Virtual photon polarization and dilepton anisotropy in relativistic nucleus-nucleus collisions

Enrico Speranza

E.S., A. Jaiswal, B. Friman, PLB **782**, 395 (2018)
B. Friman, R. Rapp, E.S., J. Wambach (in preparation)

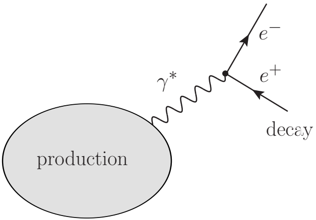
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC–TR 211

From QCD matter to hadrons

Hirschegg, January 16, 2019

The goal



Angular distribution of the dilepton

⇓

Information on the polarization states of the virtual photon

⇓

Information on the production mechanism

- Disentangle mechanisms in HIC and elementary reactions (see D. Nitt talk)
  E.L. Bratkovskaya, O.V. Teryaev, and V.D. Toneev, PLB **348**, 283
  E.S., M. Zétényi, and B. Friman, PLB **764**, 282
- Early stages and onset of thermalization in HIC
  P. Hoyer, PLB **187**, 162; E. Shuryak, arXiv:1203.1012
- Parton anisotropic momentum distributions in HIC
  G. Baym, T. Hatsuda, and M. Strickland, PRC **95**, 044907

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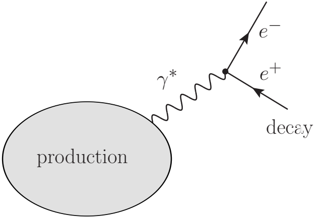

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Spin-density matrix

- **Pure state**: $|\psi\rangle = \sum_\lambda c_\lambda |\lambda\rangle$
  Expectation value of an operator $\langle O \rangle = \langle \psi | O | \psi \rangle$
- **Mixed state**: incoherent mixture of $|\psi_i\rangle$ with statistical weight a_i

$$\rho = \sum_i a_i |\psi_i\rangle\langle\psi_i| = \sum_{\lambda,\lambda'} \rho_{\lambda\lambda'} |\lambda\rangle\langle\lambda'|$$

$\rho_{\lambda\lambda'} = \sum_i a_i\, c_\lambda^{(i)} c_{\lambda'}^{(i)*}$. Expectation value: $\langle O \rangle = \mathrm{Tr}(\rho\, O)$

Example: Spin-1/2 particle (2×2 hermitian matrix):

$$\rho = \frac{1}{2}(1 + \vec{P} \cdot \vec{\sigma})$$

- Spin polarization vector: $\vec{P} = \langle \vec{\sigma} \rangle = \mathrm{Tr}(\rho\, \vec{\sigma})$

$$|\vec{P}| = 1 \quad \text{Pure state}$$
$$0 < |\vec{P}| < 1 \quad \text{Mixed state}$$
$$|\vec{P}| = 0 \quad \text{Completely unpolarized mixed state}$$

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Spin-density matrix for spin-1 particles

- Three polarization states (in rest frame)

$$\text{Transverse to } \vec{q}: \quad \epsilon(\pm 1) = \mp \frac{1}{\sqrt{2}}(0,\, 1,\, \pm i,\, 0)$$

$$\text{Longitudinal to } \vec{q}: \quad \epsilon(0) = (0,\, 0,\, 0,\, 1)$$

- **Spin-density matrix**: hermitian 3×3 matrix

$$\rho = \frac{1}{3}\left[1 + \frac{3}{2}\vec{P}\cdot\vec{S} + \sqrt{\frac{3}{2}}\sum_{i,j} T_{ij}(S_i S_j + S_j S_i)\right]$$

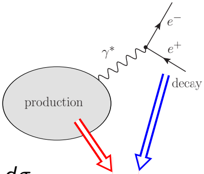
  $\vec{S}$ are the spin-1 operators

- $\mathrm{Tr}\rho = 1$ (8 parameters)
- Vector polarization: $\vec{P} = \langle \vec{S} \rangle$ (3 parameters)
- Tensor polarization: $T_{ij} = \frac{1}{2}\sqrt{\frac{3}{2}}\left(\langle S_i S_j + S_j S_i\rangle - \frac{4}{3}\delta_{ij}\right),\qquad \sum_i T_{ii} = 0$
  (5 parameters)

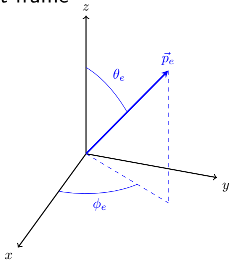
One can have tensor polarization without vector polarization

Lepton angular distribution

spin-1 $\rightarrow$ spin-$\frac{1}{2}$ + spin-$\frac{1}{2}$



Photon rest frame



$$\frac{d\sigma}{d\Omega_e} \propto \mathrm{Tr}(\,\rho\, O^{\mathrm{dec}}\,)$$

$$= \mathcal{N}\Big(1 + \lambda_\theta \cos^2\theta_e + \lambda_\phi \sin^2\theta_e \cos 2\phi_e + \lambda_{\theta\phi} \sin 2\theta_e \cos\phi_e$$

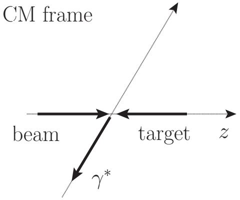
$$+ \lambda_\phi^\perp \sin^2\theta_e \sin 2\phi_e + \lambda_{\theta\phi}^\perp \sin 2\theta_e \sin\phi_e + \text{parity violating terms}\Big)$$

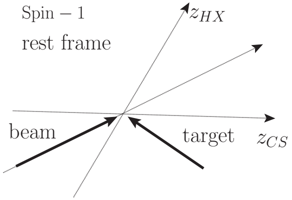
$$\lambda_\theta = \frac{\rho_T - \rho_L}{\rho_T + \rho_L}$$

- Transverse: $\rho_T = \rho_{-1-1} + \rho_{+1+1}$
  Longitudinal: $\rho_L = 2\rho_{00}$
  $(\rho = \sum_{\lambda,\lambda'} \rho_{\lambda\lambda'} |\lambda\rangle\langle\lambda'|)$
- Completely **transverse** polarized: $\lambda_\theta = +1$
  Completely **longitudinal** polarized: $\lambda_\theta = -1$
- Photon polarization reflected in angular distribution

Reference frames

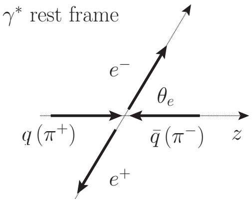
Anisotropy coefficients depend on the reference frame





- Helicity (HX): z-axis along photon momentum
- Collins-Soper (CS): z-axis along bisector between beam and target
- Different frames are related by rotation

Examples



- Drell-Yan process: $q\bar{q} \to \gamma^* \to e^+e^-$

$$\frac{d\sigma}{d\Omega_e} \sim 1 + \cos^2\theta_e$$

$\lambda_\theta = +1$. Virtual photon is completely transverse polarized along beam axis

- Pion annihilation process: $\pi^+\pi^- \to \gamma^* \to e^+e^-$

$$\frac{d\sigma}{d\Omega_e} \sim 1 - \cos^2\theta_e$$

$\lambda_\theta = -1$. Virtual photon is completely longitudinal polarized along beam axis

Virtual photon emission from a thermal medium

$$q\bar{q} \to \gamma^* \to e^+e^-$$
$$\pi^+\pi^- \to \gamma^* \to e^+e^-$$

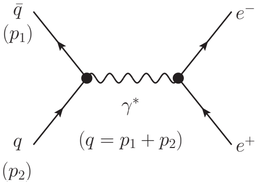
- Thermal average of initial particles momenta p through Fermi or Bose distribution
$$f(p) = \frac{1}{e^{(u\cdot p)/T} \pm 1}$$
- Fluid rest frame $u^\mu = (1,0,0,0) \Rightarrow$ Distribution is spherical symmetric

Photon momentum $\vec{q}$ breaks spherical symmetry, but not azimuthal symmetry

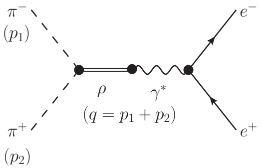
- Photons are only tensor polarized
- $|\vec{q}| \to 0 \Rightarrow$ No anisotropy $\Rightarrow$ No photon polarization

Boltzmann limit

$q + \bar{q} \to \gamma^* \to e^+ e^-$



$\pi^+ \pi^- \to \gamma^* \to e^+ e^-$

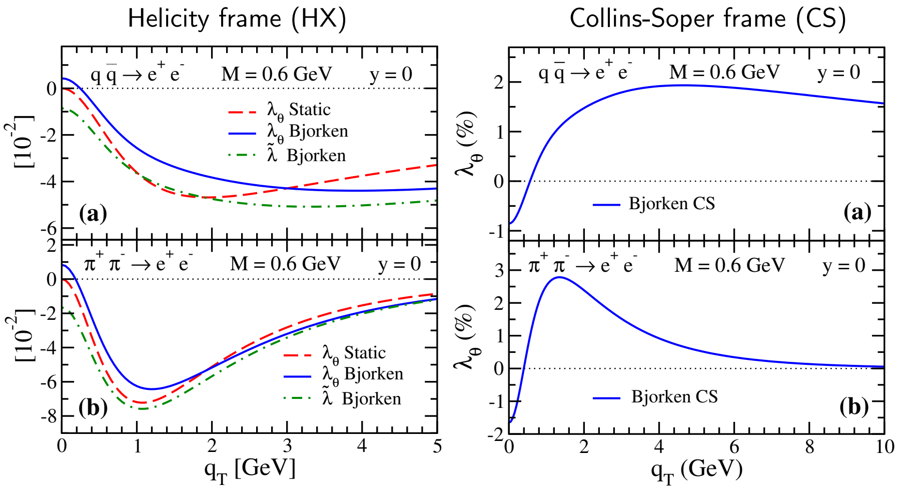


$$\int \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2} \frac{1}{e^{(u \cdot p_1)/T} \pm 1} \frac{1}{e^{(u \cdot p_2)/T} \pm 1} \sim e^{-(u \cdot q)/T} \int \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2}$$

- No photon polarization independently of photon momentum

Photon polarization is due to quantum statistics!

Results (static and Bjorken expansion)

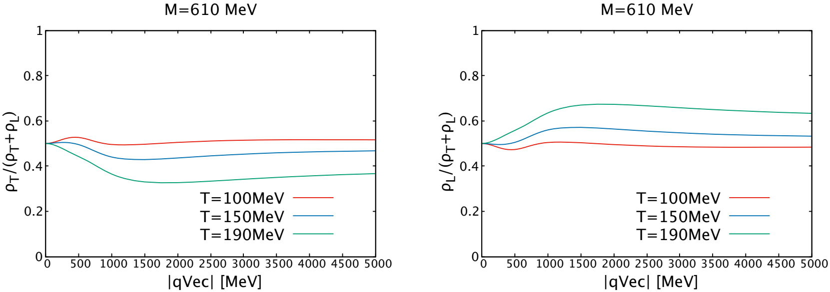


- Static case: $\lambda_\theta \to 0$ for $q_T \to 0$, and for $q_T \to \infty$ (Boltzmann limit)
- λ_θ changes sign from the Helicity to Collins-Soper frame
- Frame invariant combination: $\tilde{\lambda}_\theta = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi}$
- Experiments: sum over q_T

Realistic models (Preliminary)

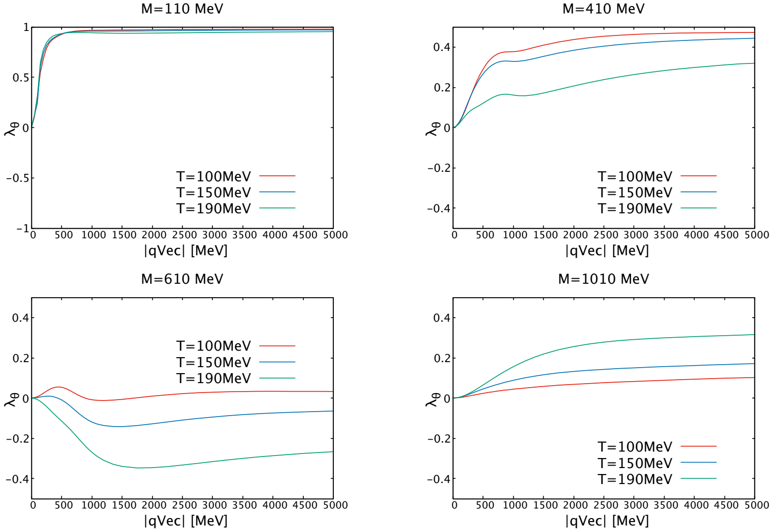
$$\frac{d\sigma}{d\Omega_e} \propto \rho^{\mu\nu} L_{\mu\nu}$$

$$\rho^{\mu\nu} = \rho_T P_T^{\mu\nu} + \rho_L P_L^{\mu\nu}$$



- ρ_T, ρ_L taken from: Rapp, Chanfray, Wambach, NPA **617**, 472; Rapp, Wambach EPJA **6**, 415; Urban, Buballa, Rapp, Wambach, NPA **673**, 357

Polarization with realistic models (Preliminary)

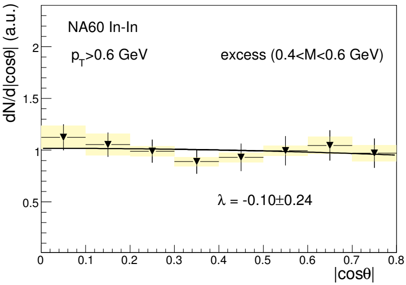


$\lambda_\theta = \frac{\rho_T - \rho_L}{\rho_T + \rho_L}$

Large polarization!

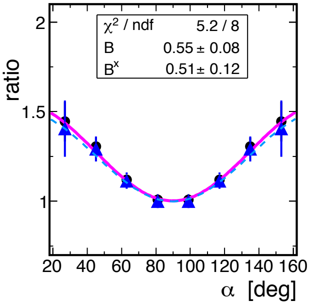
Experimental results (NA60 and HADES)

Collins-Soper frame



In-In at 158A GeV
(NA60 Collaboration), PRL **96**, 222301 (2009)

Helicity frame

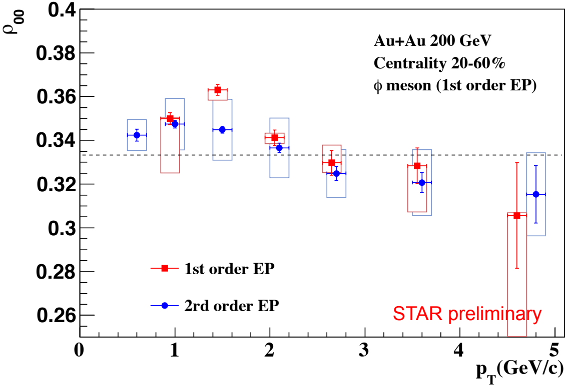


Ar-KCl at 1.76A GeV
(HADES Collaboration), PRC **84**, 014902 (2011)

- NA60: $\lambda_\theta \simeq 0$, but large error bars
- HADES: large polarization $\lambda_\theta \simeq 0.5$

ϕ meson polarization (STAR)

$$\lambda_\theta = \frac{3\rho_{00} - 1}{1 - \rho_{00}}, \qquad \rho_{00} = \rho_L$$



A. Tang, Chirality Workshop, Florence 2018; C. Zhou QM2018

- Noncentral collisions: large global angular momentum $\Rightarrow$ Vorticity $\Rightarrow$ Particle polarization
- Vorticity? Thermalized medium?

Conclusions

Summary

- Anisotropy coefficients as a tool to understand heavy-ion collisions
- Virtual photons from (unpolarized) thermal sources are polarized
- Collective flow affects shape of anisotropy coefficients
- Realistic models give large polarization

Outlook

- Analyze different elementary reactions
- Anisotropic momentum distributions $\Rightarrow$ nonequilibrium
- Effect of vorticity and magnetic field (polarized medium)

BACKUP

Diagonal form of the spin-density matrix

$$\rho = \frac{1}{3}\begin{pmatrix} 1+\frac{3}{2}P_z+\sqrt{\frac{3}{2}}T_{zz} & 0 & 0 \\ 0 & 1-\sqrt{6}T_{zz} & 0 \\ 0 & 0 & 1-\frac{3}{2}P_z+\sqrt{\frac{3}{2}}T_{zz} \end{pmatrix}$$

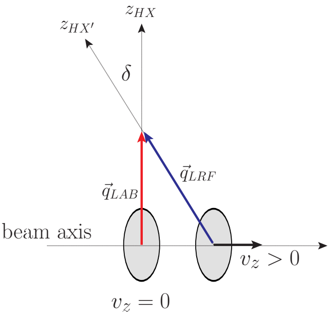
- In unpolarized system $P_z = 0$, but often $T_{zz} \neq 0$,
  i.e., no vector but tensor polarization!
- In general vector and tensor polarization axes can be different

Medium and flow

Static uniform medium

- Photon rest frame: fluid velocity $\vec{v}$ opposite to photon "direction"
- Only $\lambda_\theta \neq 0$

Longitudinal Bjorken expansion



- $v_z = z/t$ along beam axis
- Photon polarized along $z_{HX'}$ defined by its momentum in local rest frame
- Rotation δ between $z_{HX'}$ and z_{HX} (Wick helicity rotation) $\Rightarrow \lambda_\theta, \lambda_\phi, \lambda_{\theta\phi} \neq 0$
- Frame invariant combination:

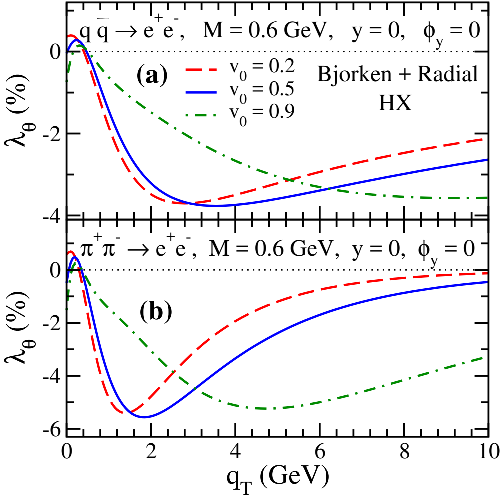
$$\tilde{\lambda}_\theta = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi}$$

Longitudinal Bjorken + Radial expansion

- All coefficients $\lambda_\theta, \lambda_\phi, \lambda_{\theta\phi}, \lambda_\phi^\perp, \lambda_{\theta\phi}^\perp \neq 0$

Results (Bjorken + radial expansion)

Helicity frame (HX)



- $v_\perp = v_0 r_\perp / R_0$ (Transverse to beam axis)
- The position of the minimum shifts towards higher q_T as v_0 increases

Comparison with NA60 – Integrated λ's

- Photon kinematics:
  $0.4\,\text{GeV} < M < 0.9\,\text{GeV}$, $0.6\,\text{GeV} < q_T < 2\,\text{GeV}$, $0.3 < y < 1.3$
- Drell-Yan
  - Helicity: $\lambda_\theta^{HX} \simeq -0.008$, $\lambda_\phi^{HX} \simeq -0.009$
  - Collins-Soper: $\lambda_\theta^{CS} \simeq 0.002$, $\lambda_\phi^{CS} \simeq -0.012$
  - Frame invariant: $\tilde{\lambda} \simeq -0.034$
- Pion annihilation
  - Helicity: $\lambda_\theta^{HX} \simeq -0.014$ and $\lambda_\phi^{HX} \simeq -0.016$
  - Collins-Soper: $\lambda_\theta^{CS} \simeq 0.007$, $\lambda_\phi^{CS} \simeq -0.023$
  - Frame invariant: $\tilde{\lambda} \simeq -0.061$