Chiral limit of (2+1)-flavor QCD

Frithjof Karsch
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- Critical behavior in the limit of vanishing light quark masses
- Finite size scaling and chiral limit
- The chiral PHASE TRANSITION temperature

BEST COLLABORATION
CRC-TR 211 Strong-interaction matter under extreme conditions
DFG Deutsche Forschungsgemeinschaft

The chiral PHASE TRANSITION temperature

R. D. Pisarski, F. Wilczek,
Remarks on the chiral phase transition in chromodynamics,
Phys. Rev. D 29 (1984) 338(R)

Abstract:
The phase transition restoring chiral symmetry at finite temperatures is considered in a linear σ model. For three or more massless flavors, the perturbative ϵ expansion predicts the phase transition is of first order. At high temperatures, the UA(1) symmetry will also be effectively restored.

– since 35 years it is understood that critical behavior in strong-interaction matter is due to **chiral symmetry restoration**

– the **phase transition temperature** in the chiral limit of QCD is one of the fundamental scales in strong-interaction physics

– **neither the order of the transition in 2 or (2+1)-flavor QCD nor the value of the transition temperature have been established so far**

Critical behavior in QCD

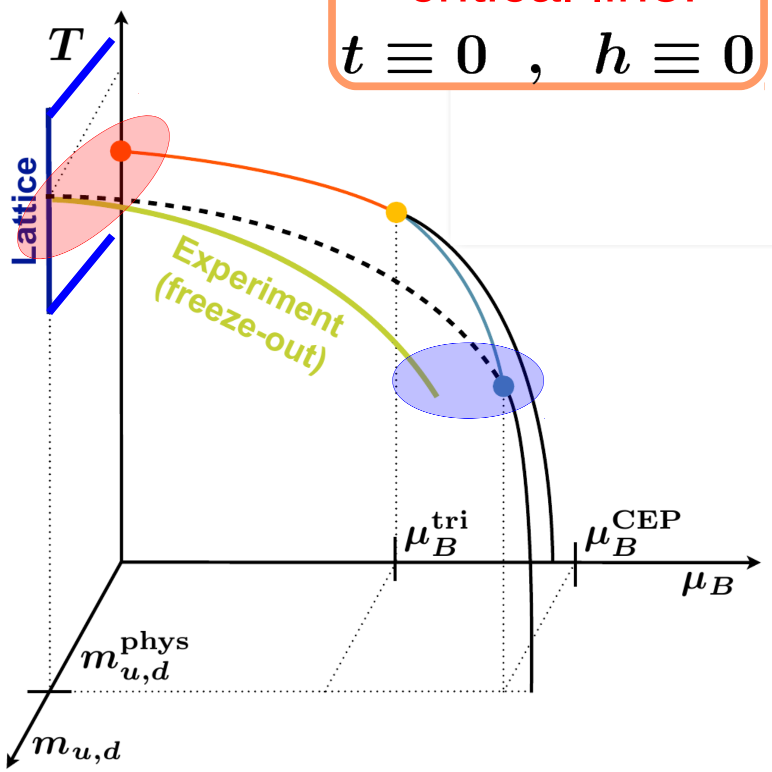
- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular | regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

critical line: $t \equiv 0 \ , \ h \equiv 0$

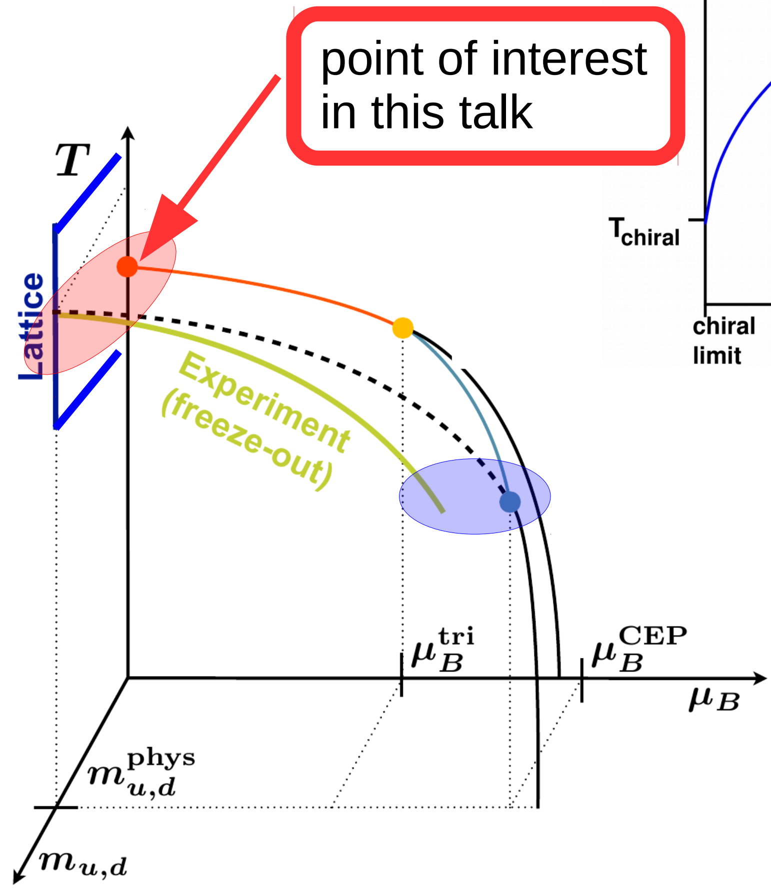
$$t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_q}{T}\right)^2 \ , \quad h \sim \frac{m_q}{T_c}$$

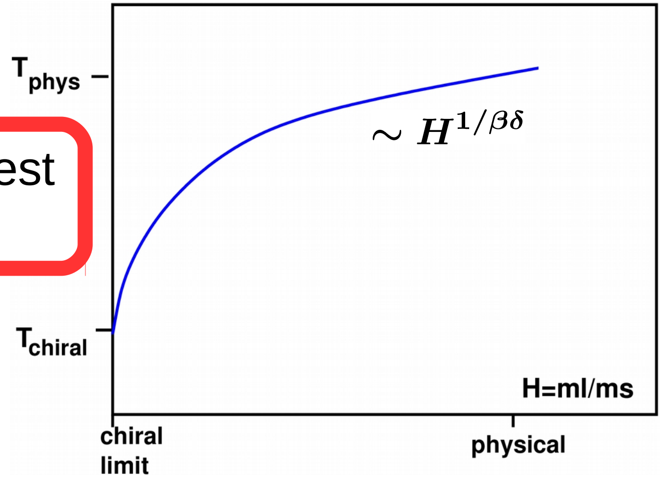


here only: $\mu_q \equiv 0$

question: Where is the chiral PHASE TRANSITION for $m_u = m_d = 0$ located?
What is its influence on observables at the pseudo-critical temperature?

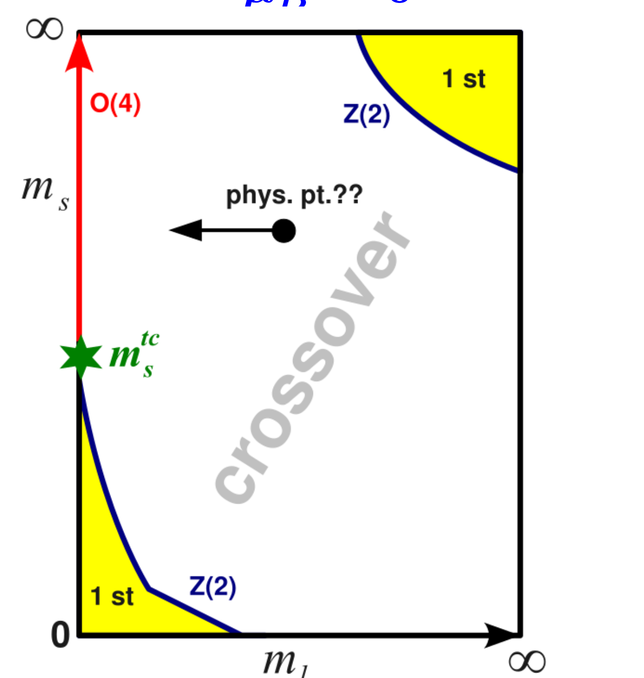
Phases of strong-interaction matter





this talk focuses on

$\mu_B = 0$



What is the influence of the chiral phase transition on observables at the pseudo-critical temperature?

-do step 1 first – determine the critical temperature

(and the order of the transition)

more on the order of the chiral phase transition at physical value of the strange quark mass → see next talk by Jishnu Goswami

Scaling in the thermodynamic (infinite volume) limit
– approaching the chiral limit –

– order parameter M and its susceptibility

$$M = h^{1/\delta} f_G(z) + f_{sub}(T,H)$$
$$\chi_M = h_0^{-1} h^{1/\delta -1} f_\chi(z) + \tilde{f}_{sub}(T,H)$$

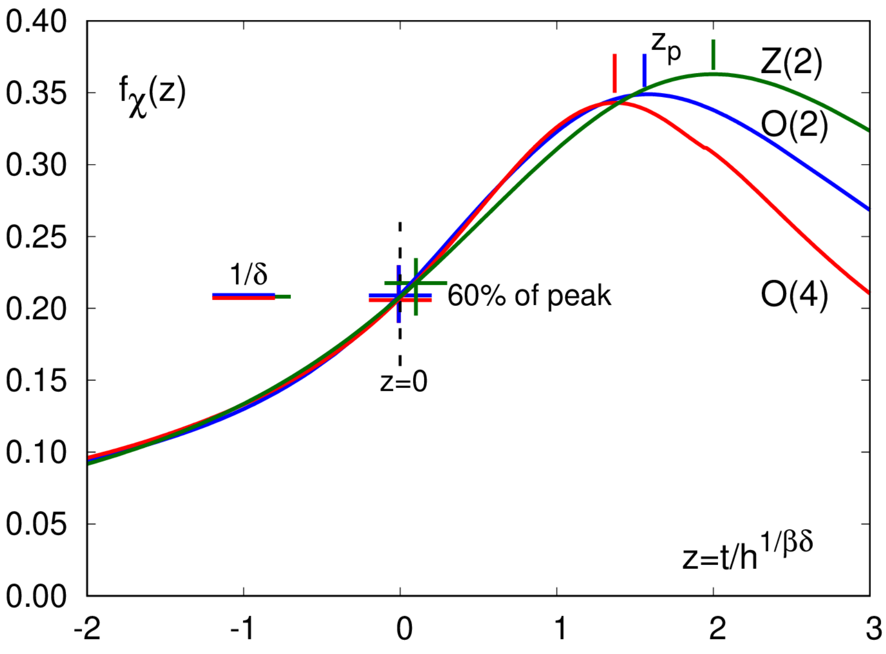
for ANY fixed z:

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z}{z_0} H^{1/\beta\delta}\right) + \text{sub leading}$$

– corrections-to-scaling
– regular terms

some definitions

$$z = \frac{t}{h^{1/\beta\delta}}$$
$$t \equiv \frac{1}{t_0}\frac{T-T_c}{T_c}$$
$$h = \frac{1}{h_0} H$$
$$H \equiv \frac{m_l}{m_s}$$
$$z_0 = h_0^{1/\beta\delta}/t_0$$



scaling functions $f_\chi(z)$ for some 3-d universality classes:

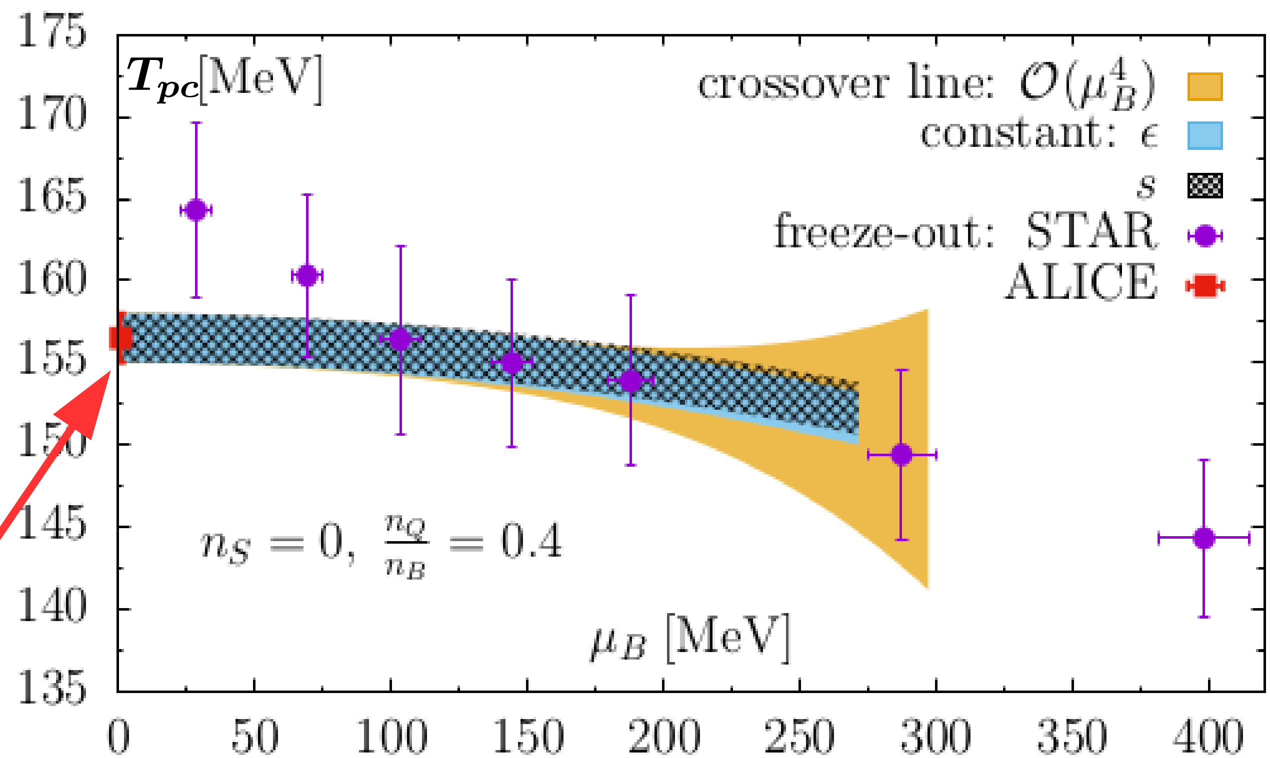
	δ	$1/\beta\delta$	z_p	z_{60}	z_δ
Z(2)	4.805	0.640	2.00(5)	0.10(1)	0
O(2)	4.780	0.599	1.58(4)	-0.005(9)	0
O(4)	4.824	0.545	1.37(3)	-0.013(7)	0

characteristic points on the scaling function $f_\chi(z)$

Phases of strong-interaction matter

$$T_{pc}(\mu_B) = T_{pc}\left(1 + \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 + \ldots\right)$$

phase diagram at physical values of the quark masses



A. Andronic et al., Nature 561 (2018) 321

$$T_{pc} = (156.5 \pm 1.5)\ \text{MeV}$$

$$\kappa_2 = 0.012(4)$$

A. Bazavov et al. [HotQCD], arXiv:1812.08235

Scaling in the thermodynamic (infinite volume) limit
– approaching the chiral limit –

– order parameter M and it's susceptibility

$$M = h^{1/\delta} f_G(z) + f_{sub}(T,H)$$
$$\chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z) + \tilde{f}_{sub}(T,H)$$

for ANY fixed z:

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z}{z_0} H^{1/\beta\delta}\right) + \text{sub leading}$$

some definitions

$$z = \frac{t}{h^{1/\beta\delta}}$$
$$t \equiv \frac{1}{t_0}\frac{T-T_c}{T_c}$$
$$h = \frac{1}{h_0} H$$
$$H \equiv \frac{m_l}{m_s}$$
$$z_0 = h_0^{1/\beta\delta}/t_0$$

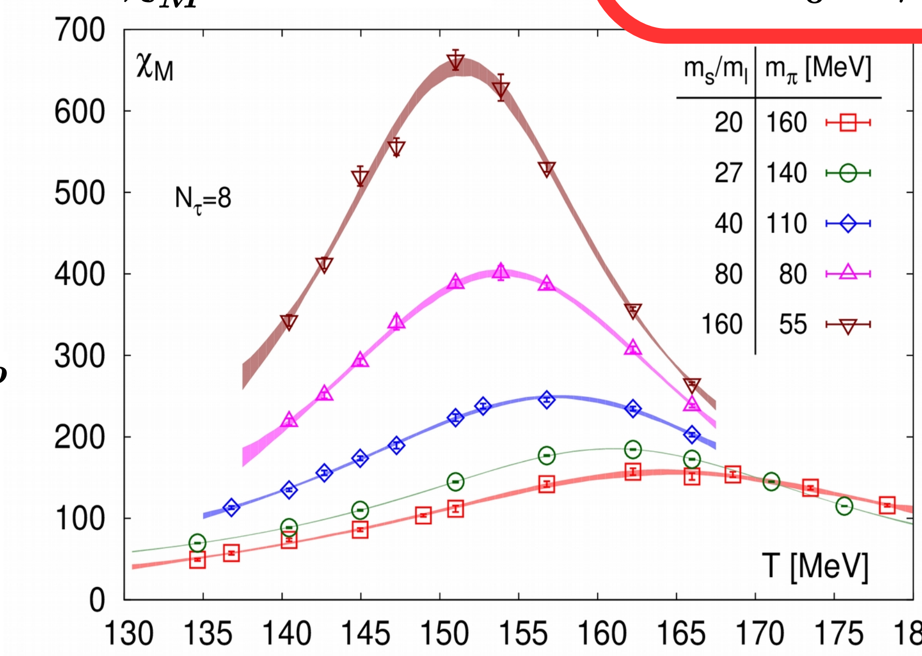
conventional steps to determine T_c^0

– choose a characteristic feature of χ_M
  → the maximum χ_M^{max}

– in the scaling regime this is located at z_p

– using the scaling ansatz for $T_{pc}(H)$ allows to extract T_c^0

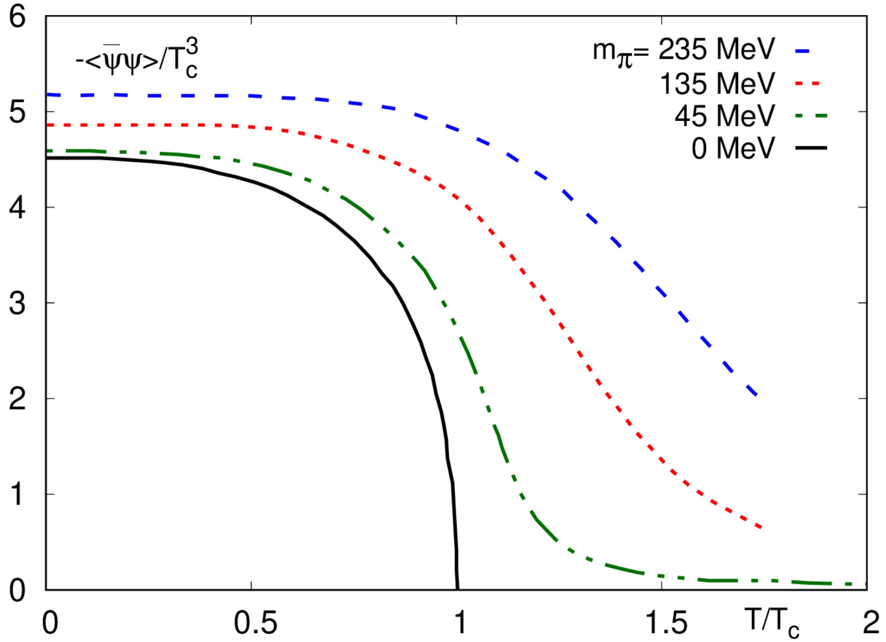
$$\chi_M^{max} \sim H^{1/\delta-1}$$



A. Lahiri et al, QM 2018, arXiv:1807.05727

Chiral extrapolation in the Quark Meson Model

$$\Gamma_{\Lambda_{UV}}[\phi] = \int d^4x \left\{ \bar{q}(\not{\partial} + g m_c) q + g\bar{q}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5) q + \frac{1}{2}(\partial_\mu \phi)^2 + U_{\Lambda_{UV}}(\phi) \right\}$$



$$\Delta T \equiv T_{pc}(m_\pi^{phys}) - T_c(0) \simeq 30 \text{ MeV}$$

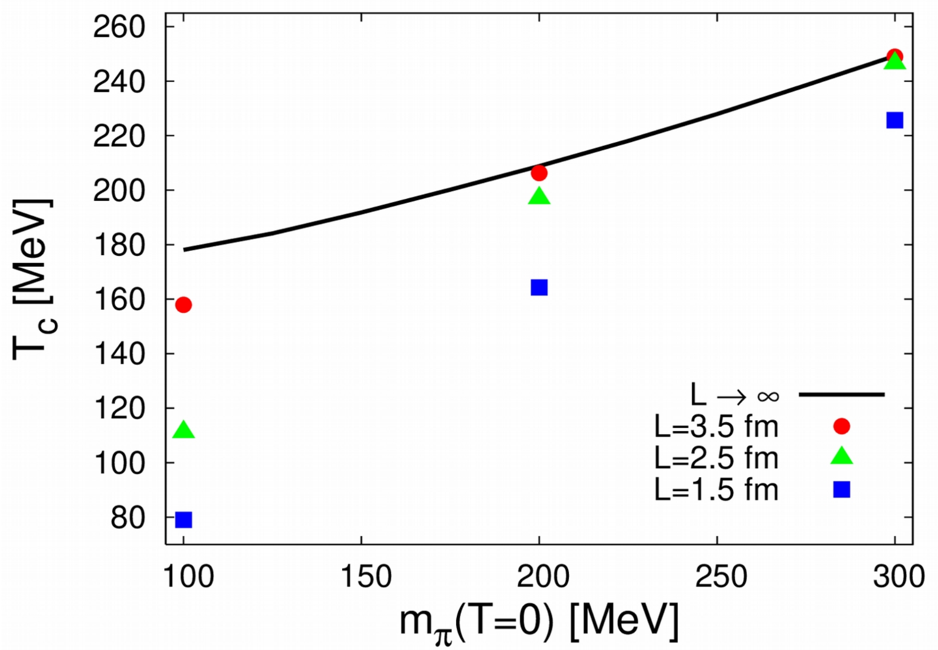
m_π[MeV]	0	45	135	230
$T_{pc}^{(1)}$[MeV]	100.7	$\simeq 110$	$\simeq 130$	$\simeq 150$
$T_{pc}^{(2)}$[MeV]	100.7	113	128	—

– strong pion mass dependence of $T_{pc}(m_\pi)$

$T_{pc}(m_\pi)$ almost linear in m_π ,
even for $m_\pi = m_\pi^{phys}$
trivial?
put O(4) in get O(4) out?

J. Berges, D.U. Jungnickel, C. Wetterich, Phys. Rev. D59 (1999) 034010

Chiral extrapolation and finite volume effects in the O(4) ϕ^4 model



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4$$

$$\phi = (\phi_1, ..., \phi_4)$$

$$\Delta T \equiv T_{pc}(m_\pi^{phys}) - T_c(0) \simeq 35\ \mathrm{MeV}$$

L [fm]	$m_\pi^{(0)} = 100$ MeV	$m_\pi^{(0)} = 200$ MeV	$m_\pi^{(0)} = 300$ MeV
1.5	79.0 MeV	164.3 MeV	225.7 MeV
2.5	111.3 MeV	197.1 MeV	246.6 MeV
3.5	157.9 MeV	206.3 MeV	249.0 MeV
∞	178.1 MeV	208.3 MeV	249.3 MeV

– increasing volume dependence with decreasing pion mass

J. Braun, B. Klein, H.-J. Pirner, A.H. Rezaeian,
Phys. Rev. D73 (2006) 074010

Finite size scaling functions of the 3-d, O(4) spin model

$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T,H,L)$$
$$\chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z, z_L) + \tilde{f}_{sub}(T,H,L)$$

$$z_L = \frac{1}{L h^{\nu_c}}$$
$$z = \frac{t}{h^{1/\beta\delta}}$$
$$\nu_c = \nu/\beta\delta = (0.5 - 0.6)$$

$$\frac{H\chi_M}{M} = \frac{f_\chi(z,z_L)}{f_G(z,z_L)} + \text{sub leading}$$

$$\lim_{L\to\infty} \left(\frac{H\chi_M}{M}\right)_{z=0} = \frac{1}{\delta}$$

volume dependence controlled by $z_L \sim 1/m_\pi^{2\nu_c} L\ ,\ 2\nu_c \simeq 1$

define $z_\delta(z_L)$ as the value z for given z_L at which $\left(\frac{H\chi_M}{M}\right)_{z_\delta(z_L)} = \frac{1}{\delta}$

$$T_\delta(H,L) = T_c^0 \left(1 + \frac{z_\delta(z_L)}{z_0} H^{1/\beta\delta}\right) + \text{ sub leading}$$

$z_\delta(0) = 0$

$z_\delta \simeq 0 \quad \Rightarrow$ weak H-dependence of T_δ even at finite H and/or L
– almost perfect estimator for T_c in the limit $H \to 0\ ,\ L \to \infty$

Finite size scaling functions of the 3-d, O(4) spin model

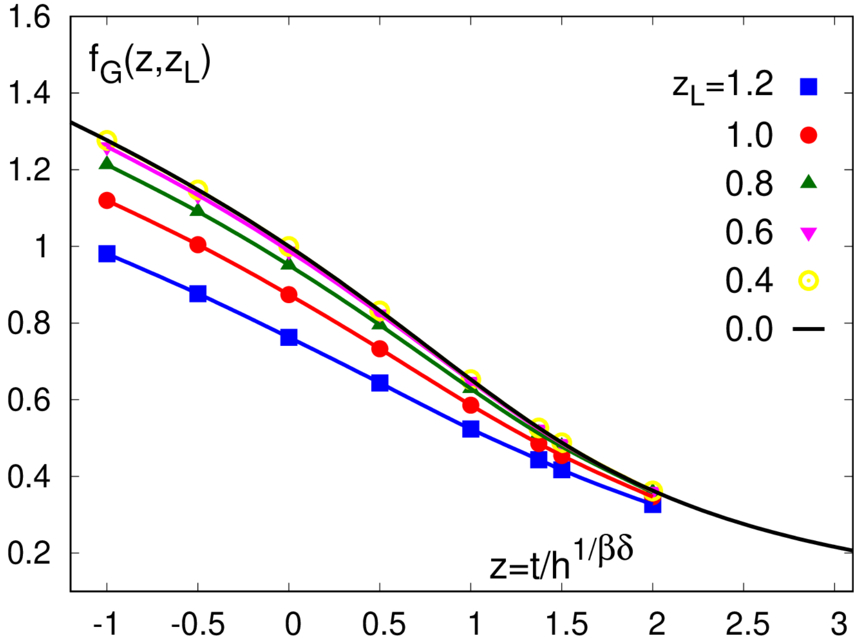
$$V \equiv L^3 \equiv (N_\sigma a)^3$$

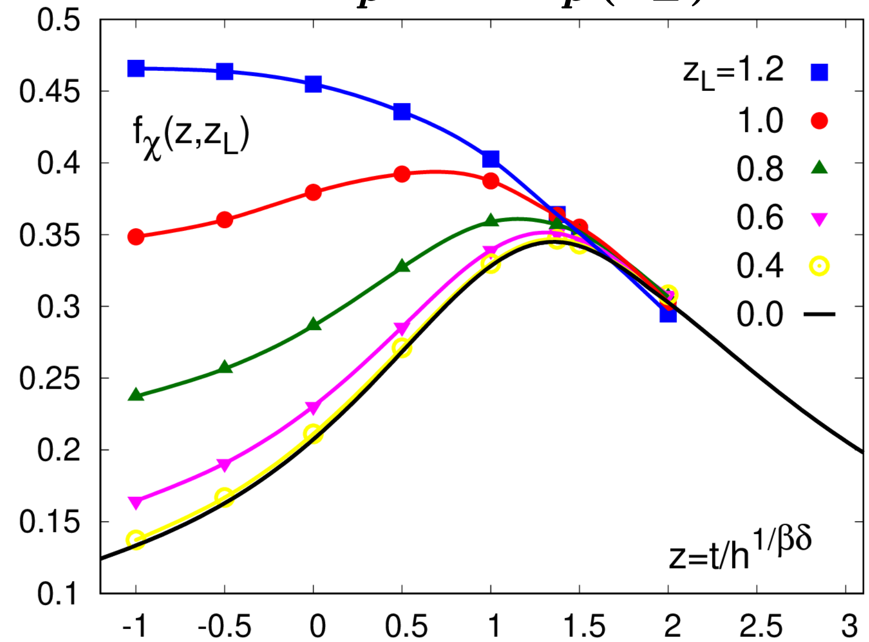
$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T,H,L)$$
$$\chi_M = h_0^{-1} h^{1/\delta - 1} f_\chi(z, z_L) + \tilde{f}_{sub}(T,H,L)$$

any "characteristic" z becomes a function of z_L :

$$T_{pc}(H,L) = T_c^0 \left(1 + \frac{z_p(z_L)}{z_0} H^{1/\beta\delta}\right) + \text{sub leading}$$

$$z_p \Rightarrow z_p(z_L)$$





J. Engels, FK, Phys. Rev. D90 (2014) 014501

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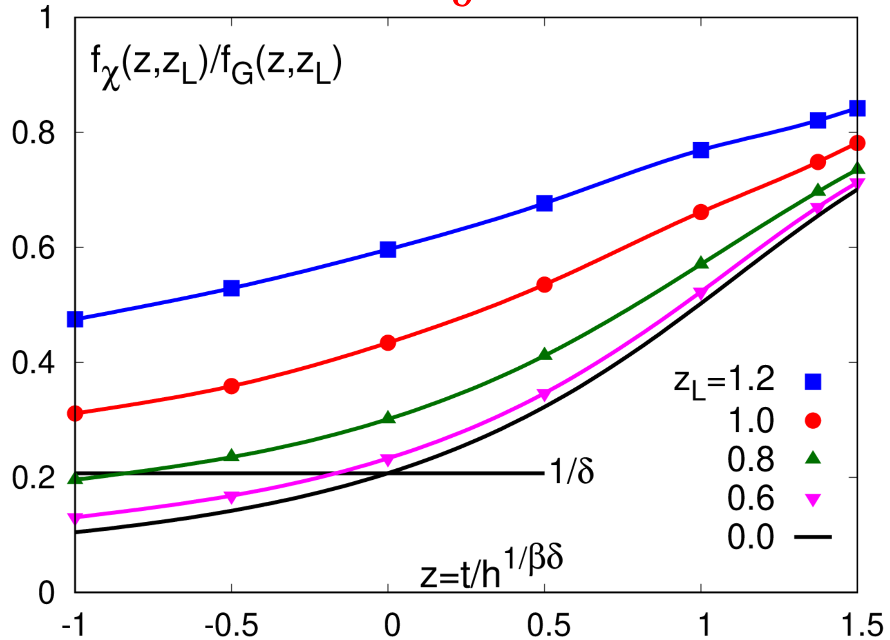
$$\chi_M = h_0^{-1} h^{1/\delta - 1} f_\chi(z, z_L) + \tilde{f}_{sub}(T,H,L)$$

$$\frac{H\chi_M}{M} = \frac{f_\chi(z,z_L)}{f_G(z,z_L)} + \text{sub leading}$$

$$\lim_{L\to\infty} \left(\frac{H\chi_M}{M}\right)_{z=0} = \frac{1}{\delta}$$

defines $z_\delta \simeq 0$



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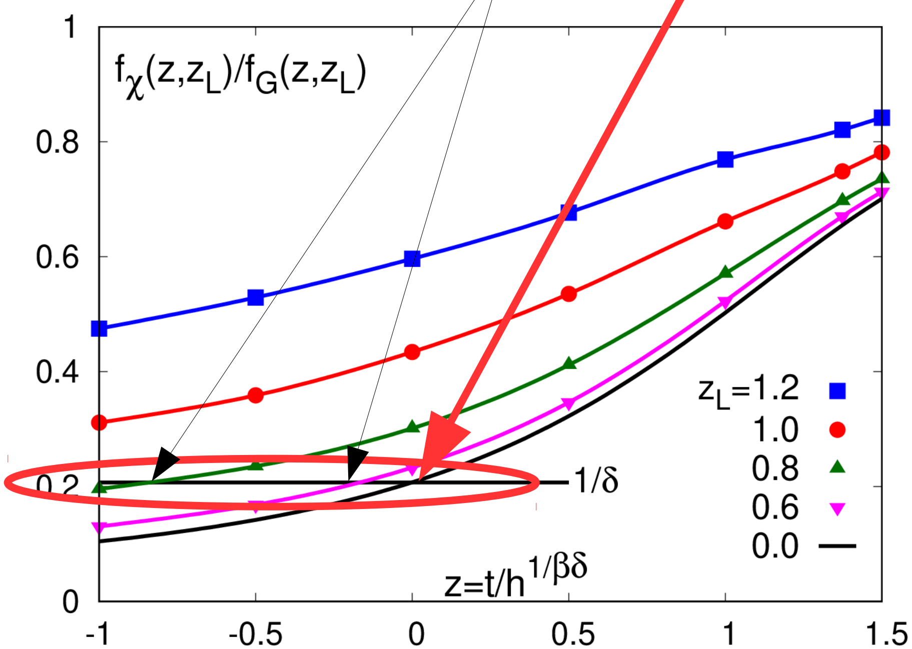
$$\chi_M = h_0^{-1} h^{1/\delta - 1} f_\chi(z, z_L) + \tilde{f}_{sub}(T,H,L)$$

$$\frac{H\chi_M}{M} = \frac{f_\chi(z,z_L)}{f_G(z,z_L)} + \text{sub leading}$$

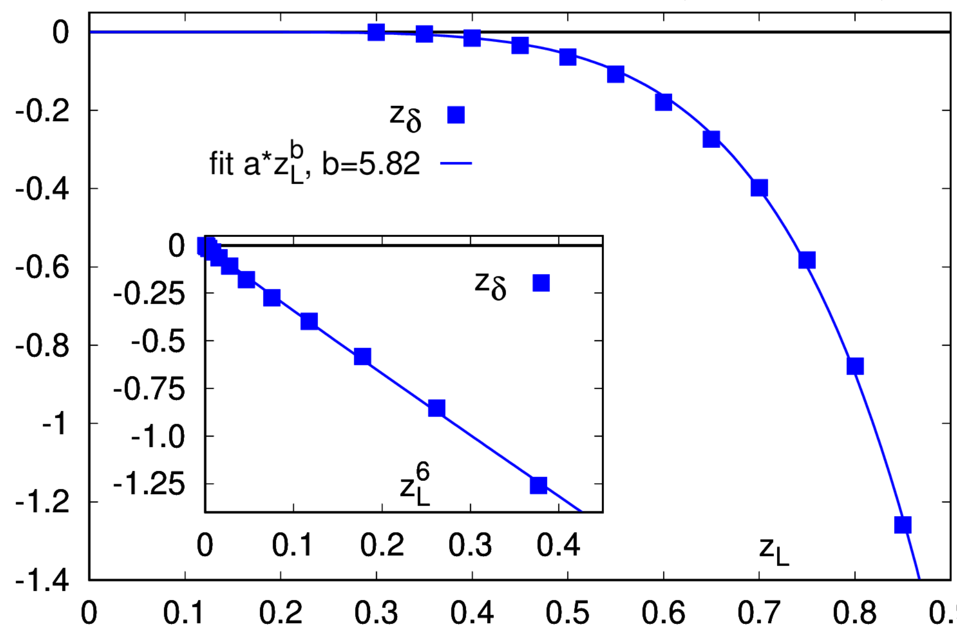
quark mass dependence arises only as a finite volume effect (+s.l.)

$$T_\delta(H,L) = T_c^0 \left(1 + \frac{z_\delta(z_L)}{z_0} H^{1/\beta\delta}\right)$$

$$\lim_{L\to\infty} T_\delta(L) = T_c^0$$

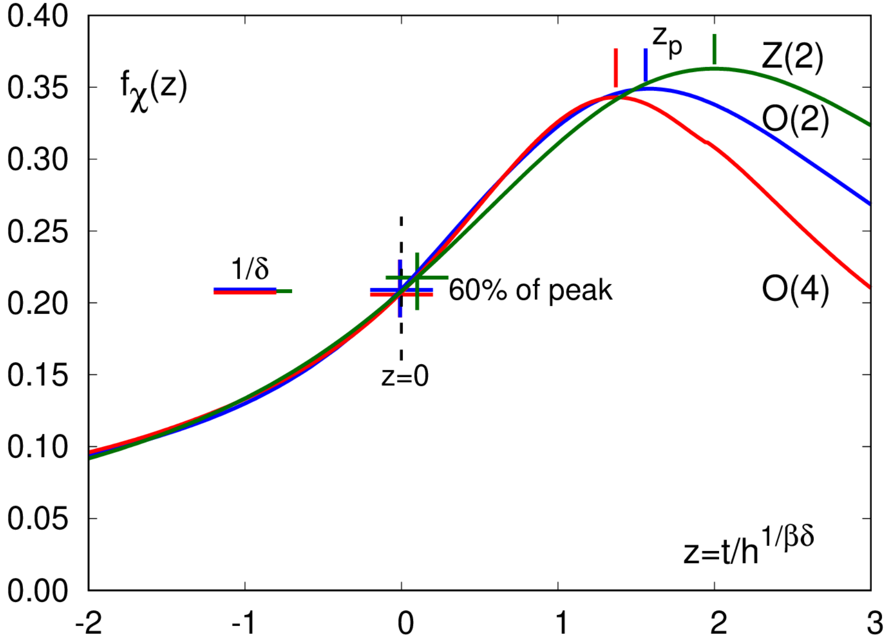


finite volume effects $\sim 1/V^2 \equiv 1/L^6$



Chiral PHASE TRANSITION in (2+1)-flavor QCD

A. Lahiri et al, QM 2018, arXiv:1807.05727
HotQCD, in preparation



– physical strange quark mass
– vary light quark mass

$$55 \text{ MeV} \le m_\pi \le 160 \text{ MeV}$$

– use new estimators for pseudo-critical temperatures

$$T_\delta,\ T_{60}$$

– control finite volume effects

$$2 \le m_\pi L \le 5$$

– extrapolate to infinite volume limit and chiral limit

$$1/aT = 6,\ 8,\ 12$$

	δ	z_p	z_{60}	$f_G(z_p)$	$f_\chi(z_p)$	$f_\chi(0)/f_\chi(z_p)$
Z(2)	4.805	2.00(5)	0.10(1)	0.548(10)	0.3629(1)	0.573(1)
O(2)	4.780	1.58(4)	-0.005(9)	0.550(10)	0.3489(1)	0.600(1)
O(4)	4.824	1.37(3)	-0.013(7)	0.532(10)	0.3430(1)	0.604(1)

Chiral PHASE TRANSITION in (2+1)-flavor QCD

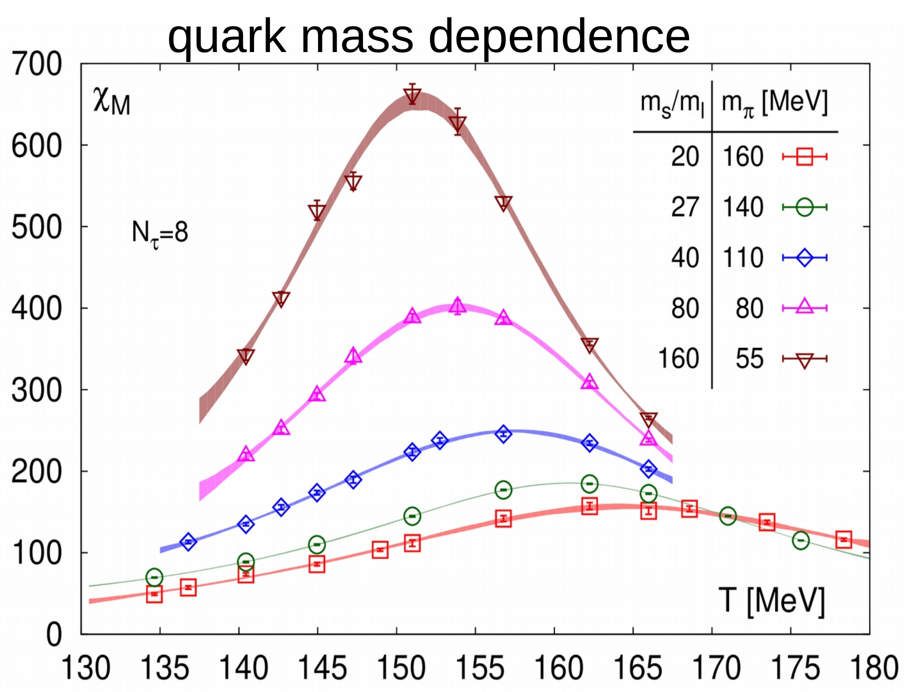
$$\langle \bar\psi\psi\rangle_f = \frac{T}{V}\frac{\partial \ln Z(T,V,m_u,m_d,m_s)}{\partial m_f}$$

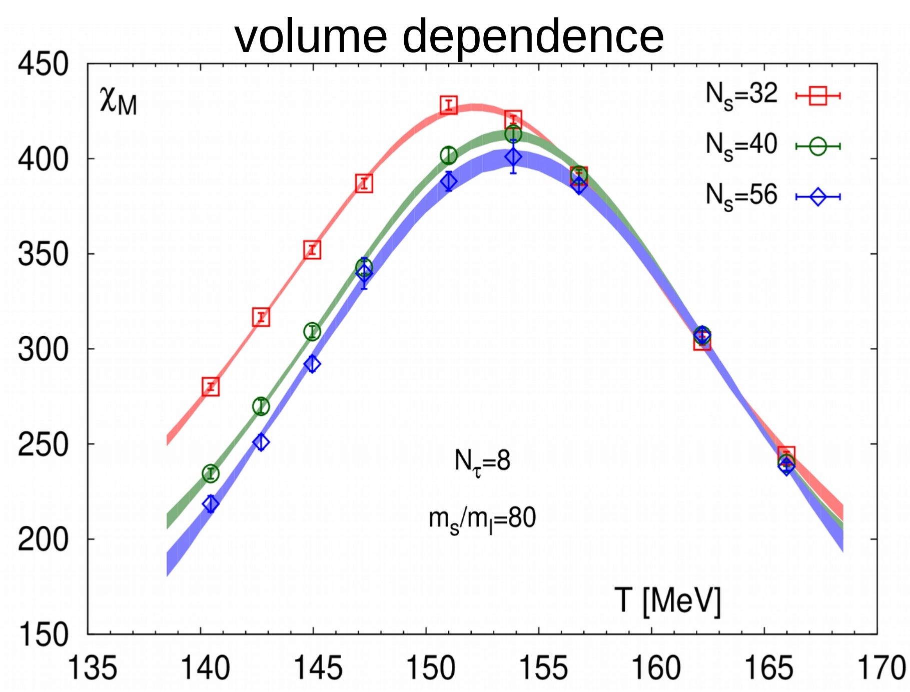
$$\langle \bar\psi\psi\rangle_l = (\langle \bar\psi\psi\rangle_u + \langle \bar\psi\psi\rangle_d)/2$$

renormalization group invariant order parameter: $M = 2\left(m_s\langle\bar\psi\psi\rangle_l - m_l\langle\bar\psi\psi\rangle_s\right)/f_K^4$

chiral susceptibility: $\chi_M = m_s(\partial_u + \partial_d)M$

lattice sizes: $N_\sigma^3 \times N_\tau \;,\quad 4 \le N_\sigma/N_\tau \le 8$





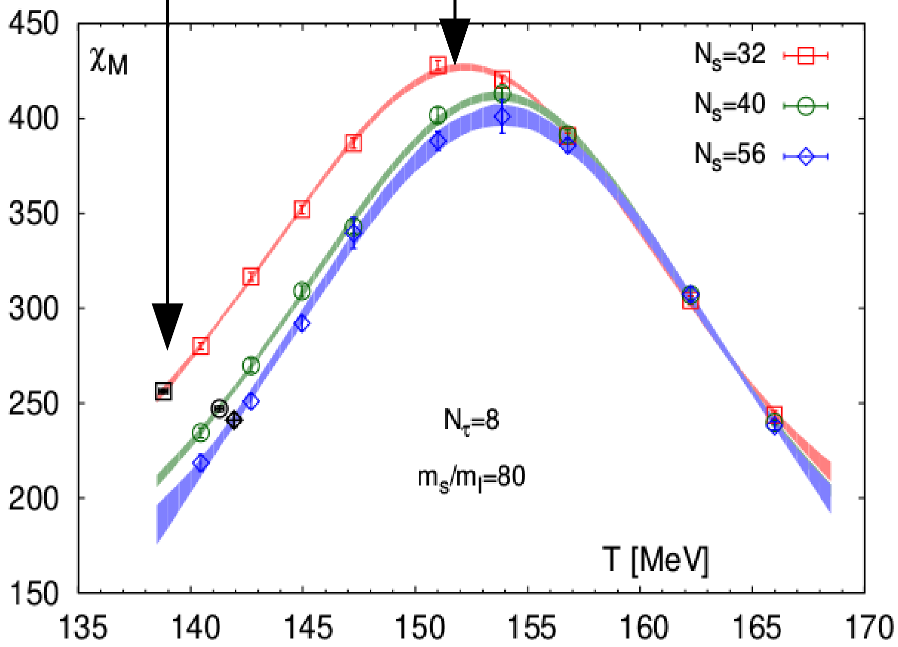
Chiral PHASE TRANSITION in (2+1)-flavor QCD

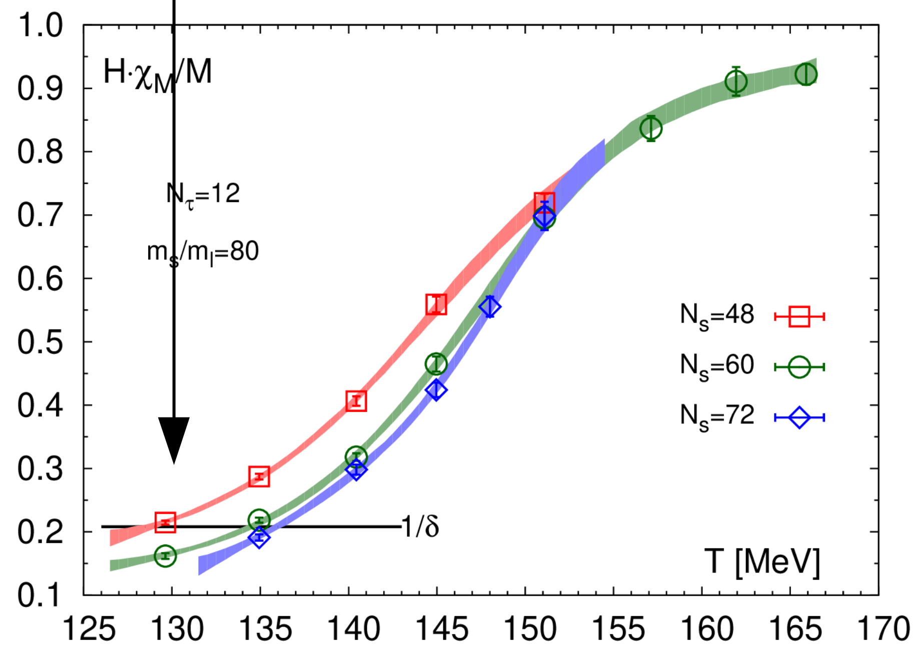
use two novel observables for the determination of the chiral PHASE TRANSITION TEMPERATURE, which in the infinite volume limit correspond to $z \simeq 0$, i.e. in the scaling regime they have almost no quark mass dependence

$$T_X(H,L) = T_c^0 \left(1 + \frac{z_X(z_L)}{z_0} H^{1/\beta\delta}\right) + \text{sub leading} \;,\; X = \delta,\; 60$$

$$\chi_{M,60} = 0.6\chi^{max} \;\Rightarrow\; T_{60}$$

$$\frac{H\chi_M}{M} = \frac{1}{\delta} \;\Rightarrow\; T_\delta$$





Finite size & and quark mass scaling

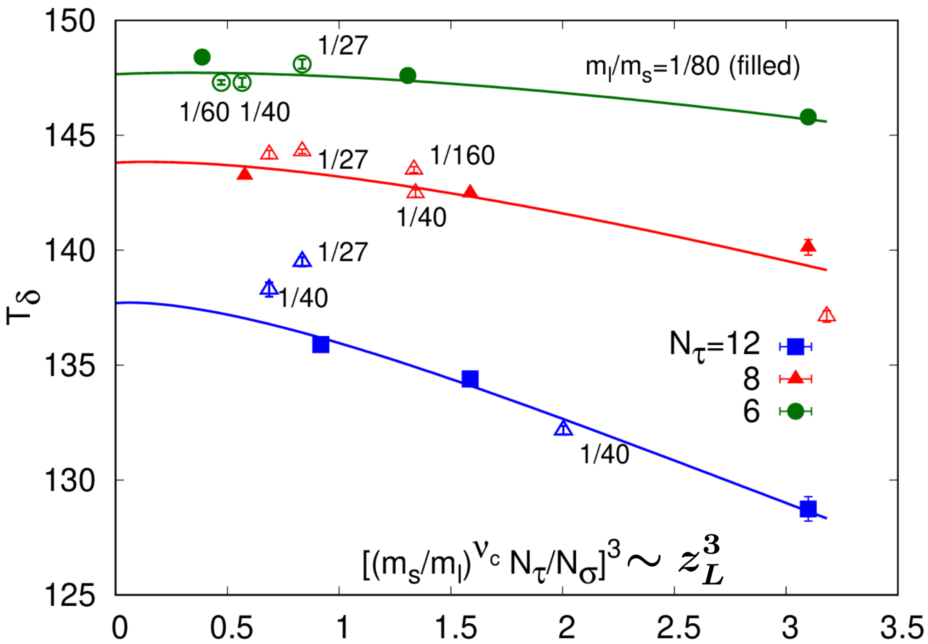
$$T_\delta(H,L) = T_c^0 \left(1 + \frac{z_\delta(z_L)}{z_0} H^{1/\beta\delta}\right) + \text{sub leading}$$

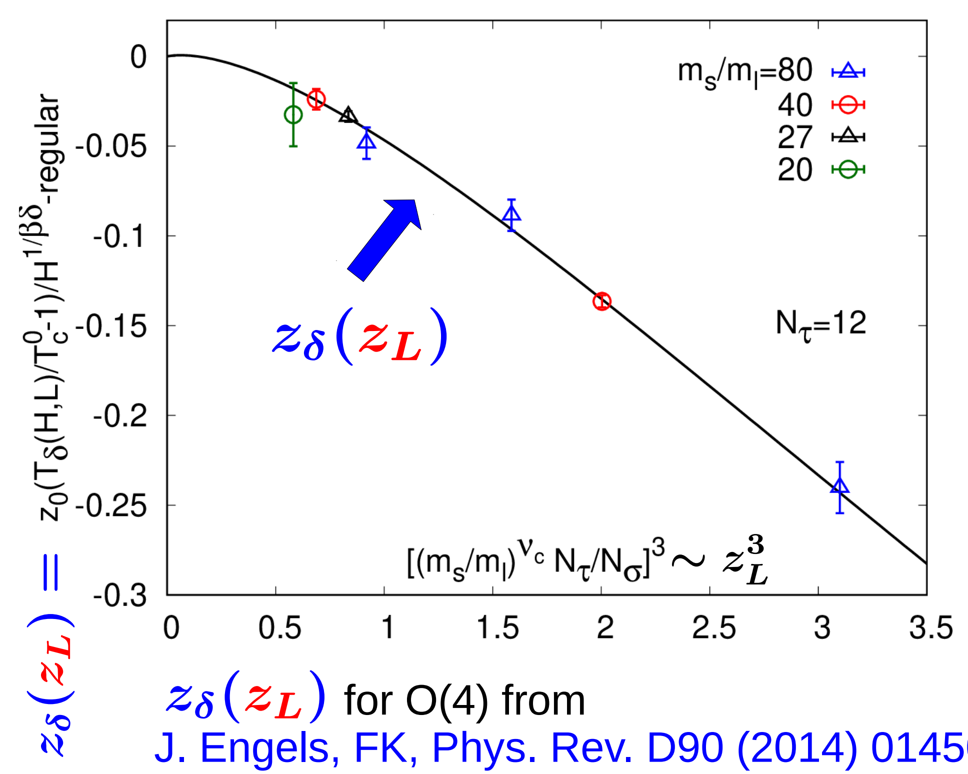
$$\left(T_\delta(H,L)/T_c^0 - 1\right) H^{-1/\beta\delta} - c_r H^{1-1/\delta} = \frac{z_\delta(z_L)}{z_0}$$

leading regular term

$$z_L = z_{L,0} \left(\frac{m_s}{m_l}\right)^{\nu_c} \frac{N_\tau}{N_\sigma}$$

$$\nu_c = \nu/\beta\delta$$





$z_\delta(z_L)$ for O(4) from

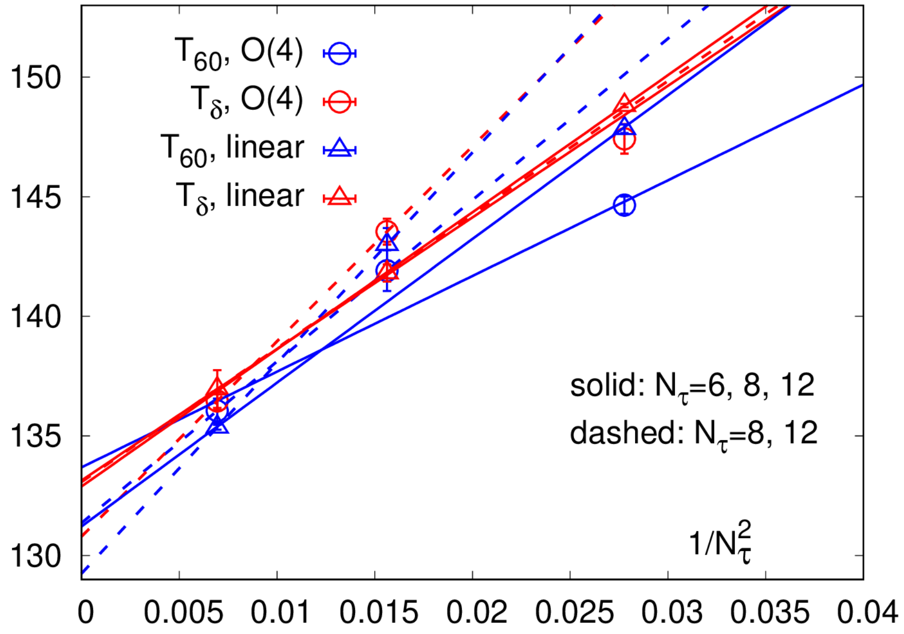
J. Engels, FK, Phys. Rev. D90 (2014) 014501

The chiral PHASE TRANSITION temperature

– using extrapolations linear in 1/V and m
as well as O(4) scaling ansatz

– extrapolations with and without data from coarsest lattice

– averaging results for T_δ and T_{60}



$$T_c = (130 - 135)\ \mathrm{MeV}$$

(HotQCD preliminary)

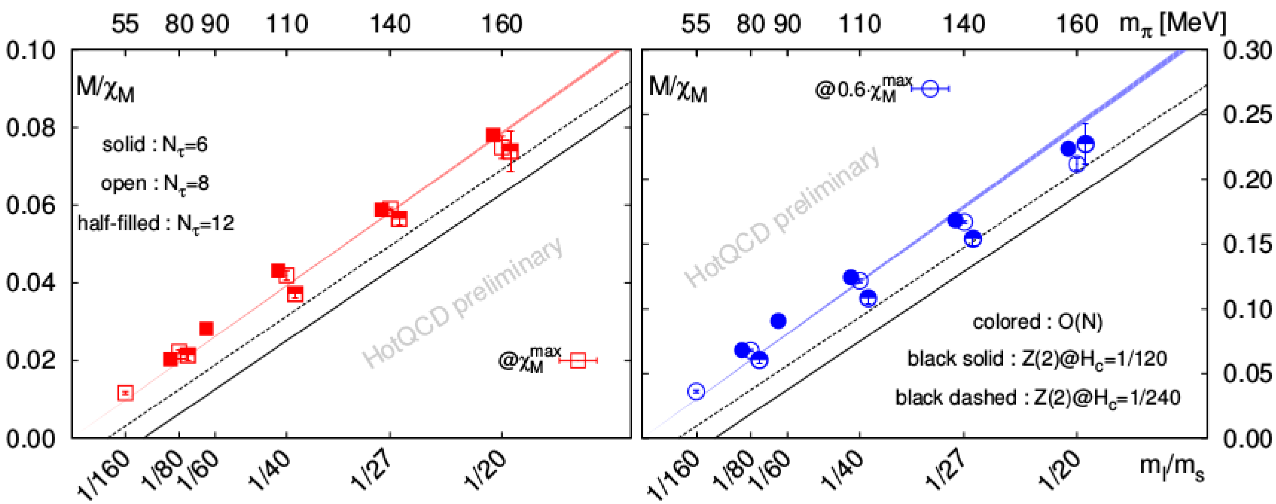
The chiral PHASE TRANSITION temperature
– evidence for a 2nd order transition in the chiral limit–

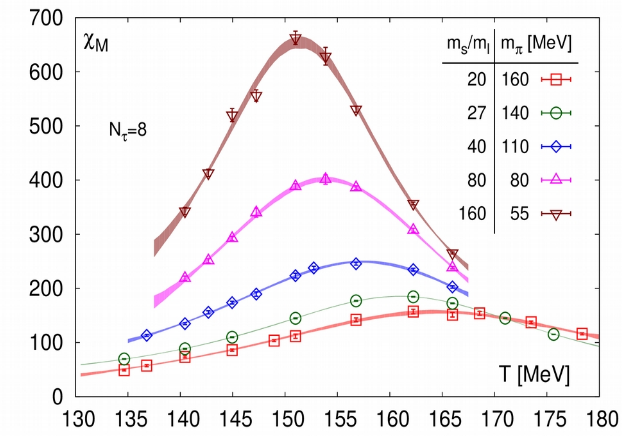
in the thermodynamic limit: suppose there occurs a 1st order transition for $H < H_c$

$$M(T,H) \sim (H-H_c)^{1/\delta} f_G(z)$$ (M is "almost" an order parameter)

$$\chi(T,H) \sim (H-H_c)^{1/\delta-1} f_\chi(z) + \ldots$$

for ANY fixed z: $\frac{M}{\chi_M} \sim (H-H_c)\,\frac{f_G(z)}{f_\chi(z)}$ ⇒ bound on H_c





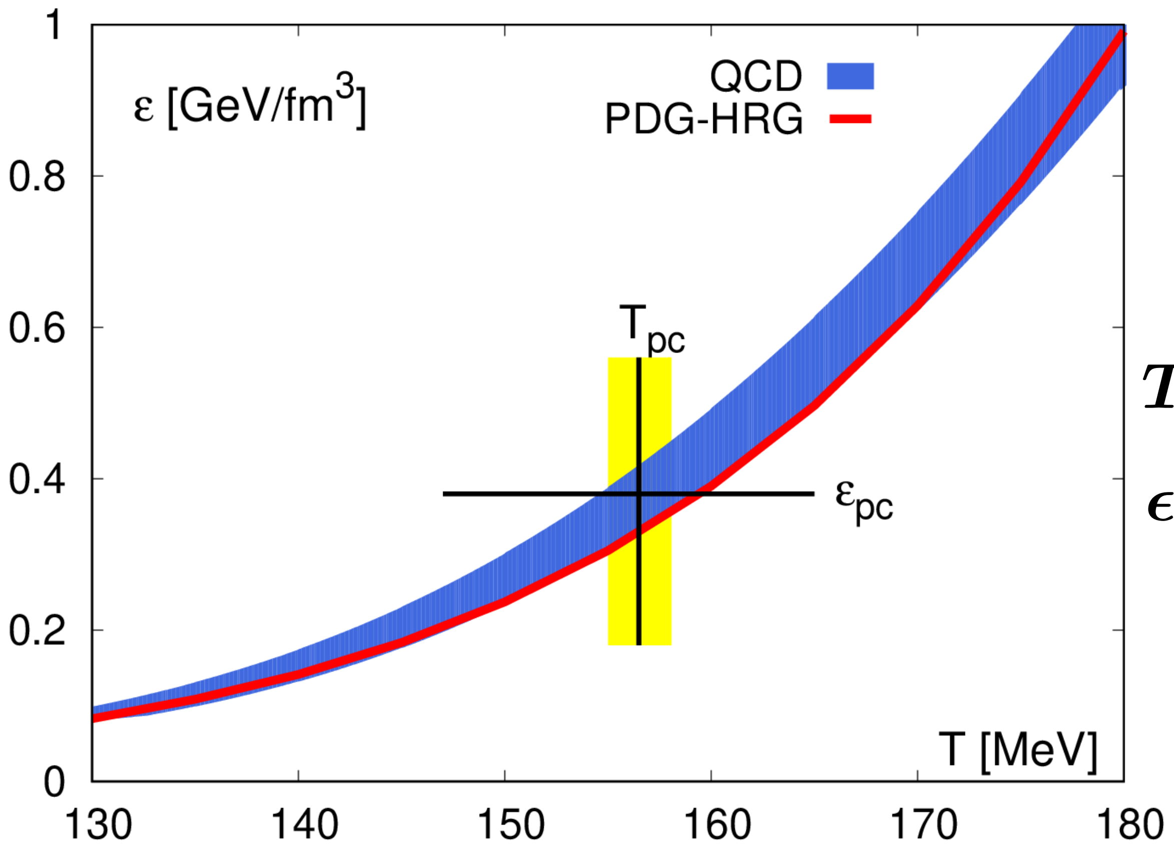
χ_M would diverge already for non-zero H_c

A. Lahiri et al, QM 2018, arXiv:1807.05727

see also next talk by Jishnu Goswami

Crossover transition parameters

PDG: Particle Data Group hadron spectrum



$$\mu_B/T = 0$$

physical quark masses

$$T_{pc} = (156.5 \pm 1.5)\ \mathrm{MeV}$$

$$\epsilon_{pc} = (0.42 \pm 0.06)\mathrm{GeV/fm}^3$$

compare with:

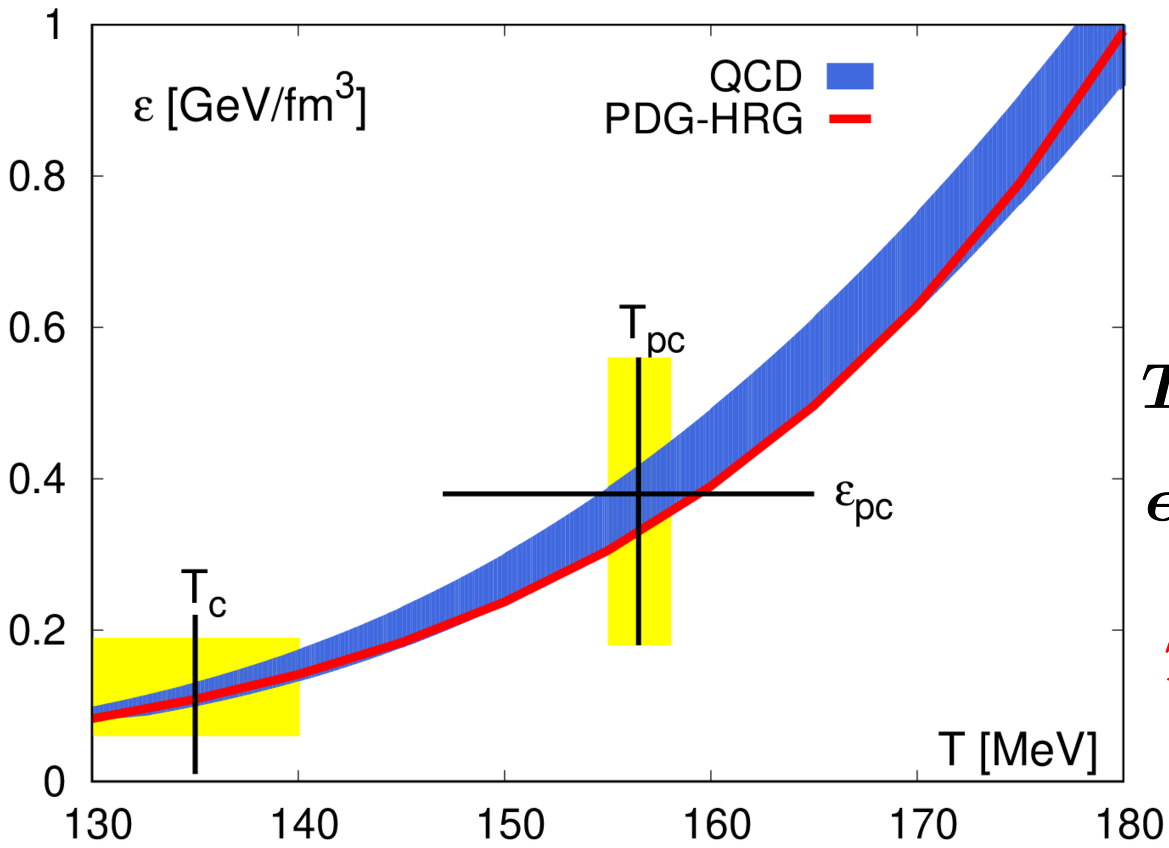
$$\epsilon^{\text{nucl. mat.}} \simeq 150\ \mathrm{MeV/fm}^3$$

$$\epsilon^{\text{nucleon}} \simeq 450\ \mathrm{MeV/fm}^3$$

A. Bazavov et al. (HotQCD) ,
Phys. Rev. D90 (2014) 094503
and arXiv:1812.08235

Crossover transition parameters
– and chiral limit –

PDG: Particle Data Group hadron spectrum



$$\mu_B/T = 0$$

physical quark masses

$$T_{pc} = (156.5 \pm 1.5)\ \mathrm{MeV}$$

$$\epsilon_{pc} = (0.42 \pm 0.06)\mathrm{GeV/fm^3}$$

chiral limit

$$T_c = (130 - 135)\ \mathrm{MeV}$$

$$\epsilon_c \simeq 0.15(5)\ \mathrm{GeV/fm^3}$$

compare with:

$$\epsilon^{\mathrm{nucl.\ mat.}} \simeq 150\ \mathrm{MeV/fm^3}$$

$$\epsilon^{\mathrm{nucleon}} \simeq 450\ \mathrm{MeV/fm^3}$$

A. Bazavov et al. (HotQCD) ,
Phys. Rev. D90 (2014) 094503
and arXiv:1812.08235

Transition parameters in the chiral limit

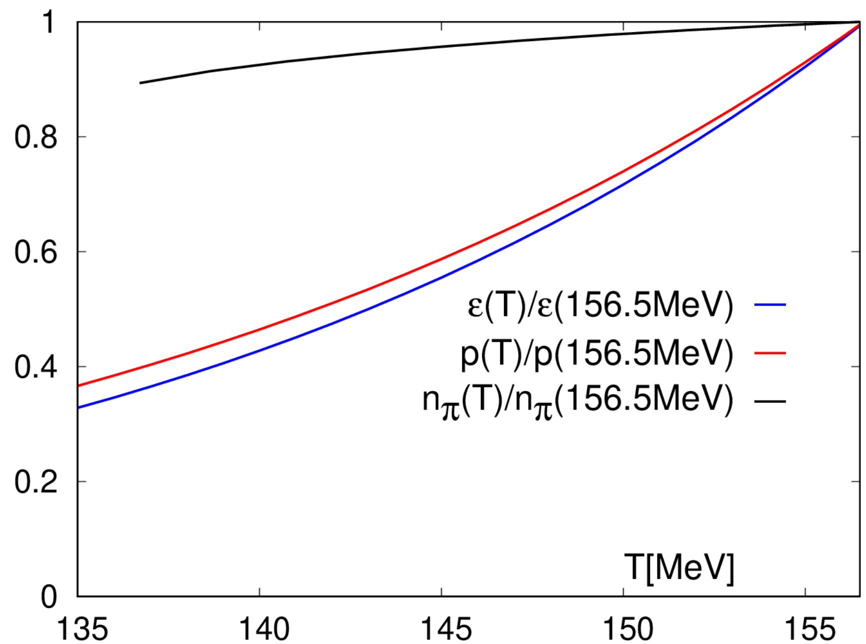
What drives the chiral transition?

– hadron resonance gas in the interval (135-156.5) MeV
– pion mass varies from 0 to its physical values

$$T \simeq (135 - 156.6)\ \mathrm{MeV}:$$

contributions to total energy density and pressure change by a factor 3
but, pion density stays roughly constant

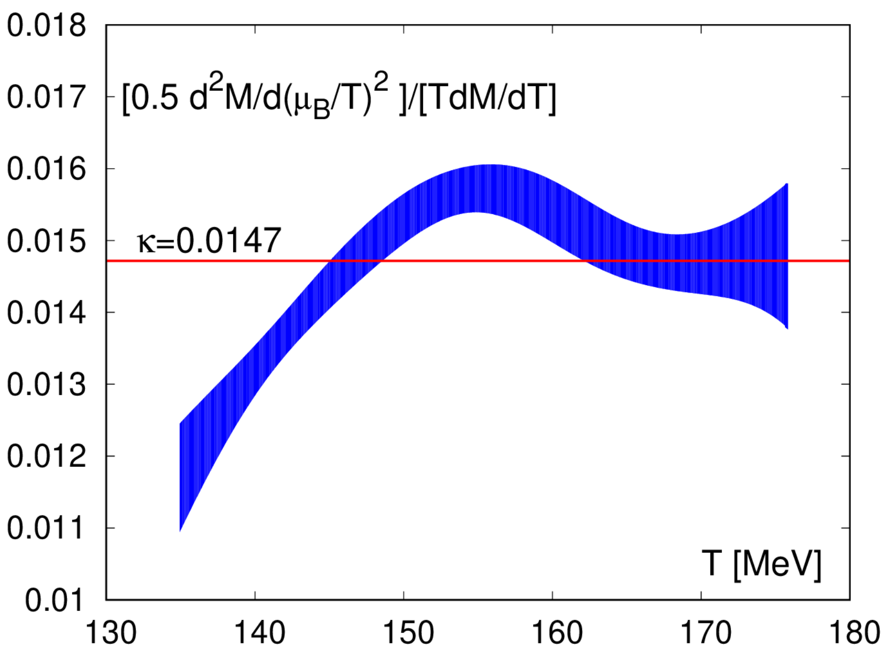
$$n_\pi \simeq 0.12/\mathrm{fm}^3$$

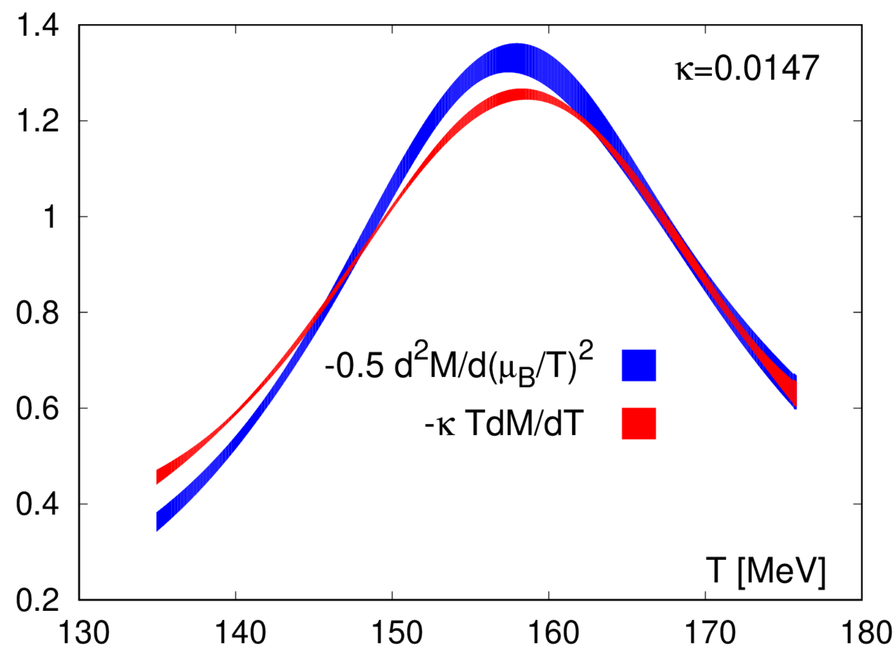


The chiral PHASE TRANSITION temperature at non-zero baryon chemical potential

$$t \sim \frac{T - T_c}{T_c} \quad \overset{\mu_B \neq 0}{\Rightarrow} \quad t \sim \frac{T - T_c}{T_c} + \kappa_2 \left(\frac{\mu_B}{T}\right)^2$$

$$M = h^{1/\delta} f_G(z_0 t / h^{1/\beta\delta}) \quad \Longrightarrow \quad T \frac{\partial M}{\partial T}\bigg|_{(T_c,0)} = \frac{1}{2\kappa_B} \frac{\partial^2 M}{\partial(\mu_B/T)^2}\bigg|_{(T_c,0)}$$





– curvature of the chiral phase transition line is compatible with that of the pseudo-critical line: $\kappa_2^B = 0.015(5)$

A. Bazavov et al. (HotQCD), arXiv1812.08235

Conclusions

– no evidence for a 1st order transition in QCD for pion masses $m_\pi \geq 55\ \mathrm{MeV}$

– the chiral phase transition in QCD is likely to be 2nd order

– the chiral phase transition is (20-25) MeV smaller than the pseudo-critical temperature for physical values of the quark masses

$$T = (130 - 135)\mathrm{MeV}$$

– the chiral phase transition occurs at a pion density

$$n_\pi \simeq 0.12/\mathrm{fm}^3$$