Hagedorn bag-like model with a crossover transition meets lattice QCD

C. Greiner

From QCD matter to hadrons,

Intnl. Workshop XLVII on Gross Properties of Nuclei and Nuclear Excitations ,

Hirschegg , January 2019

in collaboration with:

V. Vovchenko, M. Gorenstein and H. Stöcker

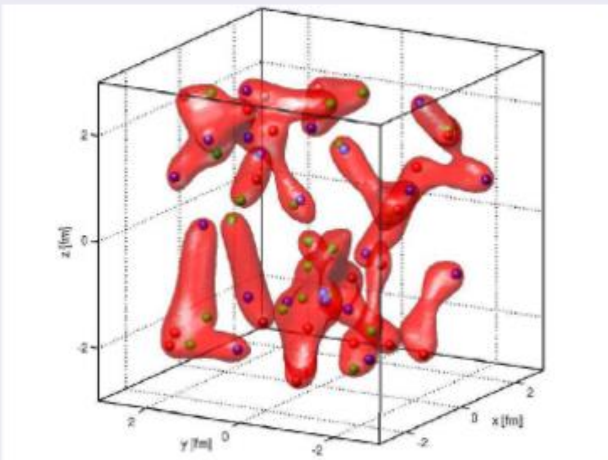
- (personal) history of Hagedorn States
- cross-over EoS with baglets within the pressure ensemble
- (higher order) baryon number and charges susceptibilities

Helmholtz International Center

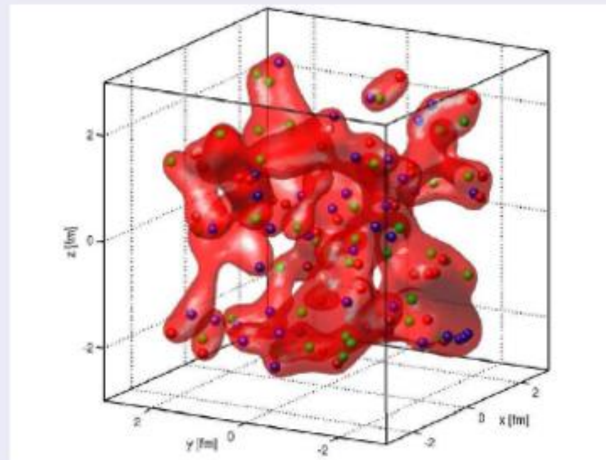
Deconfinement: transition to quark phase

G. Martens et al. Phys. Rev. D 70 / 73 (2006)

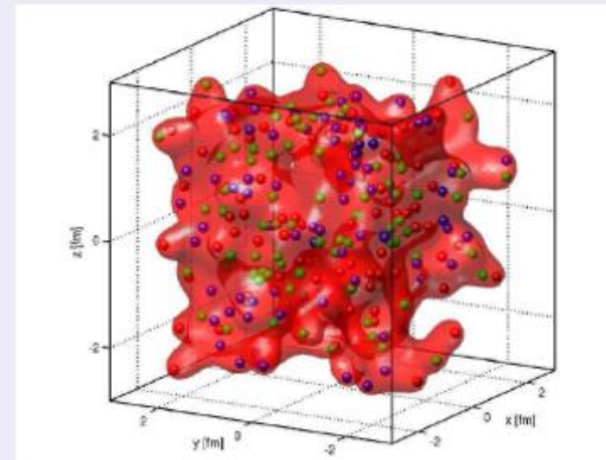
$n = 0.5\text{fm}^{-3}$



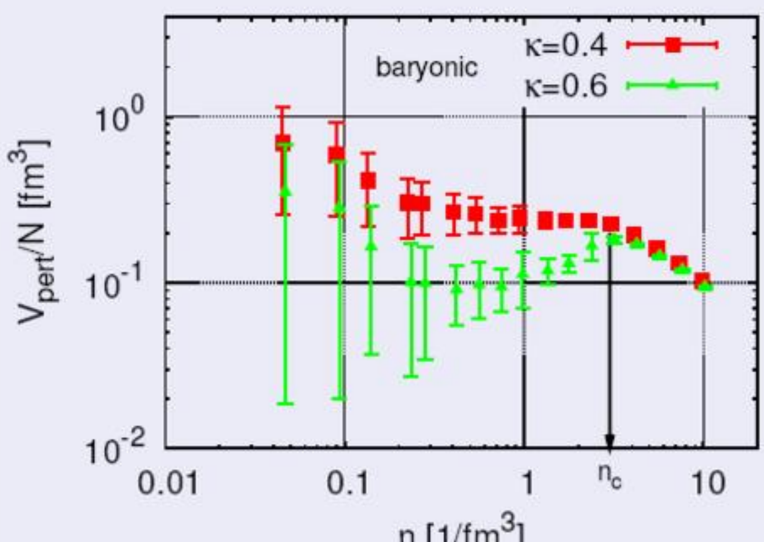
$n = 1.0\text{fm}^{-3}$



$n = 2.0\text{fm}^{-3}$



bag volume/particle

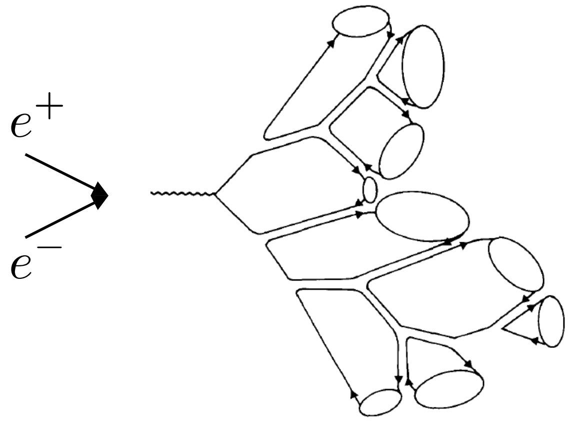


- formation of color neutral clusters at small densities
- particle number/cluster rises
- critical density at maximal overlap ($n \approx 2\text{fm}^{-3}$ or $\varepsilon \approx 1.1\text{GeV/fm}^3$)
- percolation transition

Colorless Heavy Objects

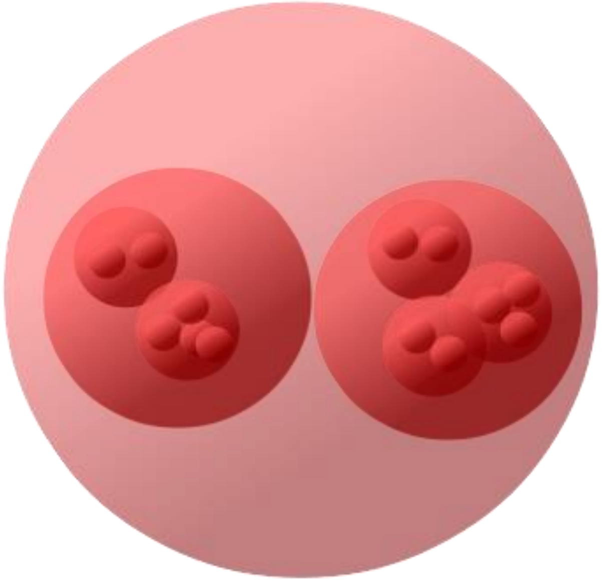
- Cluster (HERWIG)

B. Webber, Nucl.Phys.B 238 (1984) 492



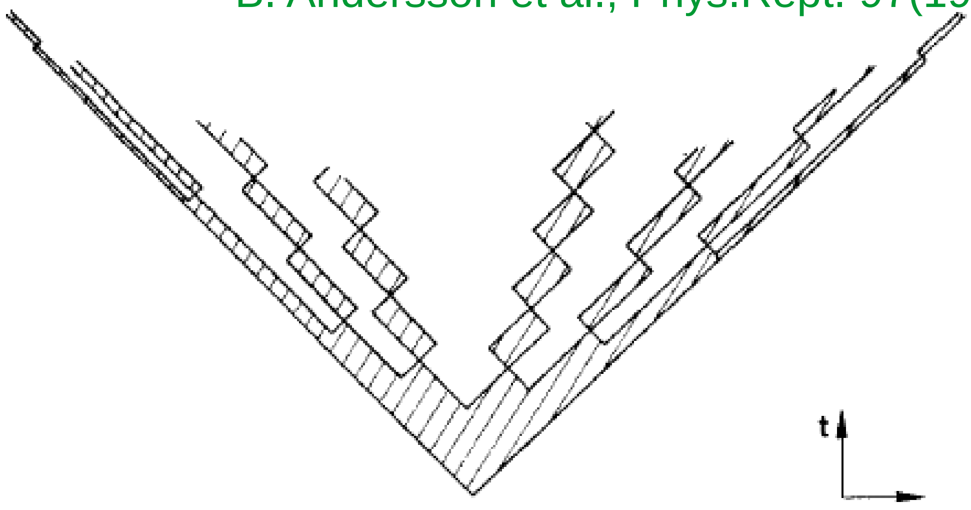
- Hagedorn states

R. Hagedorn, Nuovo Cim. Suppl. 3 (1965) 147



- Strings (Lund)

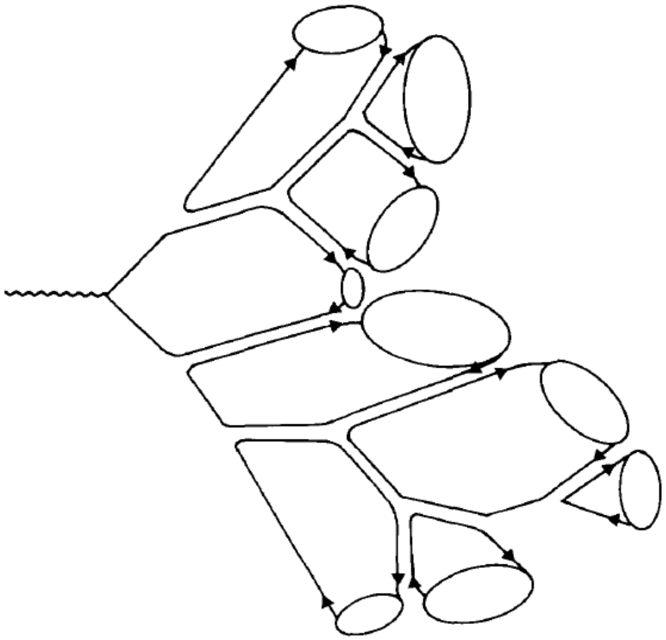
B. Andersson et al., Phys.Rept. 97(1983) 31

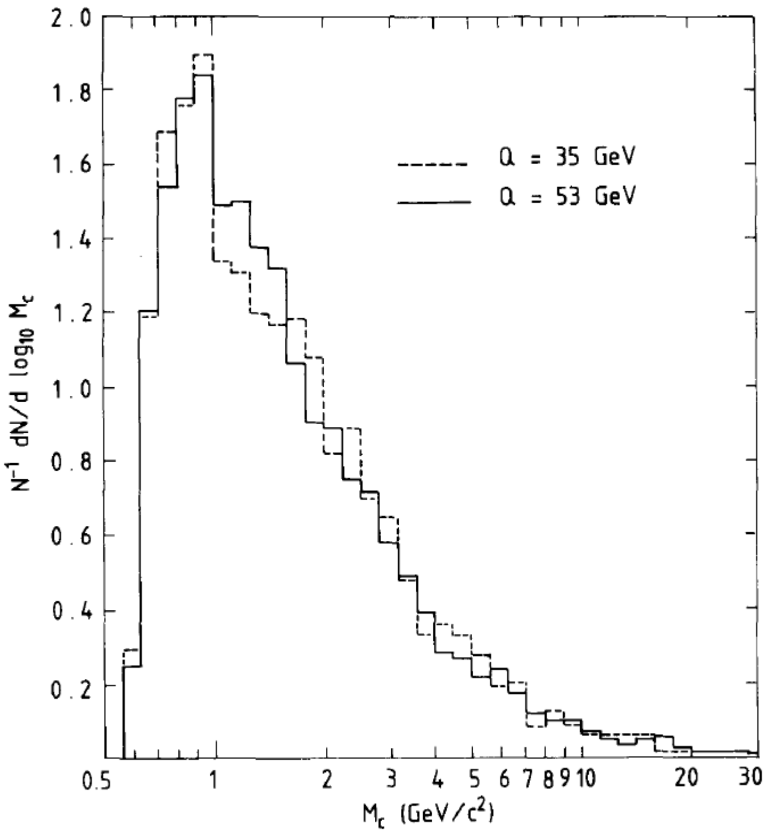


allow for
decay & recombination !

Color Singlet cluster and their distribution

B.R. Webber, Nucl. Phys. B 238 (1984)





- The blobs (right) represent colour singlet clusters as basis for hadronization
- Distribution of colour singlet cluster mass (left) in e+-e- annihilation at c.m. energies of Q=35 GeV and Q=53 GeV
- this colour singlet clusters might be identified as Hagedorn States

History

- 1965 R. Hagedorn postulated the “Statistical Bootstrap Model” before QCD

- fireballs and their constituents are the same

- nesting fireballs into each other leads to self-consistency condition (bootstrap equation)

- Euler : How many ways to subdivide an integer into different integer ? → solved in the 60ties

- solution is exponentially rising common known as Hagedorn spectrum

- slope of Hagedorn Spectrum determined by Hagedorn temperature

Maciej Sobczak – analysis of states listed in PDG2008 compilation

$$f_{FIT}(m) = log_{10}\left(\int_0^m \frac{c}{(x^2+m_0^2)^{5/4}} exp(x/T_H)\right) \qquad \rho(m) = \frac{c}{(m^2+m_0^2)^{5/4}} exp(m/T_H)$$

$$N_{exp}(m) = \sum_i g_i \Theta\left(m - m_i\right)$$

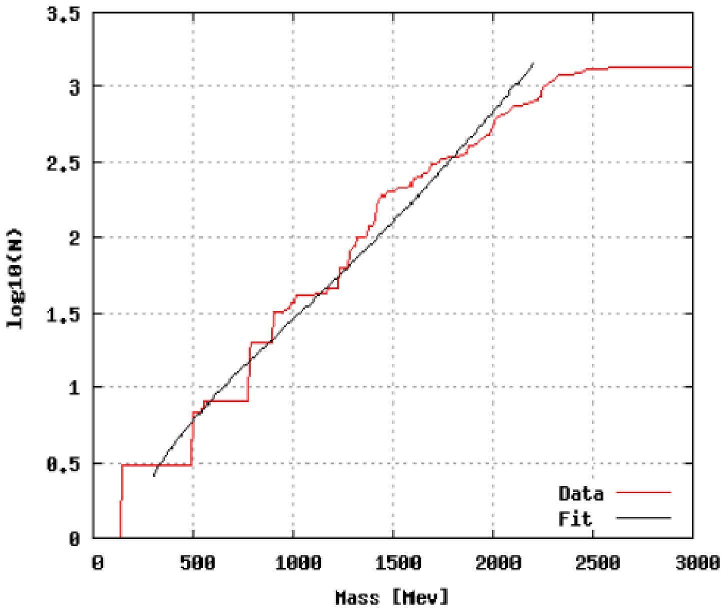


Figure 2: All mesons $T_H = 203.315$, $c = 25132.674$, range: $300 - 2200$ MeV

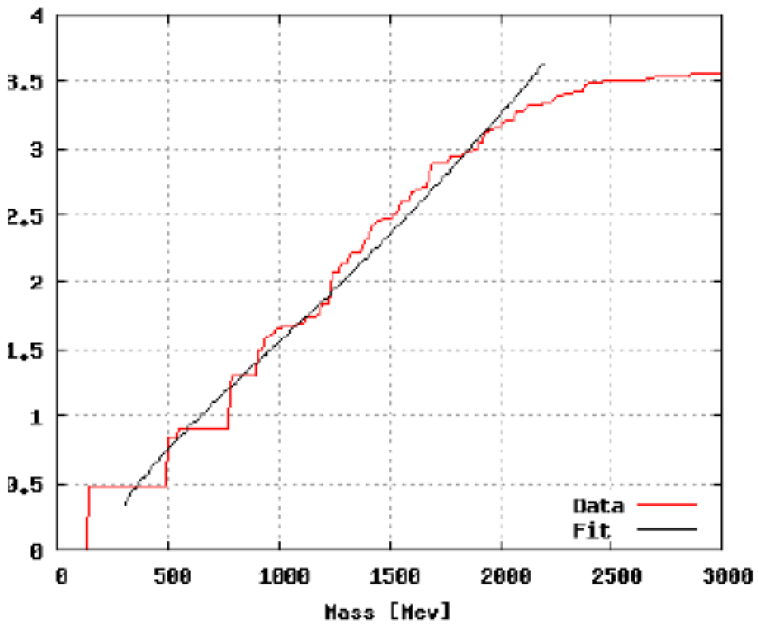


Figure 3: All hadrons $T_H = 177.086$, $c = 18726.494$, range: $300 - 2200$ MeV

Application of Hagedorn states

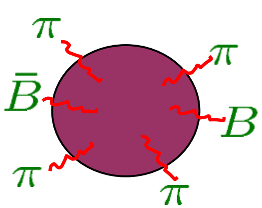
- at SPS energies chem. equil. time is 1-3 fm/c

$$n_1 \pi + n_2 K \leftrightarrow \bar{Y} + p$$

(CG, Leupold, 2000)

- at RHIC energies chem. equil. time is 10 fm/c
  with same approach

- fast chem. equil. mechanism through Hagedorn states



$$\left(n_1 \pi + n_2 K + n_3 K \leftrightarrow\right) \; HS \leftrightarrow \bar{B} + B + X$$

- dyn. evolution through set of coupled rate equations
  leads to 5 fm/c for BB pairs

J. Noronha-Hostler et al. PRL 100 (2008)
J. Noronha-Hostler et al. J. Phys. G 37 (2010)
J. Noronha-Hostler et al. Phys. Rev. C 81 (2010)

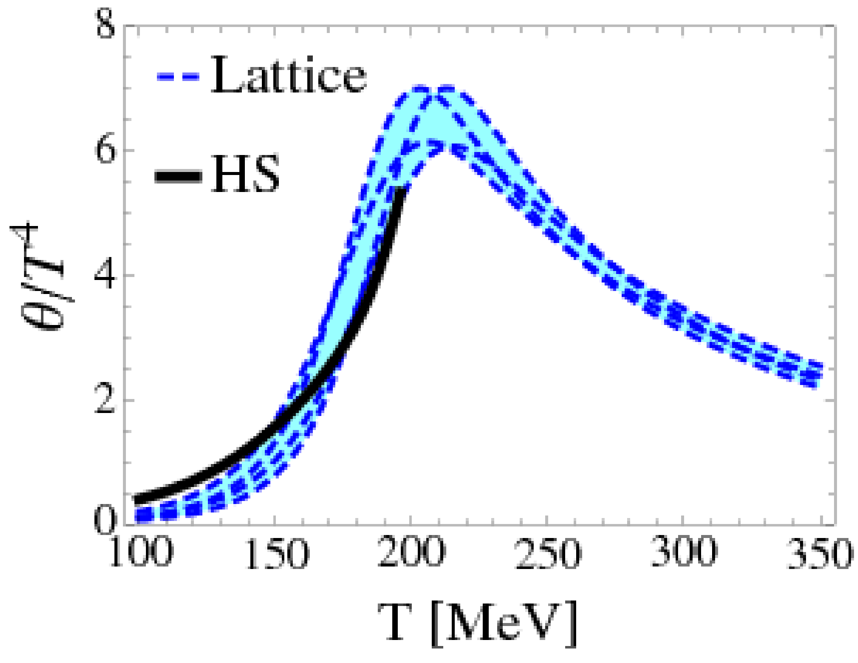
Hadron Resonance Gas with Hagedorn States and comparison to lattice QCD close to T_{critical}

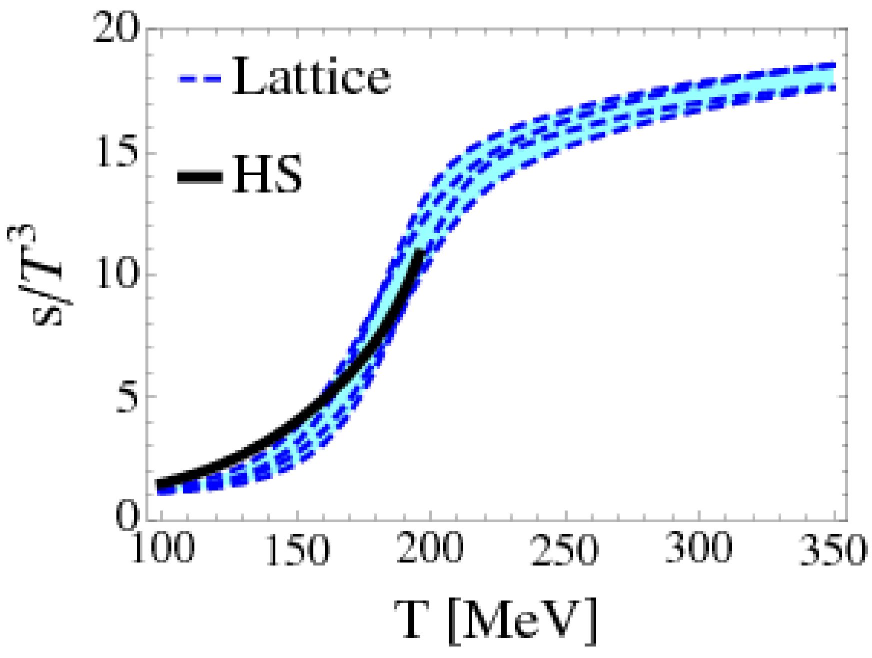
J. Noronha-Hostler, J. Noronha, CG, PRL 103 (2009), PRC 86 (2012)

- **Hagedorn spectrum**: $\rho_{HS} \sim m^{-a} \exp[m/T_H]$

$$\longrightarrow \quad \rho = \int_{M_0}^{M} \frac{A}{[m^2+m_r^2]^{\frac{5}{4}}} e^{\frac{m}{T_H}} dm$$

- **RBC collaboration**:

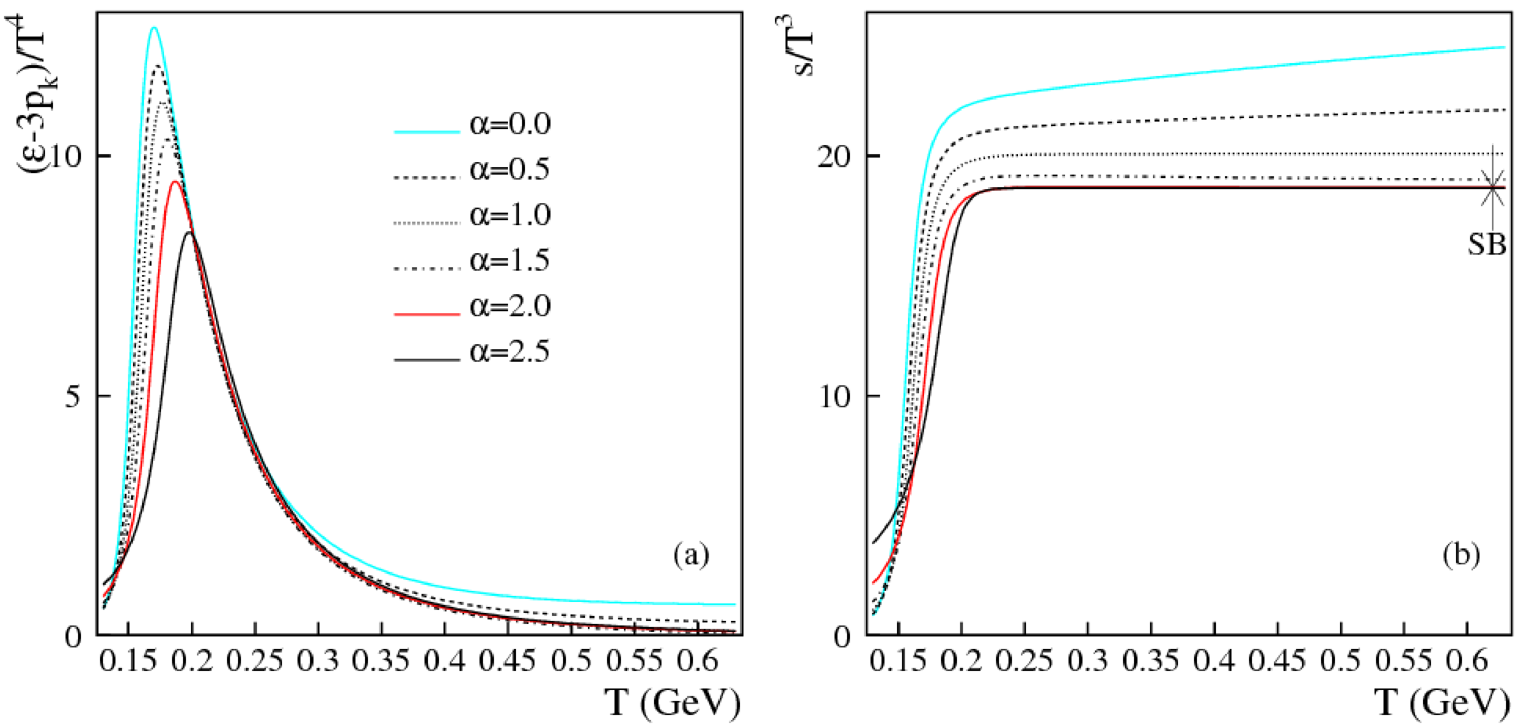




(Phase) transition in the gas of bags

- Both phases described by single partition function
- A gas of extended objects → excluded volume $V \to V - vN$
- Exponential spectrum of bags $\rho(m) = A\, m^{-\alpha}\, \exp(m/T_H)$

[Gorenstein, Petrov, Zinovjev, PLB ’81; Gorenstein, Greiner, Yang, JPG ’98;
Zakout, CG, Schaffner-Bielich, NPA ‘07]

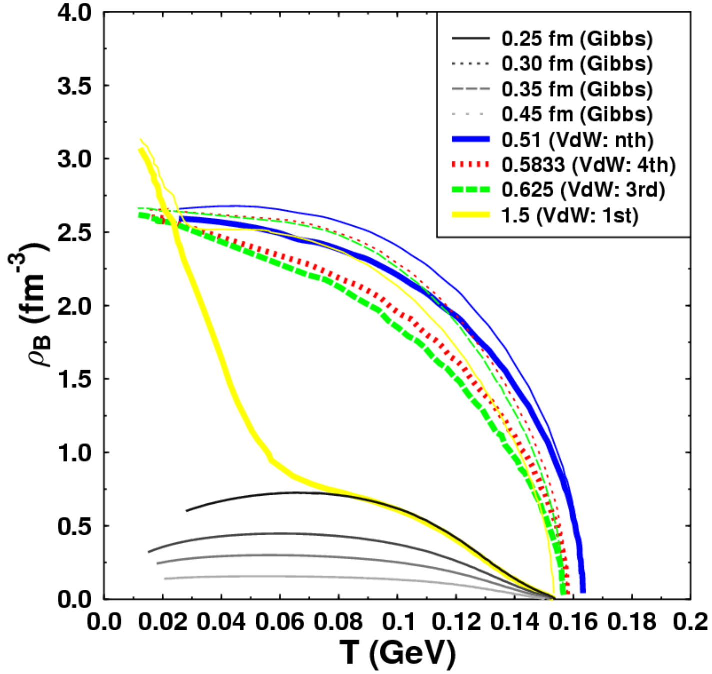


[Ferroni, Koch, PRC 79, 034905 (2009)]

Crossover transition in bag-like model qualitatively compatible with LQCD

The order and shape of QGP phase transition

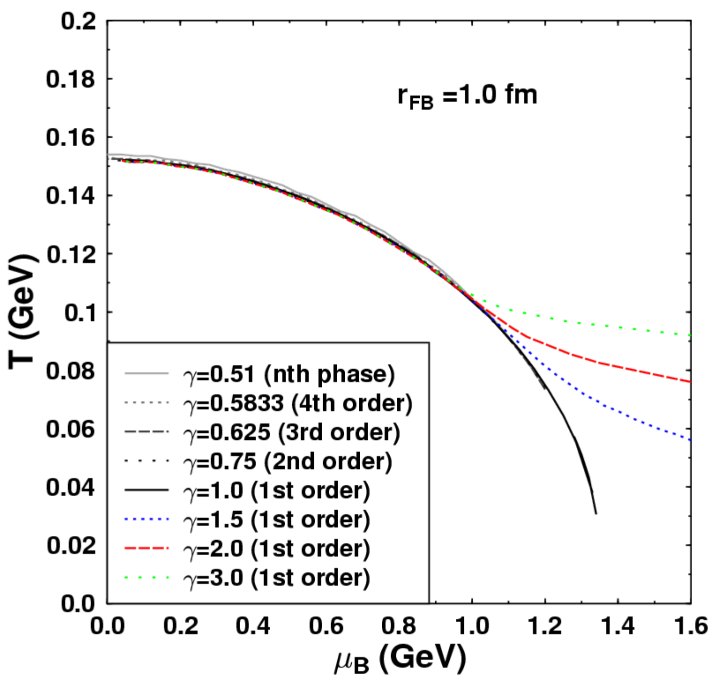
I.Zakout, CG and J. Schaffner-Bielich, NPA 781 (2007) 150, PRC78 (2008)

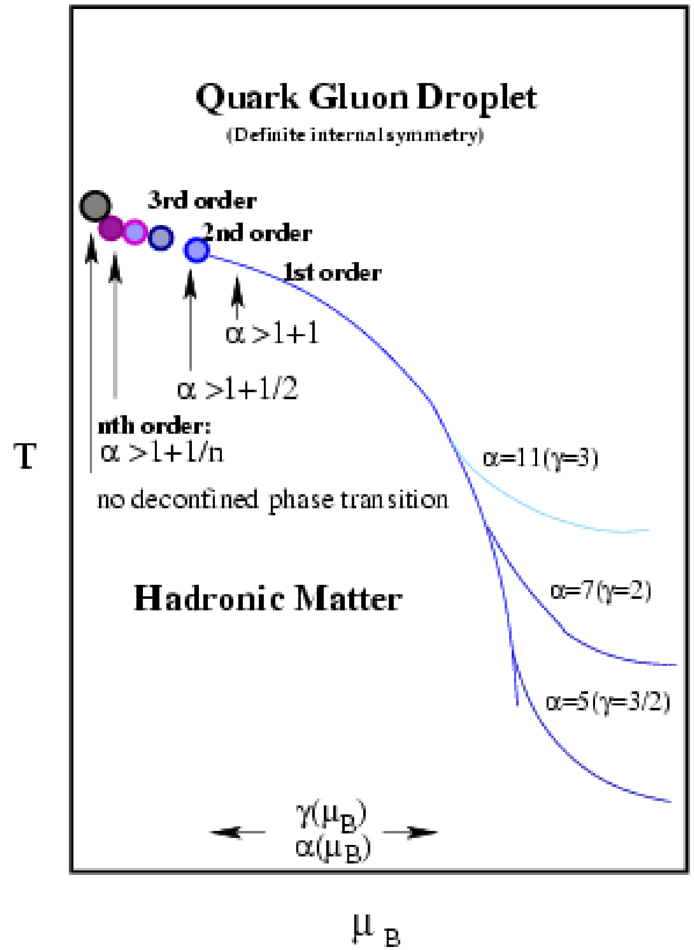


$$\} \gamma = \frac{\alpha + 1}{4}$$

density of states:

$$\rho(m,v) \sim c\, m^{-(\alpha+2)}\, e^{\frac{m}{T_{H[B]}}}\, \delta(m - 4Bv)$$

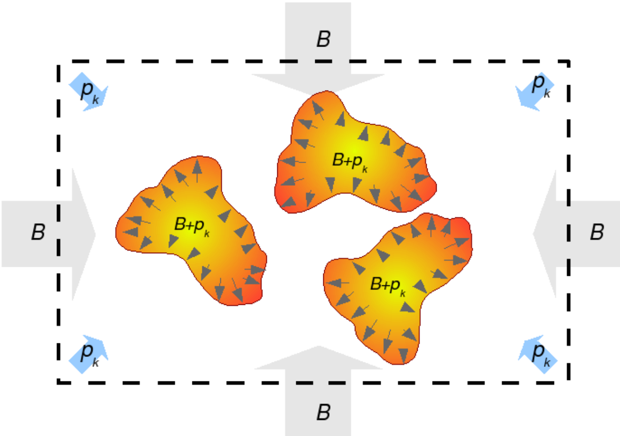




$$\alpha(\mu_B)$$

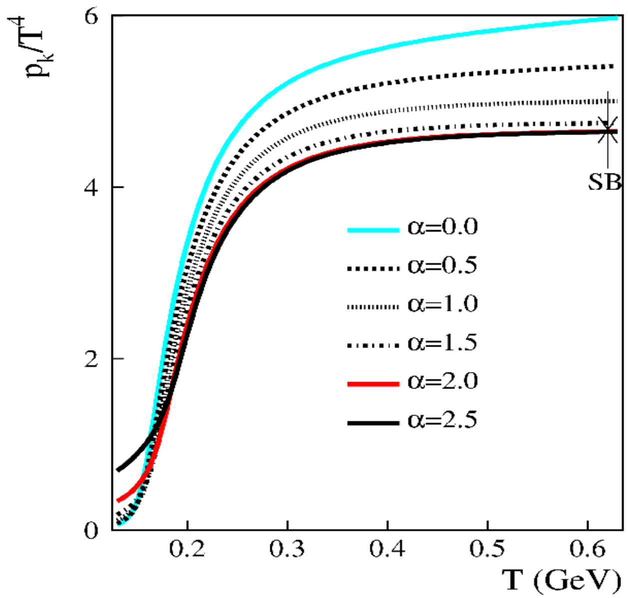
Crossover transition in bag-like models

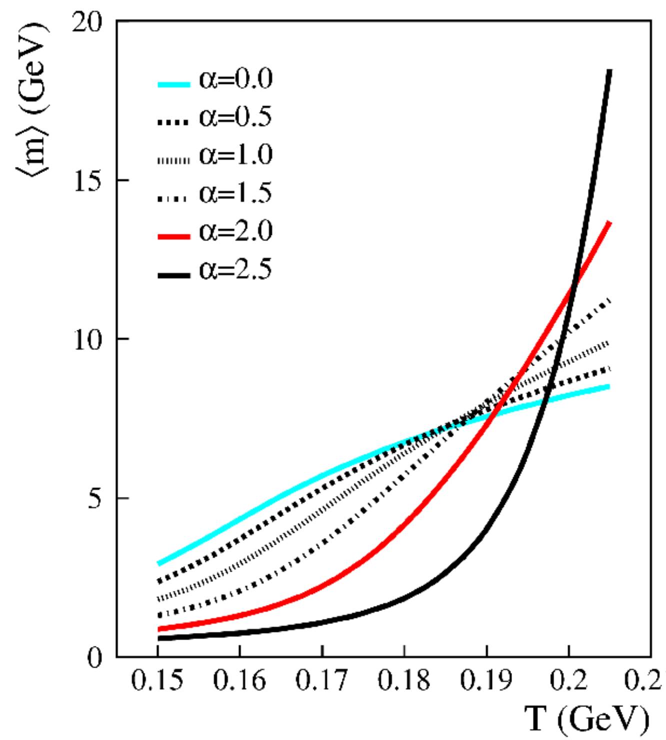
L. Ferroni and V. Koch, PRC79 (2009) 034905

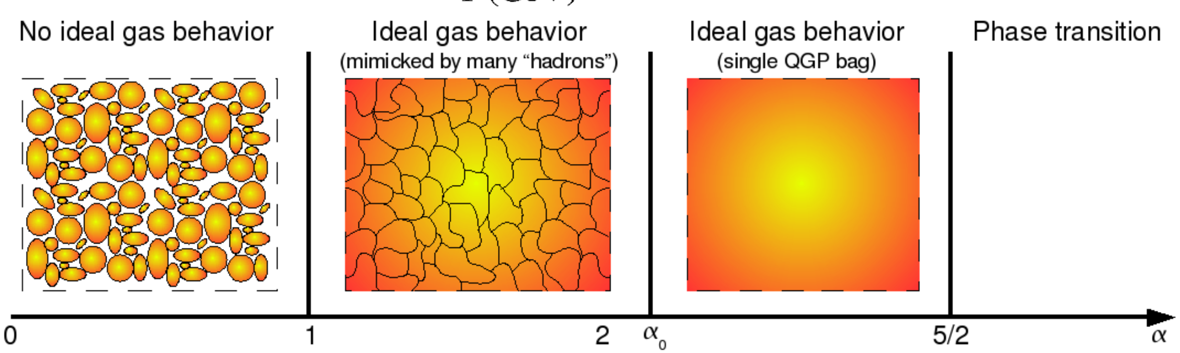


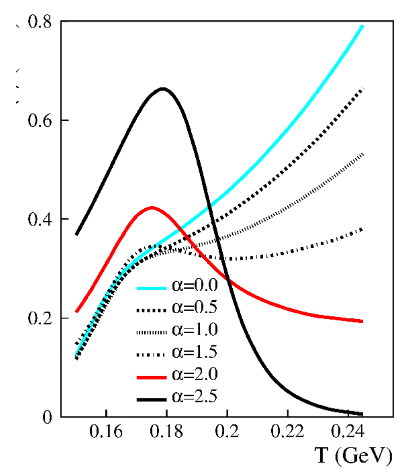
density of states:

$$\rho(m) \sim c\, m^{-(\alpha)}\, e^{\frac{m}{T_H[B]}}$$









Model formulation

Thermodynamic system of known hadrons and quark-gluon bags

Mass-volume density: $\rho(m, v; \lambda_B, \lambda_Q, \lambda_S) = \rho_H + \rho_Q$

$$\rho_H(m, v; \lambda_B, \lambda_Q, \lambda_S) = \sum_{i \in \mathrm{HRG}} \lambda_B^{b_i} \lambda_Q^{q_i} \lambda_S^{s_i} d_i \, \delta(m - m_i) \, \delta(v - v_i)$$

PDG hadrons

$$\rho_Q(m, v; \lambda_B, \lambda_Q, \lambda_S) = C \, v^\gamma \, (m - Bv)^\delta \, \exp\left\{ \frac{4}{3} [\sigma_Q v]^{1/4} \, (m - Bv)^{3/4} \right\} \theta(v - V_0) \, \theta(m - Bv)$$

Quark-gluon bags [J. Kapusta, PRC '81; Gorenstein+, ZPC '84]

Non-overlapping particles → **isobaric (pressure) ensemble**

[Gorenstein, Petrov, Zinovjev, PLB '81]

$$\hat{Z}(T, s, \lambda_B, \lambda_Q, \lambda_S) = \int_0^\infty Z(T, V, \lambda_B, \lambda_Q, \lambda_S) \, e^{-sV} \, dV = [s - f(T, s, \lambda_B, \lambda_Q, \lambda_S)]^{-1}$$

$$f(T, s, \lambda_B, \lambda_Q, \lambda_S) = \int dv \int dm \, \rho(m, v; \lambda_B, \lambda_Q, \lambda_S) \, e^{-v\,s} \, \phi(T, m)$$

The system pressure is $p = T s^*$ with s^* being the *rightmost* singularity of $\hat{Z}$

Mechanism for transition to QGP

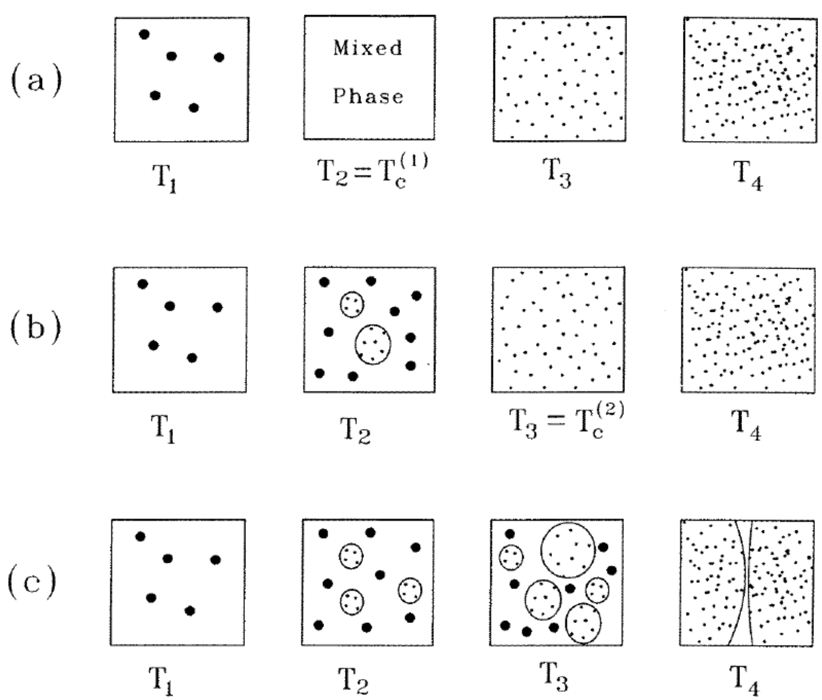
The isobaric partition function, $\hat{Z}(T, s, \lambda) = [s - f(T, s, \lambda)]^{-1}$, has

- pole singularity $s_H = f(T, s_H, \lambda)$ **"hadronic" phase**
- singularity s_B in the function $f(T, s, \lambda)$ due to the exponential spectrum

$$p_B = T\, s_B = \frac{\sigma_Q}{3} T^4 - B$$

MIT bag model EoS for QGP

[Chodos+, PRD '74; Baacke, APPB '77]



1st order PT

"collision" of singularities

$$s_H(T_C) = s_B(T_C)$$

2st order PT

crossover $s_H(T) > s_B(T)$ at all T

Crossover transition

Type of transition is determined by exponents γ and δ of bag spectrum

Crossover seen in lattice, realized in model for $\gamma + \delta \geq -3$ and $\delta \geq -7/4$

[Begun, Gorenstein, W. Greiner, JPG '09]

Transcendental equation for pressure:

$$p(T, \lambda_B, \lambda_Q, \lambda_S) = T \sum_{i \in \mathrm{HRG}} d_i \, \phi(T, m) \, \lambda_B^{b_i} \, \lambda_Q^{q_i} \, \lambda_S^{s_i} \, \exp\left(-\frac{m_i p}{4BT}\right)$$

$$+ \frac{C}{\pi} T^{5+4\delta} [\sigma_Q]^{\delta+1/2} [B + \sigma_Q T^4]^{3/2} \left(\frac{T}{p - p_B}\right)^{\gamma+\delta+3} \Gamma\left[\gamma + \delta + 3, \frac{(p - p_B) V_0}{T}\right]$$

Solved numerically

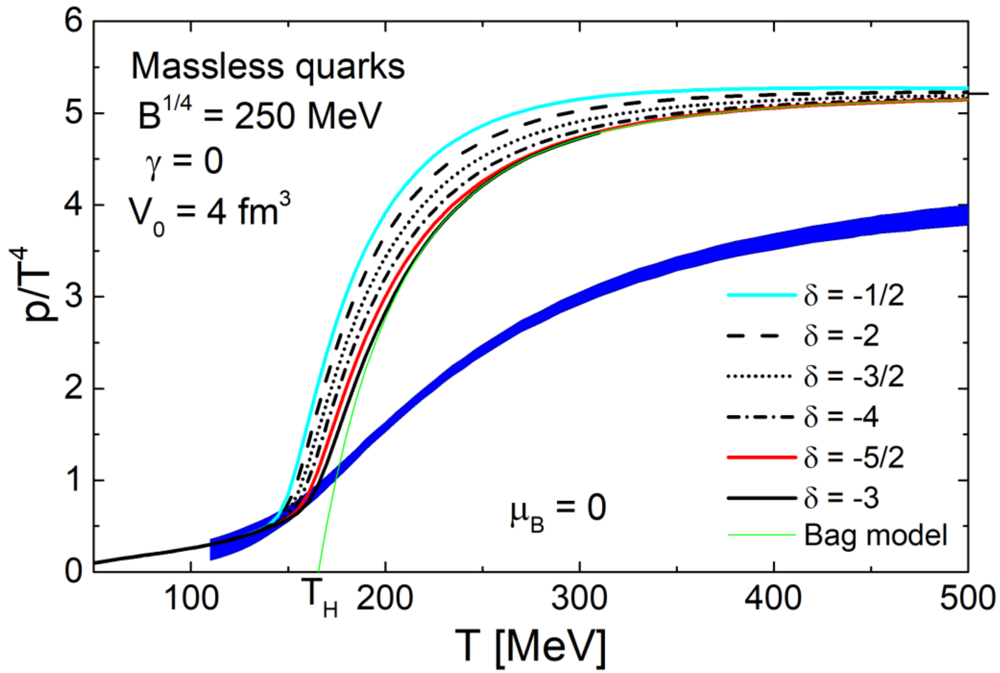
Calculation setup:

$$\gamma = 0, \quad -3 \leq \delta \leq -\frac{1}{2}, \quad B^{1/4} = 250 \text{ MeV}, \quad C = 0.03 \text{ GeV}^{-\delta+2}, \quad V_0 = 4 \text{ fm}^3$$

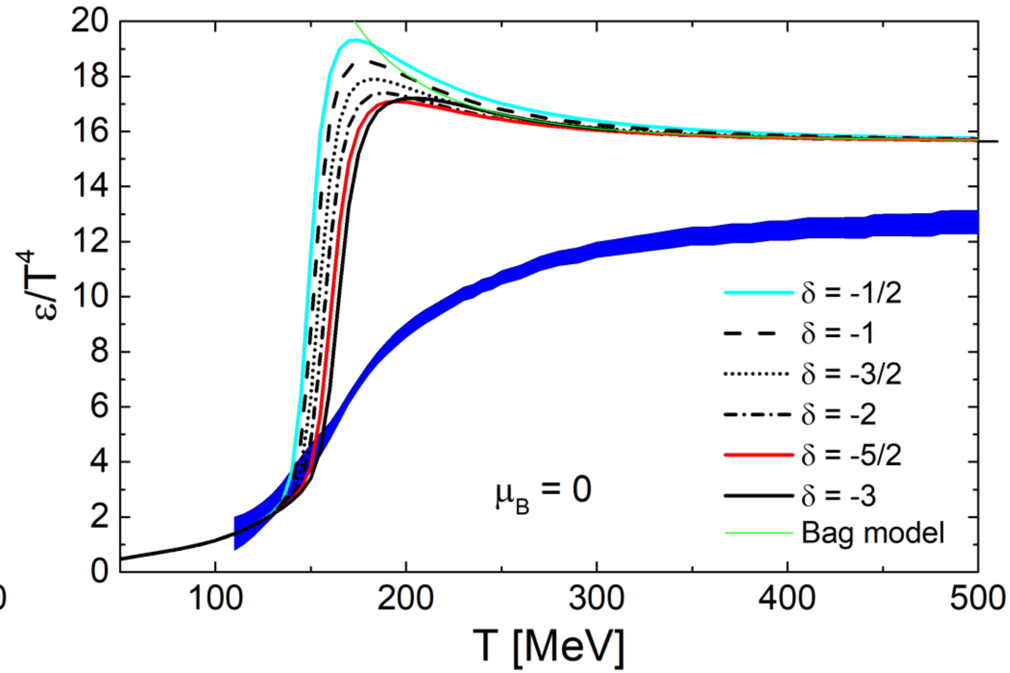
$$T_H = \left(\frac{3B}{\sigma_Q}\right)^{1/4} \simeq 165 \text{ MeV}$$

Thermodynamic functions

Pressure p/T^4



Energy density ε/T^4

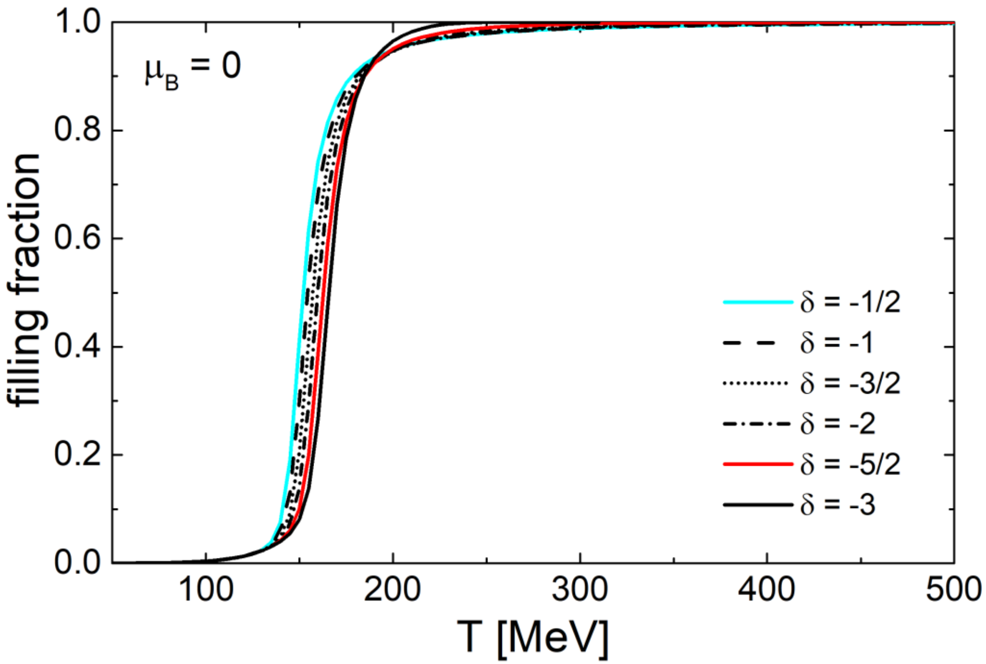


Lattice data from 1309.5258 (Wuppertal-Budapest)

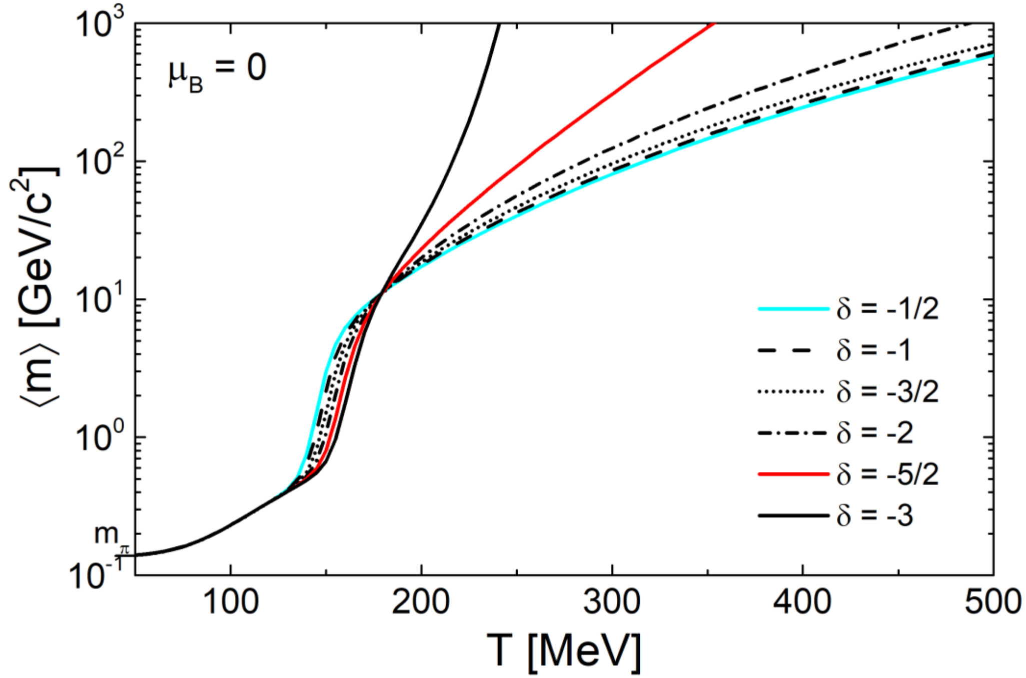
- Crossover transition towards bag model EoS
- Dependence on δ is mild
- Approach to the Stefan-Boltzmann limit is too fast
- Peak in energy density, not seen on the lattice

Nature of the transition

Filling fraction = $\frac{<V_{had}>}{V}$



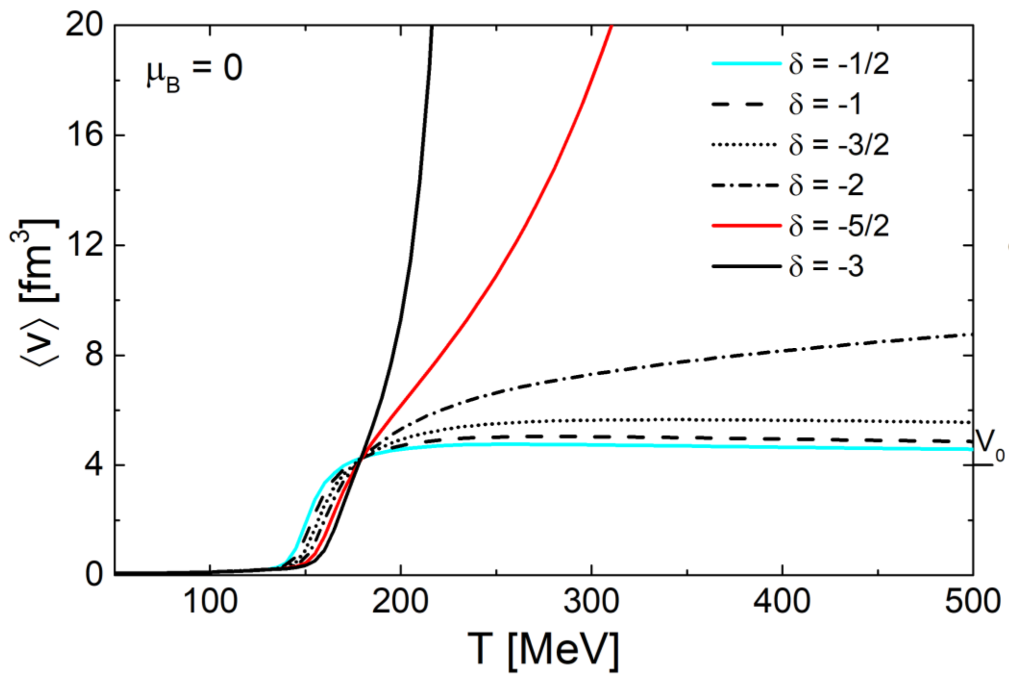
Mean hadron mass $< \mathrm{m} >$



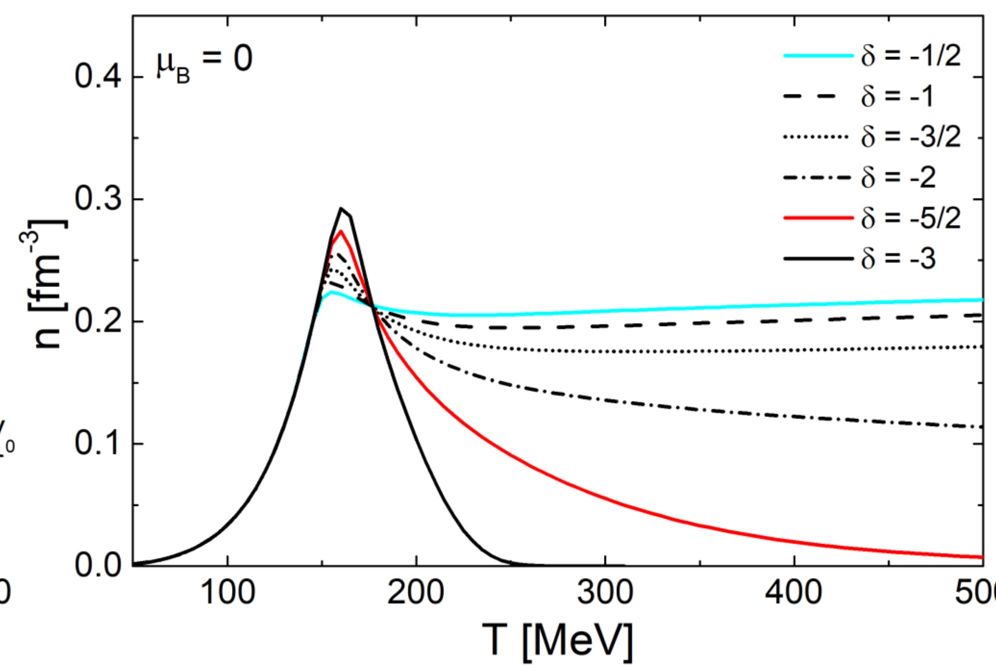
- Bags occupy almost whole space at large temperatures
- Strongest changes take place in the vicinity of T_H
- Heavy bags contribute dominantly at high temperatures

Nature of the transition

Mean hadron volume $< v >$



Hadron number density n

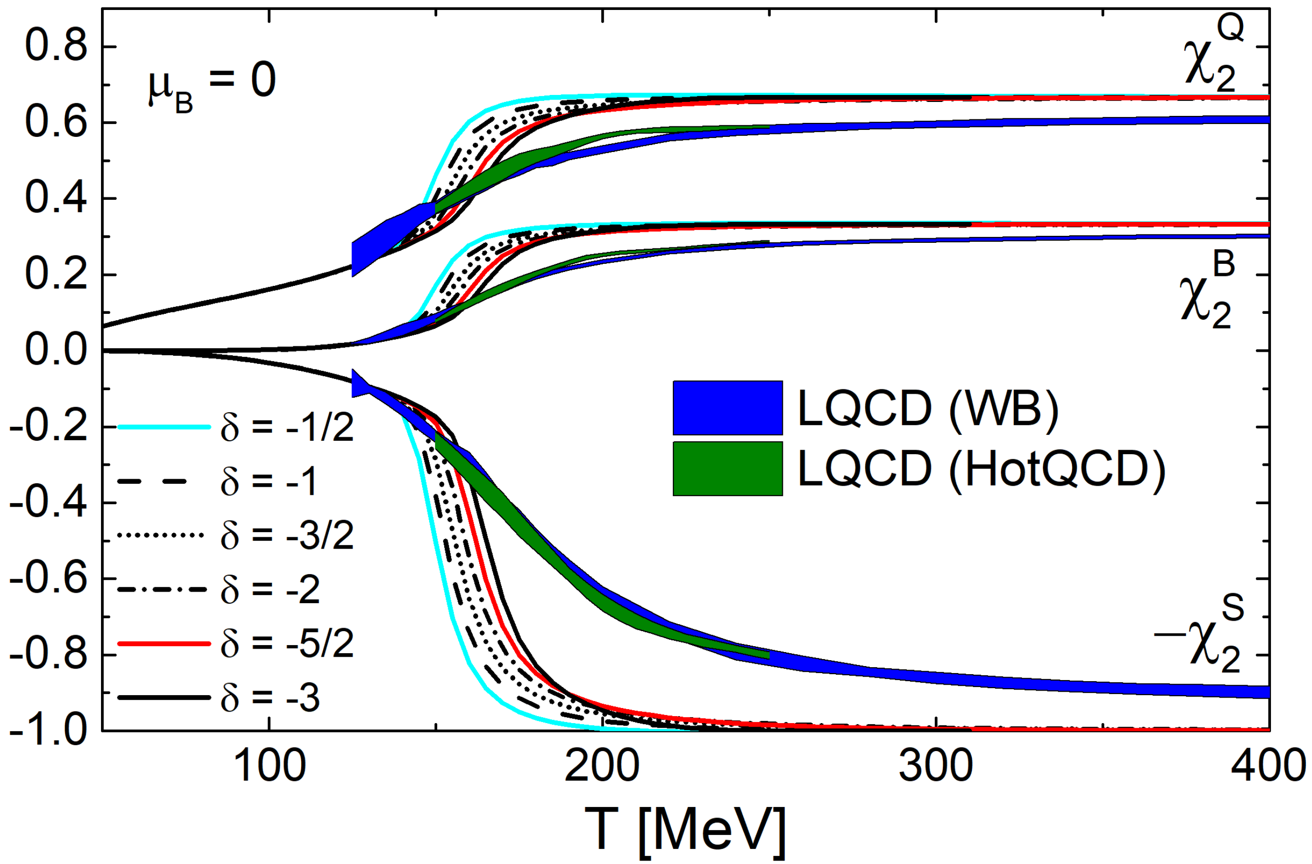


- $< v > \to \infty$ for $\delta < -7/4$ and $< v > \to V_0$ for $\delta > -7/4$
- At $\delta < -7/4$ and $T \to \infty$ whole space occupied by arbitrary large bags with QGP

Conserved charges susceptibilities

$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \, \partial(\mu_S/T)^m \, \partial(\mu_Q/T)^n}$$

Available from lattice QCD, not considered in this type of model before



Qualitatively compatible with lattice QCD

Bag model with massive quarks

Main source of quantitative disagreement comes from inaccuracy of the standard MIT bag model with massless quarks for describing QGP

Quasiparticle models suggest sizable thermal masses of quarks and gluons in high-temperature QGP [Peshier et al., PLB '94; PRC '00; PRC '02]

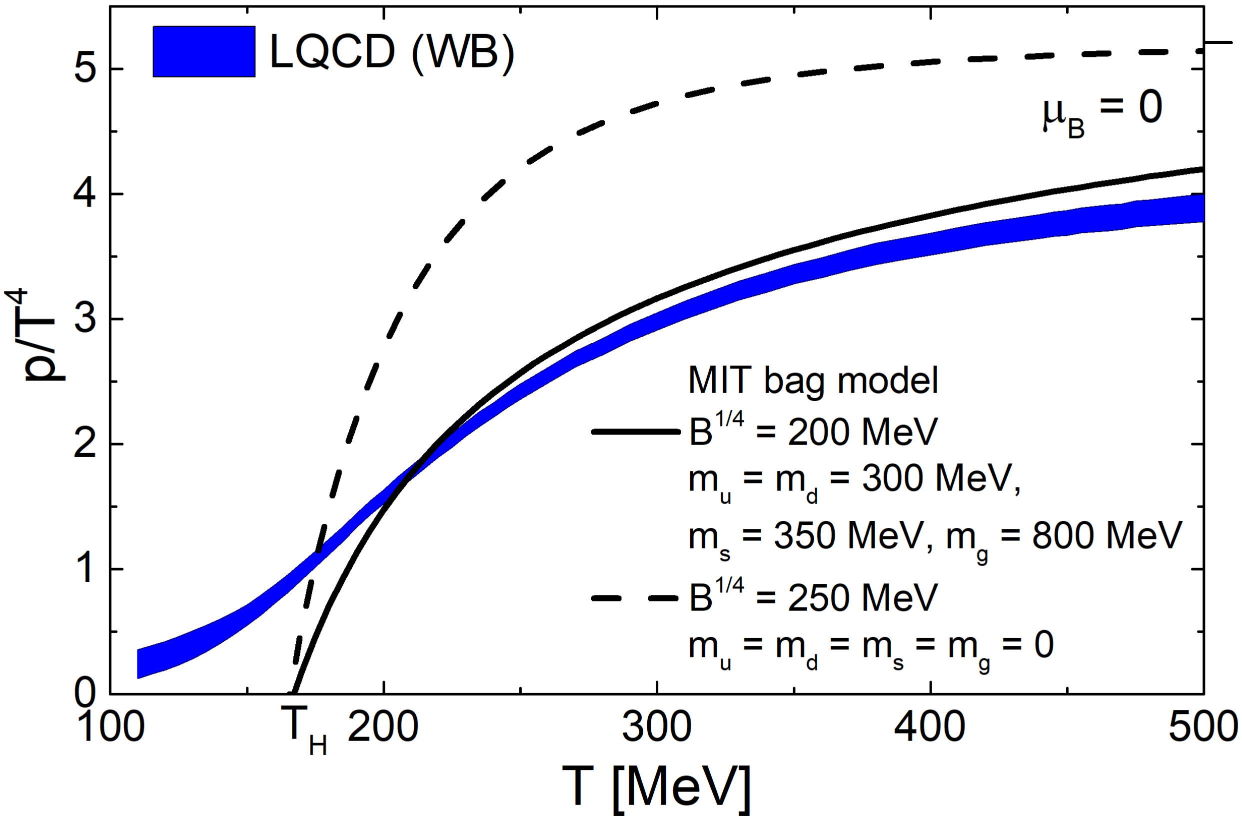
Heavy-bag model: bag model EoS with non-interacting *massive* quarks and gluons and the bag constant [Ivanov et al., PRC 72, 025804 (2005)]

Massive quarks and gluons instead of massless ones:

$$\sigma_Q(T,\lambda_B,\lambda_Q,\lambda_S) = \frac{8}{\pi^2\,T^4}\int_0^\infty dk\,\frac{k^4}{\sqrt{k^2+m_g^2}}\left[\exp\left(\frac{\sqrt{k^2+m_g^2}}{T}\right)-1\right]^{-1}$$
$$+\sum_{f=u,d,s}\frac{3}{\pi^2\,T^4}\int_0^\infty dk\,\frac{k^4}{\sqrt{k^2+m_f^2}}\left[\lambda_f^{-1}\exp\left(\frac{\sqrt{k^2+m_f^2}}{T}\right)+1\right]^{-1}$$
$$+\sum_{f=u,d,s}\frac{3}{\pi^2\,T^4}\int_0^\infty dk\,\frac{k^4}{\sqrt{k^2+m_f^2}}\left[\lambda_f\exp\left(\frac{\sqrt{k^2+m_f^2}}{T}\right)+1\right]^{-1}$$

Bag model with massive quarks

Introduction of constituent masses leads to much better description of QGP



Parameters for the crossover model:

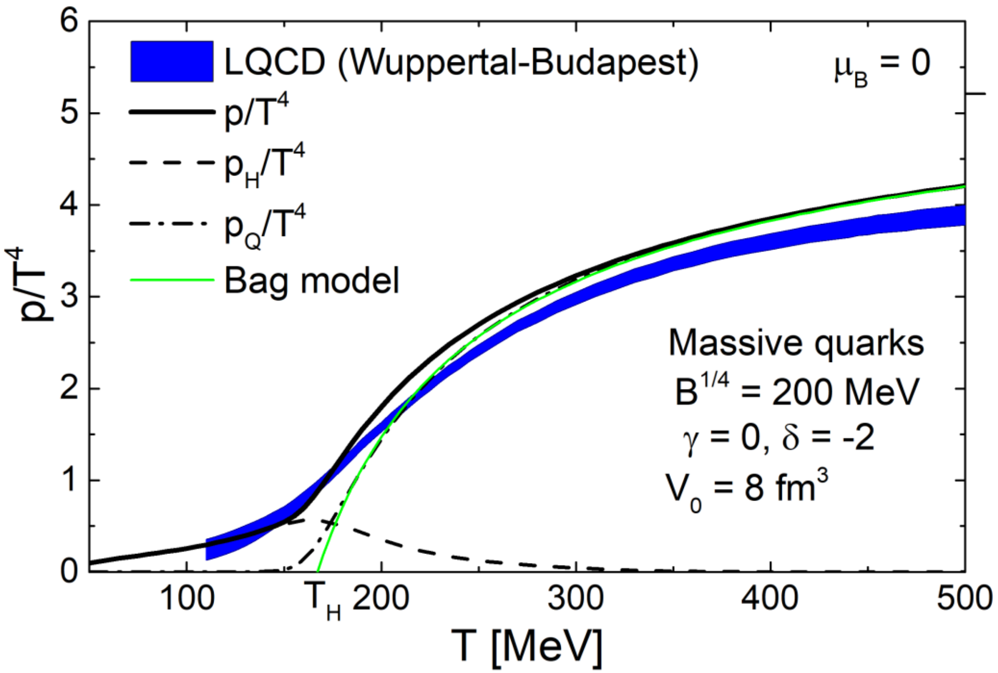
$$m_u = m_d = 300 \text{ MeV}, \quad m_s = 350 \text{ MeV}, \quad m_g = 800 \text{ MeV}, \quad B^{1/4} = 200 \text{ MeV}$$

$$\gamma = 0, \quad \delta = -2, \quad C = 0.03, \quad V_0 = 8 \text{ fm}^3$$

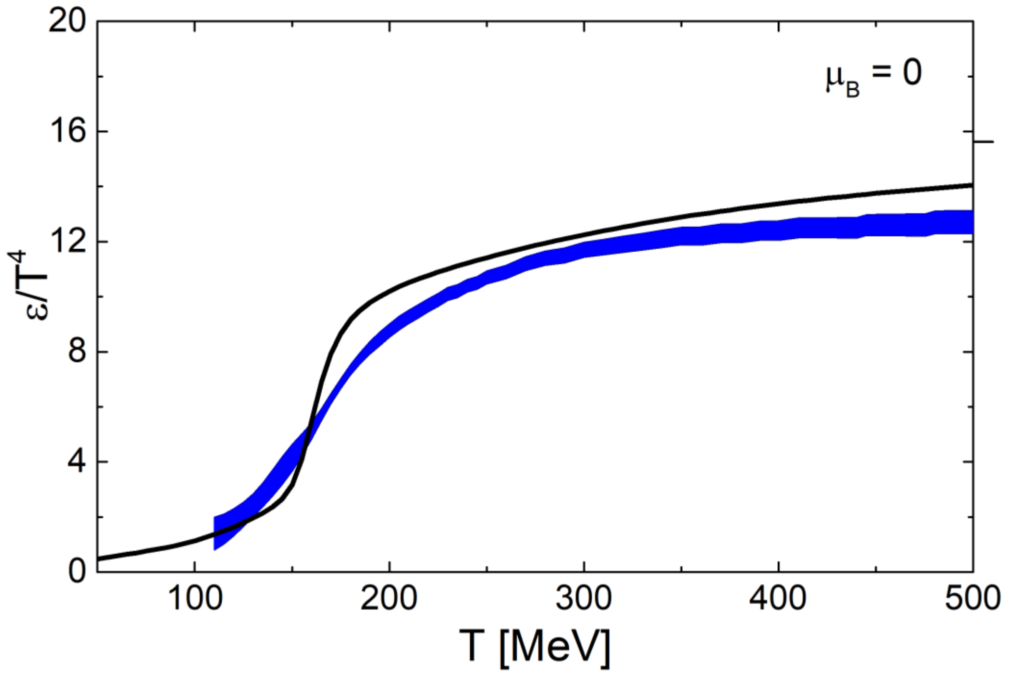
$T_H \simeq 167$ MeV

Hagedorn model: Thermodynamic functions

Pressure p/T^4



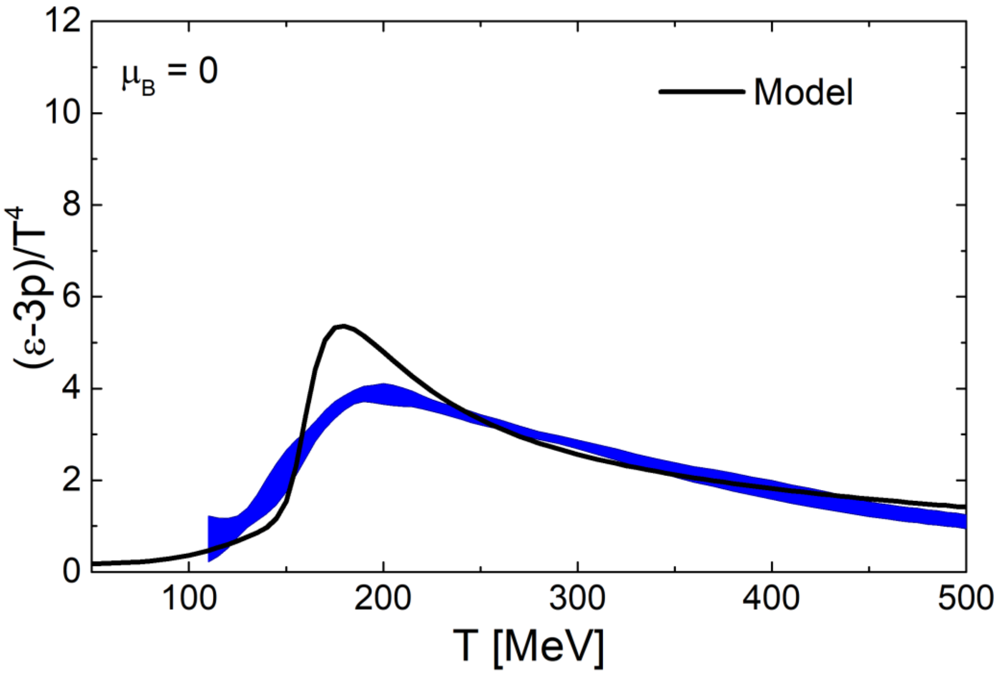
Energy density ε/T^4



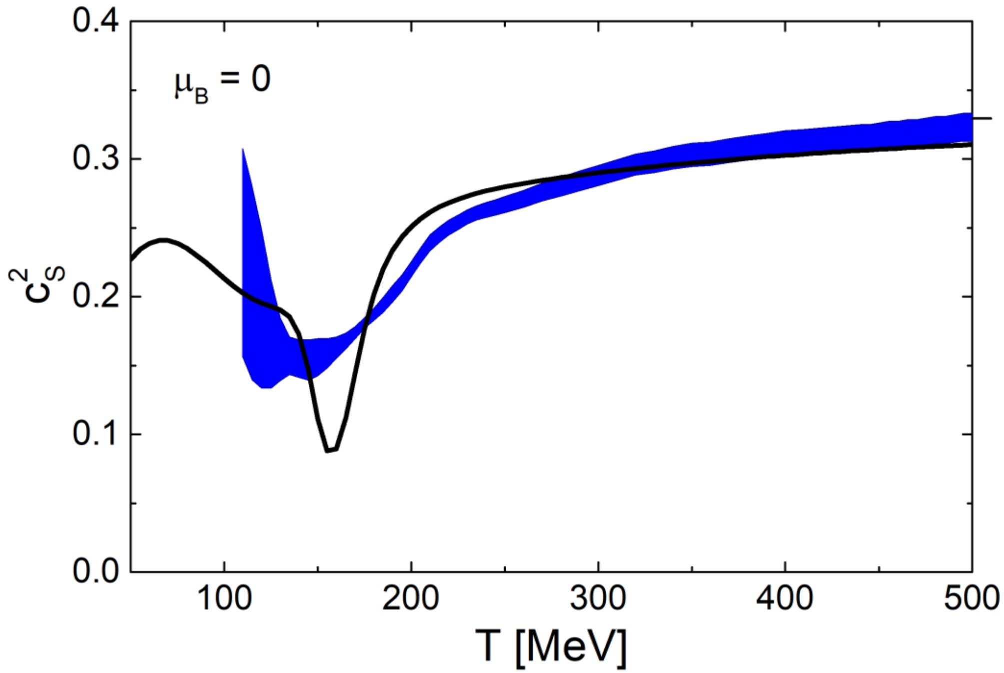
- Semi-quantitative description of lattice data
- Peak in energy density gone!

Hagedorn model: Thermodynamic functions

Trace anomaly $(\varepsilon - 3p)/T^4$

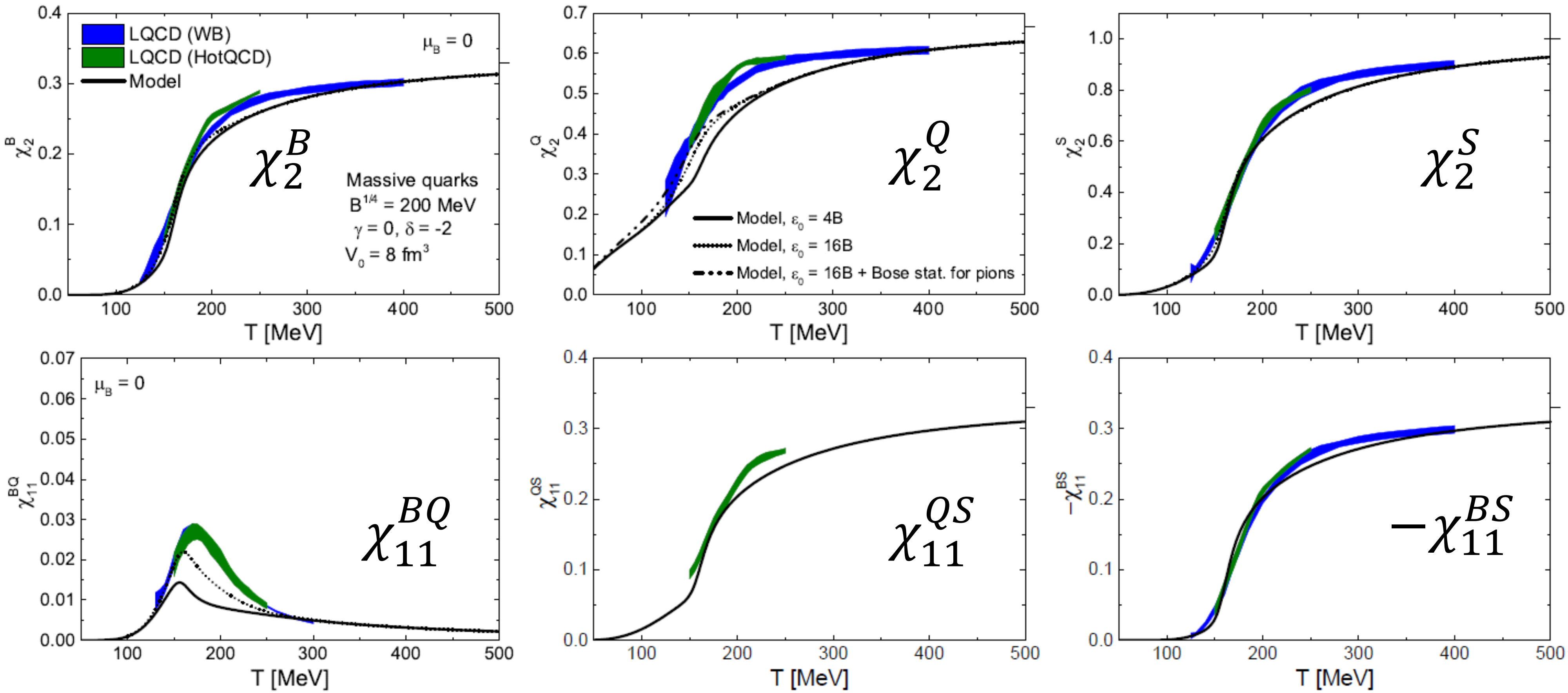


Speed of sound $c_s^2 = dp/d\varepsilon$



Hagedorn model: Susceptibilities

$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \, \partial(\mu_S/T)^m \, \partial(\mu_Q/T)^n}$$



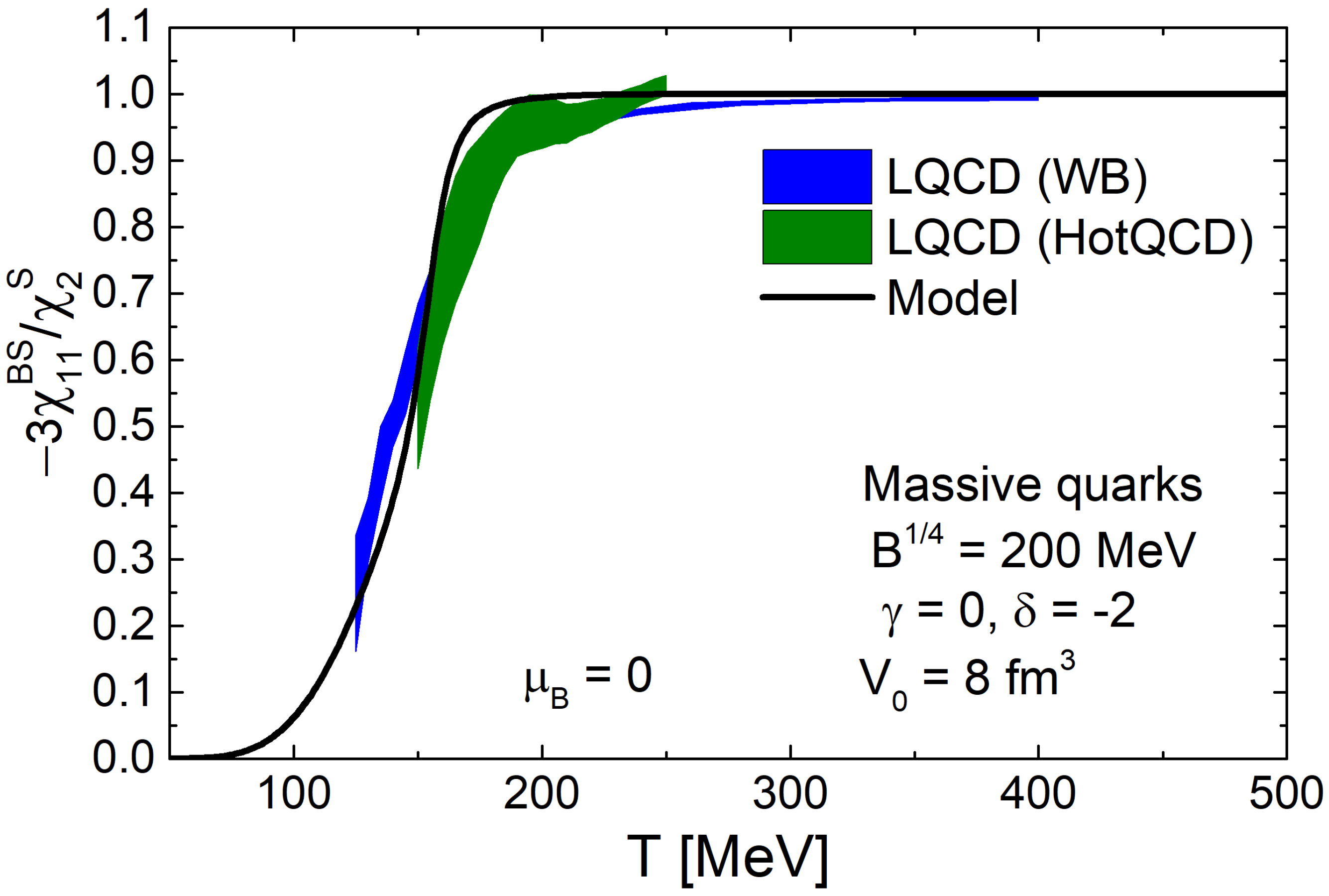
Lattice data from 1112.4416 (Wuppertal-Budapest), 1203.0784 (HotQCD)

Hagedorn model: Baryon-strangeness ratio

$$C_{BS} = -\frac{3\chi_{11}^{BS}}{\chi_2^S}$$

Useful diagnostic of QCD matter

[V. Koch, Majumder, Randrup, PRL 95, 182301 (2005)]

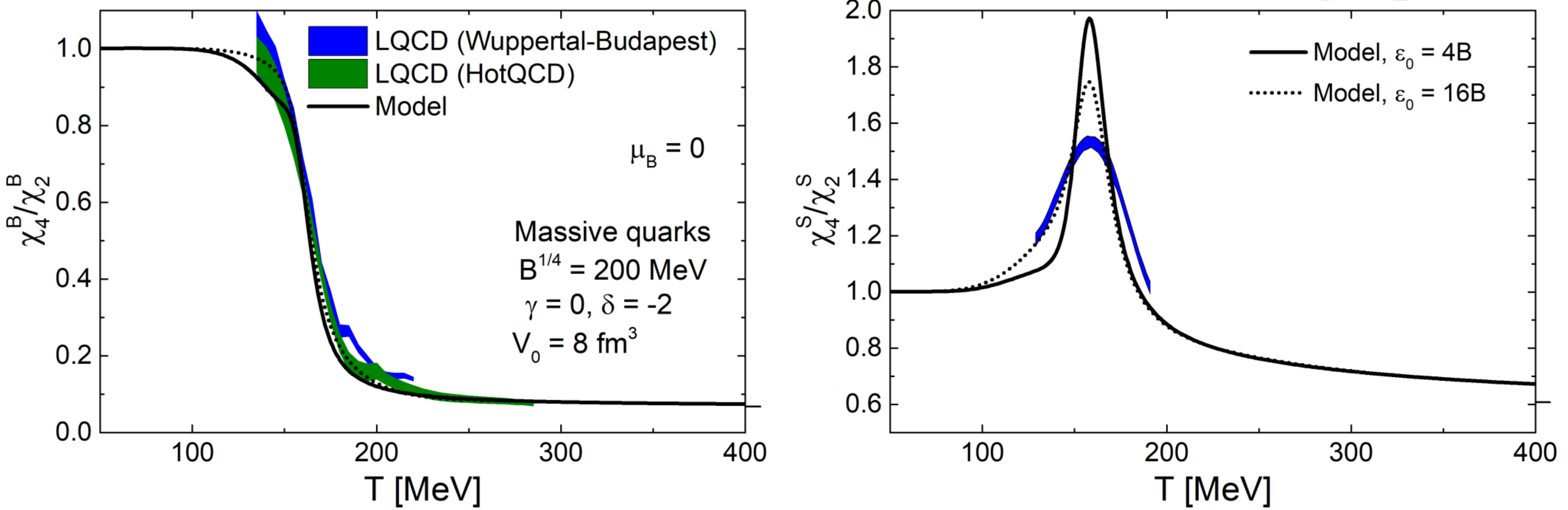


Well consistent with lattice QCD

Hagedorn model: Higher-order susceptibilities

Higher-order susceptibilities are particularly sensitive probes of the parton-hadron transition and possible remnants of criticality at $\mu_B = 0$

net baryon χ_4^B/χ_2^B net strangeness χ_4^S/χ_2^S

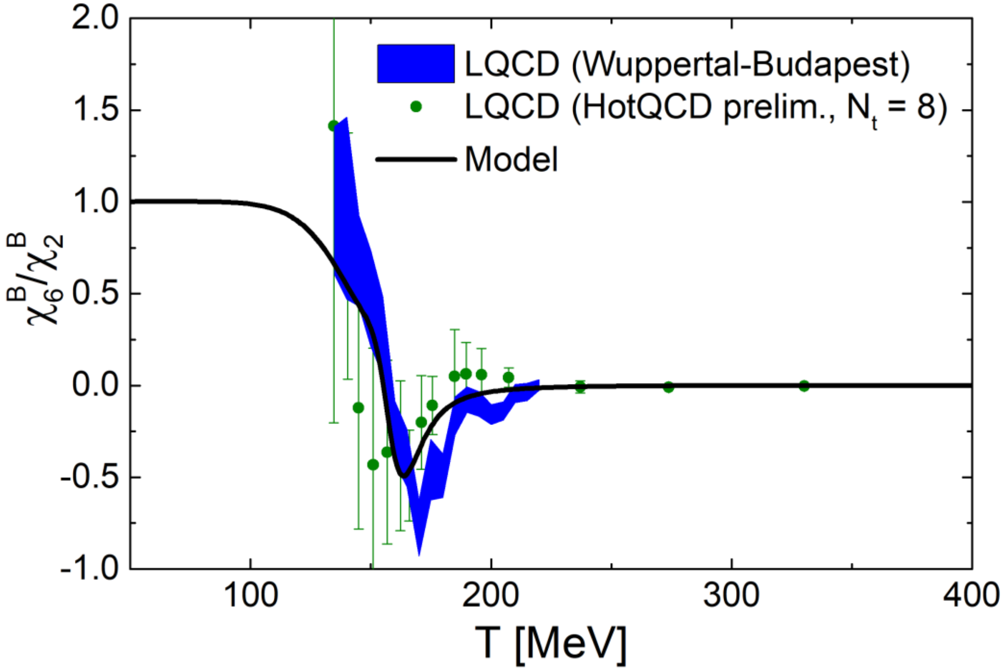


Lattice data from 1305.6297 & 1805.04445 (Wuppertal-Budapest), 1708.04897 (HotQCD)

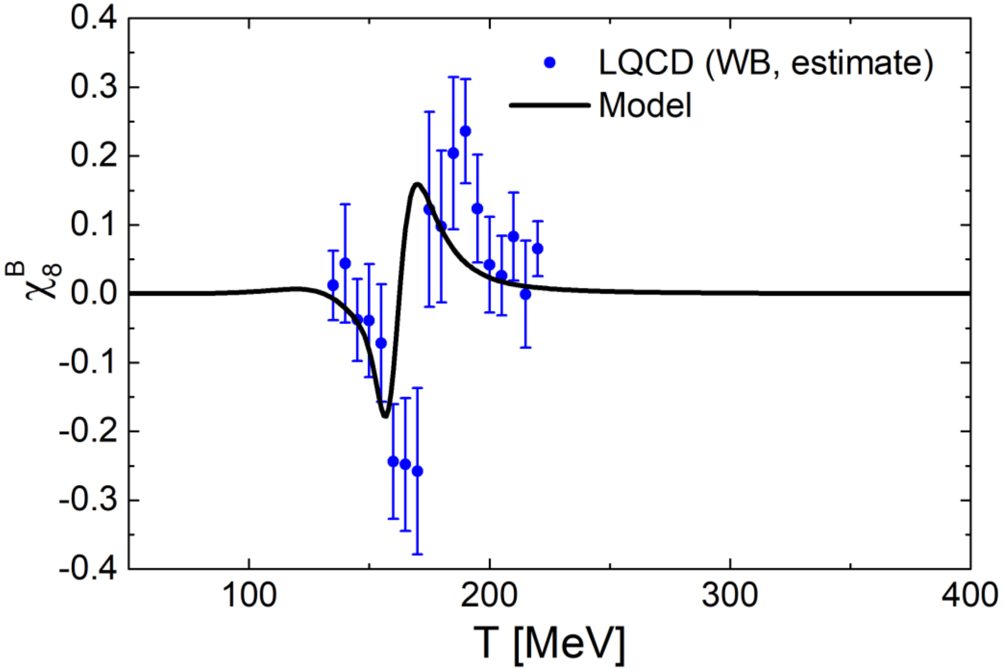
- Drop of χ_4^B/χ_2^B caused by repulsive interactions which ensure crossover transition to QGP
- Peak in χ_4^S/χ_2^S is an interplay of the presence of multi-strange hyperons and repulsive interactions

Hagedorn model: Higher-order susceptibilities

net baryon χ_6^B/χ_2^B



net baryon χ_8^B



Lattice data from 1805.04445 (Wuppertal-Budapest), 1708.04897 (HotQCD)

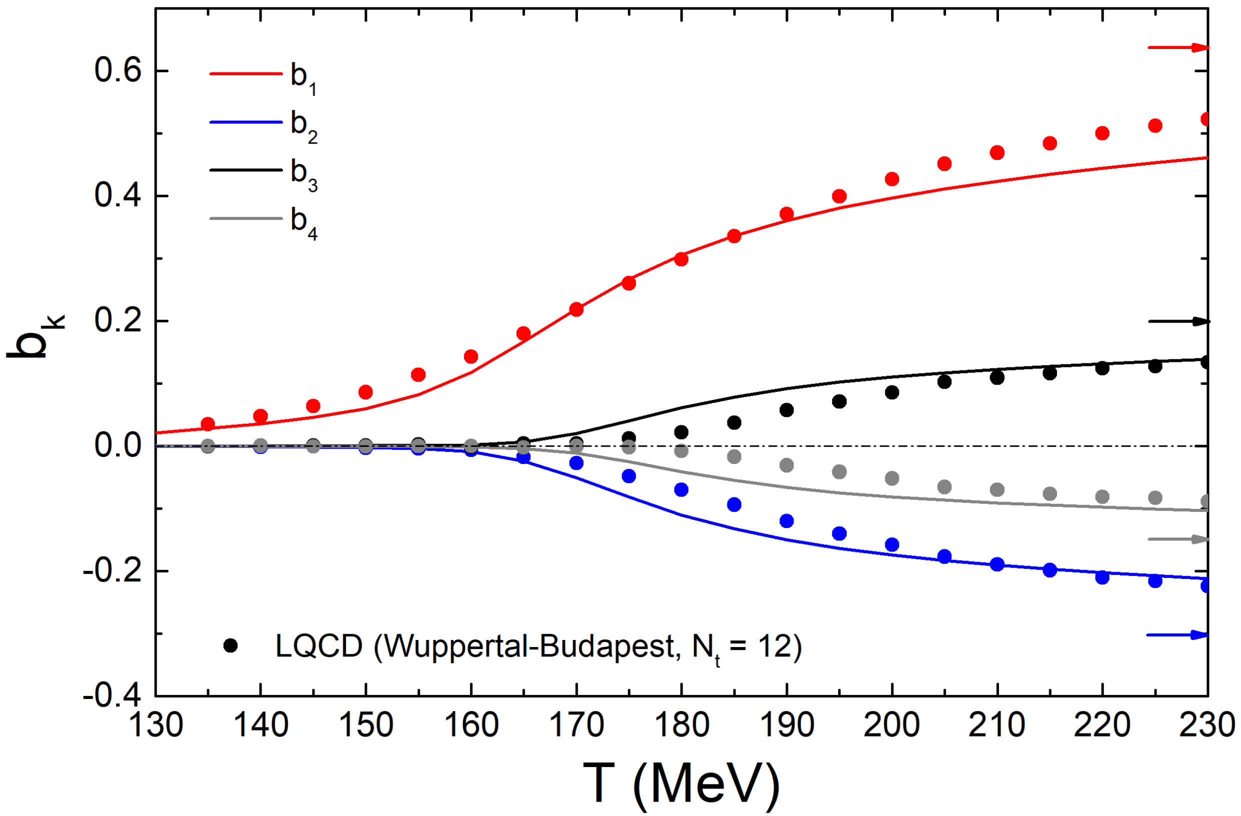
- Strong non-monotonic dependence of higher-order baryon number susceptibilities χ_6^B/χ_2^B and χ_8^B well reproduced by the crossover model
- No critical point signal in lattice data?

Fourier coefficients at imaginary μ_B

Additional model test provided by imaginary μ_B lattice data, where Fourier coefficients of the net baryon density were computed

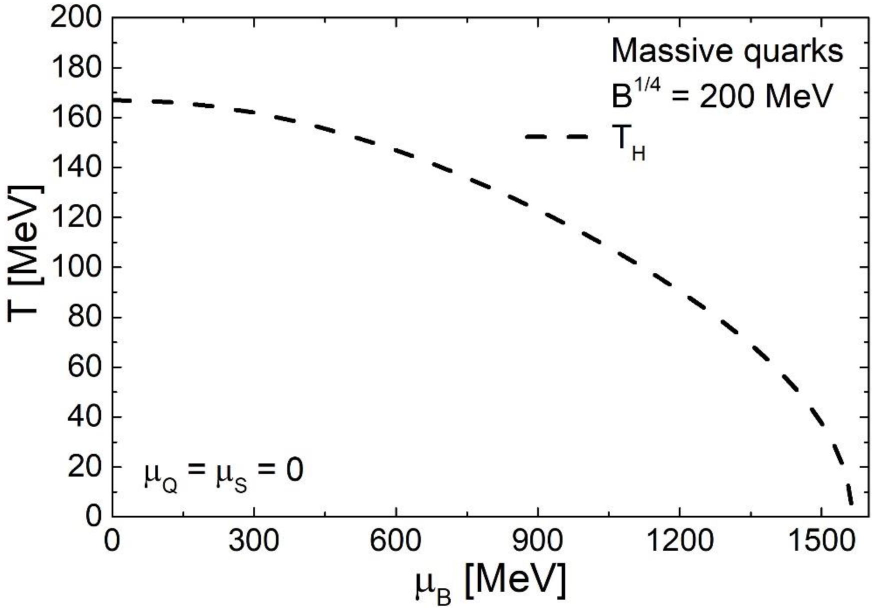
[Vovchenko, Pasztor, Fodor, Katz, Stoecker, 1708.02852]

$$\left.\frac{\rho_B(T,\mu_B)}{T^3}\right|_{\mu_B=i\theta_B T} = \sum_{k=1}^{\infty} b_k(T)\sin(k\theta_B)$$



Summary, Conclusions, Outlook

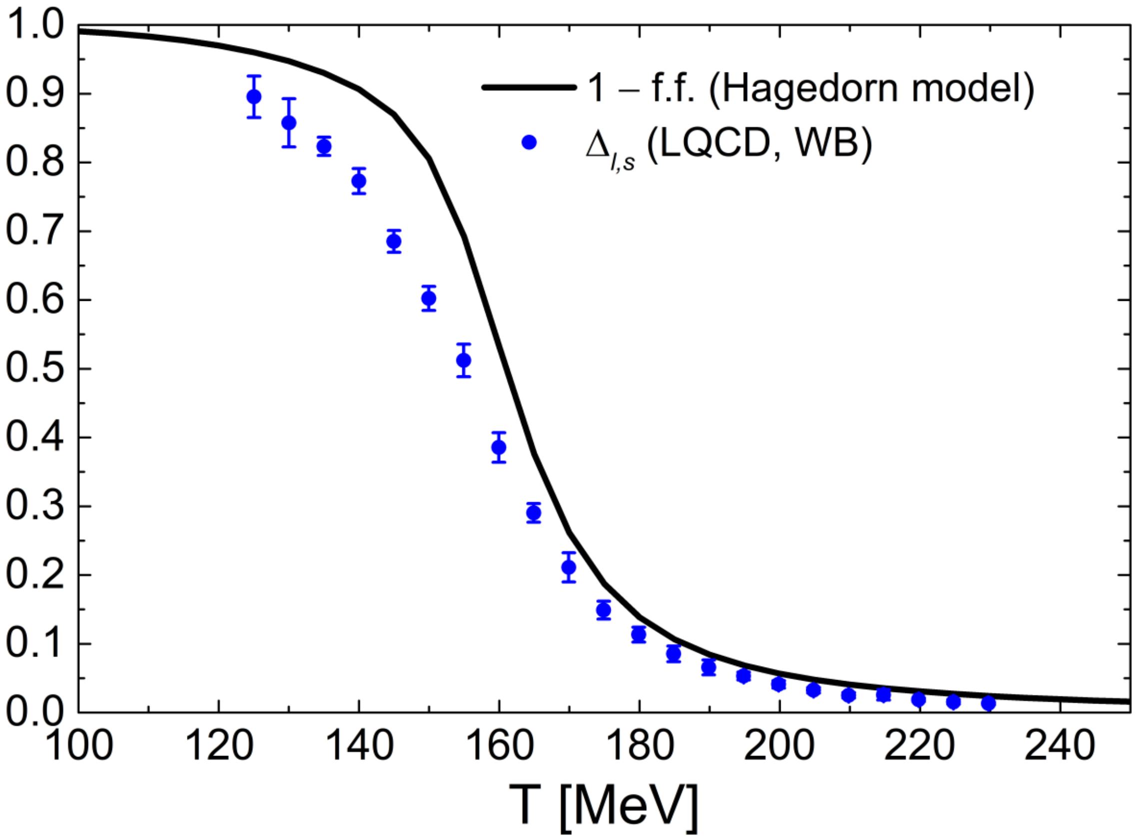
- HRG combined Hagedorn baglet model:
  Single partition function for low to high energy densities, be it a real phase transition or crossover
- Inclusion of exponentially increasing Hagedorn states as well as excluded volume corrections are in line with various high order susceptibilities of lattice QCD
- No sign of critical phenomena
- … adjusting parameters for hypothetical critical point at finite baryochemical potential to make predictions for cumulants



Chiral condensate

Picture: bag interior is chirally restored, vacuum is chirally broken

Proxy observable: $$\frac{<\psi\bar{\psi}>_{T\neq 0}}{<\psi\bar{\psi}>_{T=0}} \cong 1 - \frac{<V_{had}>}{V} = 1\ - f.f.$$



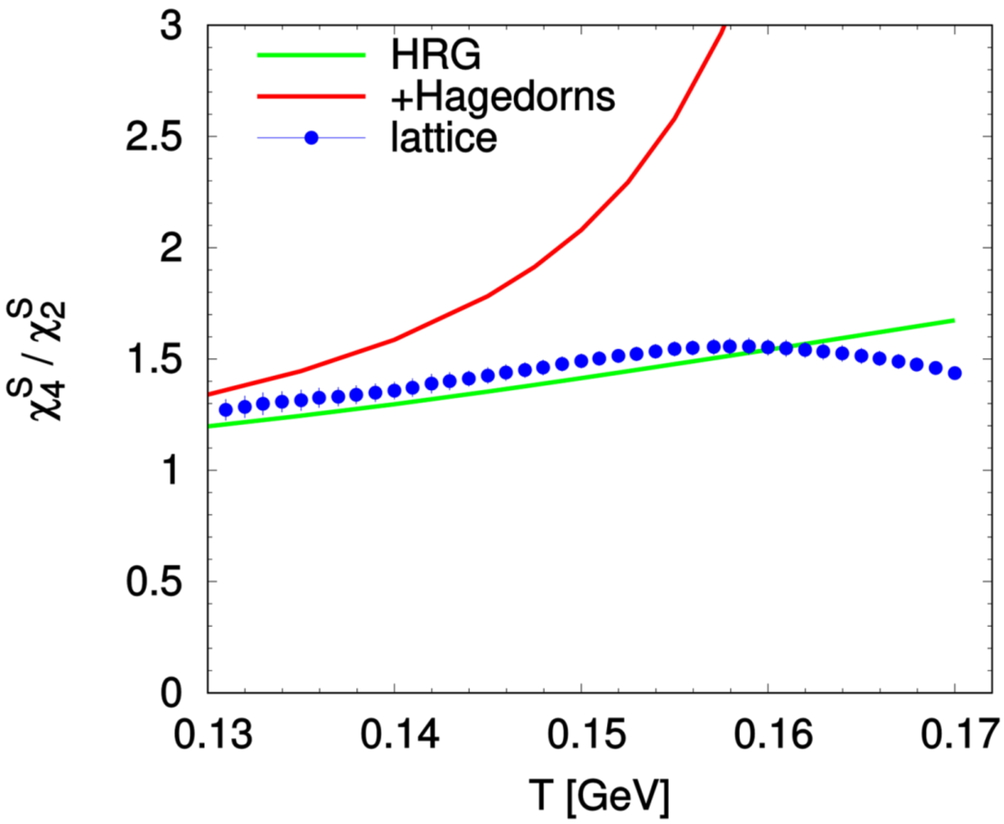
LQCD:

$$\langle\bar{\psi}\psi\rangle_q = \frac{T}{V}\frac{\partial \ln Z}{\partial m_q}$$

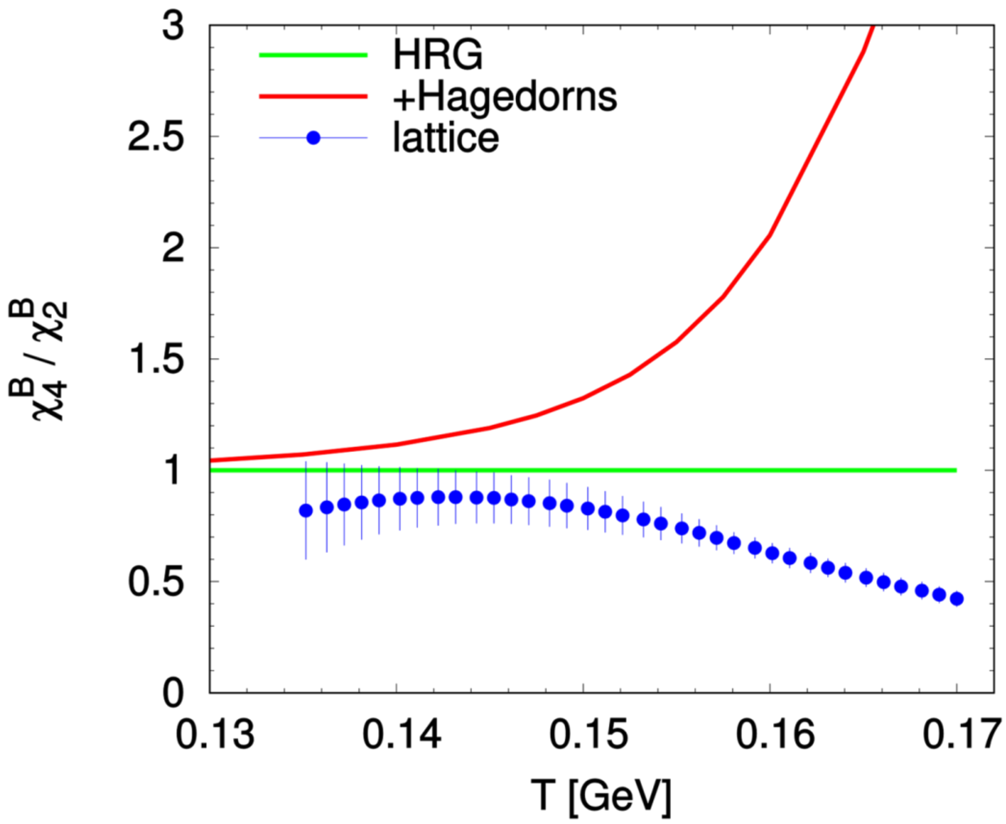
$$\Delta_{l,s} = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$

Lattice data from 1005.3508 (Wuppertal-Budapest), see also 1111.1710 (HotQCD)

Susceptibility ratios



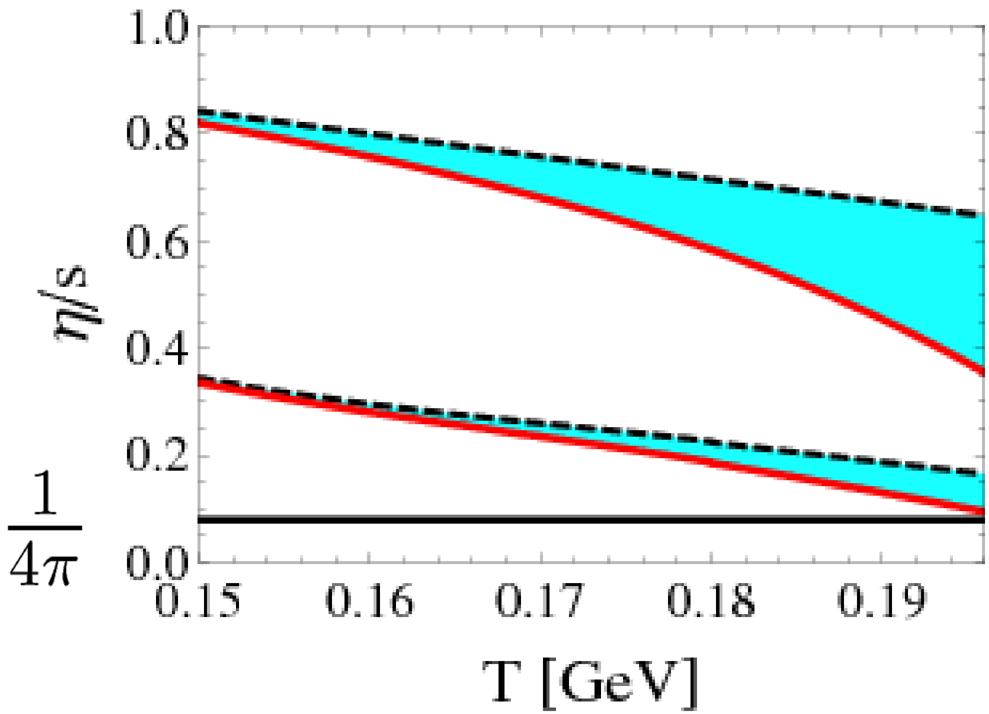
lattice: Bellwied et al., PRL 111(2013) 202302



lattice: Borsanyi et al., PRL 111(2013) 062005

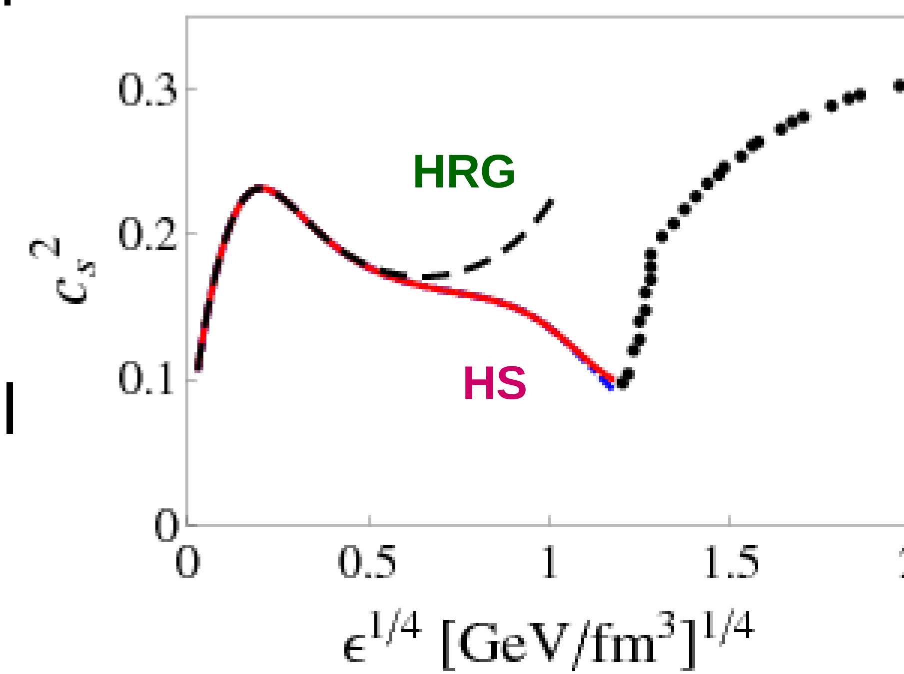
Transport Coefficients of Hadronic Matter near T_c

J. Noronha-Hostler, J. Noronha, CG,
PRL103:172302 (2009)

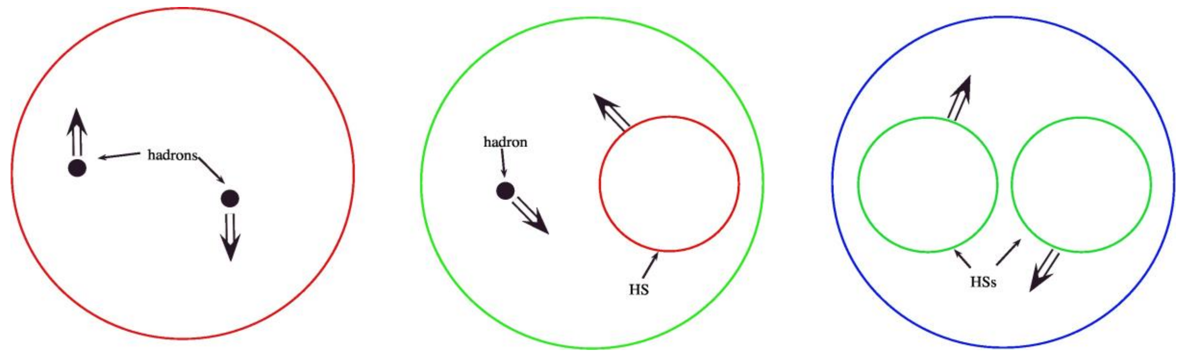


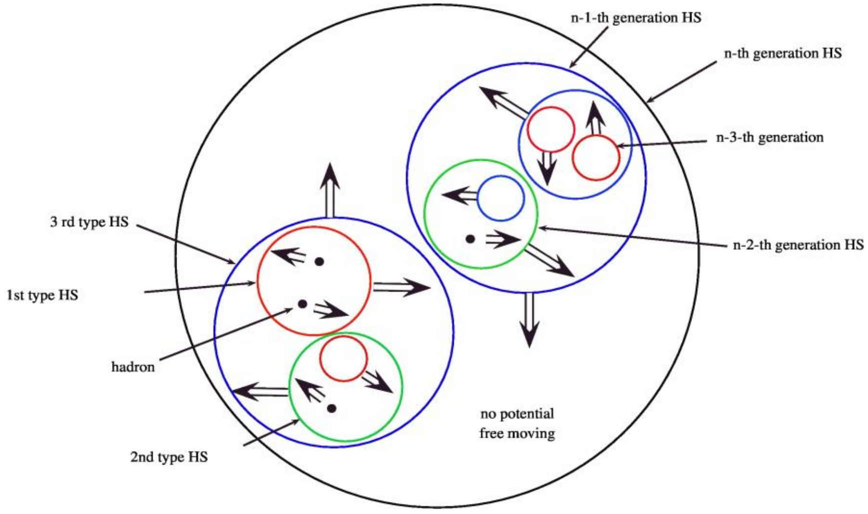
While both η (due to the small MFP of HS) and s increase with increasing T, the entropy increases quicker close to Tc, which decreases η/s.

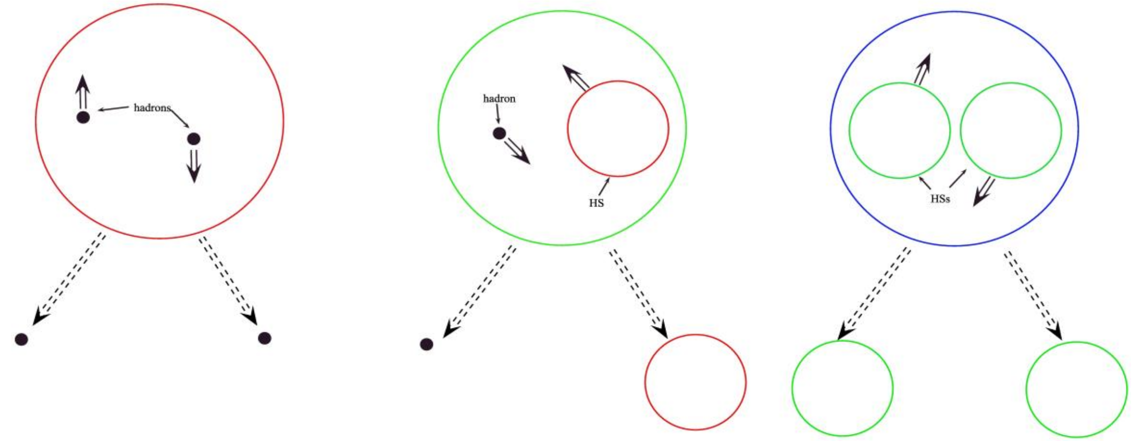
c_s^2 of a hadron gas including HS matches well with the lattice at T_c



Basics: Build up and decay of Hagedorn states







M.Beitel, K: Gallmeister, CG, PRC 90 (2014) 045203

Hagedorn Bootstrap

cf.: S. Frautschi, PRD 3 (1971) 2821
C. Hamer, S. Frautschi, PRD 4 (1971) 2125
J. Yellin, NPB 52 (1973) 583

- Assumption: only 2-body (detailed balance!)
- Input: known hadrons (UrQMD/GiBUU/PDG)
- Bootstrap equation

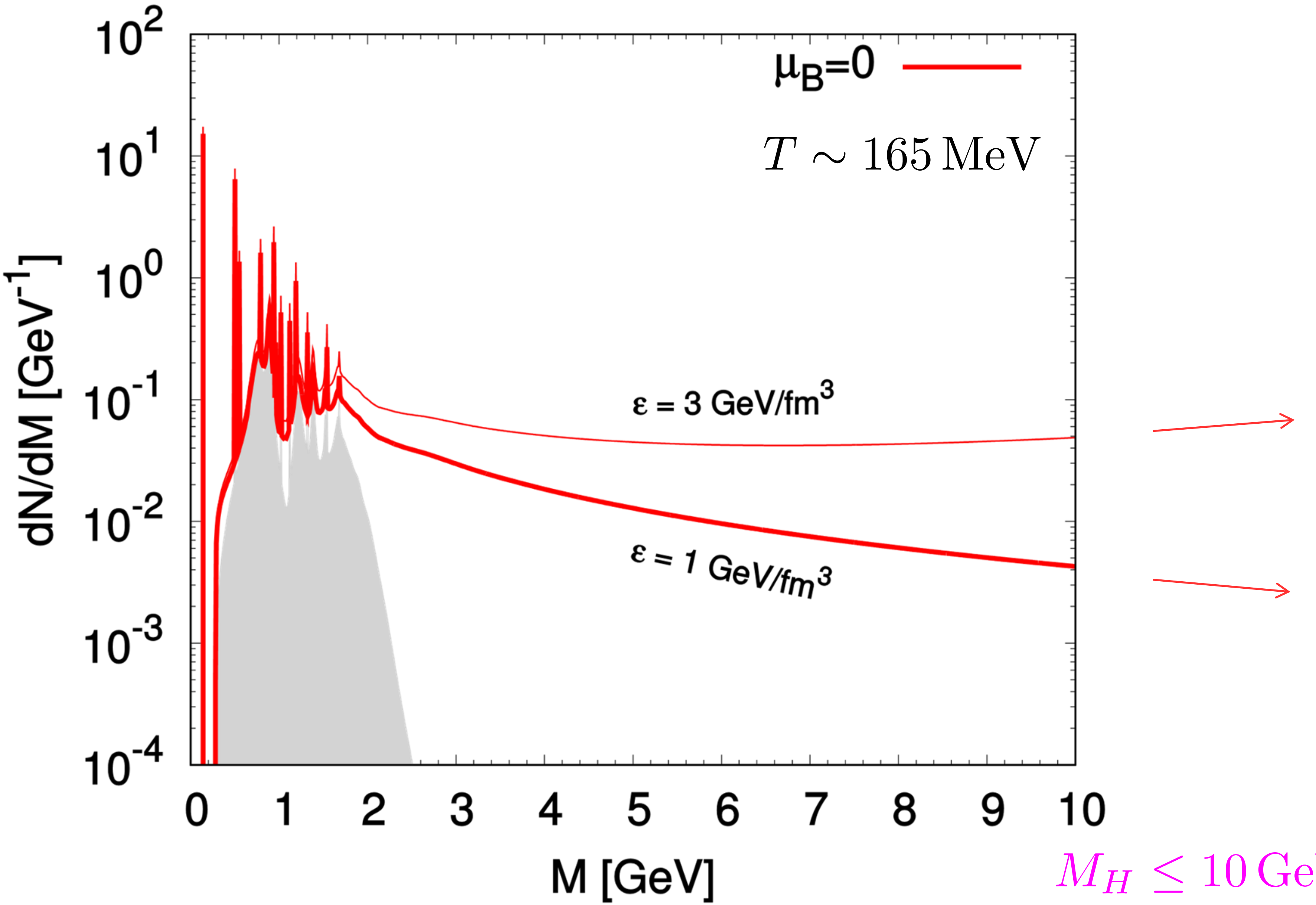
$$\tau_{\vec{C}}(m) = \tau^0_{\vec{C}}(m) + \frac{V(m)}{(2\pi)^2\,2m} \sum_{\vec{C}_1,\vec{C}_2}^{*} \iint \mathrm{d}m_1 \mathrm{d}m_2 \times \; \tau_{\vec{C}_1}(m_1)\,\tau_{\vec{C}_2}(m_2)\, m_1\, m_2\; p_{\mathrm{cm}}(m, m_1, m_2)$$

$$\vec{C} = (B, S, Q)$$

non-linear integral equation, Volterra type

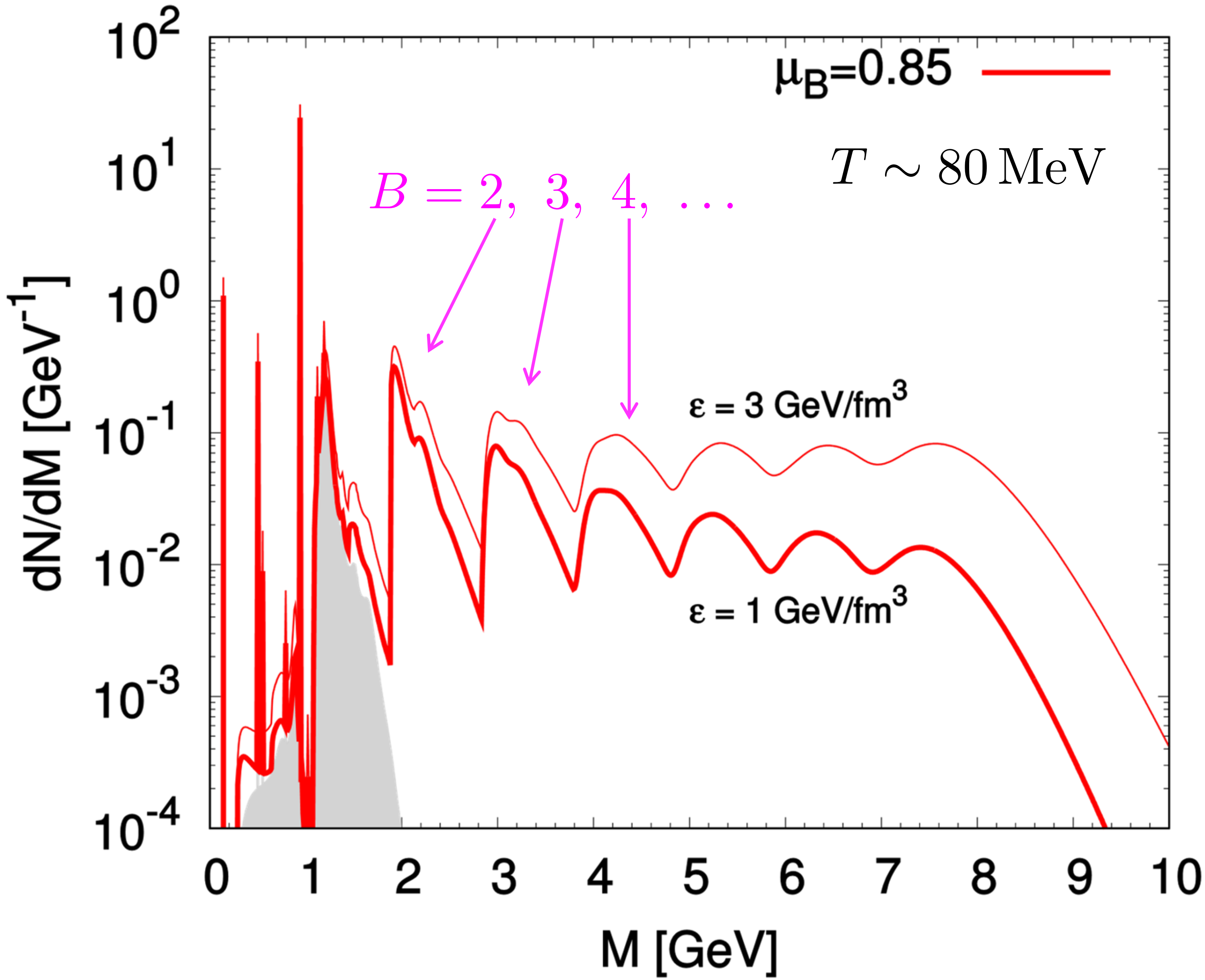
Divergence

exponential Hagedorn increase vs. thermal Boltzmann decrease



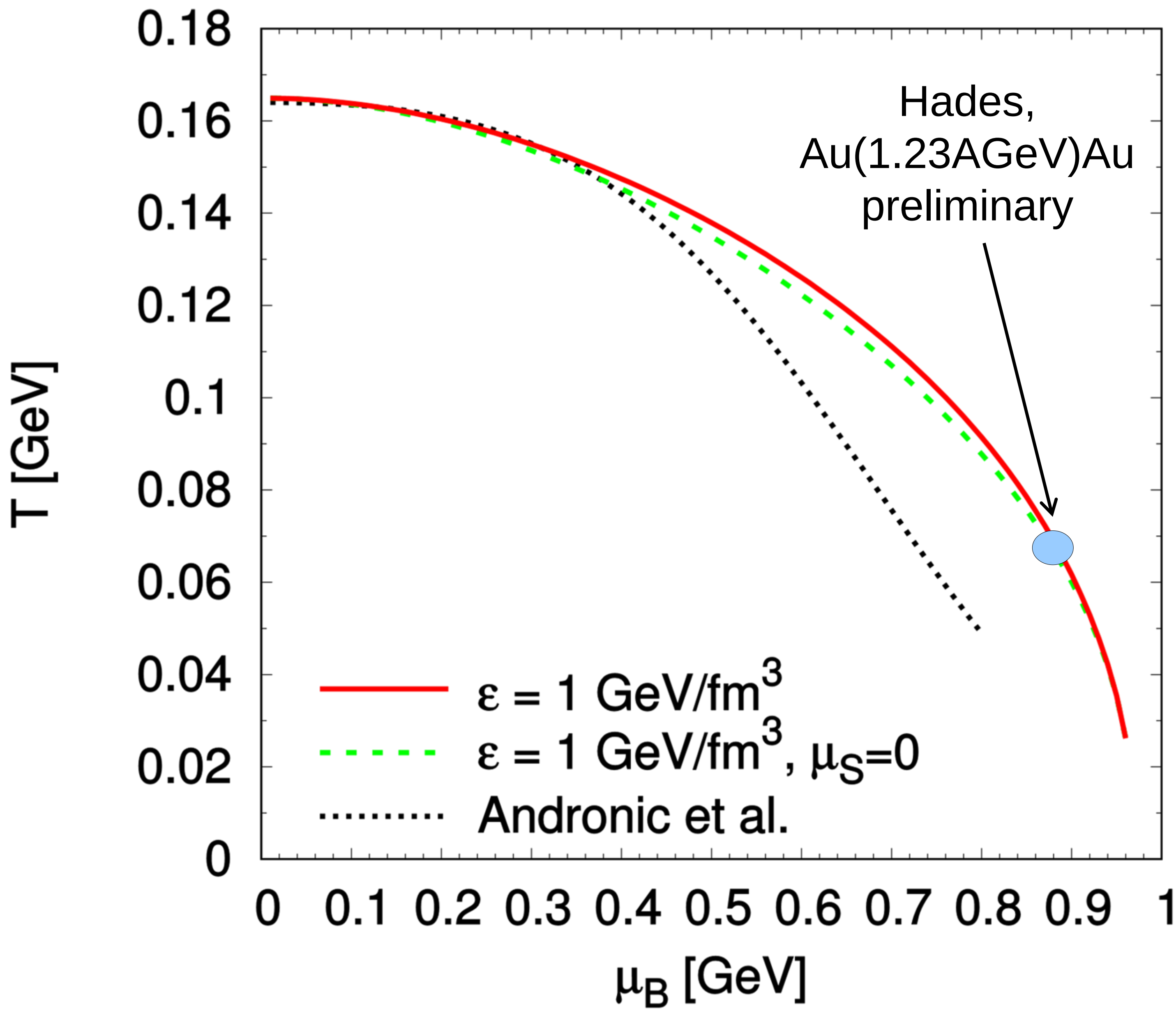
$M_H \leq 10\,\mathrm{GeV}$

Divergence



μ_B=0.85
$T \sim 80\,\mathrm{MeV}$
$B = 2,\ 3,\ 4,\ \ldots$
ε = 3 GeV/fm^3
ε = 1 GeV/fm^3
dN/dM [GeV^{-1}]
M [GeV]
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0 1 2 3 4 5 6 7 8 9 10

Divergence Boundary

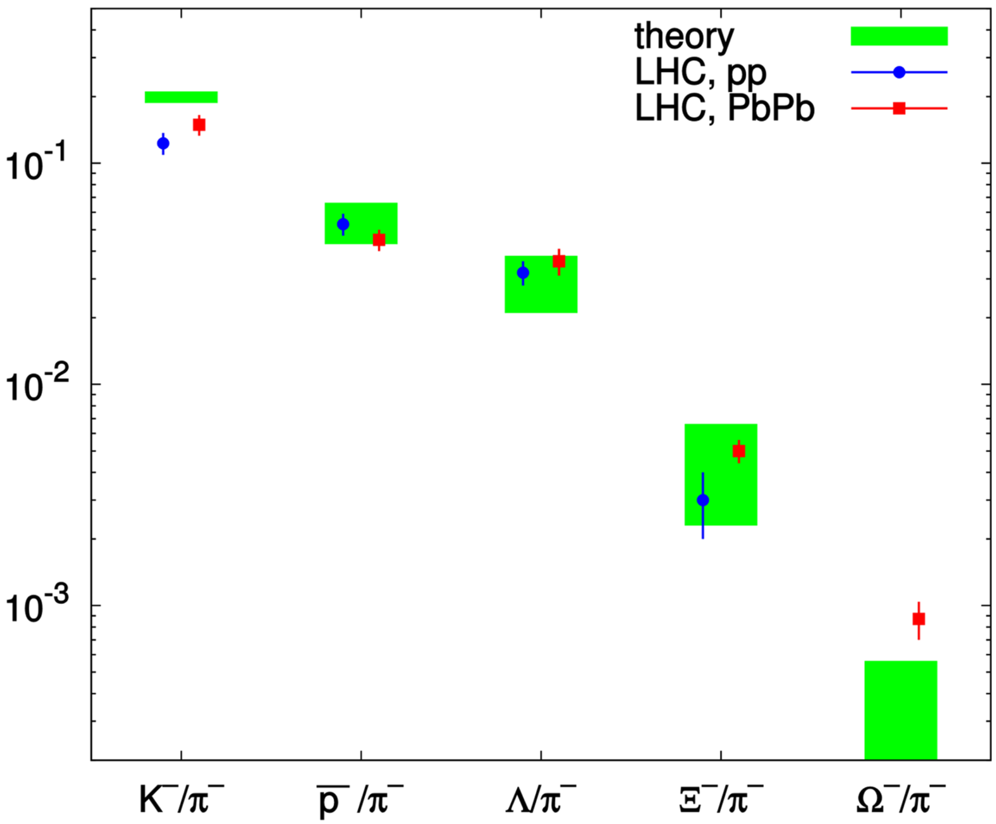


$T = 68 \pm 2\,\mathrm{MeV}$

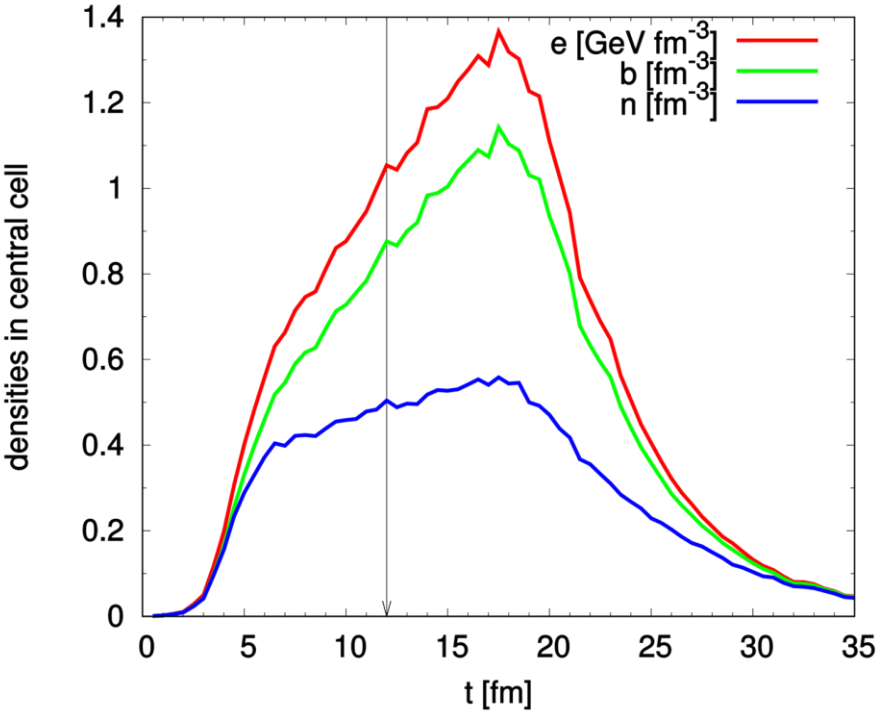
$\mu_B = 883 \pm 25\,\mathrm{MeV}$

Application of Hagedorn States

- Decay cascade



- Full dynamical calculation
  - Au(1.23 AGeV)Au
  - large densities



  - HADES, ratio phi/K-

- Box calculations
  - fast equilibration (~5fm)

Hagedorn decays yield thermal(-like) spectra!