Cluster Structure of Nuclei under Constraints on the Quadrupole Moments

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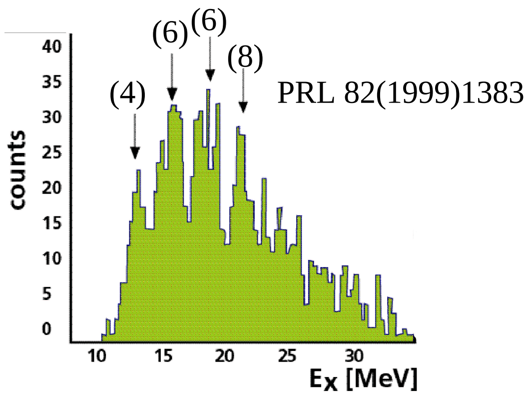
- **1. Motivation**
  - ✘ Cluster Structure of Nuclei
    - Reaction Paths
  - ✘ Nuclear Structure with FMD
    - Mean-field and multiconfiguration mixing
- **2. Constraining Quadrupole Moments**
  - ✘ The quadrupole mass tensor $\mathcal{Q}$
  - ✘ Rotationally invariant constraints
- **3. Results**
  - ✘ Break-up of light nuclei
  - ✘ Fission valleys for clusters of α's
- **4. Summary & Outlook**

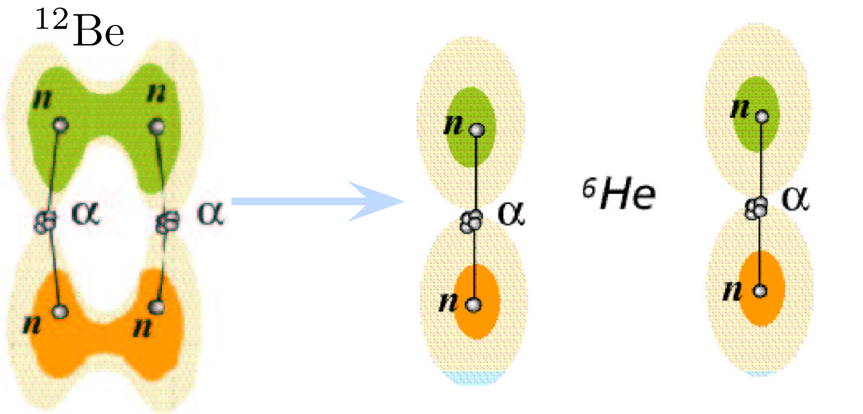
GSI

Cluster Structure of Nuclei

Experimental observation

- Collision experiments show that nuclei can have cluster structure

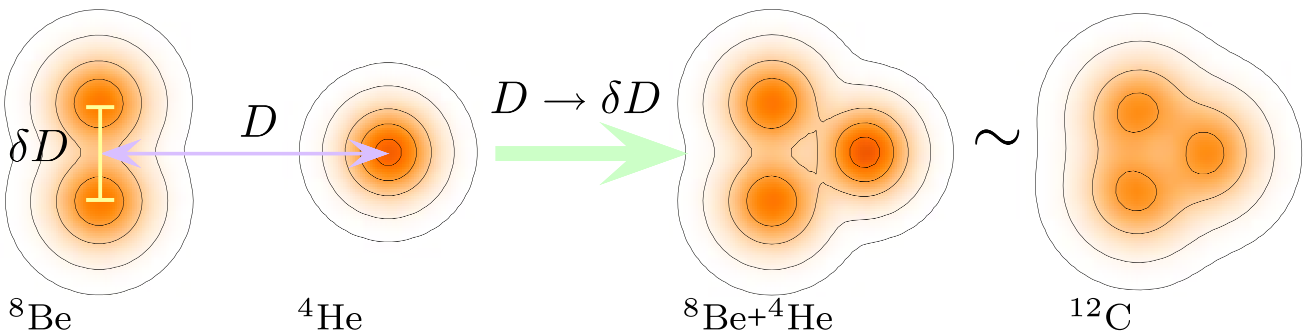


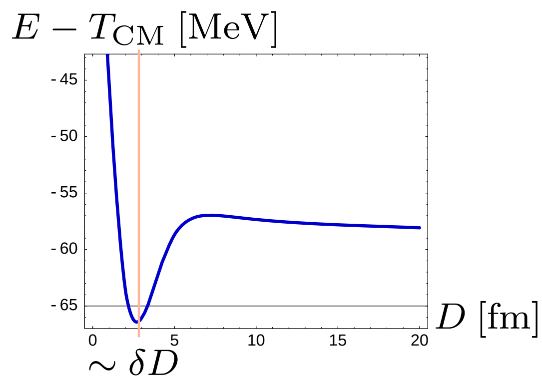


Excitation energy spectrum from decay of ^{12}Be in two ^{6}He. Molecular nature illustrated on the right. GANIL magazine, July 2001.

Description in FMD

- In Fermionic Molecular Dynamics (FMD) no cluster substructures are assumed *a priori*.





Nuclear Structure with FMD

$|\,Q\,\rangle$: *FMD Representation*

- Many-body state

$$|\,Q\,\rangle = \mathcal{A}(|\,q_1\,\rangle \otimes |\,q_2\,\rangle \otimes \ldots \otimes |\,q_A\,\rangle)$$

- Single-particle state:

$$|\,q_k\,\rangle = \sum_{i=1}^{2} |\,a_k^{(i)}, \vec{b}_k^{(i)}\,\rangle \otimes |\,\chi_k\,\rangle \otimes |\,\xi_k\,\rangle,$$

  - Coordinate space

$$\langle\,\vec{x}\,|\,a_k^{(i)}, \vec{b}_k^{(i)}\,\rangle = \exp\left\{-\frac{(\vec{x}-\vec{b}_k^{(i)})^2}{2a_k^{(i)}}\right\}$$

  - Spin space

$$|\,\chi_k\,\rangle = c_\uparrow|\,\uparrow\,\rangle + c_\downarrow|\,\downarrow\,\rangle$$

  - Isospin space

$$|\,\xi_k\,\rangle \in \{|\,p\,\rangle, |\,n\,\rangle\}$$

$\underset{\sim}{\hat{H}}$: *Correlated interaction*

- Bonn A Potential
  - ✘ Realistic NN interaction:
    - Adjusted to NN scattering and deuteron properties
  - ✘ Based on meson exchange
- Unitary Correlation Operator $\underset{\sim}{C}$
  - ✘ Slater determinants are not able to describe short-range correlations
  - ✘ UCOM implements short-range radial and tensor correlations: $|\,\hat{Q}\,\rangle = \underset{\sim}{C}|\,Q\,\rangle$
  - ✘ Correlated Hamiltonian

$$\begin{aligned}\underset{\sim}{\hat{H}} = \underset{\sim}{C}^\dagger \underset{\sim}{H}\underset{\sim}{C} &= \underset{\sim}{\hat{H}}^{[1]} + \underset{\sim}{\hat{H}}^{[2]} + \cdots \\ &= \underset{\sim}{T} + \underset{\sim}{\hat{H}}^{[2]} + \cdots\end{aligned}$$

  - ✘ Effective Hamiltonian

$$\underset{\sim}{\hat{H}}_{\text{eff}} = \underset{\sim}{T} + \underset{\sim}{\hat{H}}^{[2]} + \underset{\sim}{\hat{H}}_{\text{corr}}$$

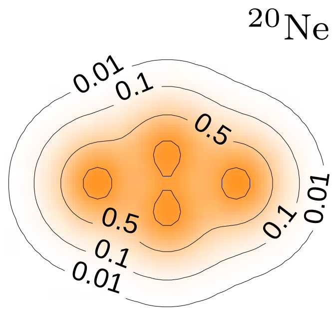
Nuclear Structure with FMD

Mean-field solution

- Find $|\,Q\,\rangle$ such that

$$\delta \frac{\langle\, Q \,|\, \underset{\sim}{\hat{H}}_{\text{eff}} - \underset{\sim}{T}_{\text{CM}} \,|\, Q \,\rangle}{\langle\, Q \,|\, Q \,\rangle} = \delta \langle\, \underset{\sim}{\hat{H}}_{\text{eff}} - \underset{\sim}{T}_{\text{CM}} \,\rangle = 0$$

- In general, it results in an intrinsically deformed state $|\,Q\,\rangle$
  - ✘ Not eigenstate of Angular Momentum



Multiconfiguration Mixing

- Diagonalize $\underset{\sim}{\hat{H}}_{\text{eff}} - \underset{\sim}{T}_{\text{CM}}$ in a set of rotated states $|\,Q_i\,\rangle$.

$$\sum_j \langle\, Q^{(i)} \,|\, \underset{\sim}{\hat{H}}_{\text{eff}} \,|\, Q^{(j)} \,\rangle c_j^\alpha = E^\alpha \sum_j \langle\, Q^{(i)} \,|\, Q^{(j)} \,\rangle c_j^\alpha$$

  - ✘ Equivalent to angular momentum projection
- Also consider vibrational degrees of freedom
  - ✘ Monopole constraints
  - ✘ Quadrupole constraints
  - ✘ Octupole constraints
  - ✘ ...

✔ Improves ground state
✔ Yields excited levels
- Rotational
- Vibrational

Constraining Quadrupole Moments

The quadrupole mass tensor $\mathcal{Q}$

- Mass quadrupole operator

$$\underset{\sim}{Q} = \{\underset{\sim}{Q}_{ij}\}, \quad i,j = 1,2,3$$

$$\underset{\sim}{Q}_{ij} = \sum_{k=1}^{A} \Big(3\big(\underset{\sim}{x}_i(k) - \underset{\sim}{X}_i\big)\big(\underset{\sim}{x}_j(k) - \underset{\sim}{X}_j\big) - \delta_{ij}\big(\underset{\sim}{\vec{x}}(k) - \underset{\sim}{\vec{X}}\big)^2 \Big),$$

with $\underset{\sim}{\vec{X}} = \frac{1}{A}\sum_k^A \underset{\sim}{\vec{x}}(k)$

- Expectation values

$$\mathcal{Q} = \{\mathcal{Q}_{ij}\}$$

$$\mathcal{Q}_{ij} = \frac{\langle\, Q \,|\, \underset{\sim}{Q}_{ij} \,|\, Q \,\rangle}{\langle\, Q \,|\, Q \,\rangle}$$

Rotationally invariant constraints

- Use properties of $\mathcal{Q}$
  - ✘ Symmetric
  - ✘ Traceless

  → 2 independent parameters: Tr $\mathcal{Q}^2$, Det Q

- Minimize with constraints

$$\langle\, \underset{\sim}{\hat{H}}_{\text{eff}} - \underset{\sim}{T}_{\text{CM}} \,\rangle + \kappa_1\big(\text{Det}\ \mathcal{Q} - D_1\big)^2 + \kappa_2\big(\text{Tr}\ \mathcal{Q}^2 - D_2\big)^2$$

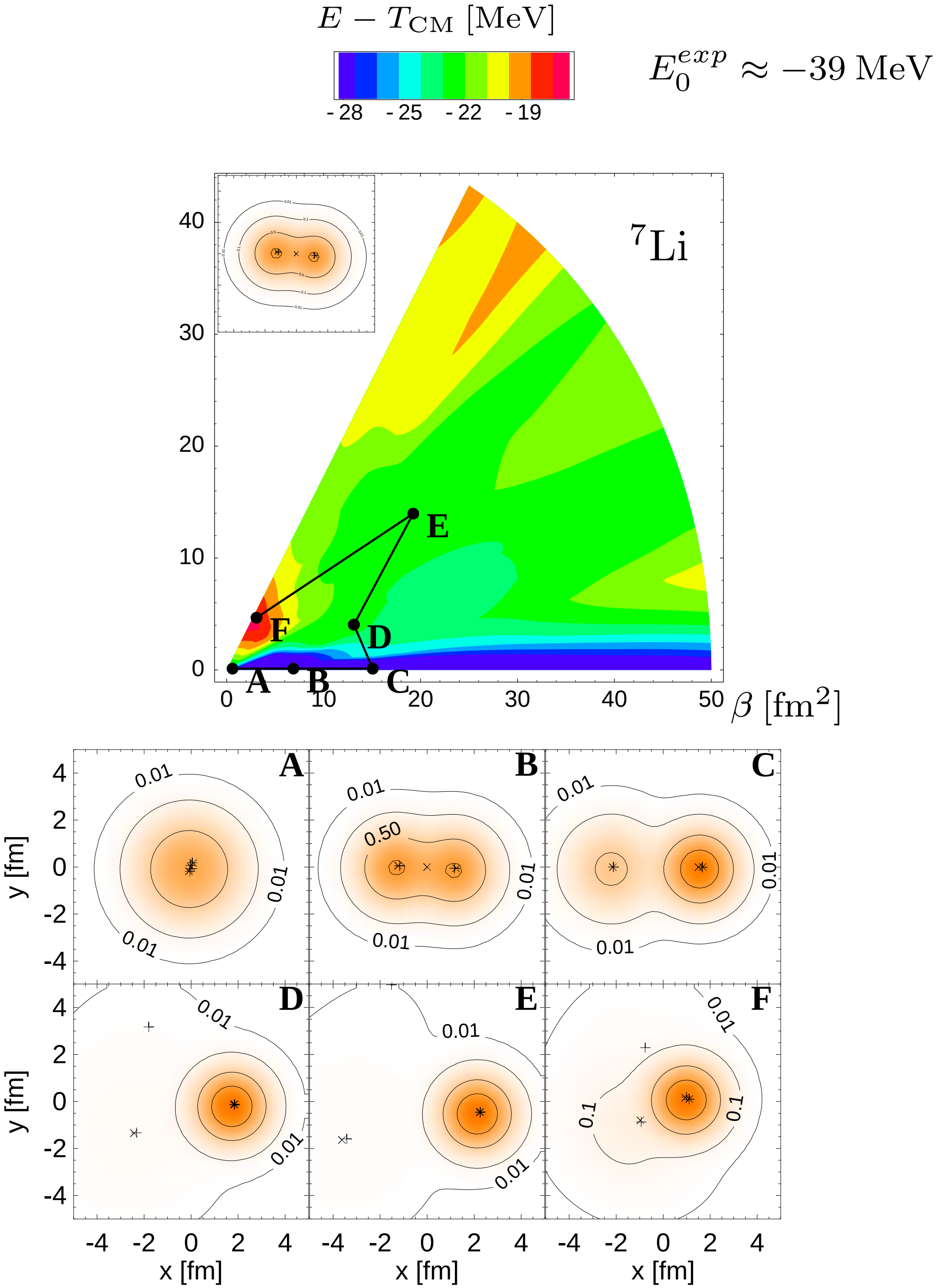
- Customary parametrization

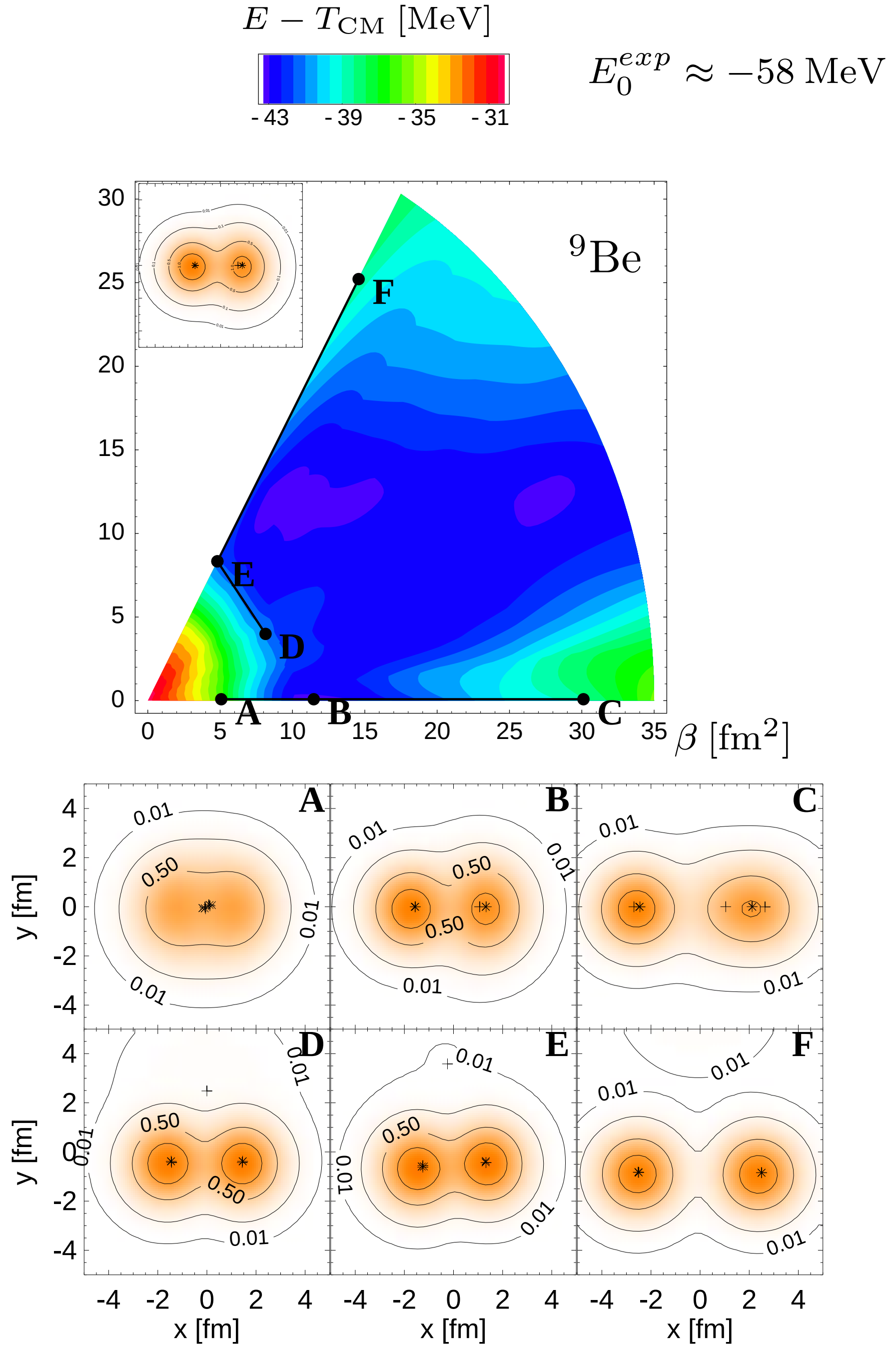
$$\gamma = \gamma(\text{Tr}\ \mathcal{Q}^2, \text{Det}\ \mathcal{Q})$$

$$\beta = \beta(\text{Tr}\ \mathcal{Q}^2)$$

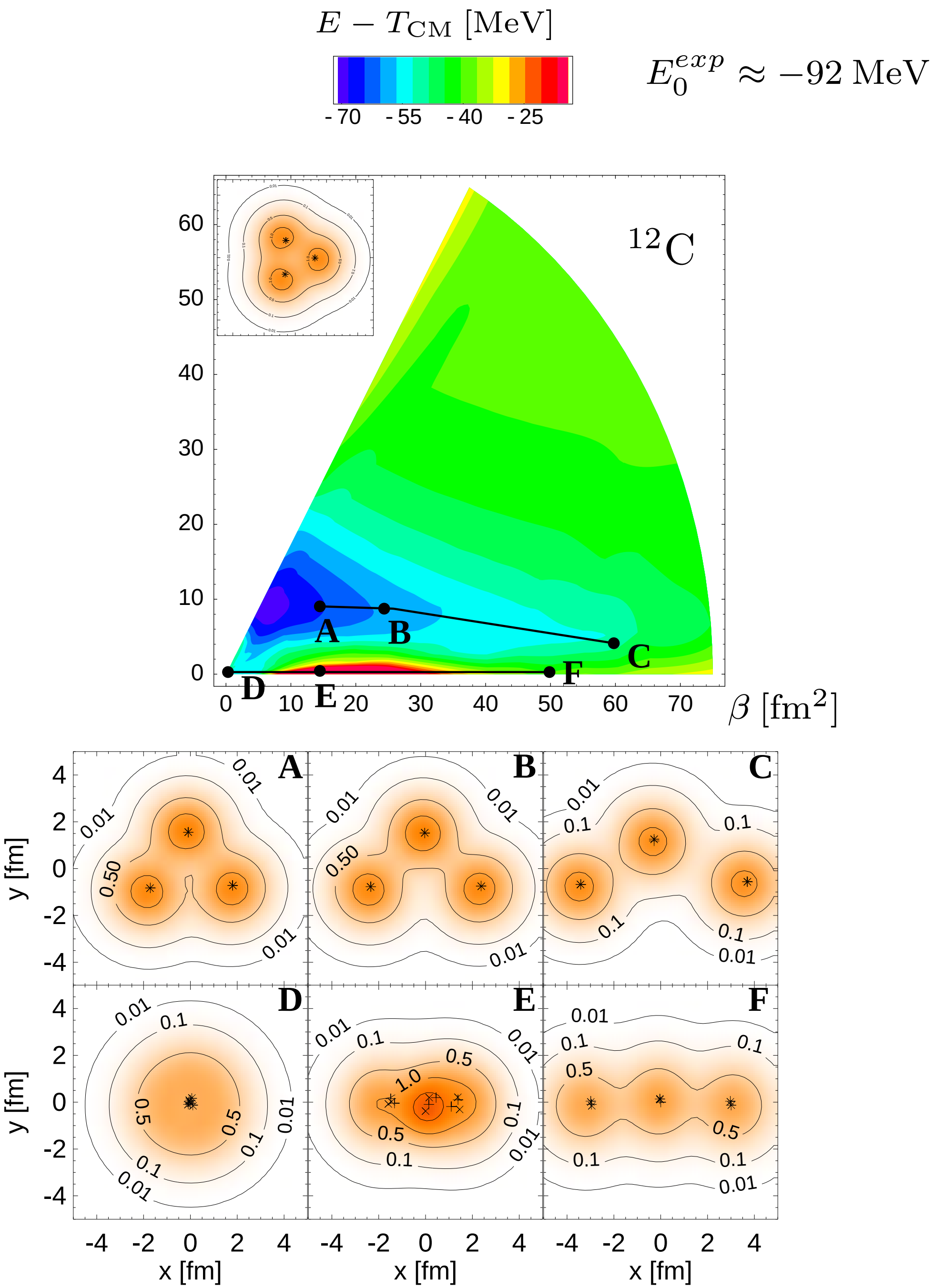
  - ✘ $\beta = 0\ \text{fm}^2$ round
  - ✘ $\beta \neq 0\ \text{fm}^2$
    - $\gamma = 0^\circ$ prolate
    - $\gamma = 60^\circ$ oblate
    - $\gamma \neq 0, 60^\circ$ triaxial

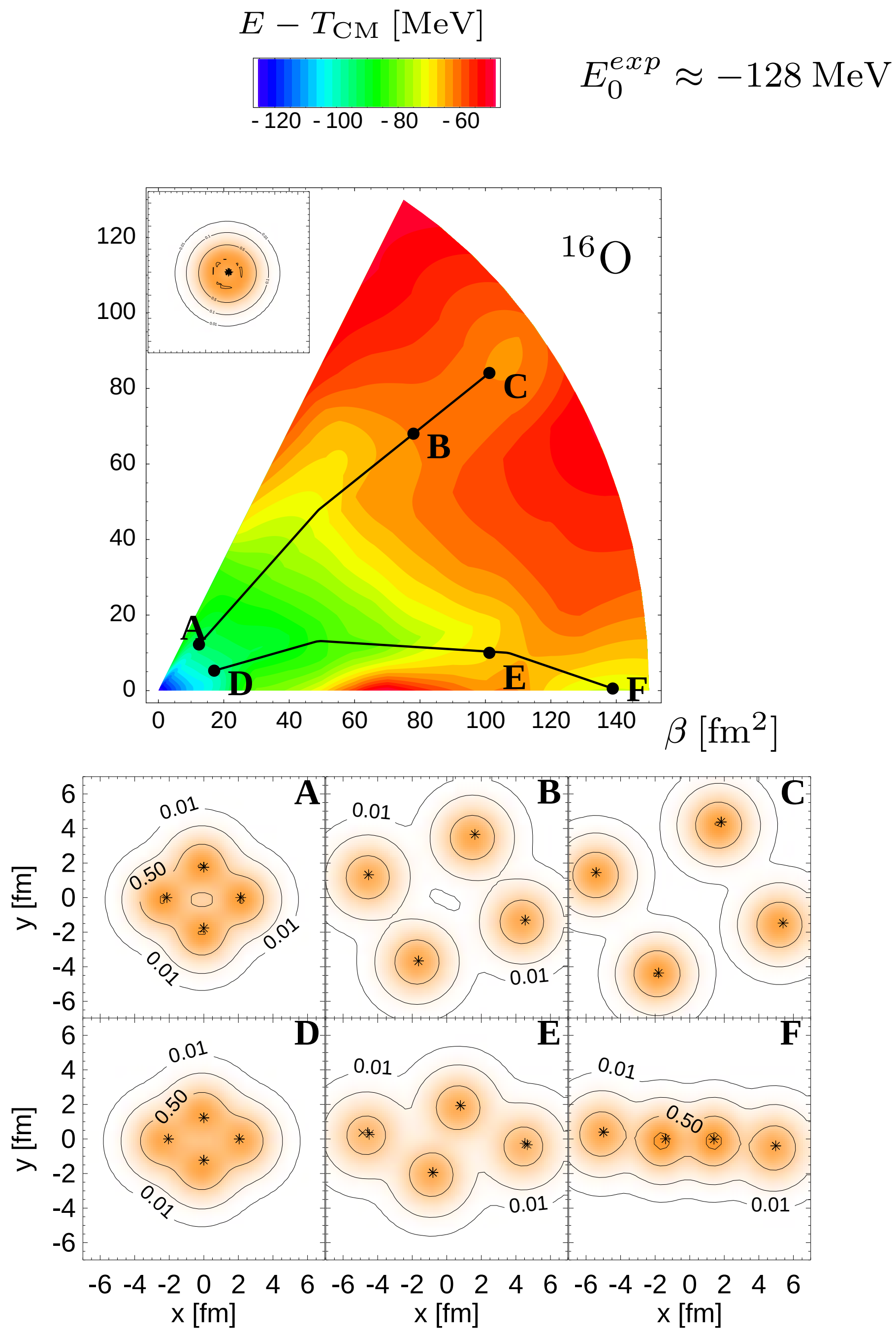
Break-up of light nuclei





Fission valleys for clusters of α's





Summary & Outlook

Summary

- Appearance of nucleon clusters without ab initio assumptions, specially α-clusters
- Energy landscapes show break-up valleys
  - ✘ Found possible path for the reaction $^{8}\mathrm{Be}(\alpha,\gamma)^{12}\mathrm{C}$
- Large β deformations at $\gamma = 0^{\circ}$ for $^{12}\mathrm{C}$ and $^{16}\mathrm{O}$ yield α-chains.
- Valence nucleons in $^{7}\mathrm{Li}$ and $^{9}\mathrm{Be}$ loosely bound

Outlook

- Multiconfiguration mixing calculations in FMD: nuclear structure
- Cross section calculations along the constrained paths
  - ✘ Reactions $^{8}\mathrm{Be}(\alpha,\gamma)^{12}\mathrm{C}$ and $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$
    → Important for stellar evolution
- Higher order constraints: octupole deformations

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