

Multi-particle reactions in hadronic transport approaches

HFHF Theory Retreat 2022

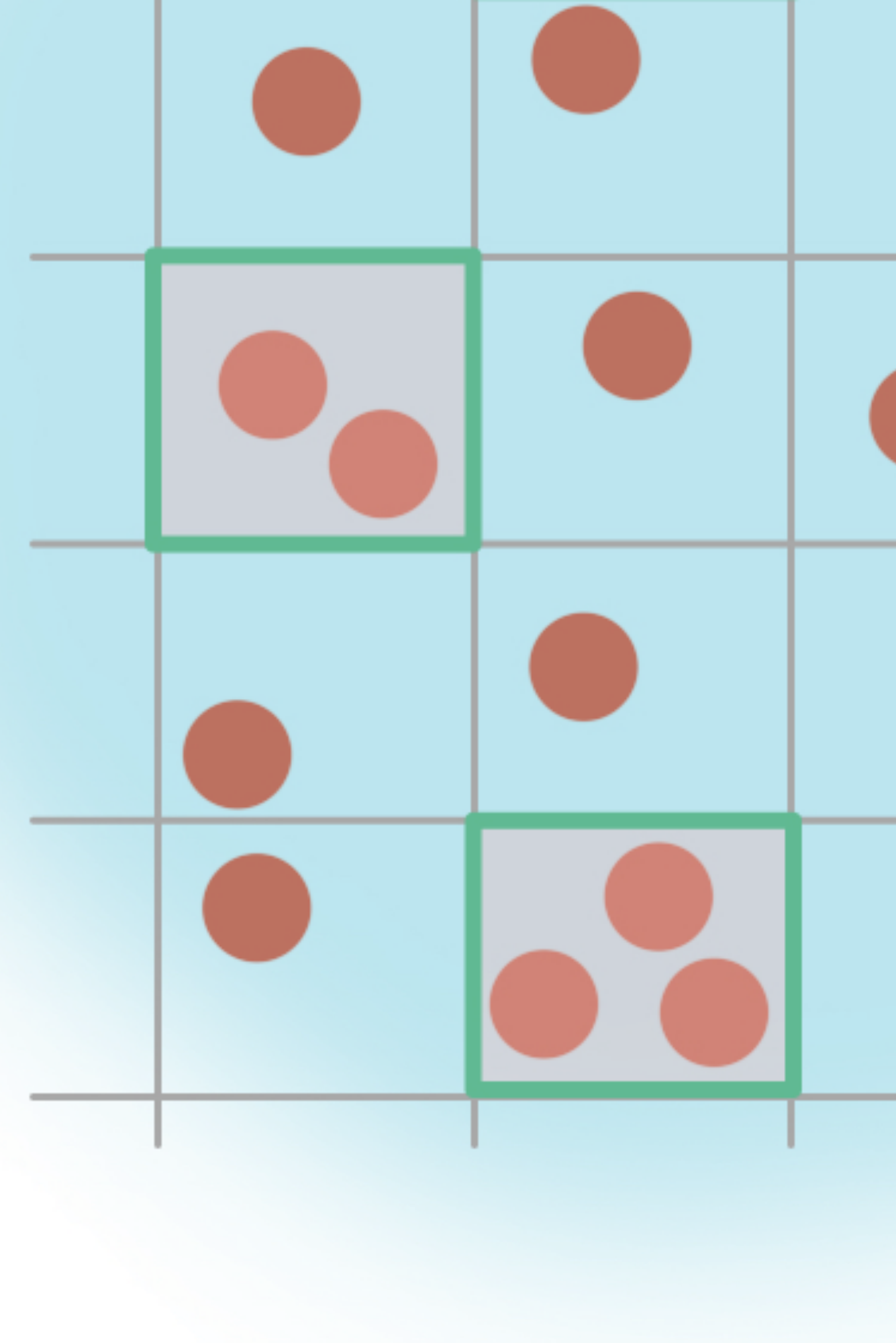
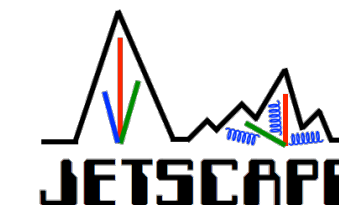
JS, D. Oliinychenko, J. M. Torres-Rincon & H. Elfner, Phys. Rev. C 104, 034908 (2021)

O. Garcia-Montero, JS, A. Schäfer, J. M. Torres-Rincon & H. Elfner, Phys. Rev. C 105, 064906 (2022)

JS PhD Thesis (2021)

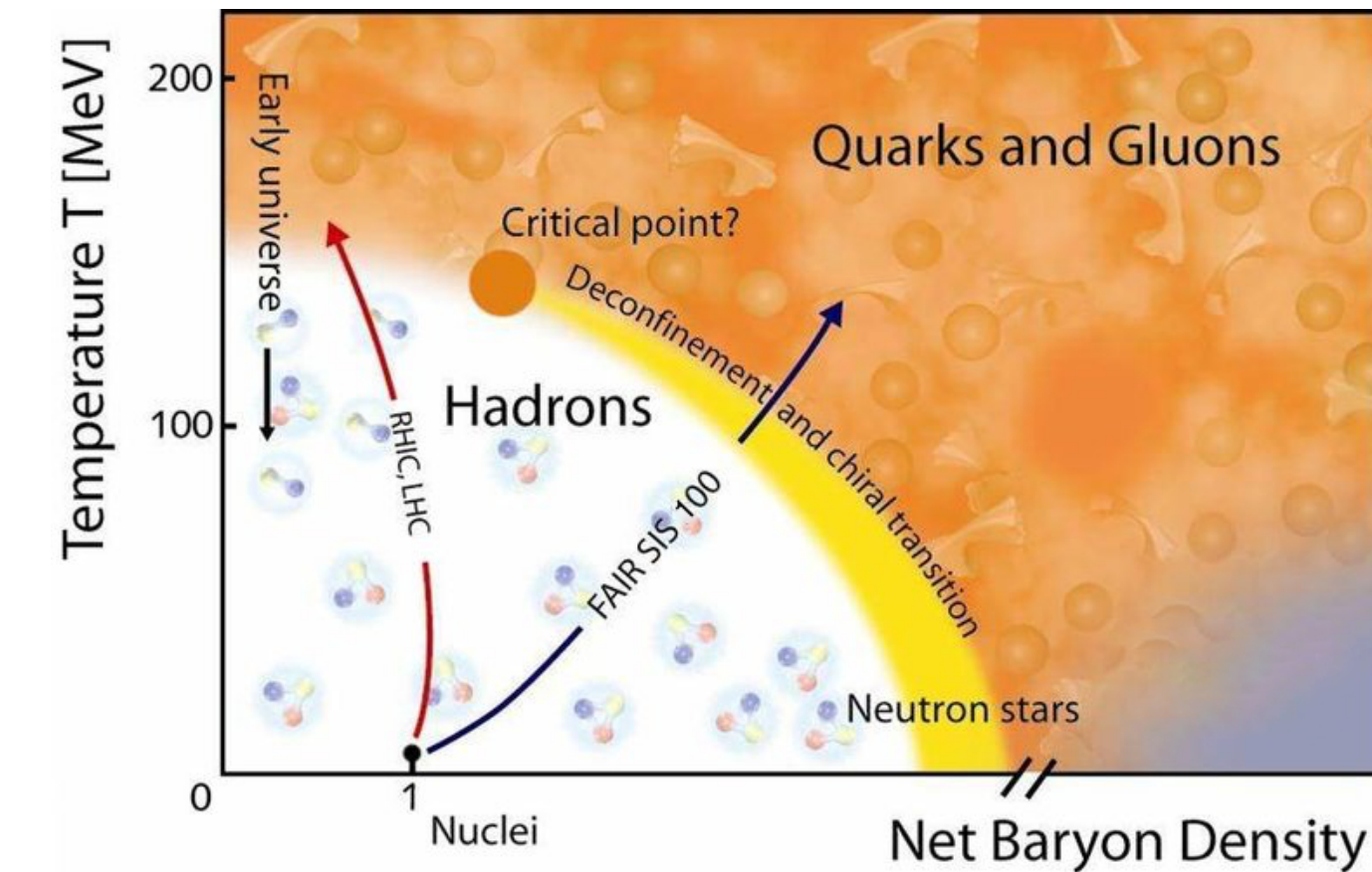
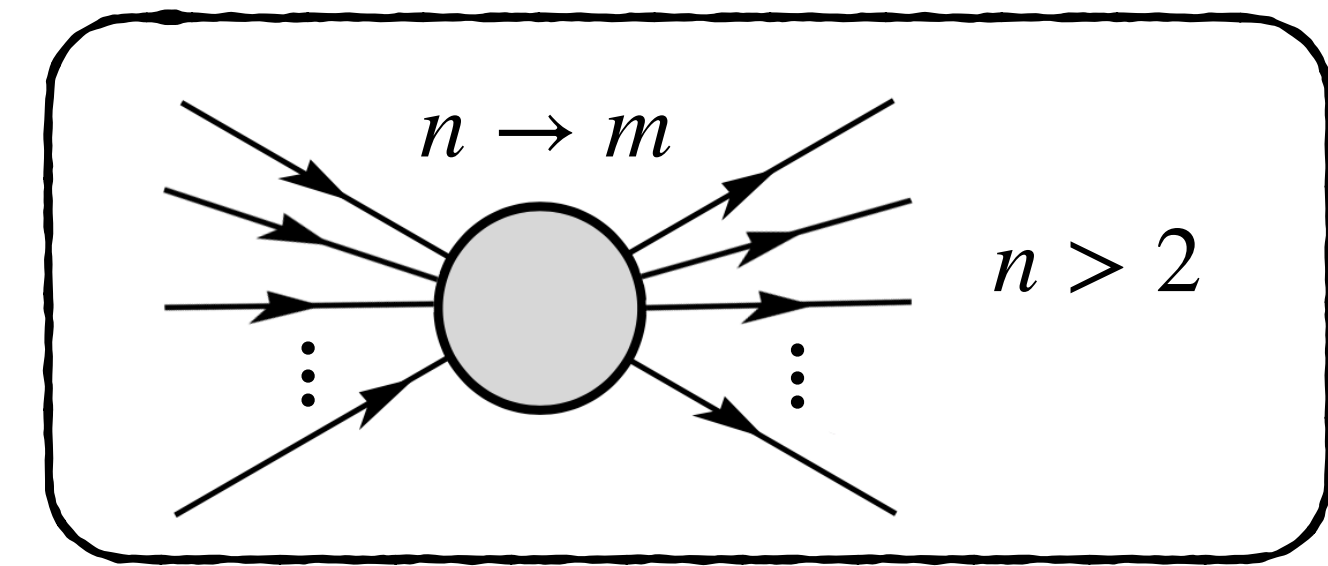
→ see also Gabriele's talk for multi-particle interactions with PHQMD

Jan Staudenmaier, September 15, 2022

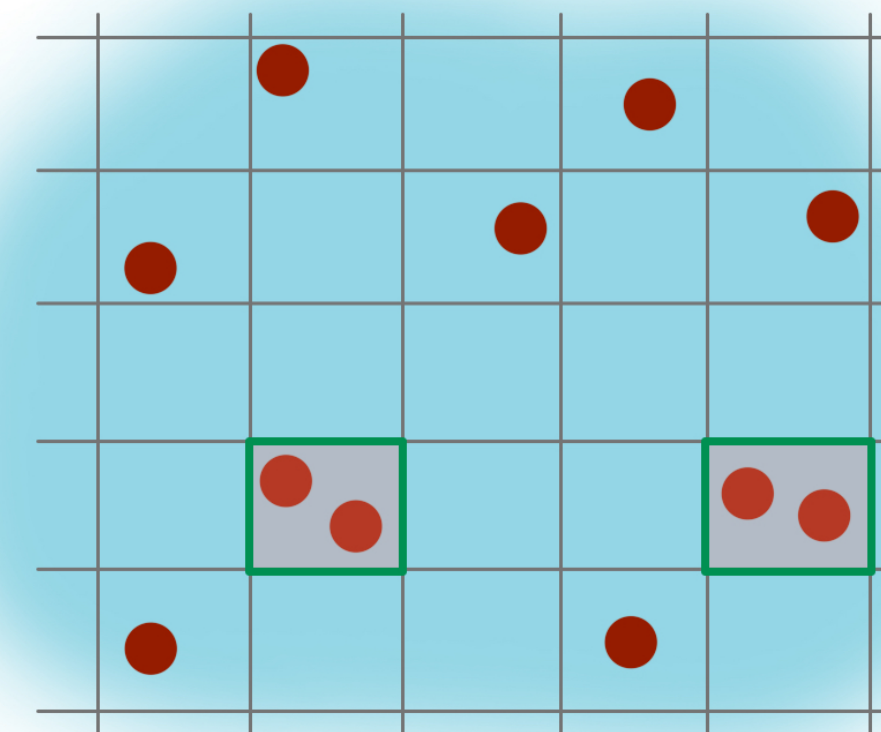


Multi-particle interactions

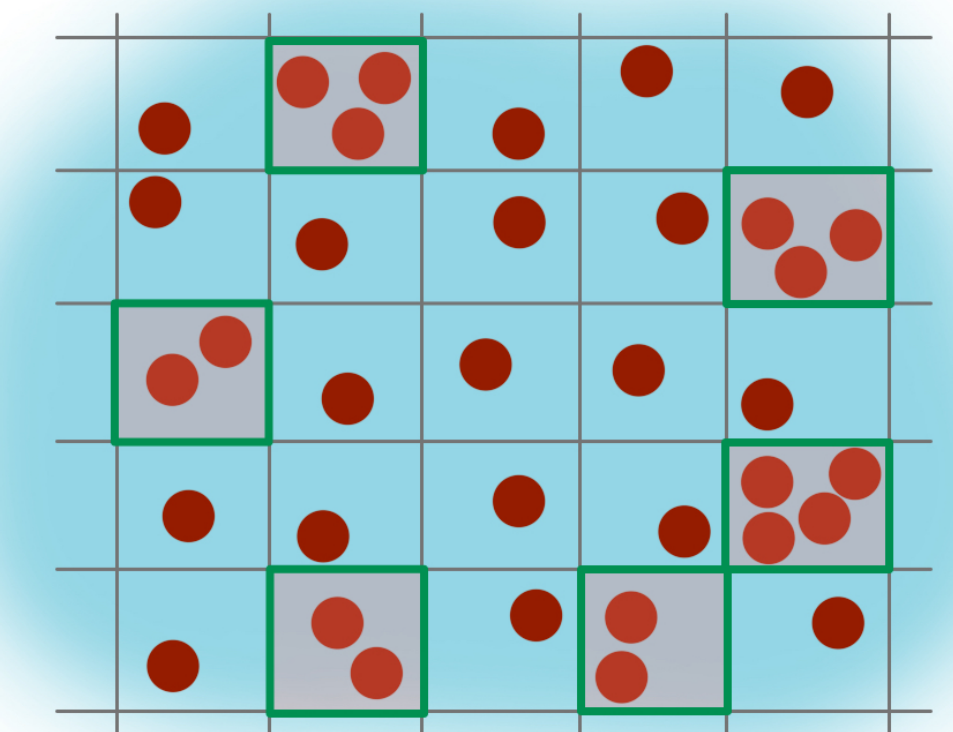
- Relevant in high-density medium as produced by FAIR, RHIC-BES, NICA, JPARC-HI → Disentangle signature of phase transition and critical endpoint
- Equilibration and thermalization of system
- Known, important multi-particle interactions
 - Deuteron catalysis: $Npn \leftrightarrow Nd, \pi pn \leftrightarrow \pi d$
P. Danielewicz and G. Bertsch, Nucl. Phys. A, vol. 533, pp. 712–748, 1991.
 K.-J. Sun, R. Wang, C. M. Ko, Y.-G. Ma, and C. Shen, 2021, 2106.12742.
 - Baryon-antibaryon annihilation: $B\bar{B} \leftrightarrow n$ mesons
W. Cassing, Nucl. Phys. A, vol. 700, pp. 618–646, 2002, nucl-th/0105069.
 E. Seifert and W. Cassing, Phys. Rev. C, vol. 97, no. 4, p. 044907, 2018, 1801.07557 and
 Phys. Rev. C, vol. 97, no. 2, p. 024913, 2018, 1710.00665.
 - Gluon Bremsstrahlung: $gg \leftrightarrow ggg$
Z. Xu and C. Greiner, Phys. Rev. C, vol. 71, p. 064901, 2005, hep-ph/0406278.
- Obey time reversal symmetry (i.e. detailed balance)
- Mostly neglected in microscopic approaches



Dilute System



Dense System



Hadronic transport approach: SMASH

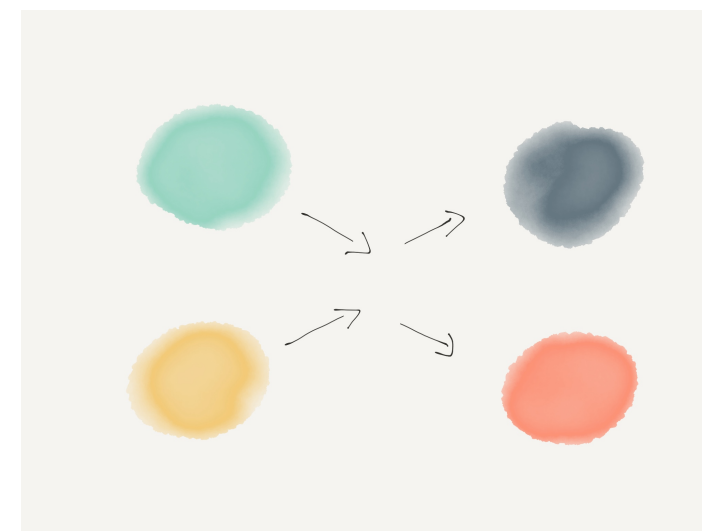
J. Weil et al., Phys. Rev. C, vol. 94, no. 5, p. 054905, 2016, 1606.06642.



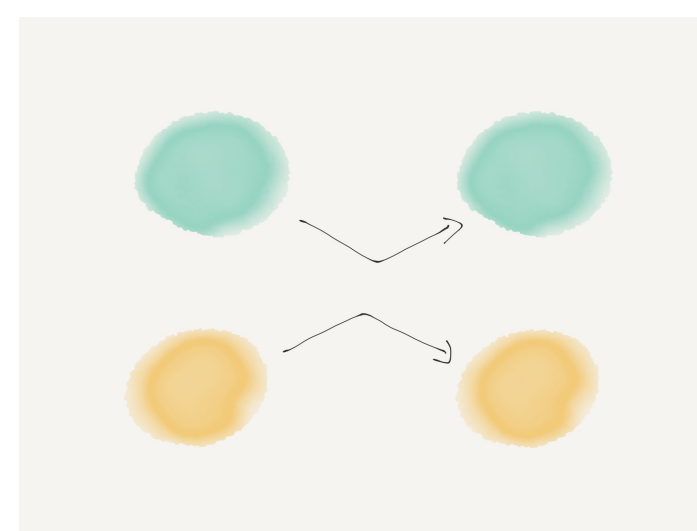
Boltzmann equation:
$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \nabla f + \vec{F} \frac{\partial f}{\partial \vec{p}} = \left(\frac{\partial f}{\partial t} \right)_{\text{coll}}$$

- Hadronic degrees of freedom → Hadronic multi-particle reactions
- Testparticle-Method: $N \mapsto NN_{\text{Test}}$ $\sigma \mapsto \sigma N_{\text{test}}^{-1}$

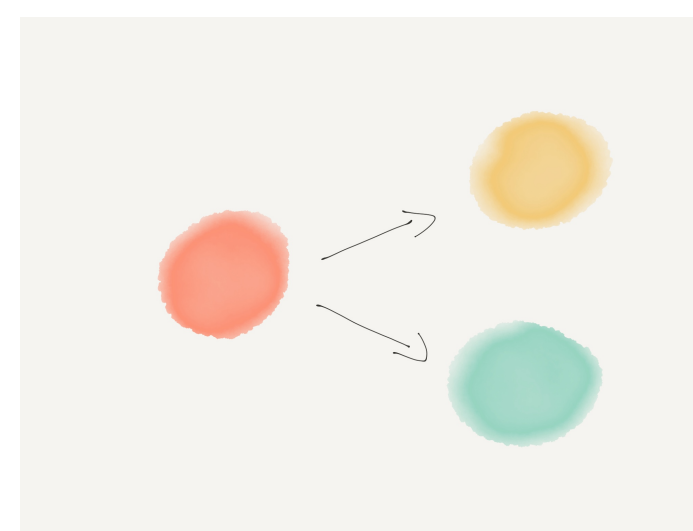
- Collision term:



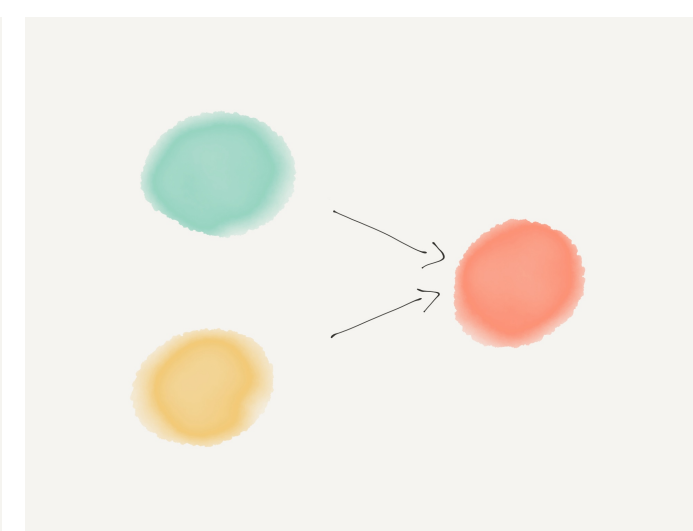
Inelastic Scattering



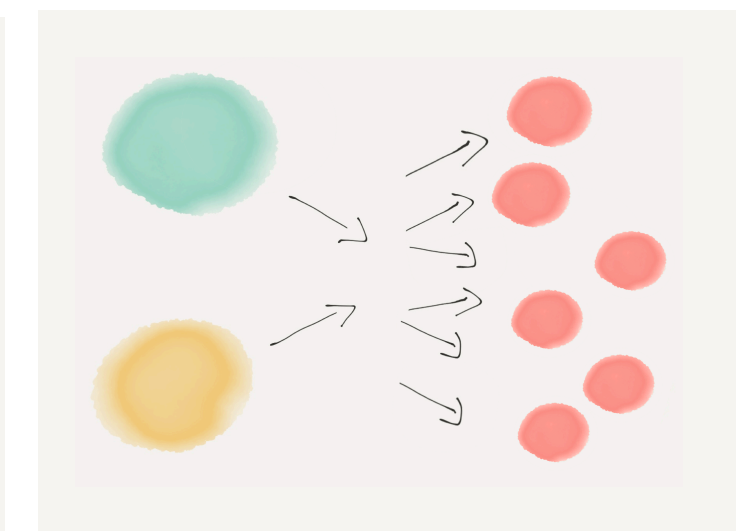
Elastic Scattering



Decays



Resonance Formation



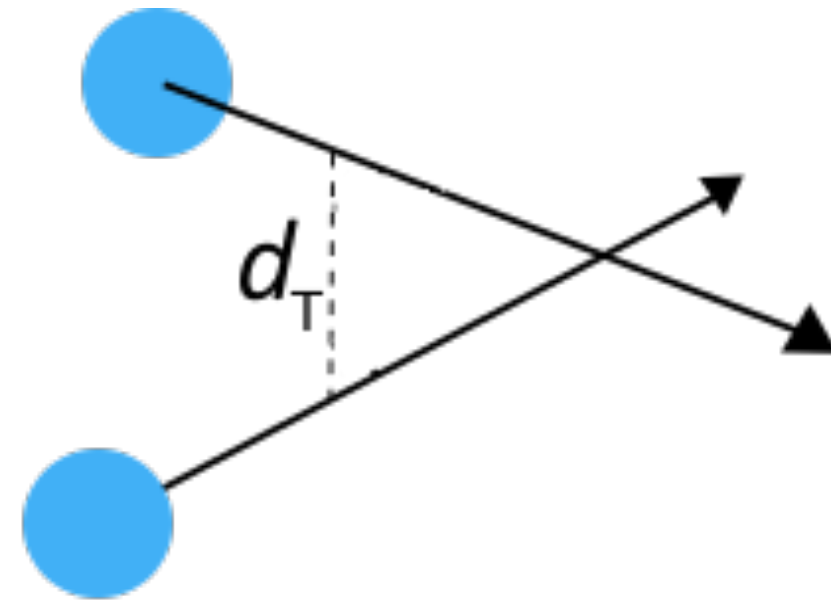
String Fragmentations

→ up to now only binary scatterings

- All new developments public → <https://github.com/smash-transport/smash>

Collision criteria in transport approaches

Geometric



$$d_T < d_{\text{int}} = \sqrt{\frac{\sigma}{\pi}}$$

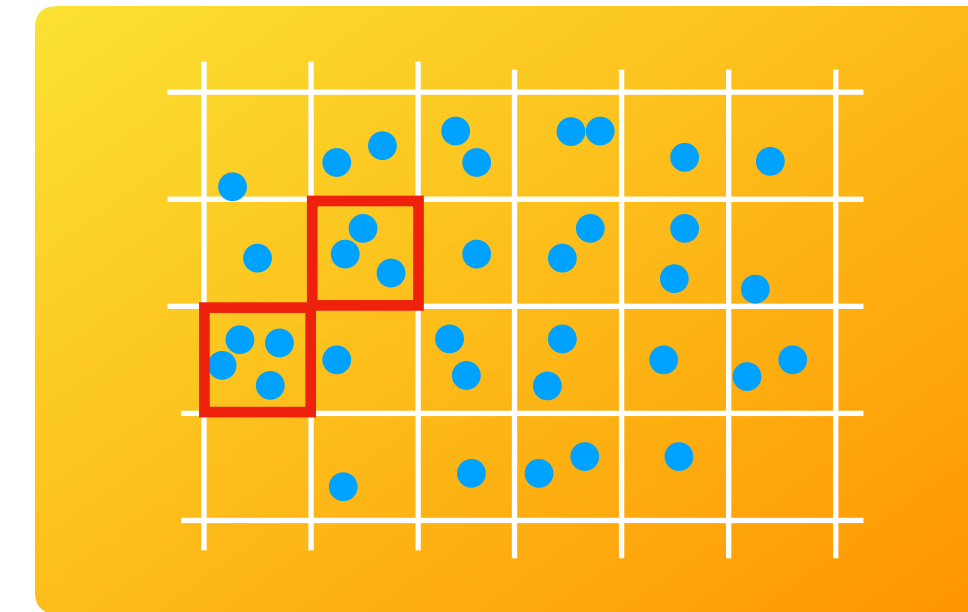
Based on transverse distance

Geometric interpretation of the total cross-section

Only binary collisions

z.B. S. A. Bass et al., Prog. Part. Nucl. Phys., vol. 41, pp. 255–369, 1998

Stochastic



$$P_{(2 \rightarrow 2)} = \frac{\Delta t}{\Delta^3 x} v_{\text{rel}} \sigma$$

Based in local collision probability

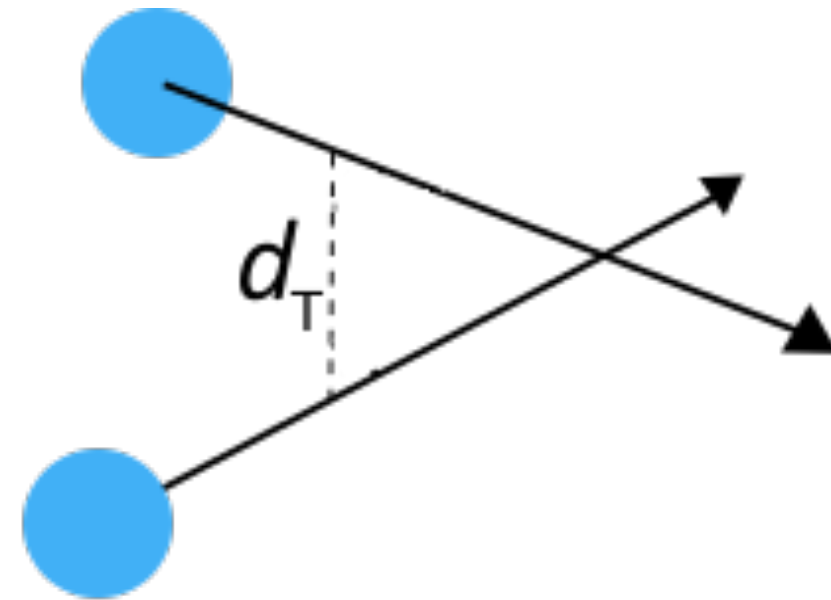
Derived from collision integral of Boltzmann equation

Natural extension to multi-particle collisions

P. Danielewicz and G. Bertsch, Nucl. Phys. A, vol. 533, pp. 712–748, 1991.
A. Lang, H. Babovsky, W. Cassing, U. Mosel, H.-G. Reusch, and K. Weber,
Journal of Computational Physics, vol. 106, no. 2, pp. 391 – 396, 1993.

Collision criteria in transport approaches

Geometric



$$d_T < d_{\text{int}} = \sqrt{\frac{\sigma}{\pi}}$$

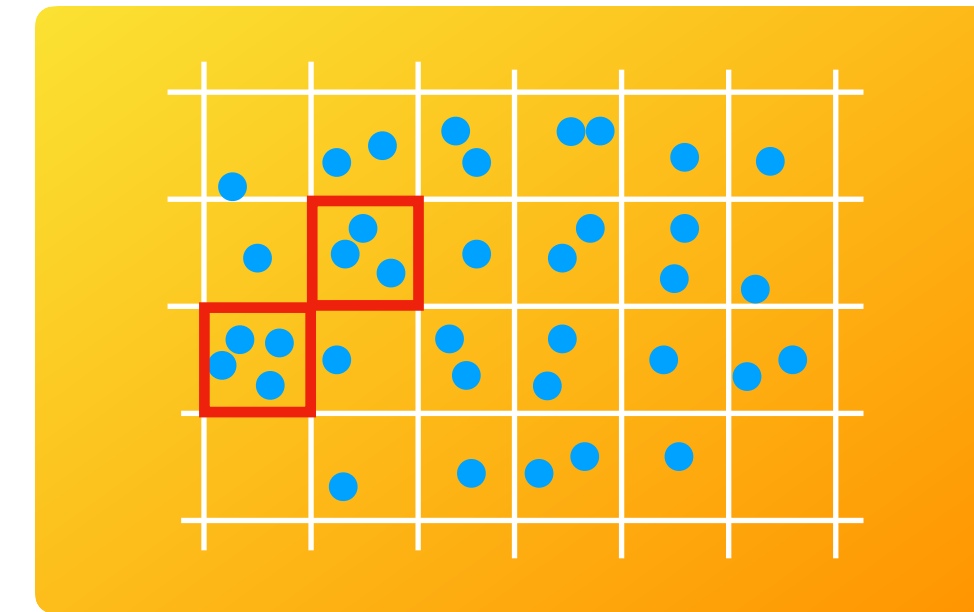
Based on transverse distance

Geometric interpretation of the total cross-section

Only binary collisions

z.B. S. A. Bass et al., Prog. Part. Nucl. Phys., vol. 41, pp. 255–369, 1998

Stochastic



$$P_{(2 \rightarrow 2)} = \frac{\Delta t}{\Delta^3 x} v_{\text{rel}} \sigma$$

Based in local collision probability

Derived from collision integral of Boltzmann equation

Natural extension to multi-particle collisions

P. Danielewicz and G. Bertsch, Nucl. Phys. A, vol. 533, pp. 712–748, 1991.
A. Lang, H. Babovsky, W. Cassing, U. Mosel, H.-G. Reusch, and K. Weber,
Journal of Computational Physics, vol. 106, no. 2, pp. 391 – 396, 1993.

Collision criteria in SMASH

- Geometric collision criterion
 - **Geometric** “UrQMD” criterion *S. A. Bass et al., Prog. Part. Nucl. Phys., vol. 41, 1998*
 - **Covariant** geometric criterion - *New default T. Hirano, Y. Nara, PTEP 2012 (2012), M.Sc. Thesis D. Mitrovic*
 - Covariant expression of the transverse distance
 - Collision time calculation in the two-particle center-of-momentum frame
- **Stochastic** collision criterion *A. Lang et al., Journal of Computational Physics, vol. 106, no. 2, 1993.*
 - Other name in literature: *Local-ensemble method* or *stochastic rates*
 - Treat usual binary reactions, plus enables multi-particle reactions
 - Use collision probability for Monte-Carlo decision
 - Time-step based evolution (Collision time random within timesteps)

$$d_T < \sqrt{\frac{\sigma}{\pi}}$$

$$\text{Rand}[0,1] \leq P_{n \rightarrow m}$$

Collision probabilities

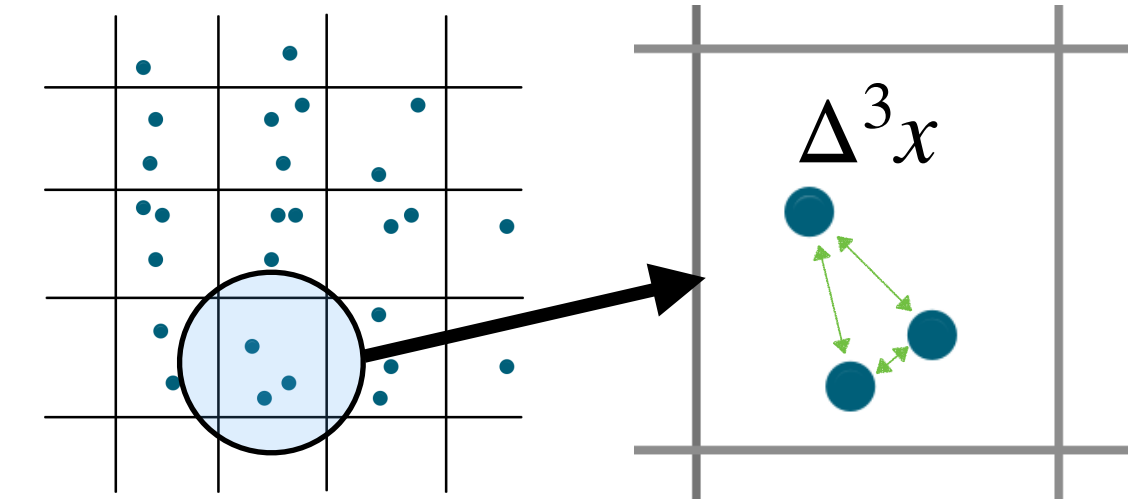
Z. Xu and C. Greiner, Phys. Rev. C, vol. 71, p. 064901, 2005 → **BAMPS**

+

W. Cassing, Nucl. Phys. A, vol. 700, pp. 618–646, 2002 → **(P)HSD**

- Probability of a collision within a cell (for Δt)

$$P_{12 \rightarrow 1'2'} = \frac{\Delta N_{\text{coll}}^{12 \rightarrow 1'2'}}{\Delta N_1 \Delta N_2} \quad P_{n \rightarrow m'} = \frac{\Delta N_{\text{coll}}^{n \rightarrow m'}}{\Delta N_1 \Delta N_2 \dots \Delta N_n}$$



- Number of particles and collision from Boltzmann equation

$$\Delta N = f(x, p, t) \Delta^3 x \frac{d^3 p}{(2\pi)^3} \quad \Delta N_{\text{coll}} = \left(\frac{\partial f}{\partial t} \right)_{\text{coll}} \Delta t \Delta^3 x \frac{d^3 p}{(2\pi)^3}$$

- Two-body scattering:

$$\left(\frac{\partial f}{\partial t} \right)_{\text{coll, loss}}^{12 \leftrightarrow 1'2'} = -\frac{1}{2E_1} \int d\Gamma_2 \frac{1}{S_{1'2'}} \int \underbrace{f_1 f_2 \overline{|\mathcal{M}_{12 \rightarrow 1'2'}|^2}}_{\text{cross section } \sigma} d\Phi_{2'}$$

$$P_{12 \rightarrow 1'2'} = \frac{\Delta t}{\Delta^3 x} v_{\text{rel}} \sigma_{12 \rightarrow 1'2'}$$

A. Lang, H. Babovsky, W. Cassing, U. Mosel, H.-G. Reusch, and K. Weber, Journal of Computational Physics, vol. 106, no. 2, pp. 391 – 396, 1993

- Straight forward to extend to multi-particle scatterings

↳ *Lorentz-invariant!*

$$P_{12 \dots n \rightarrow 1'2' \dots m'} = \frac{1}{2^n \prod_{j=1}^n E_j} I'_{nm} \frac{\Delta t}{(\Delta^3 x)^{n-1}} \quad \text{with } I'_{nm} = \frac{1}{S_{1'2' \dots m'}} \int \overline{|\mathcal{M}_{12 \dots n \rightarrow 1'2' \dots m'}|^2} d\Phi_{m'}$$

- But:** No cross section to use for more than 2 particles and matrix element in general unknown!

Collision probabilities

- Resolution: Express probability in terms of cross section of inverse process $P_{n \rightarrow 2} \propto \sigma_{2 \rightarrow n}$ or decay width $P_{n \rightarrow 1} \propto \Gamma_{1 \rightarrow n}$
- Assume matrix element not dependent on final state momenta and use detailed balance relation

$$\overline{|\mathcal{M}_{n \rightarrow m}|^2} = \frac{\prod_{j=1}^m g_k}{\prod_{j=1}^n g_j} \overline{|\mathcal{M}_{m \rightarrow n}|^2}$$

degeneracy factors

⇒ **Derived collision probabilities**

$$P_{3 \rightarrow 1} = \frac{g_R}{g_1 g_2 g_3} S_{123} \frac{\Delta t}{(\Delta^3 x)^2} \frac{\pi}{4 E_1 E_2 E_3} \frac{\Gamma_{1 \rightarrow 3}}{\Phi_3} \mathcal{A}(\sqrt{S})$$

$$P_{3 \rightarrow 2} = \frac{g'_1 g'_2}{g_1 g_2 g_3} \frac{S_{123}}{S'_{12}} \frac{1}{4 E_1 E_2 E_3} \frac{\Delta t}{(\Delta^3 x)^2} \frac{\lambda}{\Phi_3} \frac{1}{8 \pi s} \sigma_{2 \rightarrow 3}$$

$$P_{5 \rightarrow 2} = \frac{g'_1 g'_2}{g_1 g_2 g_3 g_4 g_5} \frac{S_{12345}}{S'_{12}} \frac{1}{32 E_1 E_2 E_3 E_4 E_5} \frac{\Delta t}{(\Delta^3 x)^4} \frac{\lambda}{\Phi_5} \frac{1}{4 \pi s} \sigma_{2 \rightarrow 5}$$

Collision probabilities

- Resolution: Express probability in terms of cross section of inverse process $P_{n \rightarrow 2} \propto \sigma_{2 \rightarrow n}$ or decay width $P_{n \rightarrow 1} \propto \Gamma_{1 \rightarrow n}$

- Assume matrix element not dependent on final state momenta and use detailed balance relation

$$\overline{|\mathcal{M}_{n \rightarrow m}|^2} = \frac{\prod_{j=1}^m g_j}{\prod_{j=1}^n g_j} \overline{|\mathcal{M}_{m \rightarrow n}|^2}$$

degeneracy factors

⇒ **Derived collision probabilities**

$$P_{3 \rightarrow 1} = \frac{g_R}{g_1 g_2 g_3} S_{123} \frac{\Delta t}{(\Delta^3 x)^2} \frac{\pi}{4 E_1 E_2 E_3} \frac{\Gamma_{1 \rightarrow 3}}{\Phi_3} \mathcal{A}(\sqrt{s})$$

$$P_{3 \rightarrow 2} = \frac{g'_1 g'_2}{g_1 g_2 g_3} \frac{S_{123}}{S'_{12}} \frac{1}{4 E_1 E_2 E_3} \frac{\Delta t}{(\Delta^3 x)^2} \frac{\lambda}{\Phi_3 8 \pi s} \sigma_{2 \rightarrow 3}$$

$$P_{5 \rightarrow 2} = \frac{g'_1 g'_2}{g_1 g_2 g_3 g_4 g_5} \frac{S_{12345}}{S'_{12}} \frac{1}{32 E_1 E_2 E_3 E_4 E_5} \frac{\Delta t}{(\Delta^3 x)^4} \frac{\lambda}{\Phi_5} \frac{1}{4 \pi s} \sigma_{2 \rightarrow 5}$$

⇒ **General forms for n-to-1 and n-to-2**

$$P_{n \rightarrow 1} = \frac{g_R}{\prod_{i=1}^n g_i} S_n \frac{\Delta t}{(\Delta^3 x)^{n-1}} \frac{\pi}{2^{n-1} \prod_{j=1}^n E_j} \frac{\Gamma_{1 \rightarrow n}}{\Phi_n} \mathcal{A}_R(\sqrt{s})$$

$$P_{n \rightarrow 2} = \frac{g'_1 g'_2}{\prod_{i=1}^n g_i} \frac{S_n}{S'_{12}} \frac{1}{2^n \prod_{j=1}^n E_j} \frac{\Delta t}{(\Delta^3 x)^{n-1}} \frac{\lambda(s, m_1'^2, m_2'^2)}{\Phi_n} \frac{1}{4 \pi s} \sigma_{2 \rightarrow n}$$

Newly introduced reactions in SMASH

- New collision criterion applied for all existing binary reactions with
- New multi-particle reactions ...

$$P_{2 \rightarrow 2/1} = \frac{\Delta t}{\Delta^3 x} v_{rel} \sigma_{2 \rightarrow 2/1}$$

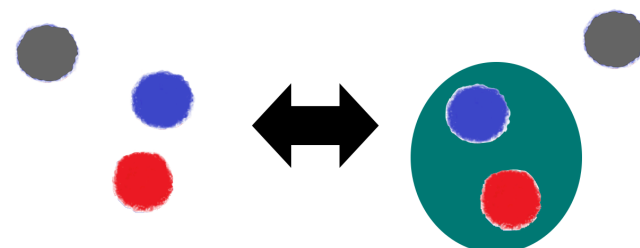
- for the **meson Dalitz decay back-reaction**

$$\hookrightarrow 3\pi \leftrightarrow \omega, 3\pi \leftrightarrow \phi, 2\pi\eta \leftrightarrow \eta'$$

$$P_{3 \rightarrow 1} = \frac{g_R}{g_1 g_2 g_3} S_{123} \frac{\Delta t}{(\Delta^3 x)^2} \frac{\pi}{4E_1 E_2 E_3} \frac{\Gamma_{1 \rightarrow 3}}{\Phi_3} \mathcal{A}(\sqrt{S})$$

- for the **deuteron pion and nucleon catalysis**

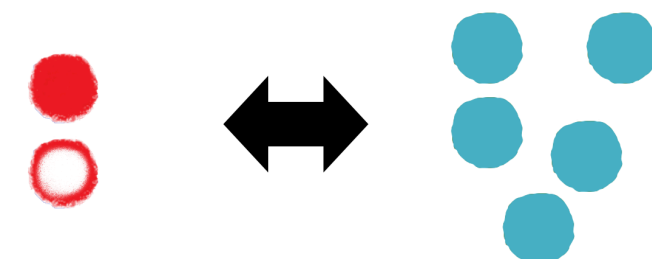
$$\hookrightarrow Npn \leftrightarrow Nd, \pi pn \leftrightarrow \pi d$$



$$P_{3 \rightarrow 2} = \frac{g'_1 g'_2}{g_1 g_2 g_3} \frac{S_{123}}{S'_{12}} \frac{1}{4E_1 E_2 E_3} \frac{\Delta t}{(\Delta^3 x)^2} \frac{\lambda}{\Phi_3 8\pi s} \sigma_{2 \rightarrow 3}$$

- for the **5-body annihilation back-reaction**

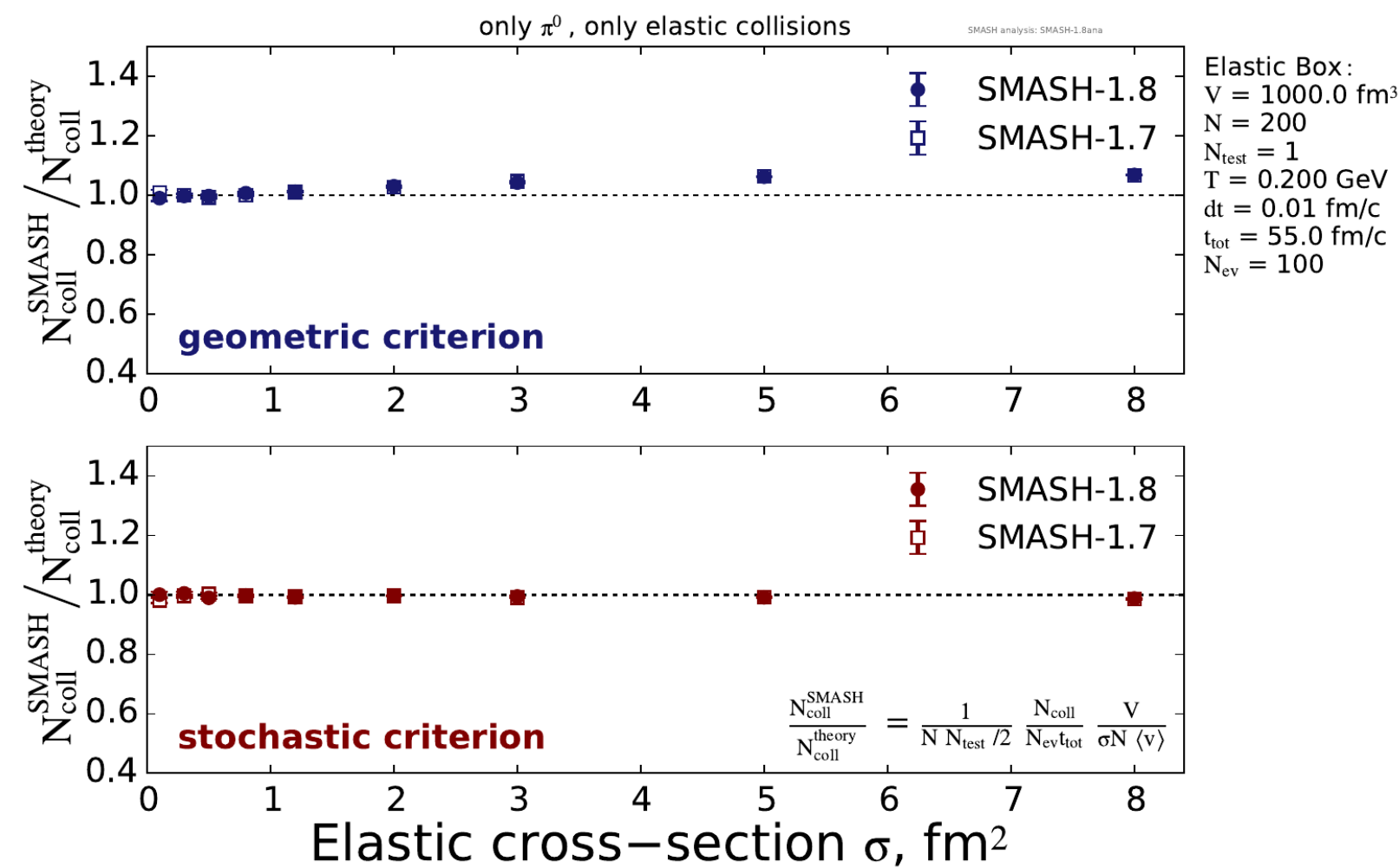
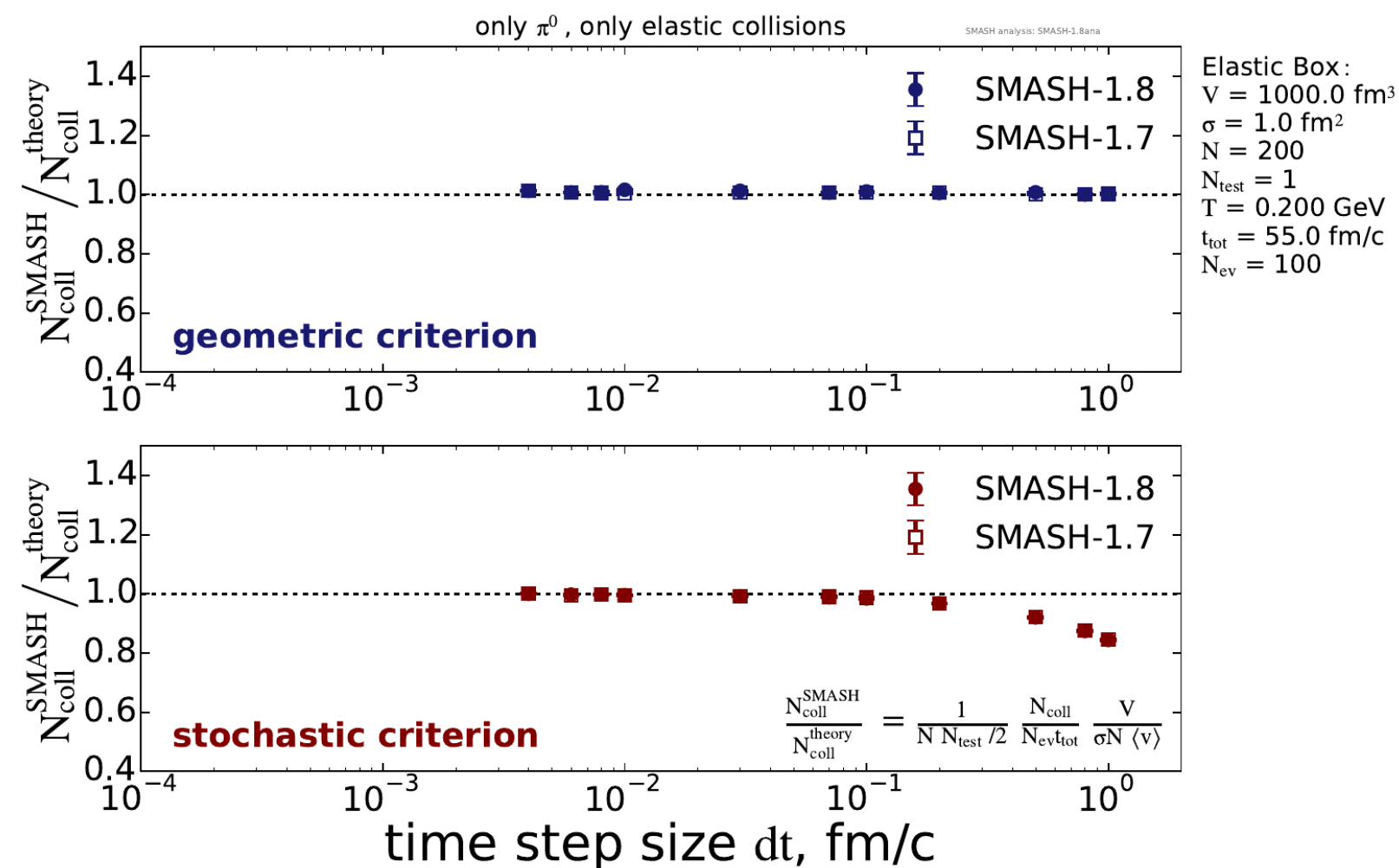
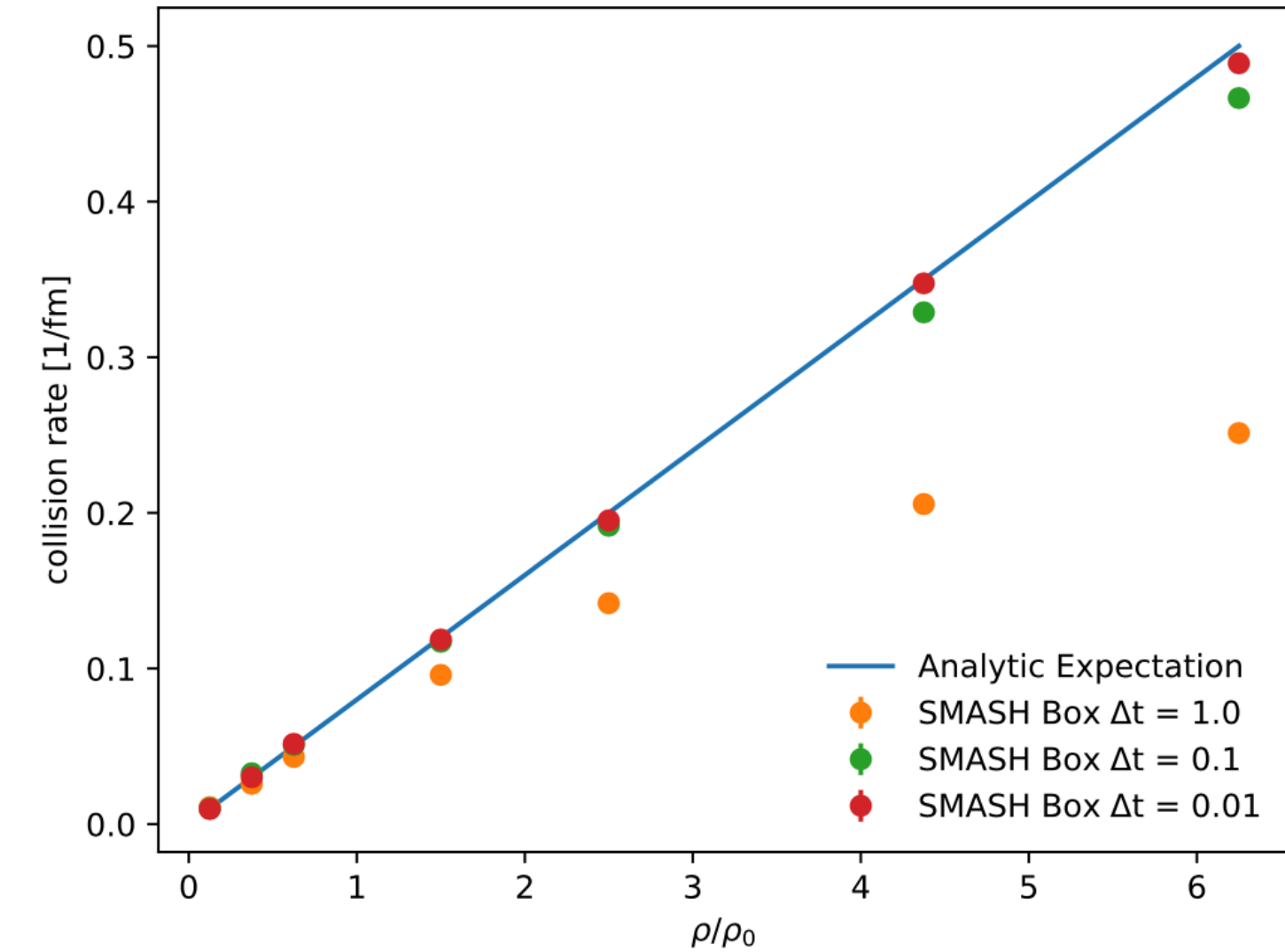
$$\hookrightarrow p\bar{p} \leftrightarrow 5\pi$$



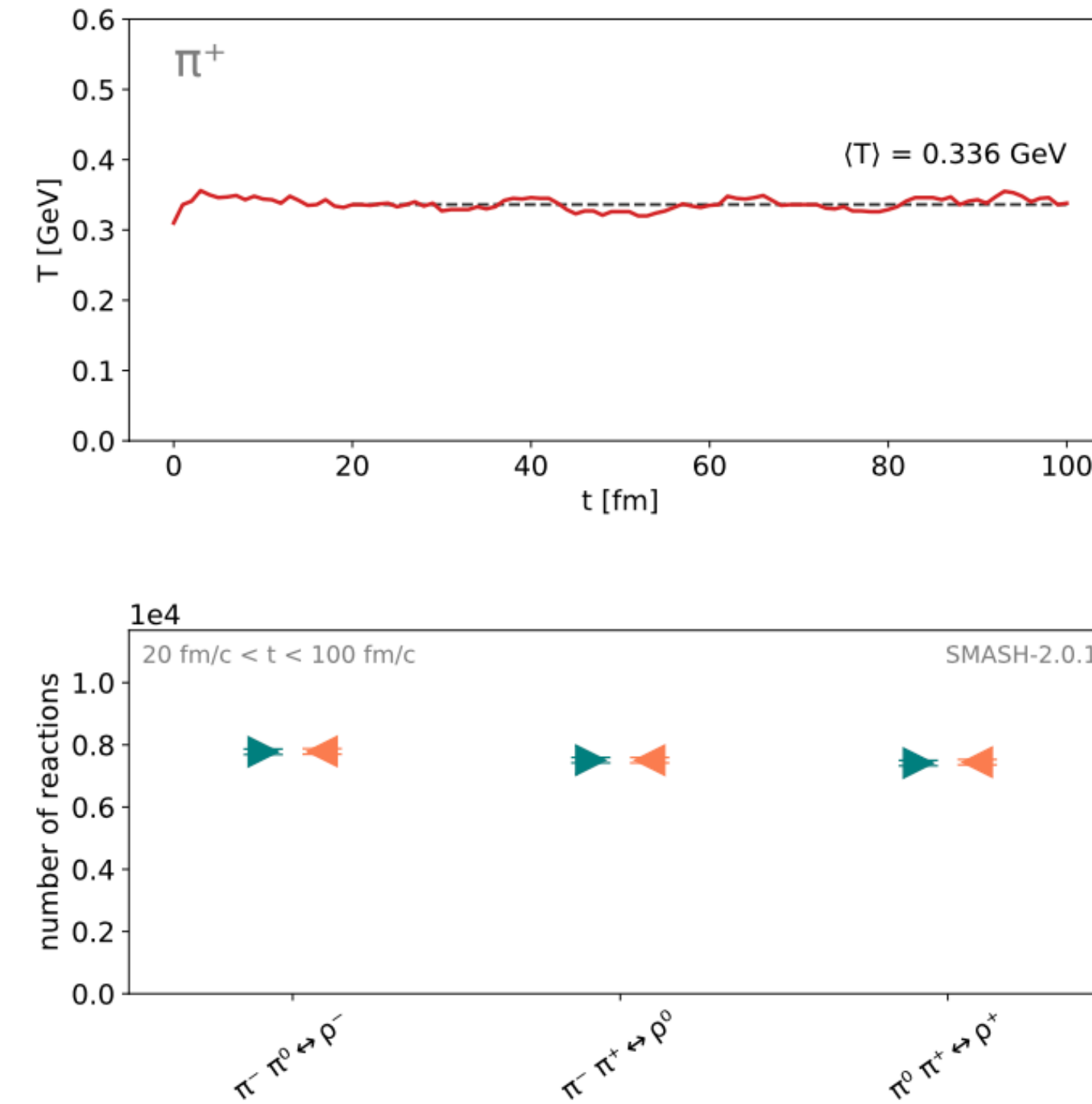
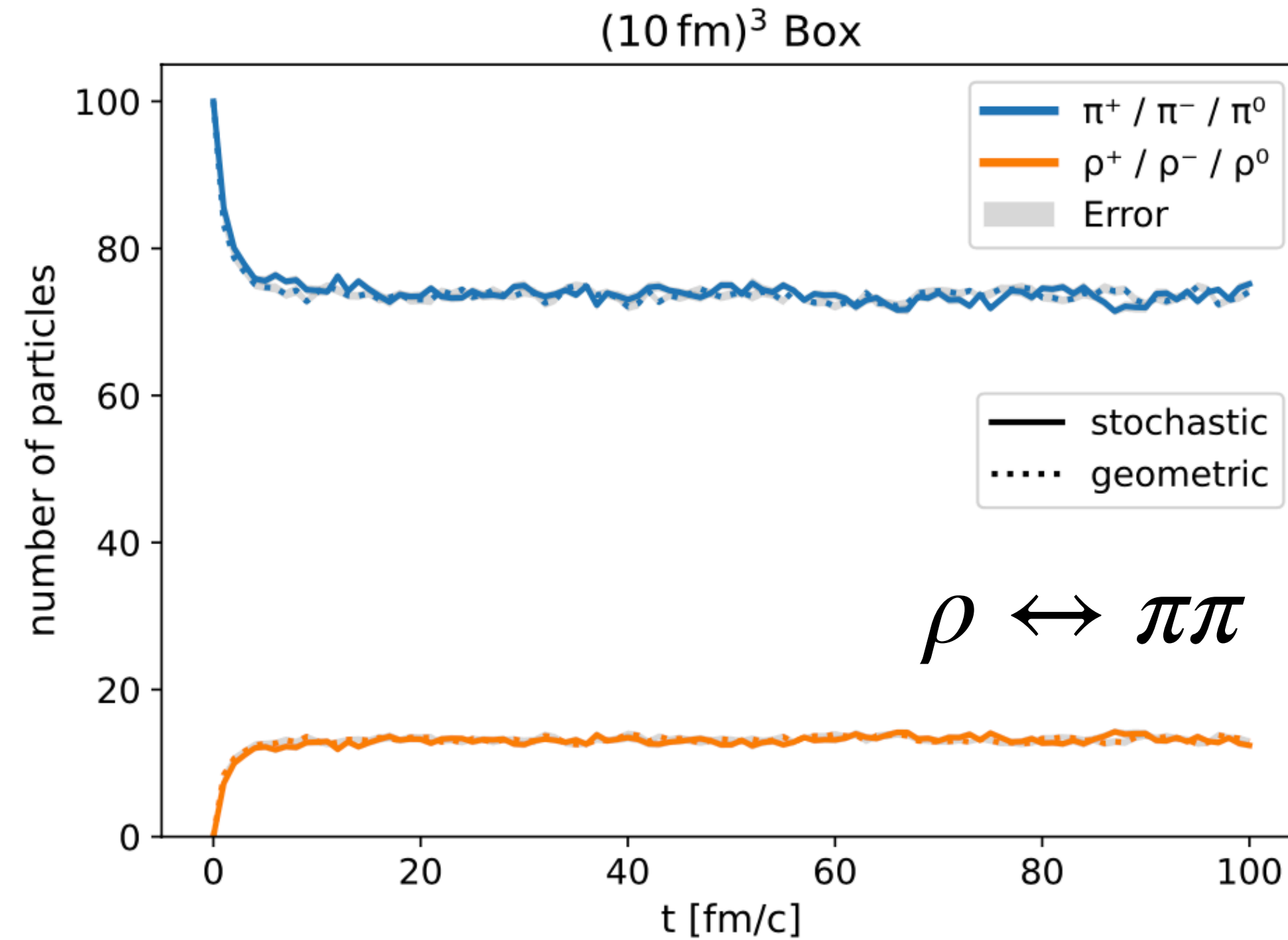
$$P_{5 \rightarrow 2} = \frac{g'_1 g'_2}{g_1 g_2 g_3 g_4 g_5} \frac{S_{12345}}{S'_{12}} \frac{1}{32E_1 E_2 E_3 E_4 E_5} \frac{\Delta t}{(\Delta^3 x)^4} \frac{\lambda}{\Phi_5} \frac{1}{4\pi s} \sigma_{2 \rightarrow 5}$$

Verification of new collision criterion

- Starting point: Check binary **scattering rate in equilibrium** in box calculations (“infinite matter”) with only elastic reactions
- Is the analytic expectation reproduced for different parameters?
 - Yes**, numerical stable for small enough timesteps
 - Small enough = 1 scattering per timestep

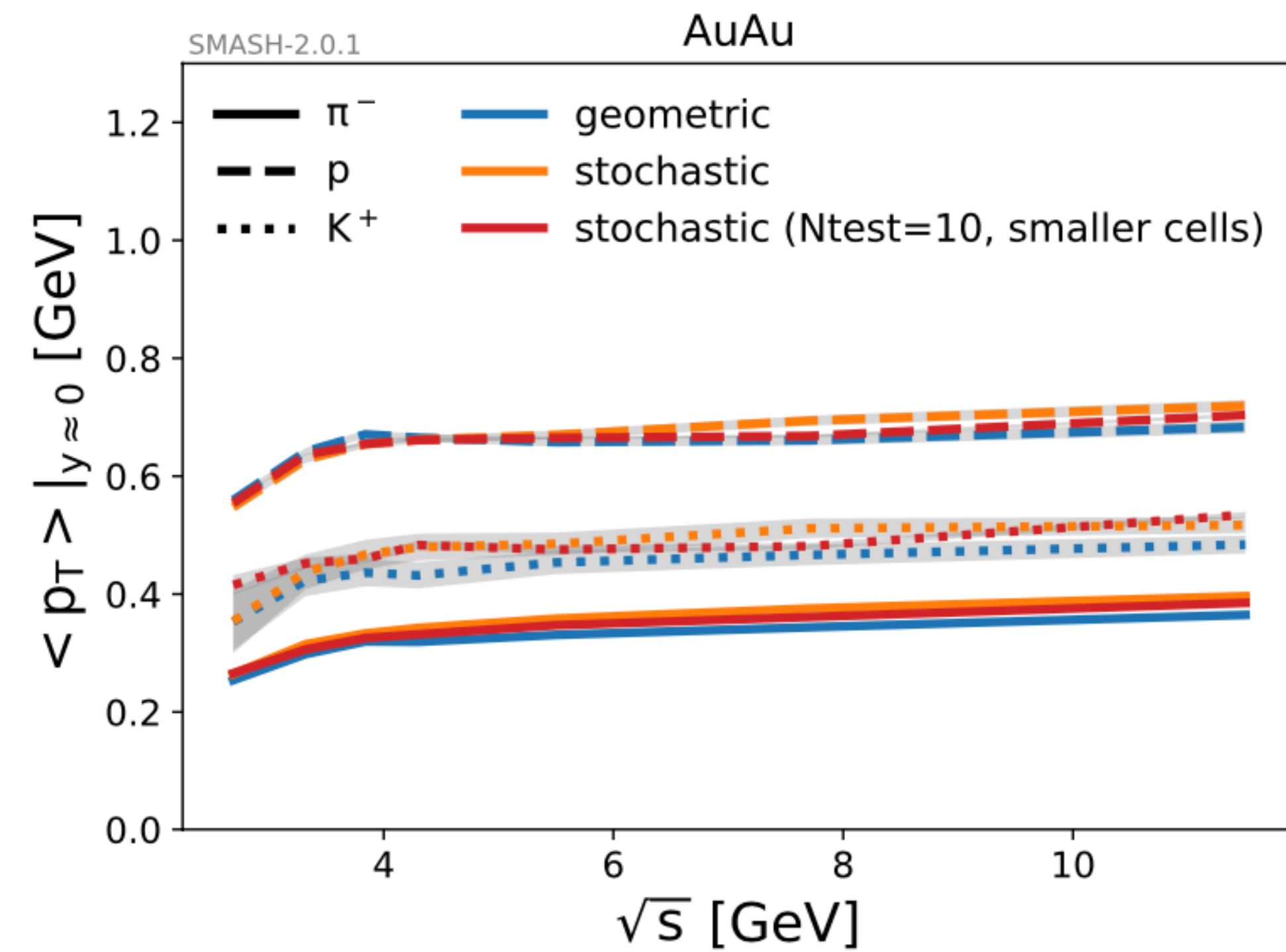
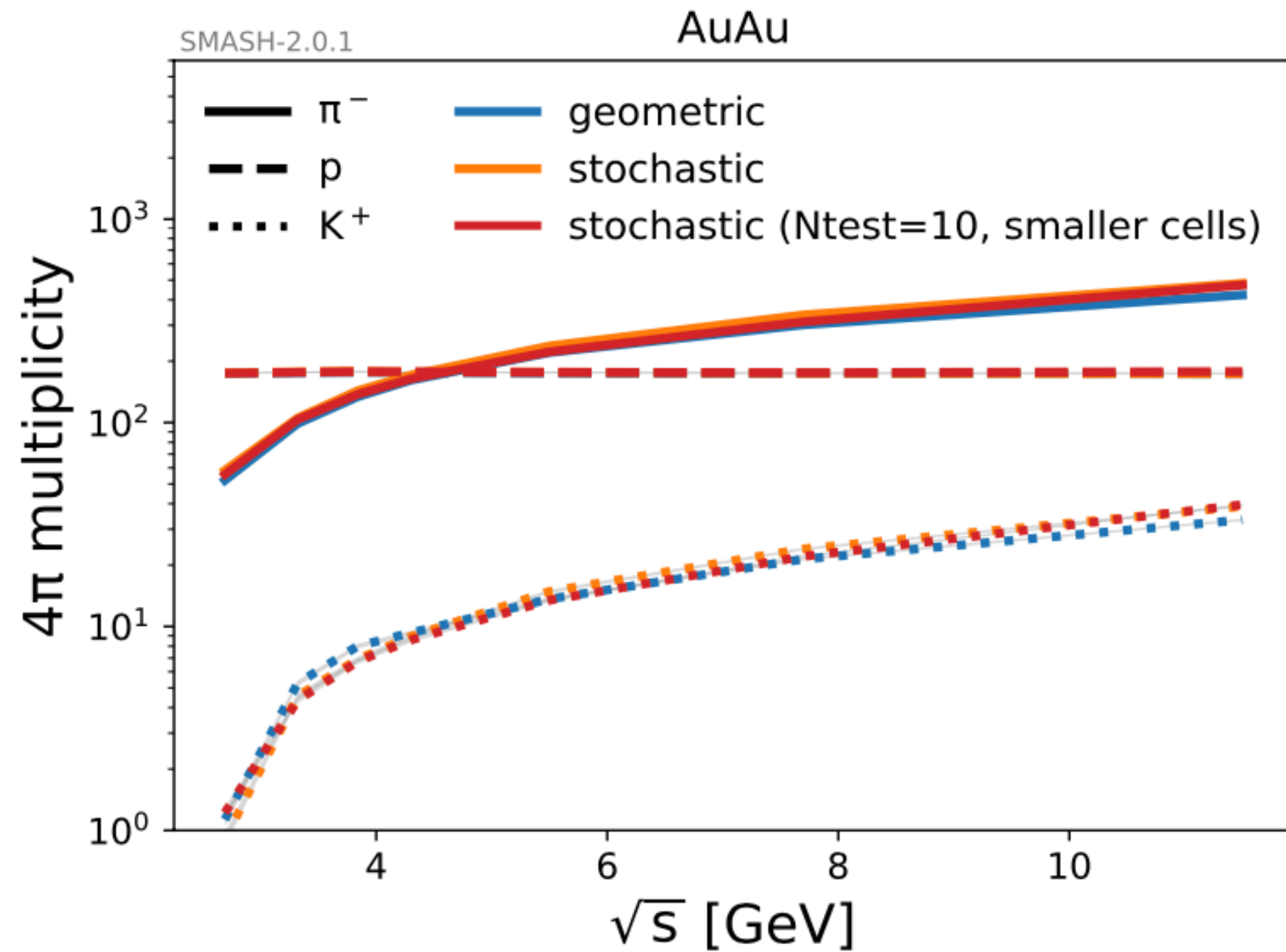


Equilibration and detailed balance



- Check equilibration in box calculations: here for $\rho - \pi$ gas with binary reactions
- System equilibrates and detailed balance is fulfilled
- Stochastic and geometric criterion matching for simple system

Comparison of yield and mean- p_T



- Multiplicity and mean- p_T for AuAu collisions at different beam energies - only binary reactions!
- Agreement between geometric and stochastic criterion (improves with more testparticles)

Equilibration for multi-particle interactions

Want to describe

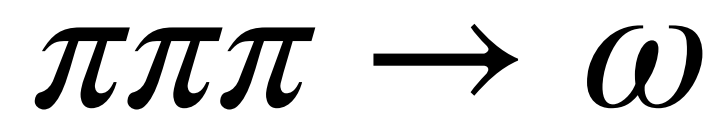


Geometric criterion

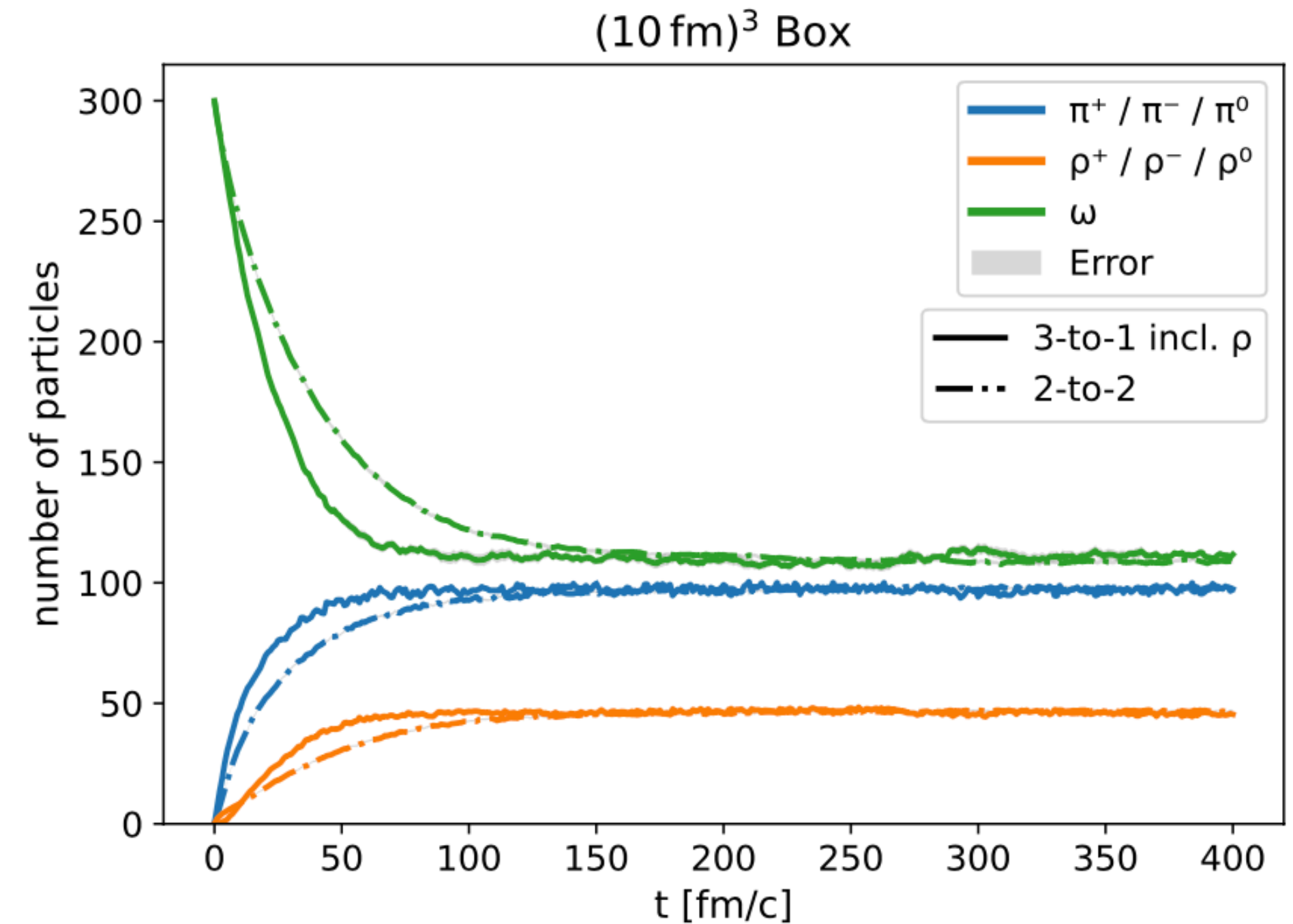


need to resort to
binary reaction chain

Stochastic criterion



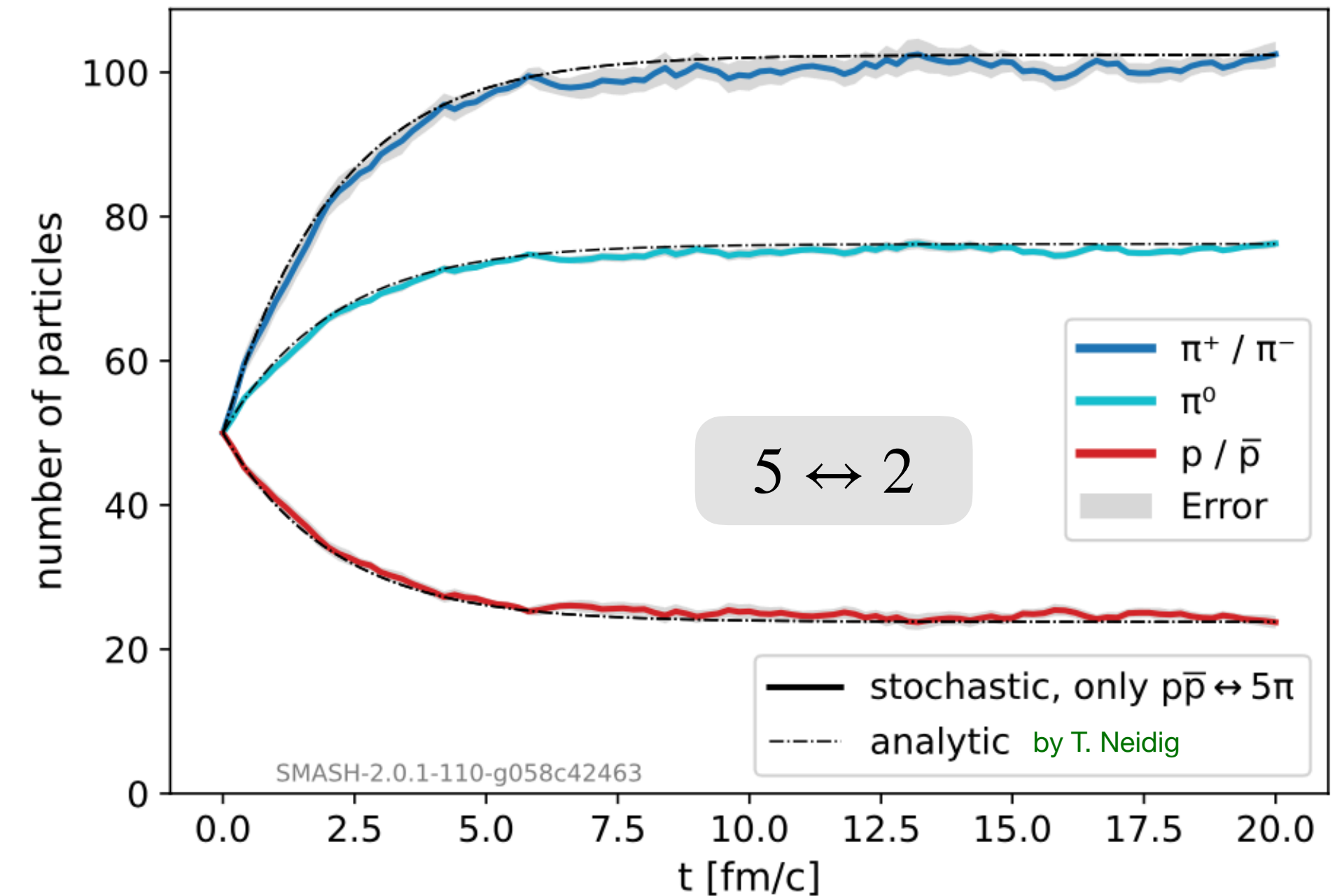
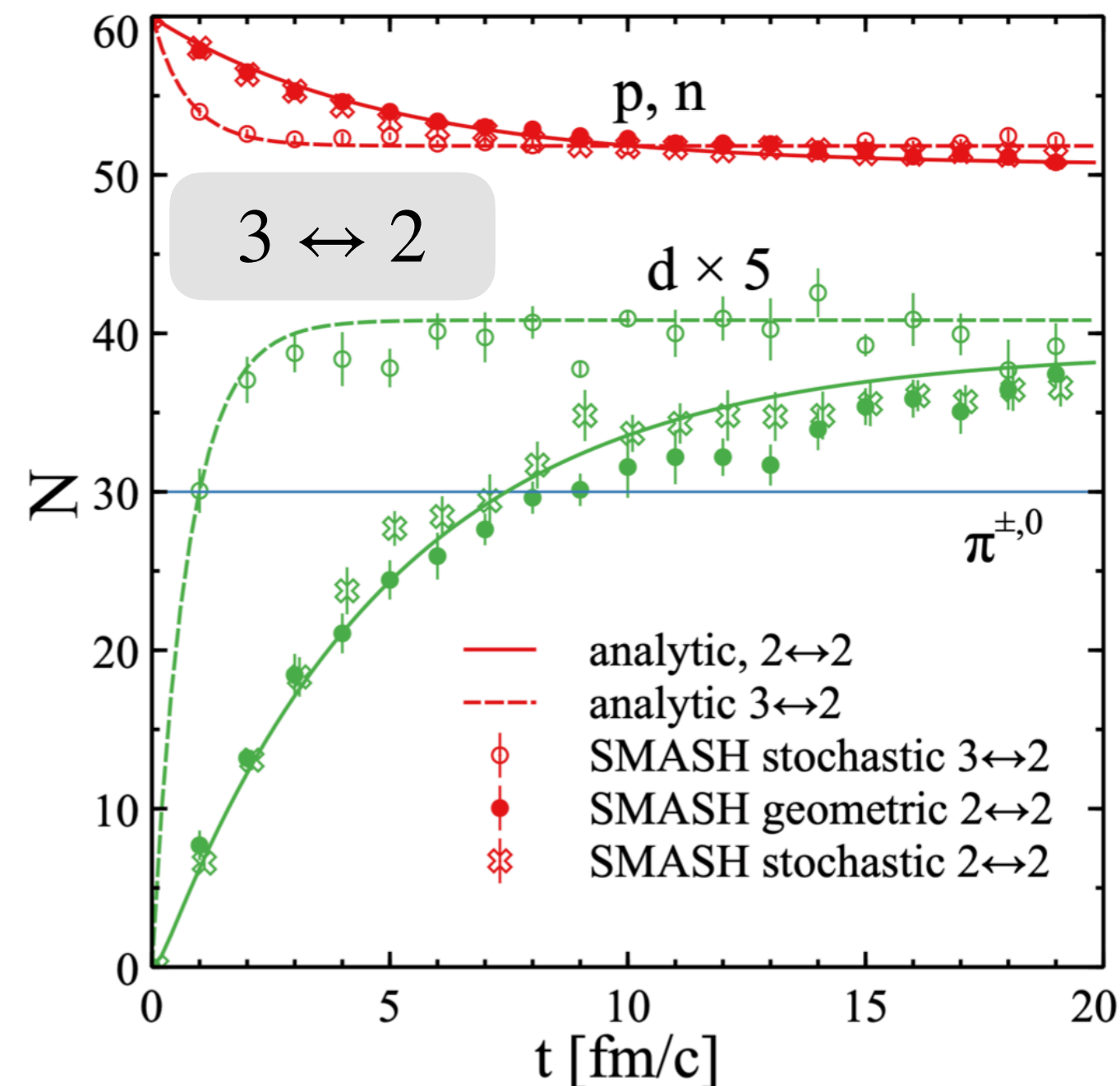
able to check reaction
chain approach



- Box calculation for $\rho - \omega - \pi$ gas \rightarrow First detailed balance for multi-particle reactions in SMASH
- Compare direct multi-particle reaction with equivalent reaction chain with intermediate resonances
- **Faster equilibration with direct multi-particle reactions** observed

+ Checked for all reactions that equil. yields match thermodynamic grand-canonical expectation

Comparison with rate equations



- Rate equations gives analytic expectation for yields over time
- **Perfect agreement** for deuteron 3-to-2 and proton 5-to-2 reactions **with rate equation** expectation
- Again faster equilibration for d multi-particle reaction

Rate equations

Y. Pan and S. Pratt, Phys. Rev. C 89, 044911. 2014.
T. Neidig, Master Thesis

Grand-canonical expectation: $N_i = V n_i^{th}(T) \lambda_i$

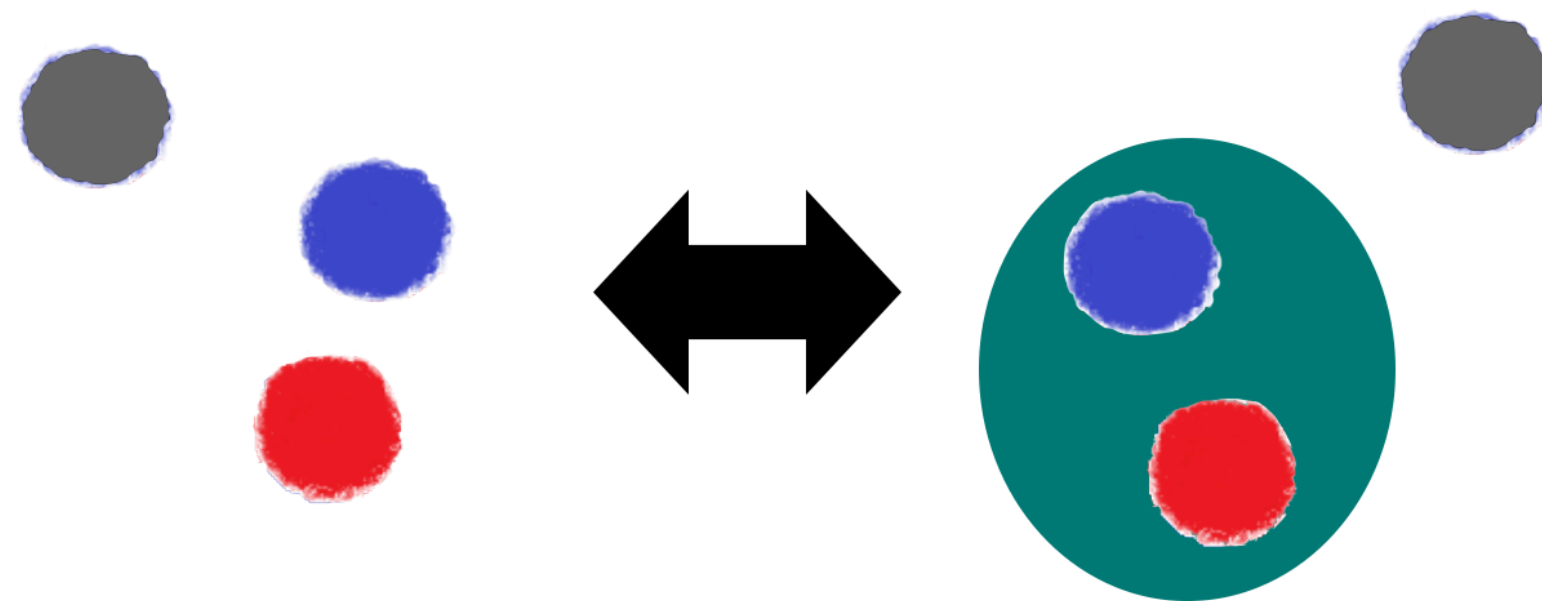
Reaction rates:

$$\frac{dN_{2 \rightarrow n}}{d^4x} = A \lambda_1 \lambda_2, \quad A \equiv \langle \sigma_{2 \rightarrow n} v_{rel} \rangle n_1^{th}(T) n_2^{th}(T).$$

$$\frac{dN_{n \rightarrow 2}}{d^4x} = A \prod_{j=1}^n \lambda_j \rightarrow \text{Formulate and solve equation system for specific reactions}$$

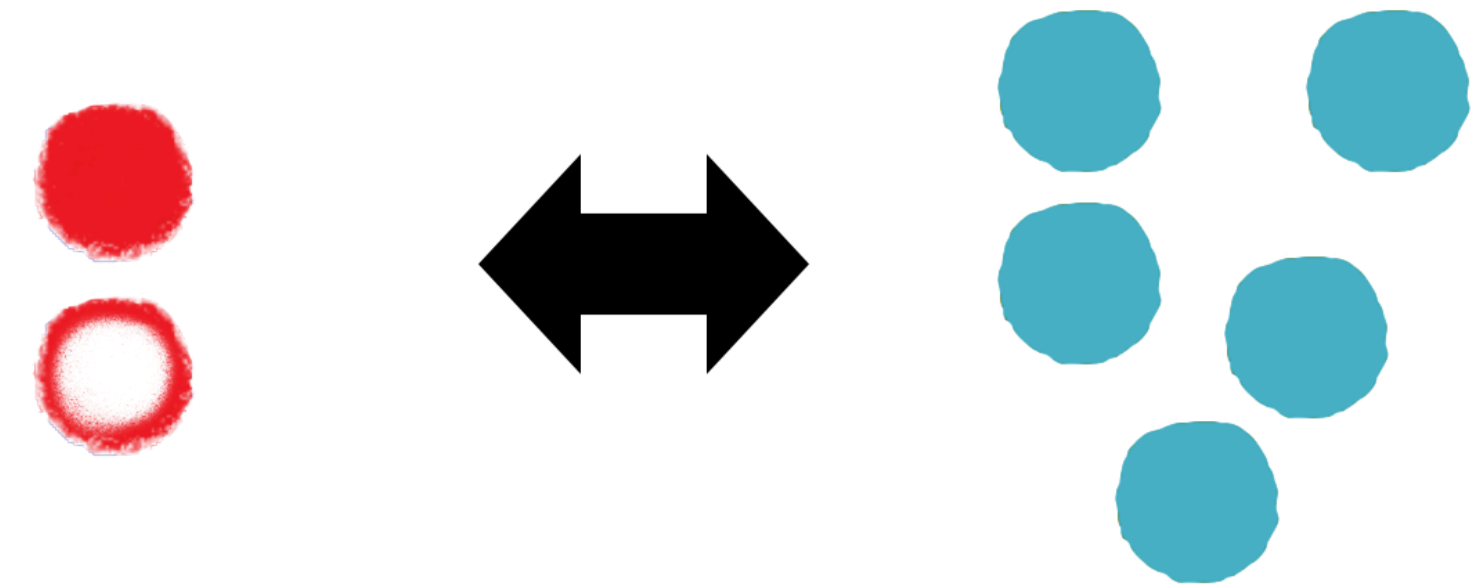
Two main studies

Deuteron catalysis



JS, D. Oliinychenko, J. M. Torres-Rincon & H. Elfner,
Phys. Rev. C 104, 034908 (2021)

Proton annihilation

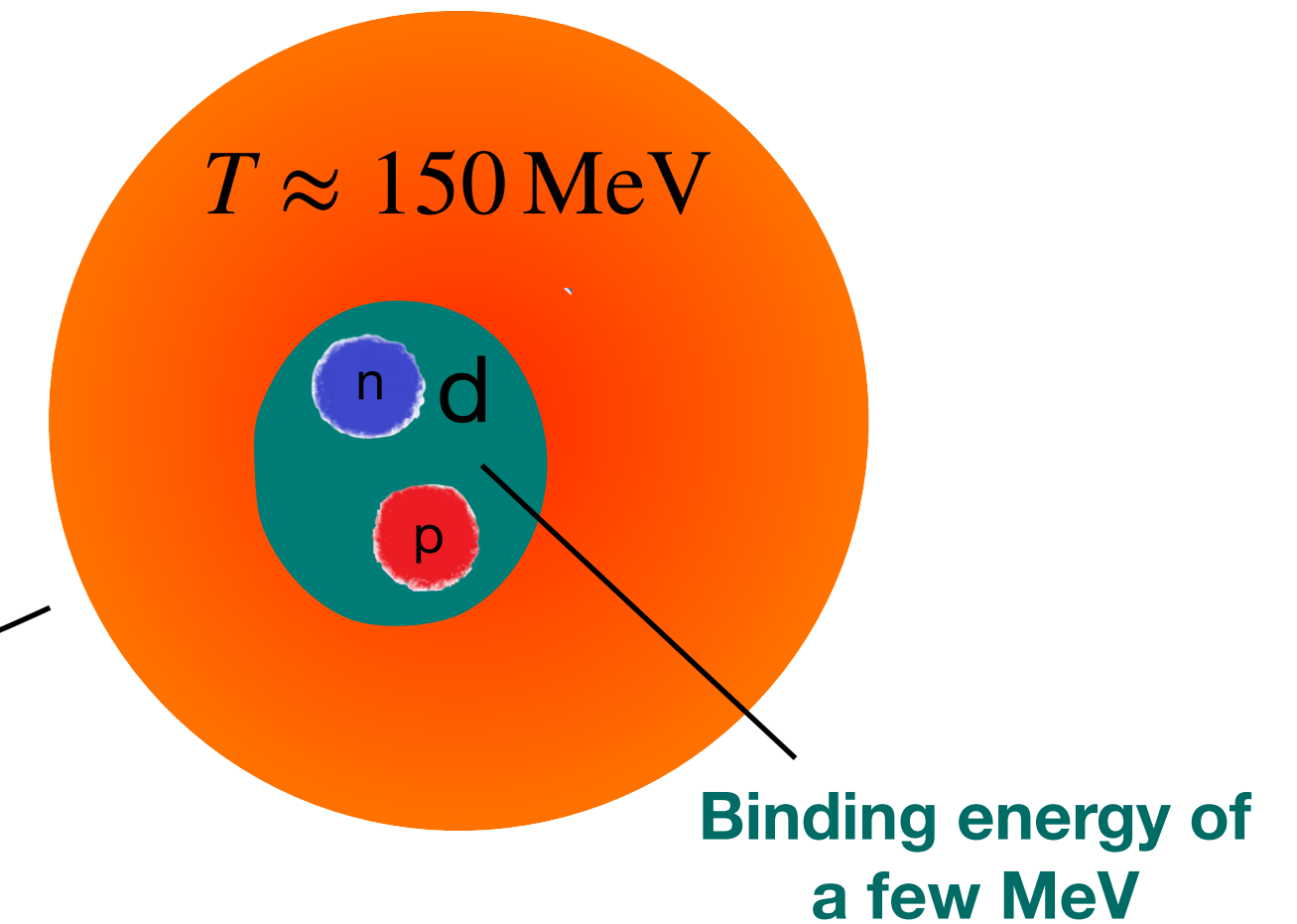
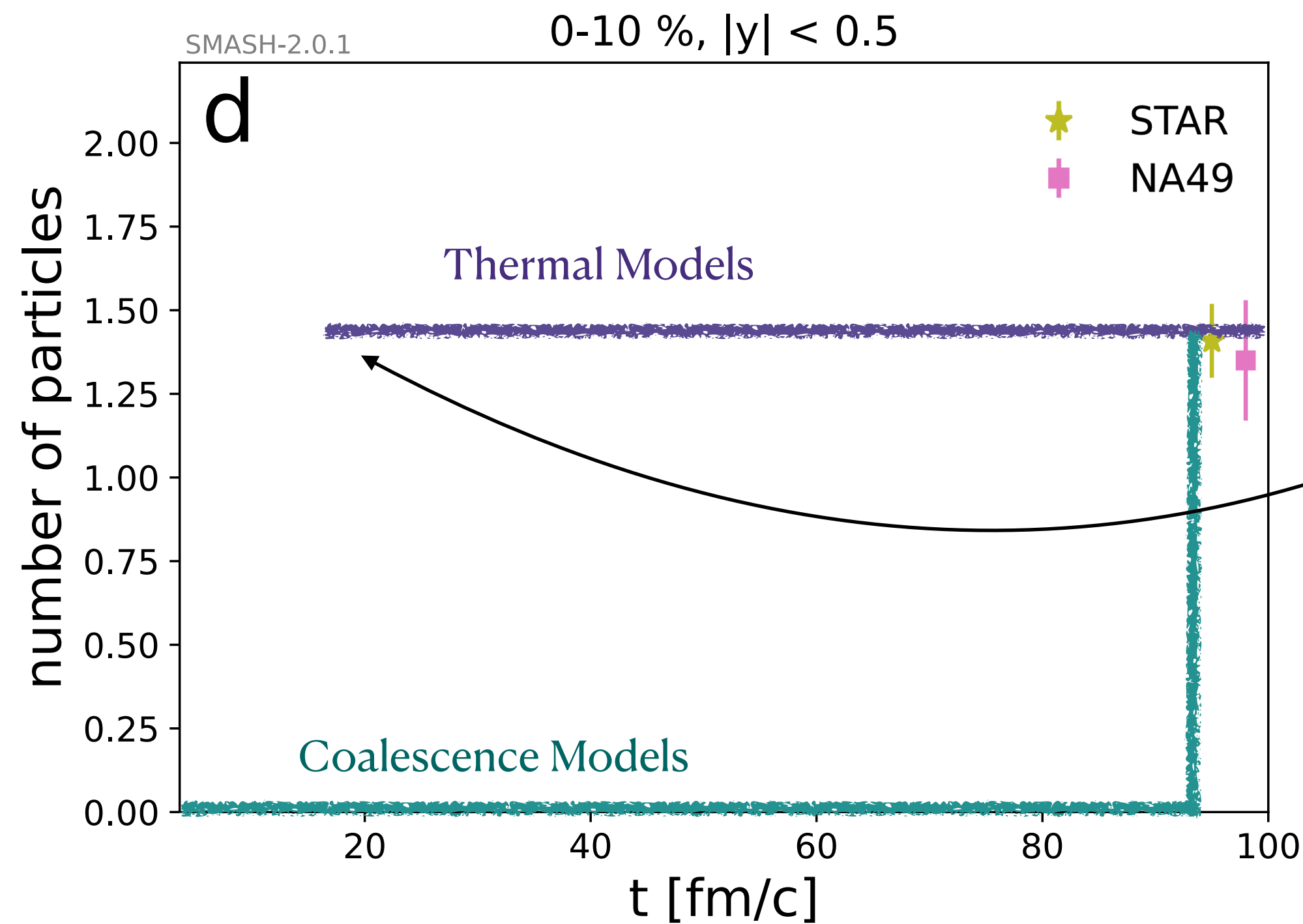


O. Garcia-Montero, JS, A. Schäfer, J. M. Torres-Rincon & H. Elfner,
Phys. Rev. C 105, 064906 (2022)

„Snowballs in hell“

How and when do
deuterons form?

see e.g. D. Oliinychenko, Nucl. Phys.
A, vol. 1005, p. 121754, 2021,
2003.05476.

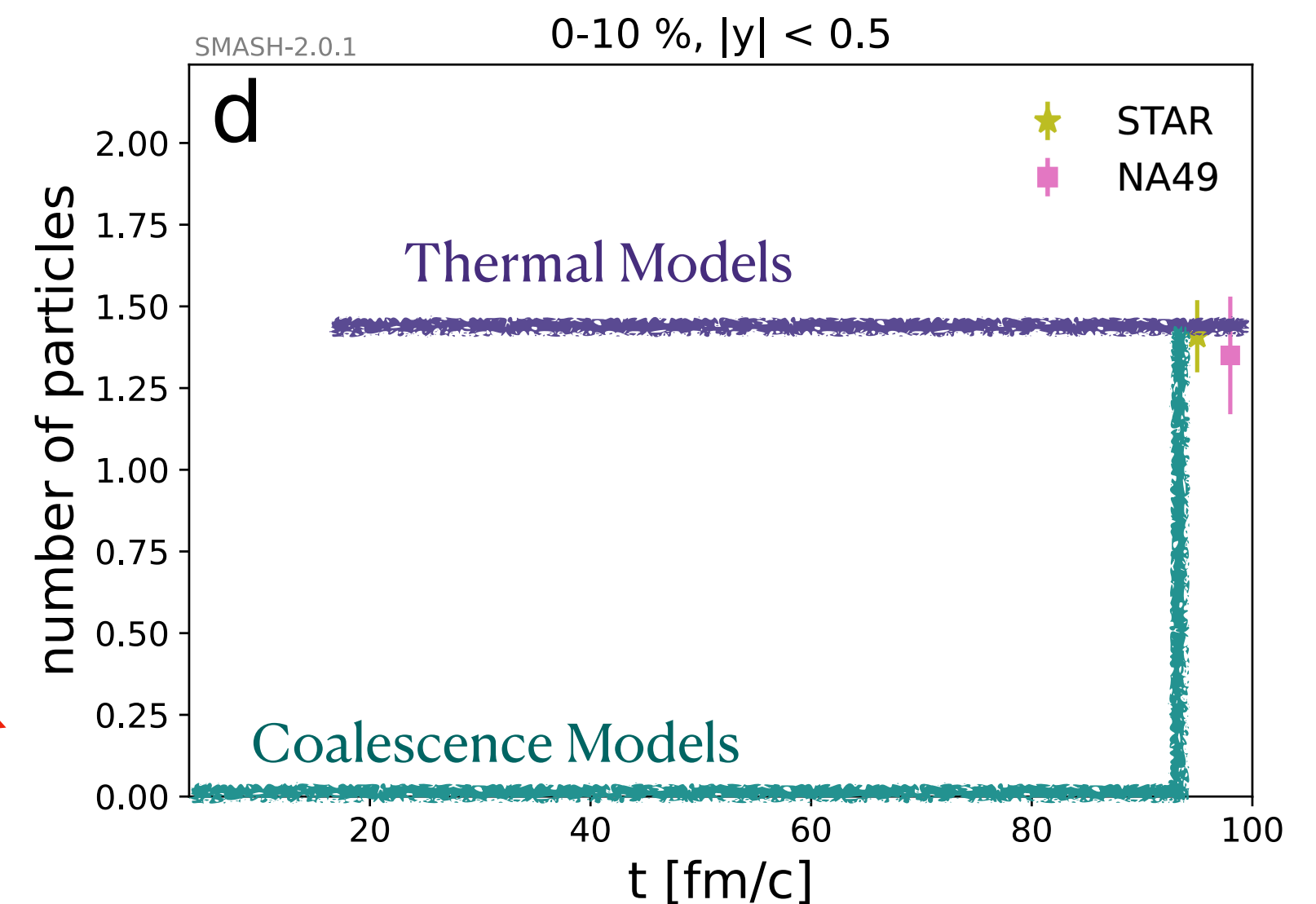
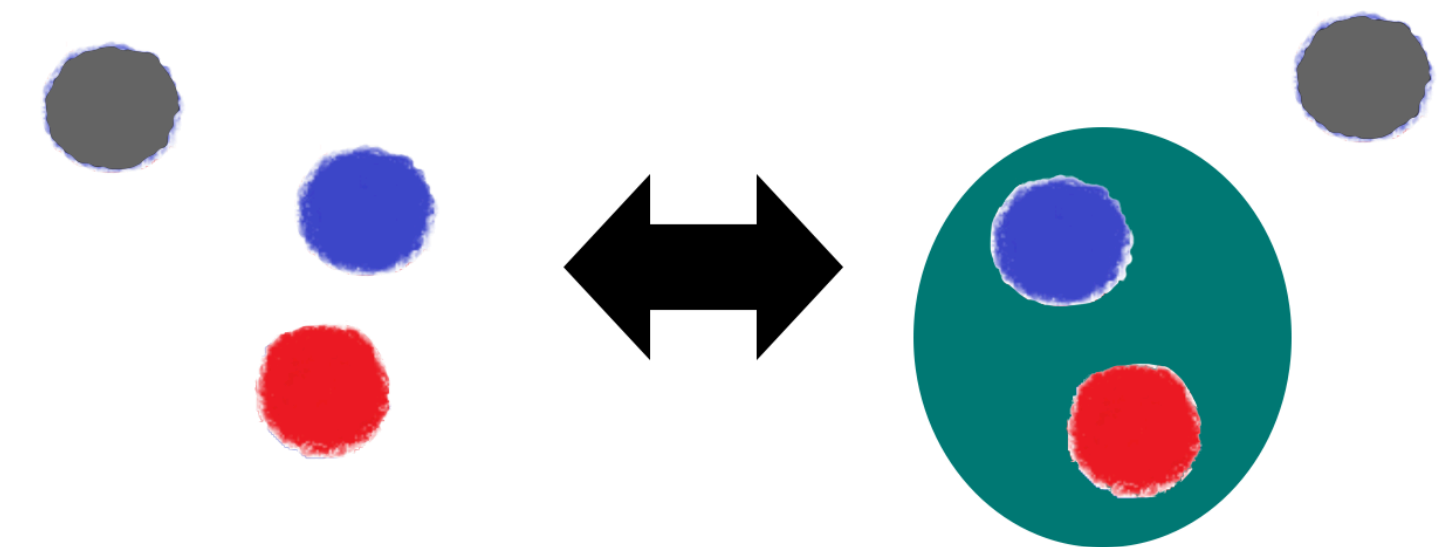
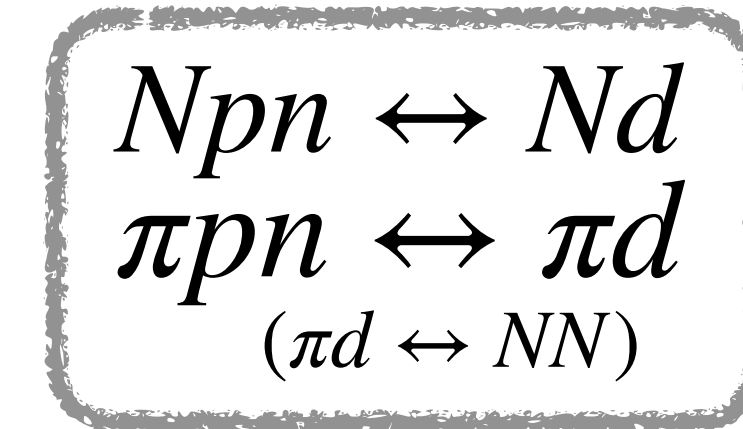


- Thermal and coalescences models are both able to describe the deuteron data with drastically different assumptions → see Tom Reichert's talk for more details
- How can deuteron be formed early if the medium has a temperature of 150 MeV?

Deuteron production: Third approach

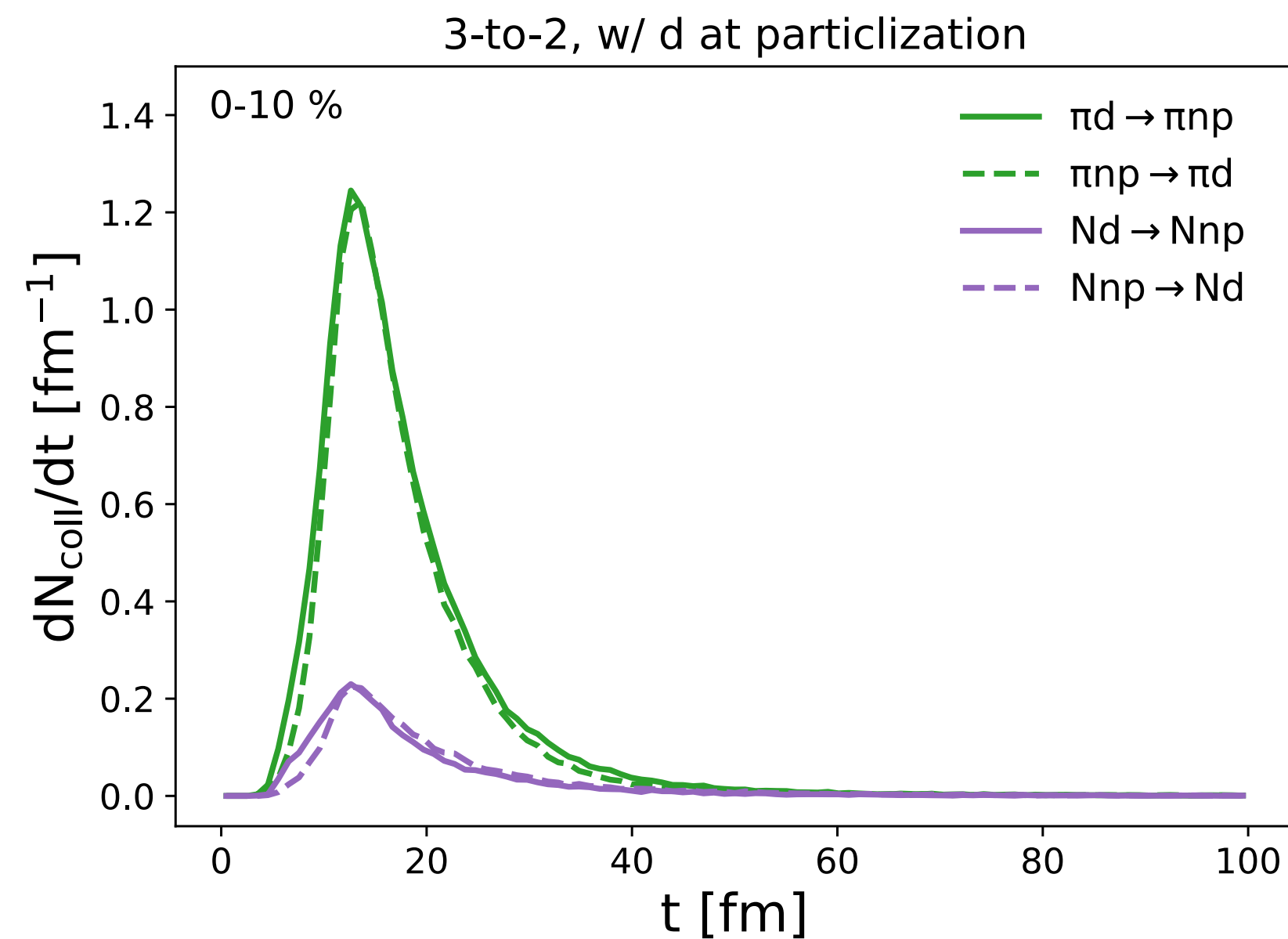
→ see also Gabriele's talk

- Dynamic production via microscopic interactions (d treated as particle with large cross section)
- “Strict” catalysis reactions with π or N
- Earlier work with geometric criterion realized catalysis with reaction chain introducing an artificial d' resonance:
 $\pi d \leftrightarrow \pi d' \leftrightarrow \pi n p$
D. Oliinychenko, C. Shen, and V. Koch, Phys. Rev. C, vol. 103, no. 3, p. 034913, 2021, 2009.01915.
- With stochastic criterion → **Direct 3-to-2 treatment**
- Use catalysis interactions in afterburner stage in hybrid model: MUSIC+SMASH, AuAu at $\sqrt{s_{NN}} = 7.7$ GeV
- Compare thermal and coalescence assumption by different particlization scenarios: with or without d

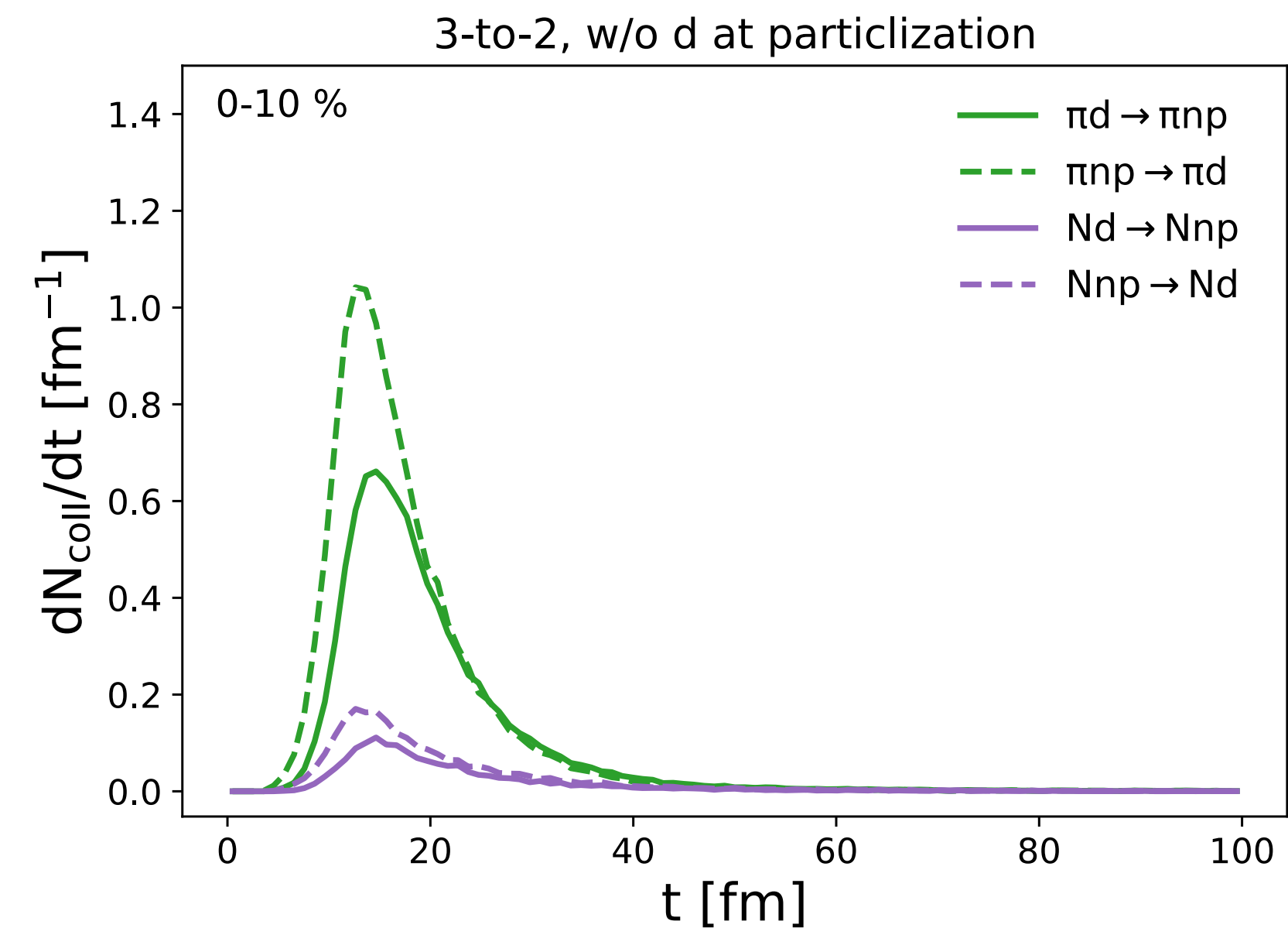


Collision rates

With d at particlization

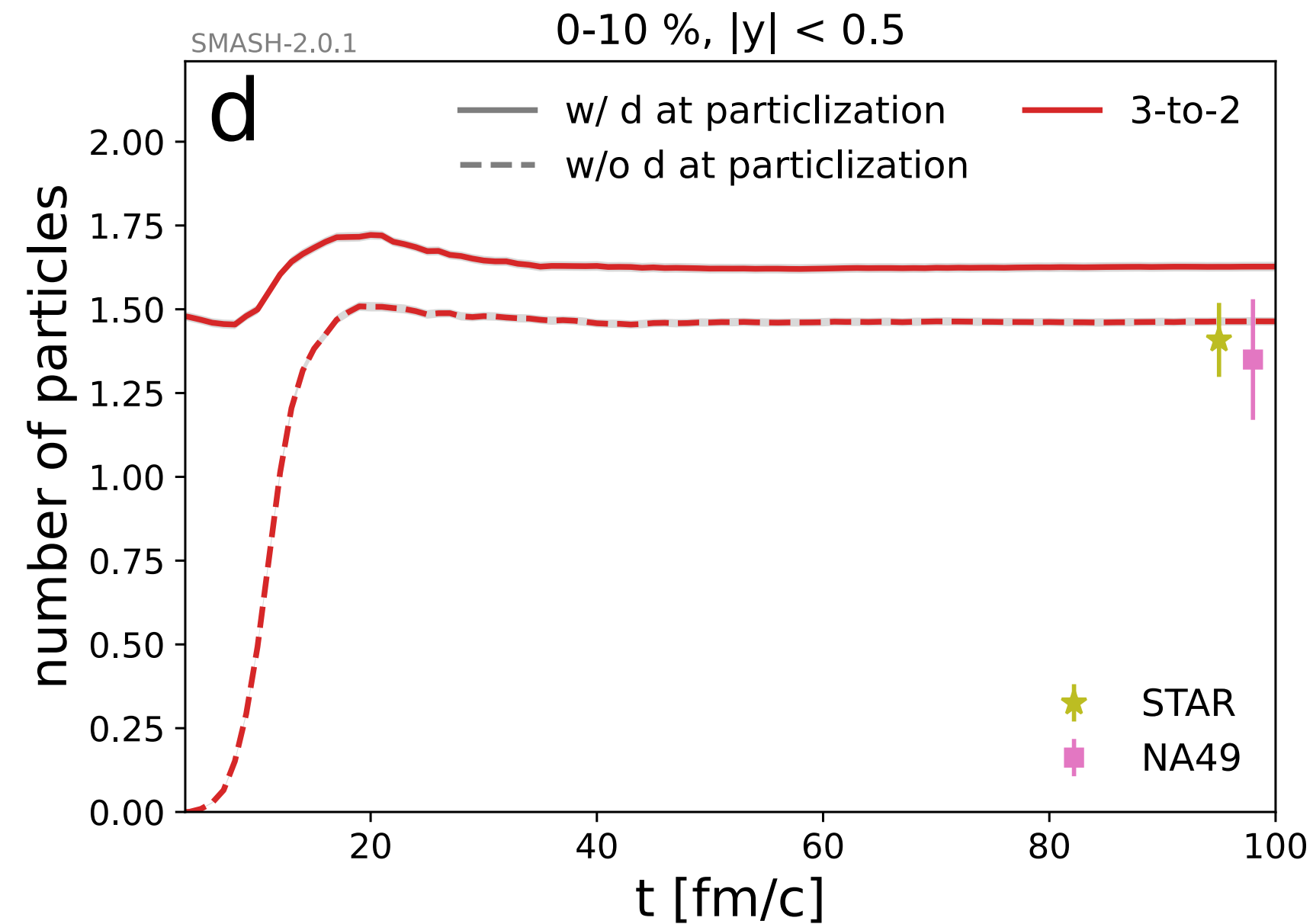


Without d at particlization



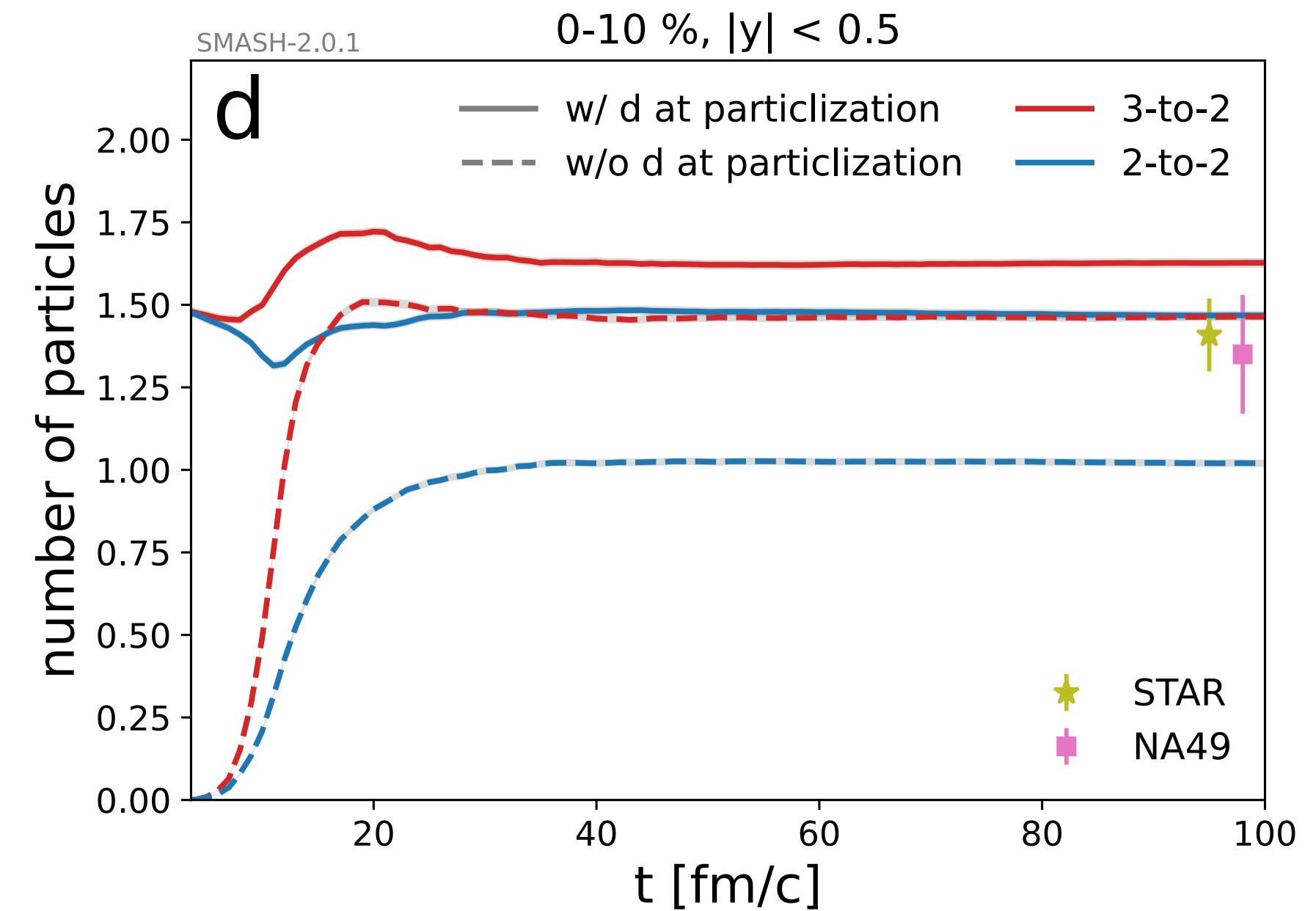
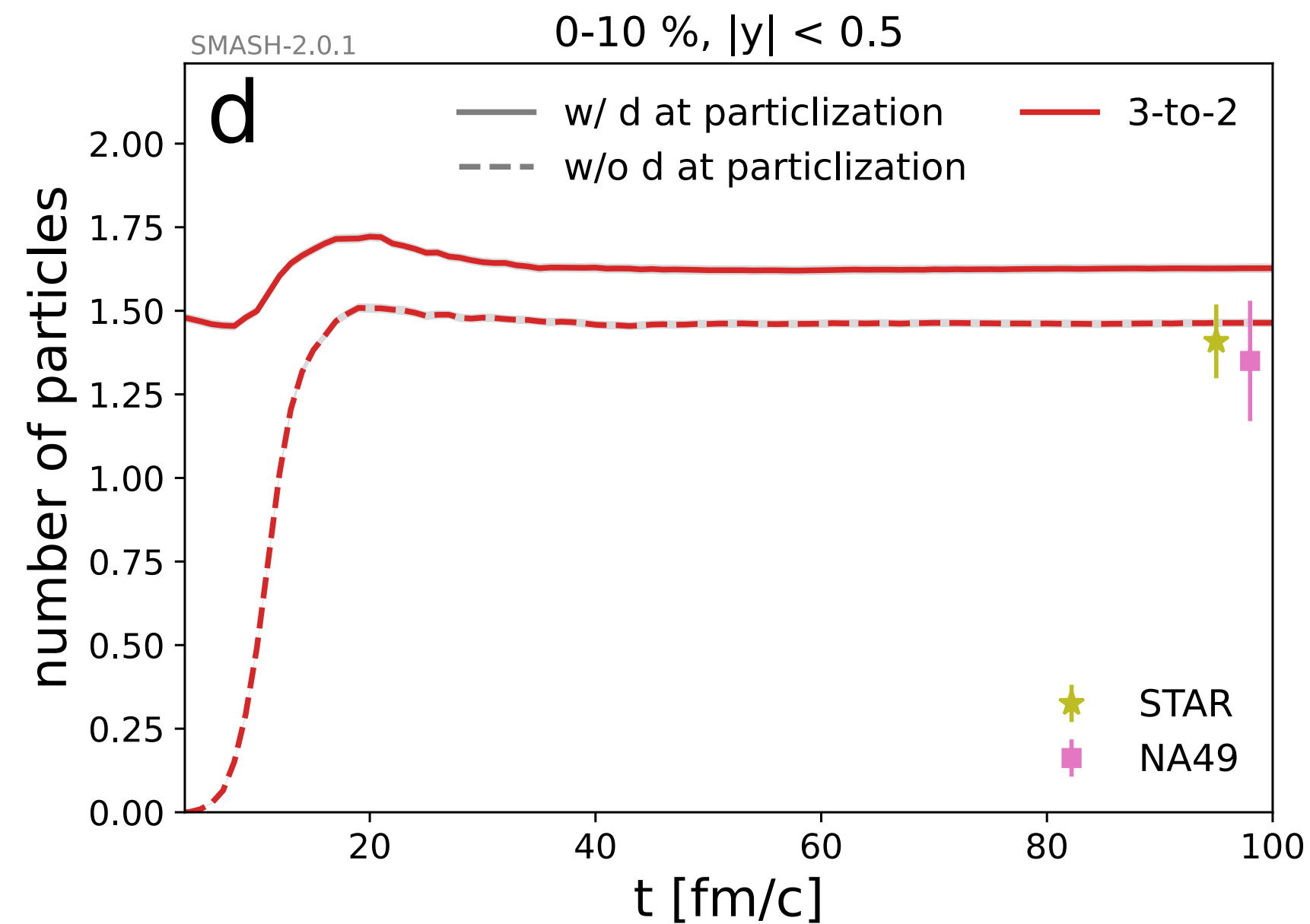
- Generation and destruction of d equilibrate quickly with d at particlization (“thermal” case \rightarrow “snowballs melt”)
- d rapidly generated in case without d at start of afterburner (“coalescence” case)

Multiplicity evolution



- Multiplicity driven to same yield independent of d number at start
- Constant yield in *thermal case*
- Agreement within experimental error bars for both scenarios → intuition why thermal and coalescence approaches describe data

Multiplicity evolution



- Multiplicity driven to same yield independent of d number at start
- Constant yield in *thermal case*
- Agreement within experimental error bars for both scenarios → intuition why thermal and coalescence approaches describe data

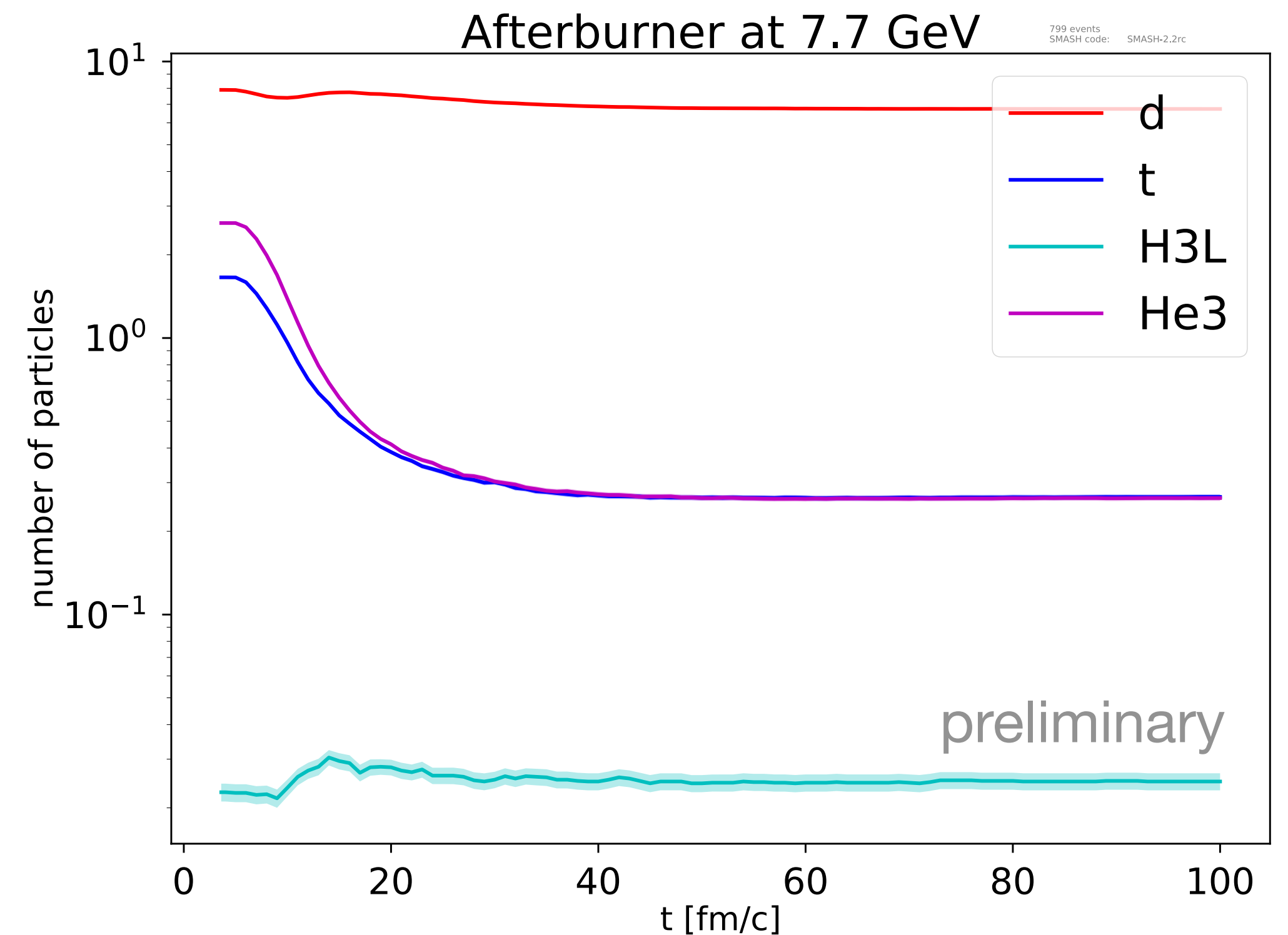
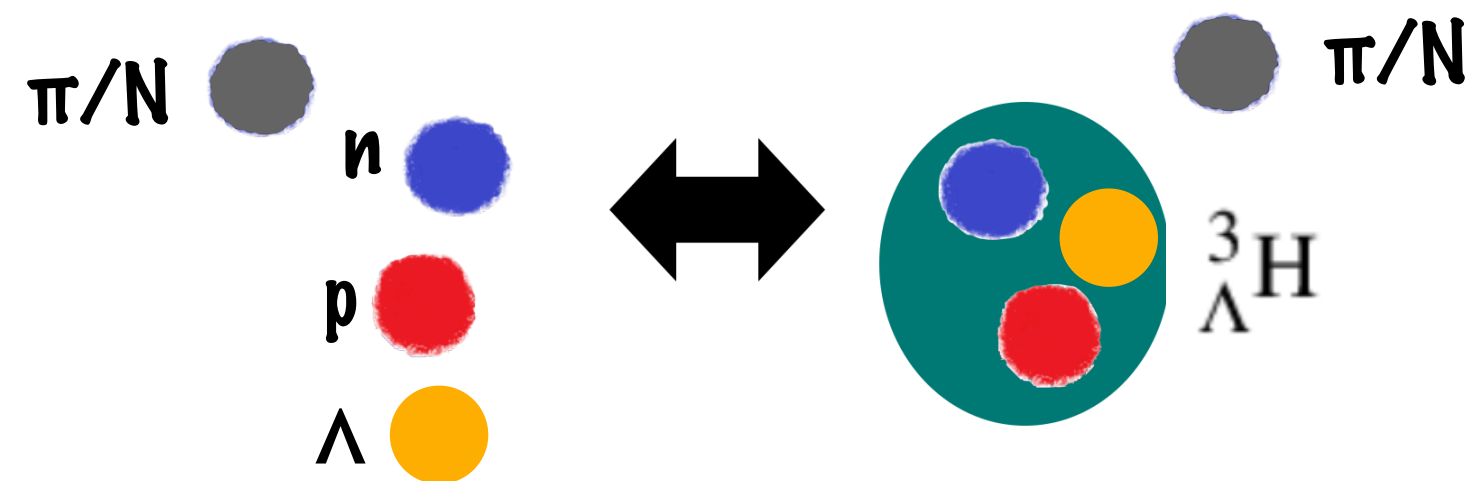
- Comparison with earlier treatment using binary reaction chain approach
 → Faster equilibration yields to more d generation before freeze-out
- Confirmation of earlier results, same interpretation

Other light ($A=3$) nuclei production Work in progress

- Extension to catalysis reactions to 4 particles incoming \rightarrow 4-to-2 reactions
- Reaction probability (neglecting symmetry and degeneracy factors)

$$P_{4 \rightarrow 2} = \frac{1}{16E_1E_2E_3} \frac{\Delta t}{(\Delta^x)^3} \frac{\tilde{\lambda}}{\Phi_4 4\pi s} \sigma_{2 \rightarrow 4}$$

- (Hyper-) Triton and Helium-3 production possible



Proton Anomaly

- LHC data was overestimated* by thermal models (= „proton anomaly“)

J. Stachel, A. Andronic, P. Braun-Munzinger, and K. Redlich, J. Phys. Conf. Ser., vol. 509, p. 012019, 2014,

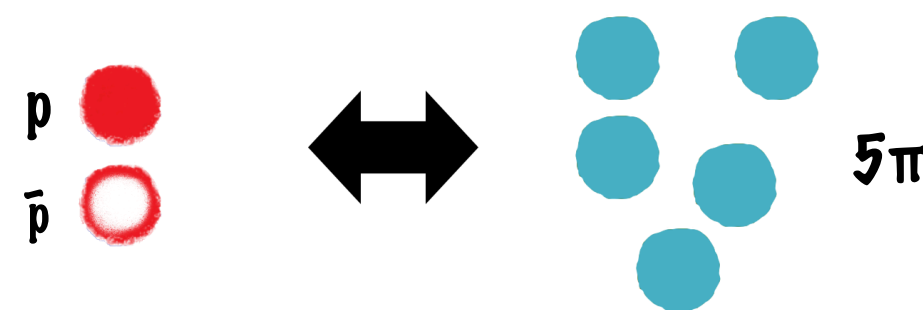
- Role of annihilations in late non-equilibrium stage $p\bar{p} \rightarrow 5\pi$

K. Werner, I. Karpenko, T. Pierog, M. Bleicher, and K. Mikhailov, Phys. Rev. C, vol. 82, p. 044904, 2010,
J. Steinheimer, J. Aichelin, and M. Bleicher, Phys. Rev. Lett., vol. 110, no. 4, p. 042501, 2013

- Relevance of annihilation back-reaction? $p\bar{p} \leftarrow 5\pi$

E. Seifert and W. Cassing, Phys. Rev. C, vol. 97, no. 4, p. 044907, 2018
Y. Pan and S. Pratt, Phys. Rev. C, vol. 89, no. 4, p. 044911, 2014.

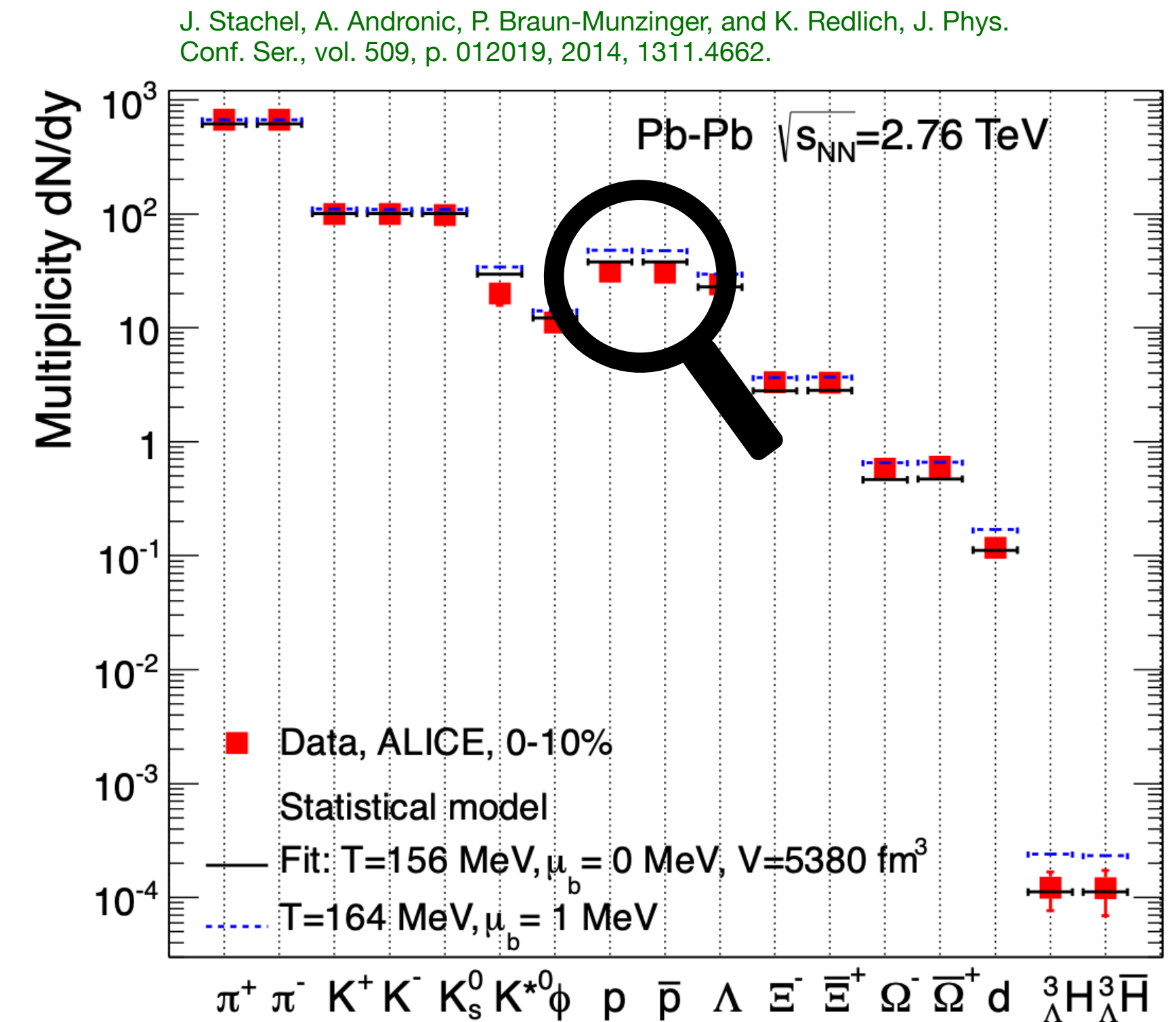
- Address this question for the first time with **direct 5-body back-reaction** (average number of π produced in $p\bar{p}$)



- SMASH-vHLLÉ-hybrid calculation with initial state from SMASH for AuAu/PbPb from 17.3 GeV up to 5.02 TeV

↳ see Hannah’s talk for more details

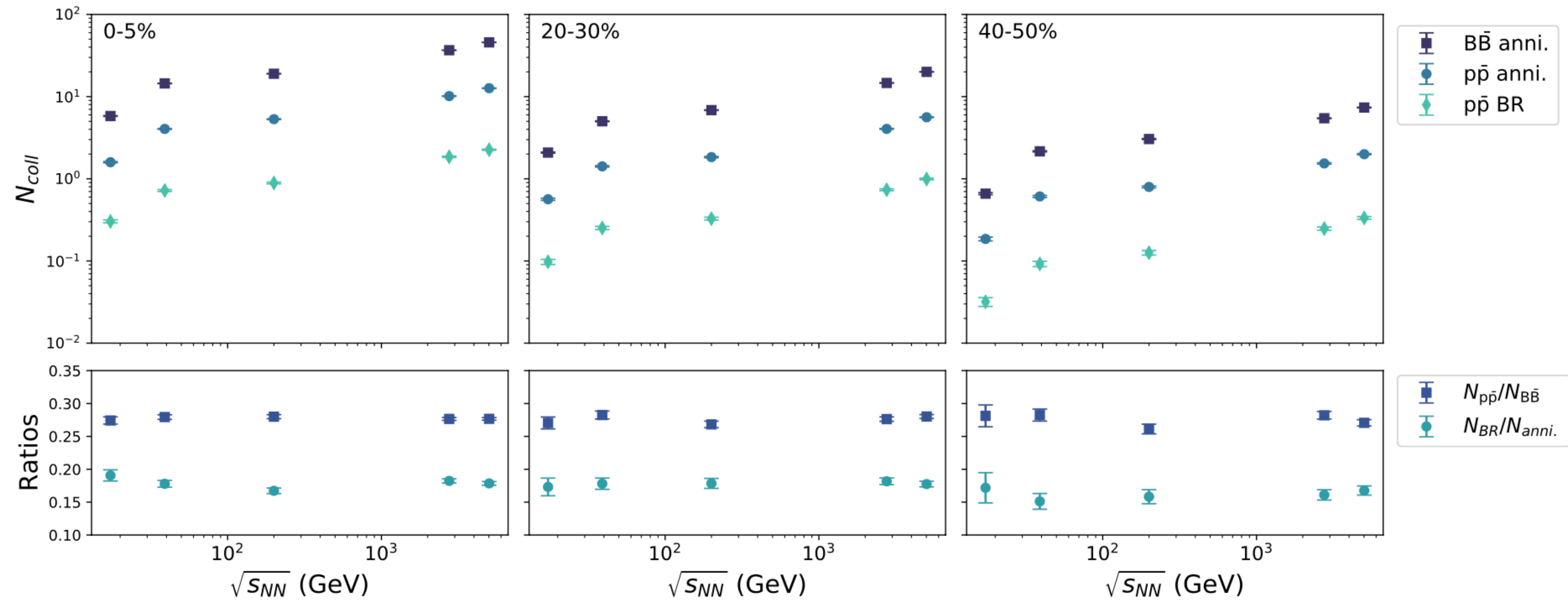
- Compare with “resonance” approach using binary reaction chain $N\bar{N} \leftrightarrow h_1\rho \leftrightarrow \rho\pi\pi\pi \leftrightarrow 5\pi$



* since alleviated by the inclusion of π -N interaction terms:
A. Andronic, P. Braun-Munzinger, B. Friman, P. M. Lo, K. Redlich and J. Stachel, Phys. Lett. B 792, 2019

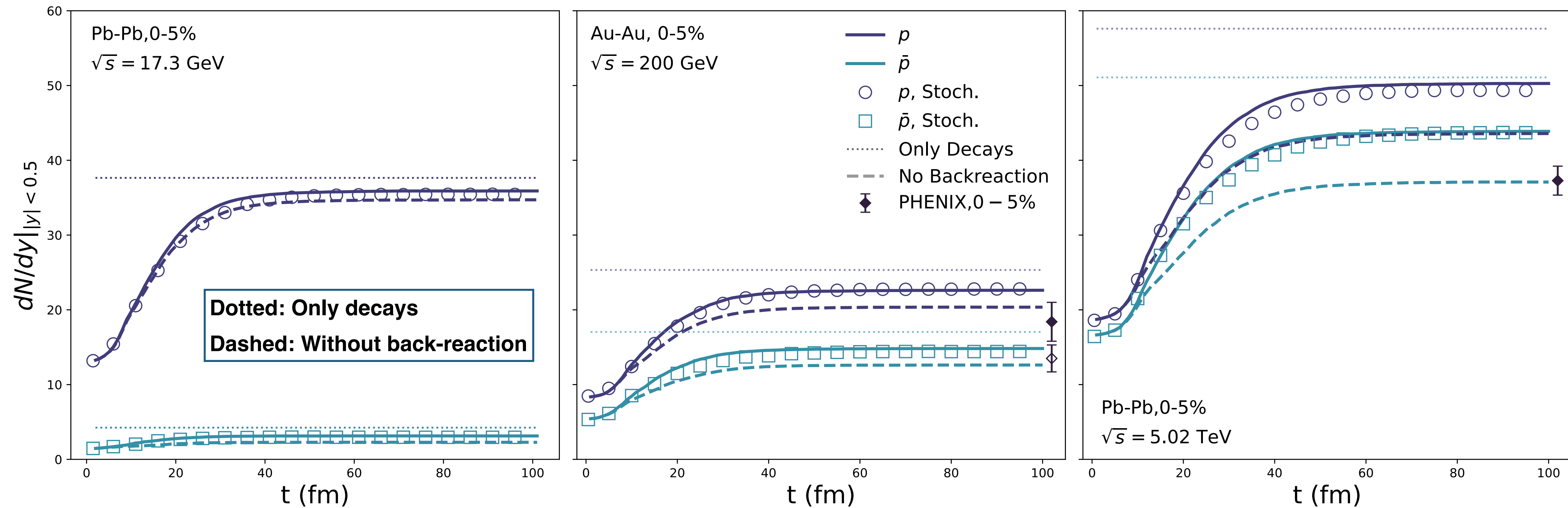
Reaction numbers

BR = annihilation back-reaction



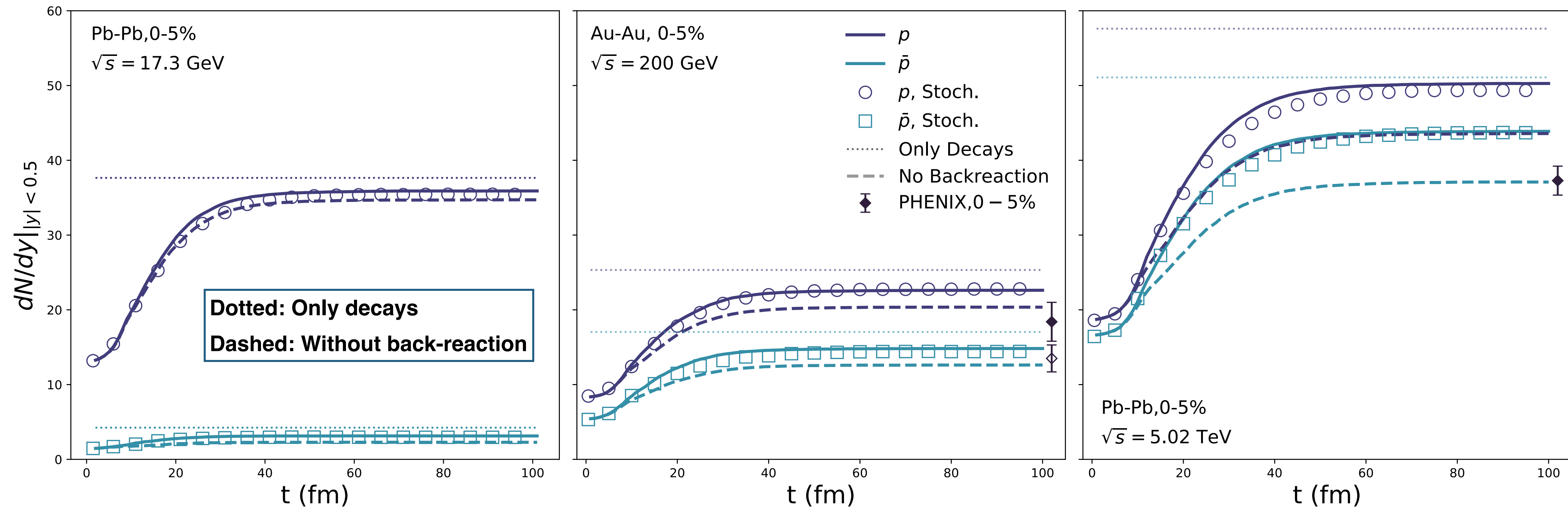
- Almost constant ratio of 5π -backreaction over annihilation for all energies and centralities (same for $B\bar{B}$ ratio)
- **20% of annihilations have a 5-body backreaction (in 4π)**

Multiplicity evolution



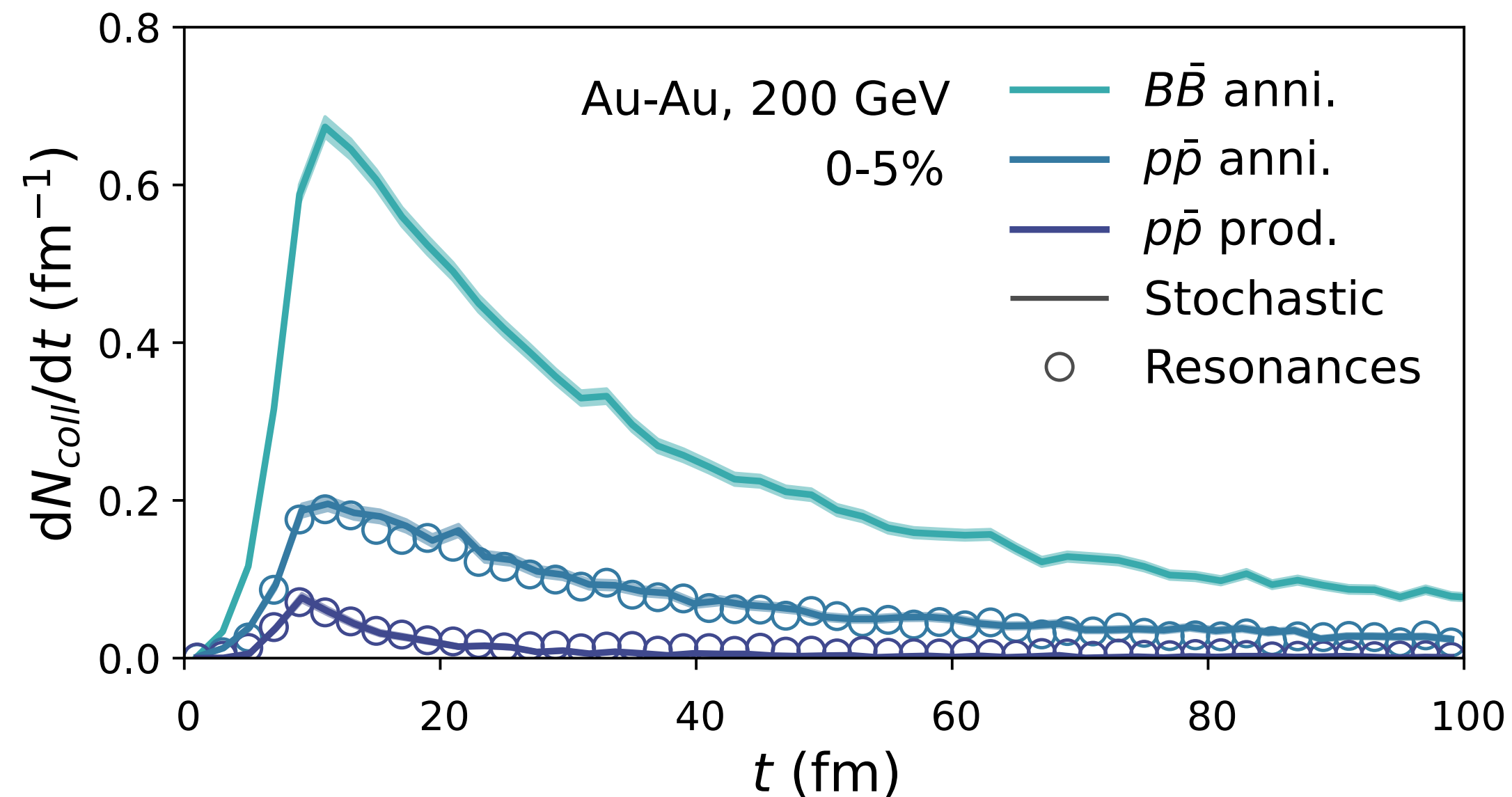
- Comparison of just including decays (“thermal model yield”), only the (forward) annihilation reaction and also accounting for the back-reaction **at mid-rapidity**
- For both reaction treatments (stochastic or resonance): **50% of $p(\bar{p})$ yield regenerated by back-reaction**

Multiplicity evolution

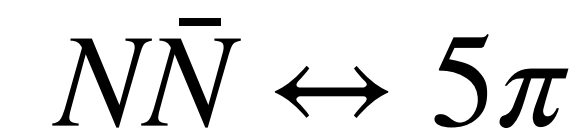


- Comparison of just including decays (“thermal model yield”), only the (forward) annihilation reaction and also accounting for the back-reaction **at mid-rapidity**
 - For both reaction treatments (stochastic or resonance): **50% of $p(\bar{p})$ yield regenerated by back-reaction**
- Interplay of annihilation and its backreaction in the late stage important for (anti-) proton yield

Comparison between reaction treatments

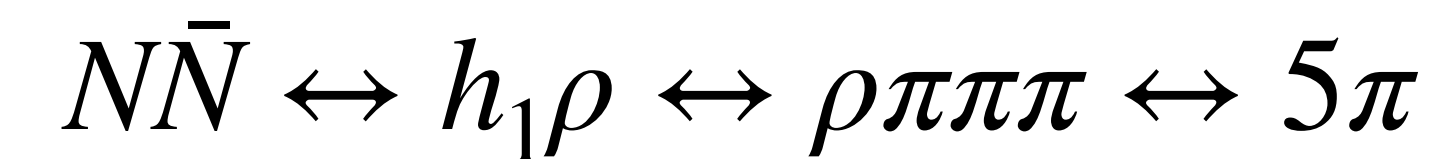


Stochastic:



vs.

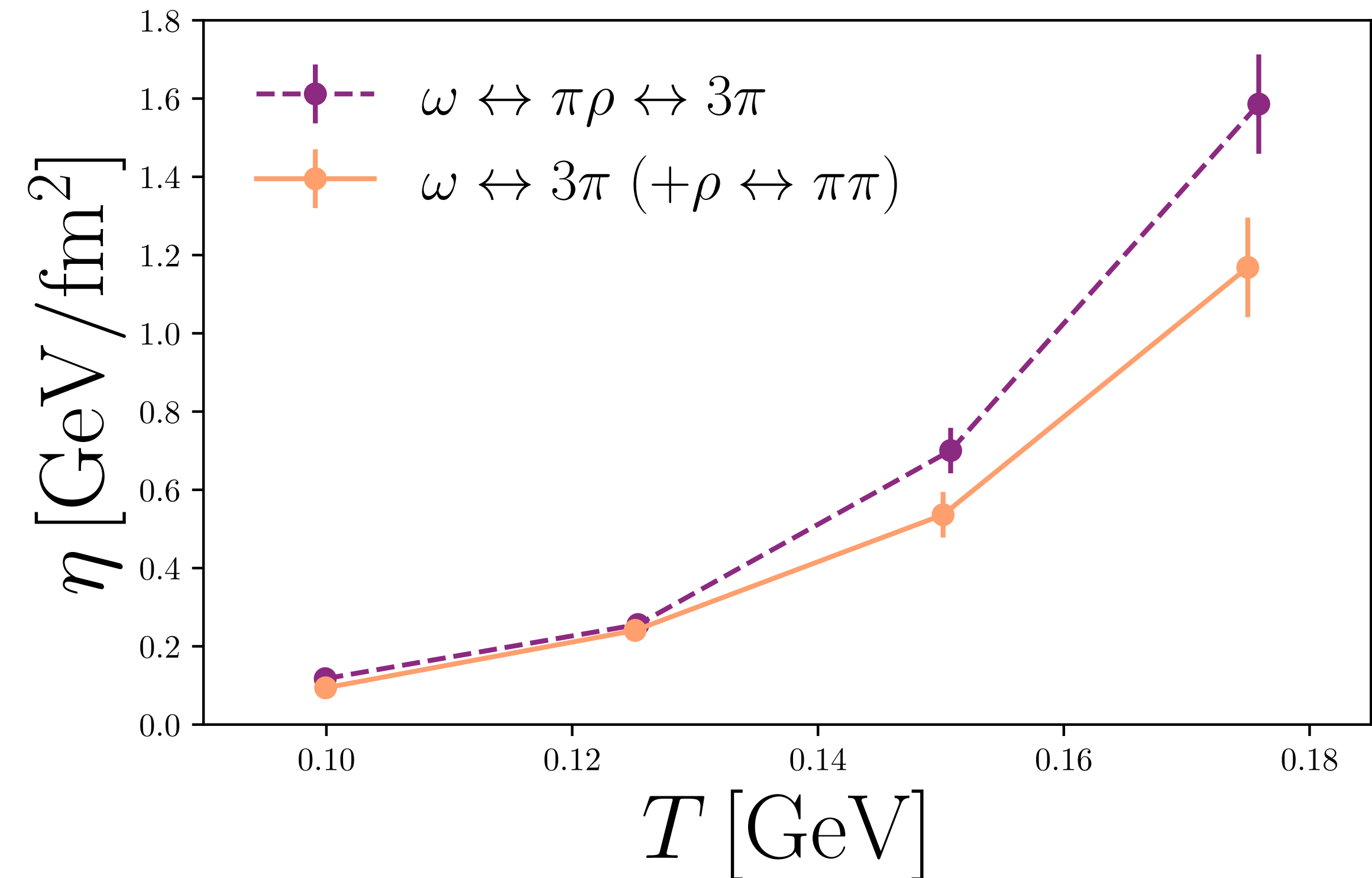
Resonance:



- Comparison of collision rate over time for $p\bar{p}$ annihilation and production
- (Slightly surprising) agreement between multi-particle and multi-step approach

Outlook: Effect on viscosity

- Box calculation with $\rho - \omega - \pi$ gas
- Comparison between multi-step and multi-particle treatment of the $3\pi \leftrightarrow \omega$ reaction
- Impact visible starting at $T \approx 150$ MeV
- **Lower viscosity with direct multi-particle reactions**



Summary

- Introduced and verified **general treatment for multi-particle reactions** in transport approach SMASH
- **Faster equilibration** with direct multi-particle reactions compared to reaction chains
- Multi-particle reactions are **significant for particle abundances** in the late hadronic collision stages within hybrid approaches
 - Deuteron generation via catalysis reaction independent of d particlization treatment
 - 50% of annihilated protons regenerated at mid-rapidity

Outlook

- Straight-forward to extend to more reactions e.g. other light nuclei, more $B\bar{B}$ annihilations, string fragmentation back-reaction (?), ...
- Study influence of multi-particle reactions on transport coefficients like viscosity

