Interplay of Bosonic and Fermionic Fluctuations at Finite Densities

Christopher Busch

based on: Otto,Busch,Schaefer (2022), arXiv:2206.13067

HFHF Theory Retreat – 16.09.2022

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

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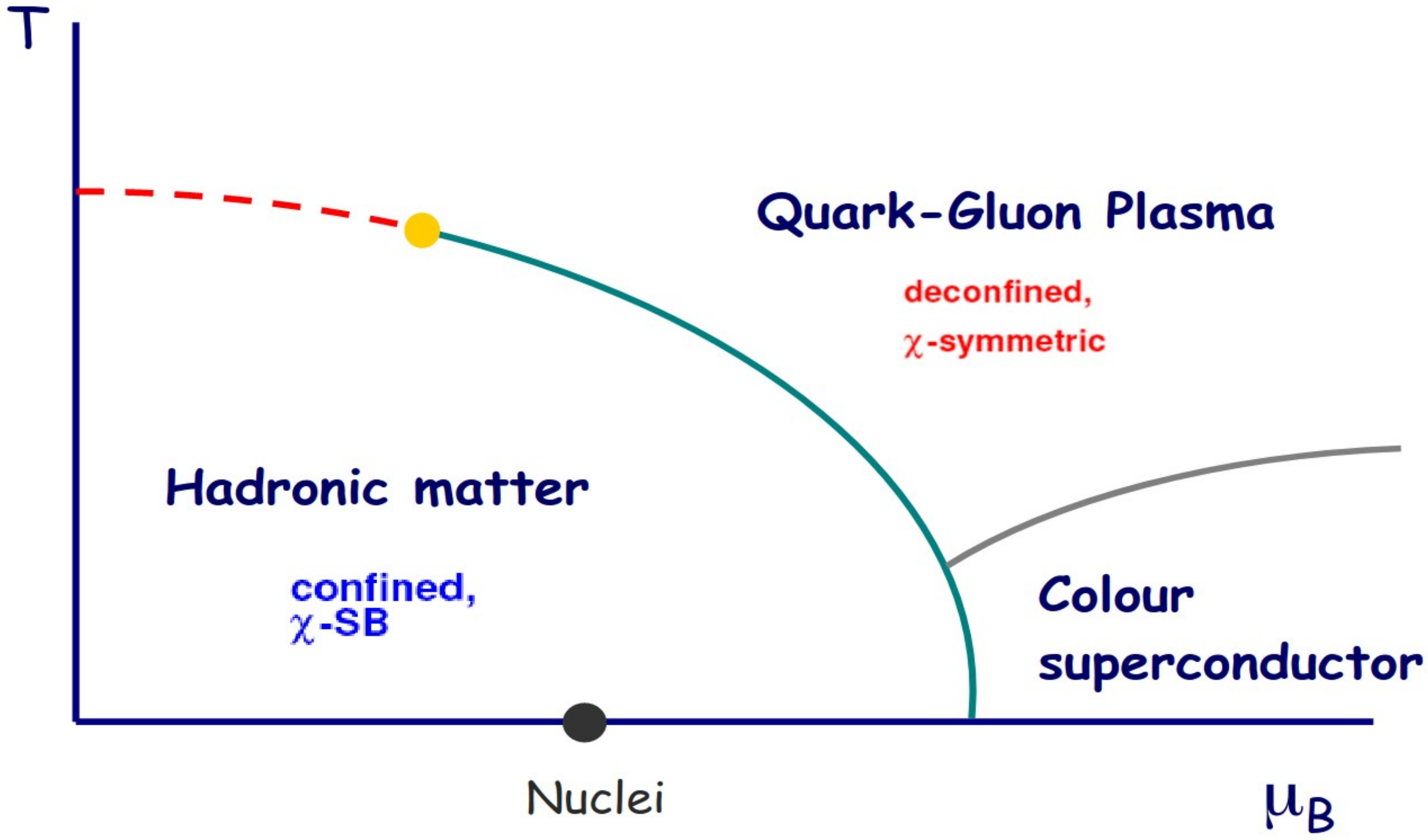
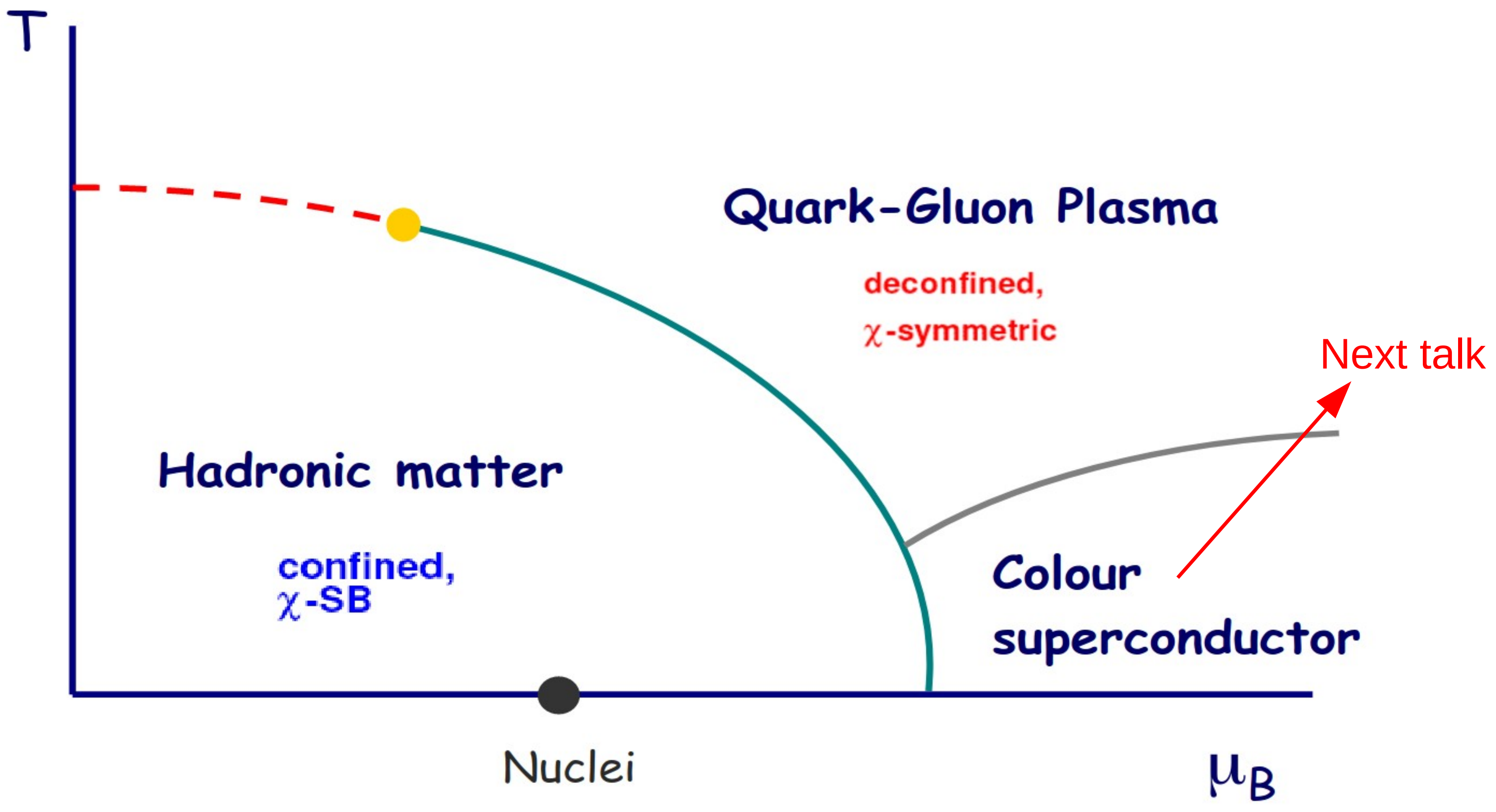
Helmholtz Graduate School for Hadron and Ion Research

Objectives



T
Quark-Gluon Plasma
deconfined,
χ-symmetric
Hadronic matter
confined,
χ-SB
Colour
superconductor
Nuclei
μB

Objectives



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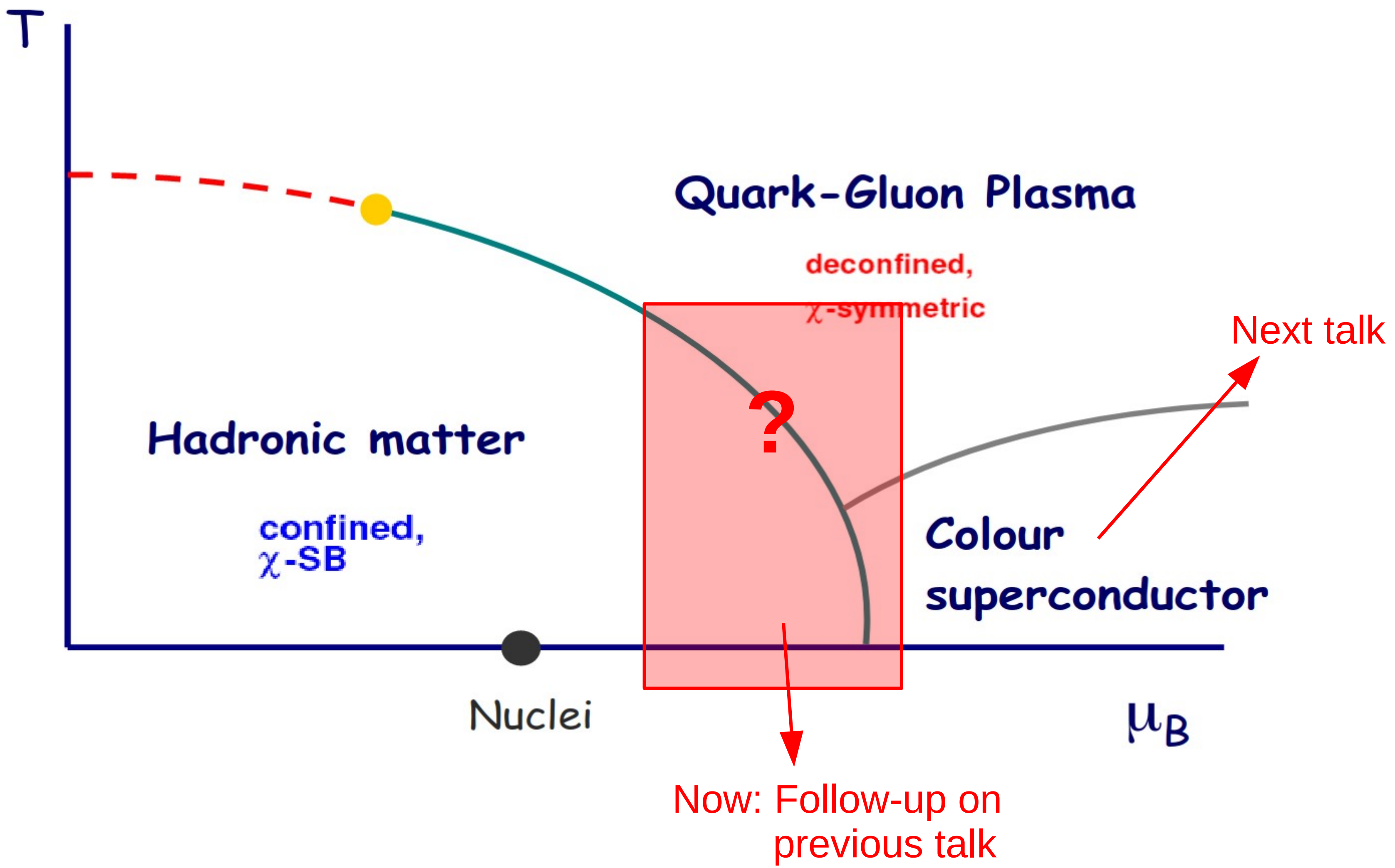
T
Quark-Gluon Plasma
deconfined,
χ-symmetric
Next talk
?
Hadronic matter
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μB
Now: Follow-up on
previous talk

Outline

1) Scheme Dependence

2) Modifications of the LPA Setup

3) Results: Regulator Dependence at Large Densities

4) Summary & Outlook

FRG and Truncation Errors

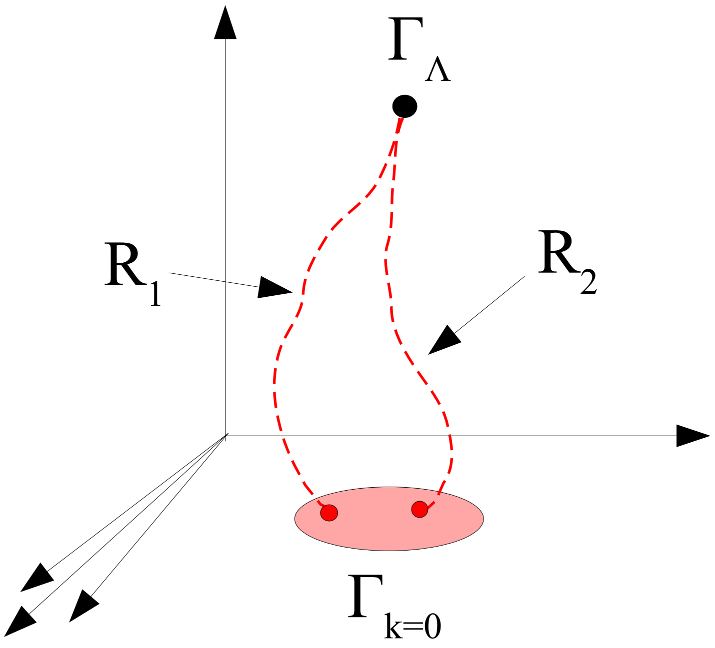
Wetterich equation: $(\partial_t = k\partial_k)$

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{(\partial_t R_k)\left(\Gamma_k^{(1,1)} + R_k\right)^{-1}\right\}$$

Requirements for regulators:

1. $\lim_{p^2/k^2 \to \infty} R_k\left(p^2\right) = 0$
2. $\lim_{p^2/k^2 \to 0} R_k\left(p^2\right) > 0$
3. $\lim_{k \to \infty} R_k\left(p^2\right) = \infty$

- Solving Wetterich eq. for full theory not possible
  - ➔ Use of effective models / truncations
- Leads to truncation errors and regulator dependence of results
  - ➔ Choice of regulator becomes relevant
    - ↳ Optimization criteria for regulators
      - Principle of minimum sensitivity
      - “Gap Criterion” [Litim(2000)]
      - “Shortest Path” [Pawlowski(2007)]



Flow Equation

(for dim. reduced regulators)

➢ Quark-Meson model in Local Potential Approximation (LPA):

$$\Gamma_k = \int \left\{ \bar{\psi} \left[\not{\partial} - \mu\gamma_0 + g\left(\sigma + i\gamma_5 \vec{\tau}\vec{\pi}\right)\right]\psi + \frac{1}{2}\left(\partial_\nu \phi\right)^2 + \mathrm{U}_k\left(\phi^2\right) - c\sigma \right\}$$

➢ General form of the (LPA) flow equation:

$$\tilde{m}_j := m_j/k$$

$$\partial_t \mathrm{U}_k = \frac{k^4}{4\pi^2}\left\{ l_0^B\left(\tilde{m}_\sigma^2\right) + 3\, l_0^B\left(\tilde{m}_\pi^2\right) - 4\mathrm{N}_f\mathrm{N}_c\, l_0^F\left(\tilde{m}_\psi^2\right)\right\}$$

- Splits into bosonic and fermionic loop contributions
- Threshold functions $l_0^{F/B}$ comprise distribution functions
- Fermions at T=0:

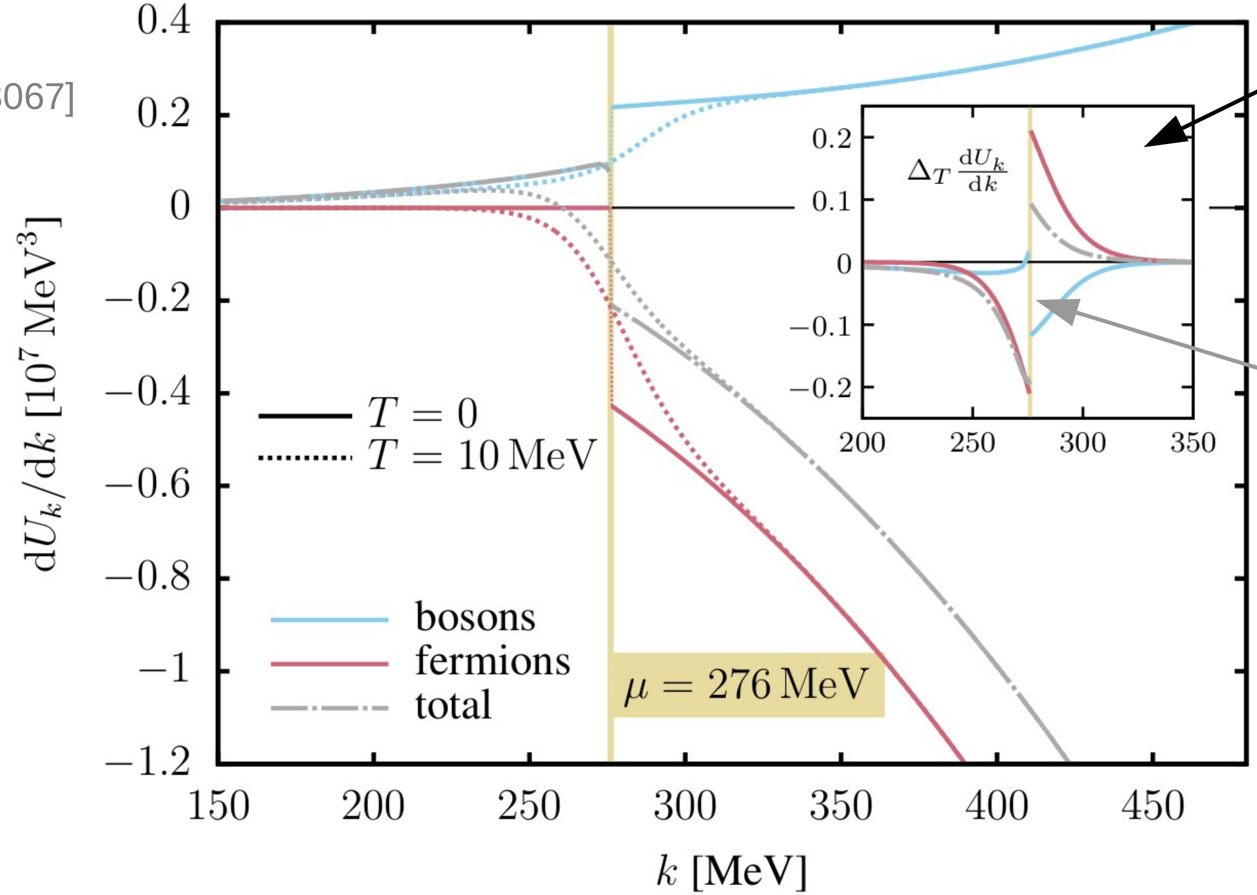
$$E_\psi(x) = \sqrt{x\left(1 + r_\psi(x)\right)^2 + \tilde{m}_\psi^2}$$

$$l_0^F\left(\tilde{m}_\psi^2\right)\big|_{T=0} = \int_0^\infty \frac{\mathrm{d}x}{2} x^{3/2} \partial_t \left(1 + r_\psi(x)\right)^2 \frac{1}{2E_\psi(x)} \left\{1 - \Theta\left(\mu - E_\psi(x)\right)\right\}$$

Fermions decouple regulator dependent

Flat Regulator: High Density Flow

[arXiv:2206.13067]



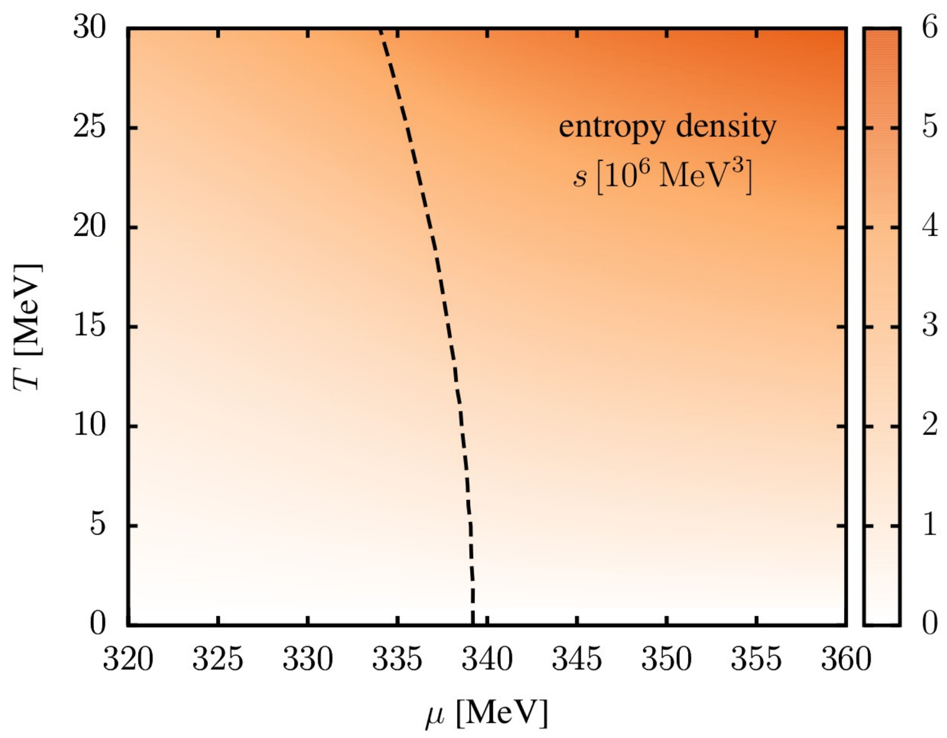
Difference
(T=10)-(T=0)

Asymmetry caused by bosons

Back-bending

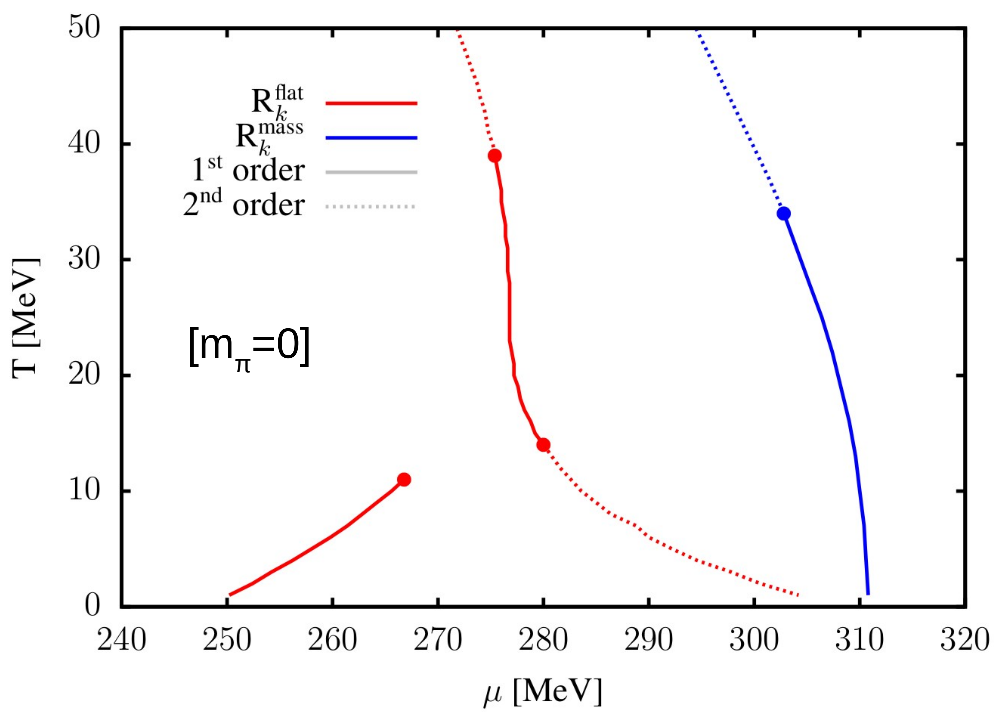
$$R_k^{\text{flat}} = \left(k^2 - \vec{p}^2\right)\,\Theta\left(k^2 - \vec{p}^2\right) \longrightarrow l_0^F \propto 1 - \Theta\left(\mu - E_\psi\right)$$

Endpoint with Mass-like Regulator



Endpoint with Mass-like Regulator

- Missing endpoint? Check chiral limit:



- Next with physical pion mass:
  - Want to vary m_σ
  - Not possible in current setup!

- Flat regulator:
  - Phase structure similar to previous findings: “triangular region”

[Schaefer, Wambach(2005)]

- Mass-like regulator:
  - No splitting of the phase transition line
  - First order transition and corresponding endpoint

↪ Existence of CEP depends on masses

How to Improve the LPA Setup?

- Limitations in the previous setup:
  - ➔ Range of possible sigma masses strongly restricted
  - ➔ Only low temperatures not affected by cutoff effects
  - ➔ Vacuum calculations not feasible for mass-like regulators

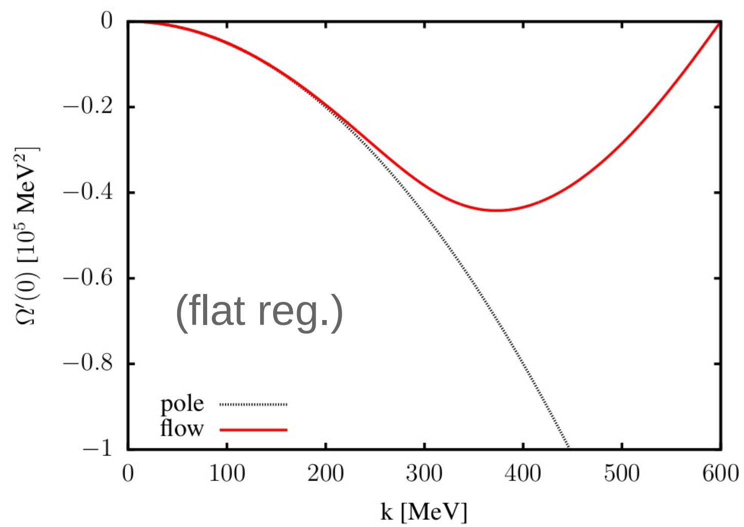
...caused by:

<u>Parameter finding procedure:</u>

$k_\phi \rightarrow k_\chi \rightarrow k_{IR}$

- k_χ and k_ϕ as only free parameters
- small UV-cutoff

<u>Proximity to pion pole:</u>



How to Improve the LPA Setup?

- <u>Idea:</u> Use two different types regulators for fermions and bosons
  - Proximity to pion pole determined by bosonic regulator

    ➔ Choose flat regulator here to stabilize calculations and allow “usual” way of parameter fixing

  - Fermion decoupling sensitive to regulator choice

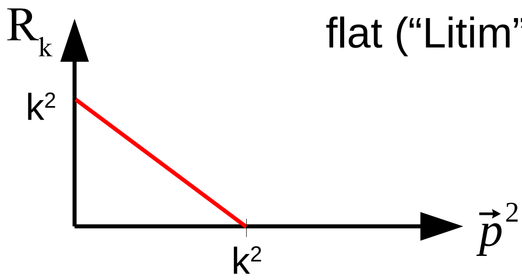
    ➔ Vary fermionic regulator:

    Check how bosons “react” to different flows

Regulator Choices: Varying the Cutoff

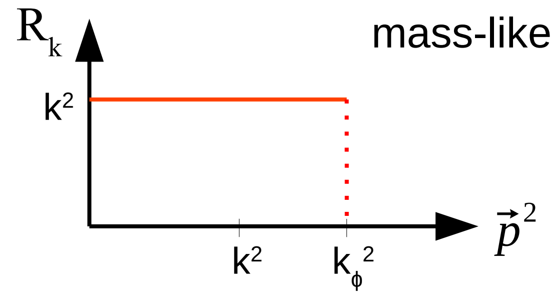
➢ Different types of (3d-) regulators, “bosonic form”:

flat (“Litim”) regulator



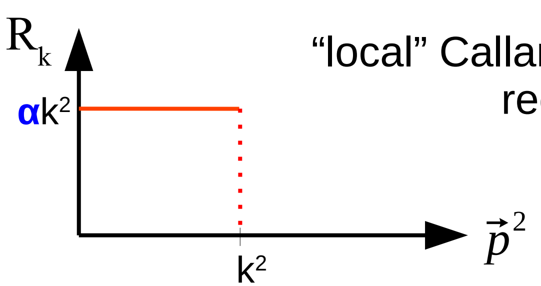
$$R_k^{\text{flat}} = \left(k^2 - \vec{p}^{\,2}\right)\,\Theta\left(k^2 - \vec{p}^{\,2}\right)$$

mass-like regulator



$$R_k^{\text{mass}} = k^2\,\Theta\left(k_\phi^2 - \vec{p}^{\,2}\right)$$

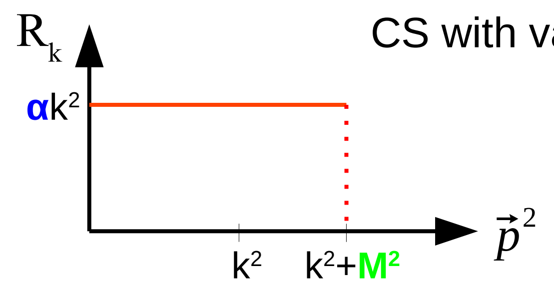
“local” Callan-Symanzik (CS) regulator



$$R_k^{\text{CS,loc}} = \boldsymbol{\alpha} k^2\,\Theta\left(k^2 - \vec{p}^{\,2}\right)$$

allows fit to R_k^{flat}-flow

CS with variable cutoff



$$R_k^{\text{CS,var}} = \boldsymbol{\alpha} k^2\,\Theta\left(k^2 + \boldsymbol{M^2} - \vec{p}^{\,2}\right)$$

see also: “Renormalized spectral flows”, (Braun et al., 2022)

Regulator Choices

$$R_k^{\text{CS,var}} = \alpha k^2 \; \Theta\left(k^2 + M^2 - \vec{p}^2\right)$$

- Parameter M:
  - Momentum boundary M, enables testing for relevant momenta
  - “Interpolation” between other regulators:

$$R_k^{\text{CS,var}} \approx \begin{cases} R_k^{CS,loc} & , k \gg M \\ R_k^{\text{mass}} & , k \ll M \end{cases}$$

- Connection between fermionic and bosonic regulators:

$$R_k\left(\vec{p}^2\right) = \vec{p}^2 \; r_\phi\left(\vec{p}^2/k^2\right)$$

$$R_k^\psi\left(\vec{p}^2\right) = i\vec{\not{p}}\, r_\psi\left(\vec{p}^2/k^2\right) \qquad \text{with} \qquad r_\psi(x) := \sqrt{1 + r_\phi(x)} - 1$$

Missmatch in Contributions

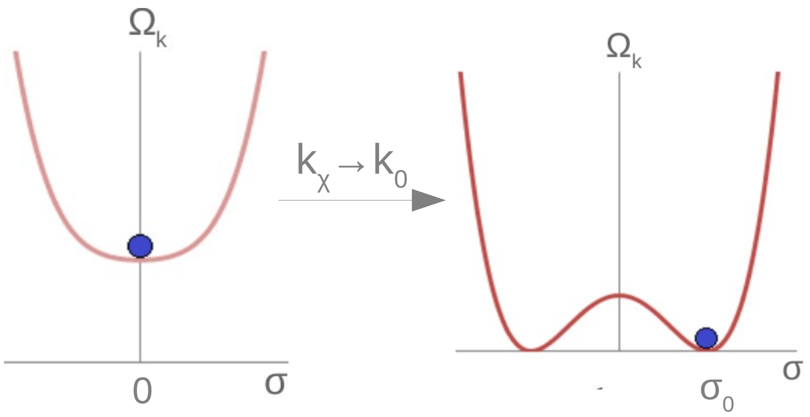
- Large differences between flows from different regulators
  - ↳ Missmatch: Different effective scales for bosons and fermions
  - ↳ partly cured by rescaling / matching vacuum flows

- Chiral sym. breaking scale k_χ:

Fermi Regulator	Λ	λ	v^2	k_χ
flat	0.9	20.9	0.355	0.605
mass	0.6	6.5	0.351	0.206
CS,loc	0.9	24.4	0.305	0.626
CS,var (M=0.3)	0.9	17.5	0.377	0.588
CS,var (M=0.5)	0.9	9.1	0.496	0.454

(in GeV units)

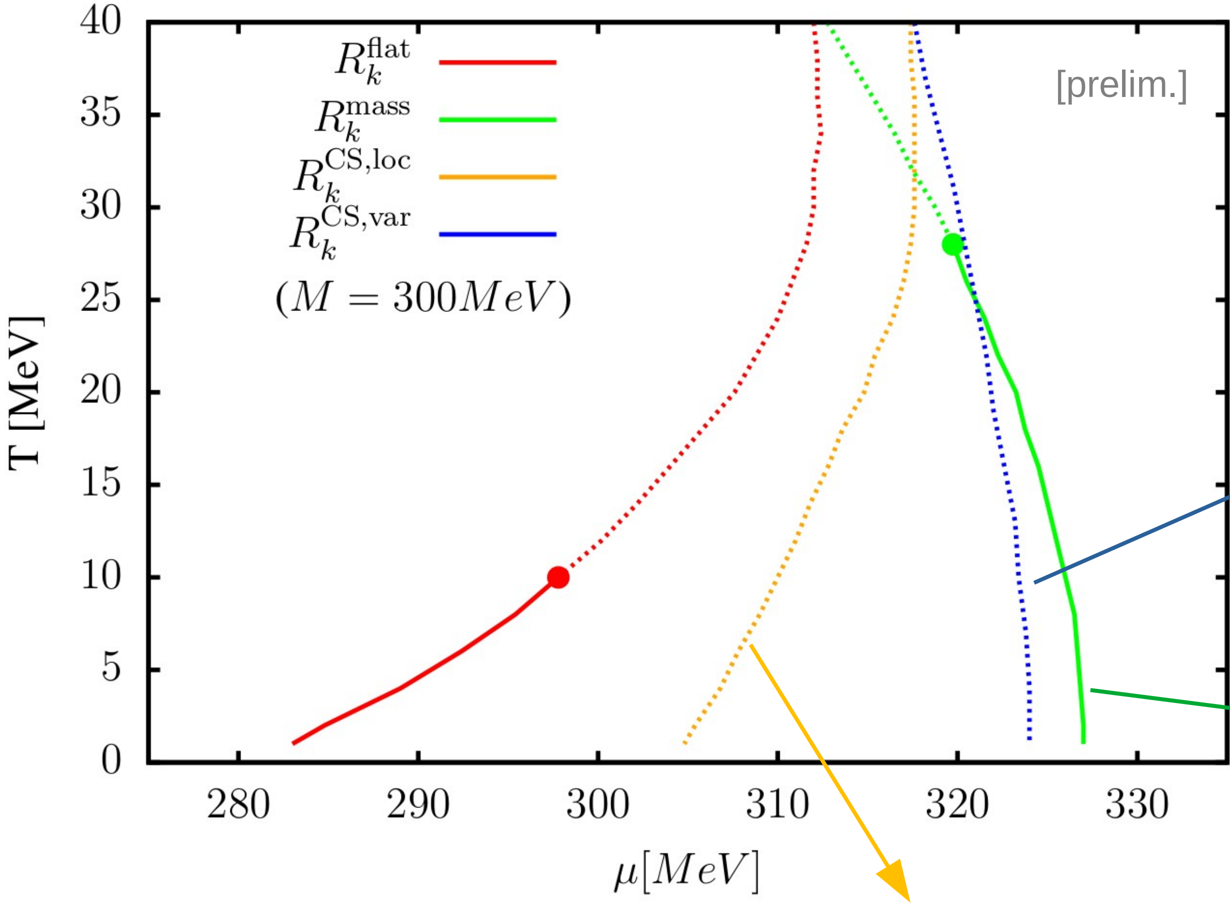
→ Fermionic contributions strongly overestimated



Numerical Results

- In the following:
  - For bosons the flat regulator is used for all results
  - Comparison of different fermionic regulators

Phase Transition with Different Regulators



$$R_k^{\text{flat}} = (k^2 - \vec{p}^2)\,\Theta(k^2 - \vec{p}^2)$$
$$R_k^{\text{mass}} = k^2\ \Theta(k_\phi^2 - \vec{p}^2)$$
$$R_k^{\text{CS,loc}} = \alpha k^2\ \Theta(k^2 - \vec{p}^2)$$
$$R_k^{\text{CS,var}} = \alpha k^2\ \Theta(k^2 + M^2 - \vec{p}^2)$$

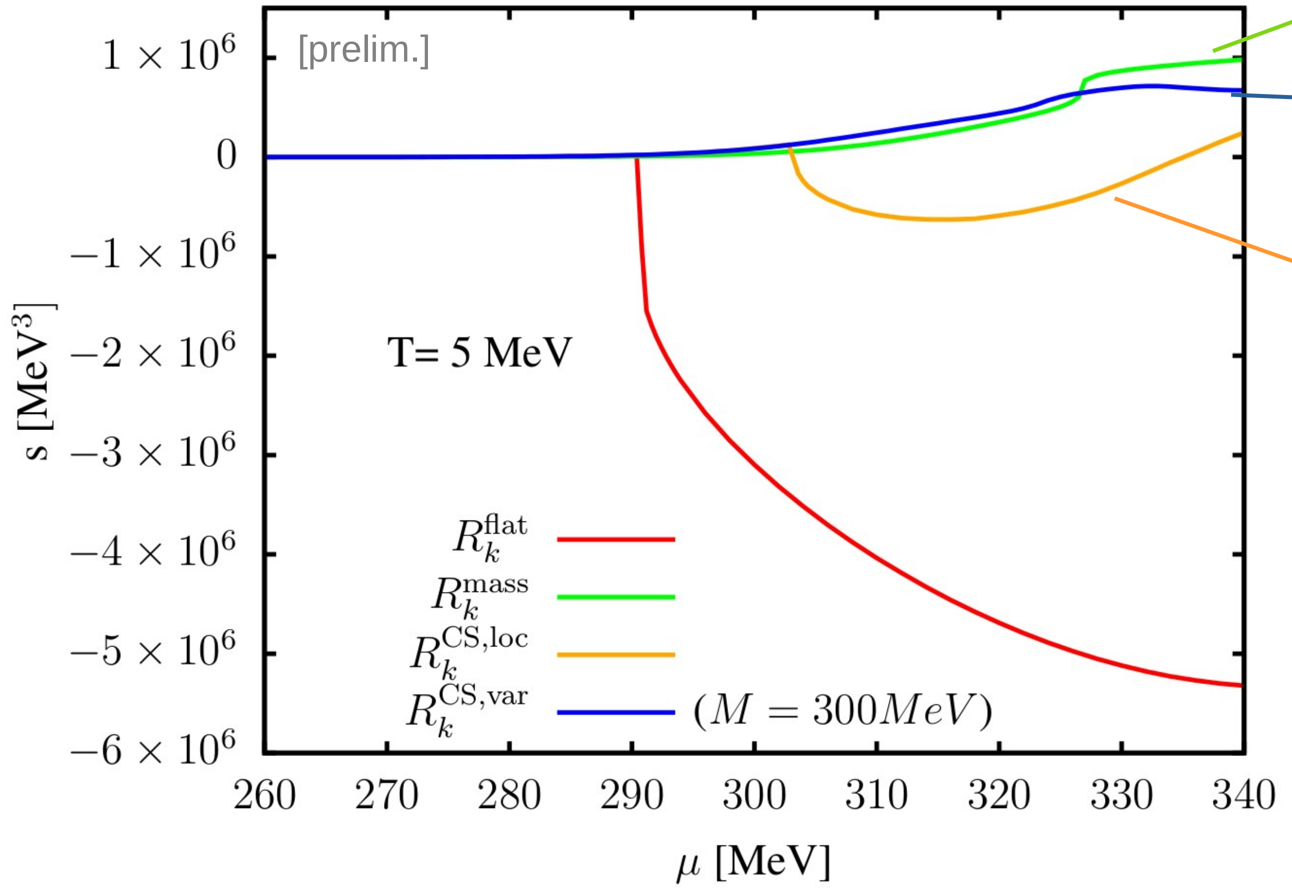
similar to mass-like regulator

first order transition due to overly large fermionic contributions

again back-bending, but reduced

$$m_\pi = 138MeV$$

Entropy Density



similar to mean field

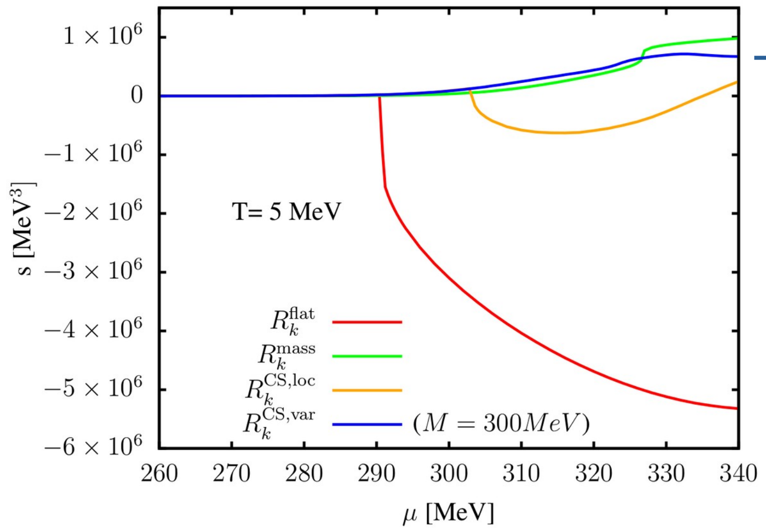
strange?

negative region strongly reduced

Clausius-Clapeyron:

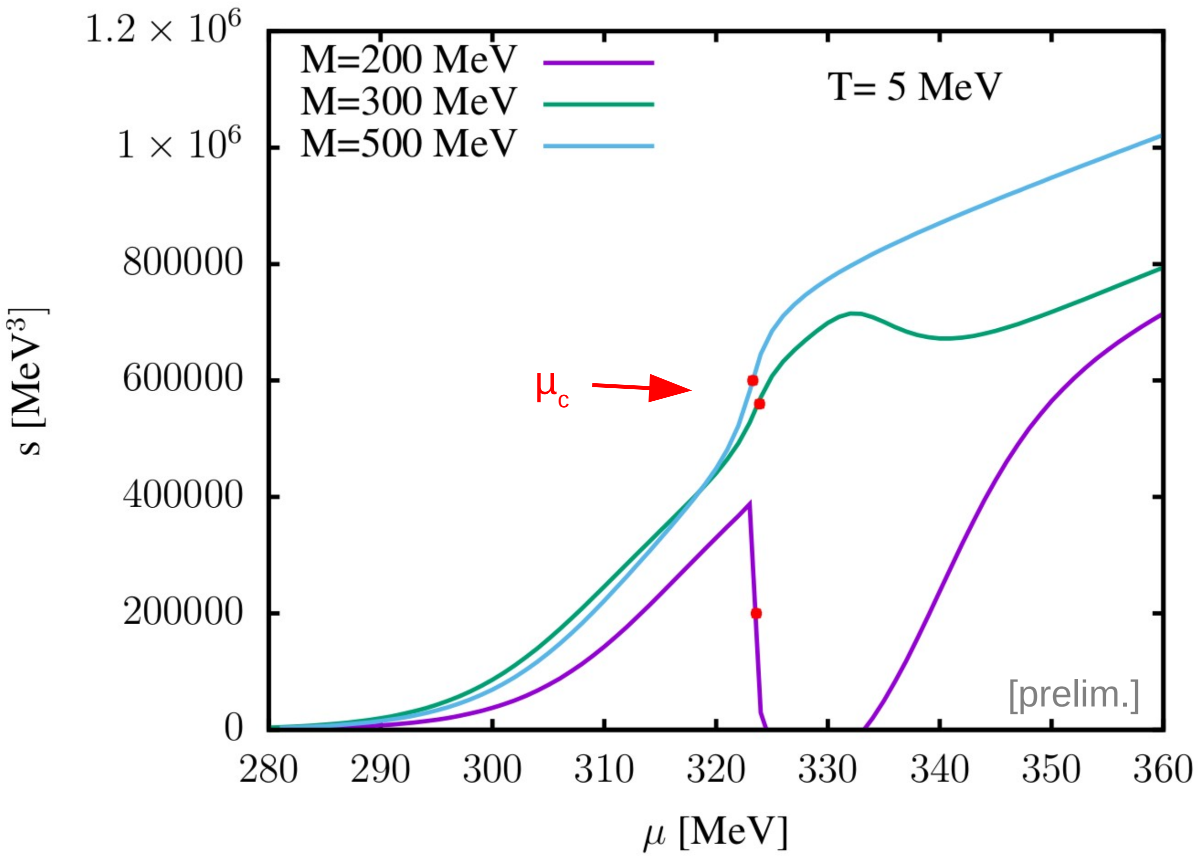
$$\frac{dT_c}{d\mu} = -\frac{\Delta n_q}{\Delta s}$$

Entropy Density



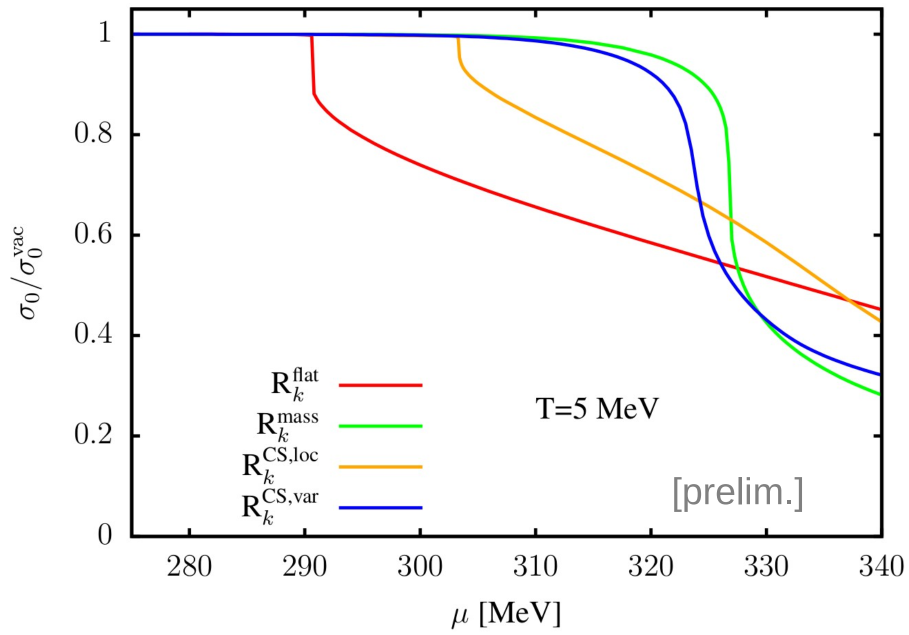
Strange behavior for $R_k^{\mathrm{CS,var}}$?

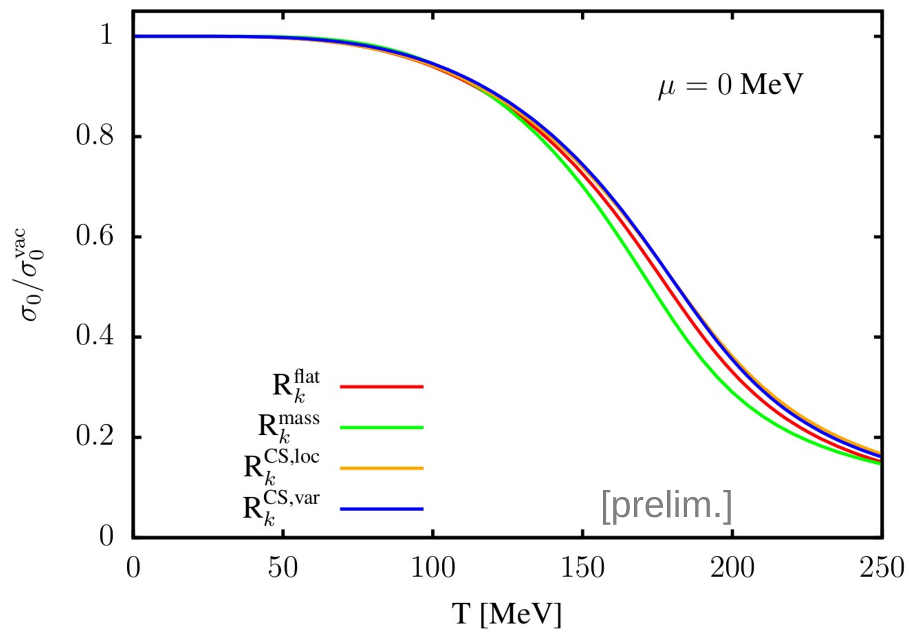
Compare different M-values:



- Negative entropy for $M \lesssim 300 MeV$
- "Dip" if $M \lesssim \mu$ as remnant away from phase transition

σ-Condensate: Comparison with μ=0-axis

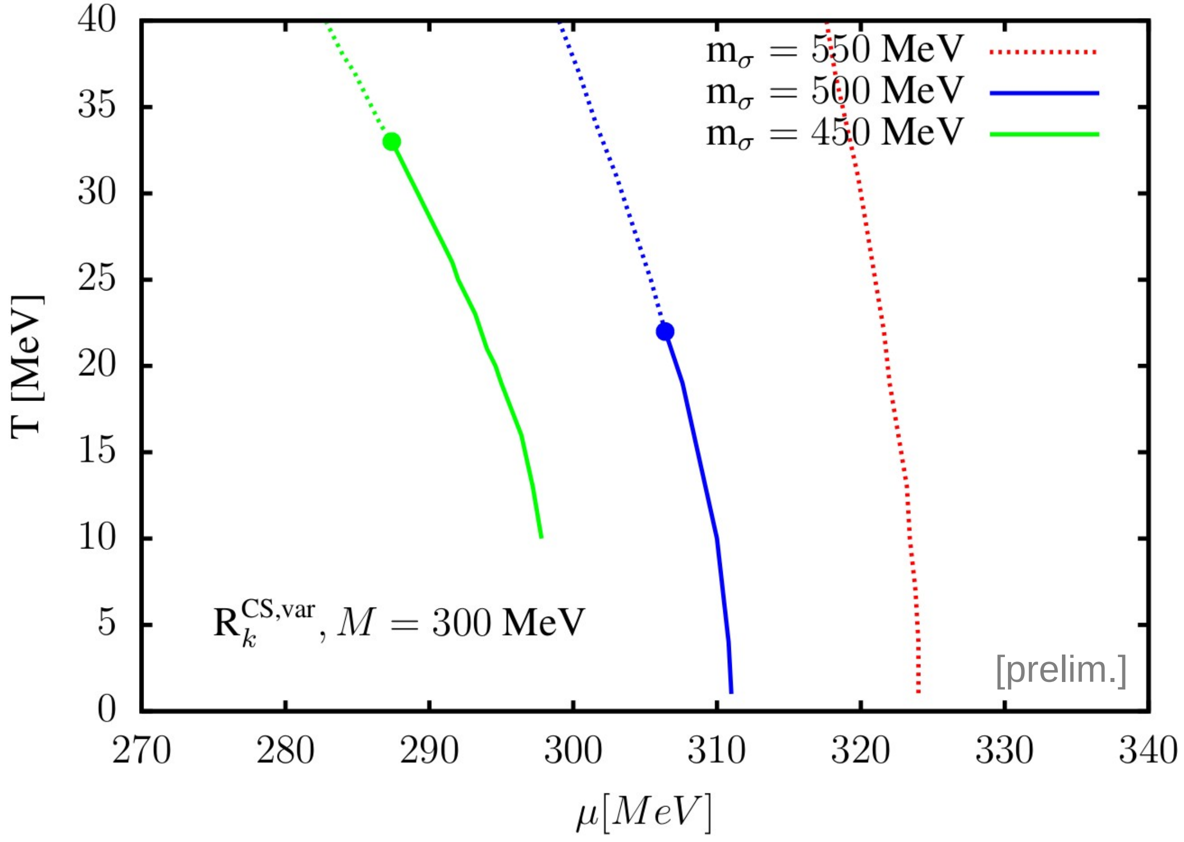




- Large distinctions for different R_k
- Back-bending: Small transition and residual condensate
- others more similar to MF result, mass-like much steeper

- Regulator effects much smaller, largest in crossover region
- Critical Temperature varies only by ~5 MeV

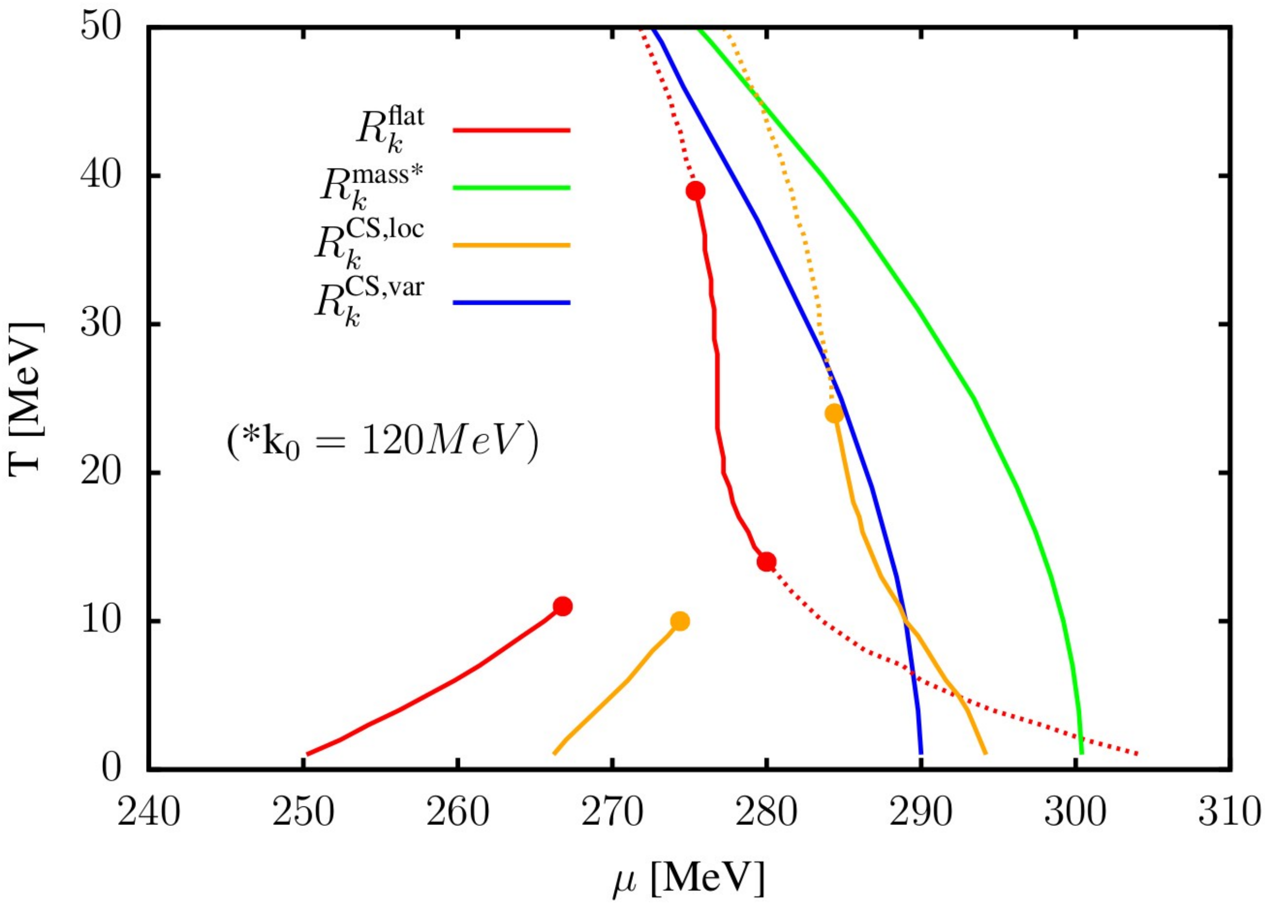
CEP for Different σ-masses



- No CEP when σ-mass similar to previous calculations
- CEP appears & crit. temperature increases when m_σ is lowered

$$R_k^{\mathrm{CS,var}} = \alpha k^2 \, \Theta\left(k^2 + M^2 - \vec{p}^2\right)$$

Chiral Limit



$$R_k^{\text{flat}} = \left(k^2 - \vec{p}^2\right)\Theta\left(k^2 - \vec{p}^2\right)$$
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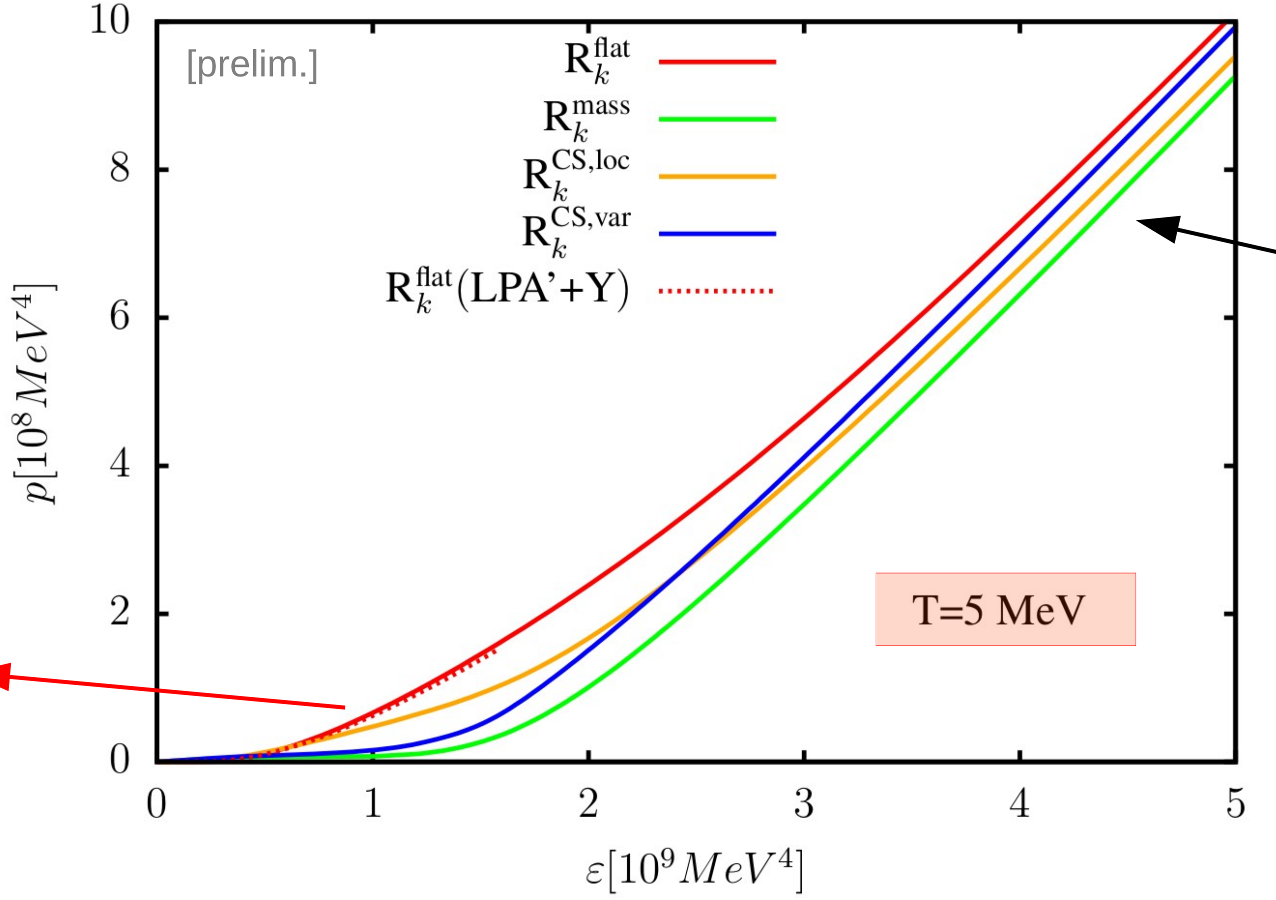
Equation of State



slope stays similar

Differences much larger than those in comparison to LPA'+Y

Summary

- Investigated truncation effects in the FRG framework

- Mass-like regulator: **No** back-bending, entropy remains **positive** but limited usability

- Tested Setup with fixed bosonic and different fermionic regulators
  - UV-cutoff of CS-type regulators crucial for results
  - artifacts found as soon as momenta $\vec{p}^2 < \mu^2$ are cut
  - only minor regulator effects at vanishing chemical potential
  - Nice testing ground but surely not “the final answer”

Outlook

- Effects on neutron star equation of state (lower T) and mass-radius relations?
- Find solutions/regulators for more advanced truncations, e.g. LPA’ or higher order derivative expansions
- Additional channels (e.g. via dynamical hadronization)
- Upgrade momentum dependencies

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- Effects on neutron star equation of state (lower T) and mass-radius relations?
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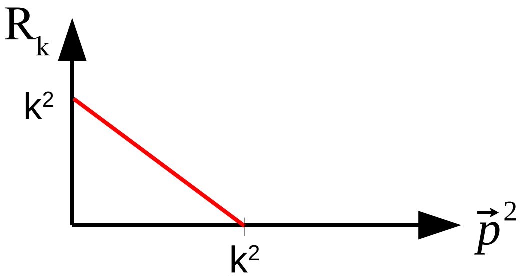
GRAZIE MILLE !

Backup

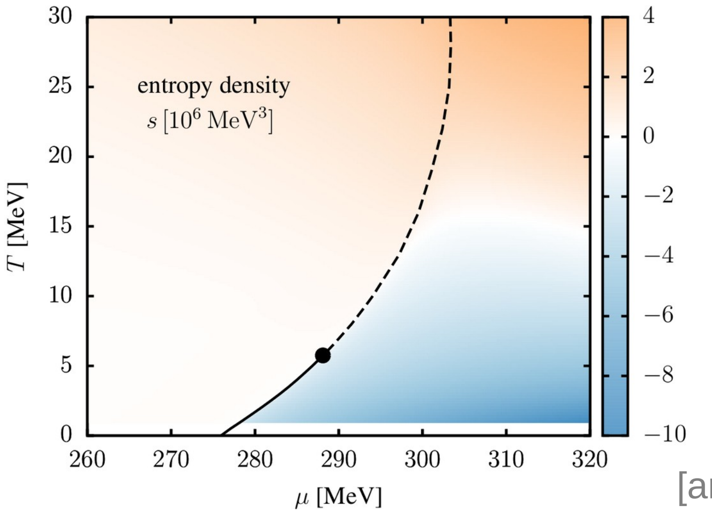
Throwback to the Previous Talk

Quark-meson model in Local Potential Approximation (LPA):

$$\Gamma_k = \int \left\{ \bar{\psi} \left[\not{\partial} - \mu\gamma_0 + g\left(\sigma + i\gamma_5 \vec{\tau}\vec{\pi}\right)\right]\psi + \frac{1}{2}\left(\partial_\nu \phi\right)^2 + U_k\left(\phi^2\right) - c\sigma \right\}$$

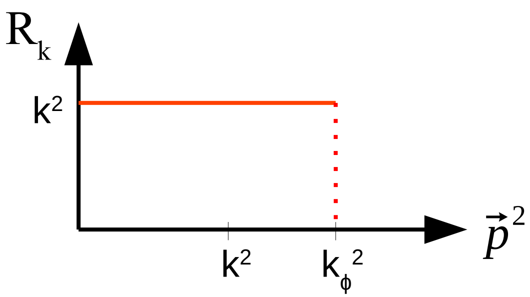


$$R_k^{\text{flat}} = \left(k^2 - \vec{p}^2\right)\,\Theta\left(k^2 - \vec{p}^2\right)$$



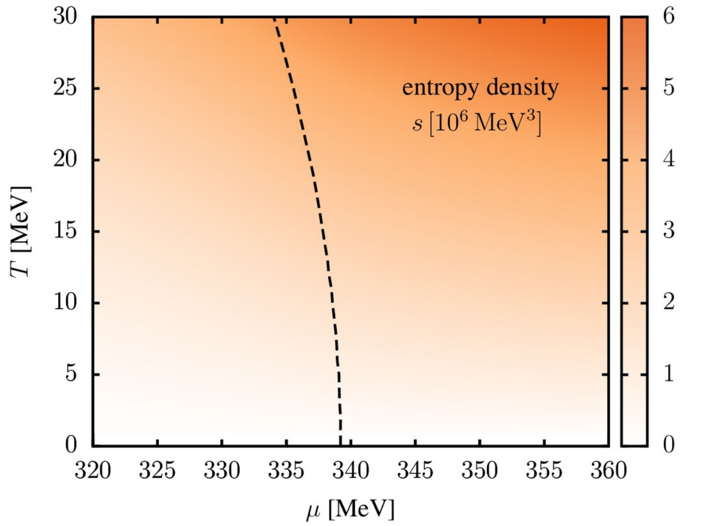
dim.reduced regulators

[arXiv:2206.13067]



$$R_k^{\text{mass}} = k^2\,\Theta\left(k_\phi^2 - \vec{p}^2\right)$$

compositeness scale



Why are we using 3d-regulators?

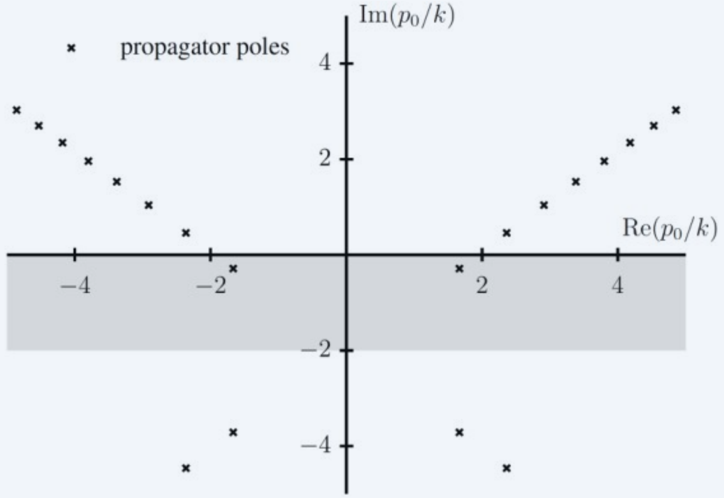
- Downside: Dimensionally reduced regulators break O(4) symmetry

- Upsides:
  - Matsubara summation can be performed analytically
  - Easiest way to prevent regulators from breaking Silver Blaze symmetry

4d fermionic regulators: $R_k^F = i\not{p}\, r^F(p^2/k^2)$

- Necessary for Silver Blaze: $R_k^F(p,\mu) = R_k^F(\tilde{p},0)$ with $\tilde{p} = (p_0 + i\mu,\, \vec{p})$

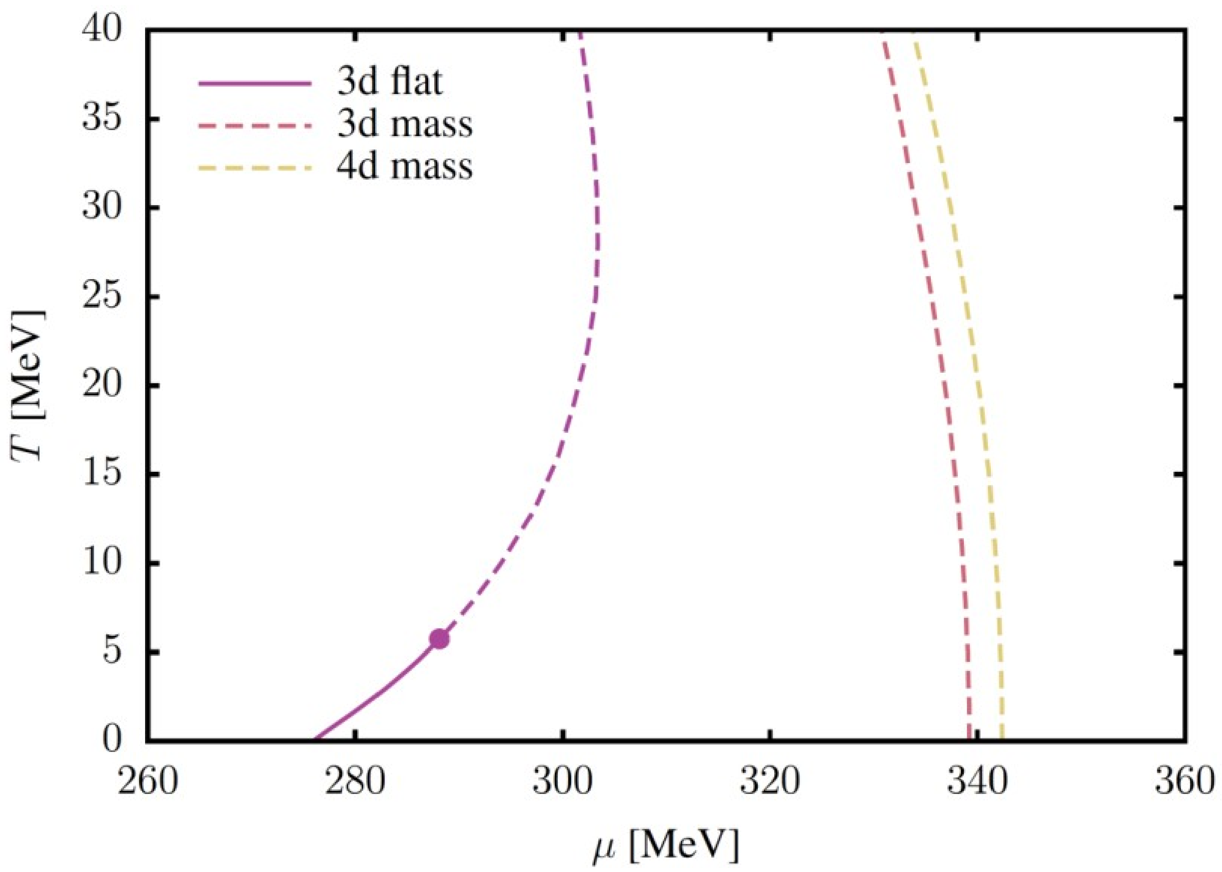
- Silver Blaze violation still possible due to regulator induced complex poles, e.g. with exponential reg.:



- Discussion and Solutions:
  [Pawlowski,Strodhoff (2015)]

Why are we using 3d-regulators?

- Mass-like regulators allow us to circumvent this problems
- Only small quantitative differences found:

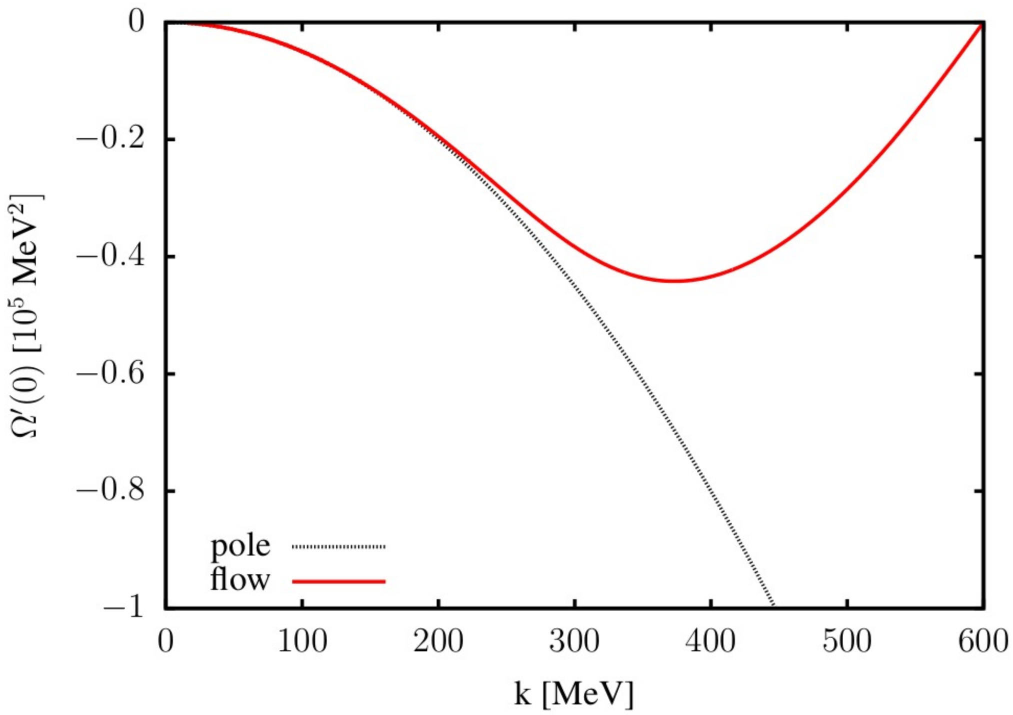


→ Minor impact of dimensional reduction on results

→ Especially: No connection between back-bending and 3d-regulators

Pole Proximity: Estimation

- Reminder: Vacuum flow runs close to the pole at E_π=0:

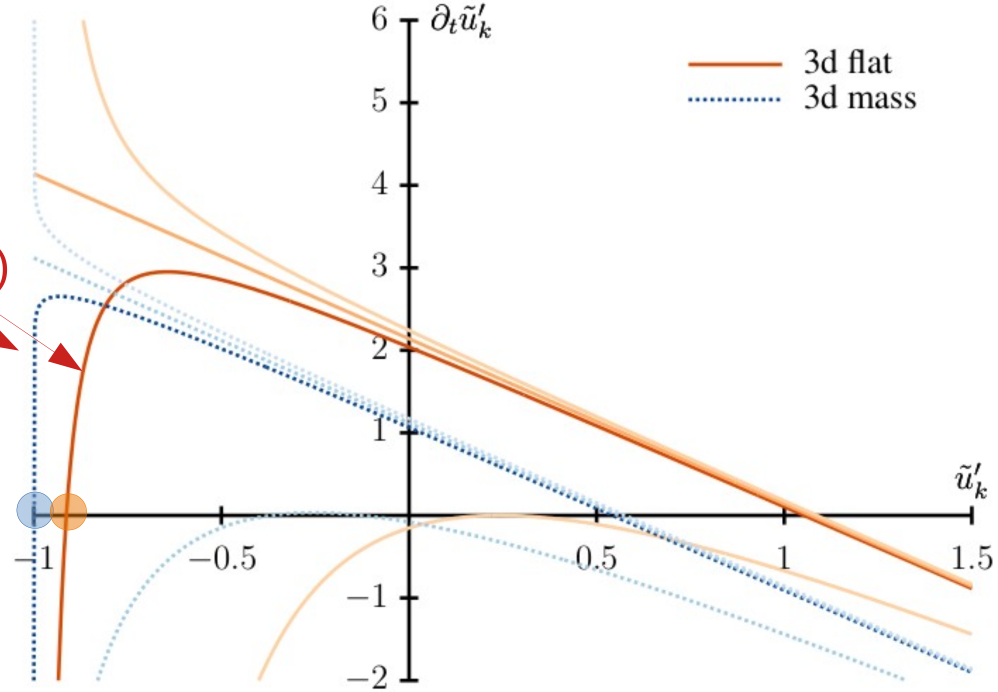


How can we estimate the distance to this pole for a given regulator?

Pole Proximity: Estimation

- For convenience define $\tilde{u}'_k = 2U'_k(0)/k^2$ ⟶ Pole at $\tilde{u}'_k = -1$
- Examine it's flow equation for different (fixed) values U'' :

Relevant cases (small U''>0)



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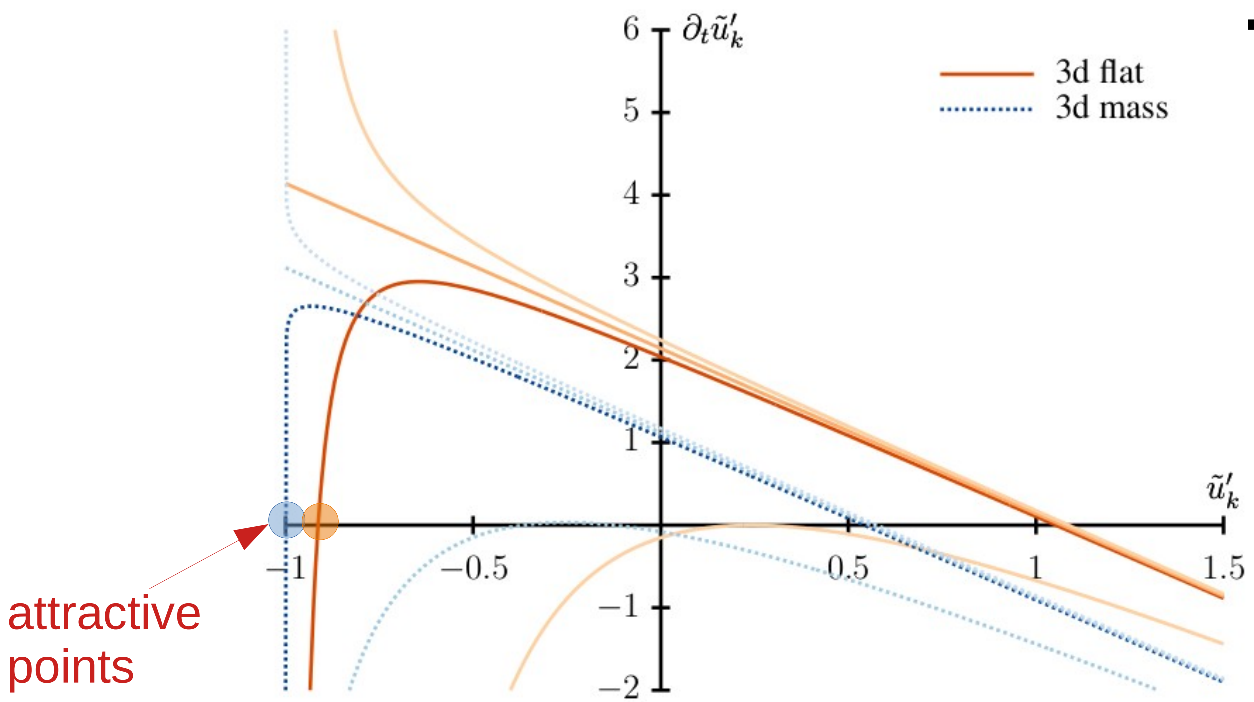

➔ Relevant cases have <u>attractive stationary point</u> near pole

↳ no fixed point (depends on U'')

↳ Explains behavior observed in the flow

↳ Setting $\partial_t \tilde{u}'_k = 0$ and expanding in $\delta u = \tilde{u}'_k + 1$ allows to approximate distance between pole and attractive point