JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Diquarks and Equation of State of Dense Quark Matter

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HFHF Theory Retreat

Castiglione della Pescaia - 16/09/2022

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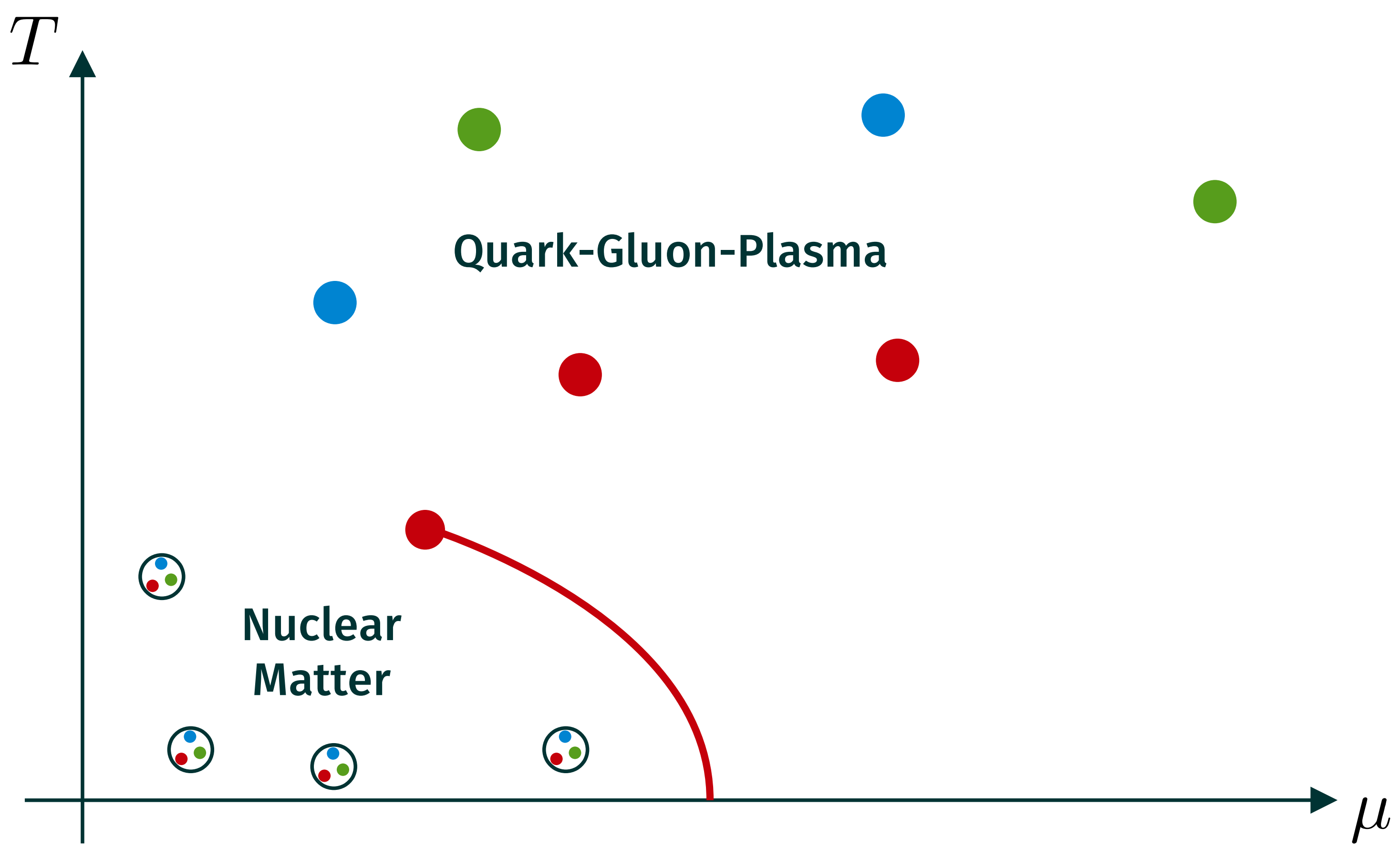
Diquark Condensation

Phase Diagram



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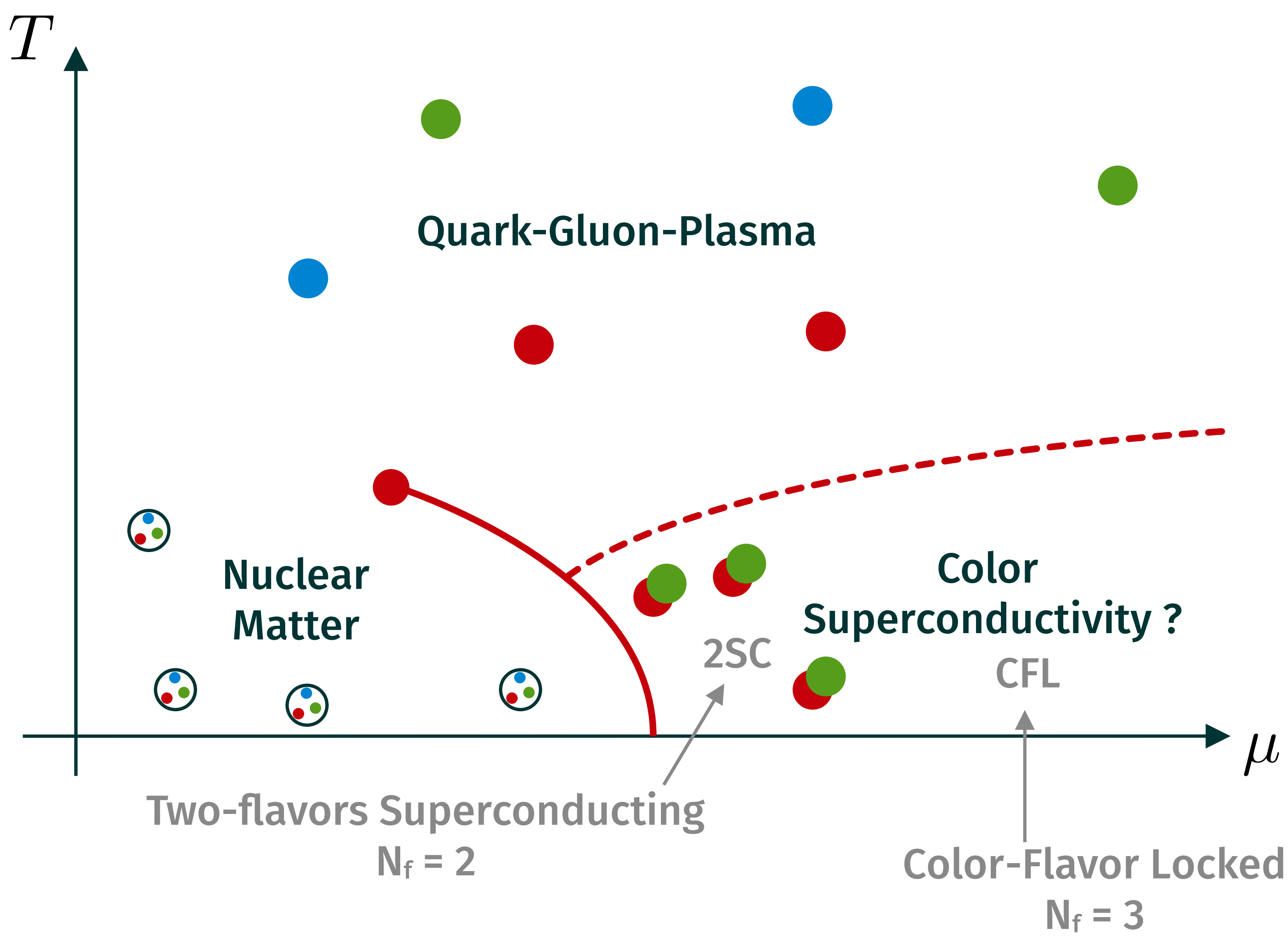

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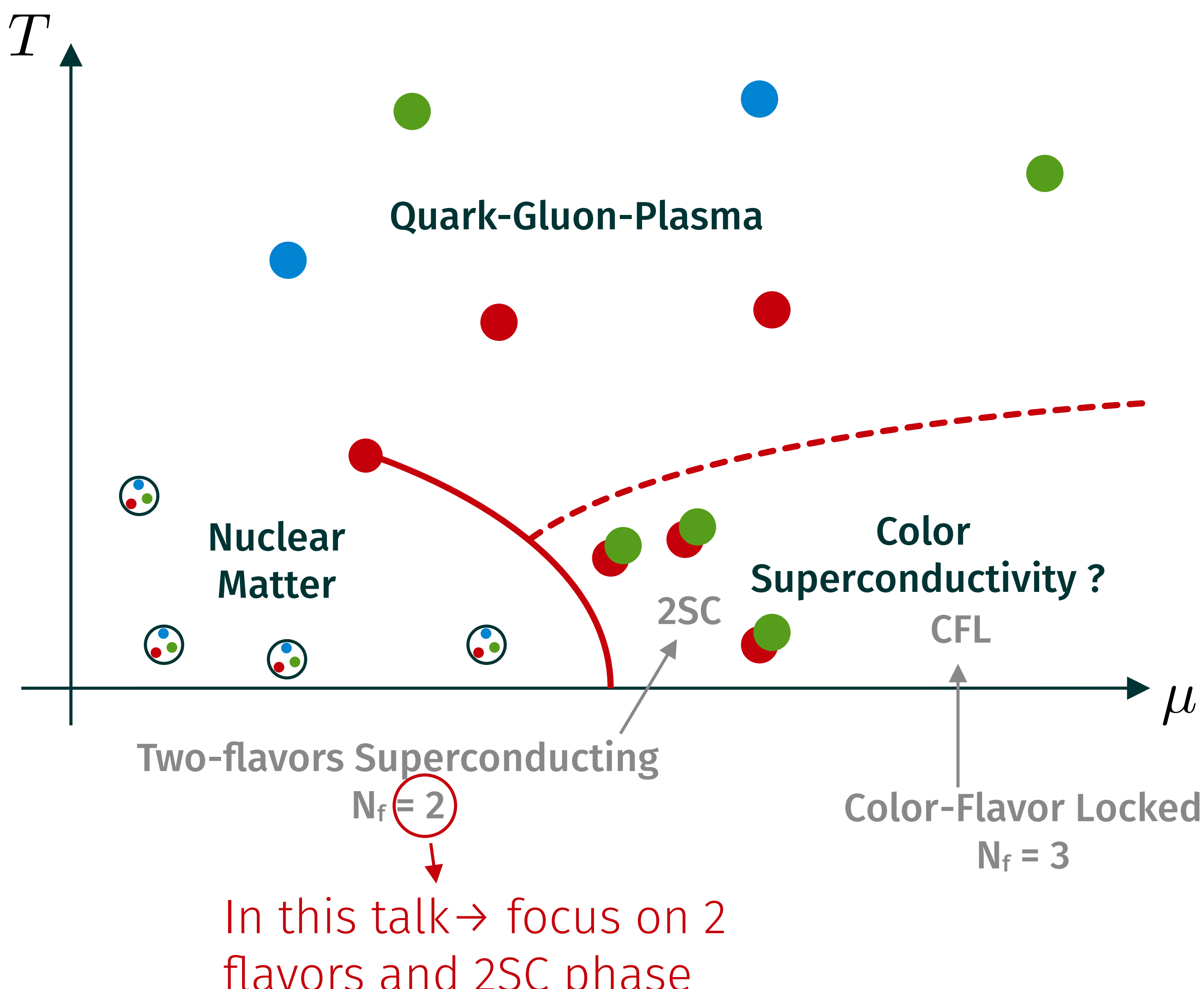

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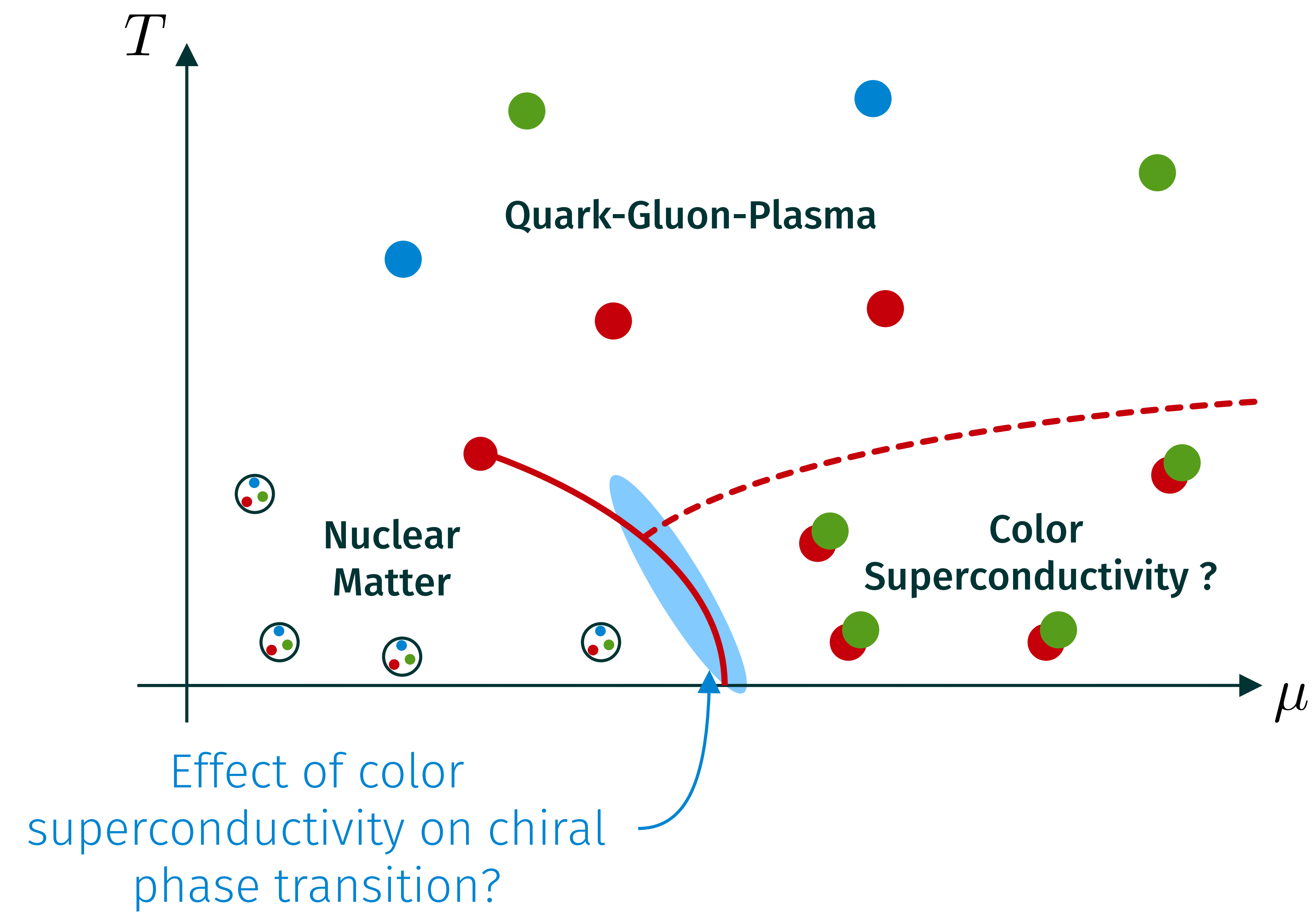

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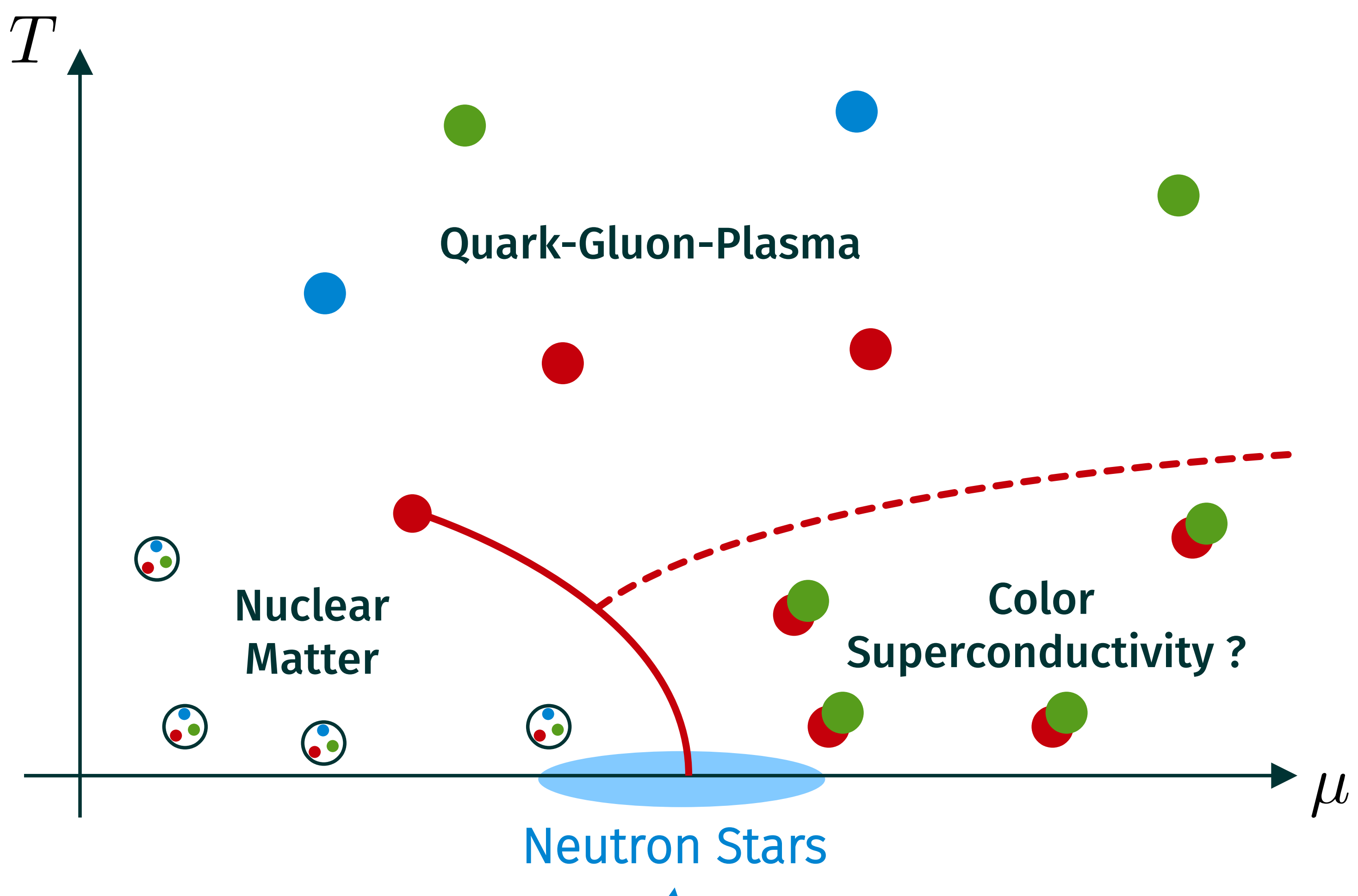
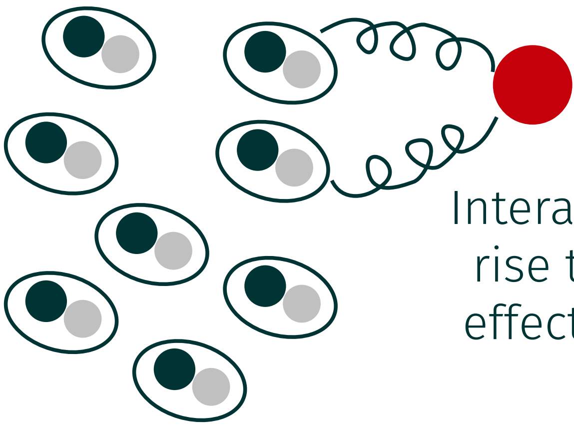


Impact on neutron stars observables?

Quark-meson model



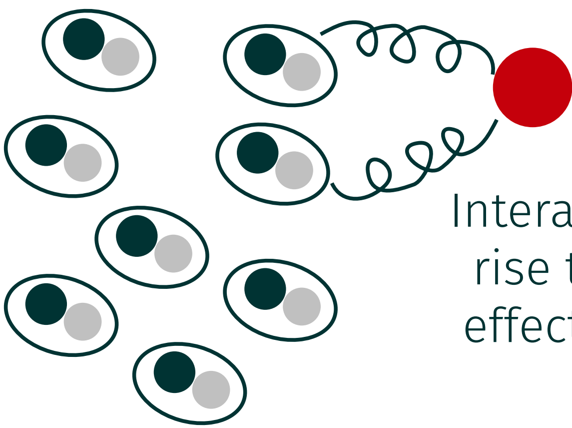
Interaction give rise to higher effective mass

$\langle \bar{q}q \rangle \neq 0$

Apparition of a **chiral condensate** at low μ

Quark-Meson-Diquark Model

Quark-meson model



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allow for a new condensate

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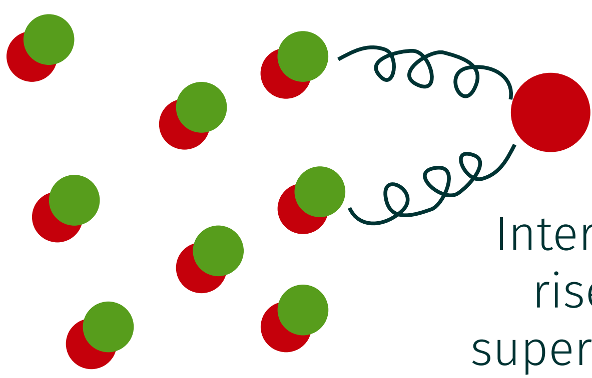

Interaction give rise to color superconductivity

$$\langle q^T q \rangle \neq 0$$

Apparition of a **diquark condensate** at high μ

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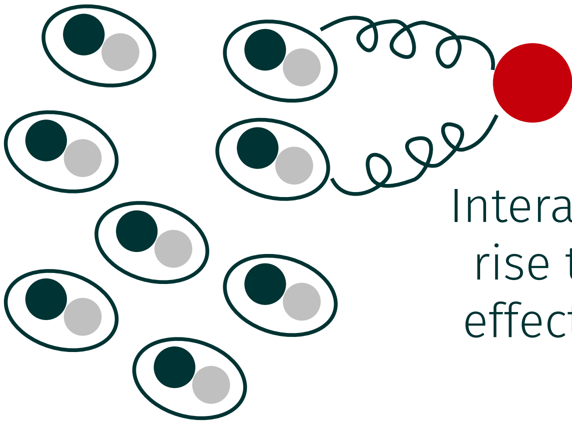

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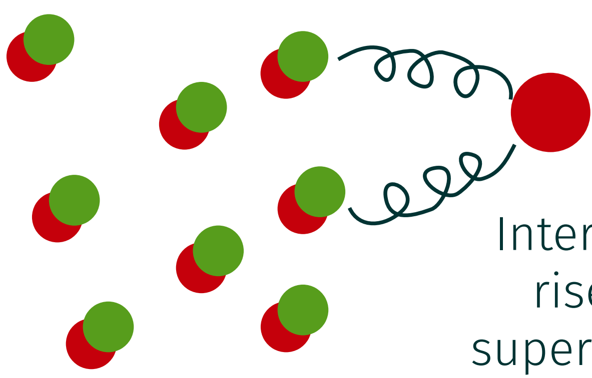


Interaction give rise to color superconductivity

$\langle q^T q \rangle \neq 0$ → Exact shape of the condensate?

Apparition of a **diquark condensate** at high μ

Shape of Diquark Condensate

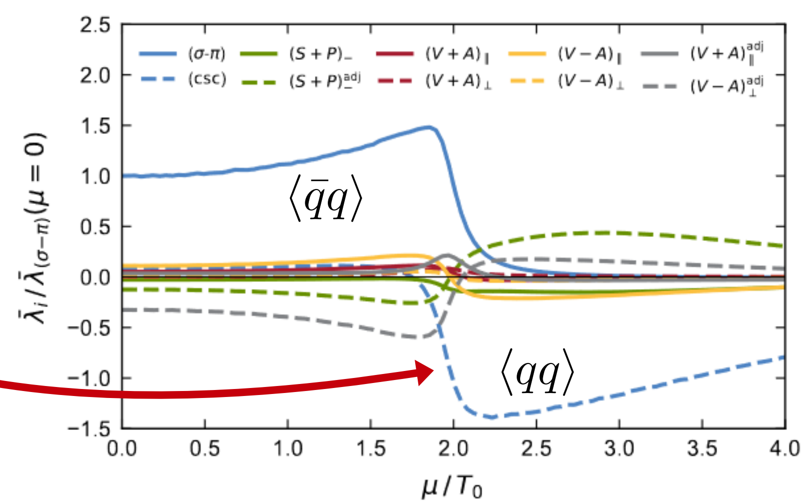
Which diquark condensate is favoured? (2 flavors)

- Argument from Pauli principle, attractive channel in color space and Lorentz scalar
- Argument from full QCD flow that diquark condensate become important at high μ [Braun, Leonhardt & Pospiech; 1909.06298]

both agree for:

$C = i\gamma^2\gamma^0$

$$\Delta = \langle q^T C \gamma_5 \tau_2 \lambda_2 q \rangle$$



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Lorentz index structure

Pauli matrix in flavor-space

→ leave chiral symmetry intact

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$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

One color do not participate in condensate

Lorentz index structure

Pauli matrix in flavor-space

Gell-Mann matrix in color-space

→ **break SU(3) color symmetry** and lead to color superconductivity

→ leave chiral symmetry intact

Quark-Meson-Diquark Model Lagrangian

quark-meson model

$\phi = (\sigma \quad \vec{\pi})^T$

$$\mathcal{L}_{\text{QMD}} = \bar{q}\left(-i\gamma^\mu \partial_\mu - \mu\gamma^0 + g_\phi(\sigma + i\gamma^5 \vec{\tau}\cdot\vec{\pi})\right) q + \frac{1}{2}(\partial_\mu \phi)(\partial_\mu \phi) + U(\phi^2 \qquad)$$

Quark-Meson-Diquark Model Lagrangian

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diquark self-interaction

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quark-diquark interaction

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diquark kinetic term

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diquark kinetic term

For homogeneous condensate we get an additional term $-4\mu^2\Delta^2$
→ suggest need for quartic term to have a **potential bounded by below** for all μ?

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Define **Nambu-Gorkov bispinor** $\Psi = \begin{pmatrix} q \\ C\bar{q}^T \end{pmatrix}$

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2×2 matrix in "Nambu-Gorkov space"

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2×2 matrix in "Nambu-Gorkov space"

What shape for the boson potential?

Potential & Parameter Fixing Strategy

small explicit chiral symmetry breaking term

Shape of the potential

$$U(\phi^2, |\Delta|^2) = \frac{1}{2}a_1\phi^2 + \frac{1}{4}a_2\phi^4 + \frac{1}{2}b_1|\Delta|^2 + \frac{1}{4}b_2|\Delta|^4 - c\sigma$$

with a_1, a_2, b_1, b_2 & c free parameters.

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Do we need b_2?
→Yes? Potential bounded from below and full QCD study.
[Braun & Schallmo; 2106.04198]

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meson parameters

Fixed to reproduce **vacuum observables**:
- pion decay constant
- pion mass
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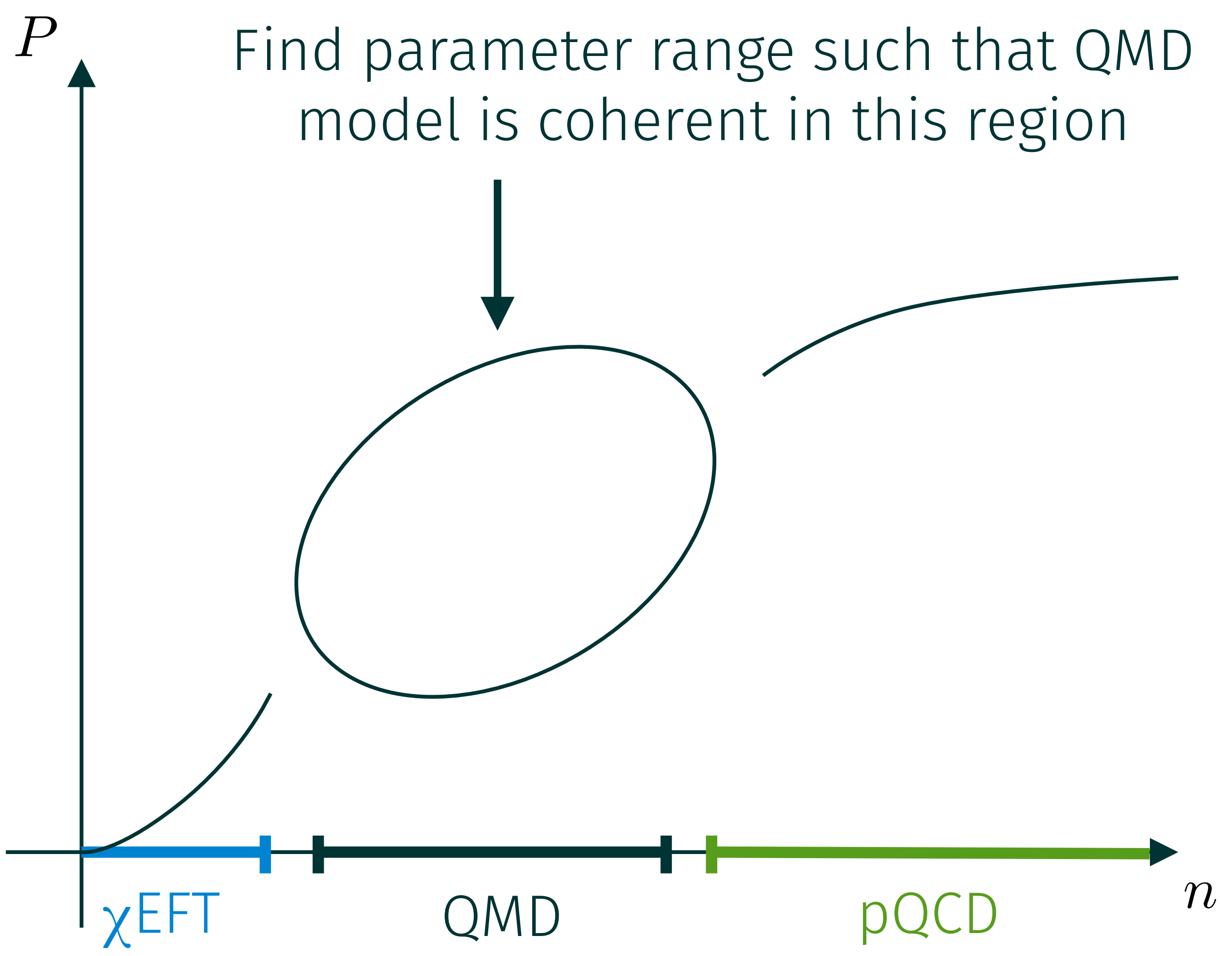
diquark parameters

In vacuum no diquark, only at high μ and low T.
→ no easy observables can be used.

Parameter Fixing Strategy

Main Problem: How to fix the diquark parameters?

→ First approach, match pQCD and χEFT with QMD model. Example with the EoS (maybe better quantity to look at?):



Thermodynamics

What to compute?

Goal: Compute the **partition function Z** of the model

$$\mathcal{D}\Psi = \mathcal{D}\bar{q}\mathcal{D}q \qquad \beta = 1/T$$

$$Z = \int \mathcal{D}\Psi \mathcal{D}\phi \mathcal{D}\Delta \mathcal{D}\Delta^* \exp\left(- \int_0^\beta d\tau \int d^3x \mathcal{L}_{\mathrm{QMD}} \right)$$

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Mean-field Approximation (MFA)

- No meson or diquark fluctuations
- Relatively simple solution

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Mean-field Approximation (MFA)
- No meson or diquark fluctuations
- Relatively simple solution

Functional renormalization group (FRG)
- Include meson and diquark fluctuations

used to adjust/check

Mean-field Theory

thermodynamic potential

mean-field approximation

$$Z = \int \mathcal{D}\Psi \mathcal{D}\phi \mathcal{D}\Delta \mathcal{D}\Delta^* \exp\left(-\int_0^\beta d\tau \int d^3x \mathcal{L}_{\text{QMD}} \right) = e^{-\beta V \Omega}$$

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Nambu-Gorkov inverse propagator

Reduced to a fermionic determinant $Z \sim \sqrt{\det S^{-1}}$

The **thermodynamic potential Ω** reads

$$\Omega = -\frac{1}{2}\frac{1}{\beta V} \mathrm{tr} \ln S^{-1}$$

trace in color, flavor, Dirac and Nambu-Gorkov space

Mean-field Theory

$$\Omega = -4\int \frac{d^3p}{(2\pi)^3}\left\{2\left(\frac{E_\Delta^+ + E_\Delta^-}{2} + T\ln\left(1+e^{-\beta E_\Delta^+}\right) + T\ln\left(1+e^{-\beta E_\Delta^-}\right)\right)\right.$$
$$\left. + \left(E_p + T\ln\left(1+e^{-\beta(E_p+\mu)}\right) + T\ln\left(1+e^{-\beta(E_p-\mu)}\right)\right)\right\}$$
$$+ U(\phi^2\ \ |\Delta|^2)$$

2 spin
×2 flavor

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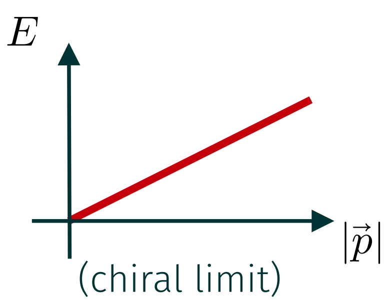

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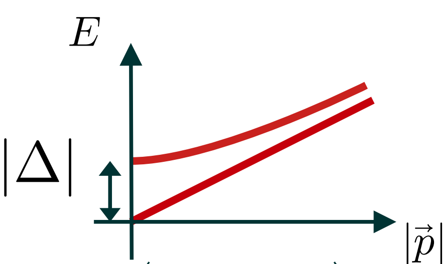

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two colors are **gapped**

$$E_\Delta^\pm = \sqrt{(E_p \pm \mu)^2 + |\Delta|^2}$$

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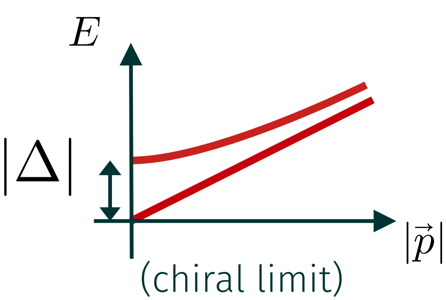

Find the **ground state** through gap-equation:

$$\frac{\partial\Omega}{\partial\sigma} = 0 \quad \& \quad \frac{\partial\Omega}{\partial\Delta} = 0$$

Functional Renormalization Group

Define an **effective action** Γ_k

- $\Gamma_{k=\Lambda}$: microscopic theory
- $\Gamma_{k=0}$: all fluctuation integrated

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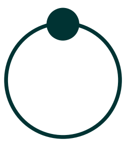
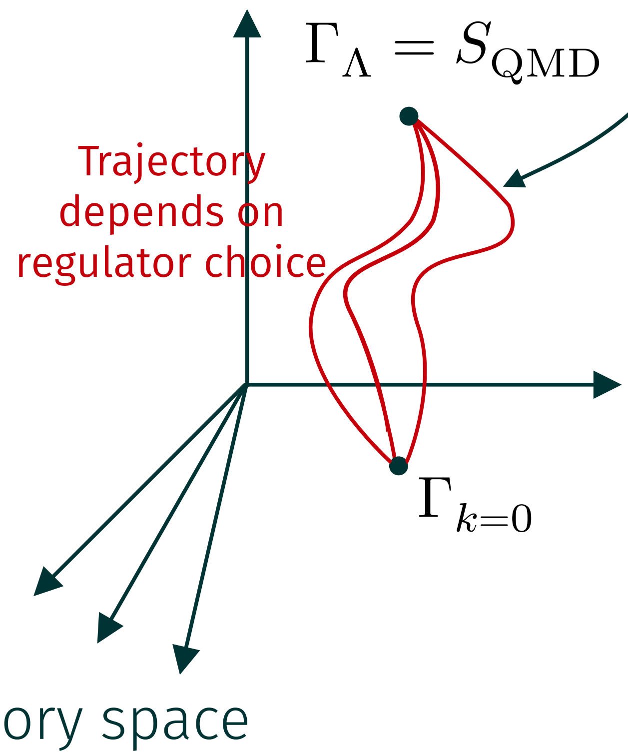
Trajectory defined by the **flow equation**

$$\partial_t \Gamma_k = \mathrm{Tr}\left[\partial_t R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right] =$$



$t = \ln k$ — regulator



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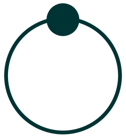

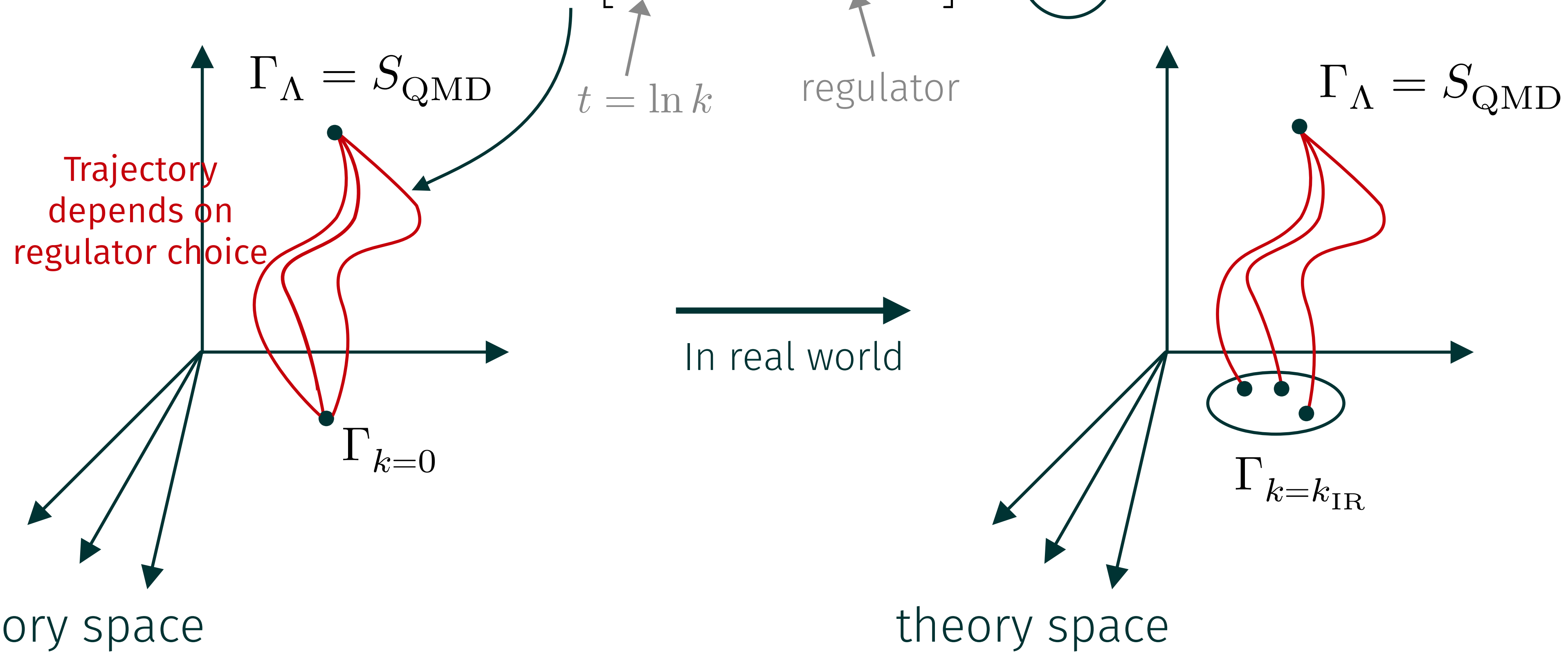

Functional Renormalization Group

Truncation

→ Local potential approximation

Only scale dependent term

$$\Gamma_k = \bar{\Psi} S^{-1} \Psi + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}[(\partial_\mu - \delta^0_\mu 2\mu)\Delta](\partial_\mu + \delta^0_\mu 2\mu)\Delta^* + U_k(\phi^2, |\Delta|^2)$$

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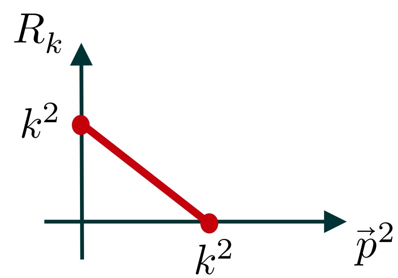

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Expect negative entropy → any effect from diquark?

[Tripolt, Schaefer, Smekal & Wambach; 1709.05991]
[Otto, Busch & Schaefer; 2206.13067]

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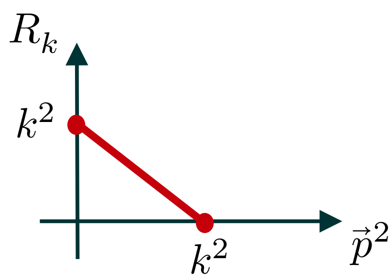

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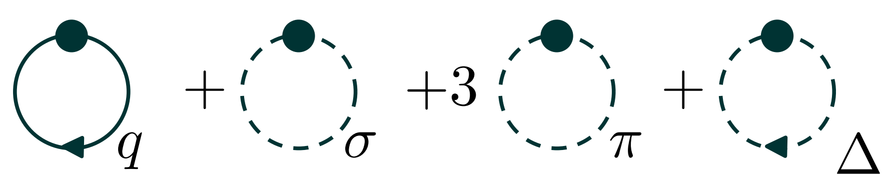
$\partial_t \Gamma_k = -$ 

Quark-meson-diquark model flow equation

$$k\partial_k U_k = -\frac{k^5}{\pi^2 E_k}\left\{\frac{2}{3}\left[\frac{E_k+\mu}{E_\Delta^+}\tanh\left(\frac{E_\Delta^+}{2T}\right)+\frac{E_k-\mu}{E_\Delta^-}\tanh\left(\frac{E_\Delta^-}{2T}\right)\right]+\frac{1}{3}\left[\tanh\left(\frac{E_k+\mu}{2T}\right)+\tanh\left(\frac{E_k-\mu}{2T}\right)\right]\right.$$

$$\left.+\frac{k^5}{24\pi^2 E_d}\left[\coth\left(\frac{E_d-2\mu}{2T}\right)+\coth\left(\frac{E_d+2\mu}{2T}\right)\right]+\frac{k^5}{12\pi^2}\left[\frac{1}{E_\sigma}\coth\left(\frac{E_\sigma}{2T}\right)+\frac{3}{E_\pi}\coth\left(\frac{E_\pi}{2T}\right)\right]\right.$$

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$$E_d = \sqrt{k^2+m_d^2} \longrightarrow m_d^2 = \frac{\partial U}{\partial|\Delta|^2}+|\Delta|^2\frac{\partial^2 U}{\partial(|\Delta|^2)^2}$$

need access to derivative of U *wrt* $|\Delta|$, σ

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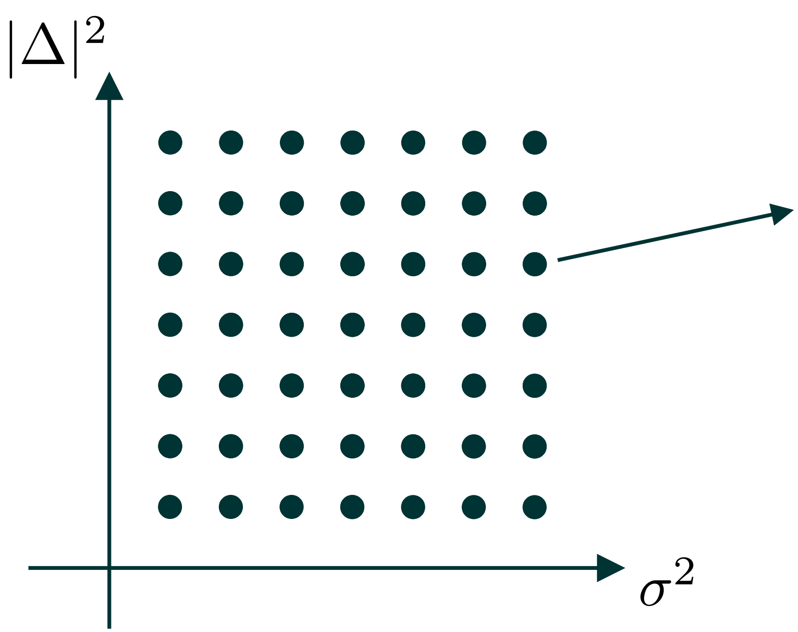
need access to derivative of U wrt $|\Delta|$, σ

Solved on 2d grid



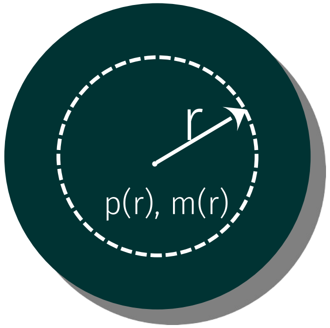
→ Solve flow equation at each grid point (extract derivative at a grid point with a cubic spline)

→ Ground state given by

$$\frac{\partial U_{k_{\mathrm{IR}}}}{\partial\sigma}=\frac{\partial U_{k_{\mathrm{IR}}}}{\partial|\Delta|}=0$$

Application to Neutron Stars

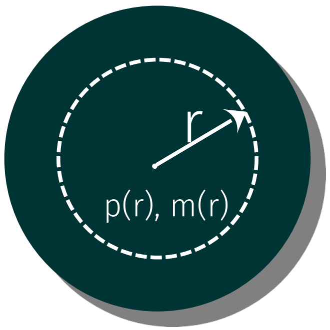
Mass-radius Relationship



$$\frac{dp}{dr} = -\frac{(\epsilon(r) + p(r))[m(r) + 4\pi r^3 p(r)]}{r(r - 2Gm(r))}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon(r)$$

Mass-radius Relationship



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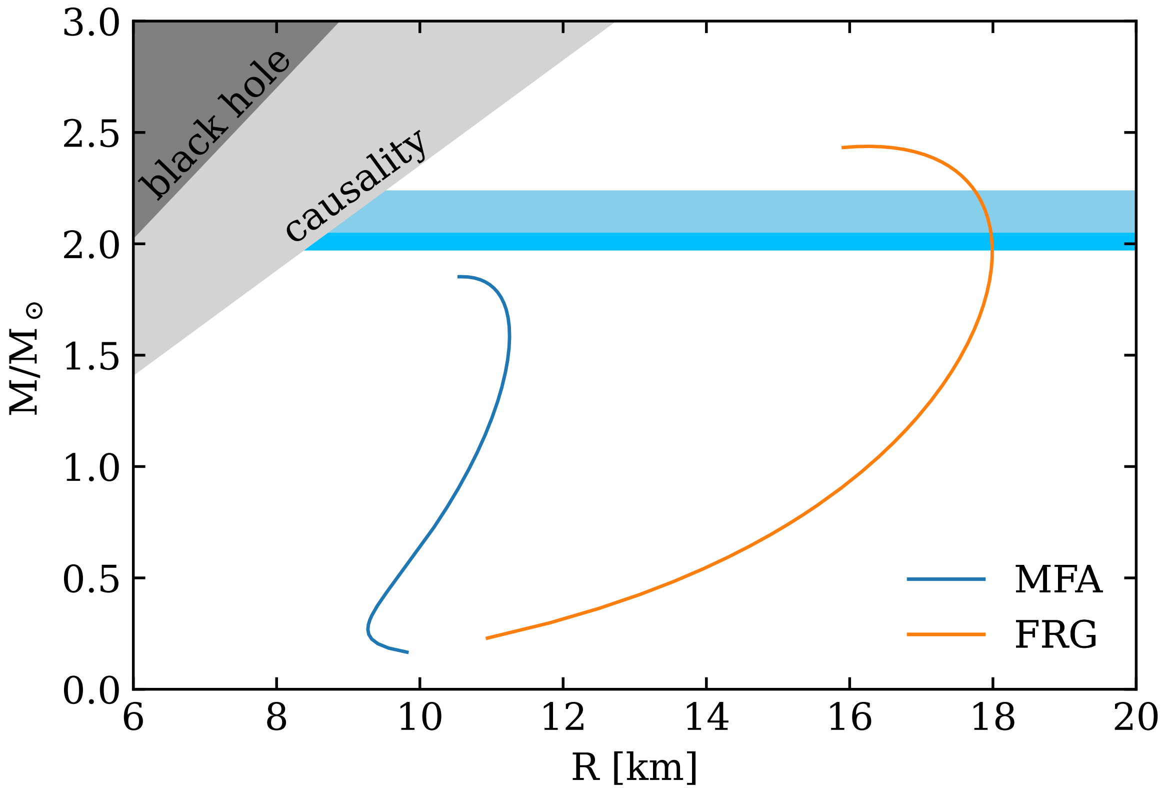
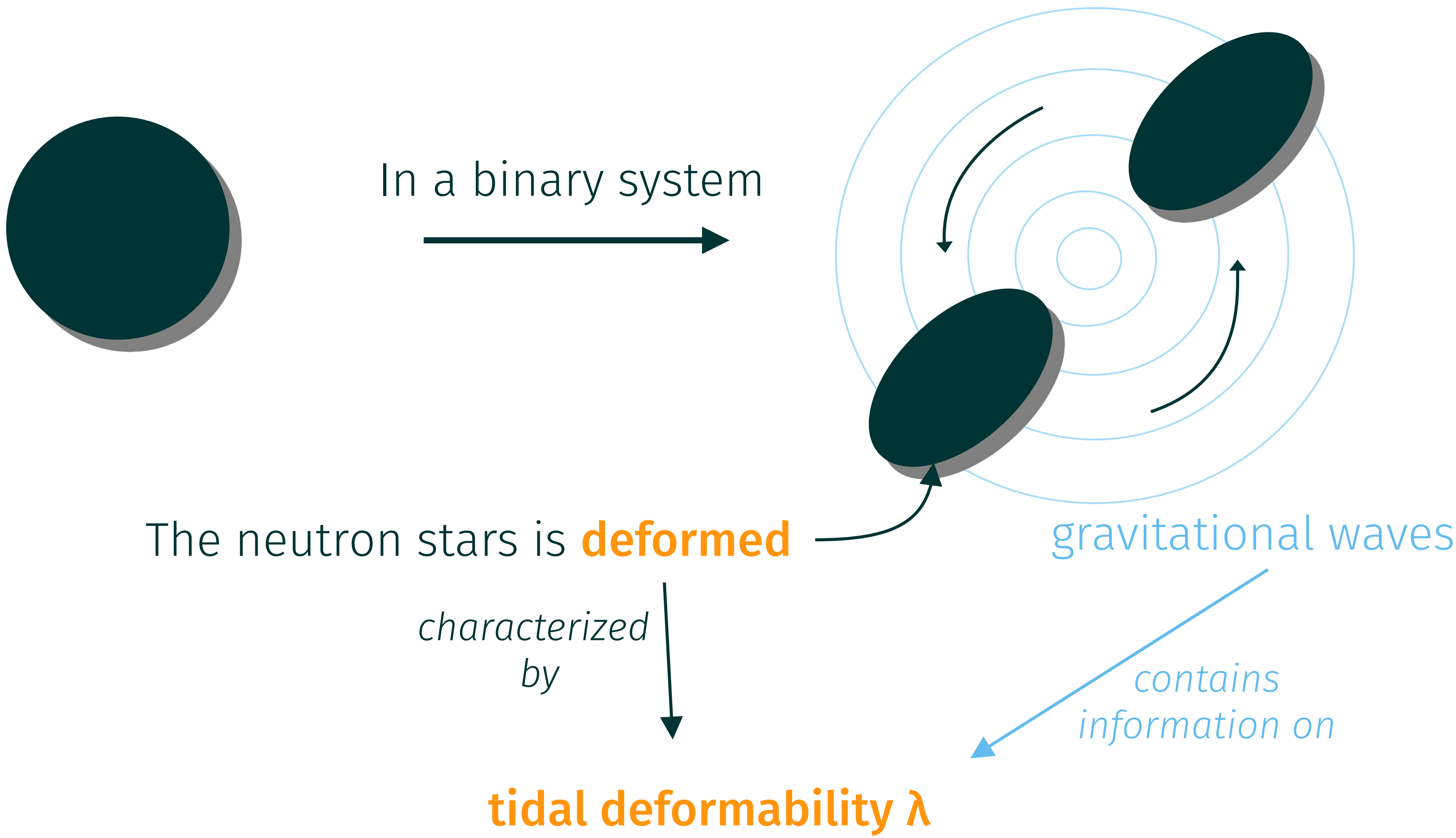
→ Solve for a variety of **central pressure** p(r=0)



Note: Results from quark-meson model

Tidal Deformability

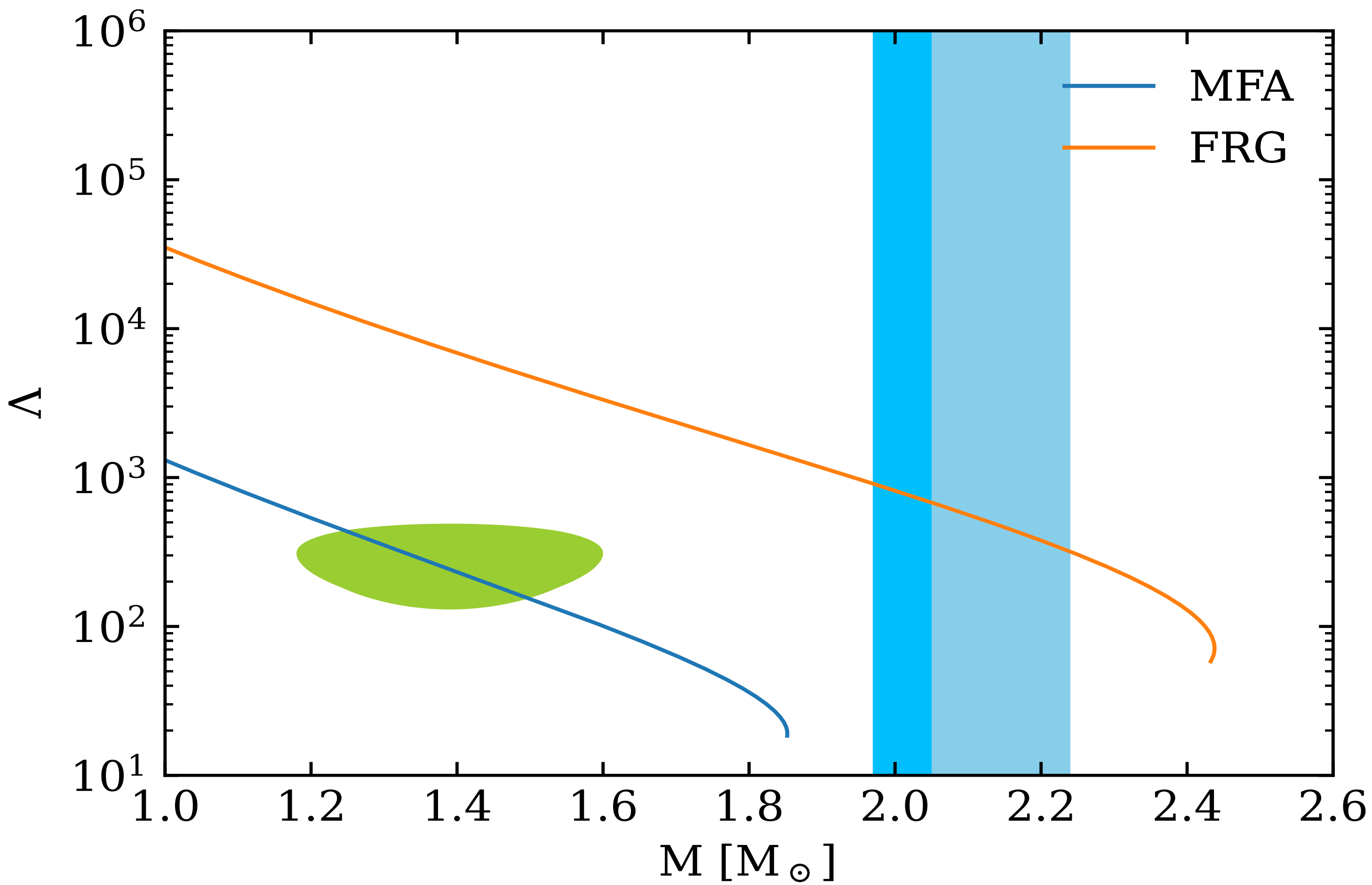


Tidal Deformability

Λ can be computed through an additional ODE in TOV equation

[Hinderer, Lackey, Lang & Read; 0911.3535]

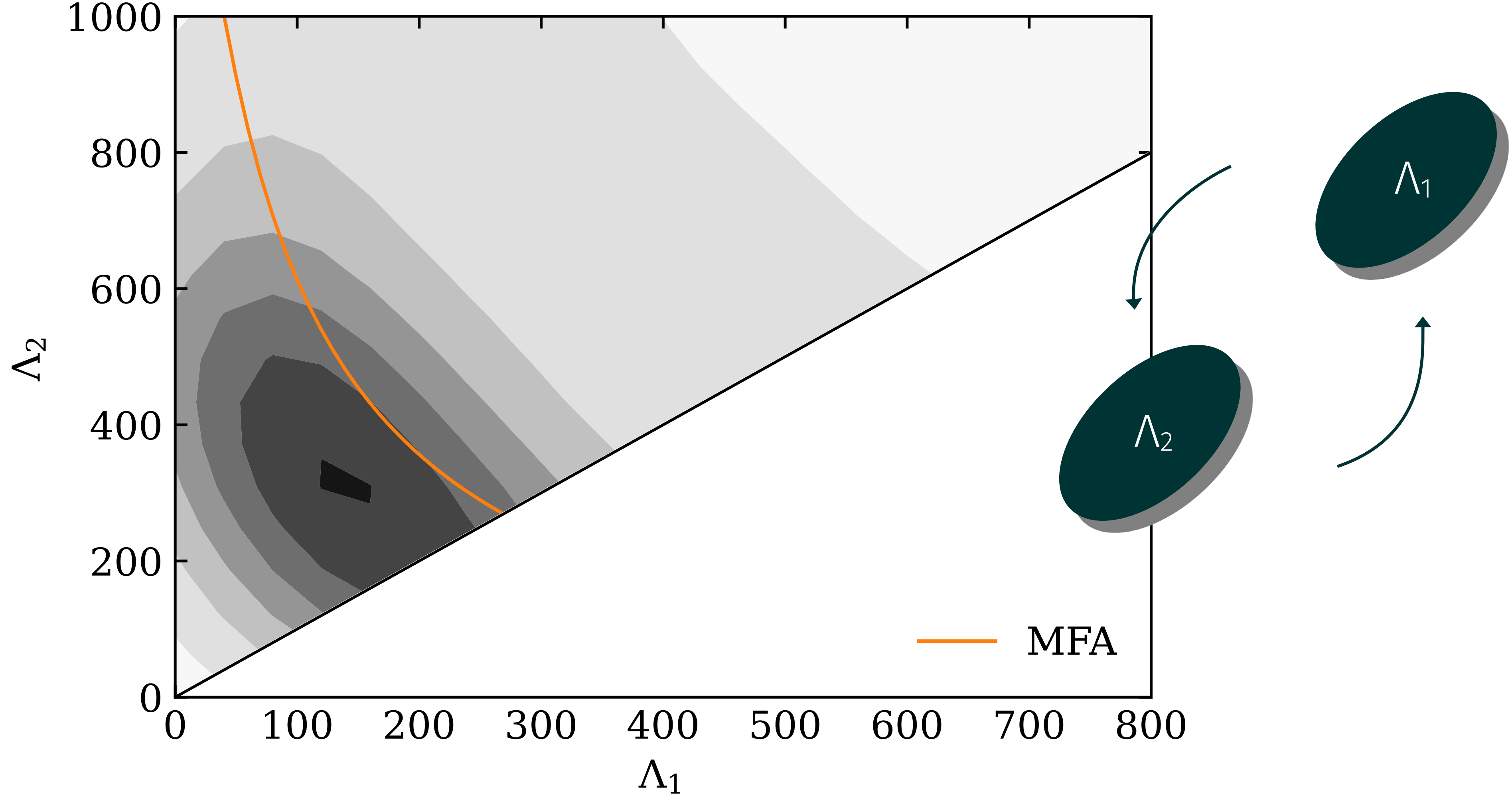
$\Lambda = \frac{\lambda}{M^5}$



Note: Results from quark-meson model

Tidal Deformability

Plot for GW170817 binary merger [Abbot et al. (LIGO Scientific, Virgo); 1805.11581]



Note: Results from quark-meson model

Conclusion and Outlook

Conclusion and Outlook

2 flavors quark-meson-diquark model and resolution with

- Mean-field approximation
- Functional renormalization

Application to neutron stars observables

Outlook:

- Find a proper way of fixing diquark parameters.
- Effect of diquark on:
  - Chiral transition?
  - Equation of state?
  - Negative entropy and Litim regulator?
- Construction of neutral matter for neutron stars:
  - 2SC phase possible in neutral matter?
  - Effect on neutron stars observables?

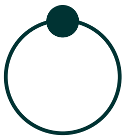
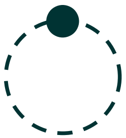
Thank you!

Backup

Potential problems

- **May be too hard numerically**
  → Need more work but seems doable at the moment.
  → If too hard, Taylor expansion in one of the direction?

- **Litim regulator**
  → expect negative entropy, maybe use different regulator?

$$\partial_t \Gamma_k = -$$  $$+ \frac{1}{2}$$ 

Quark-Meson-Diquark Model Lagrangian

$$\mathcal{L}_{\text{QMD}} = \bar{q}\left(-i\gamma^\mu\partial_\mu - \mu\gamma^0 + g_\phi(\sigma + i\gamma^5\vec{\tau}\cdot\vec{\pi})\right)q + \frac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi) + U(\phi^2, |\Delta|^2)$$
$$+ \frac{i}{2}(\Delta^*\bar{q}\gamma_5\tau_2\lambda_2 C\bar{q}^T + \Delta q^T C\gamma_5\tau_2\lambda_2 q) + [(\partial_\nu - \delta^0_\nu 2\mu)\Delta](\partial_\nu + \delta^0_\nu 2\mu)\Delta^*$$

Define **Nambu-Gorkov bispinor** $\Psi = \begin{pmatrix} q \\ q^C \end{pmatrix}$

$$\mathcal{L}_{\text{QMD}} = \bar{\Psi}S^{-1}\Psi + \frac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi) + [(\partial_\nu - \delta^0_\nu 2\mu)\Delta](\partial_\nu + \delta^0_\nu 2\mu)\Delta^* + U(\phi^2, |\Delta|^2)$$

2×2 matrix in Nambu-Gorkov space

For homogeneous condensate we get an additional term $-4\mu^2\Delta^2$
→ suggest need for quartic term to have a **potential bounded by below**

Quark-Meson-Diquark Model Lagrangian

Quark dynamic and quark-meson / quark-diquark interaction

$$\mathcal{L}_{\mathrm{QMD}} = \bar{q}\left(-i\partial - \mu\gamma_0 + g_\phi(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right)q$$
$$+ i\frac{g_\Delta}{2}\left(\bar{q}\gamma_5\tau_2\lambda_2\Delta^* q^C + \bar{q}^C\gamma_5\tau_2\lambda_2\Delta q\right)$$
$$+ \frac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi) + \frac{1}{2}m_\phi\phi^2 + \frac{1}{4}\lambda_\phi\phi^4$$
$$+ \frac{1}{2}[(\partial_\mu - \delta_\mu^0 2\mu)\Delta](\partial_\mu + \delta_\mu^0 2\mu)\Delta^* + \frac{1}{2}m_\Delta^2|\Delta|^2 + \frac{1}{4}\lambda_\phi|\Delta|^4$$

meson dynamic and self-interaction

diquark dynamic and self-interaction

$$\phi = \begin{pmatrix}\sigma \\ \vec{\pi}\end{pmatrix}$$

Mean-field Theory

2 spin
2 flavor

$$\Omega = -4 \int \frac{d^3p}{(2\pi)^3} \Bigg\{ 2 \left(\frac{E_\Delta^+ + E_\Delta^-}{2} + T \ln \left(1 + e^{-\beta E_\Delta^+}\right) + T \ln \left(1 + e^{-\beta E_\Delta^-}\right) \right)$$

blue quark

$$+ \left(E_p + T \ln \left(1 + e^{-\beta(E_p + \mu)}\right) + T \ln \left(1 + e^{-\beta(E_p - \mu)}\right)\right) \Bigg\}$$

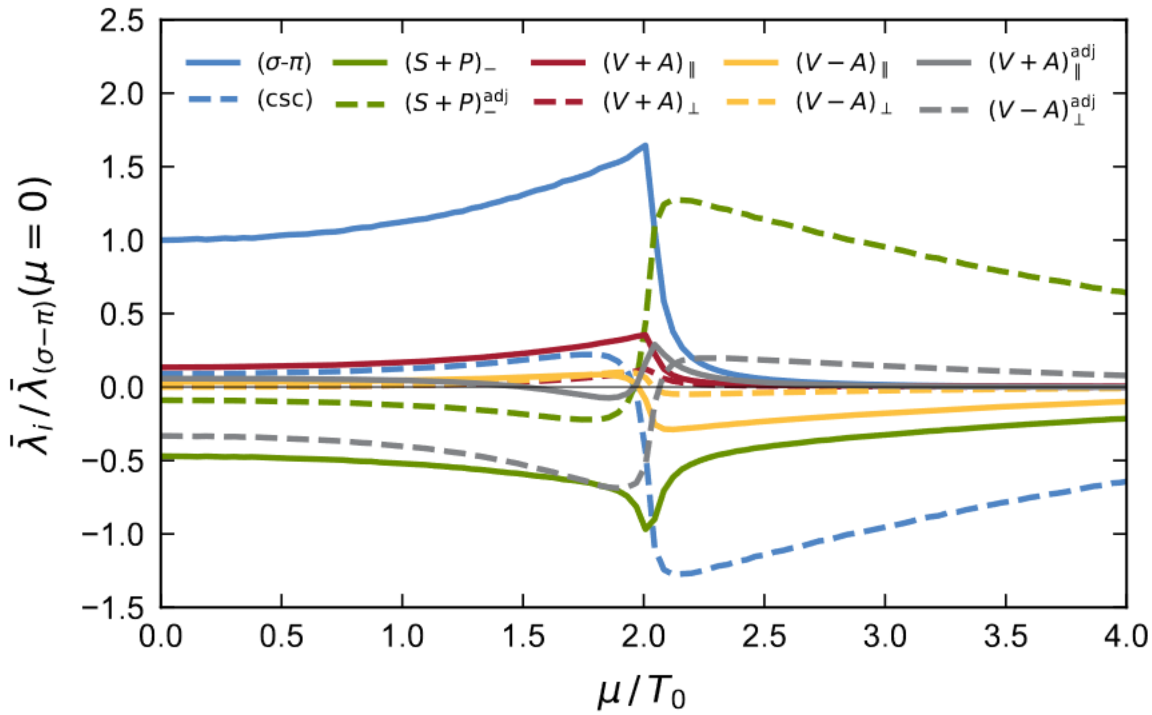
$$+ U(\phi^2, |\Delta|^2)$$

$$E_p = \sqrt{\vec{p}^2 + g_\phi^2 \sigma^2} \qquad E_\Delta^\pm = \sqrt{(E_p \pm \mu)^2 + \Delta^2}$$

Effective NJL Model

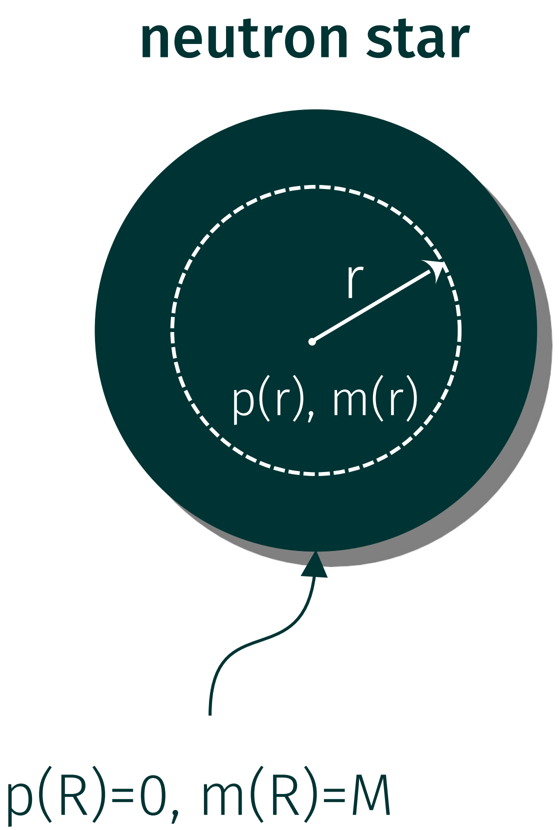
Which 4-fermion coupling to choose?
Cite J Braun

Quark-meson



Tolman-Oppenheimer-Volkoff Equation

Need the EoS ε(p) to solve



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$$\frac{dm}{dr} = 4\pi r^2 \epsilon(r)$$

Solve for a variety of **central pressure** p(r=0)

→ Mass-radius relationship M(R)

What is a Neutron Star

Most dense matter in the universe,
strong interaction becomes relevant
in the core

neutron star