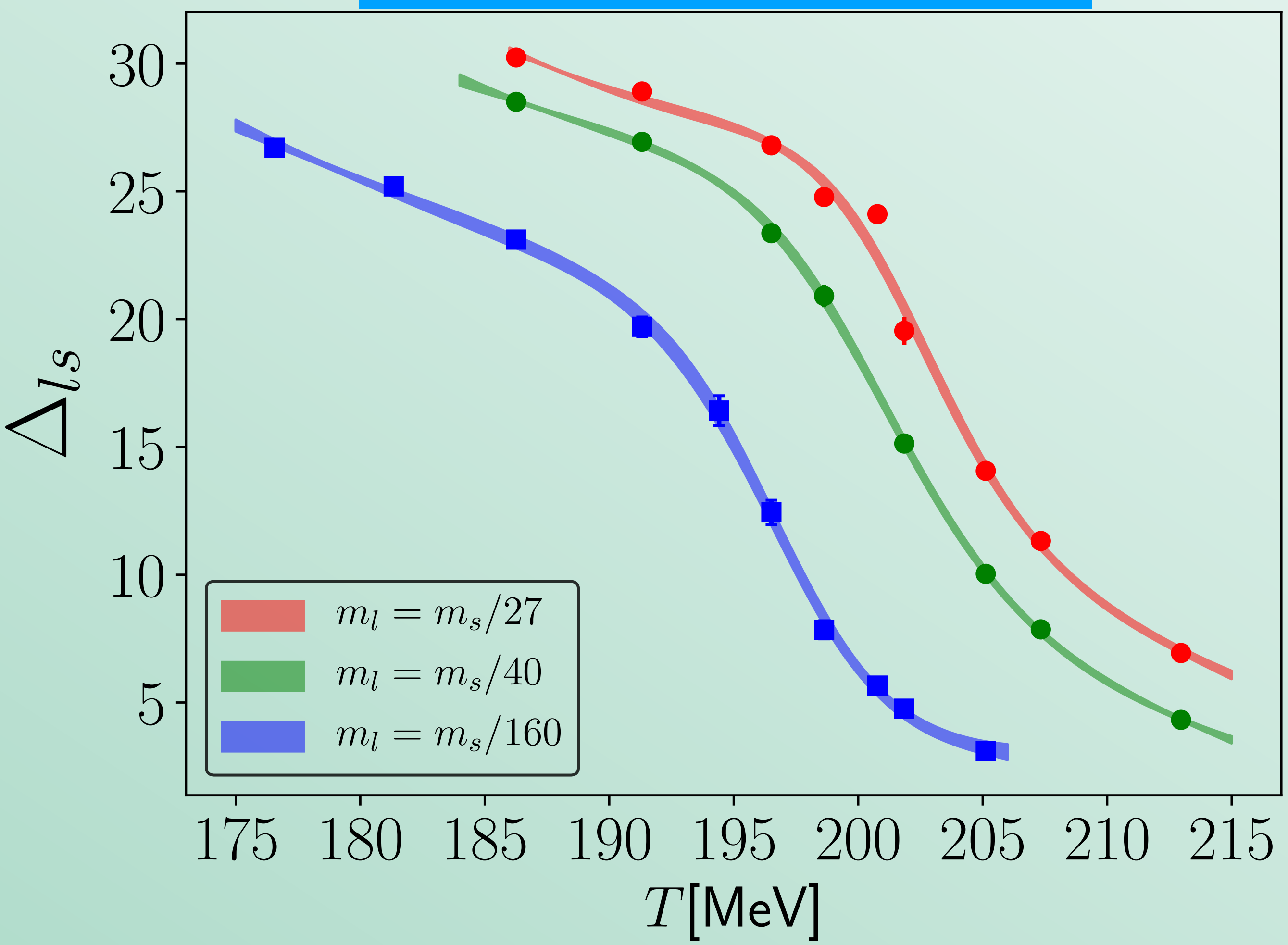
mixed chiral susceptibility sensitive to transition at the RW endpoint

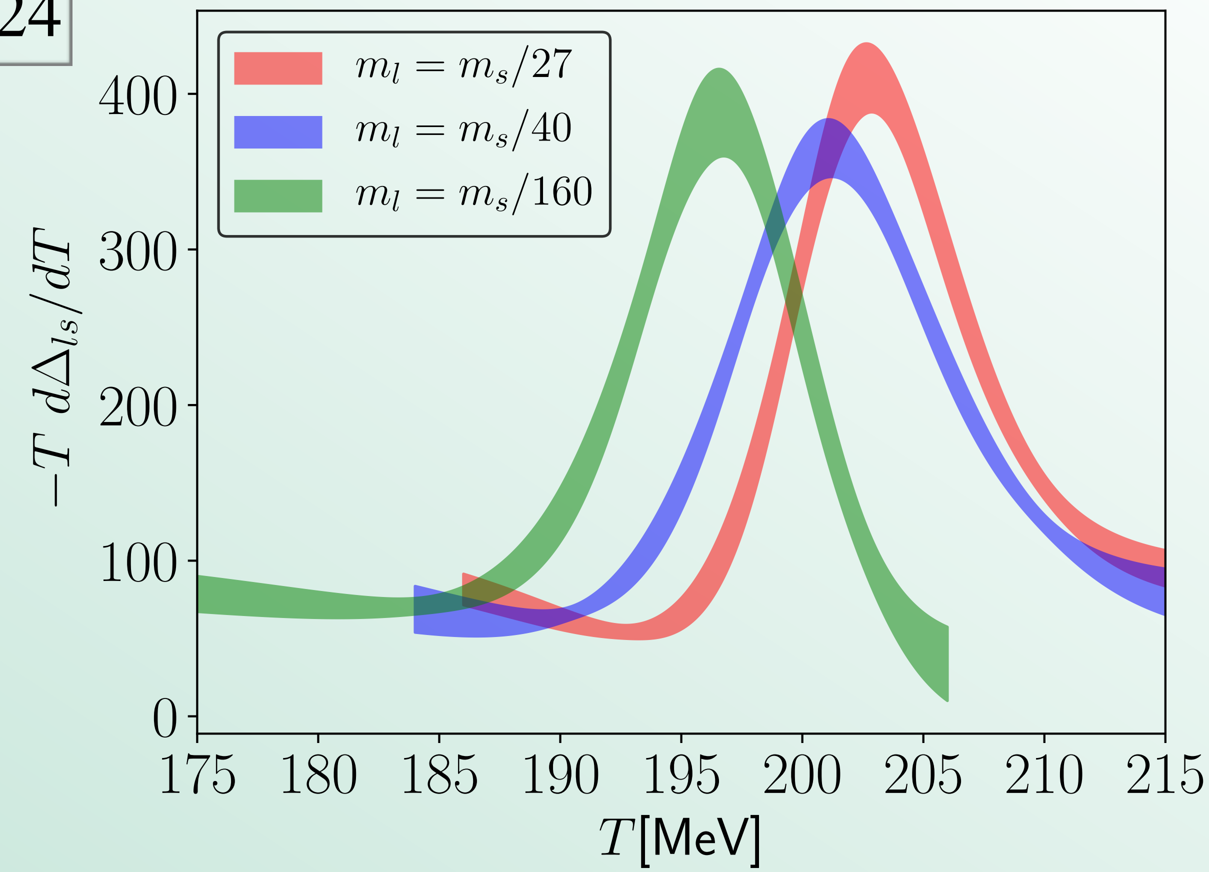


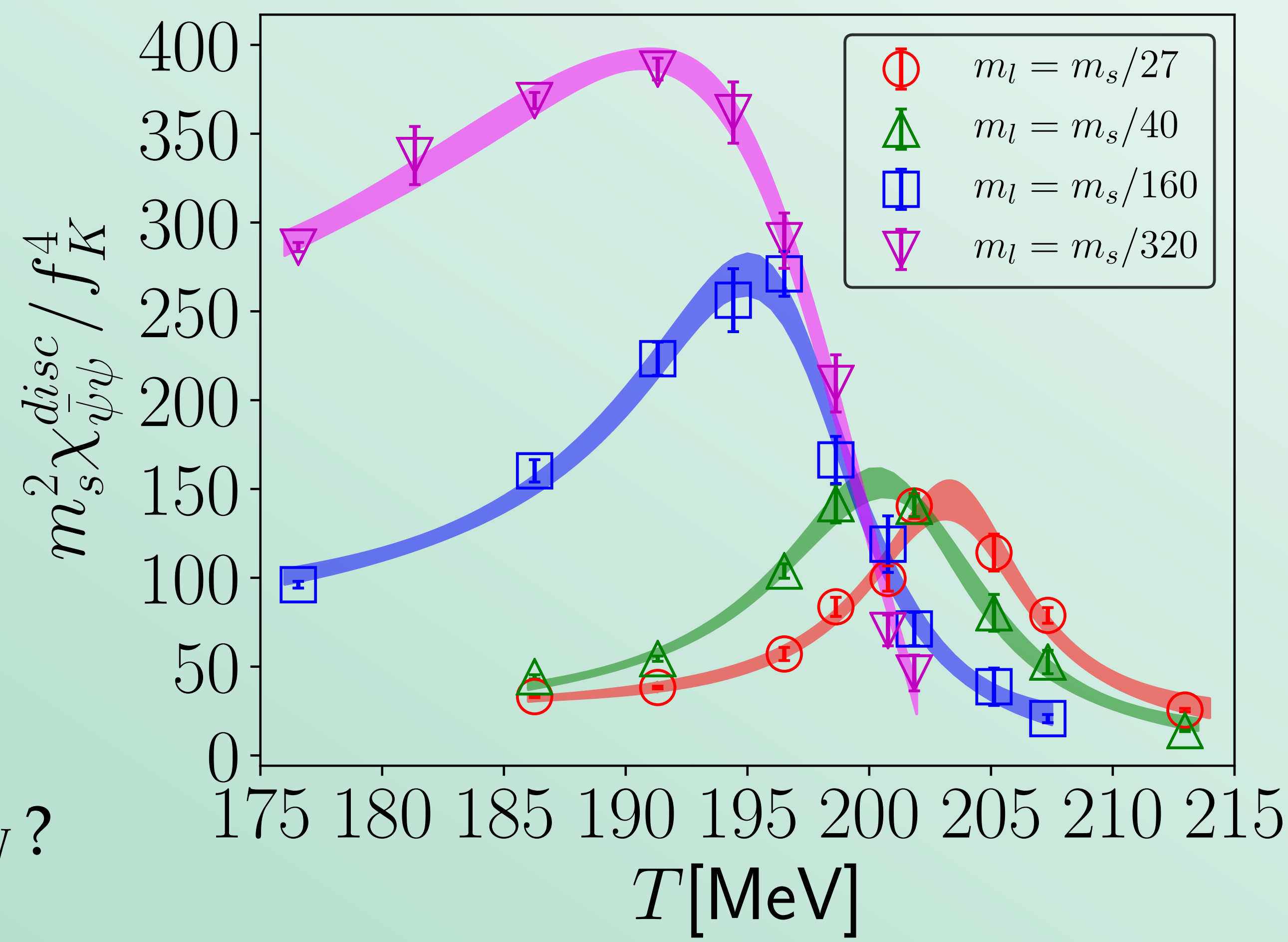
Chiral limit and RW transition

$N_\sigma = 24$

Sub. Chiral condensate







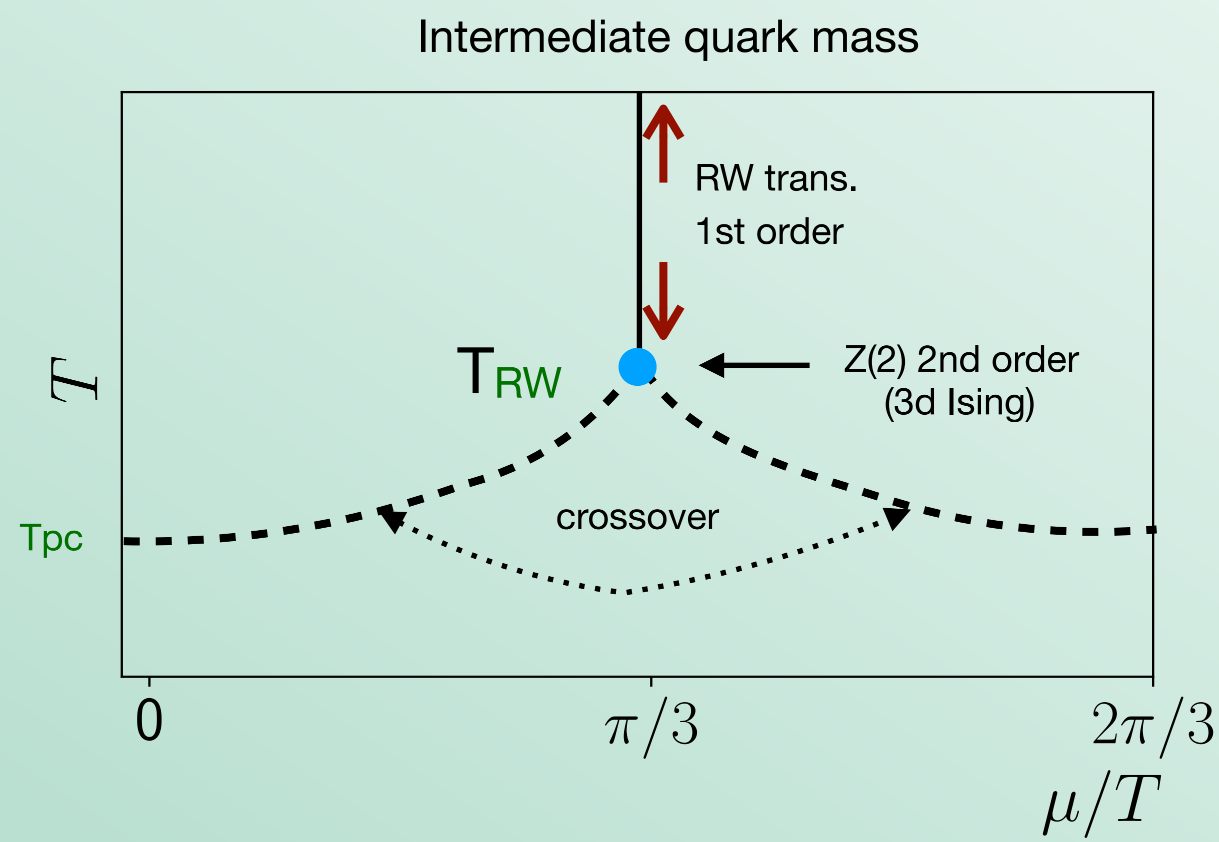
$$\langle \bar\psi\psi \rangle \sim c_0(T) + c_g(T)\ m^{1/2}$$

$$\chi^{disc}_{\bar\psi\psi} \sim m^{-1/2}$$ For, $T < T_c$

Goldstone effect (square root singularity) in $\chi^{disc}_{\bar l l}$ below T_{RW} is evident.

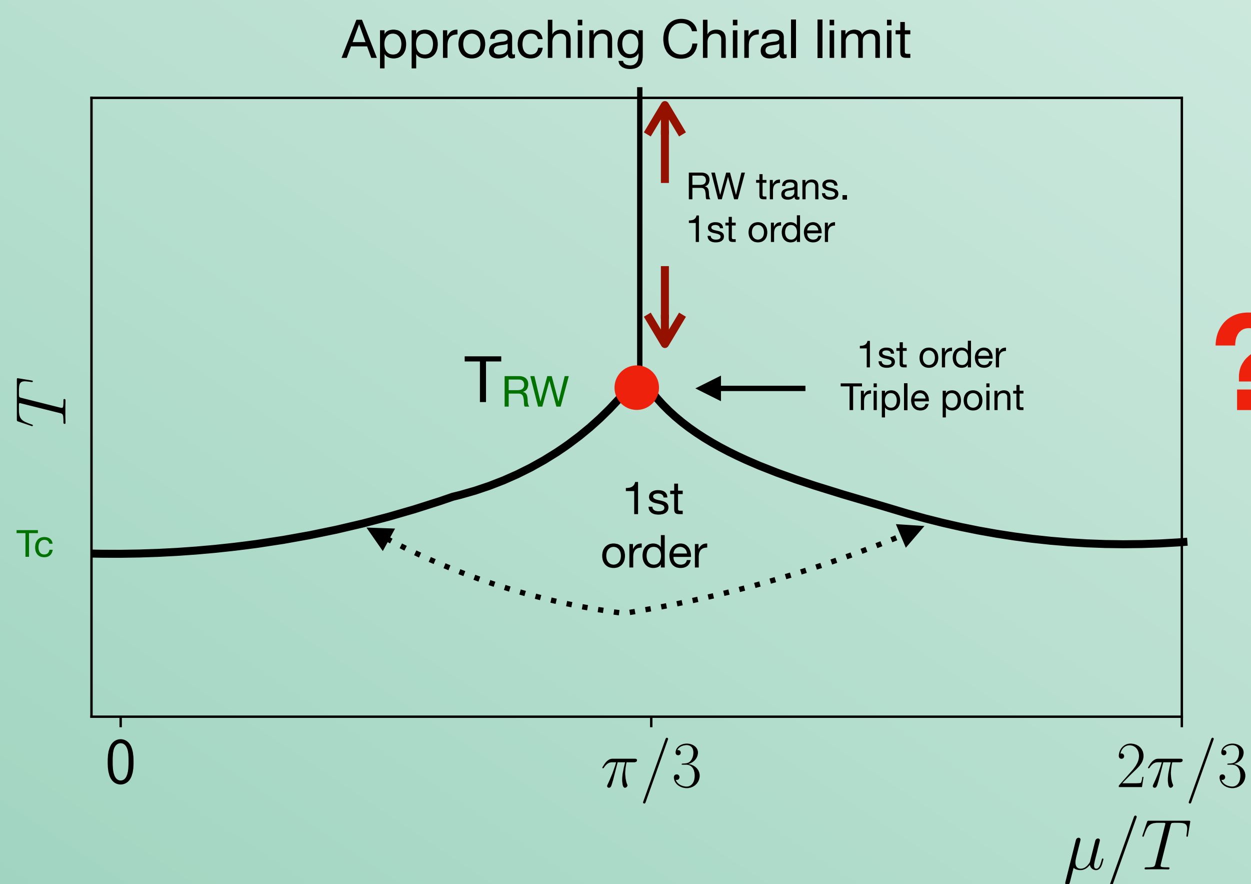
→ chiral symmetry restoration at T_{RW} ?

Outlook on the fate of the RW end point



$55 \text{ MeV} \le m_\pi \le 135 \text{ MeV}$

For, ~~intermediate~~ pion mass RW end point stays as Z(2) 2nd order.



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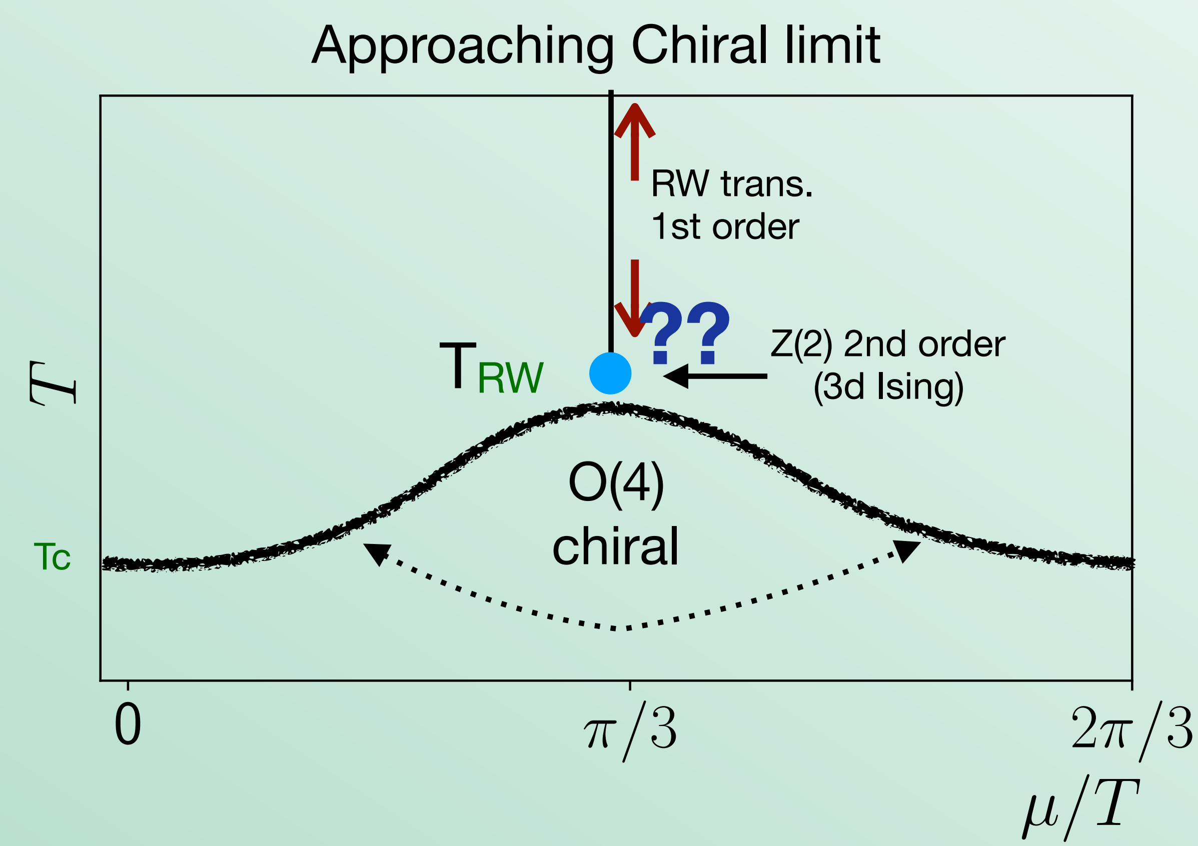

Conclusions

- Preliminary results down to $m_\pi \sim$ 40 MeV suggests that the RW end point remains as Z(2) second order.
- Nature of the chiral transition in the RW plane needs to be examined further. Favours 2nd order(O(4)) transition at μ=0.
- RW transition and chiral phase transition may coincide in the chiral limit.
- Calculations on larger lattices are ongoing.

no 1st order ≥ 40 MeV

phys. point

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Thank you for your time and attention

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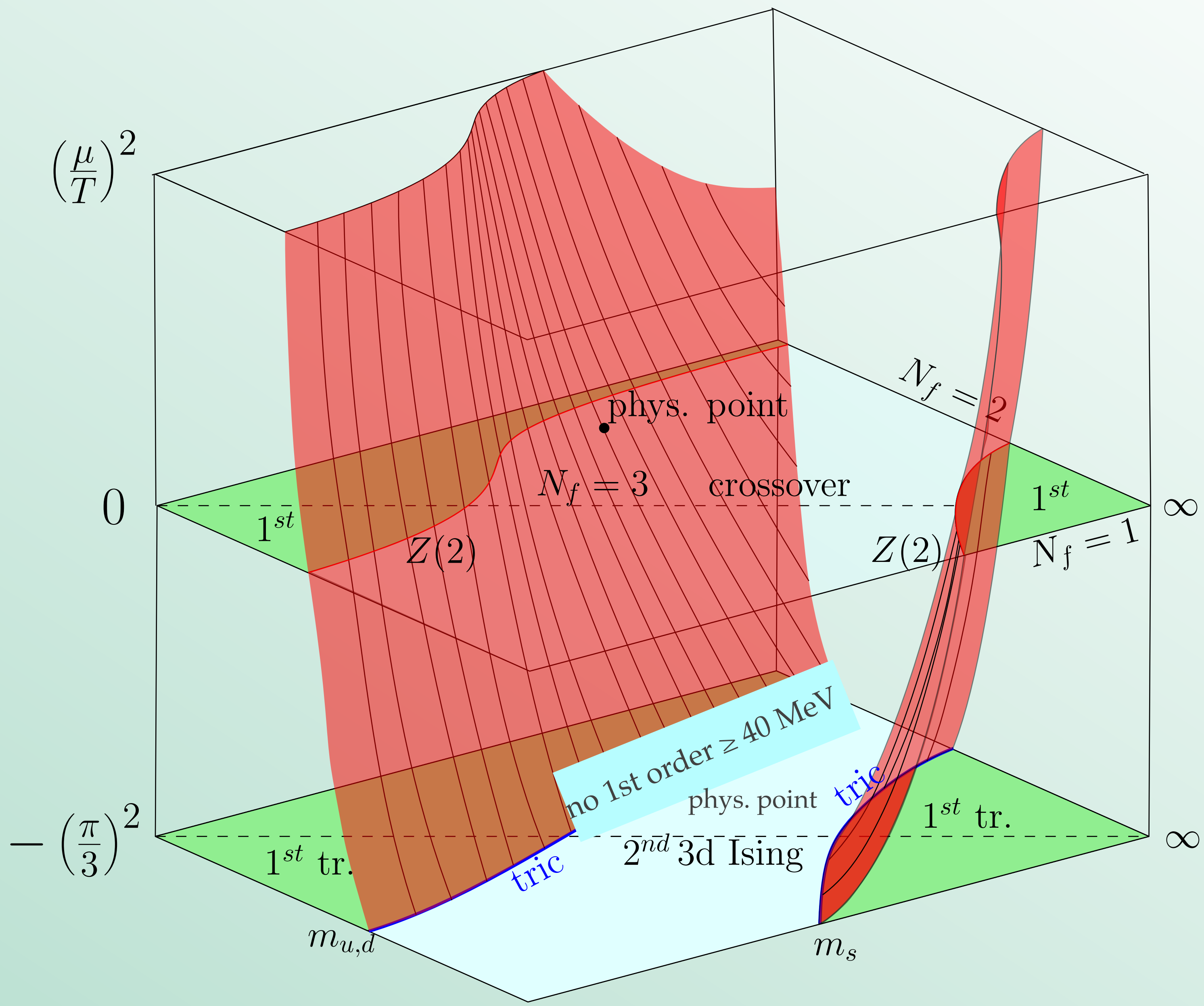

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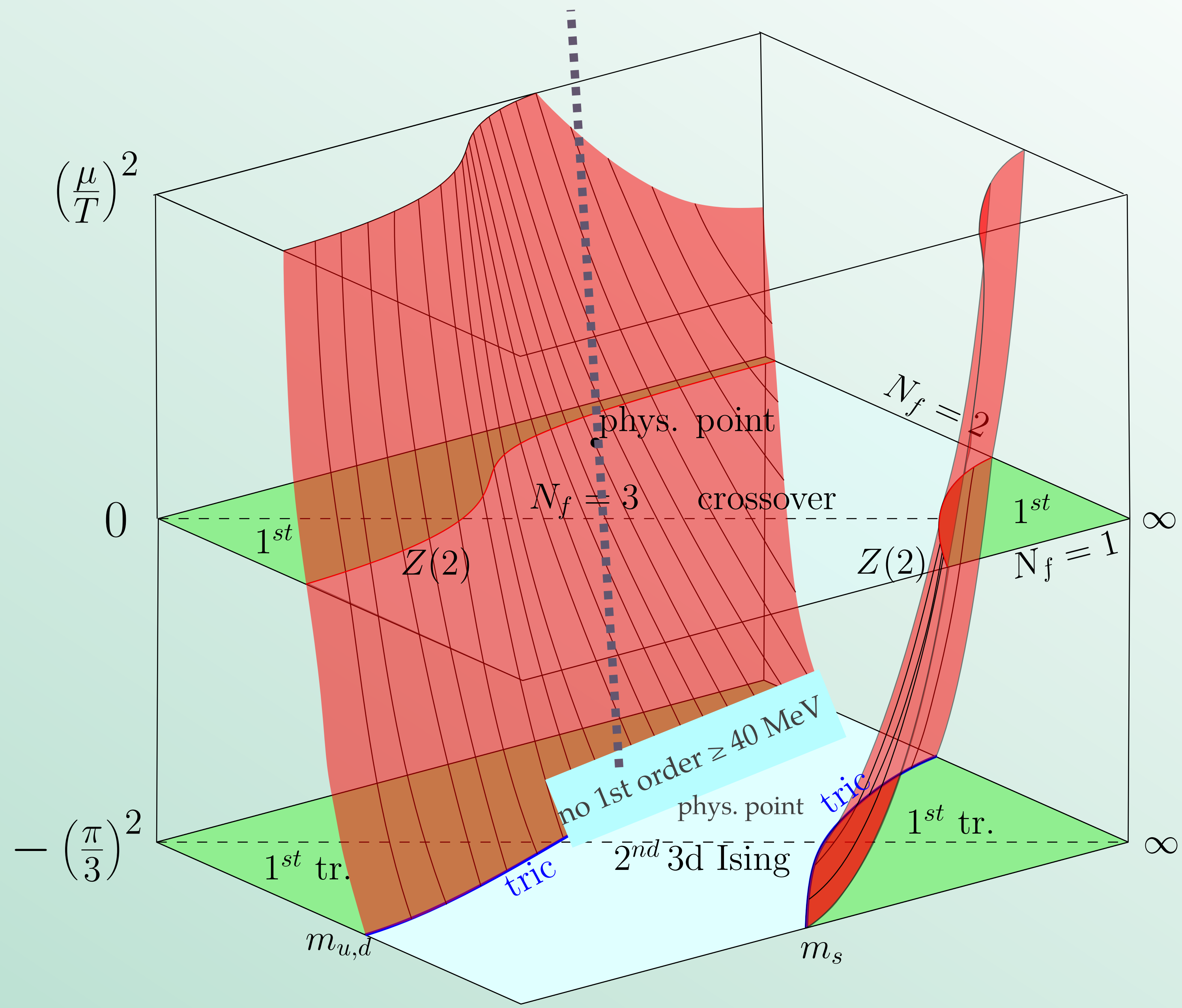

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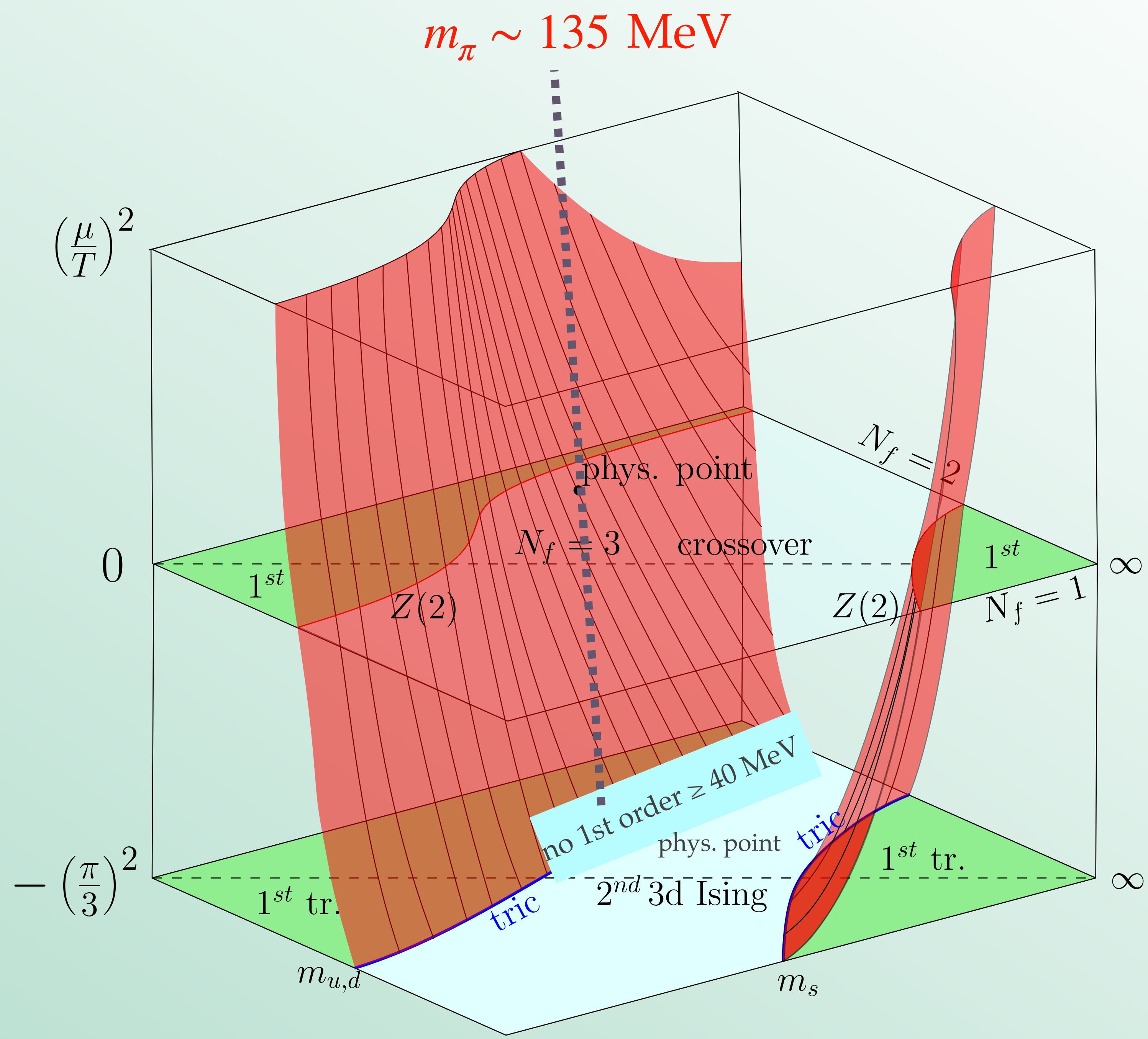

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Phases in the RW plane

- RW transition happens between two Z(3) sectors of the Polyakov loop. Hence, the order parameter can be the phase or the imaginary part of the Polyakov loop.

- In the RW plane, the 1st order region (for small mass) consists of three 1st order transitions, where high temperature RW transition meets two chiral phase transitions.

- The physical point which is crossover for $\mu=0$ can be 1st or 2nd order in the RW plane. So, our first goal is to confirm this issue and then going to the chiral limit to “search for a 1st order” transition.