Spectral Functions in Nuclear Matter

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in collaboration with

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TECHNISCHE UNIVERSITÄT DARMSTADT

HFHF Theory Retreat, Italy, 12 September 2022

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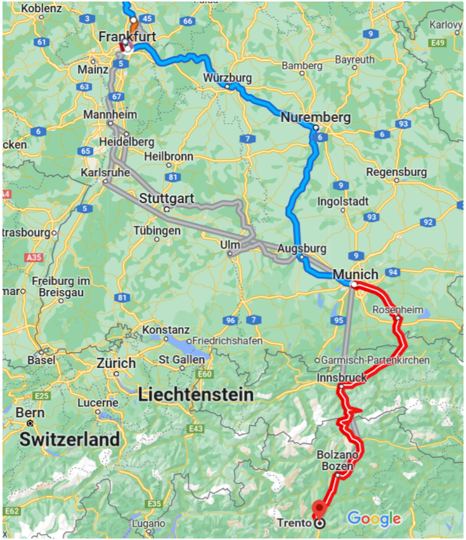
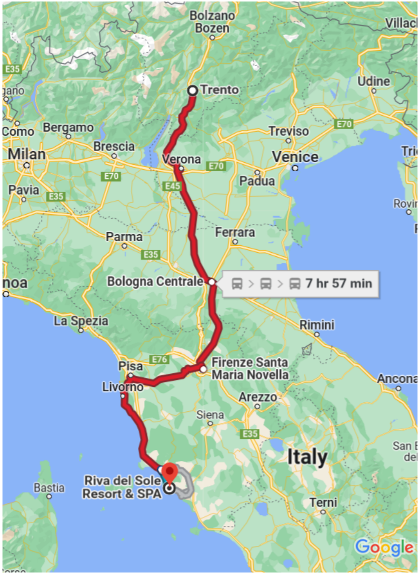
The ‘green’ travel route of the Gießen group - Day 1



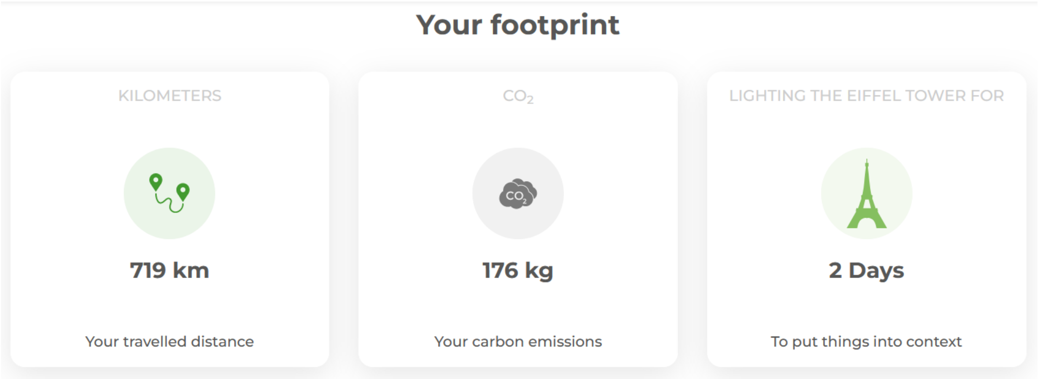
Gießen $\rightarrow$ Munich $\rightarrow$ Trento

The 'green' travel route of the Gießen group - Day 2



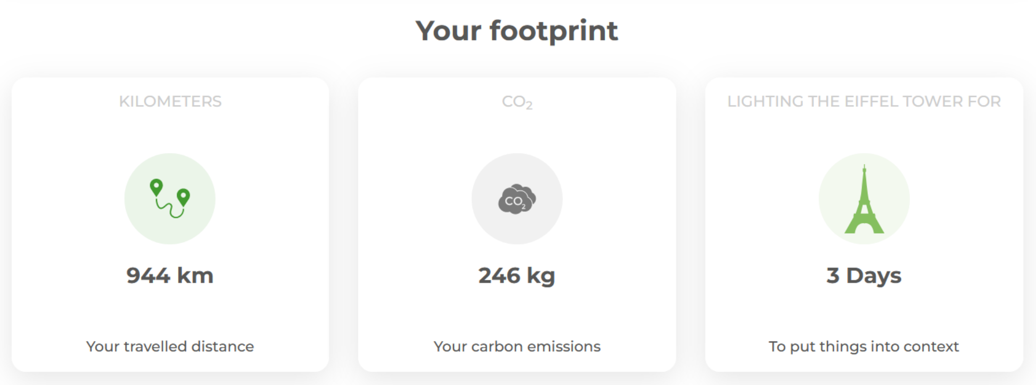
Trento → Florence → Riva del Sole

CO_2 Emissions - Airplane (per person)



[https://ecotree.green]

CO_2 Emissions - Car (1 person)



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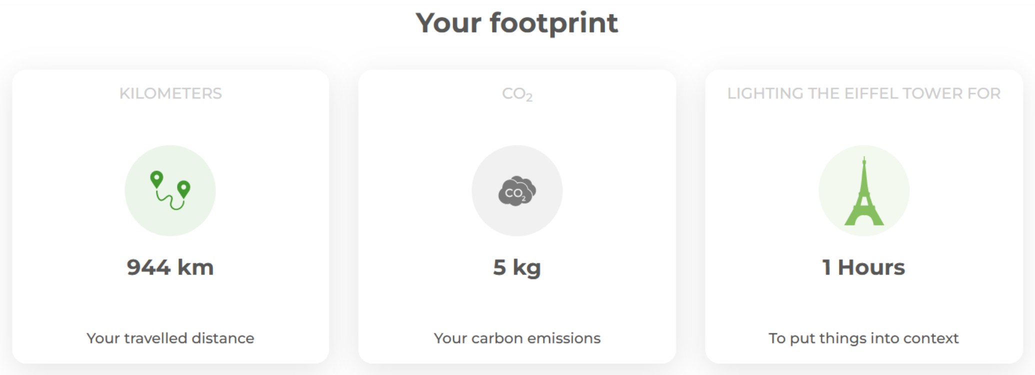
CO_2 Emissions - Train (per person)



[https://ecotree.green]

Outline

I) Introduction and motivation

- heavy-ion collisions, QCD phase diagram, chiral symmetry

II) Theoretical setup

- Functional Renormalization Group
- parity-doublet model
- spectral functions with the aFRG method

III) Results on spectral functions and dileptons

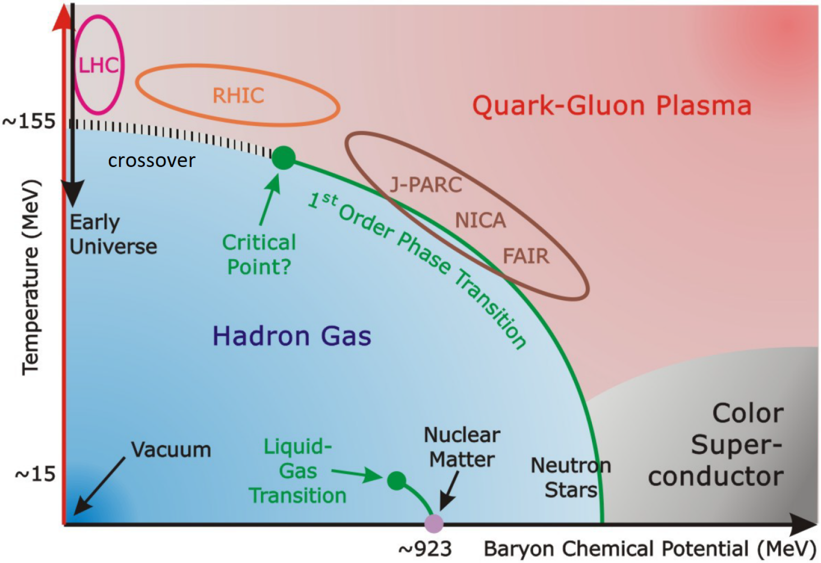
- in-medium ρ and a_1 spectral functions
- thermal dilepton rates and spectra

IV) Summary and outlook

Part I

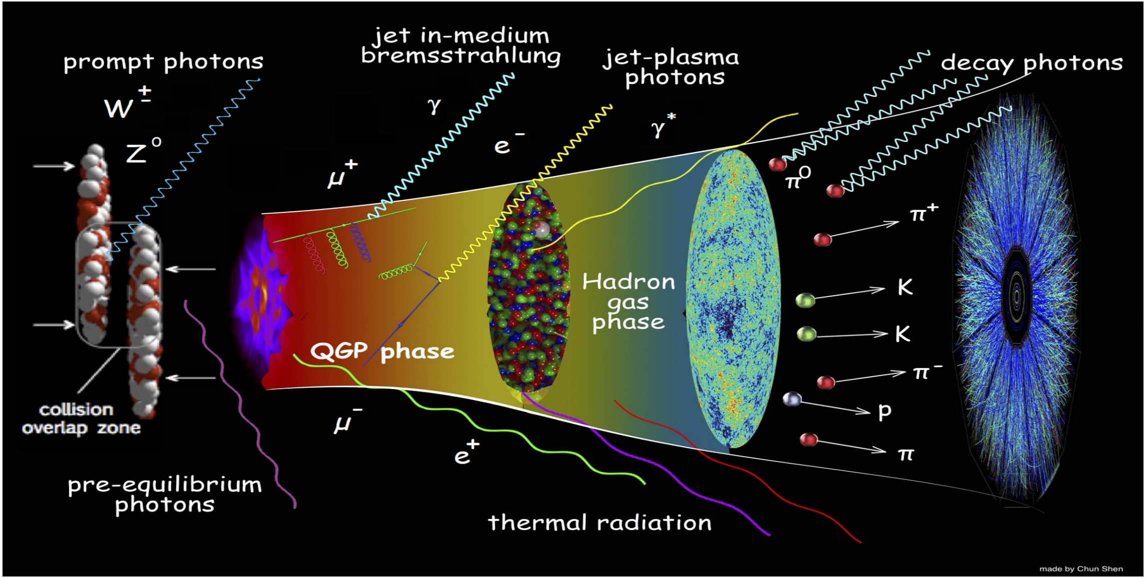
Introduction and motivation

QCD phase diagram



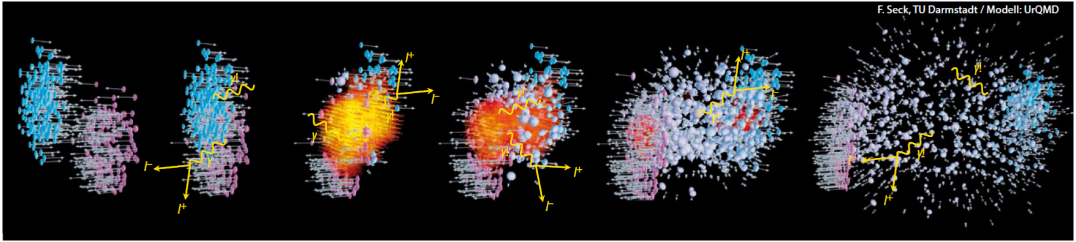
[Figure adapted from the CRC-TR 211]

Dileptons in heavy-ion collisions



[Figure by Chun Shen]

Why dileptons?



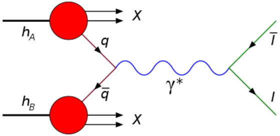
- Electromagnetic (EM) probes, i.e. photons and dileptons, don't interact 'strongly' with medium
- they have a long mean free path and can carry information from production site to detectors
- they are produced at all stages of the collision

$\rightarrow$ **dileptons are uniquely well-suited to study hot and dense matter in heavy-ion collisions!**

Dileptons in heavy-ion collisions

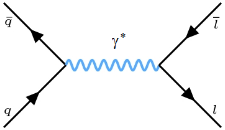
'Primordial' $q\bar{q}$ annihilation (Drell-Yan):

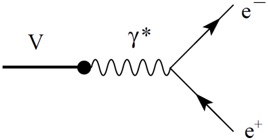
- $NN \to e^+e^-X$



Thermal radiation from QGP and hadrons:

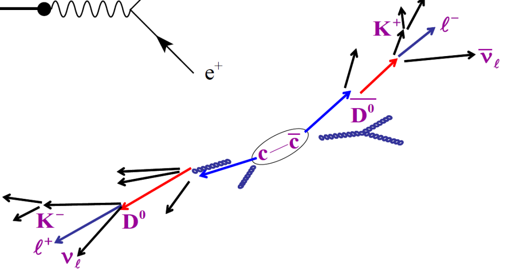
- $q\bar{q} \to e^+e^-$, ...
- $\pi^+\pi^- \to e^+e^-$, ...
- short-lived states: ρ, a_1, Δ, N^*, ...
- multi-meson reactions ('4π'): $\pi\rho$, $\pi\omega$, $\rho\rho$, πa_1, ...





Decays of long-lived mesons and baryons:

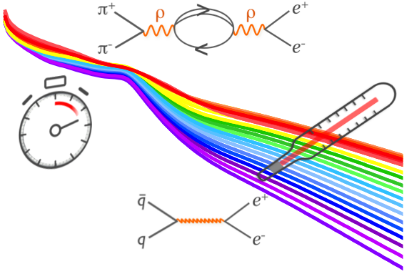
- π^0, η, ϕ, J/Ψ, Ψ', correlated $D\bar{D}$ pairs, ...



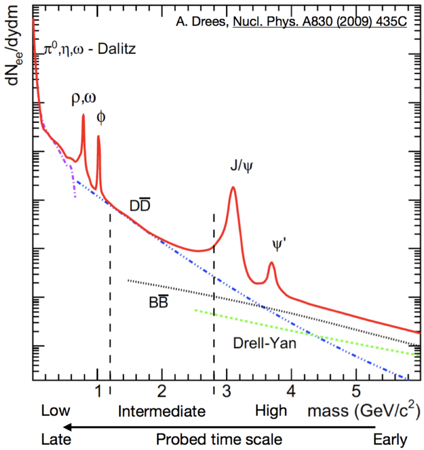
What can we learn from dileptons?

Dileptons contain information on:

- temperature, fireball lifetime, in-medium spectral functions, chiral symmetry, changes in degrees of freedom, transport coefficients (electrical conductivity), ...



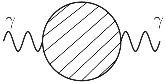
[T. Galatyuk, H. v. Hees, R. Rapp, J. Wambach, Physik Journal 17, Nr. 10 (2018)]



Dilepton production rates

Thermal field theory: Electromagnetic correlation function

$$\Pi^{\mu\nu}_{\mathrm{EM}}(M,p;\mu_B,T) = -\mathrm{i}\int d^4x\; e^{ip\cdot x}\;\Theta(x_0)\;\langle\langle [j^{\mu}_{\mathrm{EM}}(x), j^{\nu}_{\mathrm{EM}}(0)]\rangle\rangle$$



determines both **photon and dilepton rates**:

- photons: $$p_0\frac{dN_\gamma}{d^4x d^3p} = -\frac{\alpha_{\mathrm{EM}}}{\pi^2}\; f^B(p_0;T)\;\frac{1}{2}\;g_{\mu\nu}\;\mathrm{Im}\,\Pi^{\mu\nu}_{\mathrm{EM}}(M=0,p;\mu_B,T),$$
- dileptons: $$\frac{dN_{ll}}{d^4x d^4p} = -\frac{\alpha^2_{\mathrm{EM}}}{\pi^3 M^2}\;L(M)\;f^B(p_0;T)\;\frac{1}{3}\;g_{\mu\nu}\;\mathrm{Im}\,\Pi^{\mu\nu}_{\mathrm{EM}}(M,p;\mu_B,T),$$

[E.L. Feinberg, Nuovo Cim. A34, 391 (1976)], [L.D. McLerran, T. Toimela, Phys.Rev. D31, 545 (1985)]
[H.A. Weldon, Phys.Rev. D42, 2384-2387 (1990)], [C. Gale, J. Kapusta, Phys.Rev. C35, 2107 (1987) & Nucl.Phys. B357, 65-89 (1991)]

EM spectral function in the vacuum

In the vacuum, $\mathrm{Im}\,\Pi^{\mathrm{vac}}_{\mathrm{em}}$ is accurately known from e^+e^- annihilation:

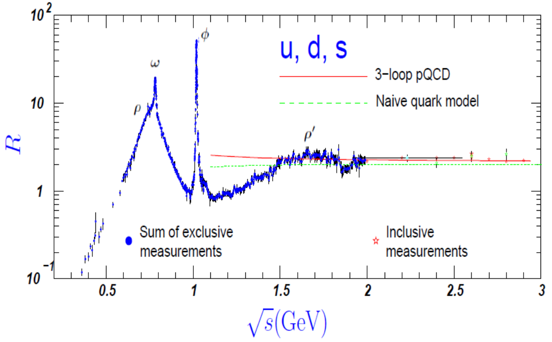
$$R = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \propto \frac{\mathrm{Im}\,\Pi^{\mathrm{vac}}_{\mathrm{em}}}{M^2}$$

In the low-mass regime (LMR: $M \leq 1$ GeV) the EM spectral function is saturated by the **spectral functions of the light vector mesons (VMD)**:

$$\mathrm{Im}\Pi^{\mathrm{vac}}_{\mathrm{EM}}(M) = \sum_{v=\rho,\omega,\phi} \left(\frac{m_v^2}{g_v}\right)^2 \mathrm{Im}D^{\mathrm{vac}}_v(M)$$

For higher energies, quark degrees of freedom:

$$\mathrm{Im}\Pi^{\mathrm{vac}}_{\mathrm{EM}}(M) = -\frac{M^2}{12\pi}\left[1 + \frac{\alpha_s(M)}{\pi} + \dots\right] N_c \sum_{q=u,d,s} (e_q)^2$$

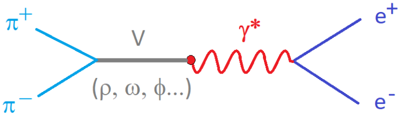


[Particle Data Group]

[J.J. Sakurai, Ann.Phys. 11 (1960) & Currents and Mesons, Chicago Lectures]
[R. Rapp, J. Wambach, Adv.Nucl.Phys. 25, 1 (2000)]
[R. Rapp, Acta Phys.Polon. B42, 2823-2852 (2011)]

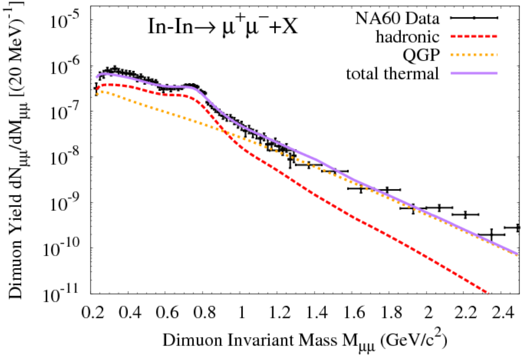
Connection between dileptons and vector mesons

Vector mesons have the same quantum numbers as photons and can decay directly into dileptons:



Excess dimuon invariant-mass spectrum as measured in In-In collisions at $\sqrt{s_{NN}} = 17.3$ GeV by the NA60 collaboration at the SPS is well described by using **vector meson dominance**:

$$\mathrm{Im}\Pi^{\mu\nu}_{\mathrm{EM}}(M) \sim \mathrm{Im}D^{\mu\nu}_{\rho} + \frac{1}{9}D^{\mu\nu}_{\omega} + \frac{2}{9}D^{\mu\nu}_{\phi}$$

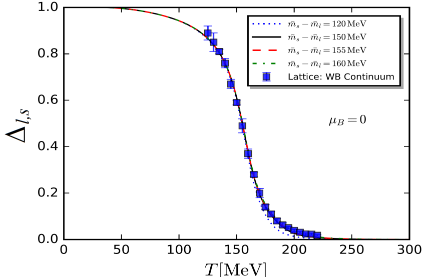


[R. Rapp, H. van Hees, Phys. Lett. B 753 (2016) 586-590]

Connection to chiral symmetry

Chiral symmetry:

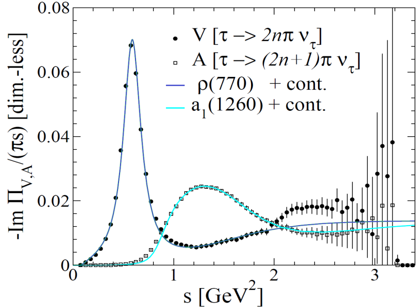
- QCD Lagrangian has chiral symmetry SU$(N_f)_L\times$ SU$(N_f)_R$ in the limit of vanishing quark masses
- chiral symmetry is broken spontaneously by dynamical formation of a quark condensate $\langle\bar{q}q\rangle \sim \Delta_{l,s}$



QCD and chiral sum rules:

$$\int_0^\infty \frac{ds}{\pi}(\Pi_V(s)-\Pi_A(s)) = m_\pi^2 f_\pi^2 = -2m_q\langle\bar{q}q\rangle$$

- sum rules connect spectral functions and condensates
- chiral restoration manifests itself through mixing of vector and axial-vector correlators!



[W.-j. Fu, J.M. Pawlowski, F. Rennecke, Phys. Rev. D 101, 054032 (2020)]
[S. Borsanyi et al. (Wuppertal-Budapest), JHEP 09, 073 (2010)]
[R. Barate, et al., (ALEPH), EPJC 4 (1998) 409-431]
[R. Rapp, J. Wambach, H. v. Hees, Landolt-Bornstein 23, 134]

Chiral Mixing

At low temperatures and densities, i.e. for a dilute pion gas, one can apply chiral reduction and current algebra to find the following 'mixing theorem' for the vector and axial-vector correlation functions:

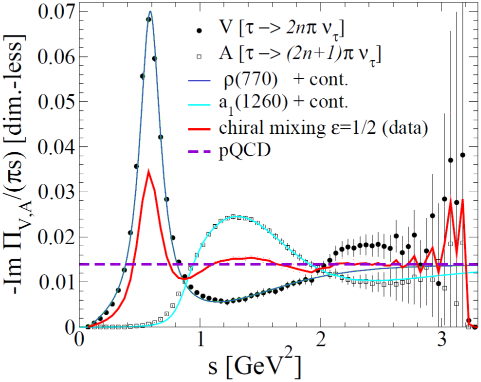
$$\Pi_V(q) = (1-\varepsilon)\,\Pi_V^0(q) + \varepsilon\,\Pi_A^0(q)$$

with mixing parameter $\varepsilon = T^2/6f_\pi^2$.

Chiral mixing has direct consequences on the thermal dilepton rate:

$$\frac{dN_{ll}}{d^4x d^4q} = \frac{4\alpha_{\mathrm{EM}}^2 f^B}{(2\pi)^2}\left\{\rho_{\mathrm{EM}} - (\varepsilon - \frac{\varepsilon^2}{2})(\rho_V - \rho_A)\right\}$$

[M. Dey et al., Phys. Lett. B 252 (1990), 620-624]
[Z. Huang, Phys. Lett. B 361 (1995) 131-136]



[R. Rapp, Acta Phys. Polon. B 42 (2011) 2823-2852]

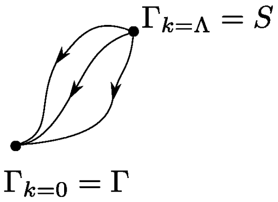
Theoretical setup

Method of choice: FRG

Functional Renormalization Group (FRG):

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\partial_k R_k \left[\Gamma_k^{(2)} + R_k\right]^{-1}\right)$$

[C. Wetterich, Phys.Lett. B301, 90 (1993)]



[wikipedia.org]

- non-perturbative continuum framework
- implements Wilson's coarse-graining idea: fluctuations integrated out
- Γ_k interpolates between bare action S in the UV and effective action Γ in the IR
- capable of describing phase transitions at finite temperature and density
- **analytically-continued FRG (aFRG) method gives access to spectral functions!**

Effective theory for nuclear matter

We use the **parity-doublet model** with $N_1 = N(938) = (n, p)$, $N_2 = N^*(1535)$:

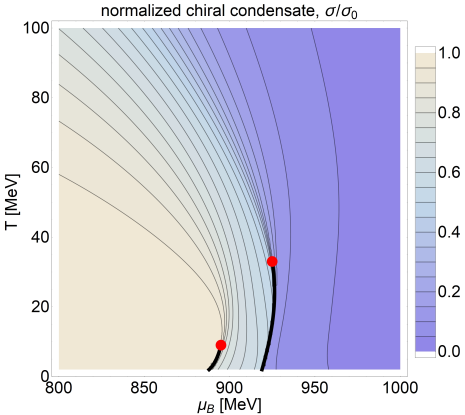
$$\Gamma_k = \int d^4x \left\{ \bar{N}_1 \left(\partial\!\!\!/ - \mu_B \gamma_0 + h_1(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma^5) \right) N_1 + \bar{N}_2 \left(\partial\!\!\!/ - \mu_B \gamma_0 + h_2(\sigma - i\vec{\tau}\cdot\vec{\pi}\gamma^5) \right) N_2 \right.$$
$$\left. + m_{0,N} \left(\bar{N}_1 \gamma^5 N_2 - \bar{N}_2 \gamma^5 N_1 \right) + U_k(\phi^2) - c\sigma \right\}$$

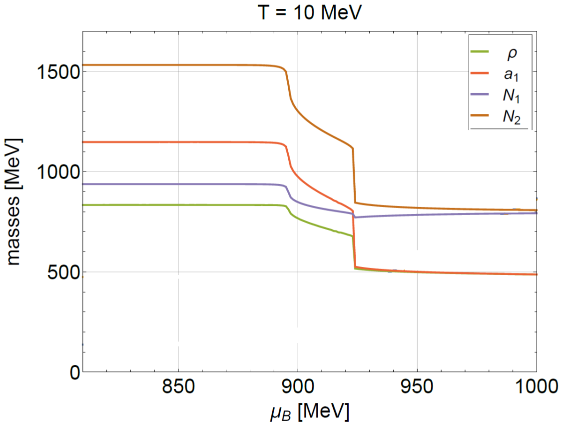
- provides a phenomenologically successful description of nuclear matter
- describes **nuclear liquid-gas transition together with a chiral phase transition**
- accounts for a **finite nucleon mass $m_{0,N}$** in a **chirally-invariant** fashion
- provides a natural description for the **parity-doubling structure** of the low-lying baryons

[C. E. Detar, T. Kunihiro, Phys. Rev. D 39, 2805 (1989)]
[R.-A. T., C. Jung, L. v. Smekal, J. Wambach, Phys. Rev. D 104, 054005 (2021)]

Parity-doublet model (I)

- describes nuclear liquid-gas transition together with a chiral phase transition:





[R.-A. T., C. Jung, L. v. Smekal, J. Wambach, Phys. Rev. D 104, 054005 (2021)]

Parity-doublet model (II)

Accounts for a **finite nucleon mass in a chirally-invariant fashion**:

- the **proton mass** can be obtained from the trace of the energy-momentum tensor of QCD

$$T \equiv T^{\mu}_{\mu} = \frac{\beta(g)}{2g} G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1 + \gamma_{m_l}) \bar{q}_l q_l$$

$$\rightarrow \quad \langle \mathbf{p}_1 | T | \mathbf{p}_2 \rangle \sim G(q^2), \qquad G(0) = M$$

  with the scalar gravitational form factor G

- only $\sim 8\%$ from chiral symmetry breaking ('sigma-term'), **rest from gluon term!**
- **mass radius of the proton** can be obtained as derivative w.r.t. momentum transfer $t = q^2$:

$$\langle R_m^2 \rangle = \frac{6}{M} \frac{dG}{dt}\Big|_{t=0}$$

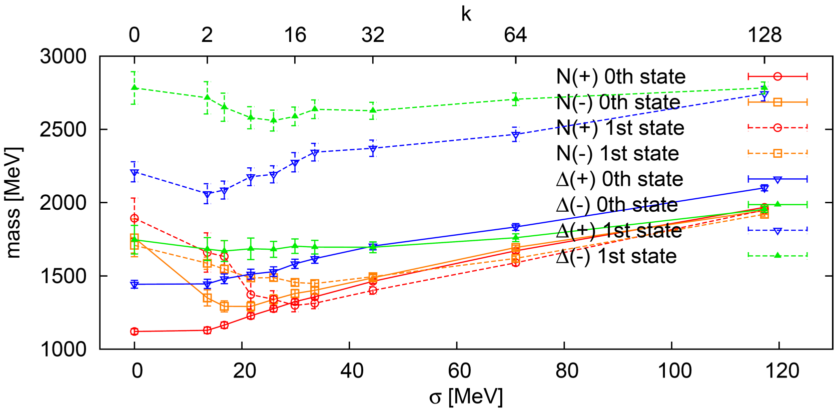
  $\rightarrow$ GlueX data leads to $R_m \approx 0.55$ fm, as opposed to $R_c \approx 0.84$ fm!

[J. C. Collins, A. Duncan, S. D. Joglekar, Phys. Rev. D 16, 438 (1977)], [N. K. Nielsen, Nucl. Phys. B 120, 212 (1977)]
[D. E. Kharzeev, Phys.Rev.D 104, 054015 (2021)], [A. Ali et al. (GlueX), Phys. Rev. Lett. 123, 072001 (2019)]

Parity-doublet model (III)

Provides a natural description for the parity-doubling structure of the low-lying baryons:

- lattice simulations show **degeneration of parity partners** as chiral symmetry is (artificially) restored by removing the k lowest Dirac modes:



[L. Ya. Glozman, C. B. Lang, M. Schröck, Phys. Rev. D 86, 014507 (2012)]

Introducing vector and axial-vector mesons

Parity-doublet model with vector mesons:

$$\Gamma_k = \int d^4x \Big\{ \bar{N}_1 \left(\not{\partial} - \mu_B \gamma_0 + h_{s,1}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,1}(\gamma_\mu \vec{\tau}\cdot\vec{\rho}_\mu + \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu}) \right) N_1$$
$$+ \bar{N}_2 \left(\not{\partial} - \mu_B \gamma_0 + h_{s,2}(\sigma - i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,2}(\gamma_\mu \vec{\tau}\cdot\vec{\rho}_\mu - \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu} \right) N_2$$
$$+ m_{0,N} \left(\bar{N}_1\gamma^5 N_2 - \bar{N}_2\gamma^5 N_1 \right) + U_k(\phi^2) - c\sigma + \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi$$
$$- \frac{1}{4}\,\mathrm{tr}\,\partial_\mu\rho_{\mu\nu}\partial_\sigma\rho_{\sigma\nu} + \frac{m_v^2}{8}\,\mathrm{tr}\,\rho_{\mu\nu}\rho_{\mu\nu} \Big\}.$$

- ρ and a_1 in terms of **anti-symmetric rank-2 tensor fields** which transform according to the (1,0) and (0,1) representations of the Euclidean O(4) group (with generators T_R and T_L):

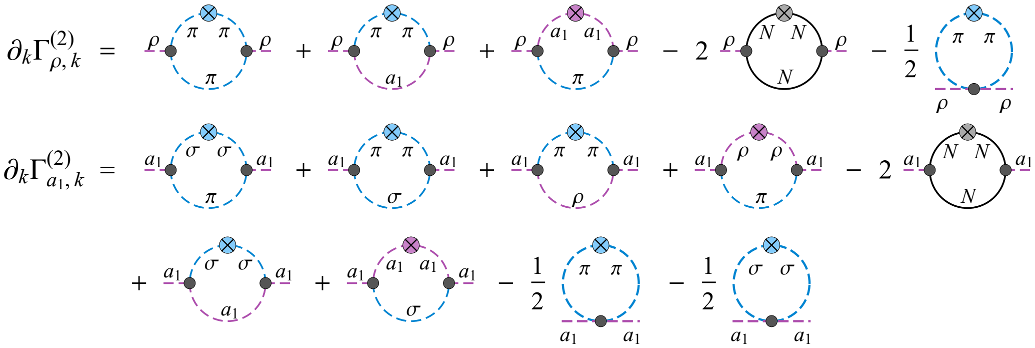
$$\rho_{\mu\nu} = \rho^+_{\mu\nu} + \rho^-_{\mu\nu} = \vec{\rho}^{\,+}_{\mu\nu}\vec{T}_R + \vec{\rho}^{\,-}_{\mu\nu}\vec{T}_L$$

- the iso-triplet **vector and axial-vector fields** are obtained as

$$\vec{\rho}_\mu = \frac{1}{2m_v}\mathrm{tr}(\partial_\sigma\rho_{\sigma\mu}\vec{T}_V), \qquad \vec{a}_{1\mu} = \frac{1}{2m_v}\mathrm{tr}(\partial_\sigma\rho_{\sigma\mu}\vec{T}_A)$$

[R.-A. T., C. Jung, L. v. Smekal, J. Wambach, Phys. Rev. D 104, 054005 (2021)], [C. Jung, L. v. Smekal, Phys. Rev. D 100, 116009 (2019)]

Flow equations for ρ and a_1 2-point functions



- vertices extracted from ansatz for the effective average action Γ_k
- **aFRG method** allows for **analytic continuation** of flow equations to real energies ω!

[R.-A. T., C. Jung, L. von Smekal, J. Wambach, Phys. Rev. D 104, 054005 (2021)]
[C. Jung, L. von Smekal, Phys. Rev. D 100, 116009 (2019)]

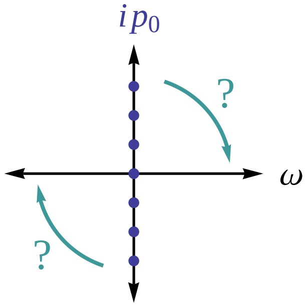
Two-step analytic continuation procedure

1) Use periodicity w.r.t. imaginary energy $ip_0 = i2n\pi T$:

$$n_{B,F}(E + ip_0) \to n_{B,F}(E)$$

2) Substitute p_0 by continuous real frequency ω:

$$\Gamma^{(2),R}(\omega, \vec{p}) = -\lim_{\epsilon \to 0} \Gamma^{(2),E}(ip_0 \to -\omega - i\epsilon, \vec{p})$$



Spectral function is then given by

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi}\mathrm{Im}\frac{1}{\Gamma^{(2),R}(\omega, \vec{p})}$$

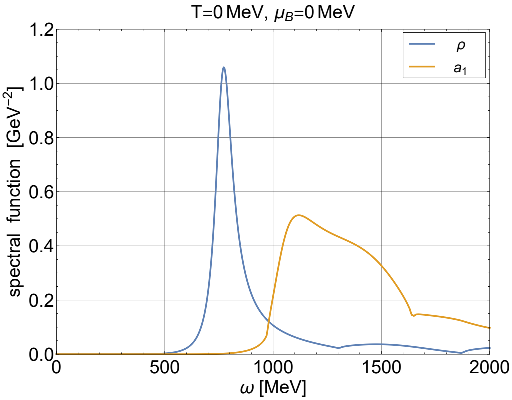
[K. Kamikado, N. Strodthoff, L. von Smekal, J. Wambach, Eur.Phys.J. C74 (2014) 2806]
[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]
[J. M. Pawlowski, N. Strodthoff, Phys. Rev. **D 92**, 094009 (2015)]
[N. Landsman and C. v. Weert, Physics Reports 145, 3&4 (1987) 141]

Part III

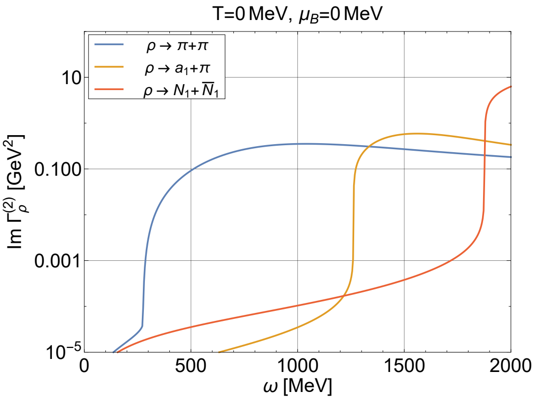
Results on spectral functions and dileptons

ρ and a_1 spectral functions in the vacuum (aFRG)

spectral functions:



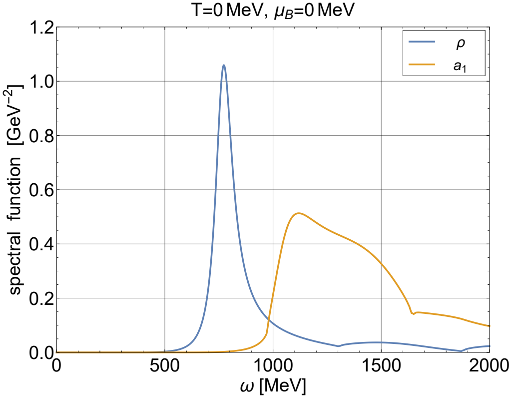
imaginary part of ρ 2-point function:



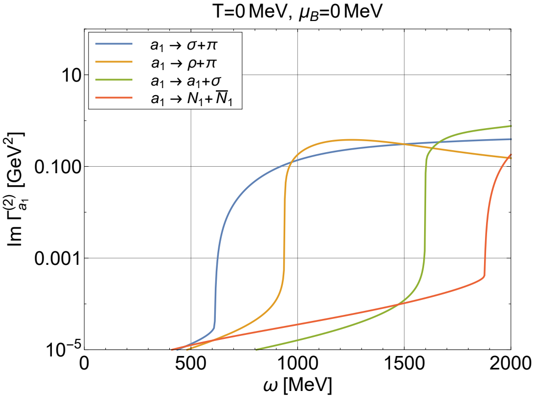
[R.-A. T., C. Jung, L. von Smekal, J. Wambach, Phys. Rev. D 104, 054005 (2021)]

ρ and a_1 spectral functions in the vacuum (aFRG)

spectral functions:



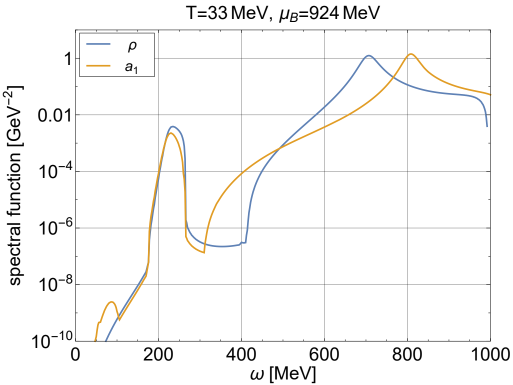
imaginary part of a_1 2-point function:



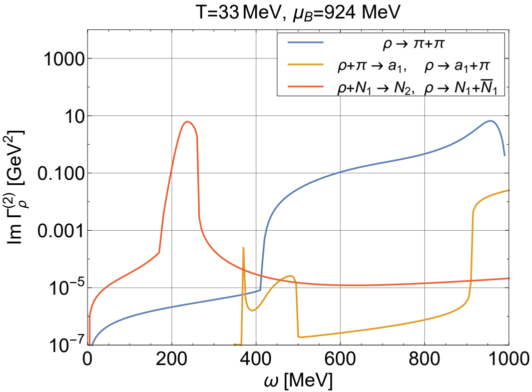
[R.-A. T., C. Jung, L. von Smekal, J. Wambach, Phys. Rev. D 104, 054005 (2021)]

ρ and a_1 spectral functions near chiral CEP (aFRG)

spectral functions:



imaginary part of ρ 2-point function:

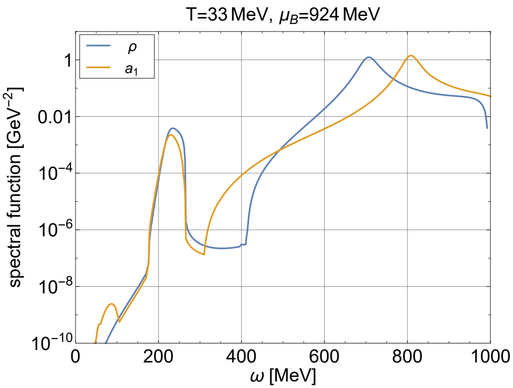


- a pronounced peak at lower energies due to the process $\rho + N_1 \to N_2$ is observed!

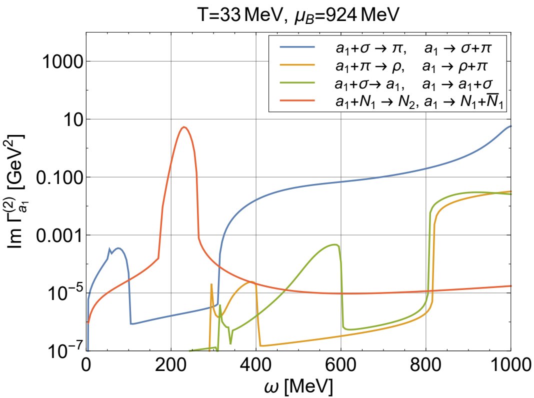
[R.-A. T., C. Jung, L. von Smekal, J. Wambach, Phys. Rev. D 104, 054005 (2021)]

ρ and a_1 spectral functions near chiral CEP (aFRG)

spectral functions:



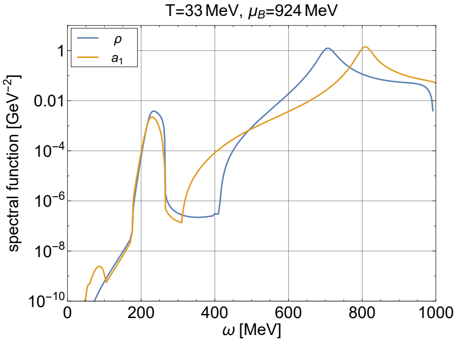
imaginary part of a_1 2-point function:

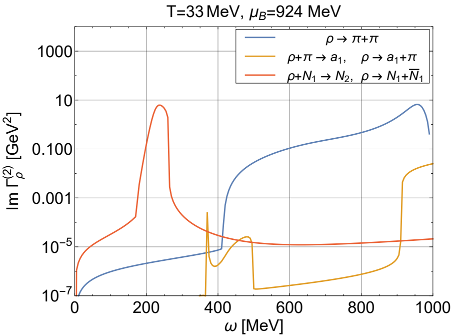


- a pronounced peak at lower energies due to the process $a_1 + N_1 \to N_2$ is observed!

[R.-A. T., C. Jung, L. von Smekal, J. Wambach, Phys. Rev. D 104, 054005 (2021)]

ρ and a_1 spectral functions near chiral CEP





- peak due to process $\rho + N \to N^*(1535)$, depends on size of ρ-N-$N^*(1535)$ coupling:

N(1535) BRANCHING RATIOS

$\Gamma(N\rho, S=1/2)/\Gamma_{total}$ **Γ_7/Γ**

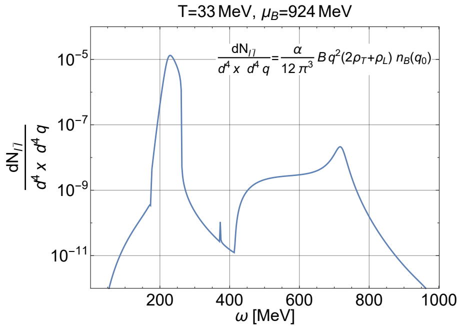
VALUE (%)	DOCUMENT ID		TECN	COMMENT
2.7±0.6	ADAMCZEW...	20	DPWA	Multichannel
14 ±2	[11] HUNT	19	DPWA	Multichannel

[R.-A. T., C. Jung, L. von Smekal, J. Wambach, Phys. Rev. D 104, 054005 (2021)], [Particle Data Group (2021)]
[M. Zétényi, D. Nitt, M. Buballa, T. Galatyuk, Phys. Rev. C 104, 015201 (2021)]

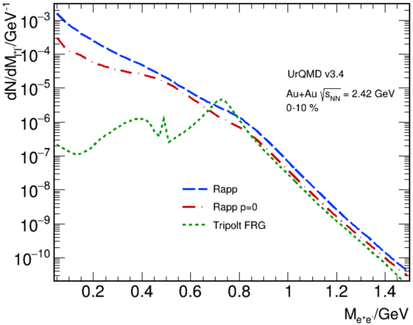
Preliminary results on dilepton rate and spectrum

The resonance-production peak in the ρ spectral function due to the process $\rho + N \to N^*(1535)$ directly translates into an **enhancement of the thermal dilepton rate**:

dilepton rate from Weldon formula:



dilepton spectrum from UrQMD and coarse-grainig (Max Wiest):



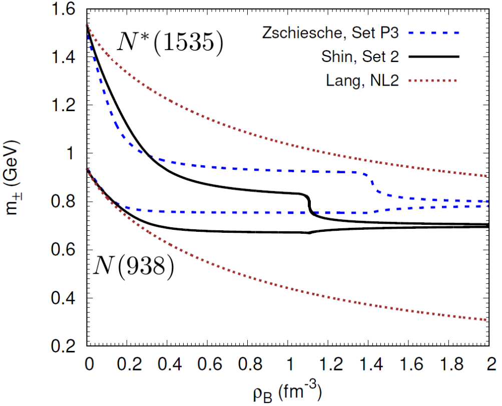
- unique **prediction of the parity-doublet model**!
- detection would yield strong evidence in support of the parity-doubling scenario as providing the mechanism for chiral symmetry restoration in dense nuclear matter!

Transport simulation with parity doubling

Parity-doublet model (PDM) mean fields for the nucleon, $N(938)$, and its parity partner, $N^*(1535)$, were included in the GiBUU microscopic transport model:

- red-dotted line: Walecka mean fields (NL2)
- black and blue-dashed lines: PDM mean fields (Set 2 and P3)
- mass of the $N^*(1535)$ resonance decreases quickly with increasing baryon density ρ_B for the PDM fields

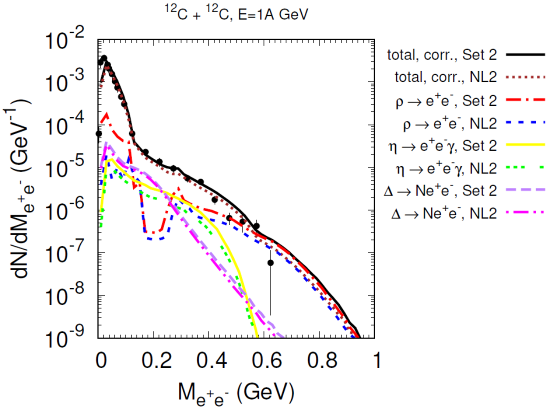
$\rightarrow$ **leads to enhancement of $N^*(1535)$ production in the intermediate stages of central heavy-ion collisions at 1 AGeV!**

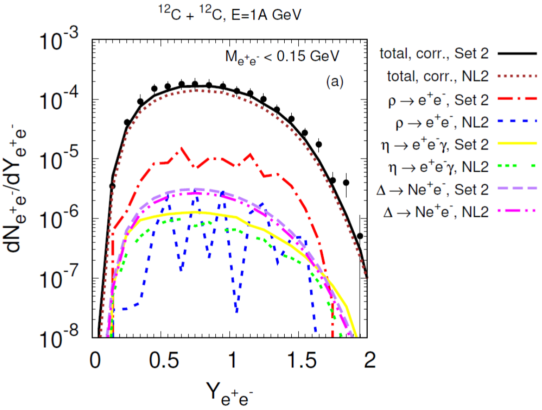


[A. B. Larionov, L. von Smekal, Phys. Rev. C 105, 034914 (2022)]

Transport simulation with parity doubling

Invariant-mass and rapidity distributions of dileptons in C+C collisions at 1 AGeV with GiBUU:





→ **PDM mean fields lead to enhanced $\rho \to e^+e^-$ and $\eta \to e^+e^-\gamma$ signals!**

[A. B. Larionov, L. von Smekal, Phys. Rev. C 105, 034914 (2022)]

Summary and Outlook

We computed ρ and a_1 spectral functions in nuclear matter:

- based on the parity-doublet model and the aFRG method
- effects of chiral symmetry restoration lead to peak in spectral functions at low energies
- might be observed experimentally in terms of increased dilepton yield!

Outlook:

- include repulsive effect ($\sim \omega$) for realistic desciption of nuclear matter
- include isospin-chemical potential to describe neutron-rich matter
- compute equation of state and thermal neutrino rates