Phase transitions in cold dense QCD matter

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agenda

- **Hybrid and quark star matter based on a nonperturbative equation of state**

  Konstantin Otto (Giessen U.), Micaela Oertel (LUTH, Meudon), Bernd-Jochen Schaefer (Giessen U.)

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- **Nonperturbative quark matter equations of state with vector interactions**

  Konstantin Otto (Giessen U.), Micaela Oertel (LUTH, Meudon), Bernd-Jochen Schaefer (Giessen U.)

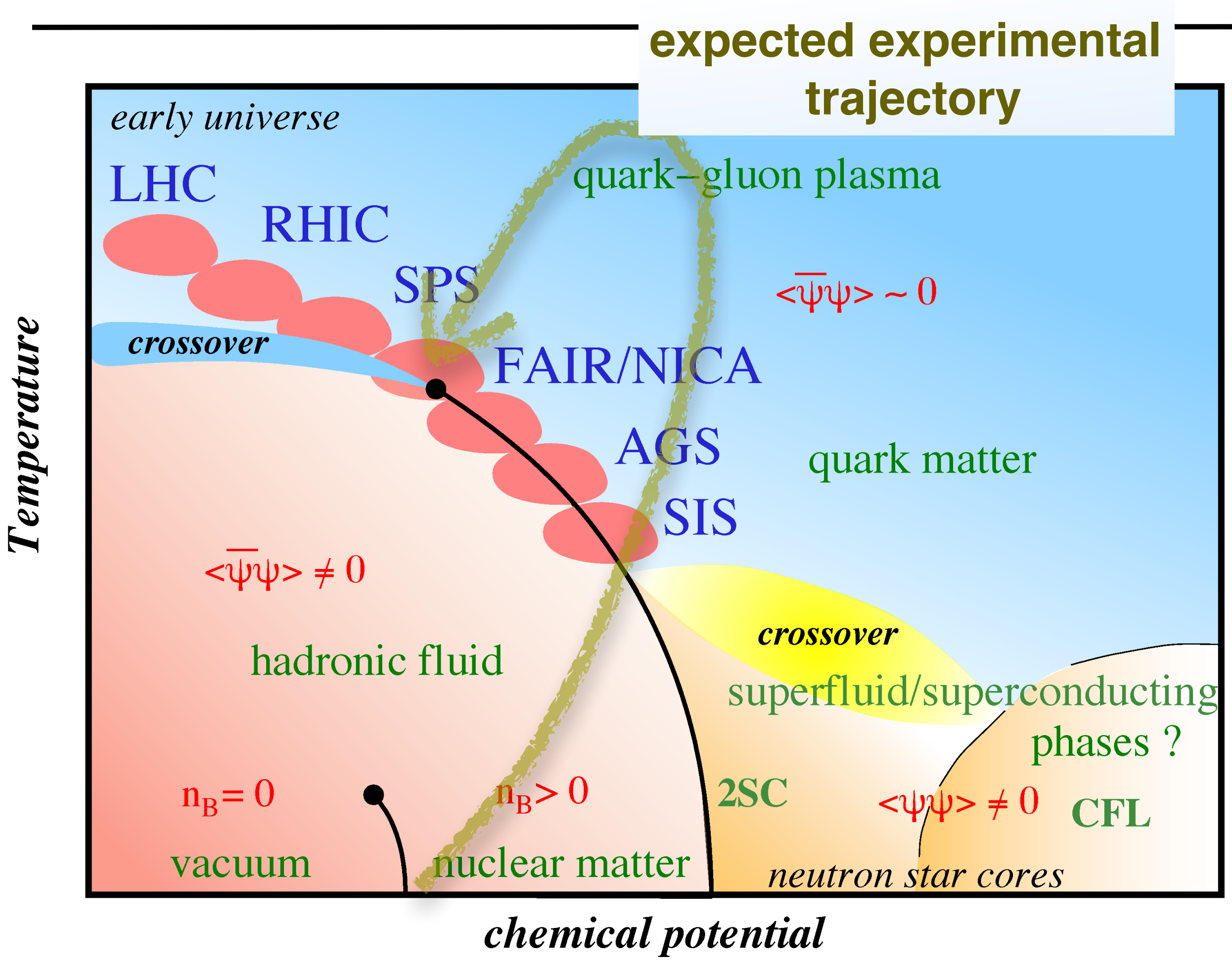
  Published in: *Eur.Phys.J.ST* 229 (2020) 22-23, 3629-3649 • e-Print: 2007.07394 [hep-ph]

- **Regulator scheme dependence of the chiral phase transition at high densities**

  Konstantin Otto (Giessen U.), Christopher Busch (Giessen U.), Bernd-Jochen Schaefer (Giessen U.)

  e-Print: 2206.13067 [hep-ph]

conjectured QC$_3$D phase structure

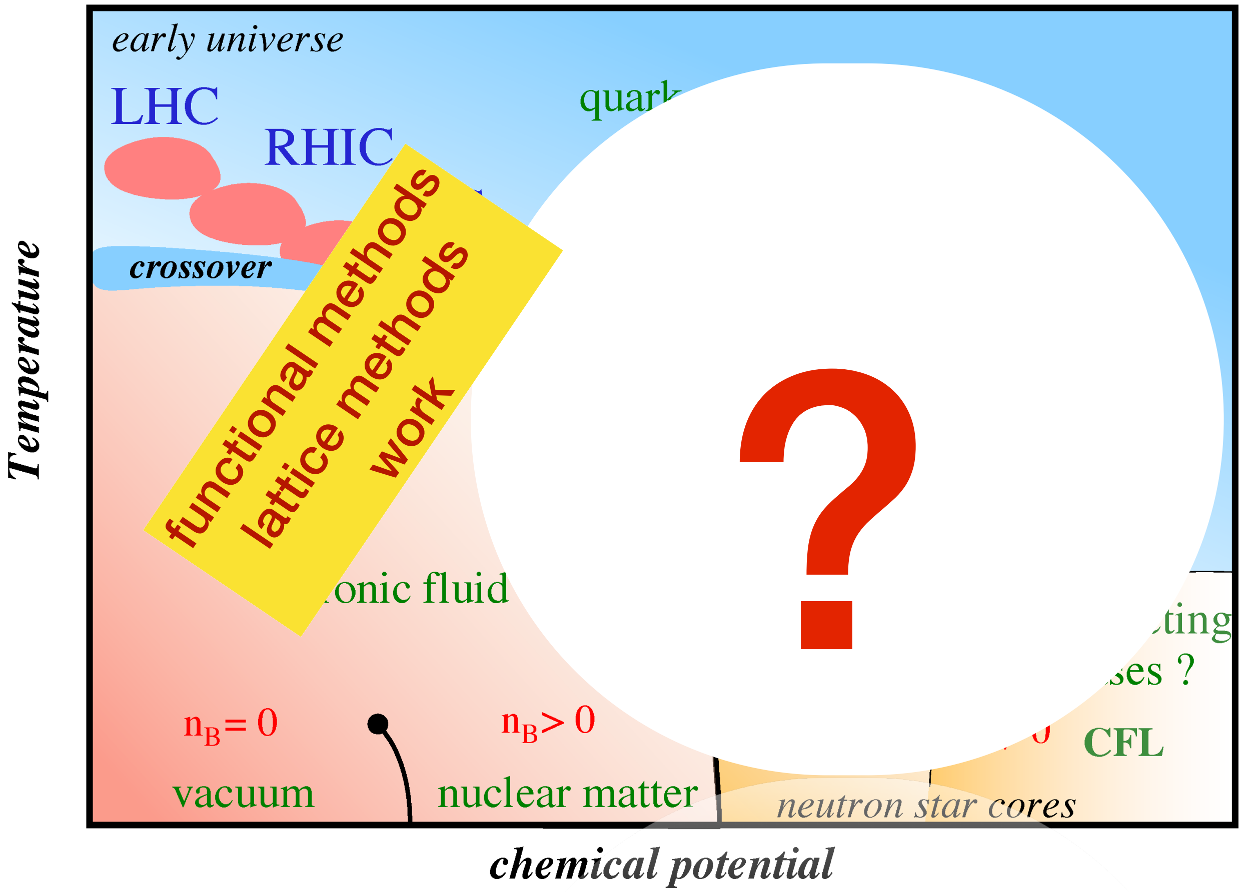


Open issues

- **Critical endpoint (CEP)?** chiral ⇔ deconfinement?
- **CS symmetry / Quarkyonic phase/s?**
- **inhomogeneous phase/s?**
- **axial anomaly restoration?**
- **finite volume effects?**
- **role of fluctuations?**
- **experimental signatures?**
- ....

assumptions: equilibrium, homogeneous phases, infinite volume, ...

conjectured QC₃D phase structure



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- ….

basically only corners known from first principle QCD

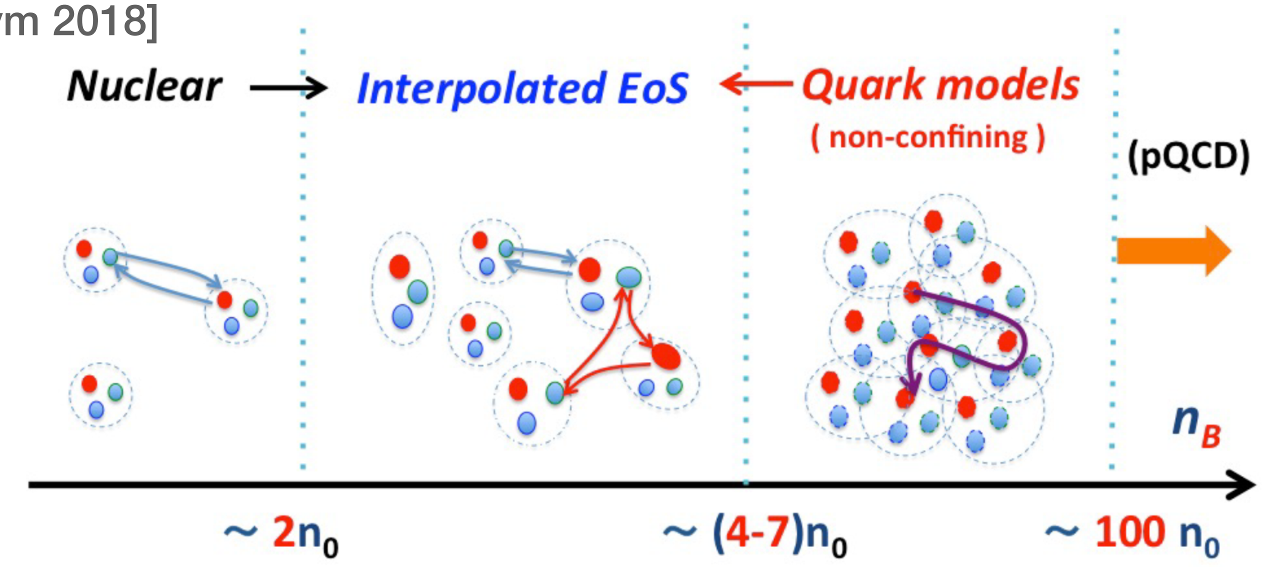
alternative to HIC to probe cold dense QCD matter ➜ massive neutron star (e.g. J0348+0432 ,…)

cold dense QCD matter: only effective low-energy realisation of QCD: e.g. (P)QM models

deconfined quarks very likely not a realistic description of neutron stars

Equation of State (EoS) for dense matter

[Baym 2018]



Nuclear phase:
1-2 meson/quark exchanges

restrictions on EoS @low densities from nuclear physics

interpolated EoS
many meson/quark exchanges

system gradually changes from hadronic to quark matter
- diquarks, colored quarks virtually …

- role of strangeness / hyperons

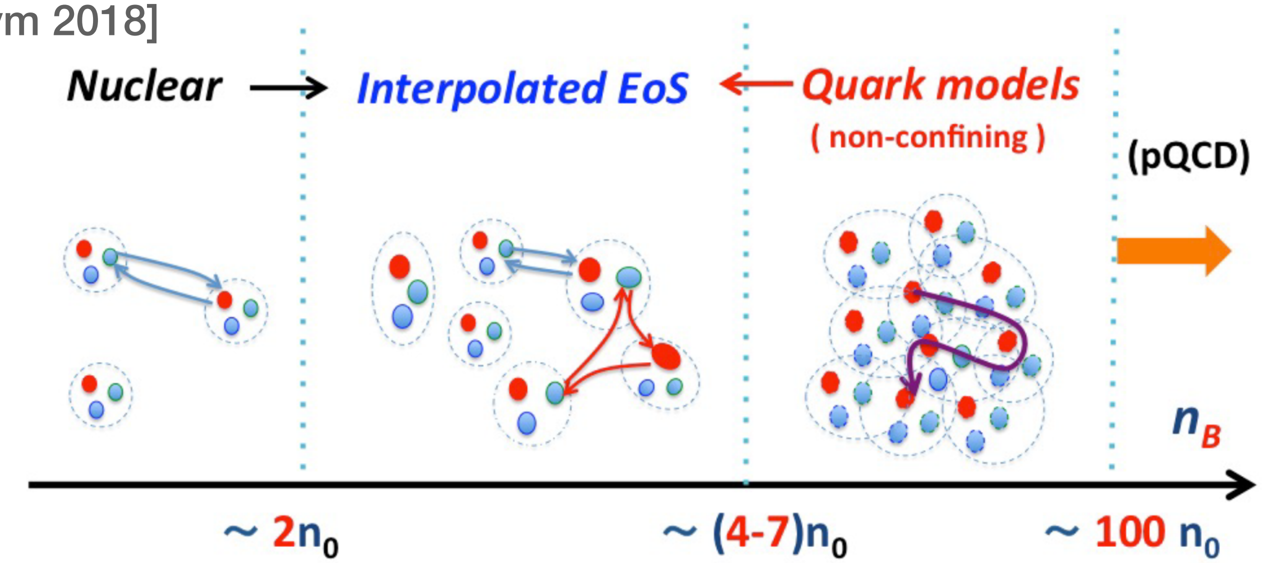
Quark phase:
quarks no longer specific to baryons

mostly mean-field investigations like NJL-type or phenomenological models

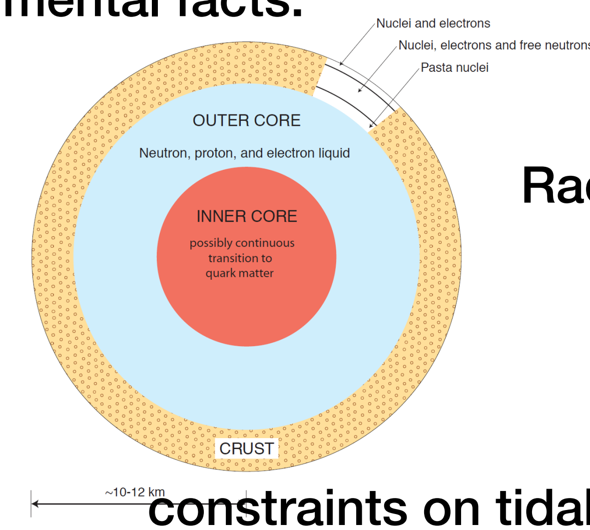
[Hebeler, Lattimer, Pethick,Schwenk et al. 2010]
[Schaffner-Bielich et al. 2008]
[Blaschke, Fischer, Oertel et al. 2018]

Transition from hadronic to quark matter

[Baym 2018]



Experimental facts:



Radius ~ 10 - 13 km

Mass > 2 Msol

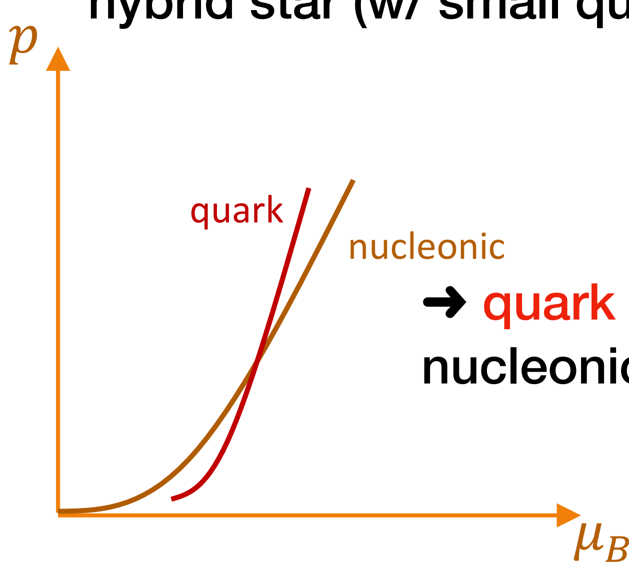
constraints on tidal deformability

combination: Maxwell construction or continuous interpolation

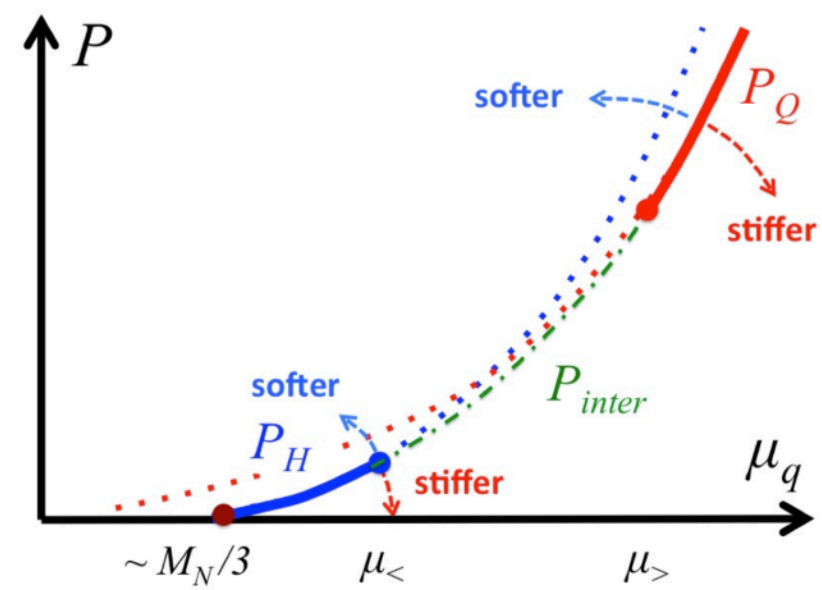
first-order transition

quark-hadron continuity

hybrid star (w/ small quark core)



➜ quark EoS softer than nucleonic EoS @ higher densities



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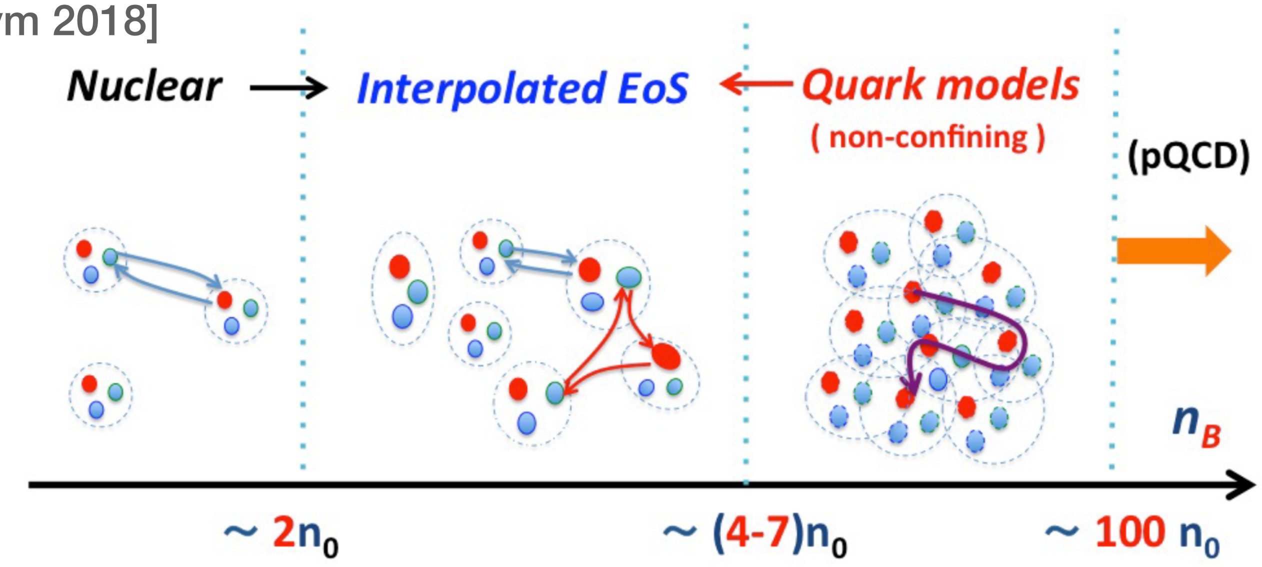

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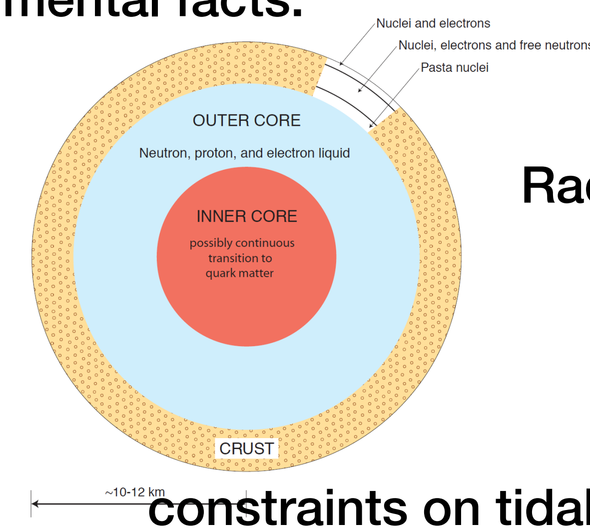

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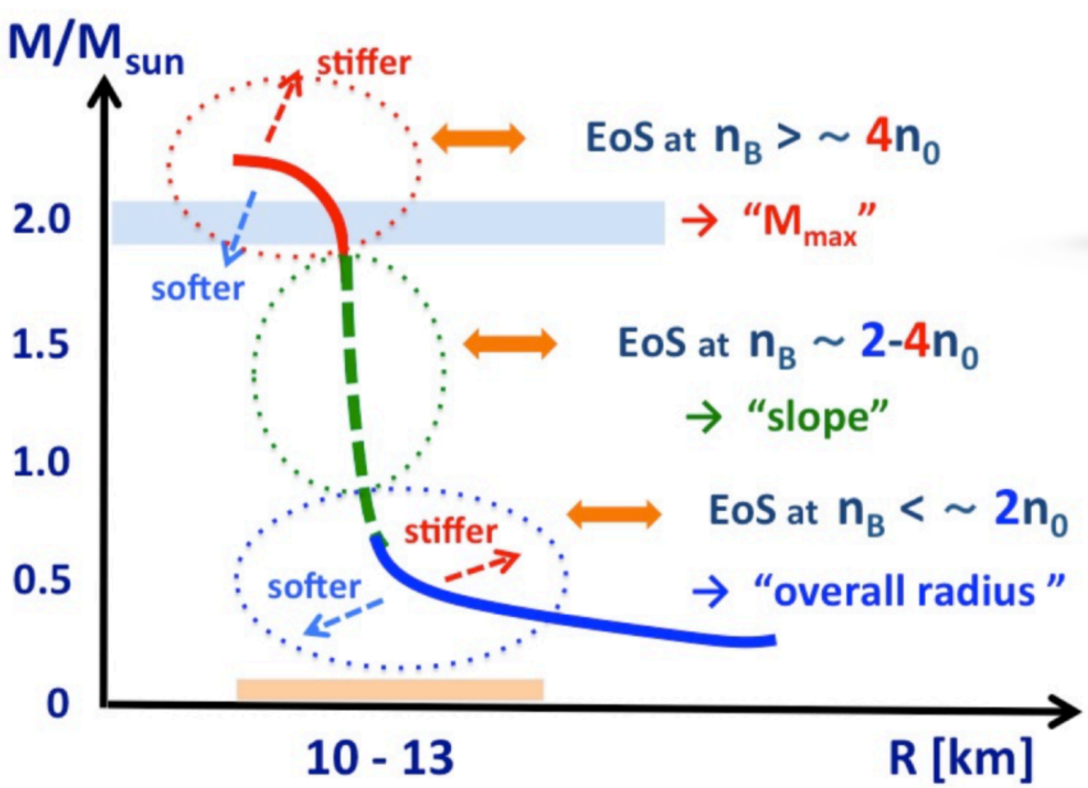


EoS ↔ TOV equation ↔ M-R relation (observables)

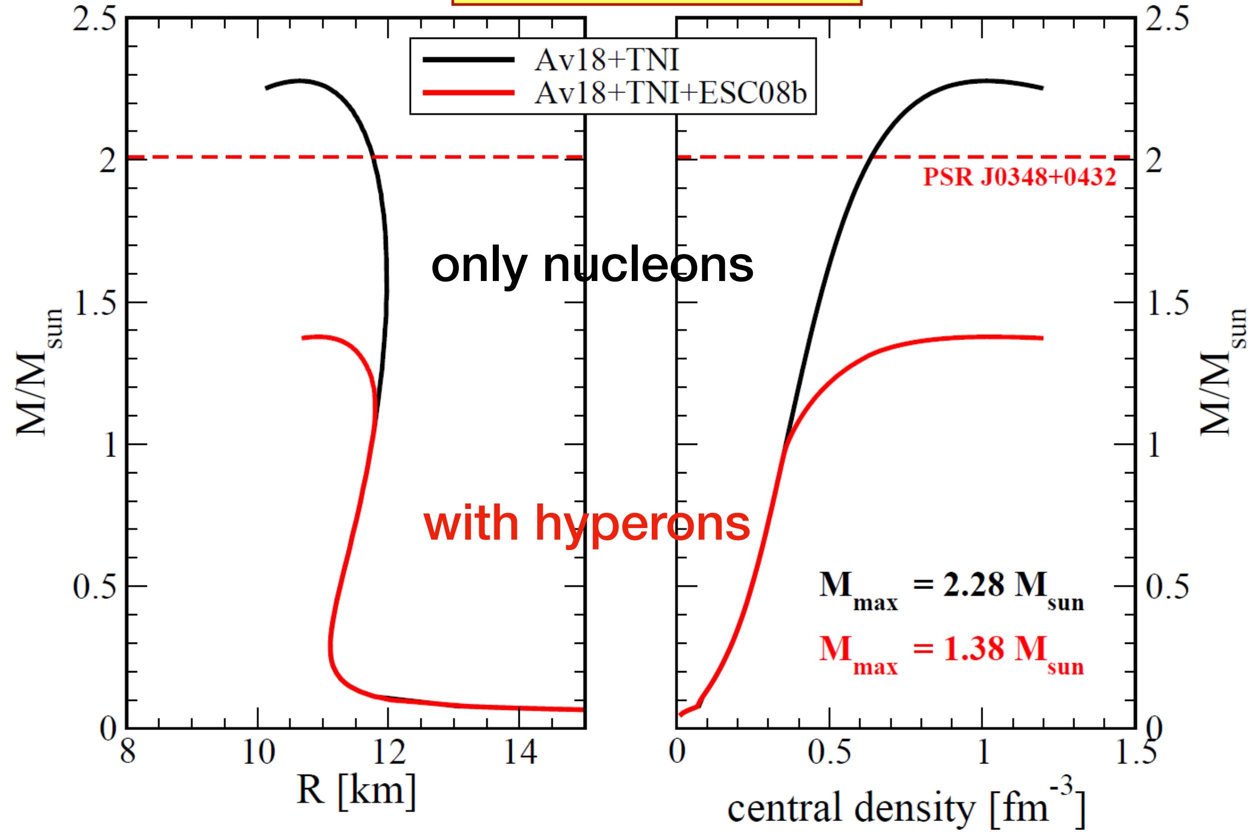
<u>three constraints on the EoS:</u>

1. stiff enough (@high density) ➜ 2M☉
2. soft enough (@low density) ➜ Radius
3. speed of sound < 1

open issues

[Bombaci 2016]

hyperons puzzle



Further constraints:

causality

charge neutrality: $n_p = n_e + n_\mu$

beta equilibrium: $\mu_n = \mu_p + \mu_e$

simplification:

➜ electrons and muons as free Fermi gas in EoS

General problems (physical theory input required):

➜ hyperon puzzle — onset of strangeness in hadronic phase or quark phase ➜ soften EoS [Djapo, BJS, Wambach 2010]

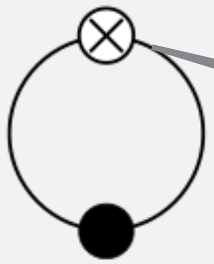
➜ masquerade problem — many EoS look similar ➜ hybrid stars have similar M-R relation as Neutron Stars
increasing #dof soften EoS,
repulsive interactions stiffen EoS [Alvarez-Castillo, Blaschke 2014]

Functional Renormalization Group

- Wetterich Equation (average effective action)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$



R_k regulators

[Wetterich 1993]

$$t = \ln(k/\Lambda)$$

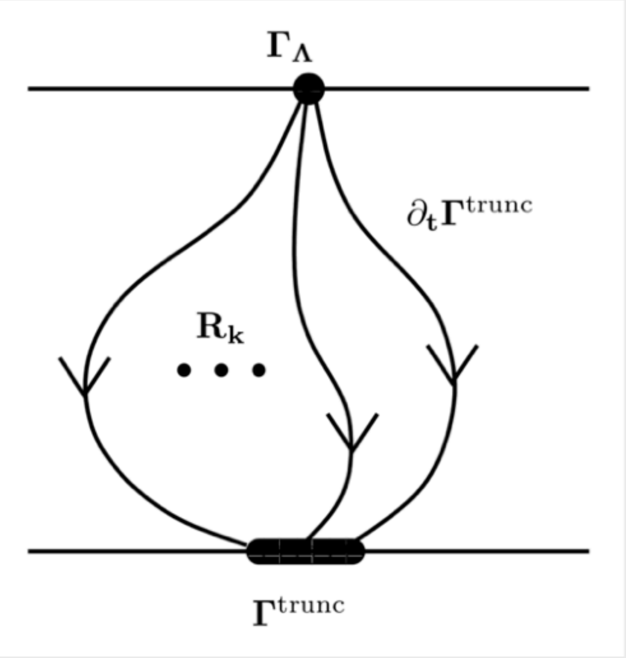
$$\Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

in practise: several truncations ...

shape function conditions:

$$R_k(p^2) = p^2 r(p^2/k^2)$$

- $\lim_{p^2/k^2 \to \infty} R_k(p^2) = 0$
- $\lim_{p^2/k^2 \to 0} R_k(p^2) > 0 \ (= k^2)$
- $\lim_{k \to \infty} R_k(p^2) \to \infty$



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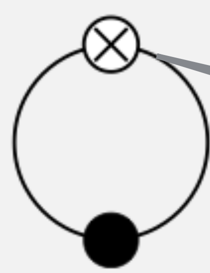

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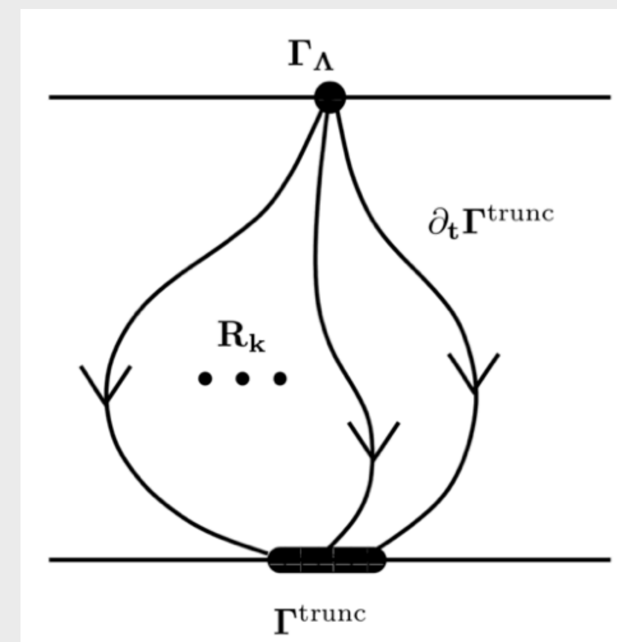


■ Ansatz effective action Quark-Meson truncation in LPA (LO derivative expansion)

$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + V_k(\phi^2)$$

$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

arbitrary potential

Impact of fluctuations on EoS

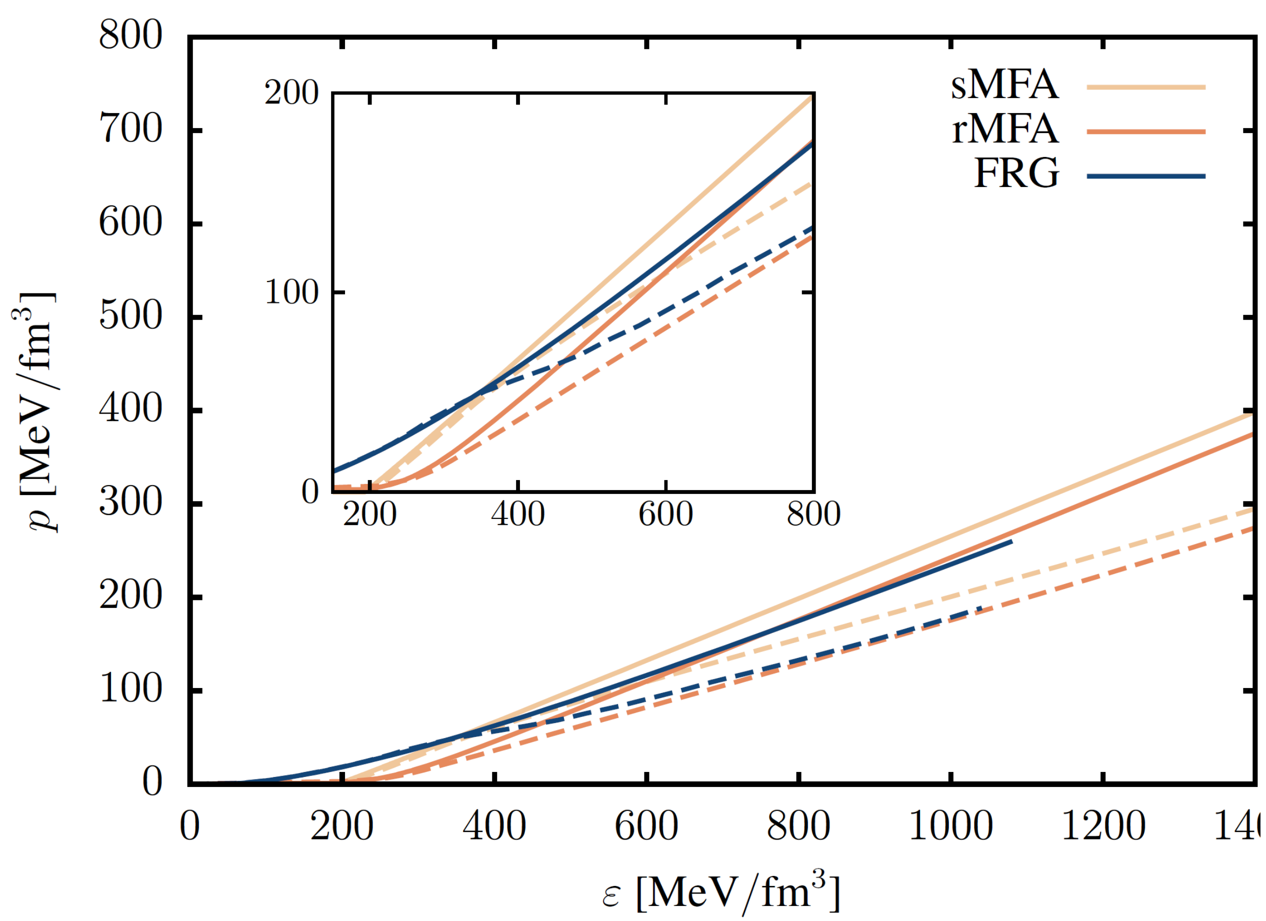
[Otto, Oertel, BJS 2020]

Impose beta equilibrium and charge neutrality conditions

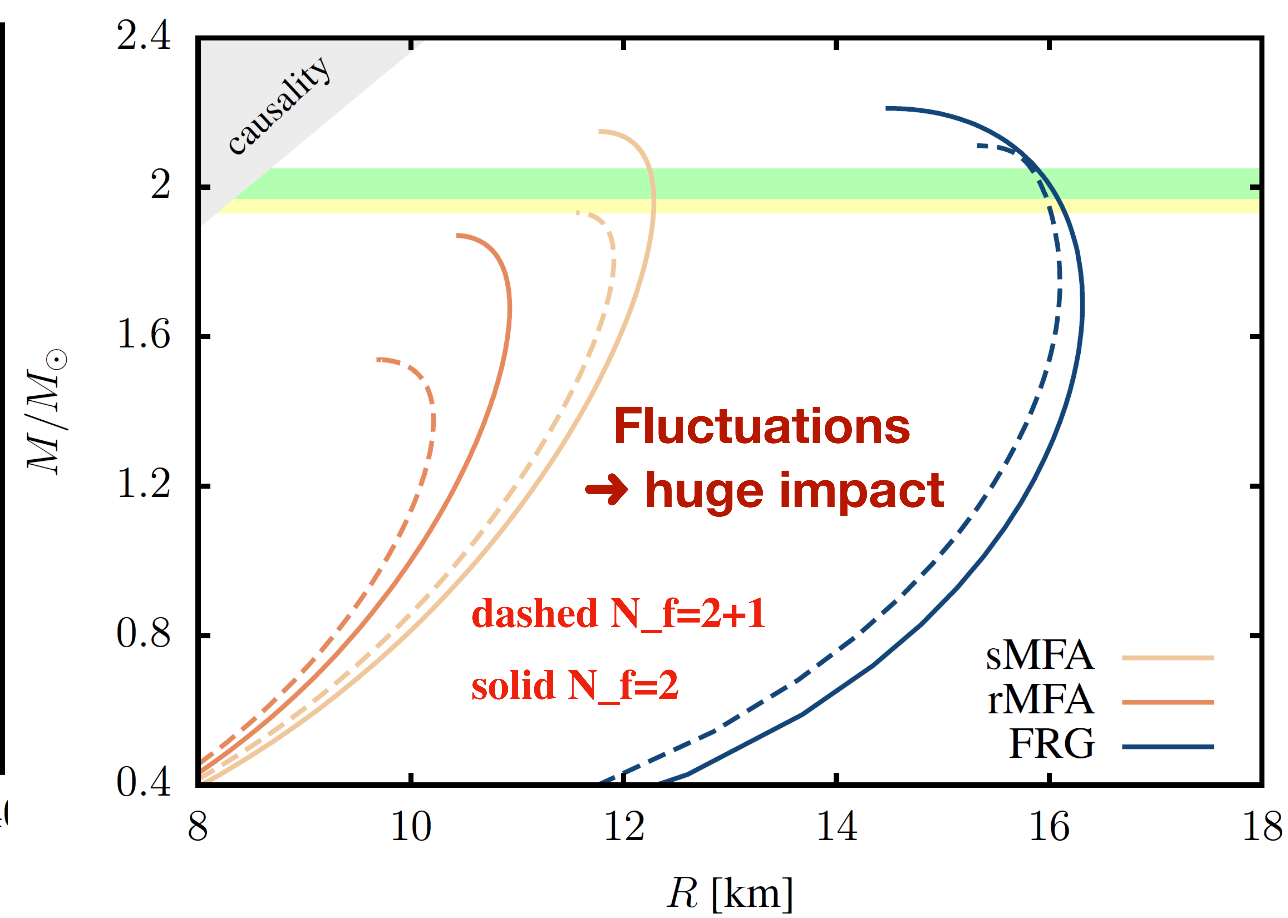
$$\mu_u = \mu_q - \frac{2}{3}\mu_e$$
$$\mu_d = \mu_q + \frac{1}{3}\mu_e$$
$$\mu_s = \mu_q + \frac{1}{3}\mu_e$$

Tolman-Oppenheimer-Volkoff equations

EoS



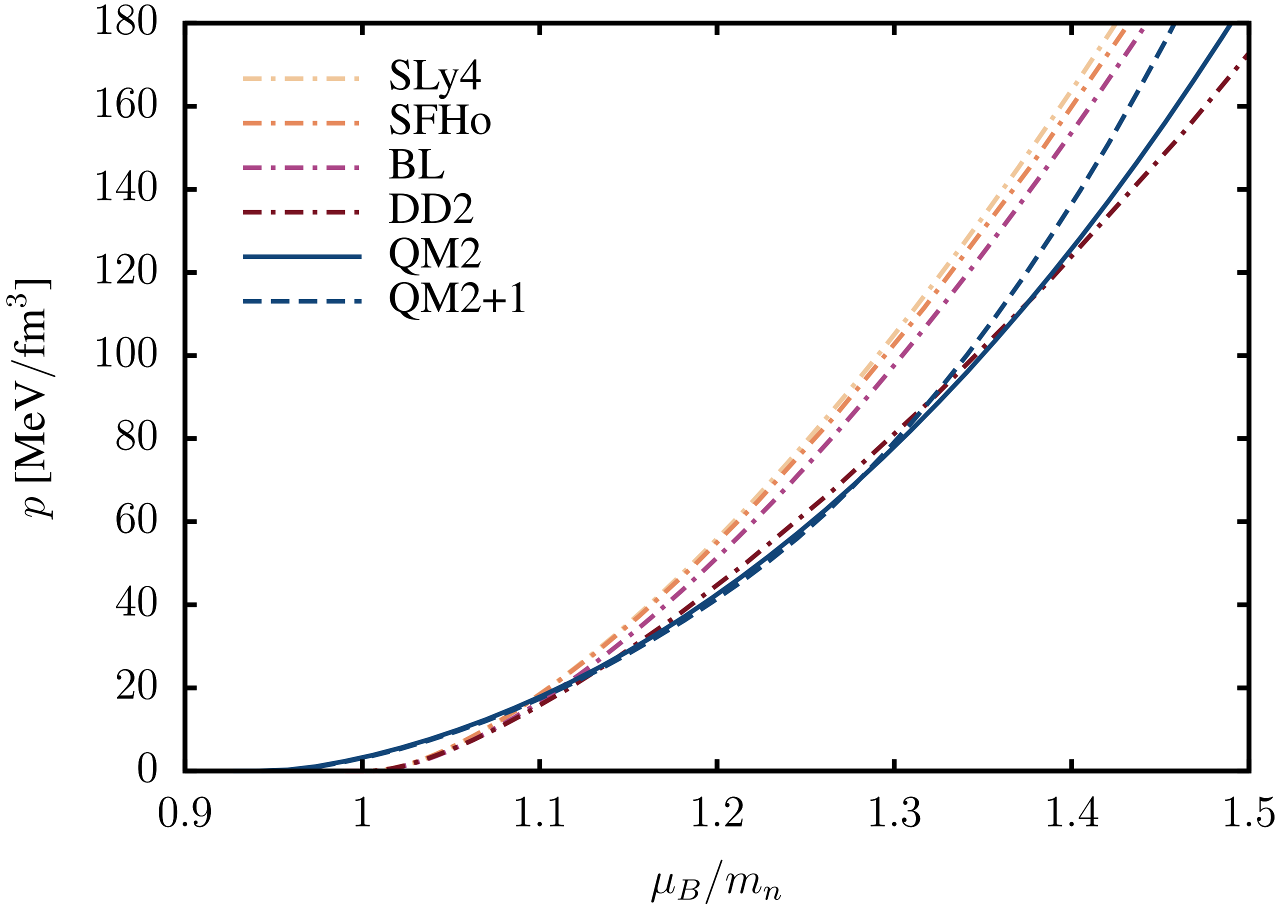
Mass-Radius relation

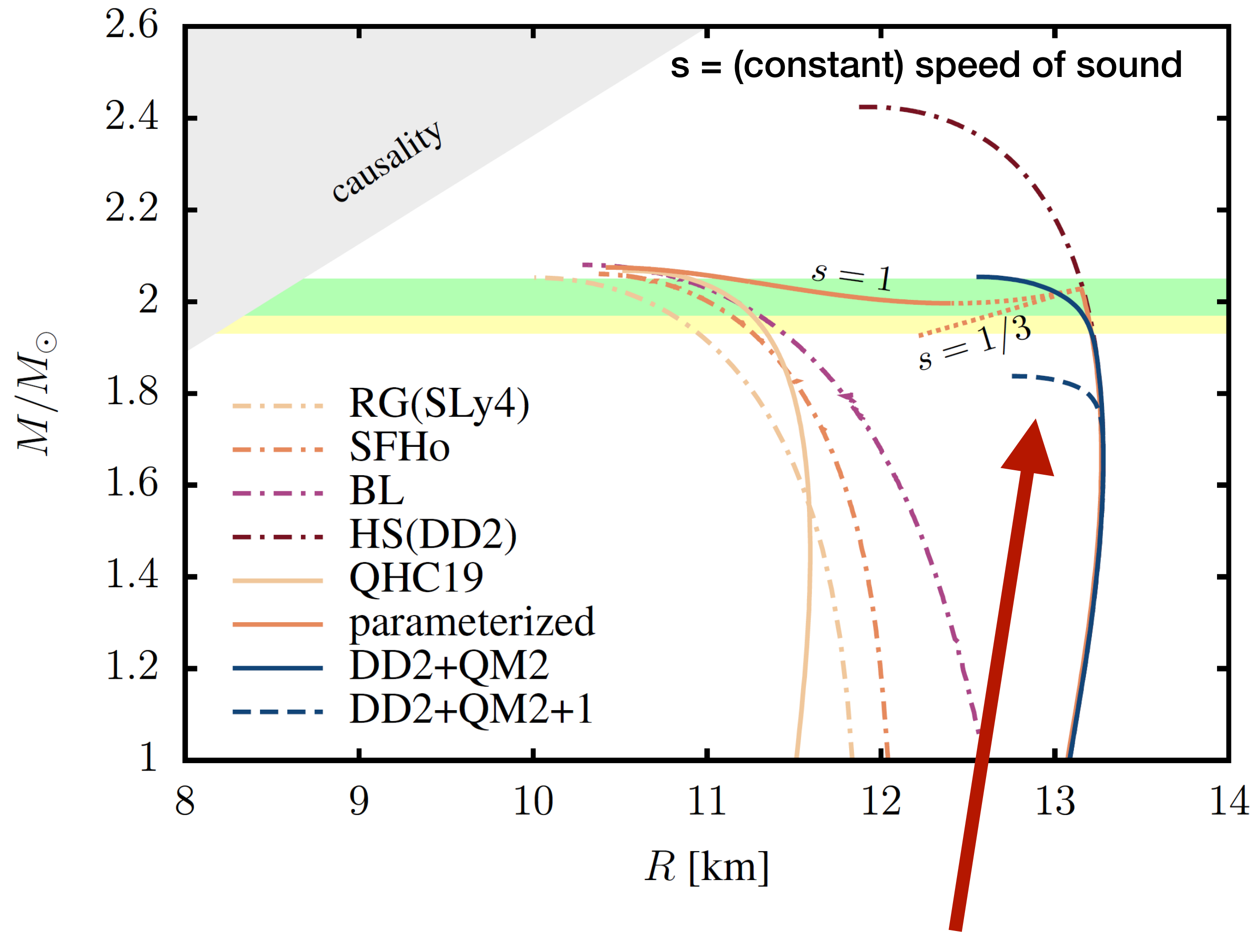


Hybrid star construction possible? - yes

[Otto, Oertel, BJS 2020]

combine nuclear EoS (DD2) with FRG QM truncation ➜ continuous nuclear-hybrid branch





2 $M_\odot$ limit violated for N_f= 2+1

can a repulsive vector interaction remedy this behavior?

Vector mesons to the FRG EoS

[Otto, Oertel, BJS 2020]

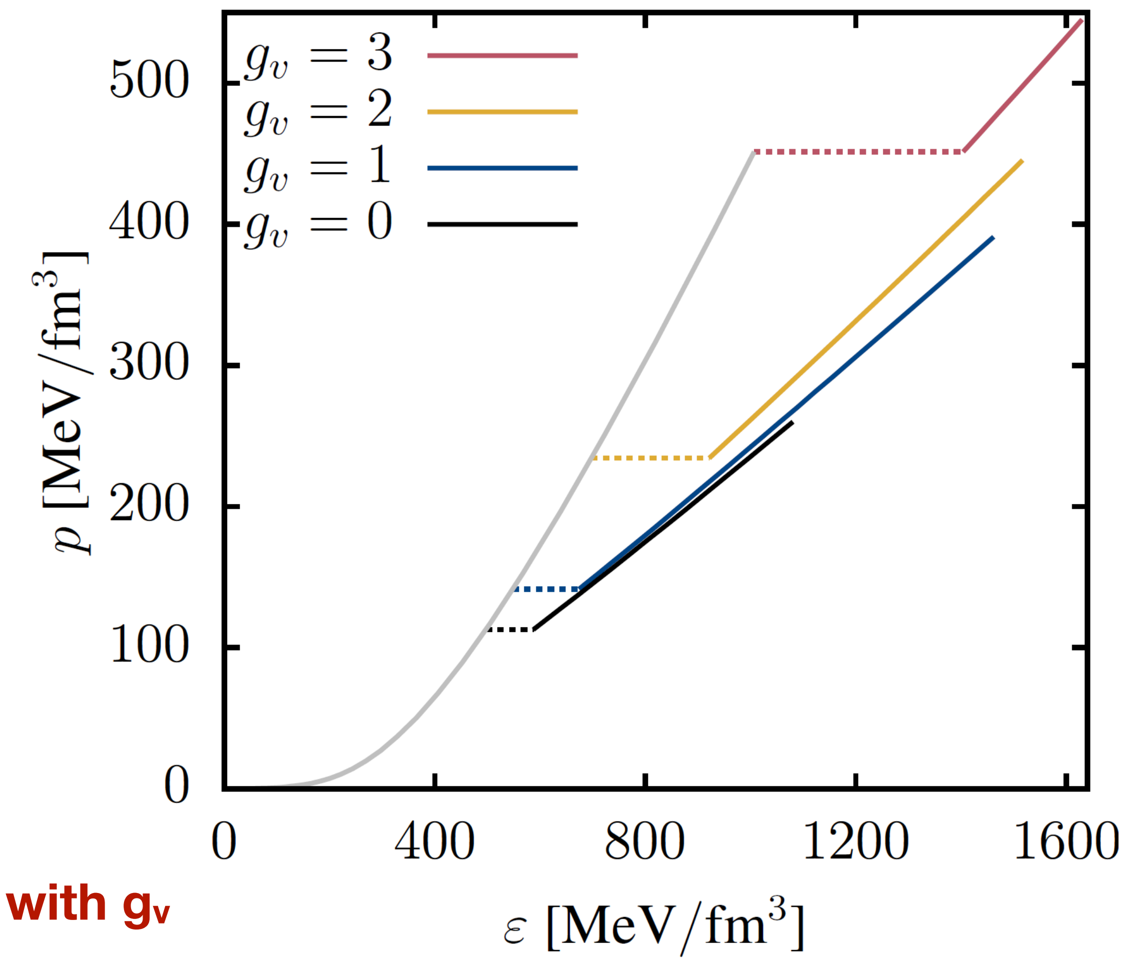
[Rennecke 2015]
[Pereira, Stiele, Costa 2020]

- Yukawa type interaction of temporal component and mean-field potential

$$\Gamma_{\text{vec}} = \int_x \left[\frac{g_v}{2}\, \bar{q}\, \gamma_0\, \text{diag}_f(\omega, \omega, \sqrt{2}\phi)\, q - \frac{1}{2}\left(m_\omega^2 \omega^2 + m_\phi^2 \phi^2\right) \right]$$

- effectively shifts the chemical potentials:

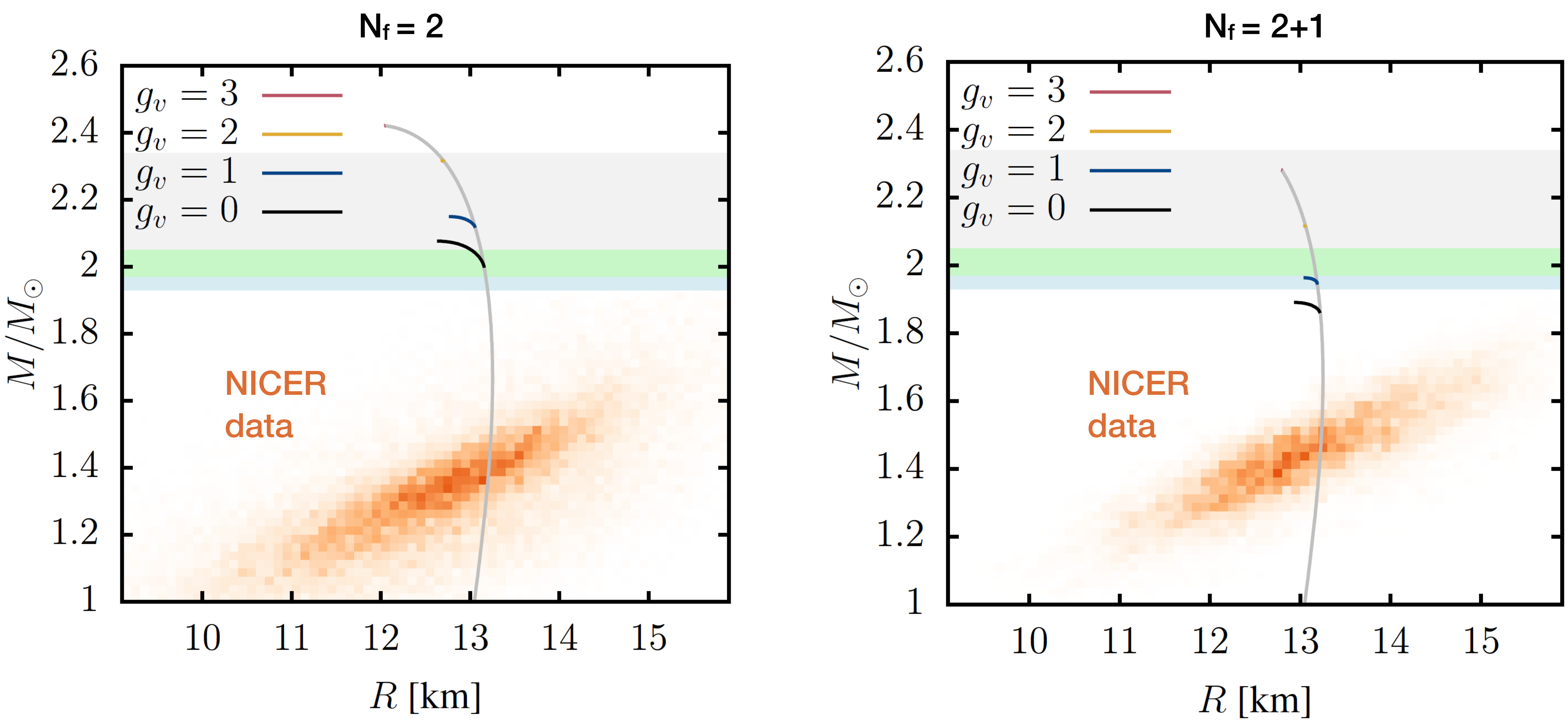
$$\tilde{\mu}_u = \mu_q - \frac{2}{3}\mu_e - \frac{g_v}{2}\omega$$
$$\tilde{\mu}_d = \mu_q + \frac{1}{3}\mu_e - \frac{g_v}{2}\omega$$
$$\tilde{\mu}_s = \mu_q + \frac{1}{3}\mu_e - \frac{g_v}{\sqrt{2}}\phi$$



➜ energy gap and transition pressure increases with g_v

Mass-Radius relations

[Otto, Oertel, BJS 2020]



➜ including strange quarks: **finite vector coupling is needed to achieve $2M_\odot$ limit**

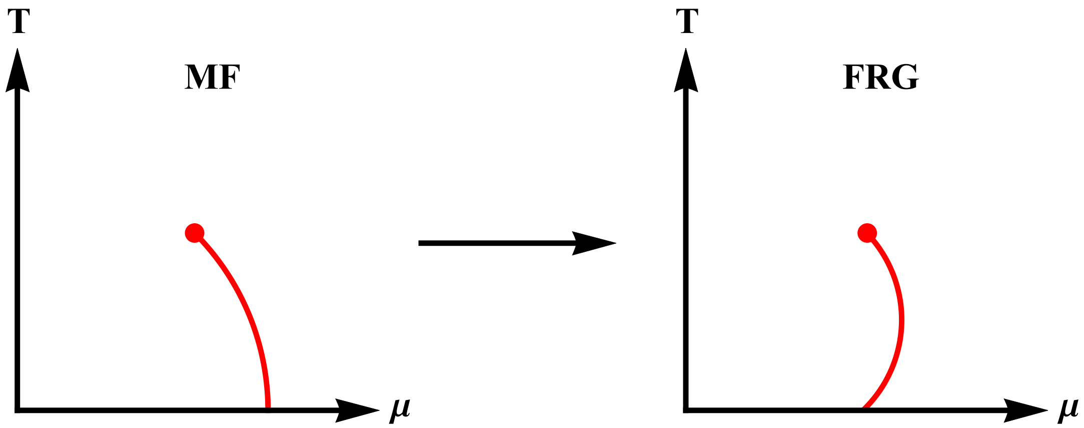
➜ at the same time: **larger vector coupling lead to smaller quark cores!**

so far so good … BUT

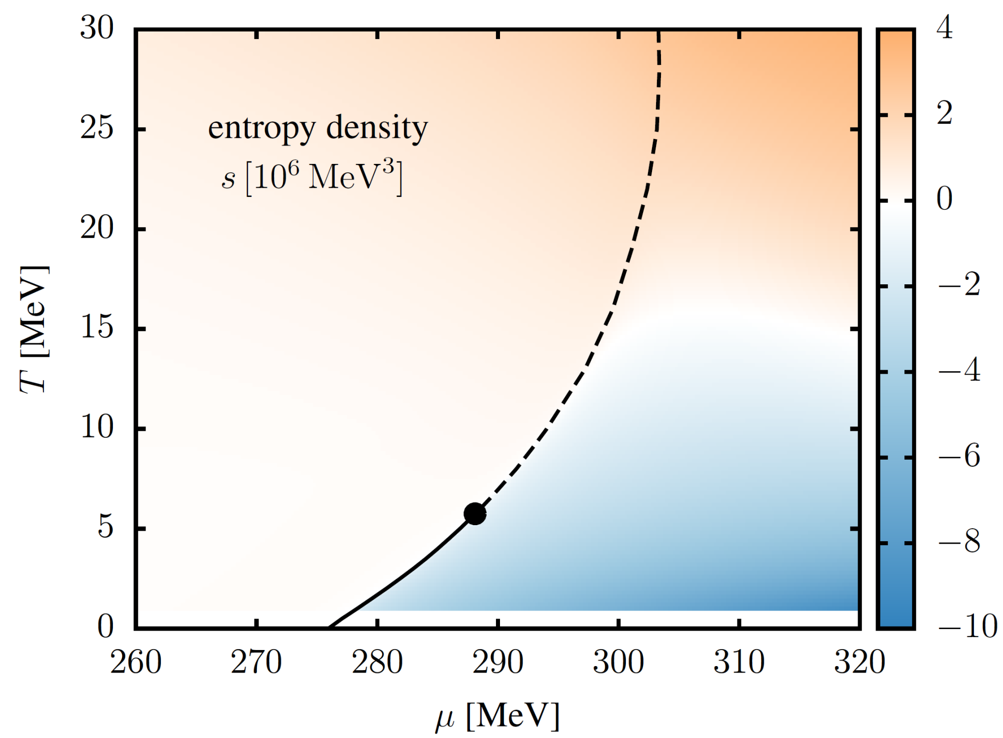
back-bending / negative entropy density

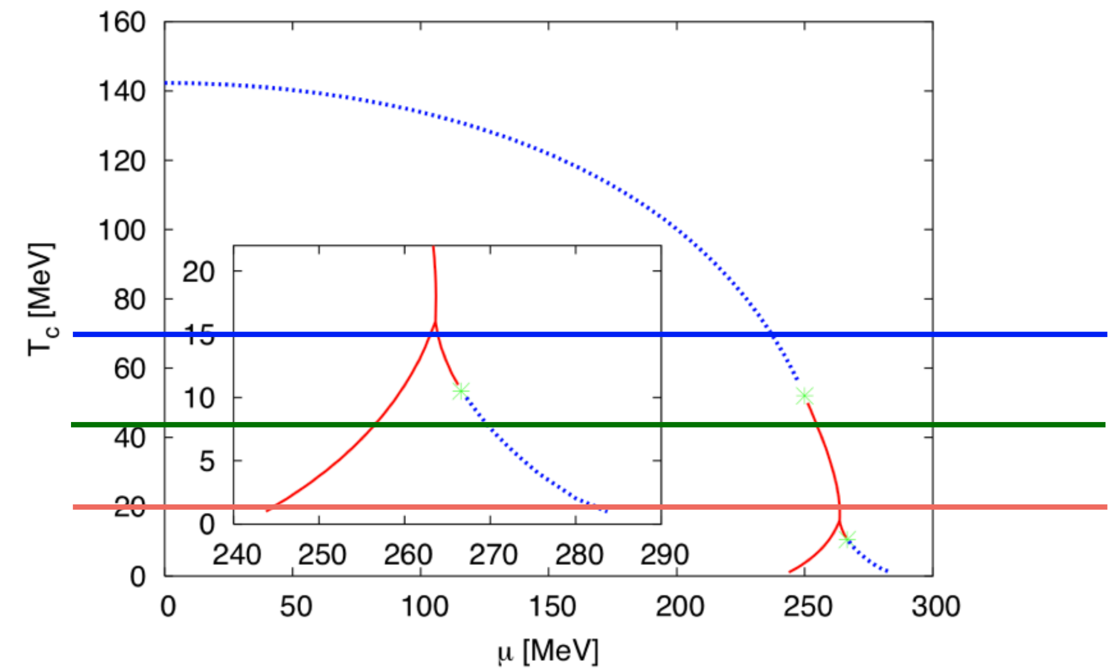
[R-A Tripolt, BJS, L von Smekal, J Wambach 2018]

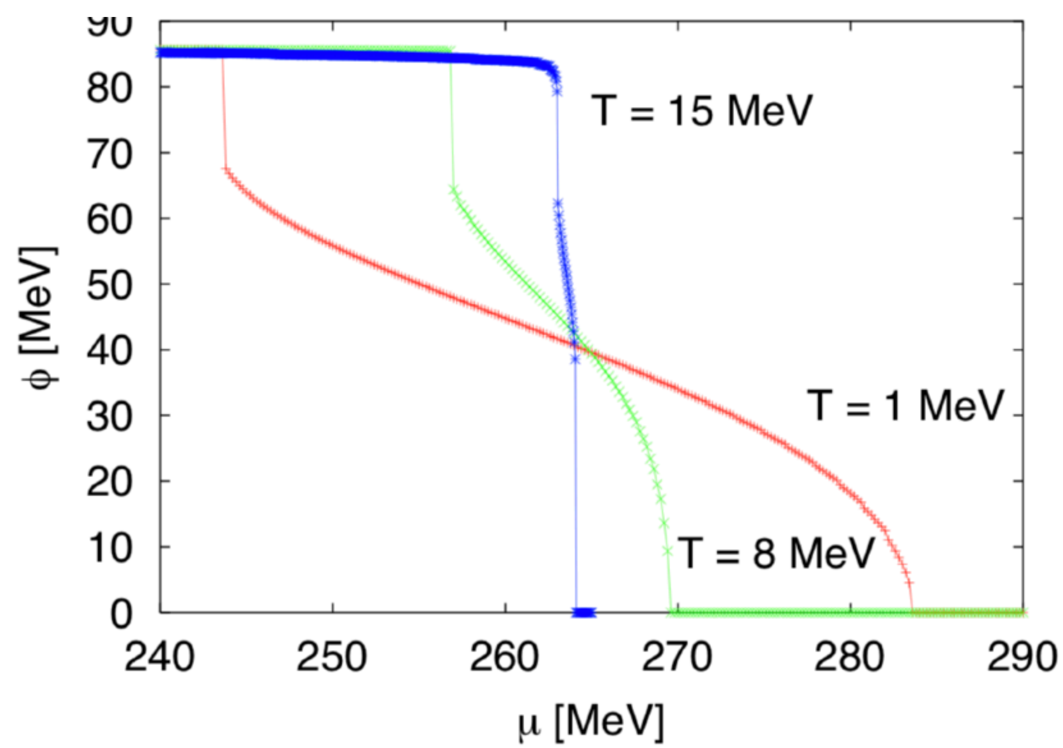
[BJS, Wambach 2005]



- Phase diagram quark-meson model
- Entropy density: s/T^3







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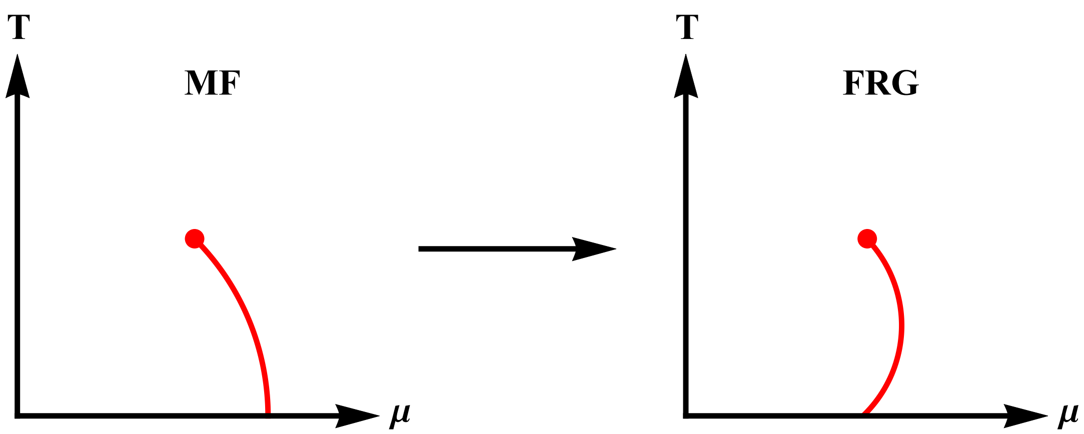

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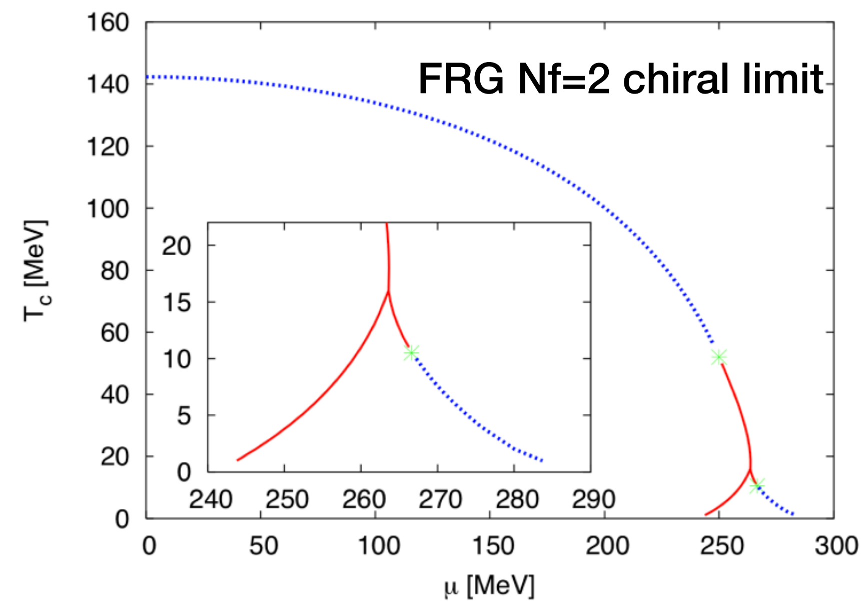

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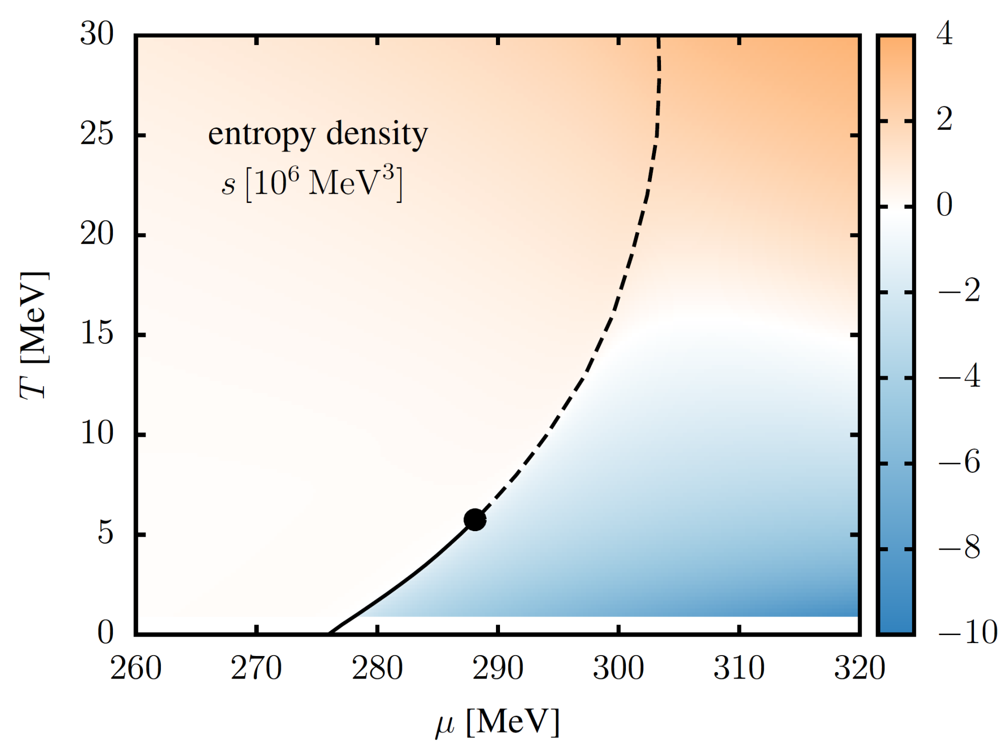

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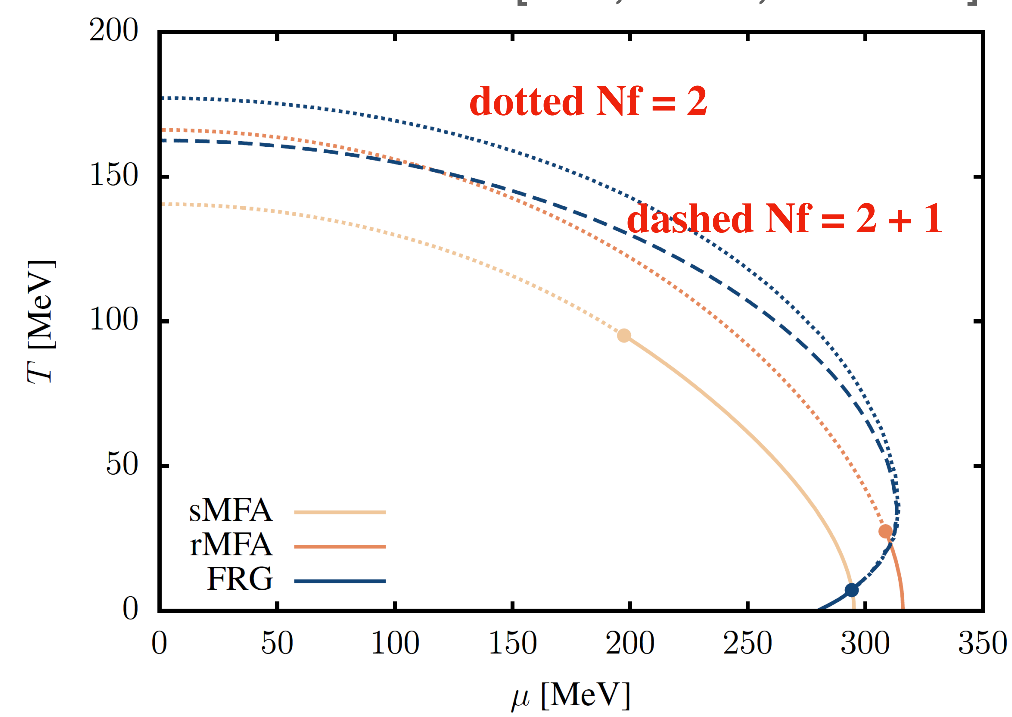

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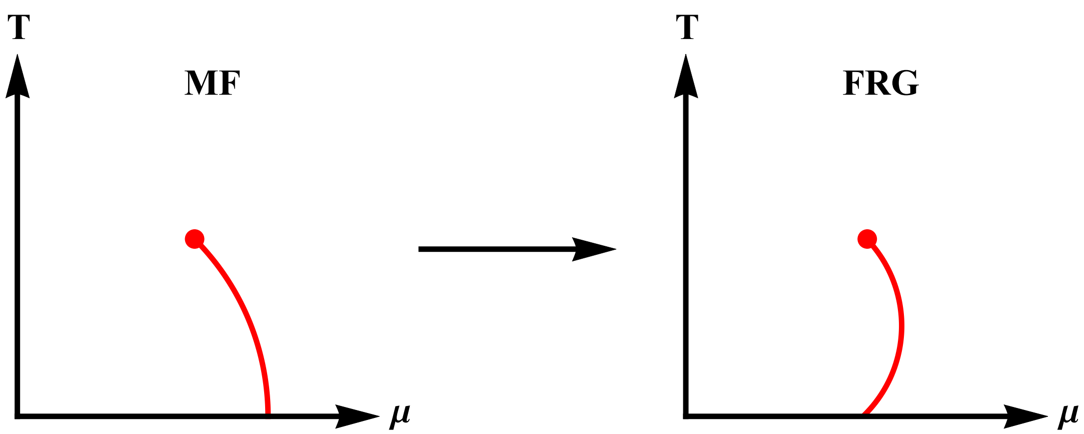

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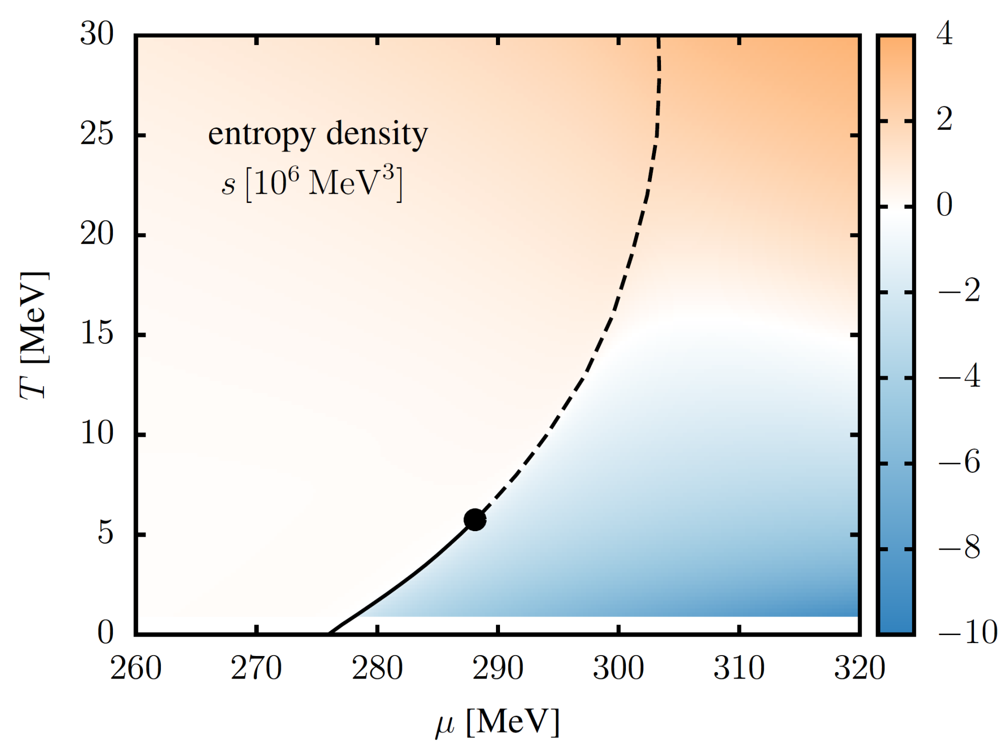
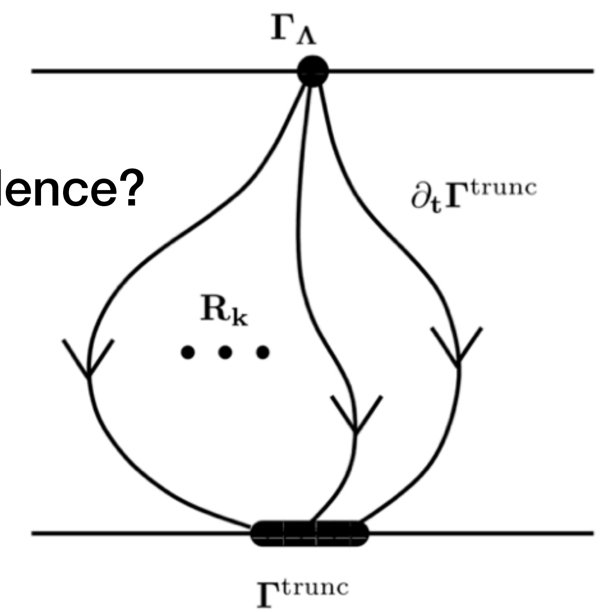


➜ Regulator scheme dependence?

cf talk by Sebastian Töpfel



- fermionic regulator

$$R_k^F(p,\mu) = R_k^F(\tilde{p},0) \qquad \tilde{p} = \begin{pmatrix} p_0 + \mathrm{i}\mu \\ \vec{p} \end{pmatrix}$$

shift required to preserve **Silver Blaze** property (T=0)

(necessary but not sufficient)

- example:

exponential regulator fulfills SB but

➜ additional (unphysical)

poles that break Silver Blaze

Callan-Symanzik type regulators

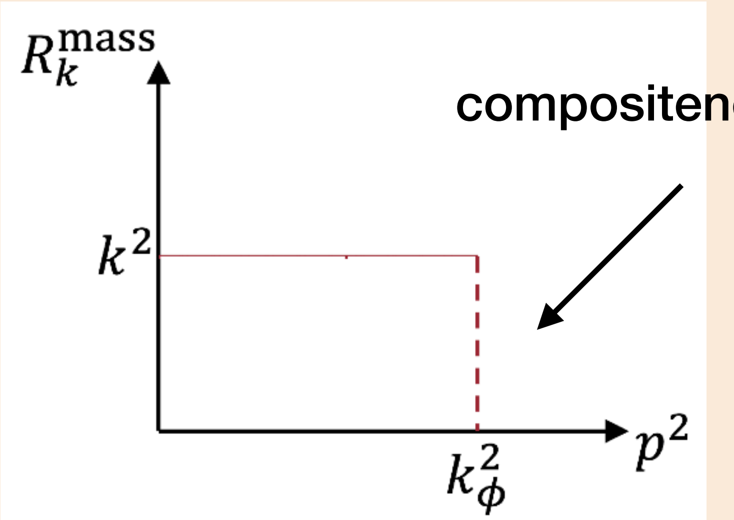
- **solution:** CS regulator: $R_k(p^2) = k^2$ UV divergent

  3dim regulators: $R_k(\vec{p}^2)$

- **comparison:**

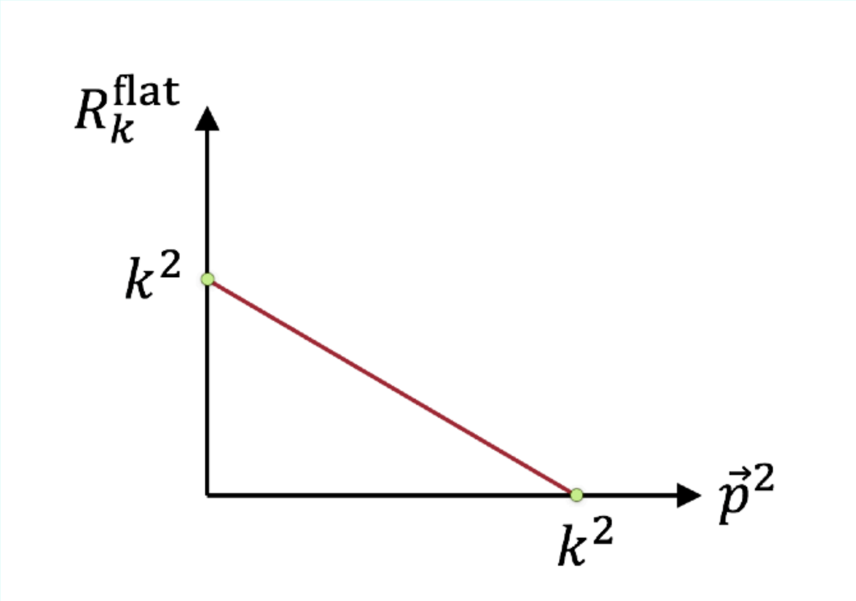
CS (mass-like) regulators

- 4dim: $R_k^{\text{mass}}(p^2) = k^2\,\Theta(k_\Phi^2 - p^2)$
- 3dim: $R_k^{\text{mass}}(\vec{p}^2) = k^2\,\Theta(k_\Phi^2 - \vec{p}^2)$



3dim flat (Litim) regulators

$$R_k^{\text{flat}}(\vec{p}^2) = (k^2 - \vec{p}^2)\Theta(k^2 - \vec{p}^2)$$



Partial IR fixed point

- **usual UV procedure not applicable :**

CS regulator: $R_k(p^2) = k^2$ UV divergent

k_Φ

approximated flow in LPA'
down
to chiral sym breaking scale k_χ

➜ partial IR fixed point

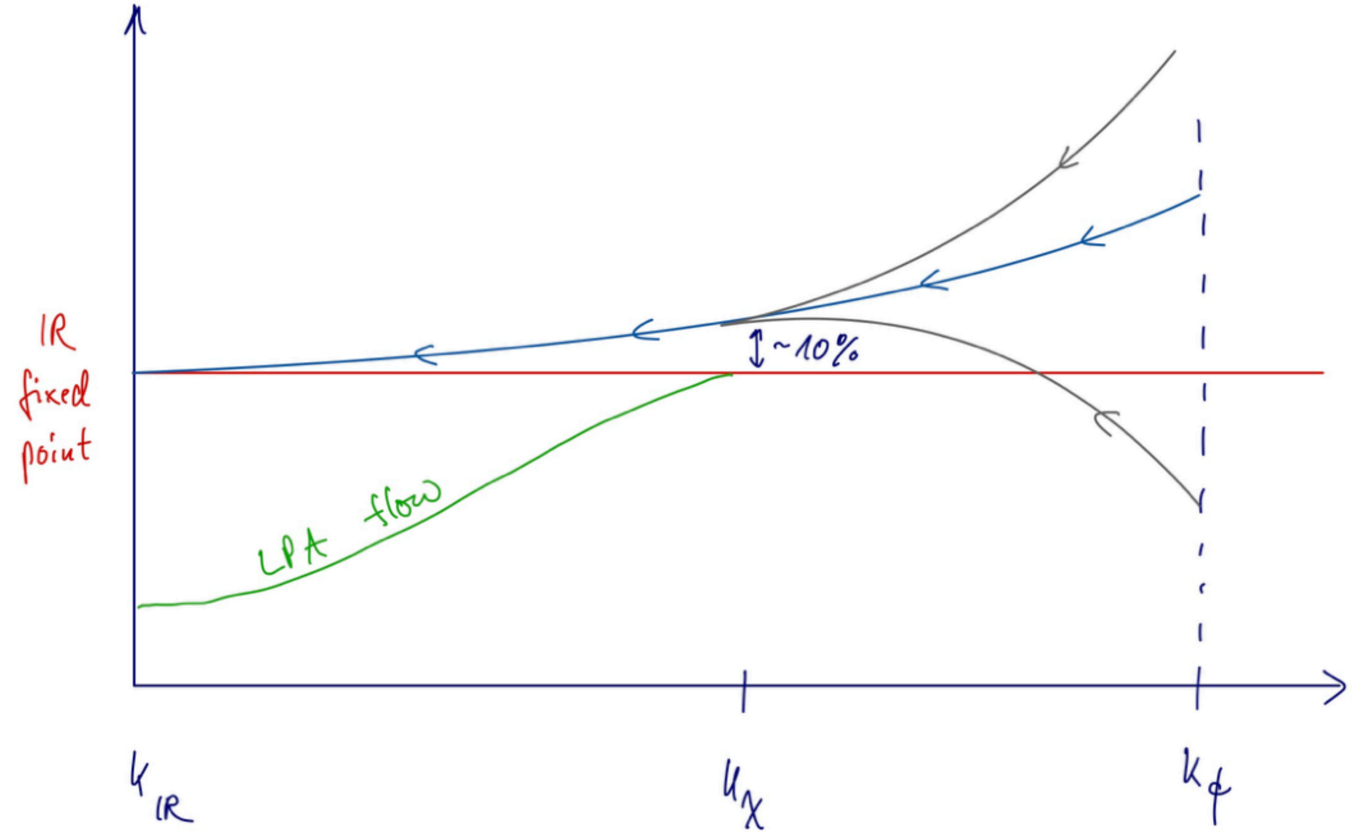
k_χ

solve flow in LPA
starting
at
fixed point

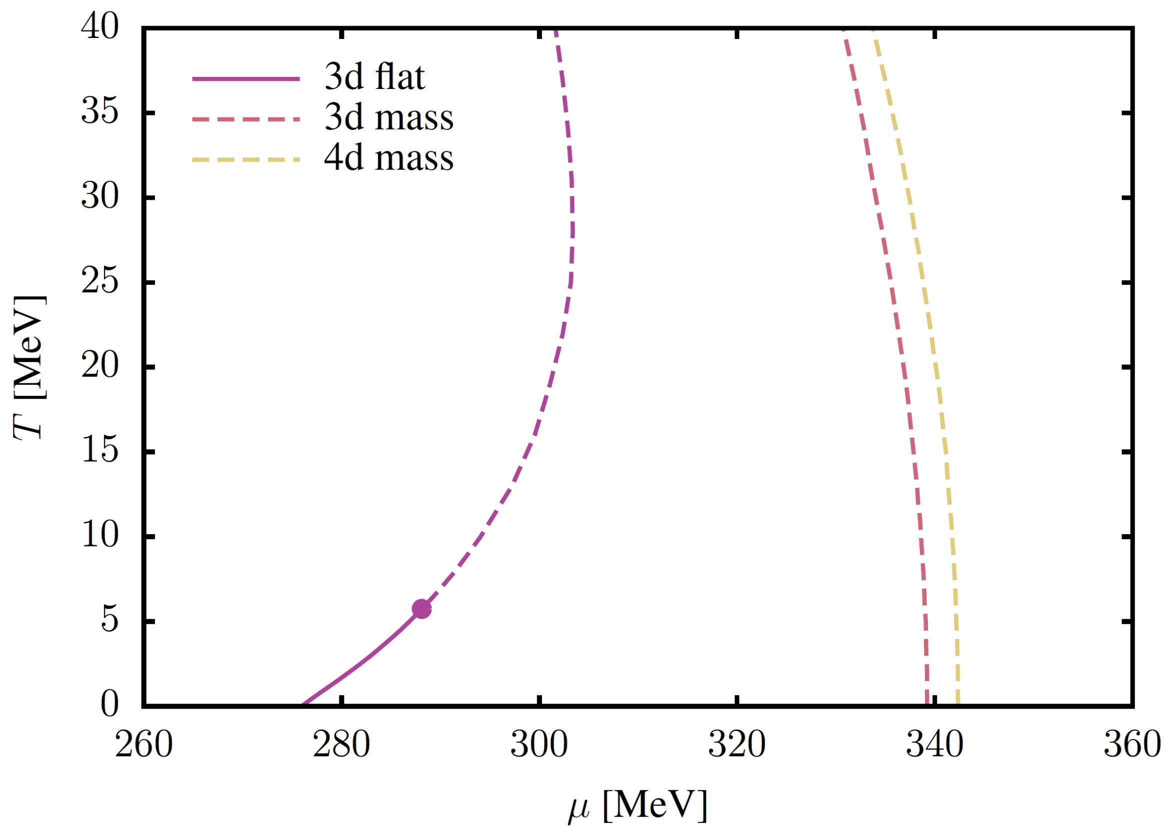
$k_{\rm IR}$

adjust IR values by variation of k_Φ and k_χ

- scales around compositeness scale:
  - neglect mesons, quarks dominate
  - simple flows for
    - ➡ Yukawa coupling
    - ➡ mesonic wave function renormalization
  - field expansion of (dimless) potential
    - ➡ IR fixed point for coeffients n>1



Chiral transition at low temperature



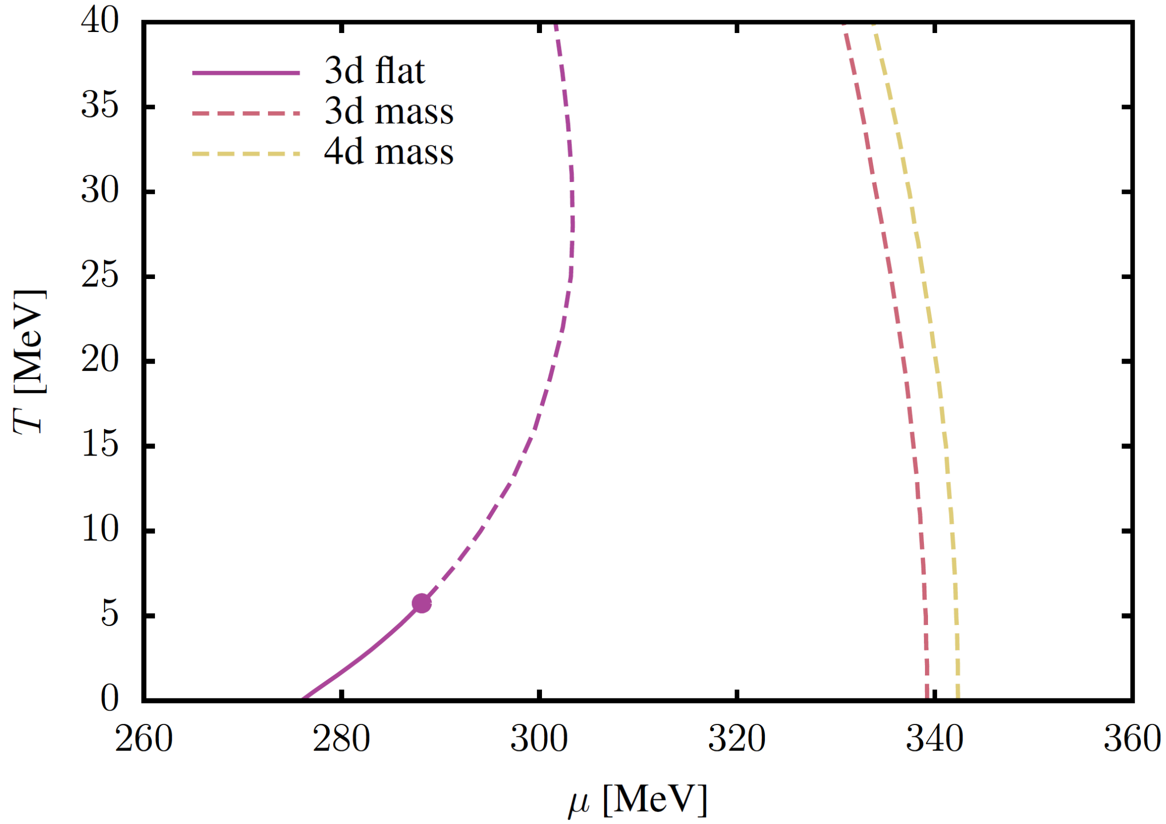
no back-bending with CS mass-like regulator

small differences between 3d and 4d regulators

purely crossover ➜ pseudocritical μ_c larger than m_q

could finite IR cutoff play a role?

Chiral transition at low temperature



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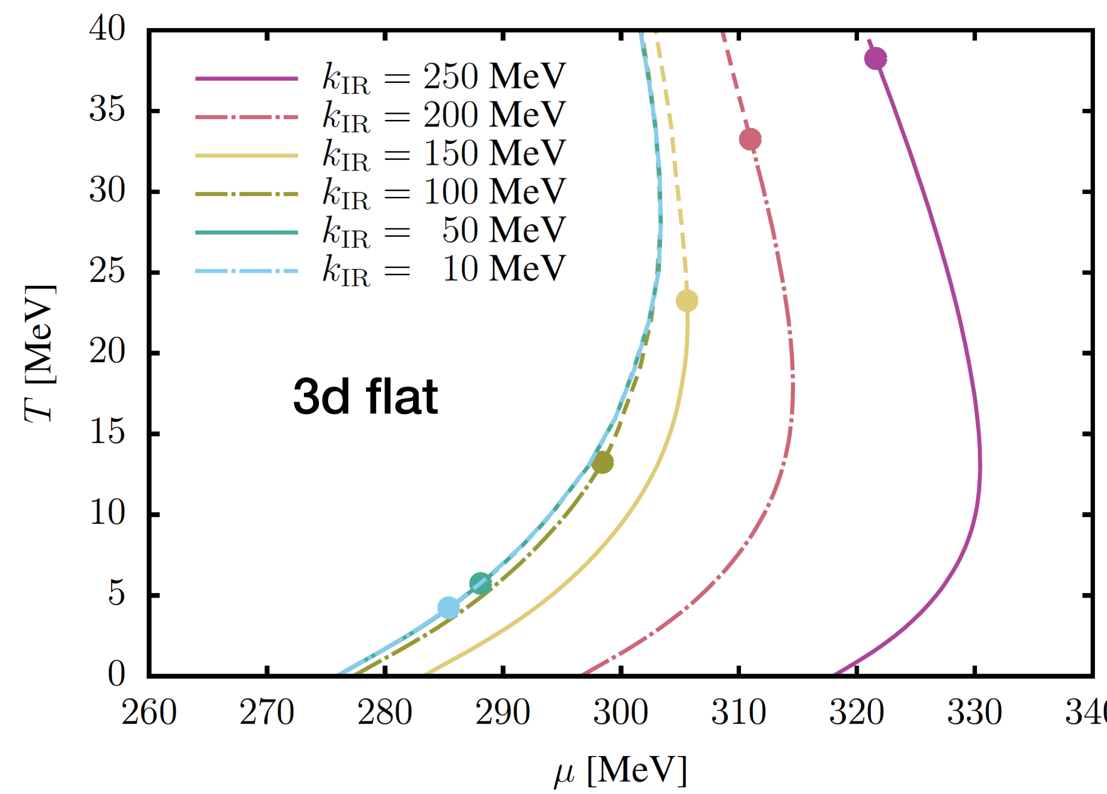
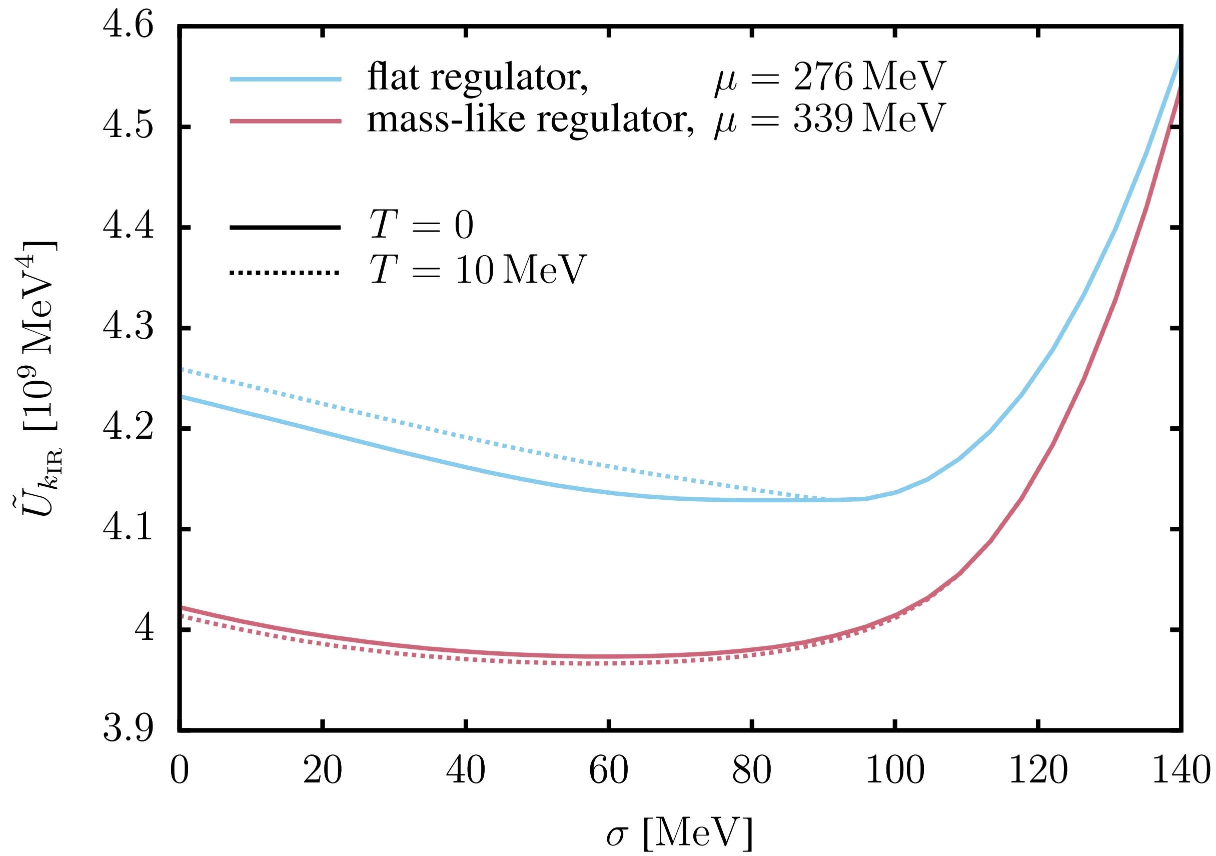
➜ No

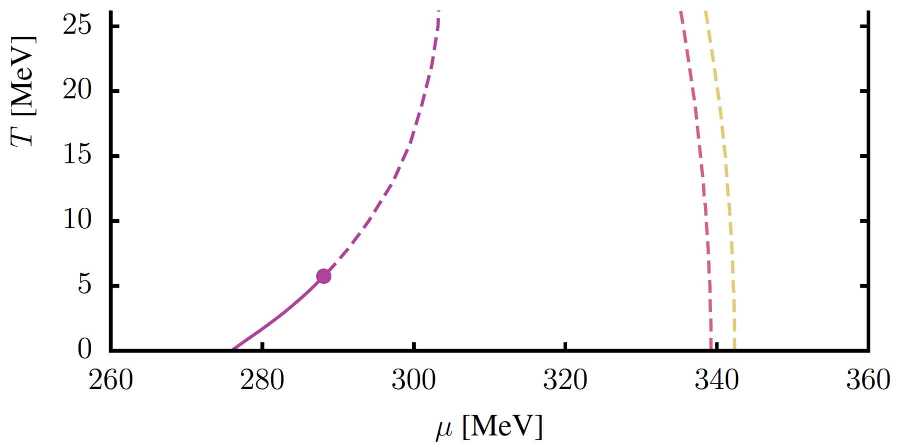


transition line shifts and CEP moves down

but back-bending over large k_{IR} range

Origin of back-bending





flat (Litim) regulator

larger variations between two temperatures

potential moves **upwards**

➜ chiral symmetry breaking

➜ back-bending

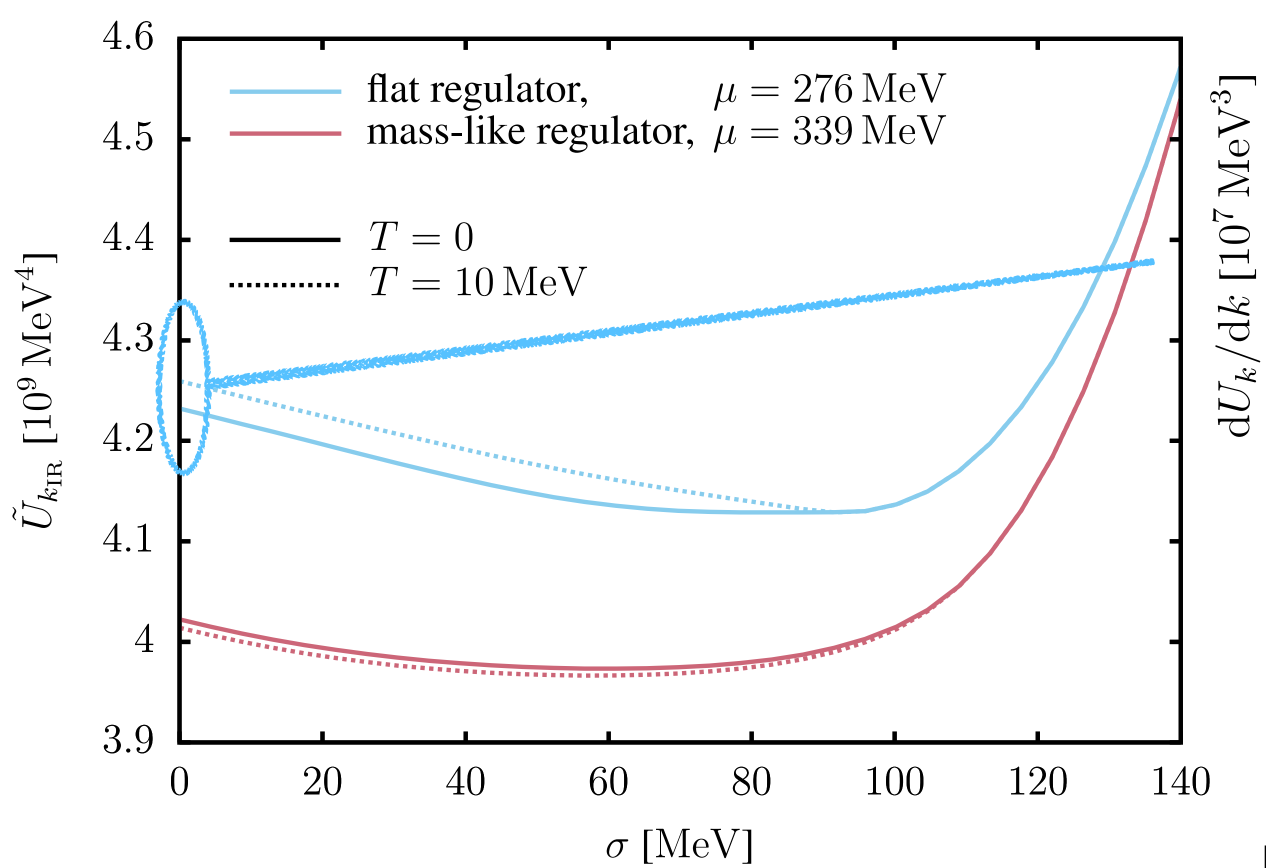
CS mass-like regulator

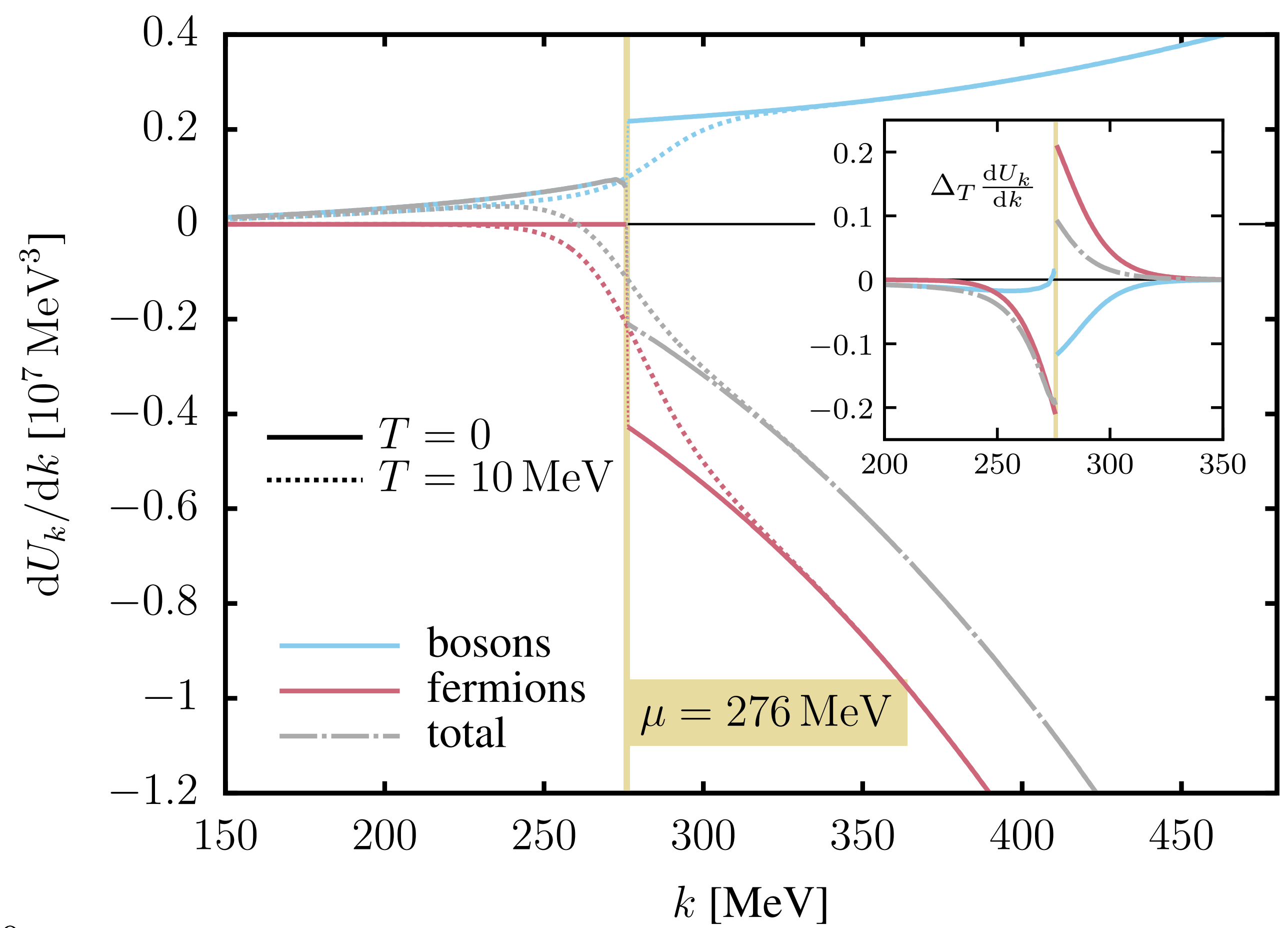
smaller variations between two temperatures

potential moves **downwards**

➜ chiral symmetry restoration

Origin of back-bending





Fermi distribution not integrated out ➜ discontinuity

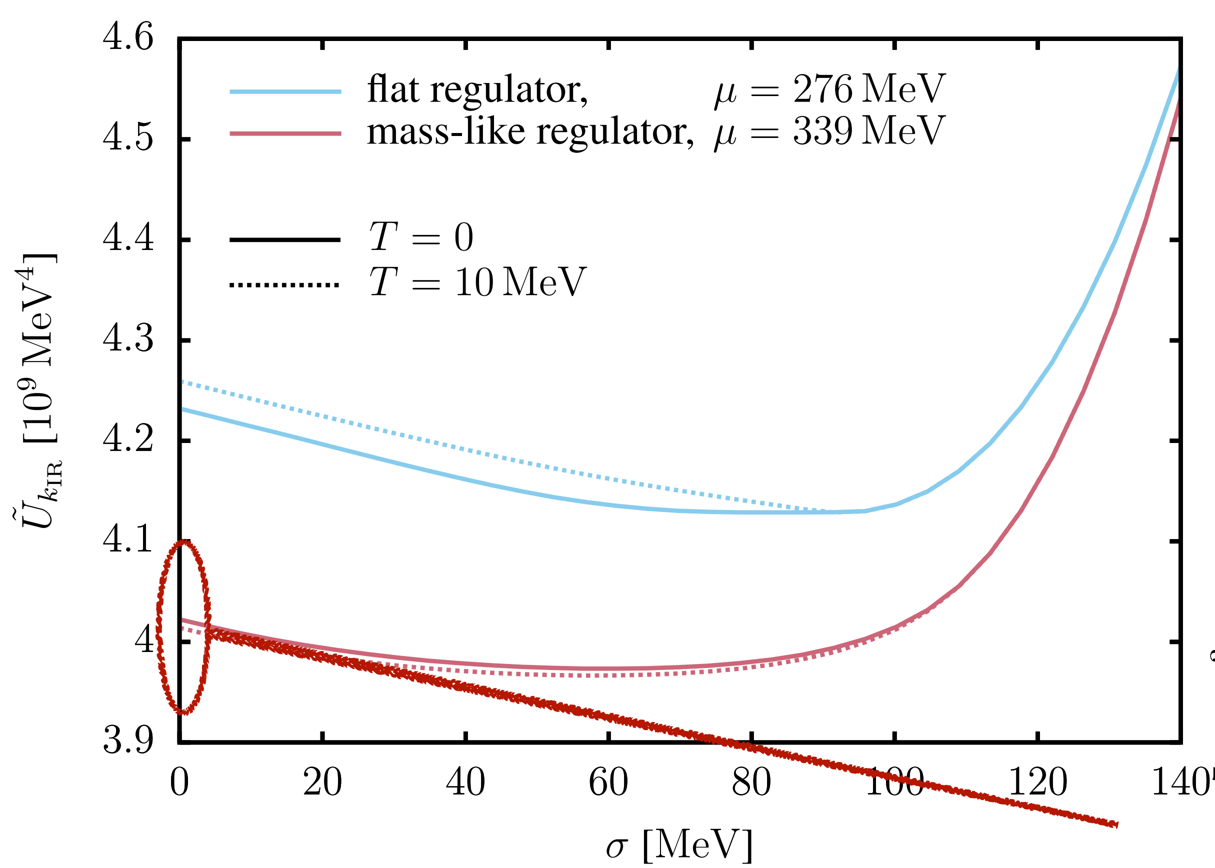
fermionic fluctuations decoupling at $k = \sqrt{\mu^2 - m_q^2}$

mean-field approximation works because net integral of relative flow for fermions close to zero

bosons cannot mimic this behavior, become weaker at finite T

$$\partial_t U_k^{F,\mathrm{flat}} = -\frac{N_c N_f k^2}{\pi^2 E_q}\,\Theta(E_q - \mu)\int_0^k \mathrm{d}p\, p^2 = -\frac{N_c N_f k^5}{3\pi^2 E_q}\Theta(E_q - \mu)$$

Origin of back-bending

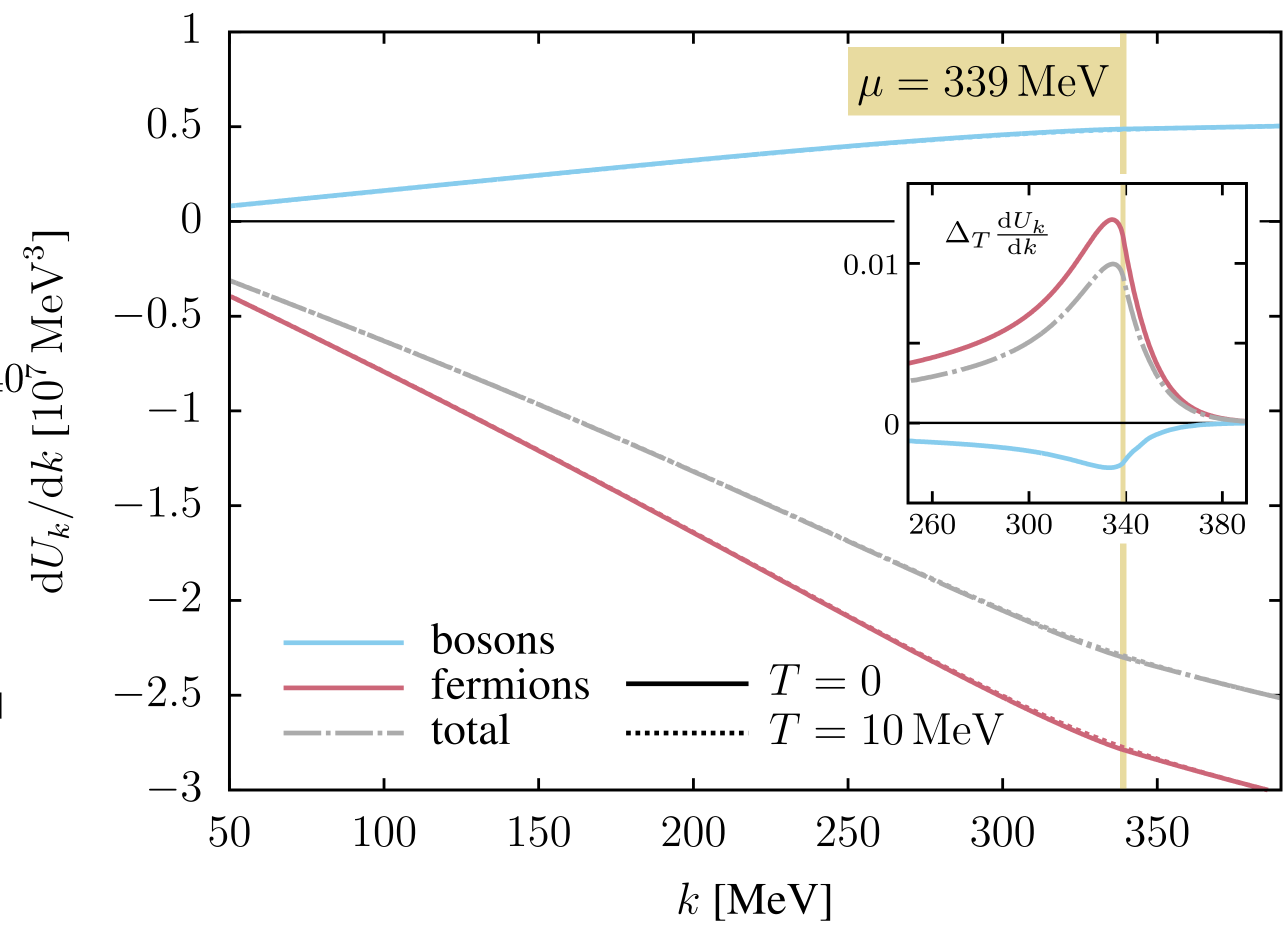


fermionic fluctuations decouple for increasing mu

Fermi distribution integrated out in the loop

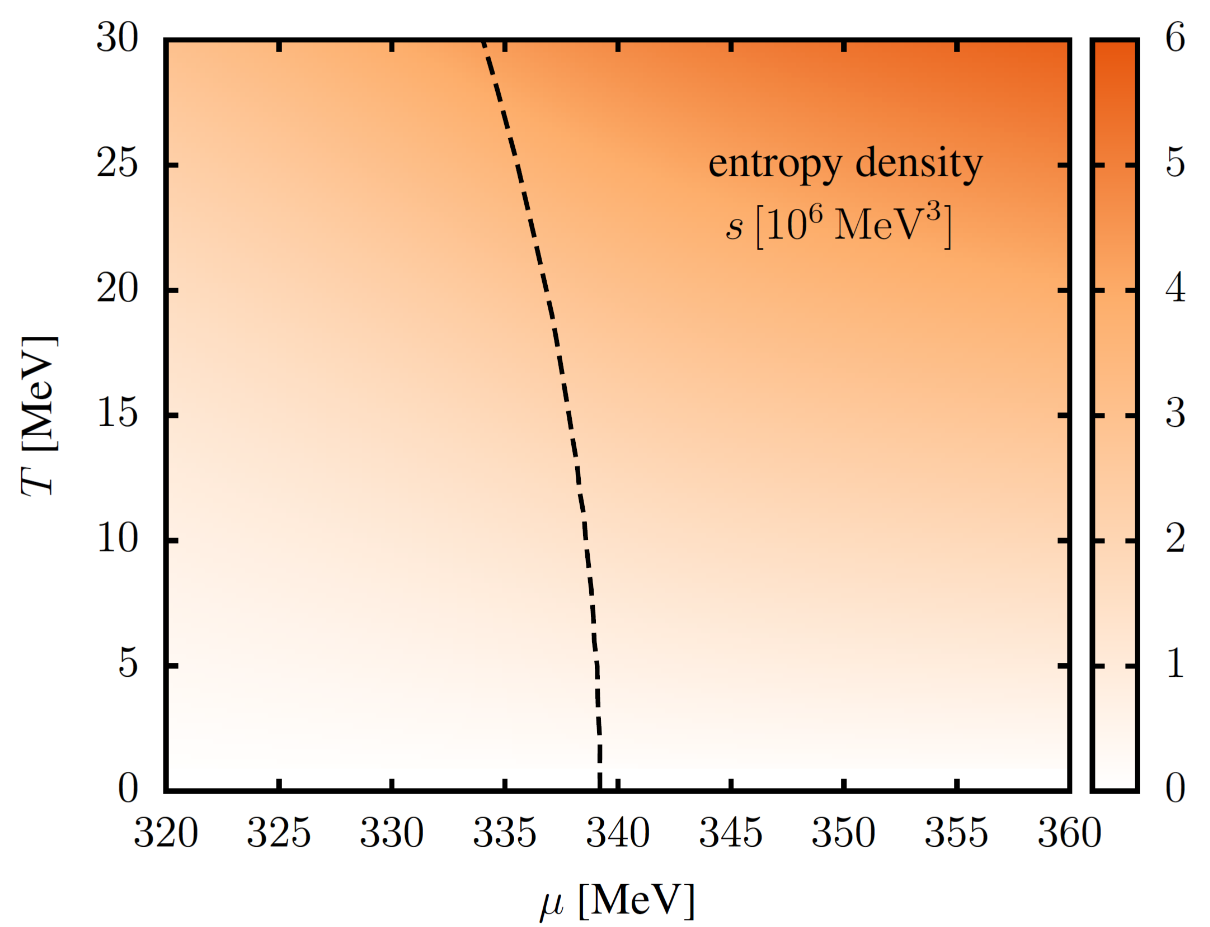
$$\partial_t U_k^{F,\text{mass}} = -\frac{N_c N_f k^2}{\pi^2} \int_{p_F}^{k_\phi} \mathrm{d}p \, \frac{p^2}{\sqrt{p^2 + k^2 + m_q^2}}$$

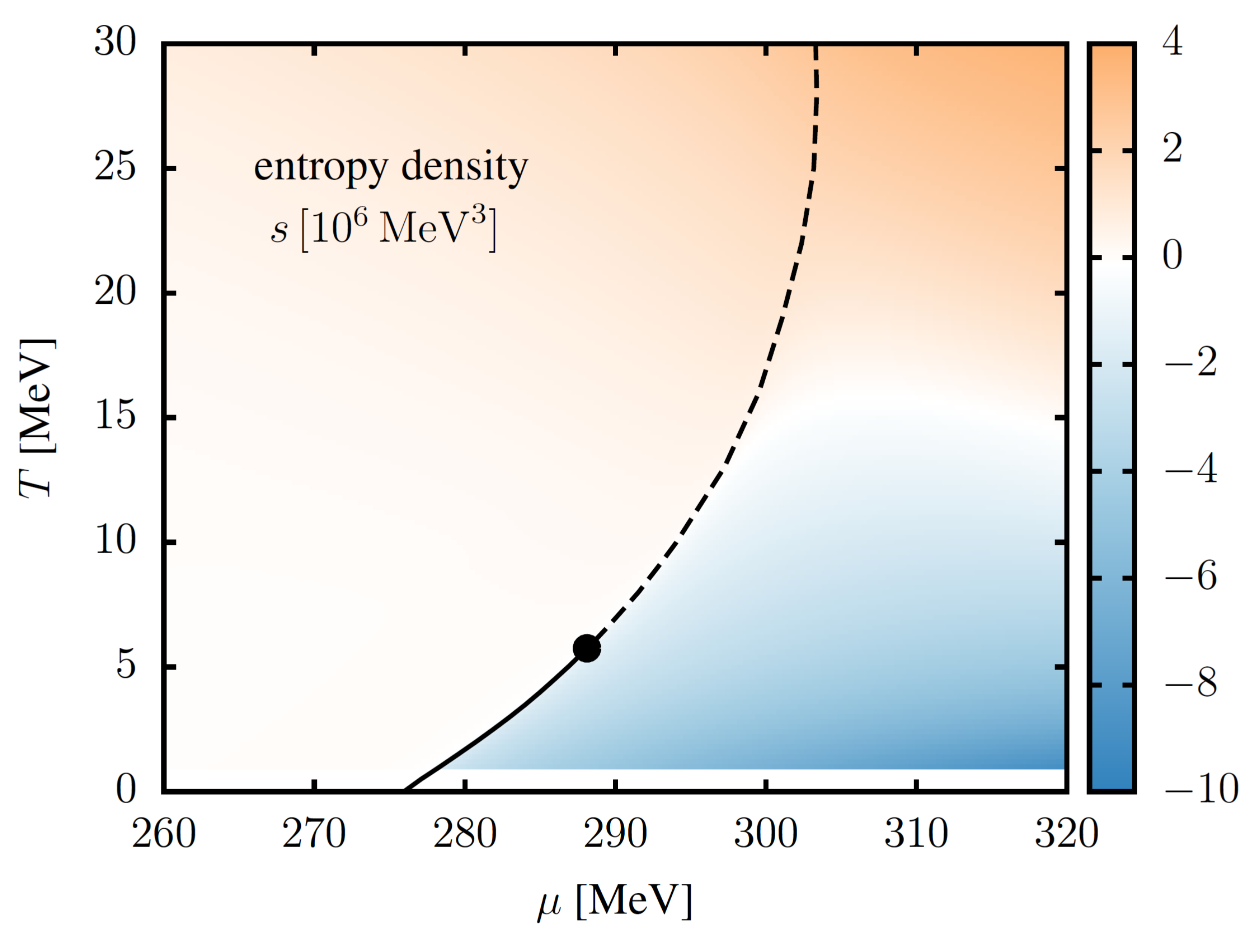
$$p_F := \begin{cases} \sqrt{\mu^2 - k^2 - m_q^2}, & \mu^2 > k^2 + m_q^2 \\ 0, & \text{else} \end{cases}$$



Chiral transition at low temperature

➜ no negative entropy density anymore for CS mass-like regulator





why are flows with CS mass-like regulators hard to solve in vacuum?

Pole proximity of vacuum flow

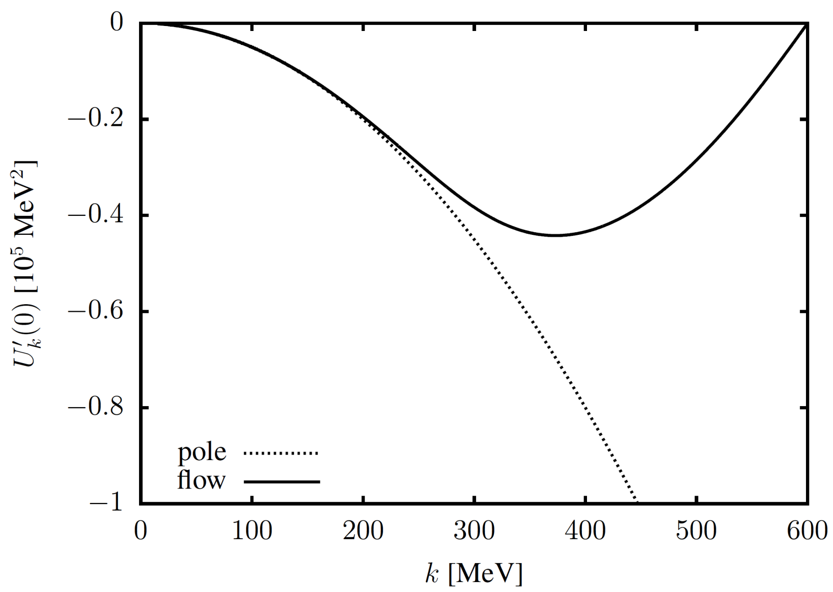
why are flows with CS mass-like regulators hard to solve in vacuum?

example: flat regulator (no problem)

$$\partial_t U_k^{\text{vac,flat}}(0) = \frac{k^5}{12\pi^2}\left(\frac{4}{E_\pi} - \frac{4N_cN_f}{k}\right)$$

$$E_\pi = \sqrt{k^2 + 2U_k'(0)}$$

pion pole



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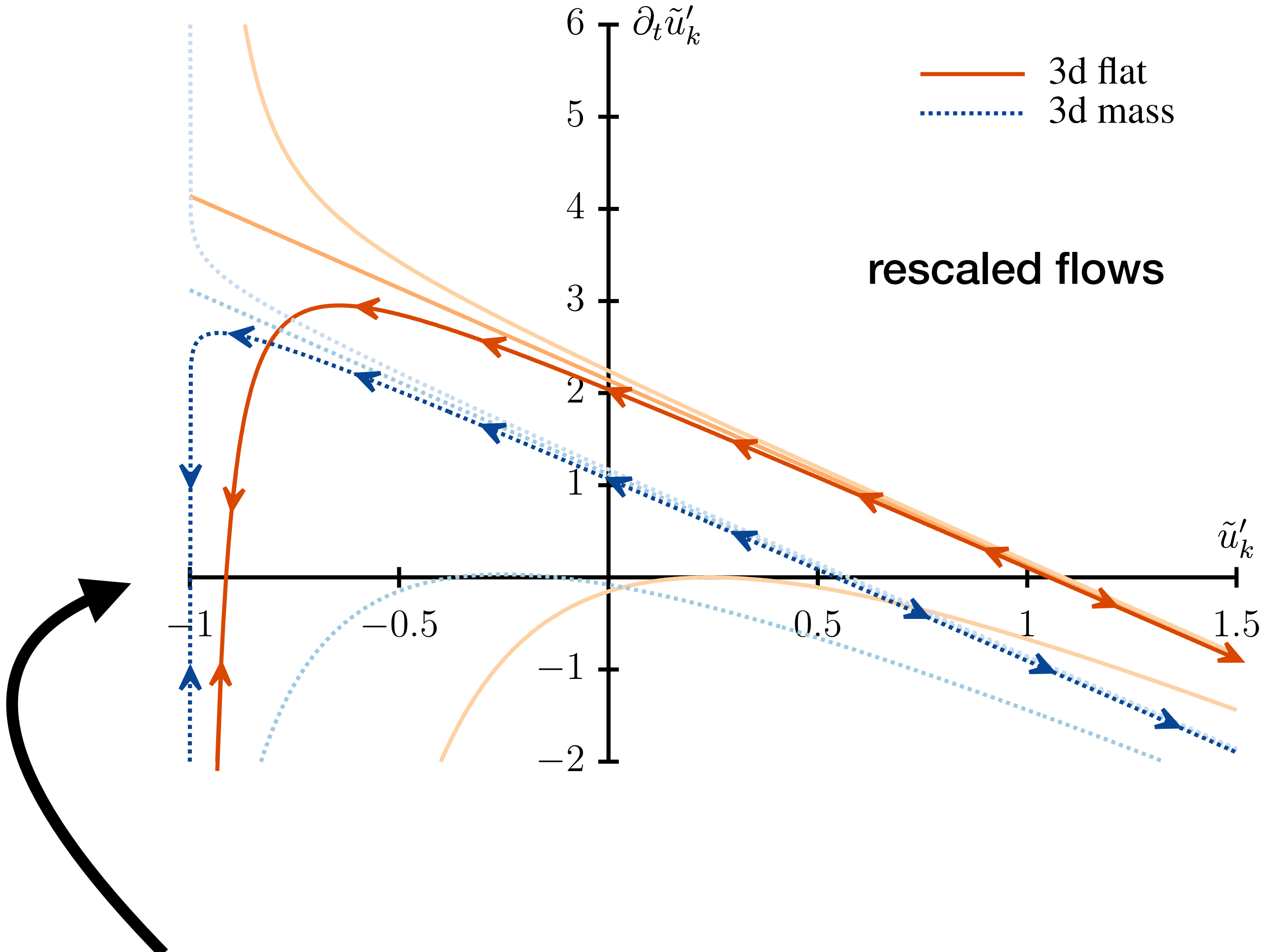

CS regulator run much closer to pole

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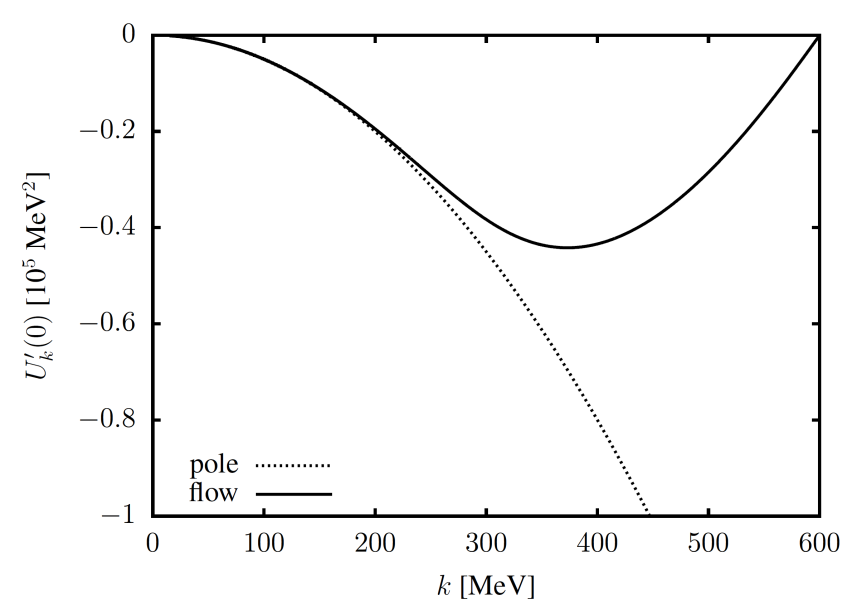

Pole proximity of vacuum flow

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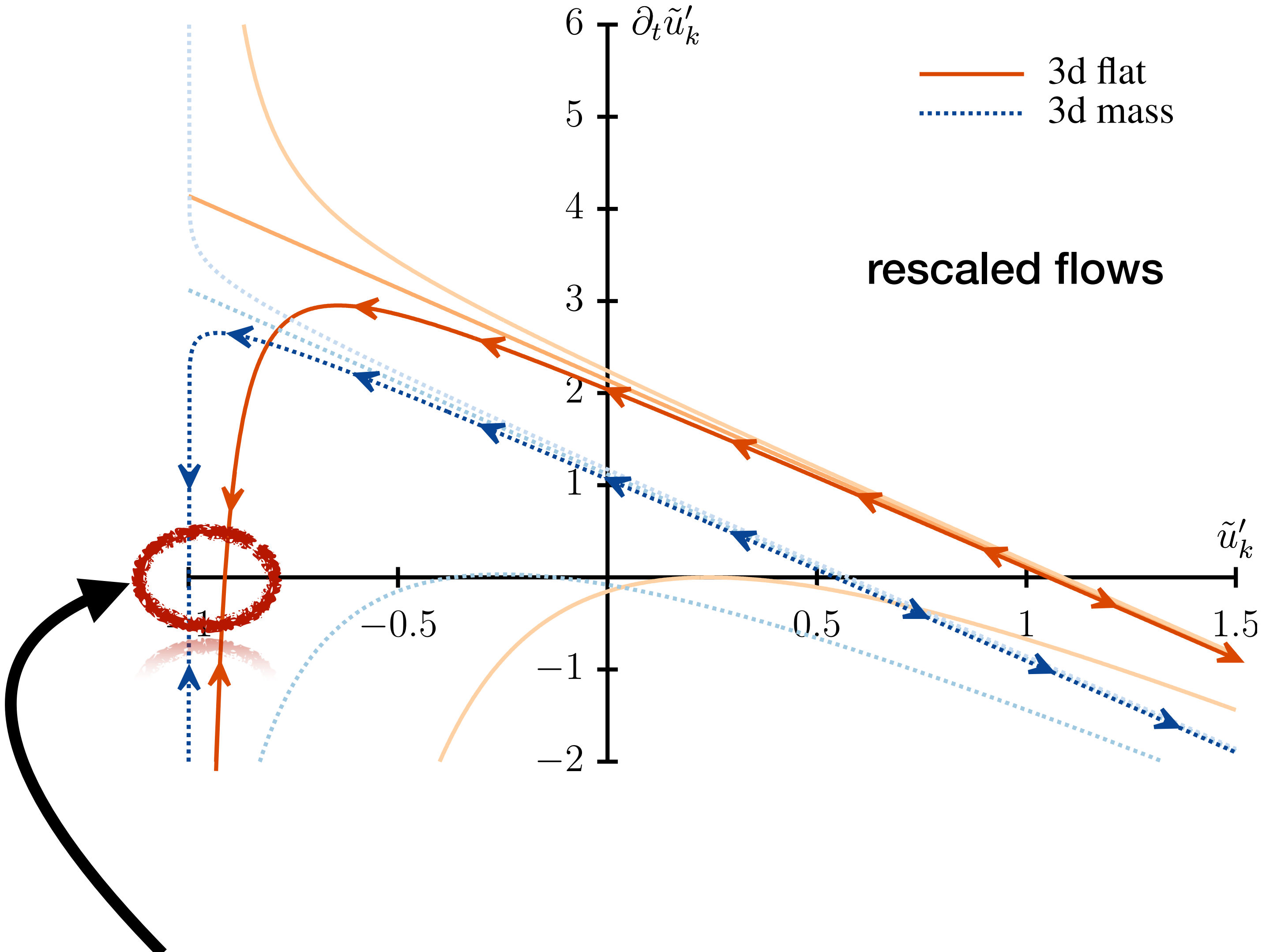

CS regulator run much closer to pole

distance to pole: useful as general criterion?

example: flat regulator (no problem)

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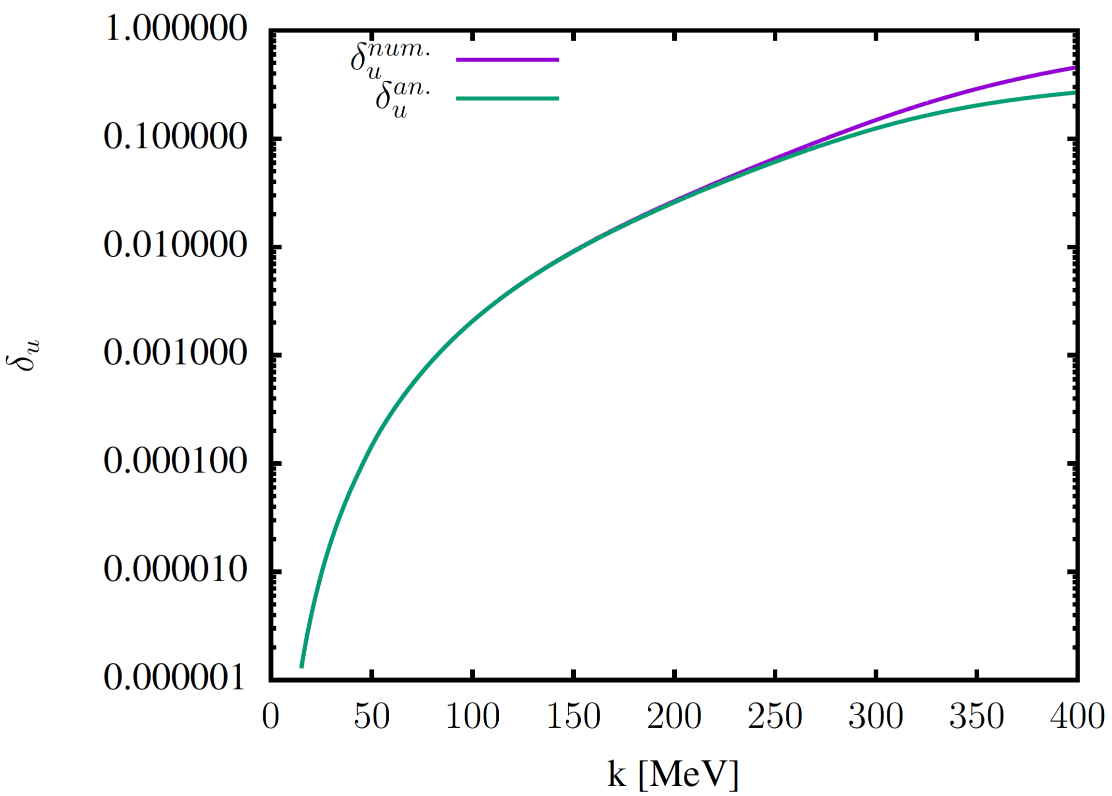
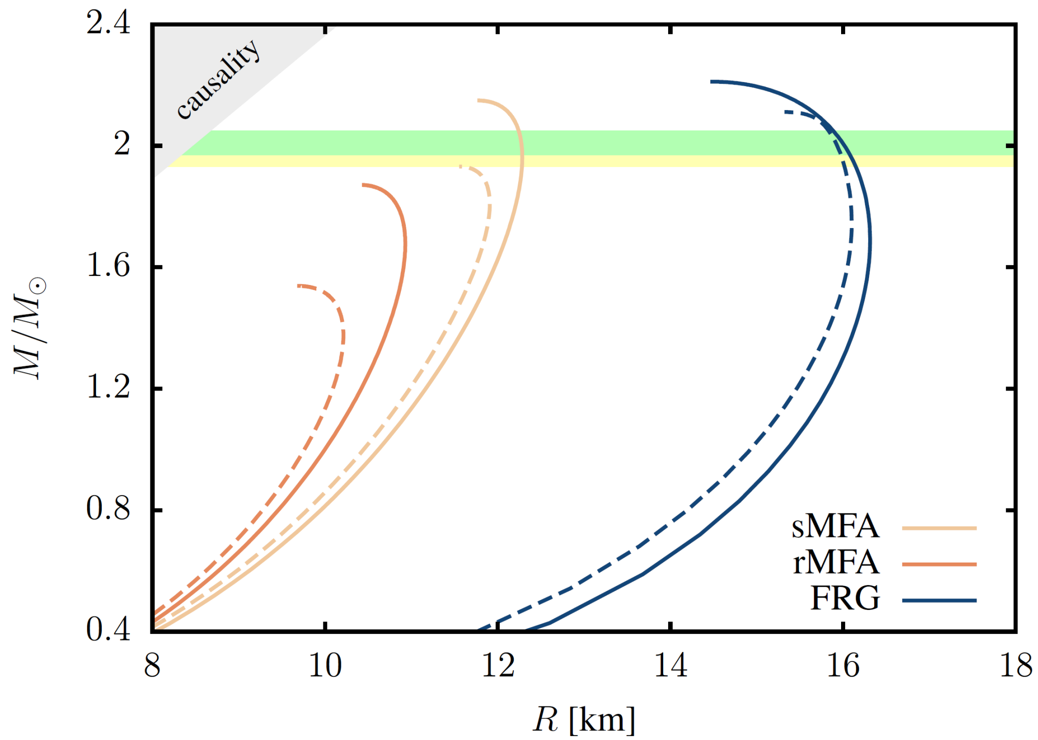


Summary

Three home messages:

1. EoS with the FRG for two and three quark flavor:

   ➜ significant impact of fluctuations
   on M-R relation for NSs



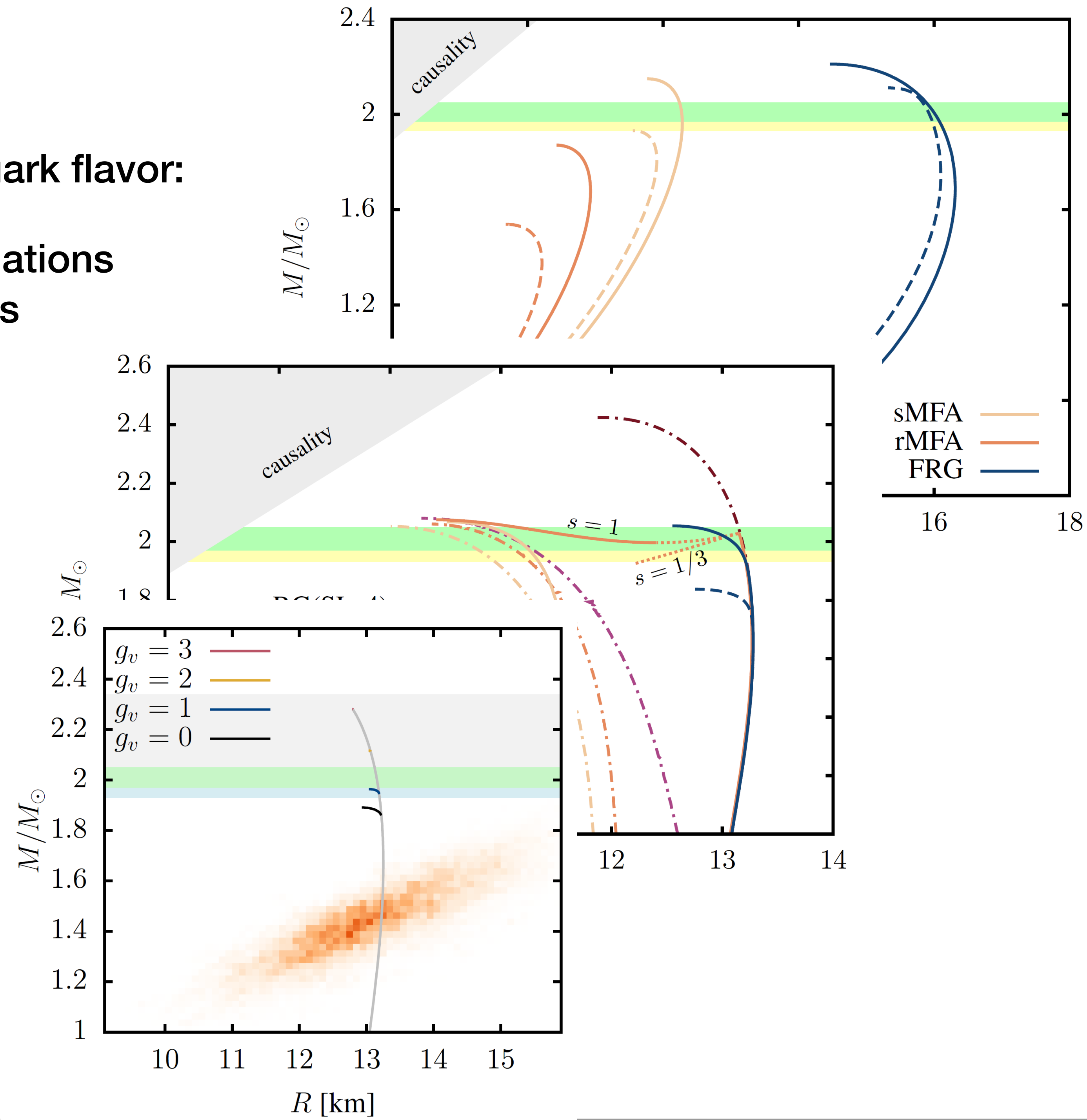
Summary

Three home messages:

1. EoS with the FRG for two and three quark flavor:

   ➜ significant impact of fluctuations
   on M-R relation for NSs

2. hybrid stars are possible



Non-zero vector coupling needed
➜ to reach 2 M⊙ with strangeness

Summary

3. in LPA no back-bending / negative entropy density
for
CS mass-like regulators

CS type regulators closer to

poles compared to flat regulator

➜ (vacuum) flows numerically harder