Spectral functions and transport coefficients with the FRG

Ralf-Arno Tripolt

In collaboration with:

Chris Jung, Fabian Rennecke, Naoto Tanji,

Lorenz von Smekal, Jochen Wambach, Johannes Weyrich

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JOHANN WOLFGANG GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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Outline

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II) Theoretical setup

- Functional Renormalization Group (FRG)
- QCD effective model
- Analytic continuation procedure

III) Results

- Quark and (pseudo-)scalar meson spectral function
- Shear viscosity and electrical conductivity
- (Axial-)vector meson spectral functions
- Electromagnetic spectral function and dilepton rates

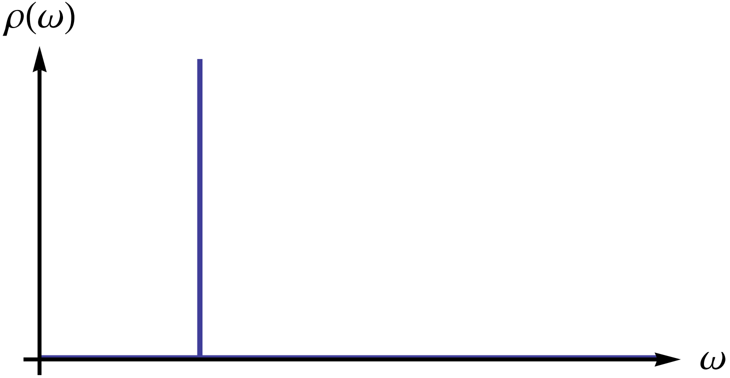
IV) Summary and outlook

I) Introduction and motivation

[courtesy L. Holicki]

What is a spectral function?

$$\rho(\omega) = -\frac{1}{\pi}\mathrm{Im}D^R(\omega)$$



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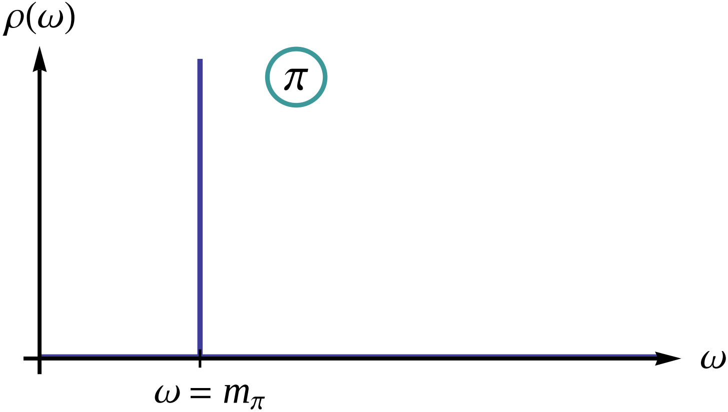

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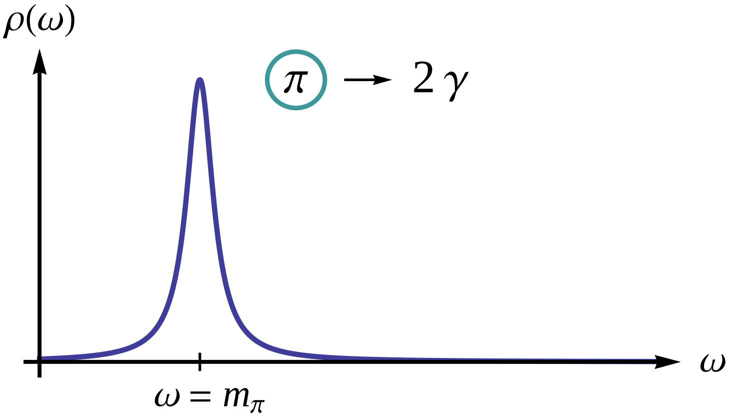

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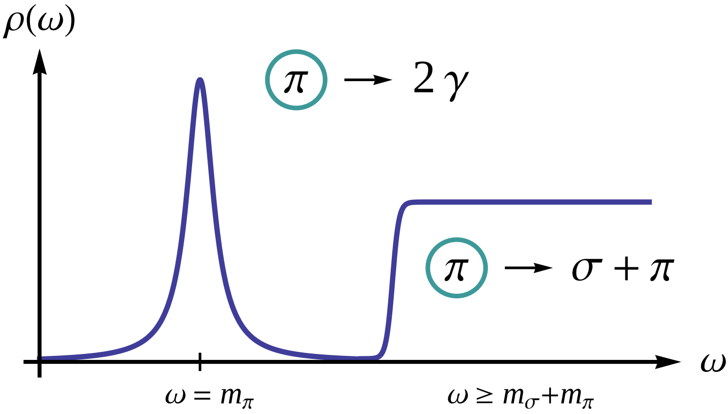


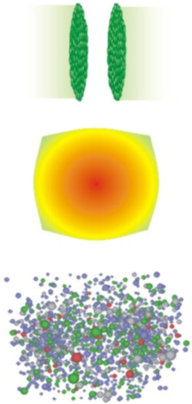
Why are spectral functions interesting?

Spectral functions determine both
real-time and imaginary-time propagators,

- $$D^R(\omega) = -\int d\omega' \frac{\rho(\omega')}{\omega' - \omega - i\varepsilon}$$
- $$D^A(\omega) = -\int d\omega' \frac{\rho(\omega')}{\omega' - \omega + i\varepsilon}$$
- $$D^E(p_0) = \int d\omega' \frac{\rho(\omega')}{\omega' + ip_0}$$

and thus allow access to many observables,
e.g. transport coefficients like the shear viscosity:

- $$\eta = \frac{1}{24} \lim_{\omega\to 0} \lim_{|\vec{p}|\to 0} \frac{1}{\omega} \int d^4x \; e^{ipx} \left\langle \left[T_{ij}(x), T^{ij}(0) \right] \right\rangle$$



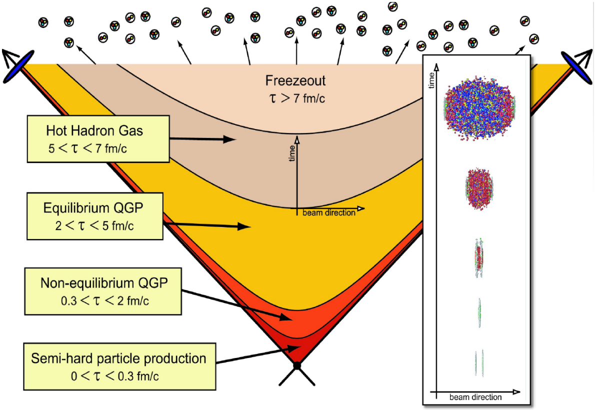
[B. Mueller, arXiv: 1309.7616]

Heavy-ion collisions and electromagnetic probes

compared to the fireball photons and dileptons have a long mean free path

$\rightarrow$ leave the interaction zone undisturbed

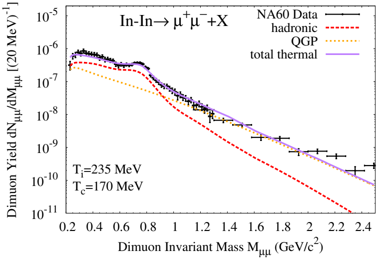
E. Feinberg 1976, E. Shuryak 1978



[M. Strickland, Acta Phys.Polon. B45 (2014) no.12, 2355-2394]

Dilepton rate and vector meson spectral functions

- Vector mesons (ρ, ω, ϕ...) can decay directly into lepton pairs
- Lifetime $\tau_\rho \approx 1.3$ fm/c, smaller than lifetime of fireball (≈ 10 fm/c)



[Rapp, van Hees, Phys.Lett. B **753** (2016) 586-590]

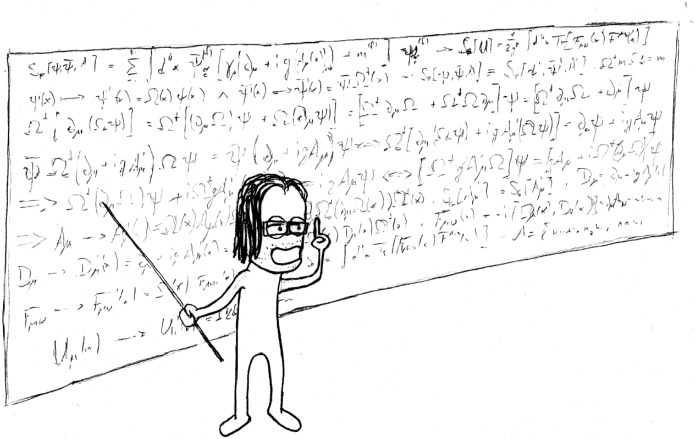
Dilepton rate:

$$\frac{dN_{ll}}{d^4x d^4q} \sim \mathrm{Im}\Pi^{\mu\nu}_{\mathrm{em}}(M, q; \mu, T)$$

For low-energy regime $M \leq 1$ GeV (VMD):

$$\mathrm{Im}\Pi^{\mu\nu}_{\mathrm{em}} \sim \mathrm{Im}D^{\mu\nu}_{\rho} + \frac{1}{9}\mathrm{Im}D^{\mu\nu}_{\omega} + \frac{2}{9}\mathrm{Im}D^{\mu\nu}_{\phi}$$

II) Theoretical setup



[courtesy L. Holicki]

Consistent theoretical framework

How are in-medium modifications of hadrons related to the change of the vacuum structure of QCD? (deconfinement and chiral symmetry restoration,...)

$\rightarrow$ want a theoretical framework for computing the thermodynamic and the spectral properties (analytic continuation) of QCD matter on the **same footing!**

Requirements:

- thermodynamic consistency
- preservation of symmetries and their breaking pattern

Candidates:

- mean-field theory
- Functional Renormalization Group (FRG)
- ...

FRG includes both **thermal** and **quantum** fluctuations and hence properly deals with phase transitions!

Functional Renormalization Group

Euclidean partition function for a scalar field:

$$Z[J] = \int \mathcal{D}\varphi \, \exp\left(-S[\varphi] + \int d^4x \, J(x)\varphi(x)\right)$$

Wilson's coarse-graining: split φ into low- and high-frequency modes

$$\varphi(x) = \varphi_{q\leq k}(x) + \varphi_{q>k}(x)$$

only include fluctuations with $q > k$

$$Z[J] = \int \mathcal{D}\varphi_{q\leq k} \underbrace{\int \mathcal{D}\varphi_{q>k} \, \exp\left(-S[\varphi] + \int d^4x J(x)\varphi(x)\right)}_{Z_k[J]}$$

Functional Renormalization Group

Scale-dependent partition function can be defined as

$$Z_k[J] = \int \mathcal{D}\varphi \, \exp\left(-S[\varphi] - \Delta S_k[\varphi] + \int d^4x J(x)\varphi(x) \right)$$

by introducing a regulator term that suppresses IR modes

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} \, \varphi(-q) R_k(q) \varphi(q)$$

Switch to scale-dependent effective action ($\phi(x) = \langle \varphi(x) \rangle$):

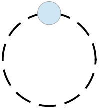
$$\Gamma_k[\phi] = \sup_J \left(\int d^4x \, J(x)\phi(x) - \log Z_k[J] \right) - \Delta S_k[\phi]$$

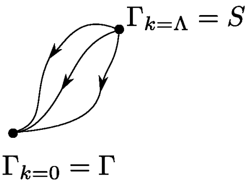
Functional Renormalization Group

Flow equation for the effective average action Γ_k:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left(\partial_k R_k \left[\Gamma_k^{(2)} + R_k \right]^{-1} \right)$$

[C. Wetterich, Phys. Lett. **B 301** (1993) 90]

$$\partial_k \Gamma_k \;=\; \frac{1}{2}$$ 



[wikipedia.org/wiki/Functional_renormalization_group]

- Γ_k interpolates between bare action S at $k = \Lambda$ and effective action Γ at $k = 0$
- regulator R_k acts as a mass term and suppresses fluctuations with momenta smaller than k
- the use of 3D regulators allows for a simple analytic continuation procedure

Gauged linear-sigma model with quarks

- $SU(2)_L \times SU(2)_R$: corresponds to chiral symmetry of two-flavor QCD
- Additional gauge symmetry $U(1)$ to include photon field

Ansatz for the effective average action $\Gamma_k \equiv \Gamma_k[\sigma, \pi, \rho, a_1, \psi, \bar{\psi}, A_\mu]$:

$$\Gamma_k = \int d^4x \Big\{ \bar{\psi} \left(\not{D} - \mu\gamma_0 + h_S \left(\sigma + \mathrm{i}\vec{\tau}\vec{\pi}\gamma_5 \right) + \mathrm{i}h_V \left(\gamma_\mu \vec{\tau}\vec{\rho}^\mu + \gamma_\mu\gamma_5\vec{\tau}\vec{a}_1^\mu \right) \right) \psi + U_k(\phi^2)$$

$$- c\sigma + \frac{1}{2} \left| (D_\mu - \mathrm{i}gV_\mu)\Phi \right|^2 + \frac{1}{8} \mathrm{Tr}\left(V_{\mu\nu}V^{\mu\nu} \right) + \frac{1}{4} m_{V,k}^2 \mathrm{Tr}\left(V_\mu V^\mu \right) \Big\}$$

with

$$V_{\mu\nu} = D_\mu V_\nu - D_\nu V_\mu - \mathrm{i}g \left[V_\mu, V_\nu \right], \quad D_\mu \psi = \left(\partial_\mu - \mathrm{i}eA_\mu Q \right) \psi,$$

$$D_\mu V_\mu = \partial_\mu V_\nu - ieA_\mu [T_3, V_\nu], \quad \phi \equiv (\vec{\pi}, \sigma), \quad V_\mu \equiv \vec{\rho}_\mu \vec{T} + \vec{a}_{1,\mu} \vec{T}^5$$

Flow of the effective potential at $\mu = 0$ and $T = 0$

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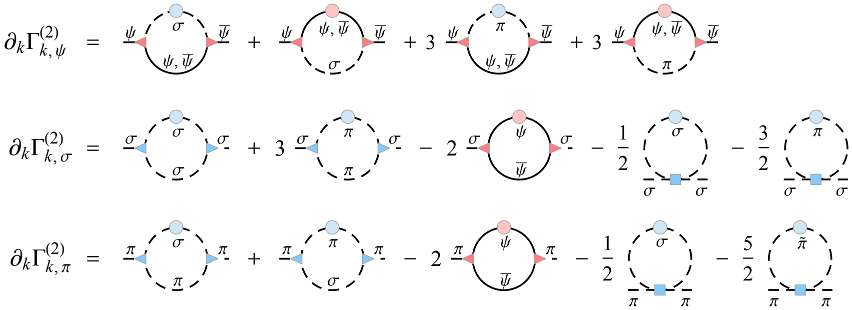
Flow equations for two-point functions

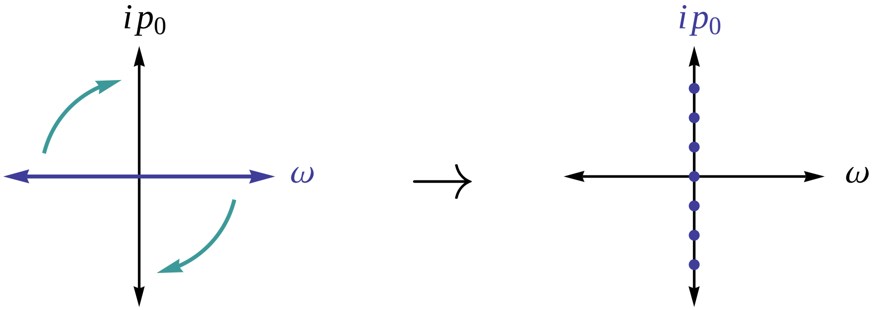


- quark-meson vertices are given by $\Gamma^{(3)}_{\bar\psi\psi\sigma} = h,\ \Gamma^{(3)}_{\bar\psi\psi\vec\pi} = ih\gamma^5\vec\tau$
- mesonic vertices from scale-dependent effective potential: $U^{(3)}_{k,\phi_i\phi_j\phi_m}$, $U^{(4)}_{k,\phi_i\phi_j\phi_m\phi_n}$
- one-loop structure and 3D regulators allow for a simple analytic continuation!

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

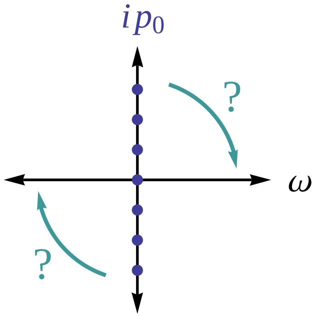
The analytic continuation problem

Calculations at finite temperature are often performed using imaginary energies:



The analytic continuation problem

Analytic continuation problem: How to get back to real energies?



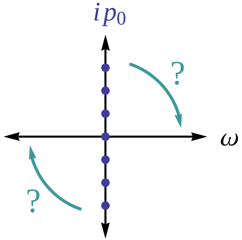
Two-step analytic continuation procedure

1) Use periodicity w.r.t. imaginary energy $ip_0 = i2n\pi T$:

$$n_{B,F}(E + ip_0) \to n_{B,F}(E)$$

2) Substitute p_0 by continuous real frequency ω:

$$\Gamma^{(2),R}(\omega, \vec{p}) = -\lim_{\epsilon \to 0} \Gamma^{(2),E}(ip_0 \to -\omega - i\epsilon, \vec{p})$$



Spectral function is then given by

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \text{Im} \frac{1}{\Gamma^{(2),R}(\omega, \vec{p})}$$

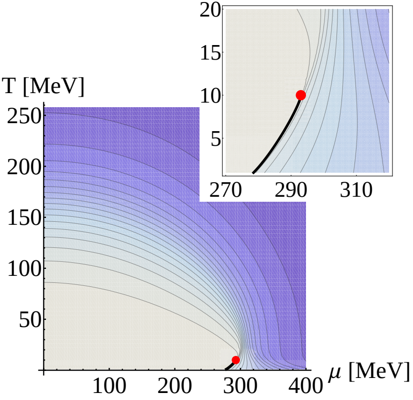
[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]
[J. M. Pawlowski, N. Strodthoff, Phys. Rev. **D 92**, 094009 (2015)]
[N. Landsman and C. v. Weert, Physics Reports 145, 3&4 (1987) 141]

III) Results

[courtesy L. Holicki]

Phase diagram of the quark-meson model

- chiral order parameter σ_0 decreases towards higher T and μ
- a crossover is observed at $T \approx 175$ MeV and $\mu = 0$
- critical endpoint (CEP) at $\mu \approx 292$ MeV and $T \approx 10$ MeV
- we will study spectral functions along $\mu = 0$ and $T \approx 10$ MeV



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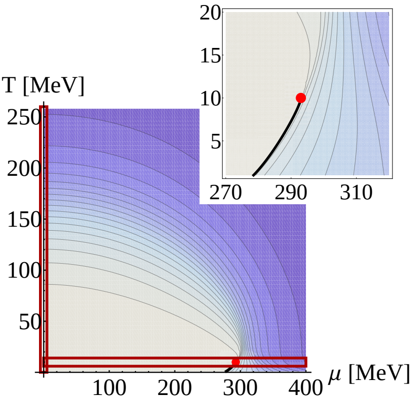

[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]

Flow of quark spectral function at $\mu = T = 0$

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Quark spectral function for $T > 0$

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Flow of σ and π spectral function at $\mu = T = 0$

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σ and π spectral function for $T > 0$ at $\mu = 0$

(Loading movie...)

σ and π spectral function for $\mu > 0$ at T_c

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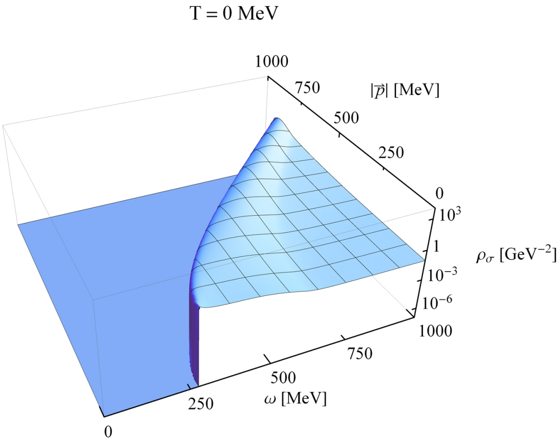
σ spectral function vs. ω and $\vec{p}$ at $\mu = T = 0$

- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes



σ spectral function vs. ω and $\vec{p}$ for $T > 0$, $\mu = 0$

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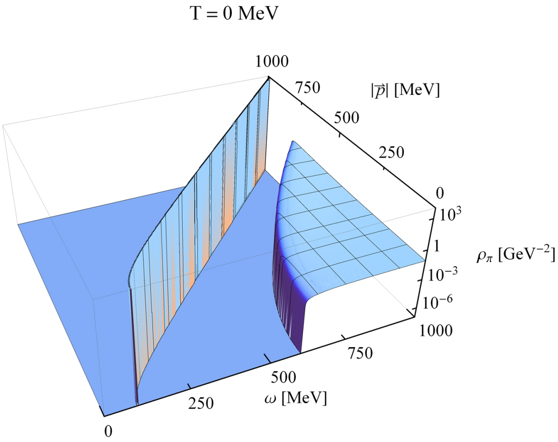
π spectral function vs. ω and $\vec{p}$ at $\mu = T = 0$

- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- capture process $\pi^* + \pi \to \sigma$ is suppressed at large $\vec{p}$
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes



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Shear viscosity

Applying the Green-Kubo formula for the shear viscosity

$$\eta = \frac{1}{24} \lim_{\omega \to 0} \lim_{|\vec{p}| \to 0} \frac{1}{\omega} \int d^4x \ e^{ipx} \left\langle \left[T_{ij}(x), T^{ij}(0) \right] \right\rangle$$

to the quark-meson model with energy-momentum tensor

$$T^{ij}(x) = \frac{i}{2} \left(\overline{\psi} \gamma^i \partial^j \psi - \partial^j \, \overline{\psi} \gamma^i \psi \right) + \partial^j \sigma \partial^i \sigma + \partial^j \vec{\pi} \partial^i \vec{\pi}$$

gives

$$\eta_{\sigma,\pi} \propto \int \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} \ p_x^2 \, p_y^2 \, n_B'(\omega) \, \rho_{\sigma,\pi}^2(\omega, \vec{p})$$

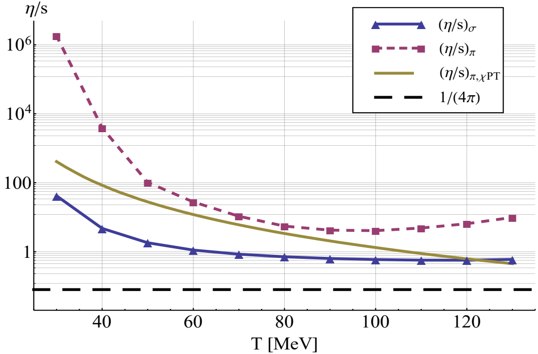
[R.-A. Tripolt, L. von Smekal, and J. Wambach, Int.J.Mod.Phys. E26 (2017) no.01n02, 1740028]

Shear viscosity over entropy density η/s at $\mu = 0$

- $\eta_{\pi,\chi\text{PT}}$: result from chiral perturbation theory

[Lang, Kaiser, Weise, EPJ **A 48**, 109 (2012)]

- large shear viscosity at low temperatures due to small width of pion peak $\rightarrow 4\pi$ processes missing
- η/s is always larger than the AdS/CFT limiting value of $\eta/s \geq 1/4\pi$



[R.-A. Tripolt, L. von Smekal, and J. Wambach, Int.J.Mod.Phys. E26 (2017) no.01n02, 1740028]

Electrical conductivity

Applying the Green-Kubo formula for the electrical conductivity

$$\sigma_{el} = \frac{1}{6} \lim_{\omega \to 0} \lim_{|\vec{p}| \to 0} \frac{1}{\omega} \int d^4x \;\, e^{ipx} \left\langle \Big[J_i(x), J^i(0) \Big] \right\rangle$$

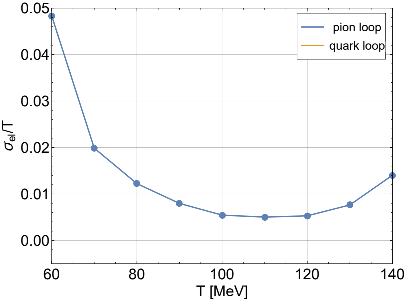
to the quark-meson model with the EM current

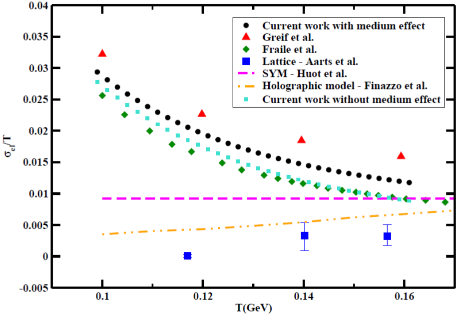
$$J^\mu(x) = i \frac{\partial}{\partial A_\mu(x)} \Gamma = e \bar{\psi} \gamma^\mu Q \psi + ie \frac{m_\rho^2}{m_{a_1}^2} (\pi^1 \partial^\mu \pi^2 - \pi^2 \partial^\mu \pi^1) + ie^2 A^\mu [(\pi^1)^2 + (\pi^2)^2]$$

gives

$$\sigma_{el,\pi} \propto \int \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} \;\, \vec{p}^{\,2} \; n_B'(\omega) \; \rho_\pi^2(\omega, \vec{p})$$

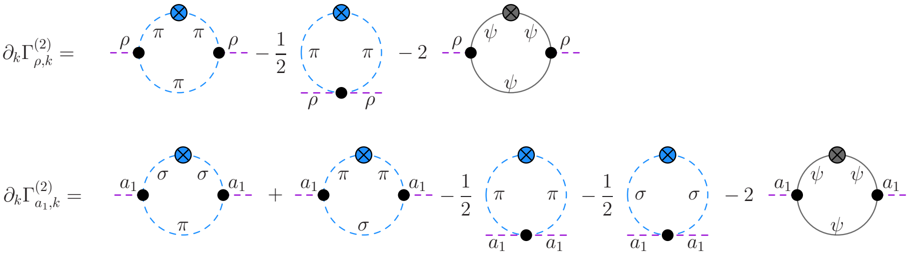
Electrical conductivity vs. T at $\mu = 0$ - Preliminary





[S. Ghosh, S. Mitra, and S. Sarkar, Nucl. Phys. A 969, 237 (2018)]

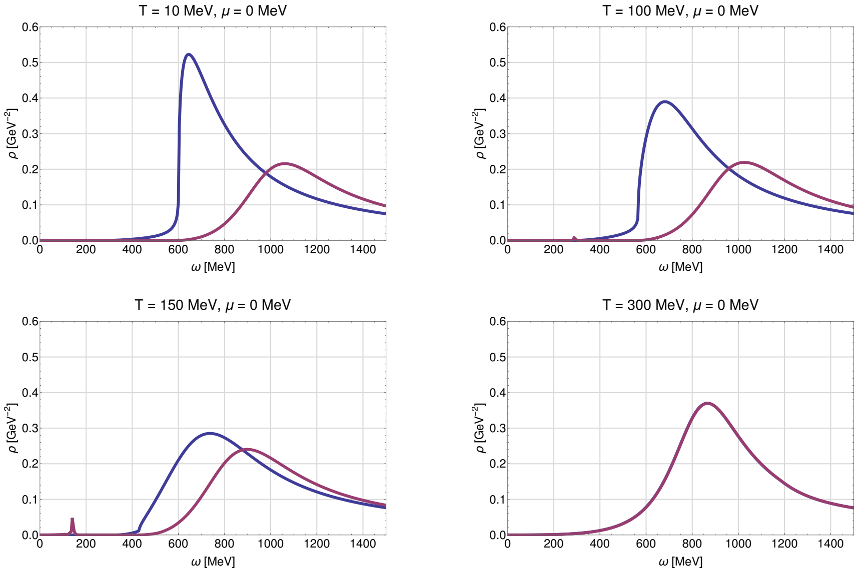
Flow equations for ρ and a_1 2-point functions



- neglect vector mesons inside the loops
- vertices extracted from ansatz for the effective average action Γ_k
- tadpole diagrams give ω-independent contributions

[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

T-dependence of ρ and a_1 spectral functions



[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

T-dependence of ρ and a_1 spectral functions

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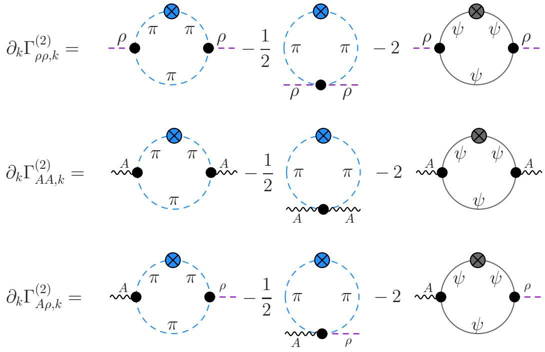
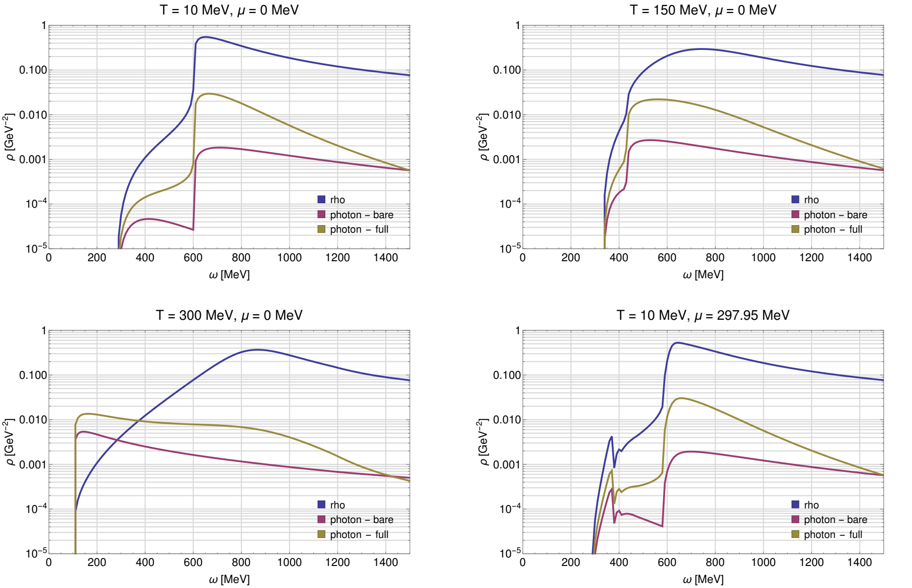
Electromagnetic (EM) spectral function

$$\begin{pmatrix} \Gamma^{(2)}_{AA} & \Gamma^{(2)}_{A\rho} \\ \Gamma^{(2)}_{\rho A} & \Gamma^{(2)}_{\rho\rho} \end{pmatrix} \xrightarrow{\text{diagonalize}} \begin{pmatrix} \tilde{\Gamma}^{(2)}_{AA} & 0 \\ 0 & \tilde{\Gamma}^{(2)}_{\rho\rho} \end{pmatrix}, \quad \tilde{\Gamma}^{(2)}_{AA} = \overbrace{\Gamma^{(2)}_{AA} - \frac{\Gamma^{(2)}_{A\rho}\Gamma^{(2)}_{\rho A}}{\Gamma^{(2)}_{\rho\rho}}}^{\mathcal{O}(e^2)} + \mathcal{O}(e^4)$$



[R.-A. T., C. Jung, N. Tanji, L. von Smekal, and J. Wambach, arXiv:1807.04952]

EM spectral function - preliminary



[R.-A. T., C. Jung, N. Tanji, L. von Smekal, and J. Wambach, arXiv:1807.04952]

Calculation of dilepton rates

- We use the Weldon formula for the thermal dilepton rate:

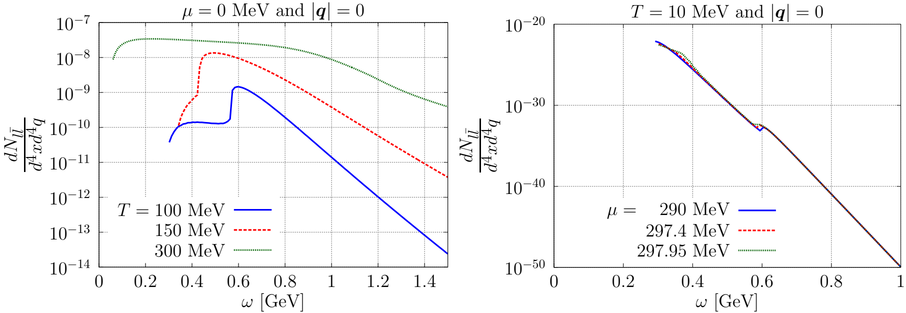
$$\frac{d^8 N_{l\bar{l}}}{d^4x d^4q} = \frac{\alpha}{12\pi^3}\left(1+\frac{2m^2}{q^2}\right)\left(1-\frac{4m^2}{q^2}\right)^{1/2} q^2(2\rho_T+\rho_L)n_B(q_0)$$

- in the following we assume $m = 0$ and set the external spatial momentum to zero, such that $\rho_T = \rho_L = \rho_{\tilde{A}\tilde{A}}$

[H. A. Weldon, Phys. Rev. D42, 2384 (1990)]

[R.-A. T., C. Jung, N. Tanji, L. von Smekal, and J. Wambach, arXiv:1807.04952]

Dilepton rates - preliminary



- clear changes are visible with increasing temperature
- no distinct signatures for the critical endpoint yet $\rightarrow$ improve truncation

[R.-A. T., C. Jung, N. Tanji, L. von Smekal, and J. Wambach, arXiv:1807.04952]

Summary and Outlook

- analytically continued flow equations for quark and (vector-)meson spectral functions using effective models for QCD within a consistent FRG framework
- degeneracy of ‘parity partners’ due to restoration of broken chiral symmetry in the QCD medium

Outlook:

- quark spectral function at finite density and temperature
- improve truncation (include baryons and more decay channels) to calculate realistic dilepton rates and identify signatures of phase transitions
- transport coefficients like the shear viscosity and the electrical conductivity