



Zooming in on the QCD phase structure with functional approaches

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Universität Heidelberg & ExtreMe Matter Institute

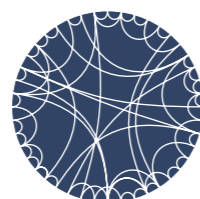
HFHF Theory Retreat, September 12-16th 2022

for the fQCD collaboration



GEFÖRDERT VOM

Bundesministerium
für Bildung
und Forschung



STRUCTURES
CLUSTER OF
EXCELLENCE



fQCD collaboration

**Braun, Chen, Fu, Gao, Geissel, Horak, Huang, Ihssen, JMP, Rennecke,
Sattler, Schallmo, Tan, Töpfel, Turnwald, Wen, Wessely, Wink, Yin**

Dalian, Beijing, Darmstadt, Heidelberg, Gießen

Hessen

Braun, Geissel, Rennecke, Schallmo, Töpfel, Turnwald, Wink

Outline

- QCD phase structure: *Where do we stand?*

- QCD phase structure: observables, estimates & systematics

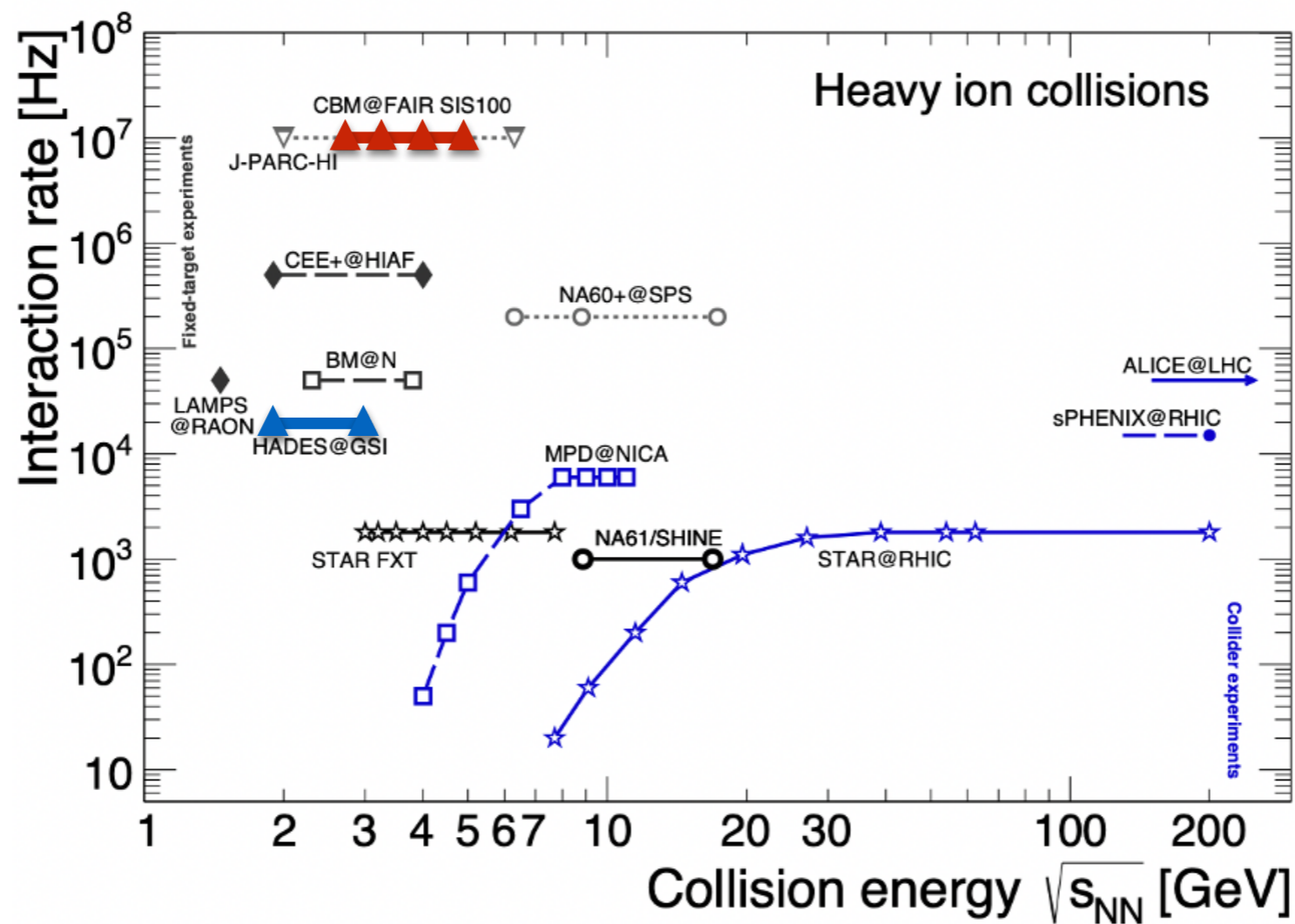
- Chiral dynamics & phenomenological applications

- Summary & outlook

Experimental landscape

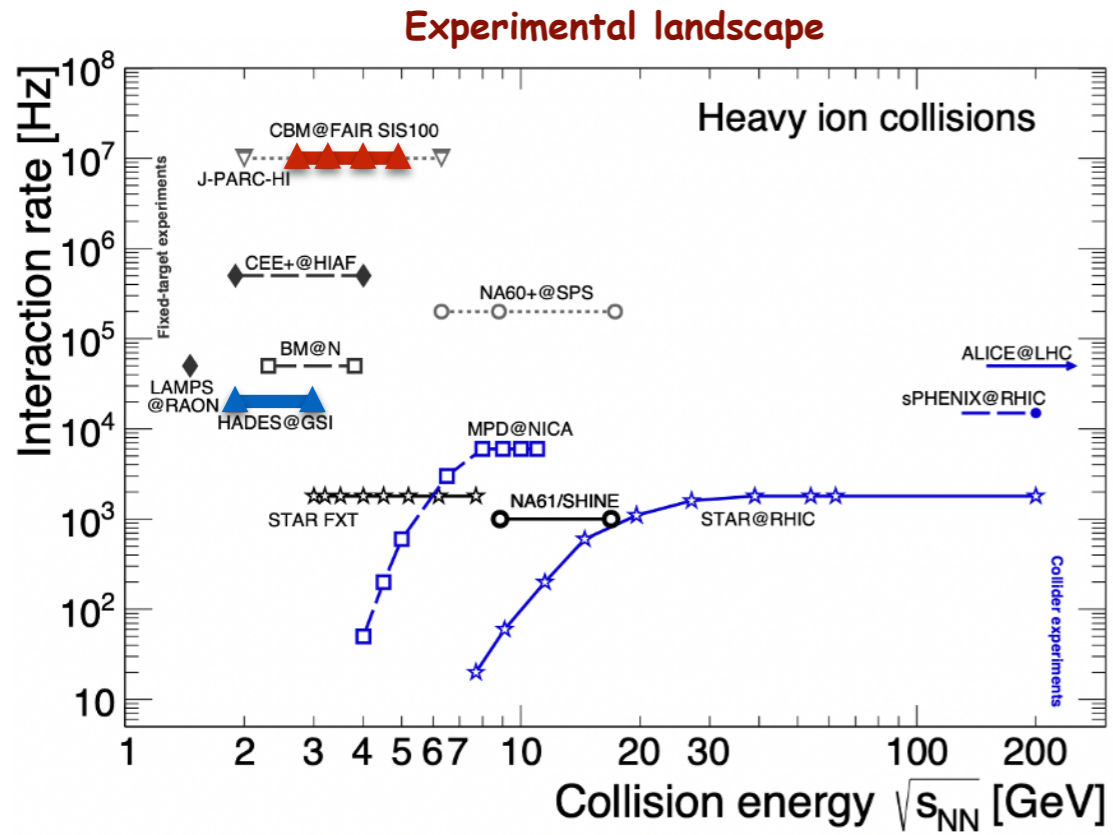
'The (experimental) future is bright'

Tetyana Galatyuk, Erice 2021



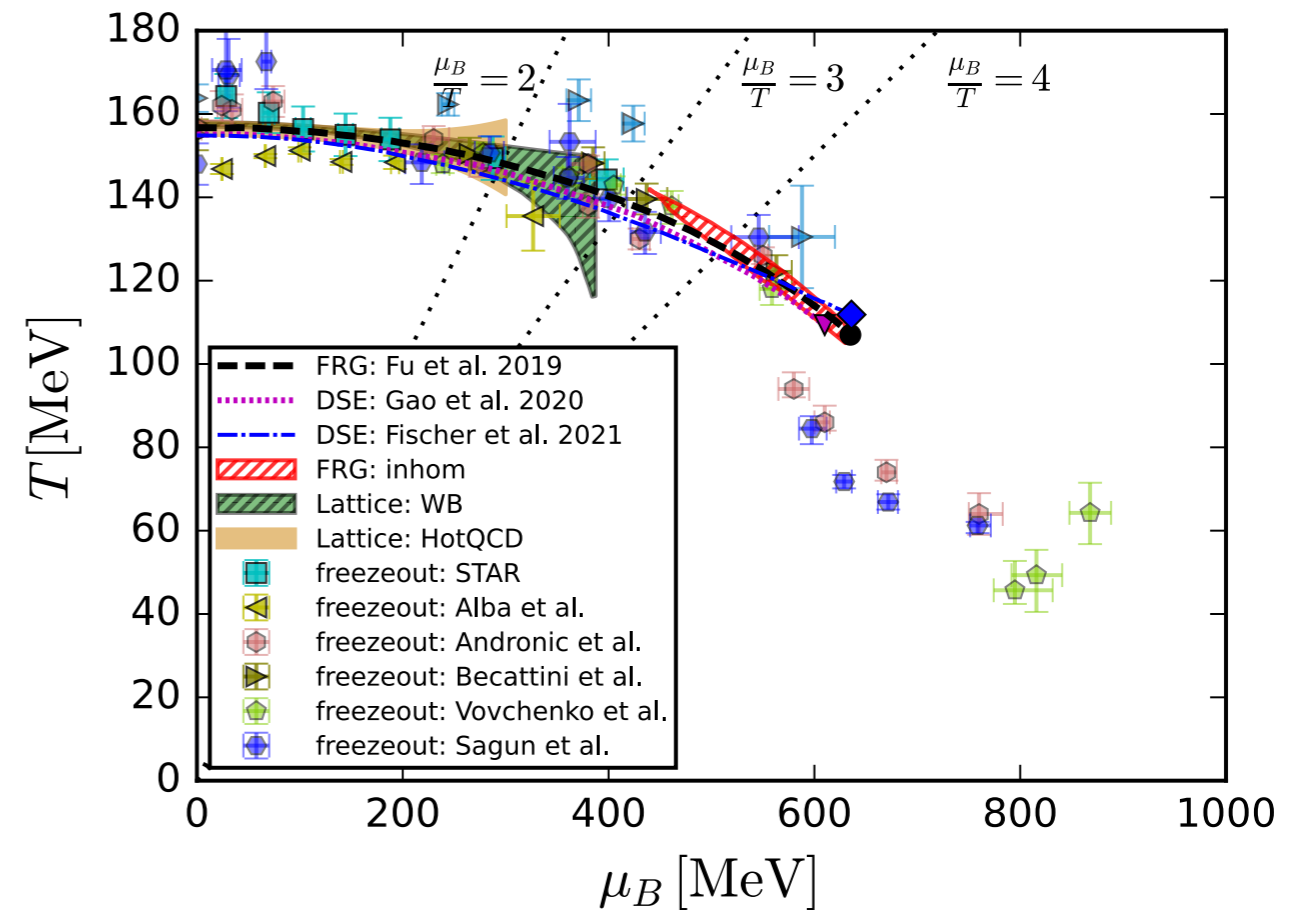
Galatyuk, A982 (2019) update 2021; CBM, EPJA 53 3 (2017) 60

How bright does it get?



Galatyuk, A982 (2019) update 2021
CBM, EPJA 53 3 (2017) 60

Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)



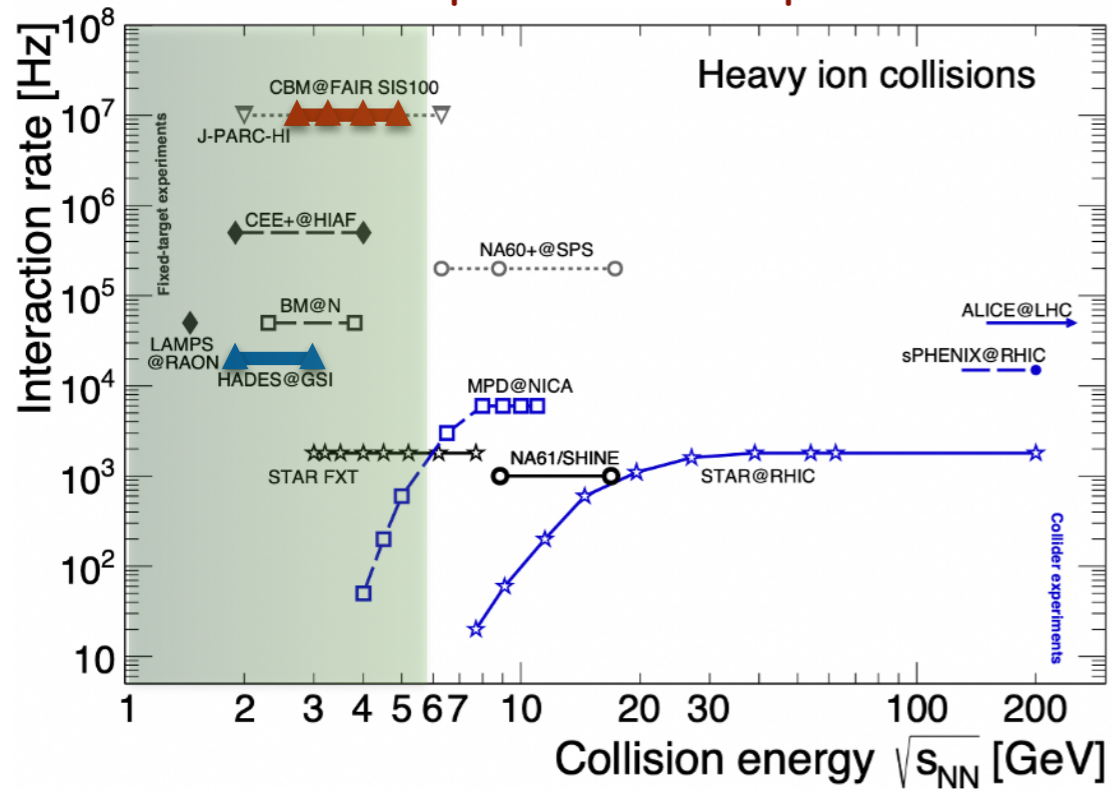
Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

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Experimental landscape

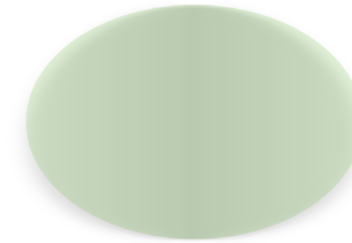


Galatyuk, A982 (2019) update 2021
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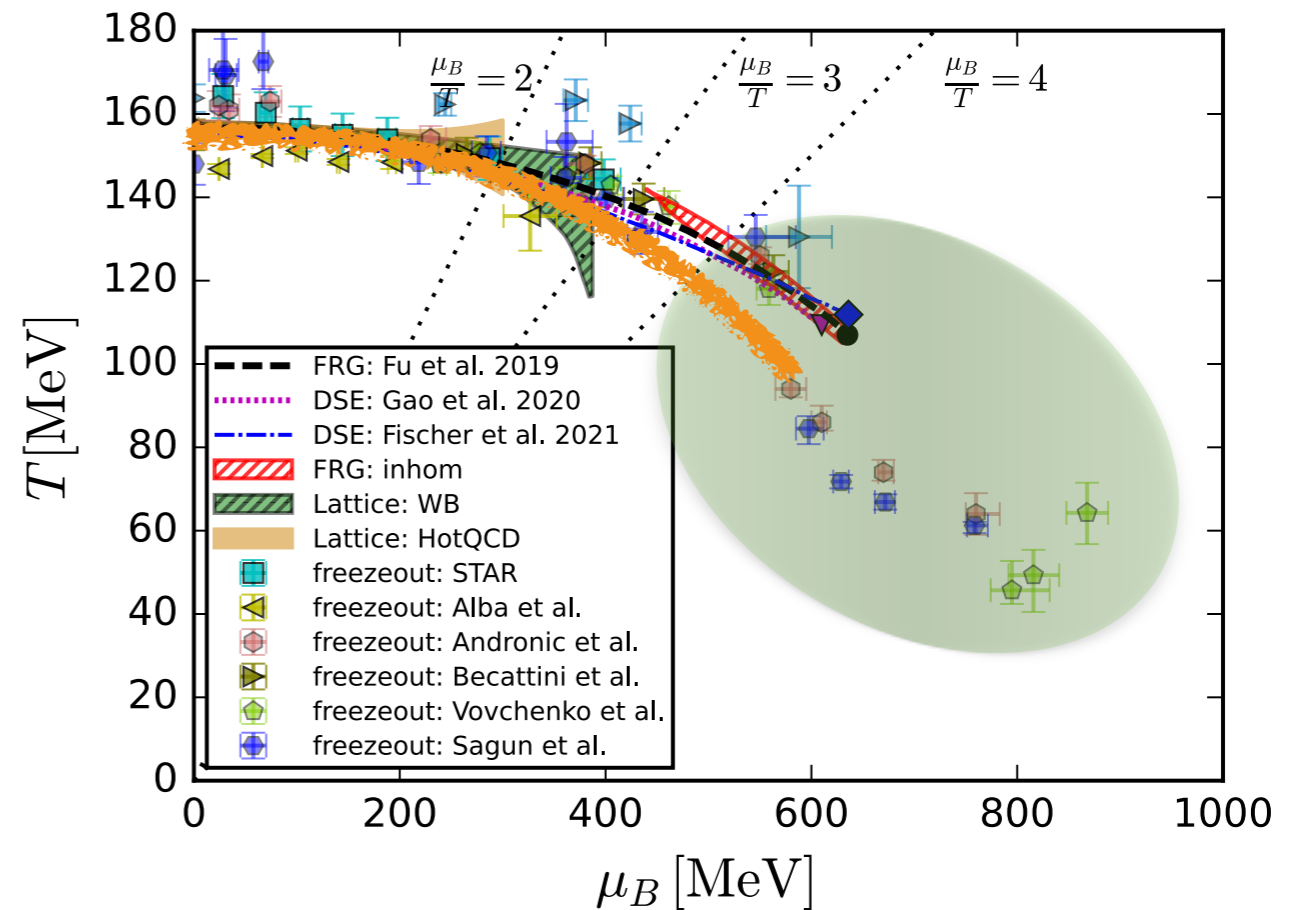
Freezeout curve

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

CEP or other NEW physics/phases



Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)



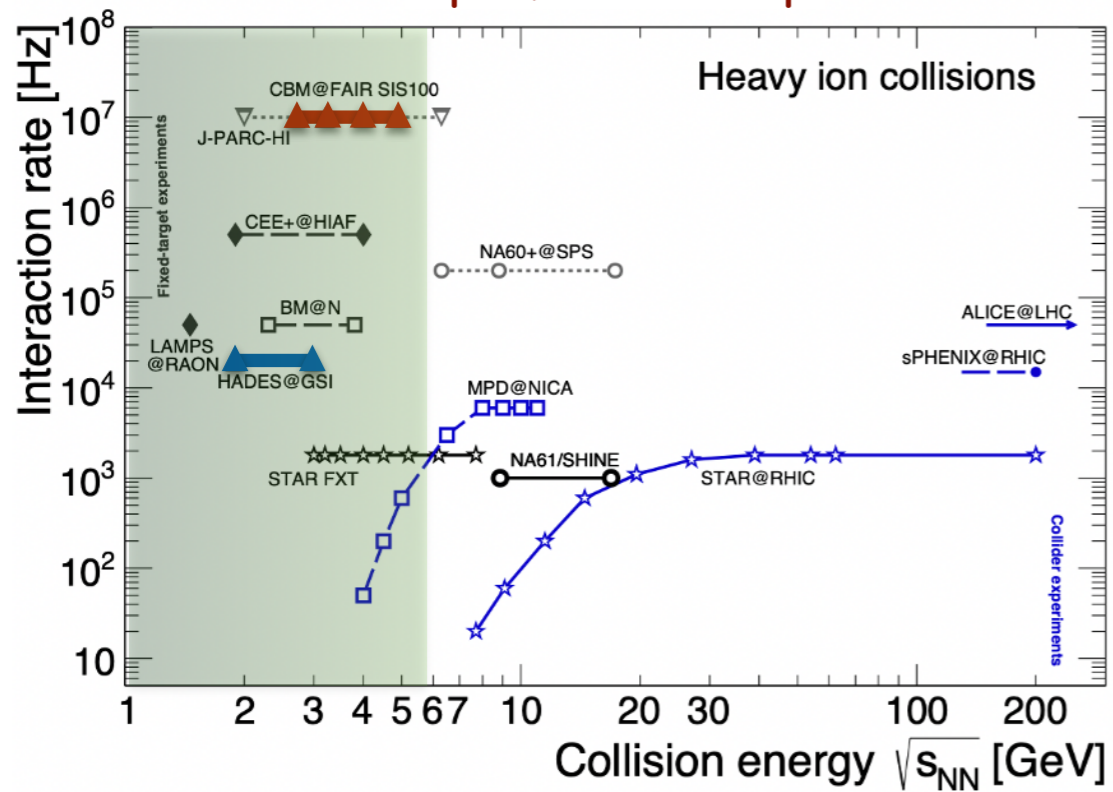
Fu, JMP, Rennecke, PRD 101 (2020) 054032

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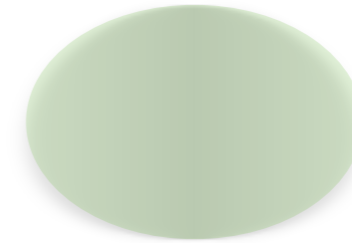
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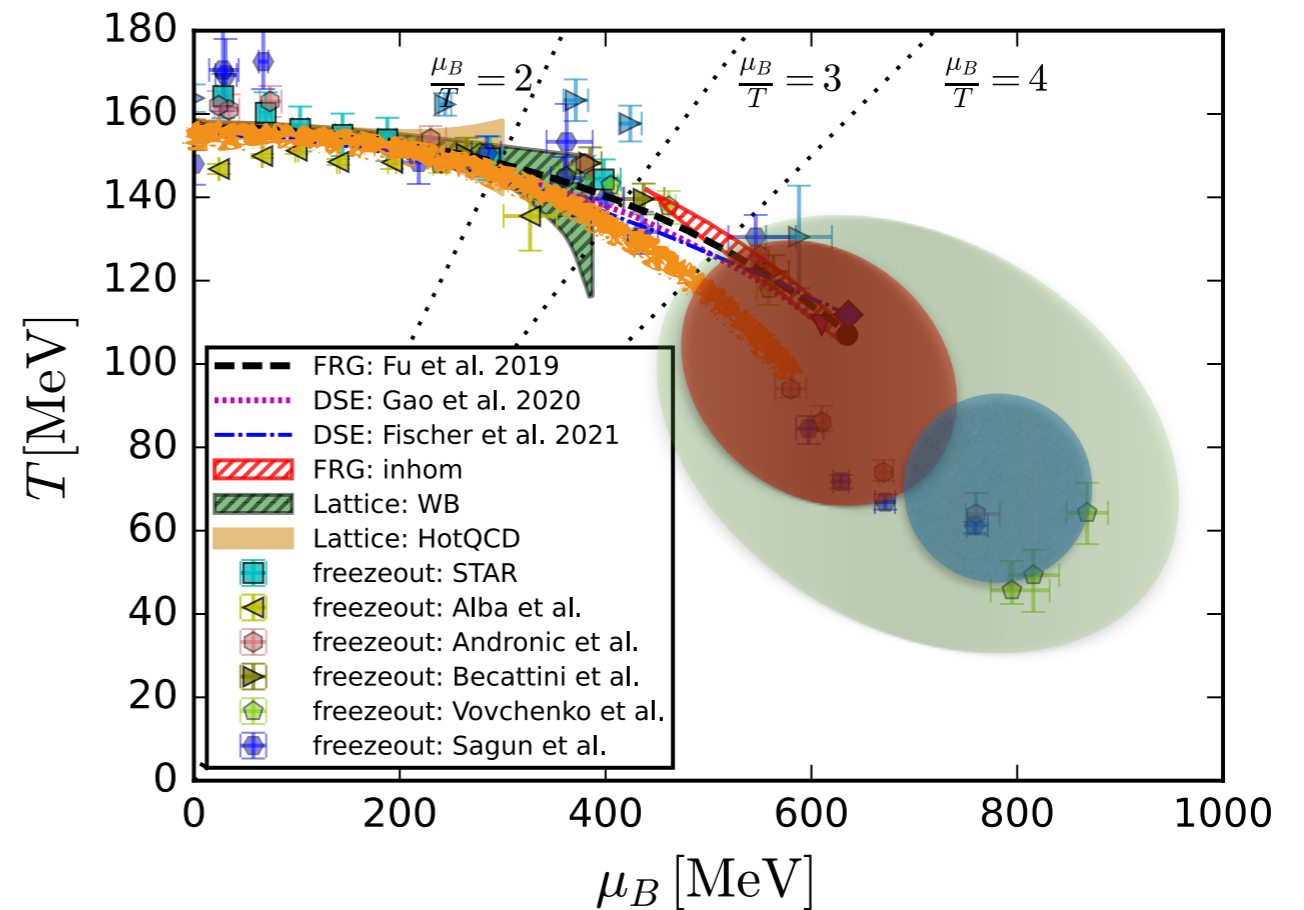
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9



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Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)



Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

QCD with Functional Approaches

ab initio & closed form

Diagrammatic functional relations for the free energy $\Gamma[\phi]$

functional RG:

free energy/
grand potential

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{glue quantum fluctuations} - \text{quark quantum fluctuations} + \frac{1}{2} \text{hadronic quantum fluctuations}$$

glue quantum fluctuations

quark quantum fluctuations

hadronic quantum fluctuations

RG-scale $k: t = \ln k$

functional DSE :

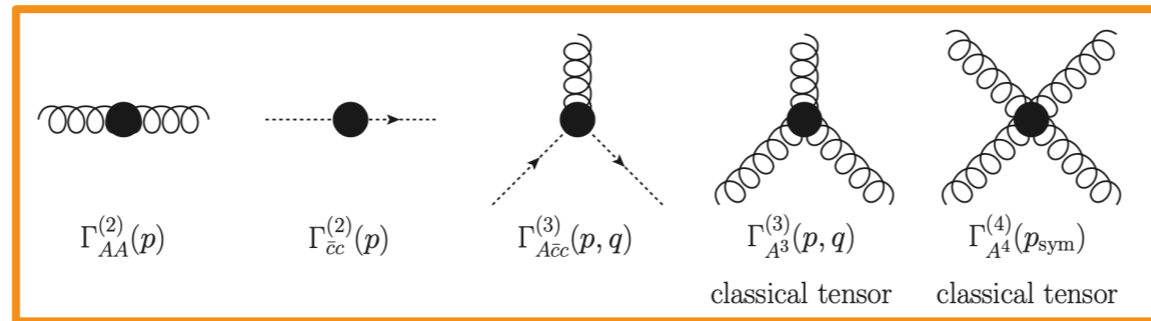
A_0 : background field

$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2} \text{glue} - \text{quark} - \frac{1}{6} \text{glue} + \text{quark}$$

fRG: Fu, 2205.00468
Dupuis et al, Phys.Rept. 910 (2021) 1

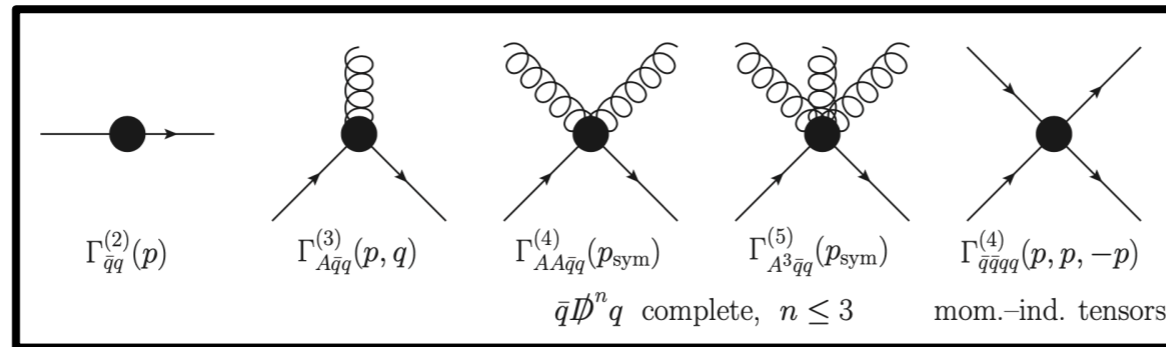
DSE: Fischer, PPNP 105 (2019) 1

vacuum fQCD: current set of correlation functions

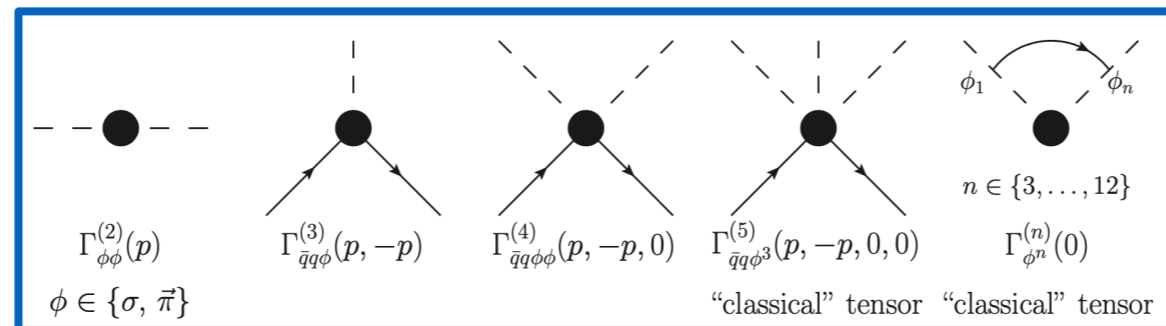


glue sector

FRG

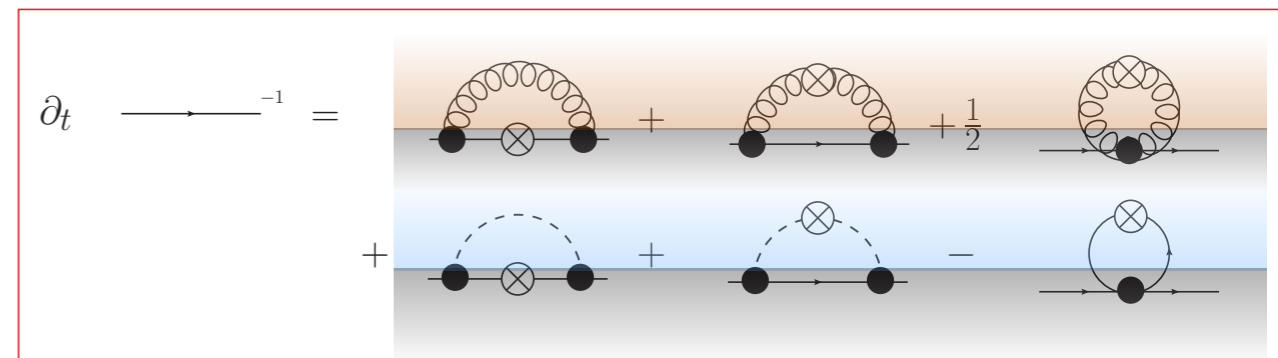


quark-gluon sector



quark-meson sector

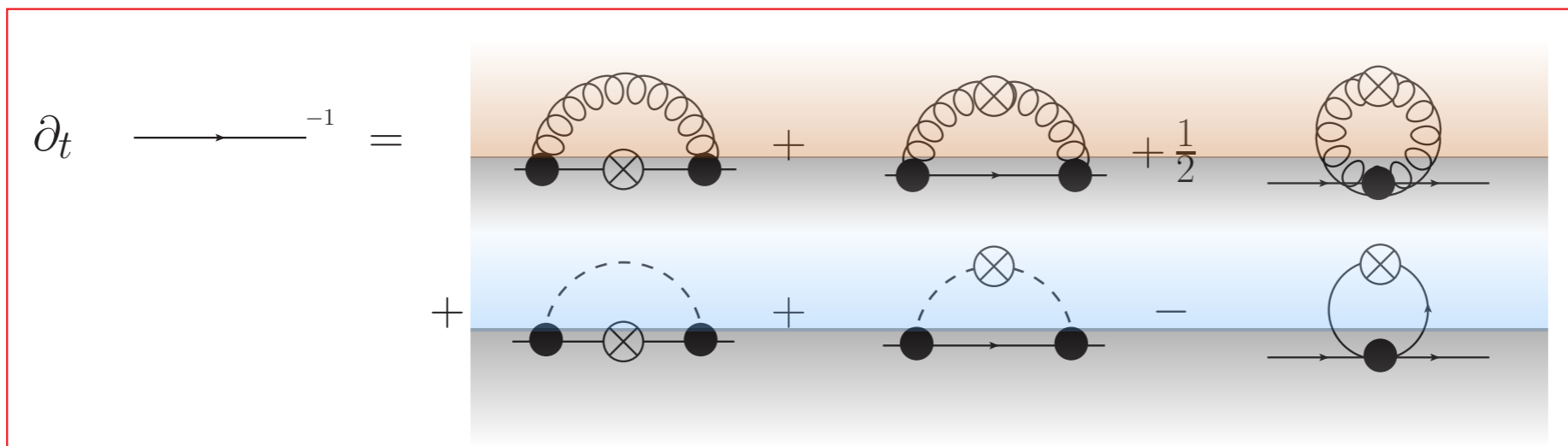
Aiming at apparent convergence



Analyticity considerations at finite density

- Self-consistent truncations to functional relations define analytic functions in μ_B , eg:

$$\partial_t \langle q(x) \bar{q}(y) \rangle(\mu_B) = \text{Loop} \left[\langle q(x) \bar{q}(y) \rangle(\mu_B), \langle q(x) A_\mu(y) \bar{q}(z) \rangle(\mu_B), \dots; \mu_B \right]$$

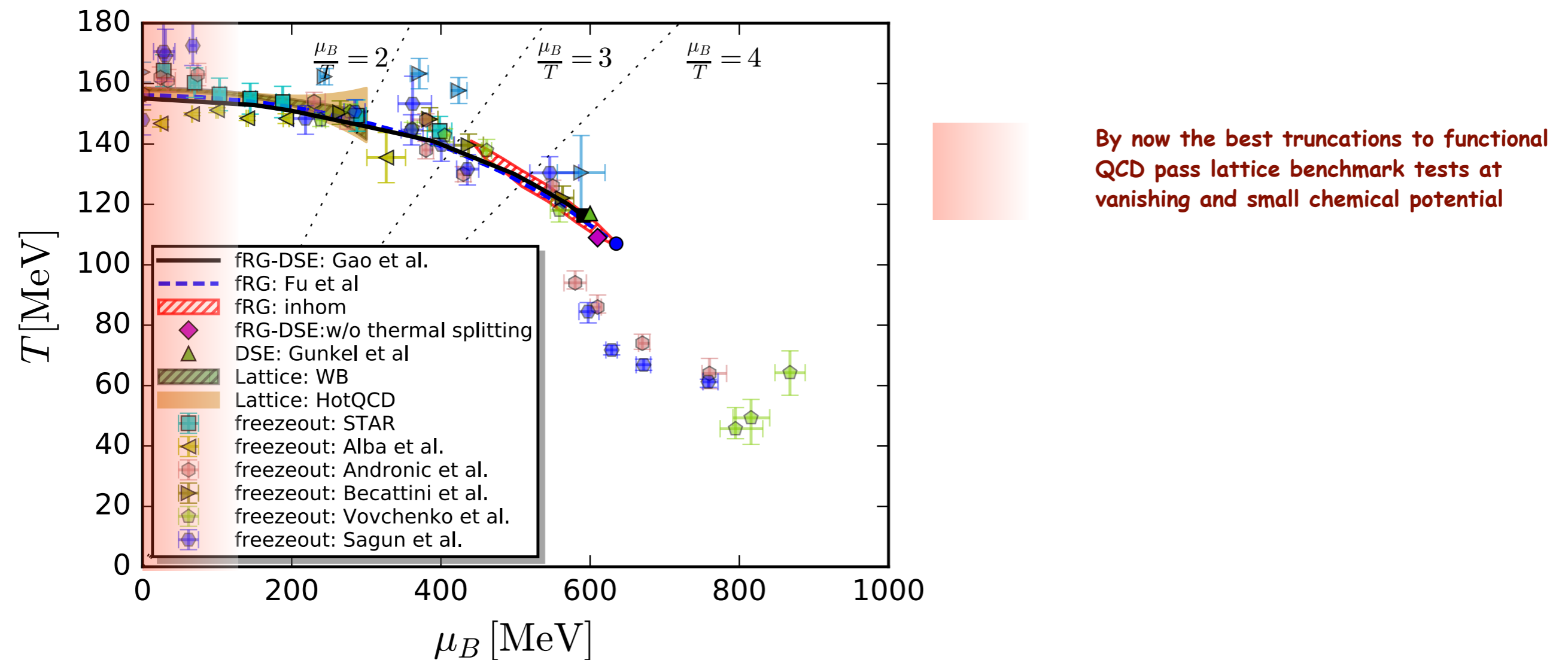


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- Consequences for functional QCD predictions at finite density

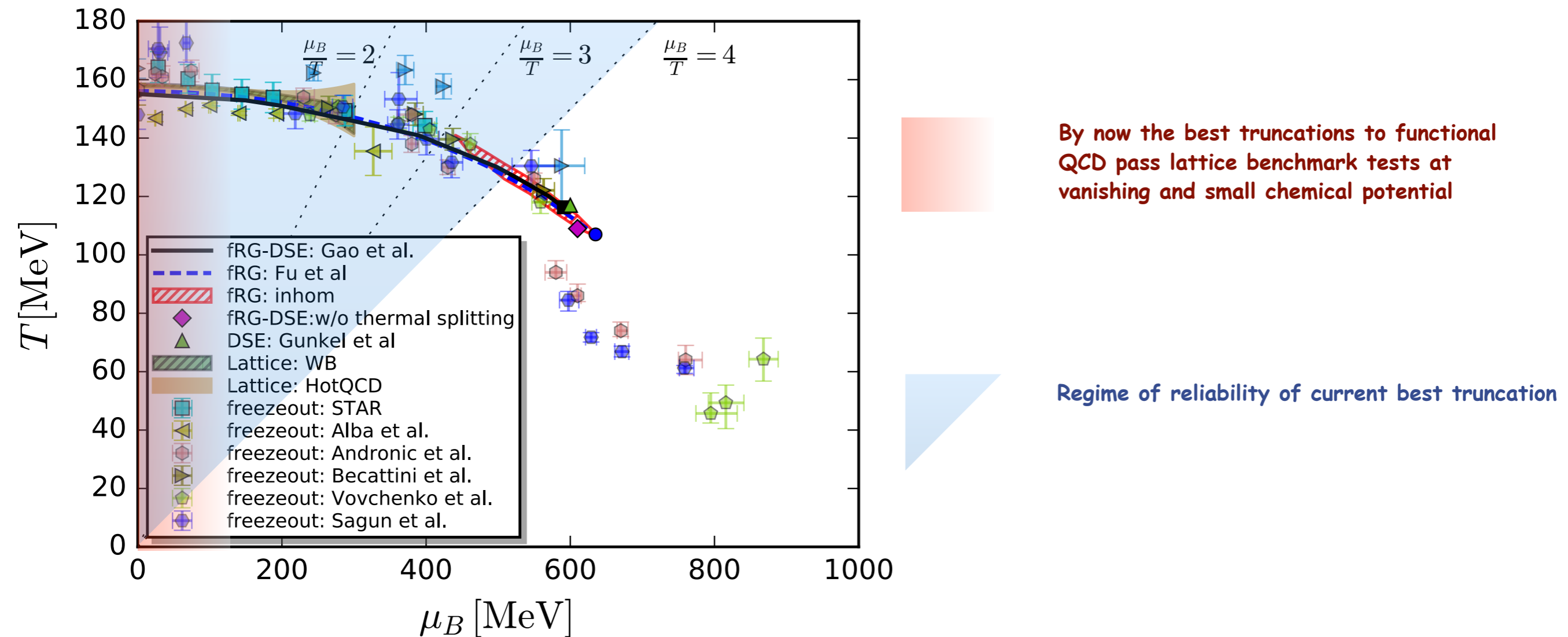


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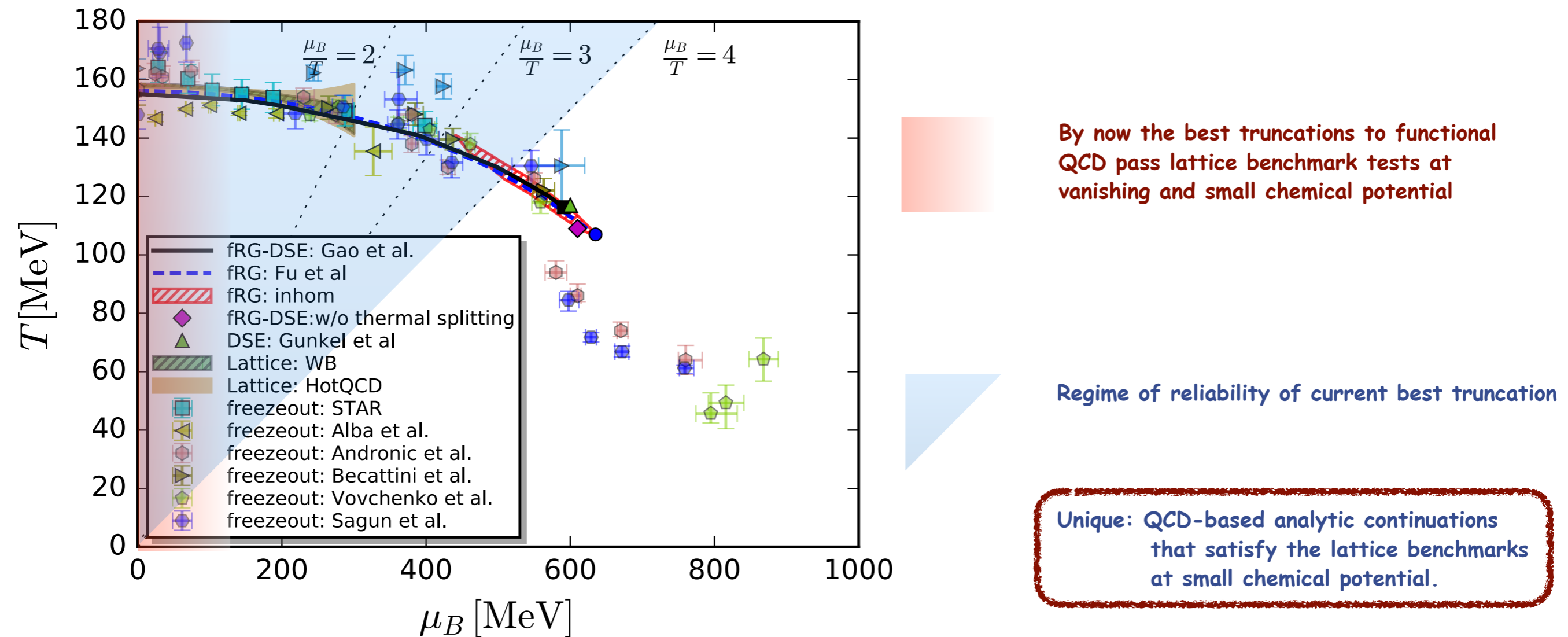


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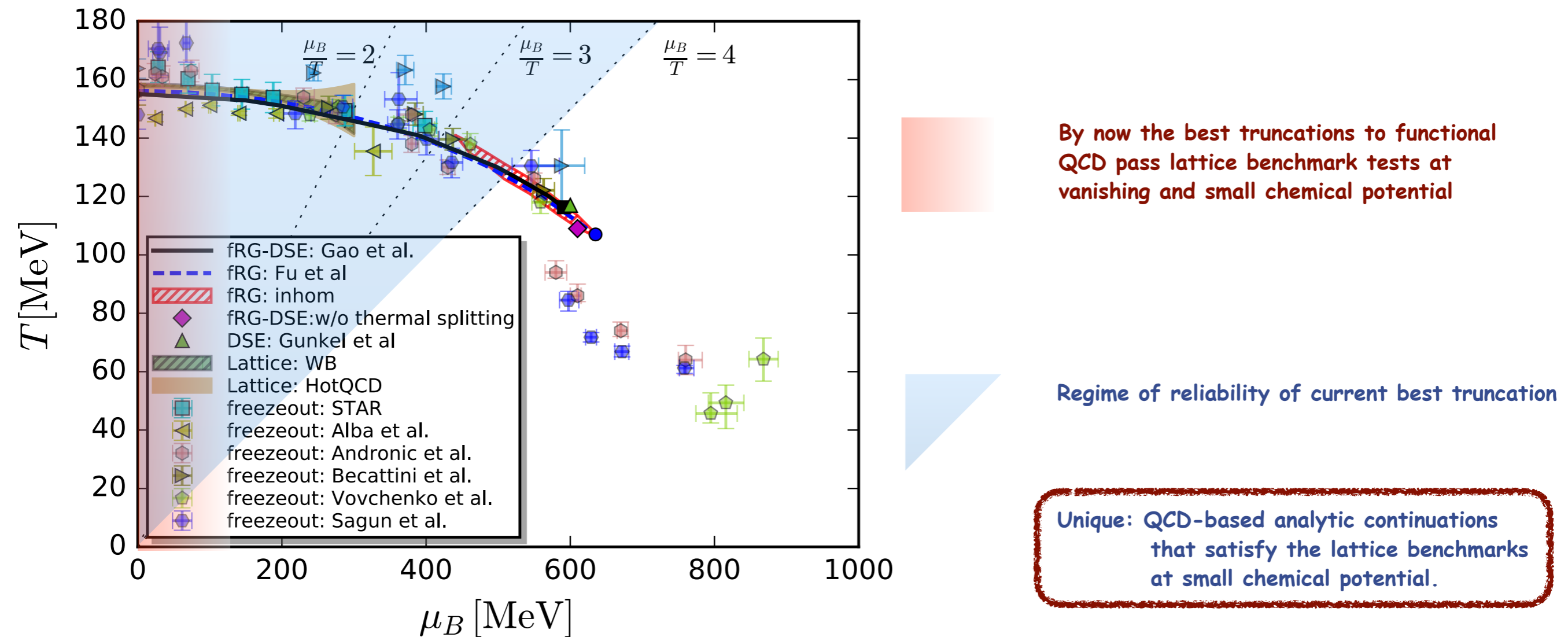


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- Consequences for functional QCD predictions at finite density



Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Outline

- QCD phase structure: Where do we stand?

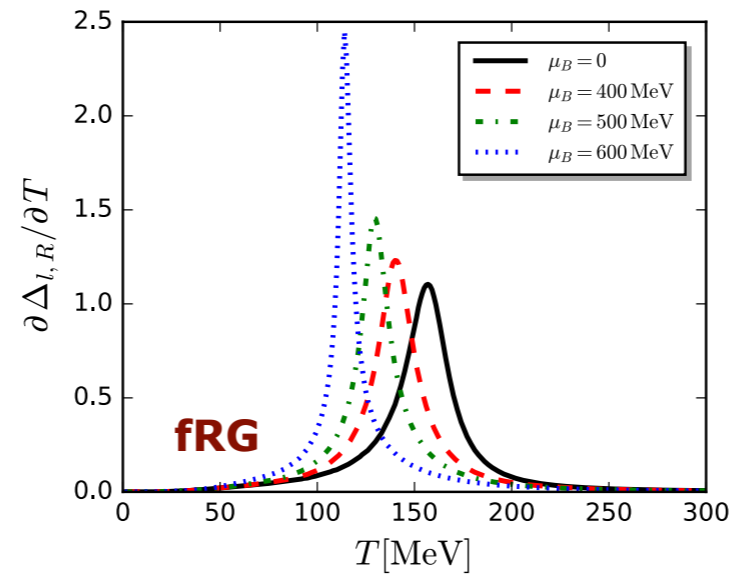
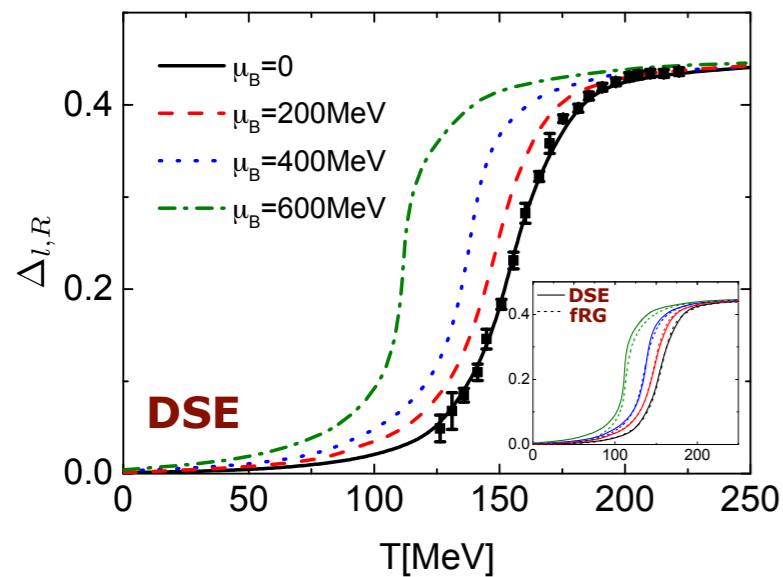
- QCD phase structure: observables, estimates & systematics

- Chiral dynamics & phenomenological applications

- Summary & outlook

Chiral condensates

renormalised condensate



$$\Delta_{l,R}(T, \mu_B) \simeq \Delta_l(T, \mu_B) - \Delta_l(0, 0)$$

$$\Delta_q(T, \mu_B) = \frac{T}{\mathcal{V}} m_q^0 \int_x \langle \bar{q}(x)q(x) \rangle$$

lattice: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010)

DSE: quark condensates

See also

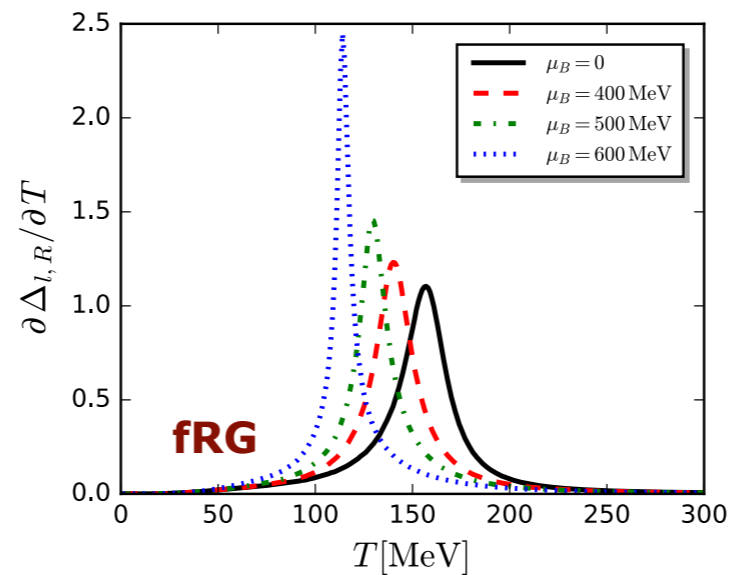
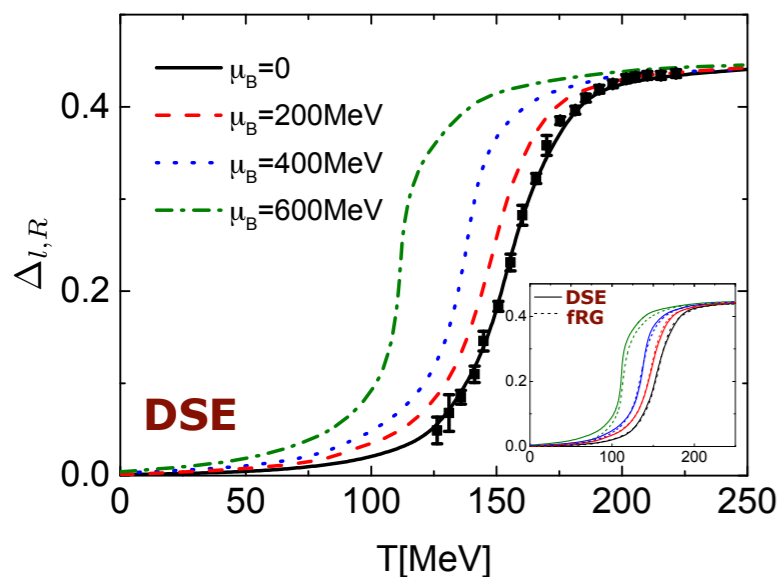
Fischer, Luecker, PLB 718 (2013) 1036

Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

Chiral condensates

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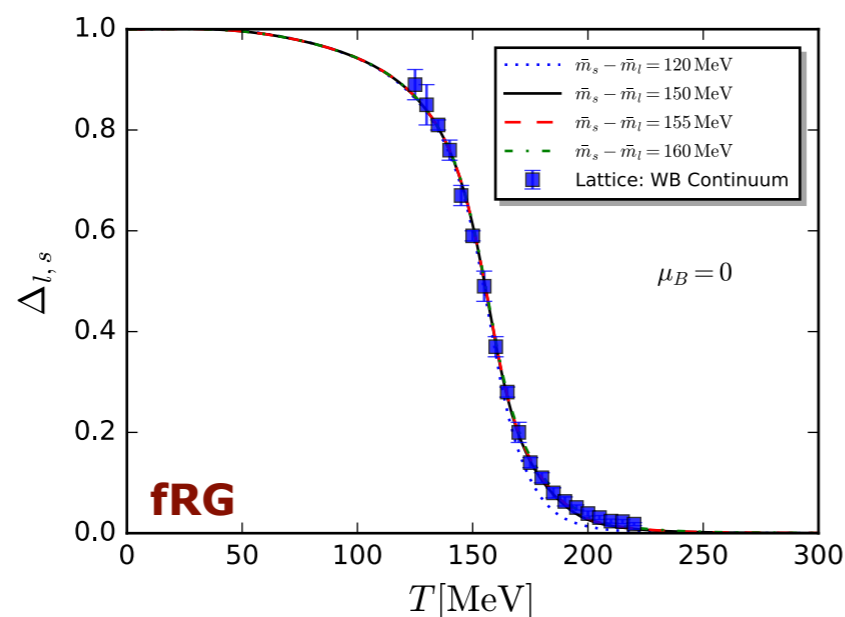


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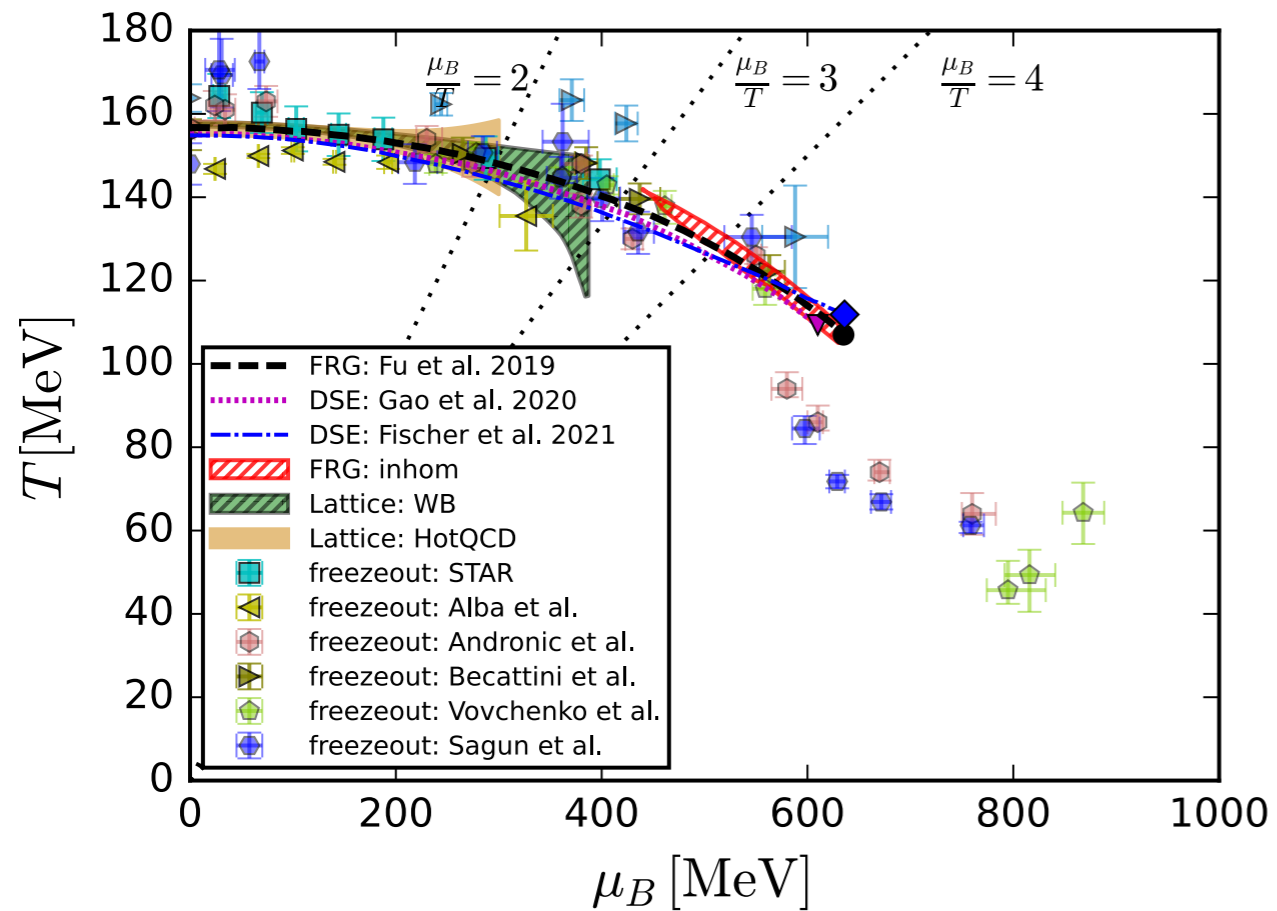
reduced condensate



$$\Delta_{l,s}(T, \mu_B) = \frac{\Delta_l(T, \mu_B) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T, \mu_B)}{\Delta_l(0, 0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0, 0)}$$

fRG: Fu, JMP, Rennecke, PRD 101 (2020) 054032
DSE: Gao, JMP, PLB 820 (2021) 136584

Curvature of the chiral transition line



fQCD

$$\kappa_{\text{FRG}} = 0.0142(2)$$

$$\kappa_{\text{DSE}} = 0.0147(5)$$

lattice

$$\kappa_{\text{WB}} = 0.0149(21)$$

WB, PLB 751 (2015) 559

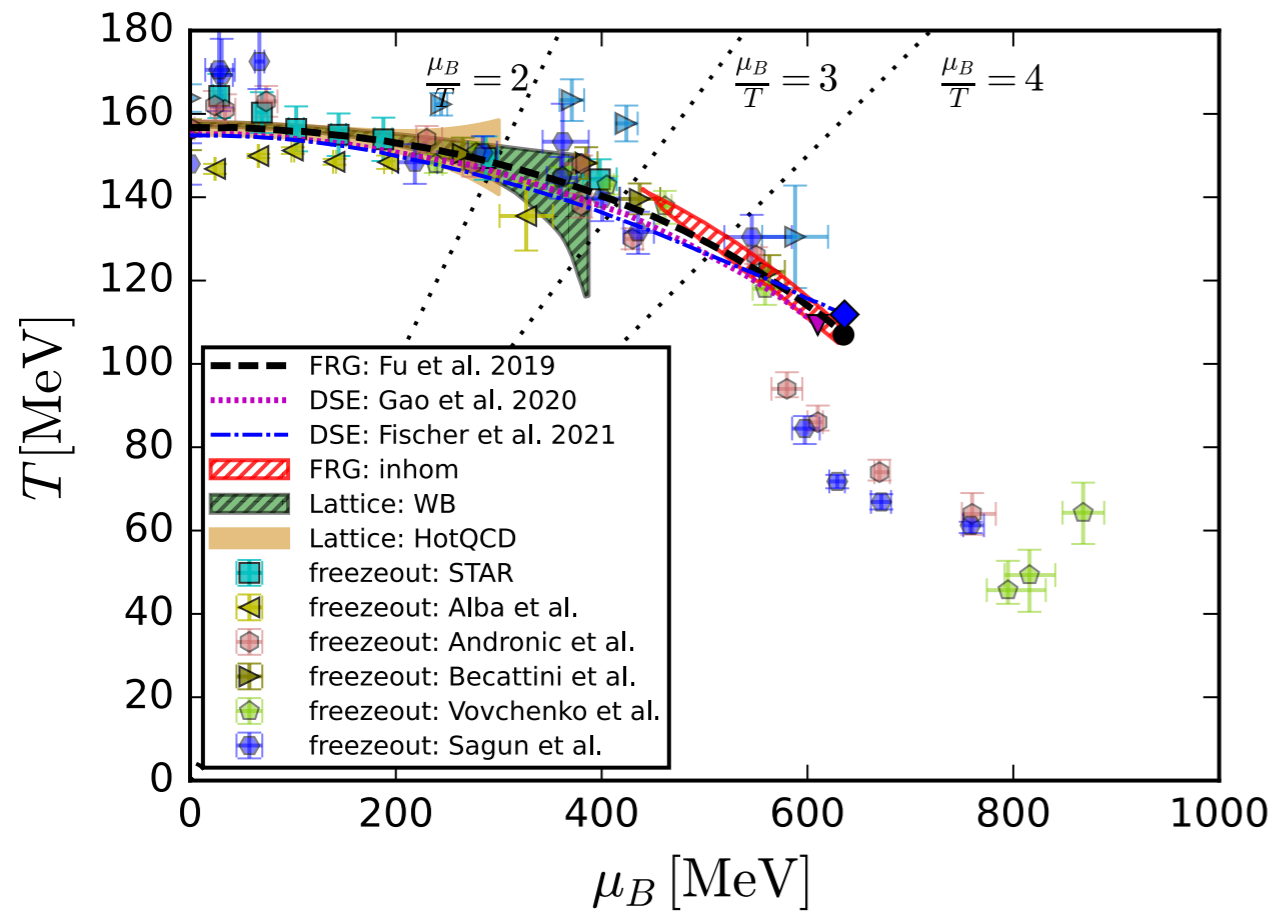
$$\kappa_{\text{hotQCD}} = 0.015(4)$$

hotQCD, PLB 795 (2019) 15

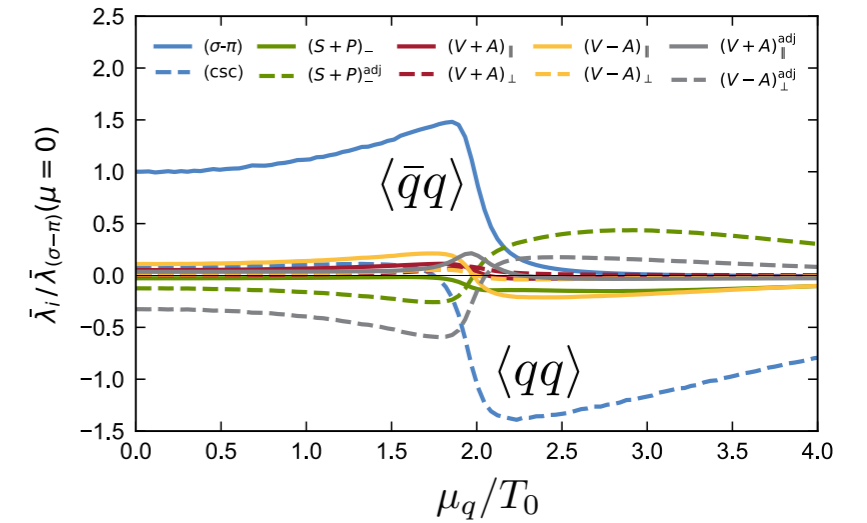
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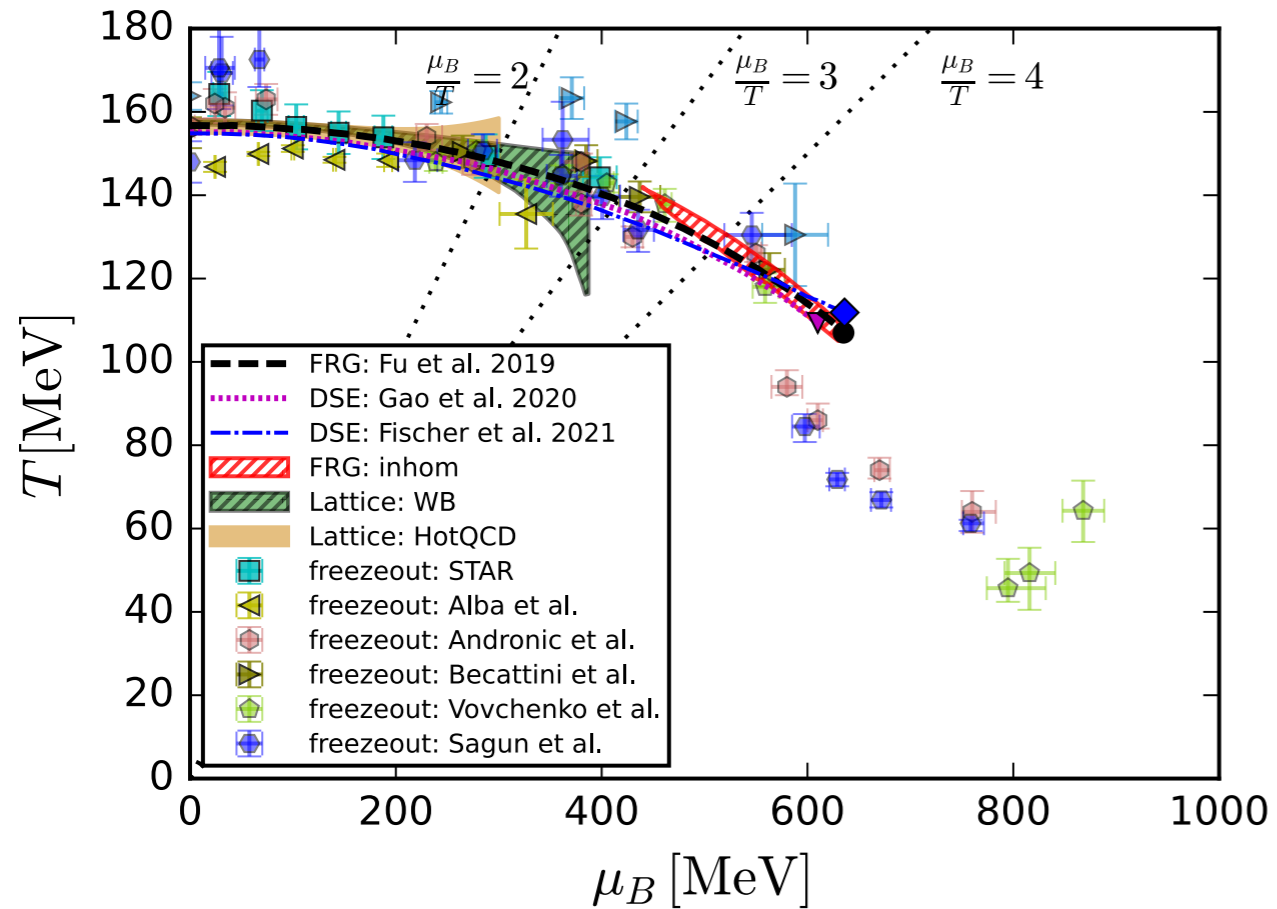
Systematic error control (CEP)



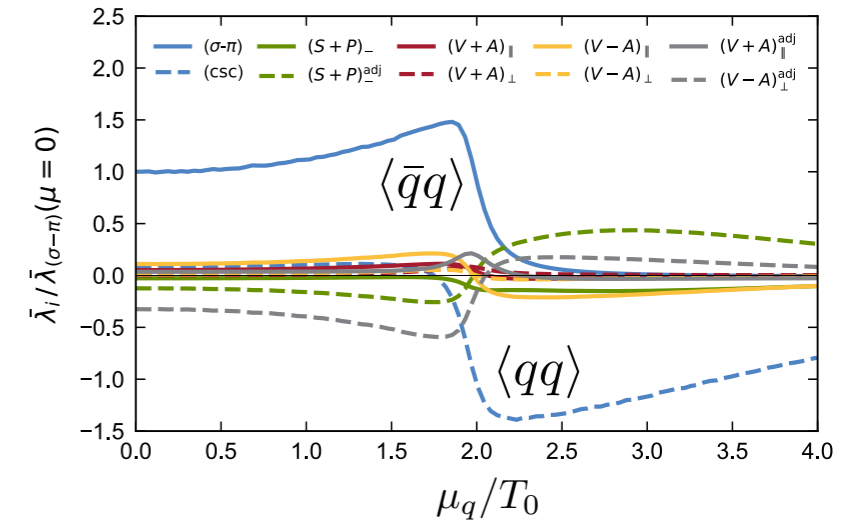
Dominant channels I (fRG)
Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



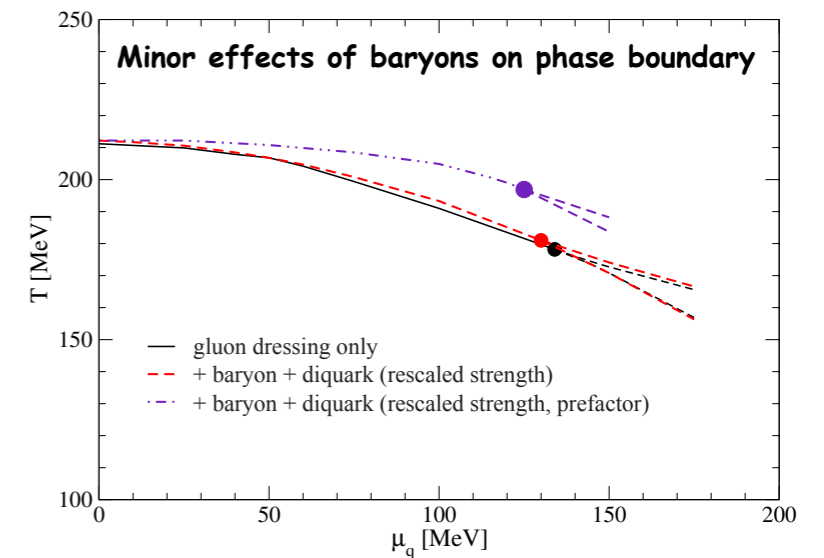
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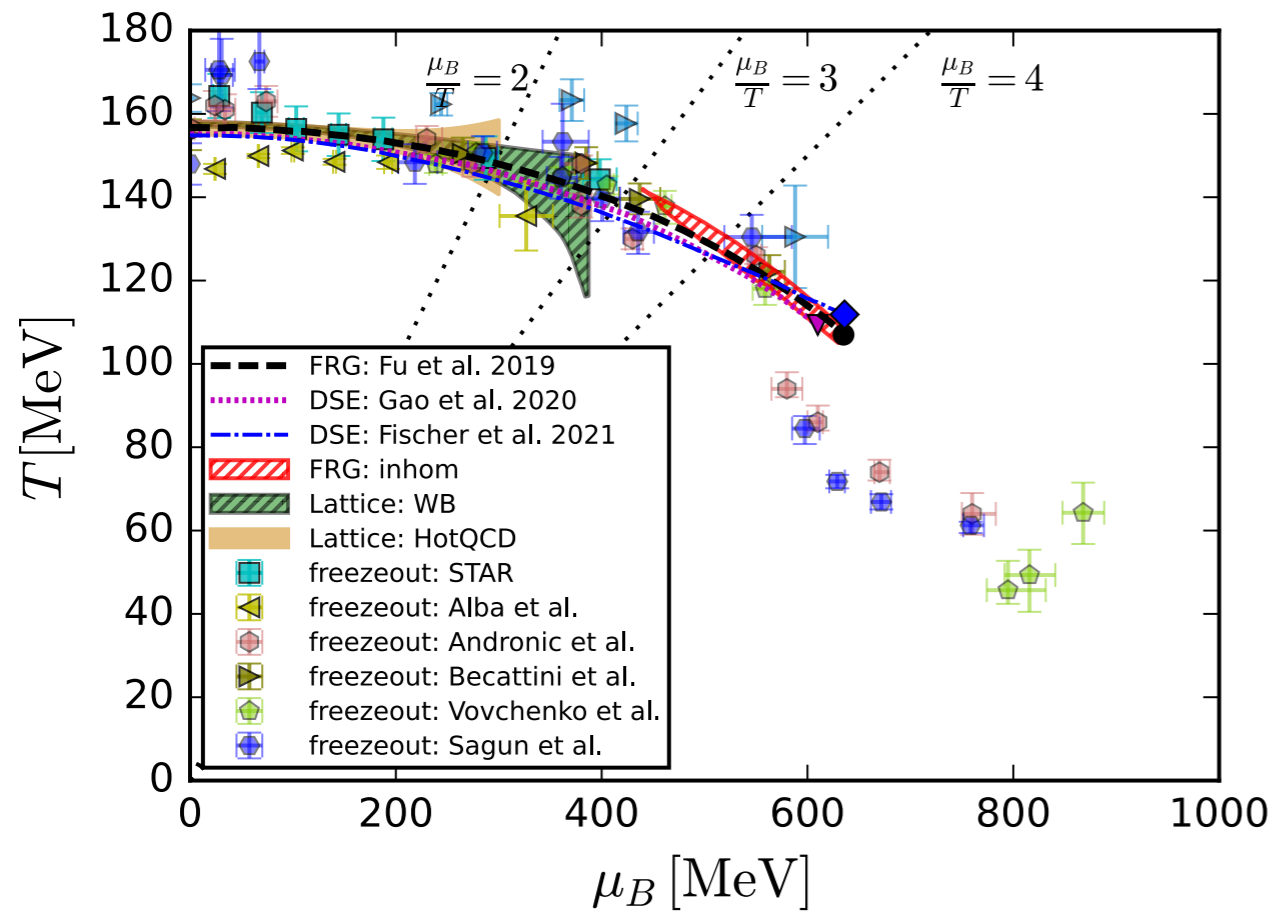


Dominant channels II (DSE)
Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013

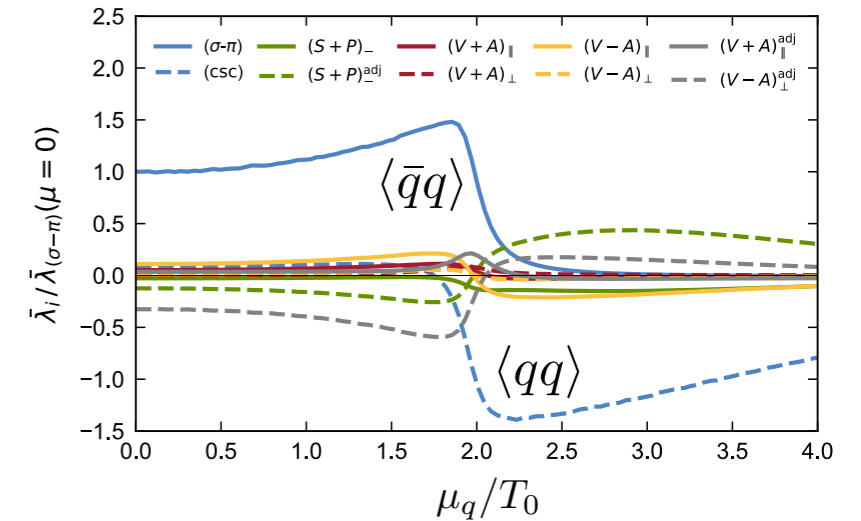


Minor effects of mesons
Gunkel, Fischer, PRD 104 (2021) 054022

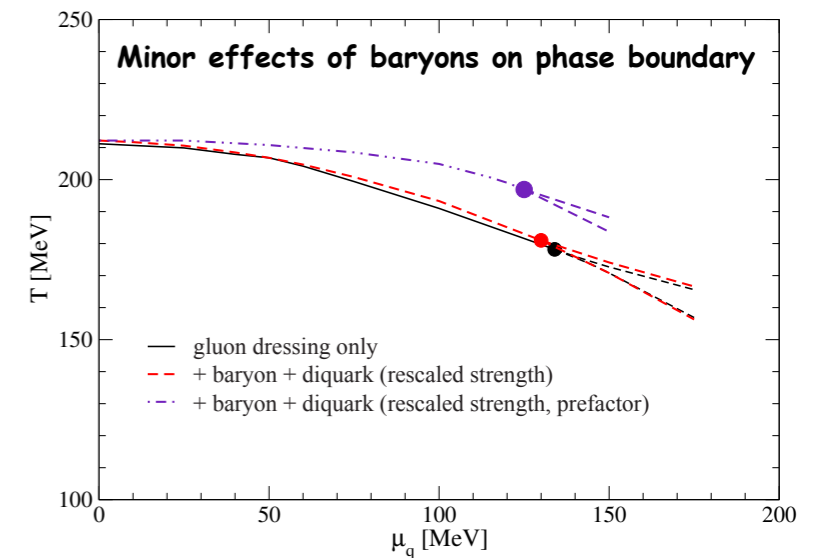
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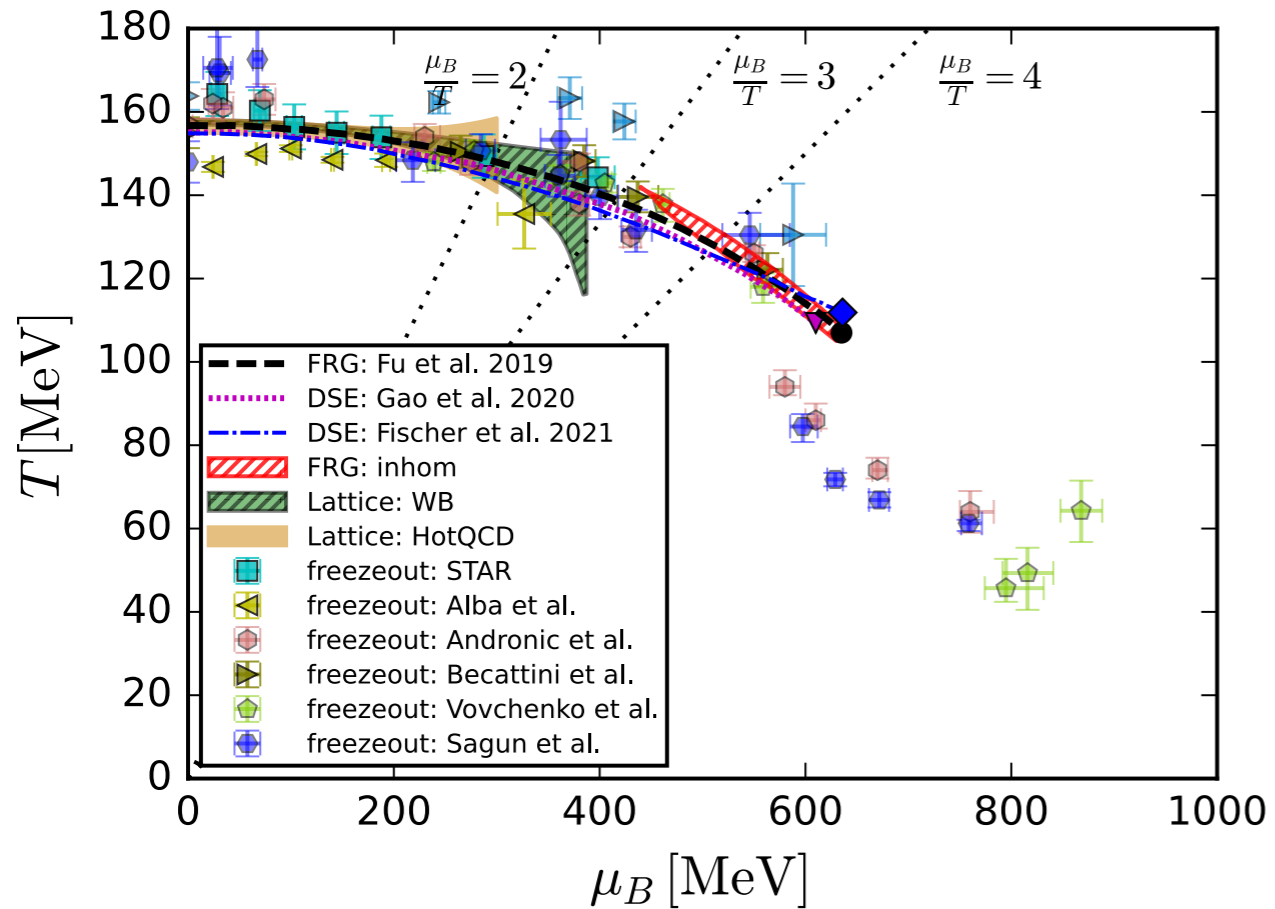


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Systematic error control (CEP)



Moat and 'inhomogeneous' phases



Pion dispersion has minimum at non-vanishing spatial momentum & sizeable chiral condensate

Fu, JMP, Rennecke, PRD 101 (2020) 054032

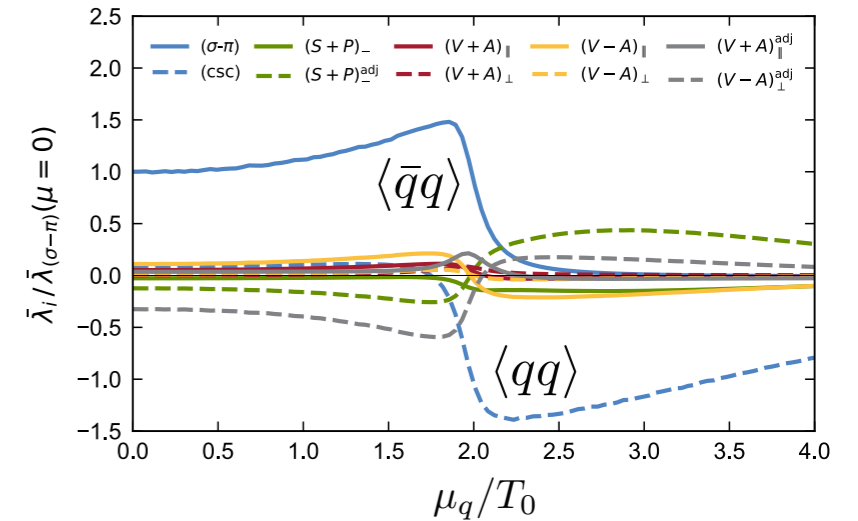
Moat: Pisarski, Rennecke, PRL 127 (2021) 152302



Non-trivial background

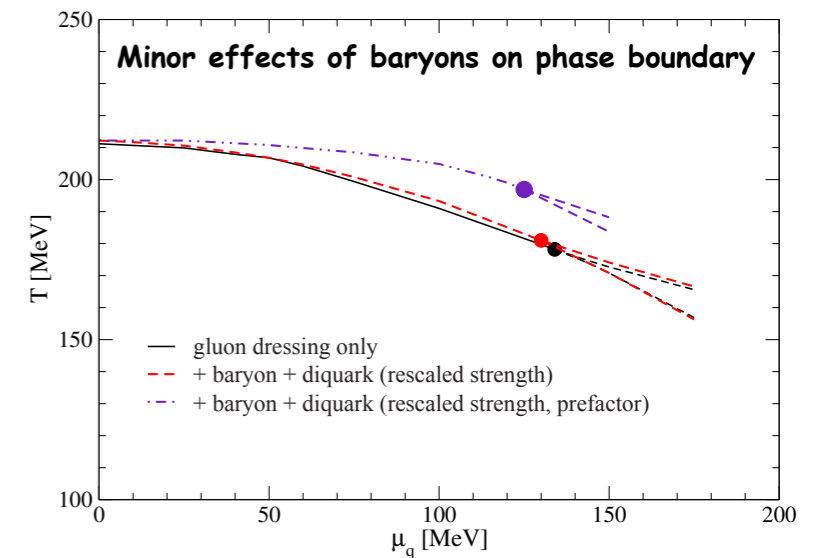
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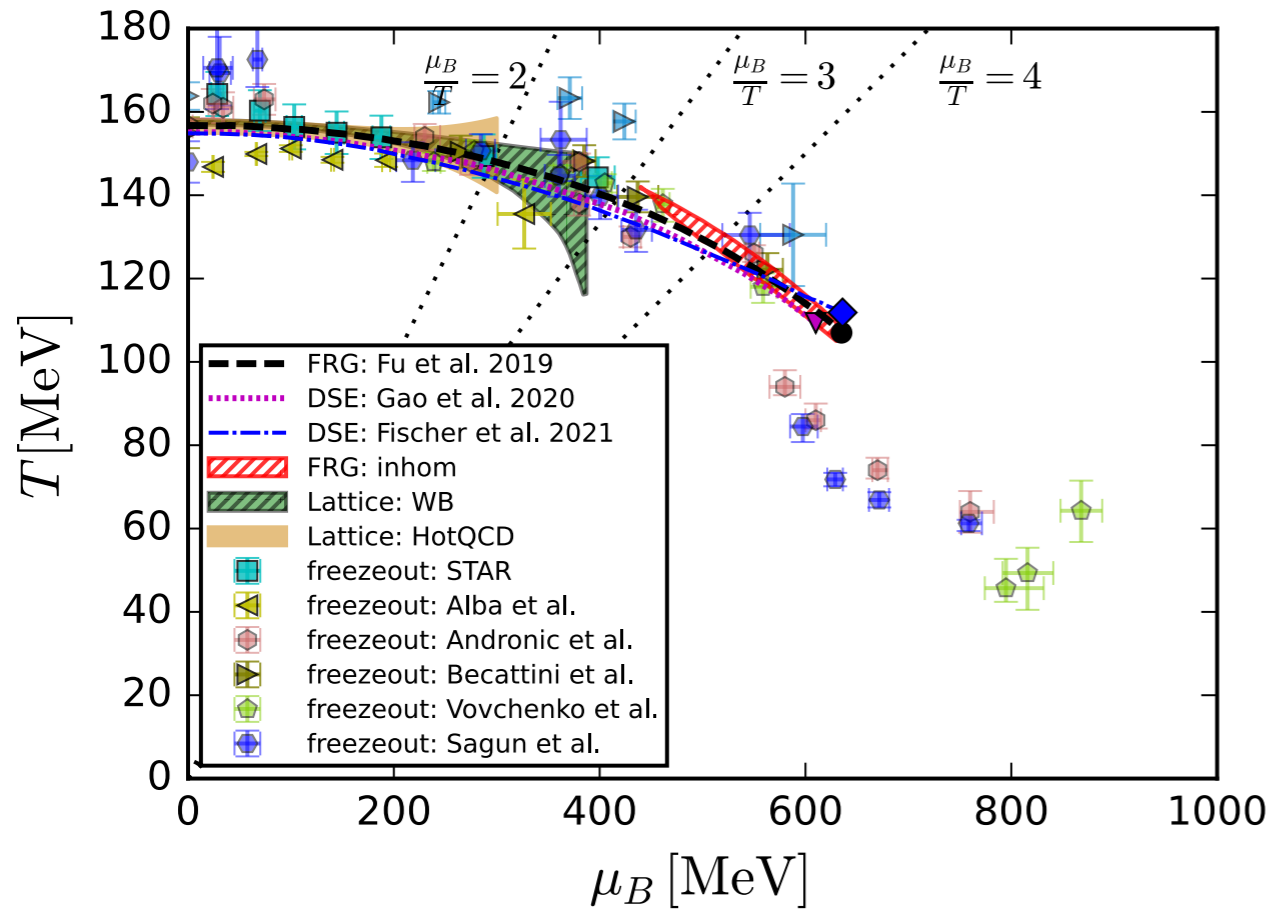
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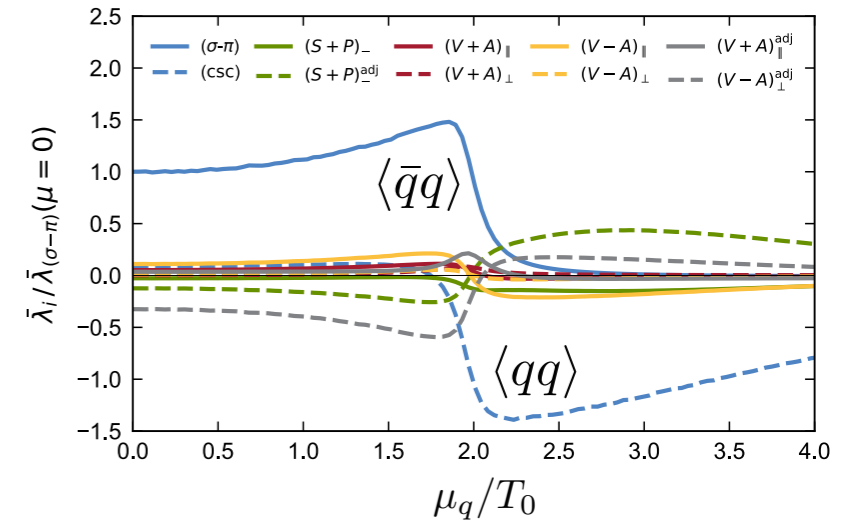
Fierz-complete computation

Systematic error control (CEP)

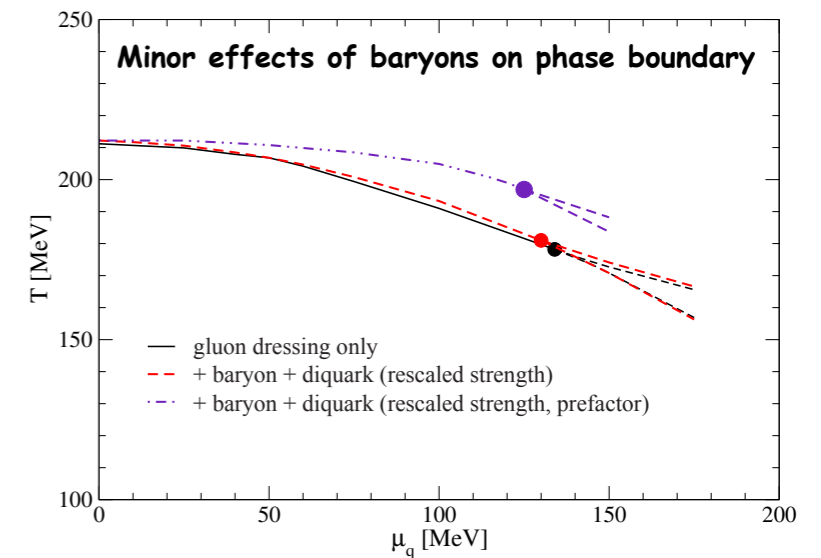


$\mu_B/T \lesssim 4$

Dominant channels I (fRG)
 Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



Dominant channels II (DSE)
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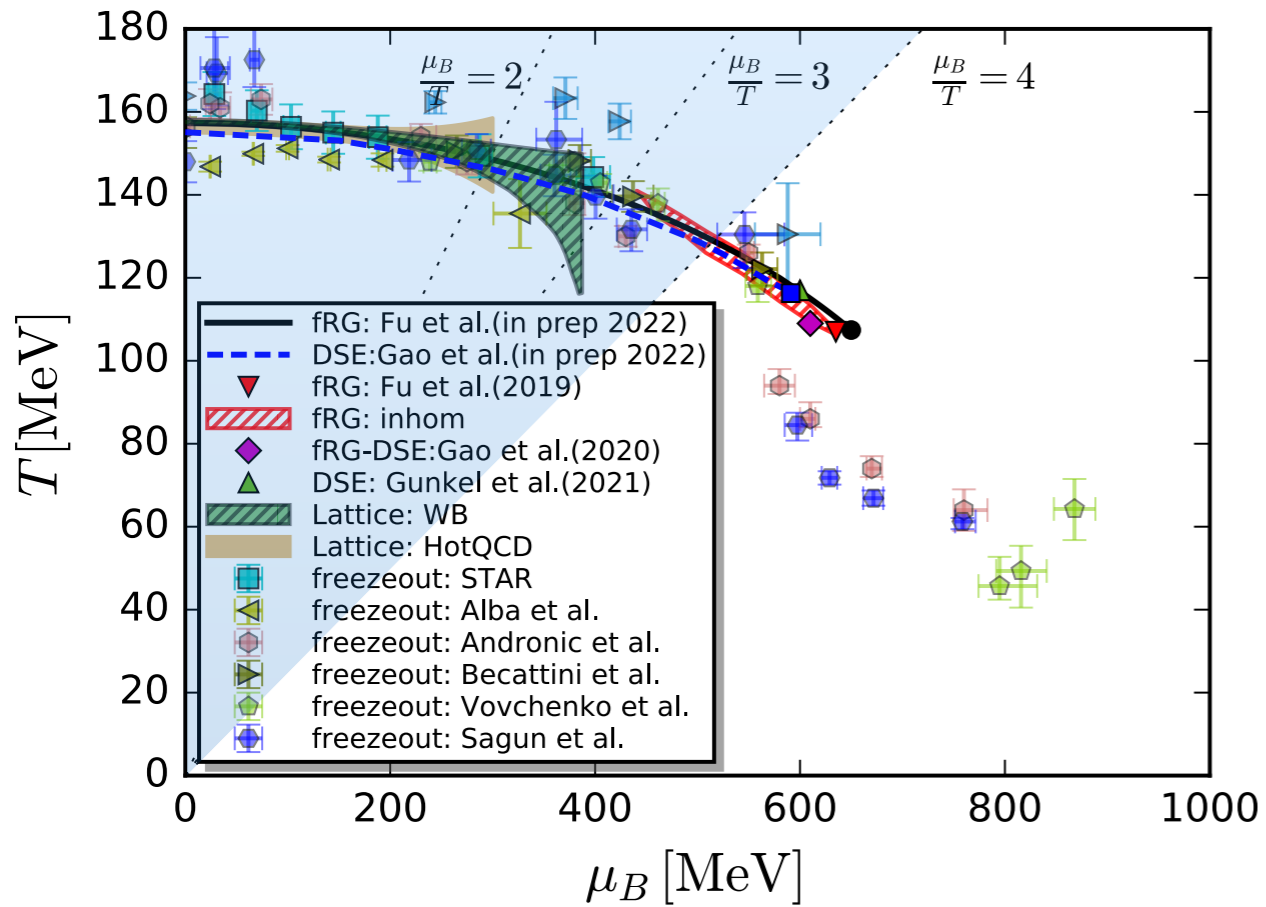
Moat: Pisarski, Rennecke, PRL 127 (2021) 152302

➔ Non-trivial background

I+II ➔ Fierz-complete computation

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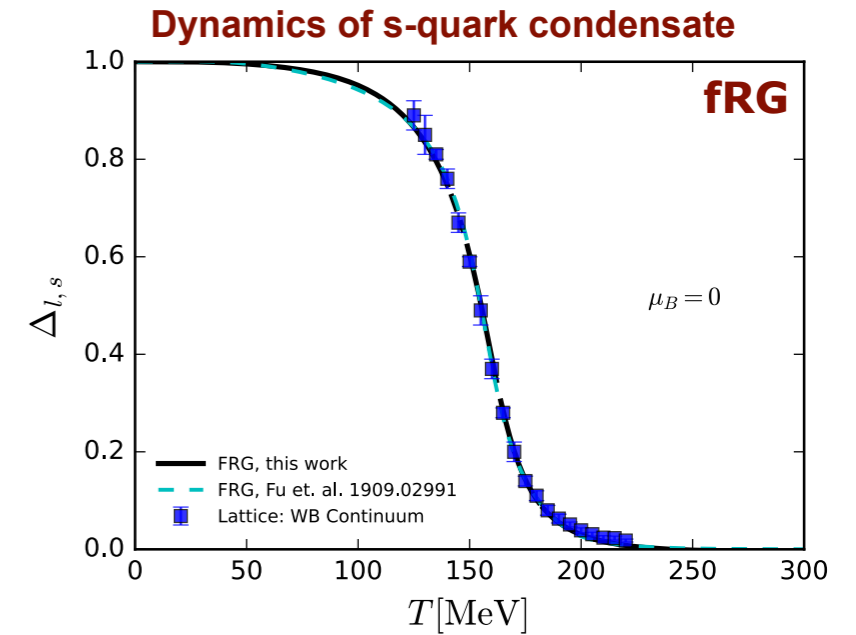
Improving the approximation & apparent convergence



Regime of reliability of current best truncation

fQCD matter sector approximation
-unique properties-

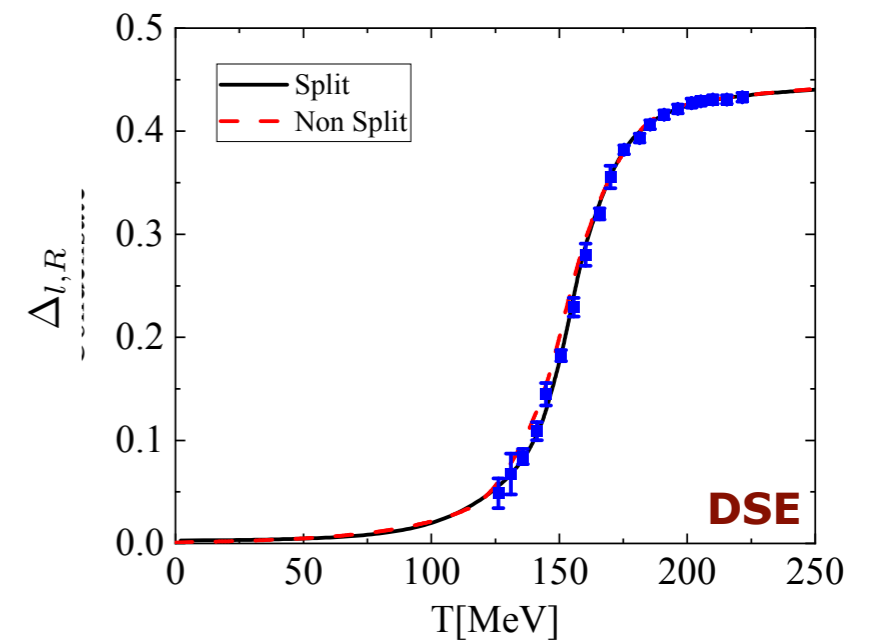
quark-gluon tensor structures (DSE full, fRG partial)
&
multi-scatterings of scalar-pseudoscalar channel/mesons (fRG)



Fu, JMP, Rennecke, Wen, Yin, in prep.

in preparation

Chromo-magnetic & electric vertex splittings



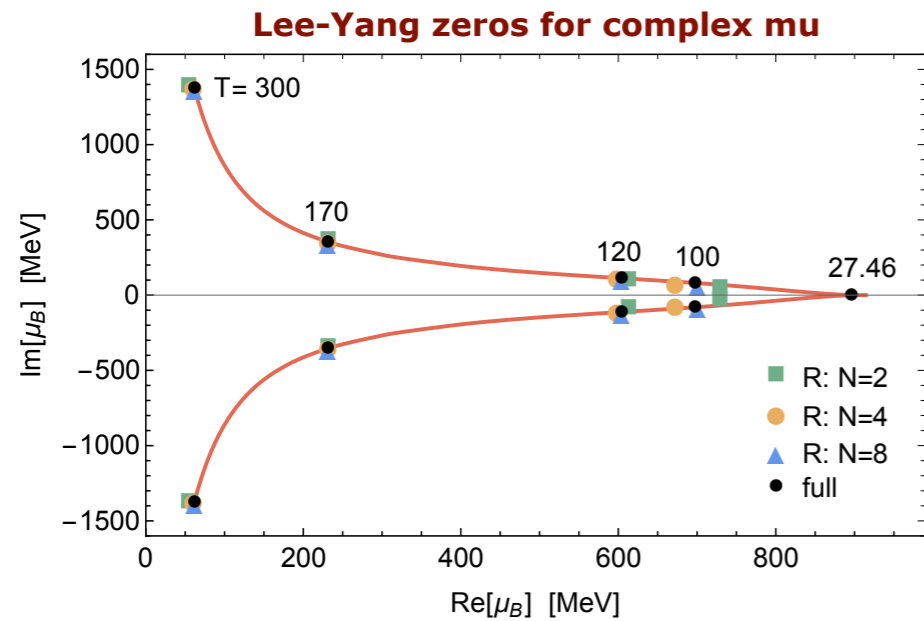
Gao, JMP, Schneider, in prep.

Systematic error control (CEP)

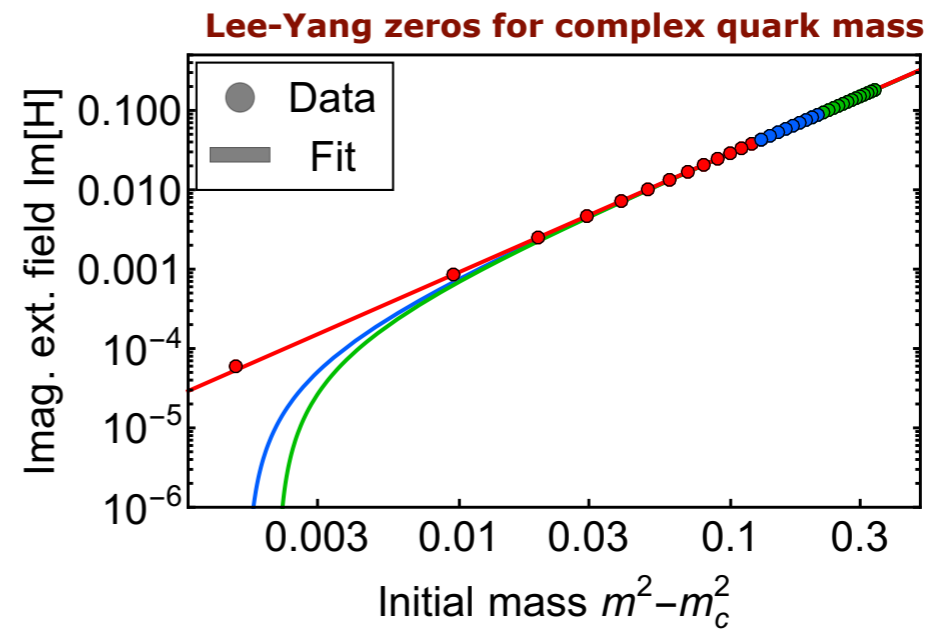
Lee-Yang zeros

Connelly, Johnson, Rennecke, Skokov, PRL 125 (2020) 191602

Idea: extrapolate to CEP & benchmark with lattice



Mukherjee, Rennecke, Skokov, PRD 105 (2022) 014026



Ihssen, JMP, 2207.10057

Mukherjee, Rennecke, Skokov, PRD 105 (2022) 014026

Rennecke, Skokov, AP 444 (2022) 169010

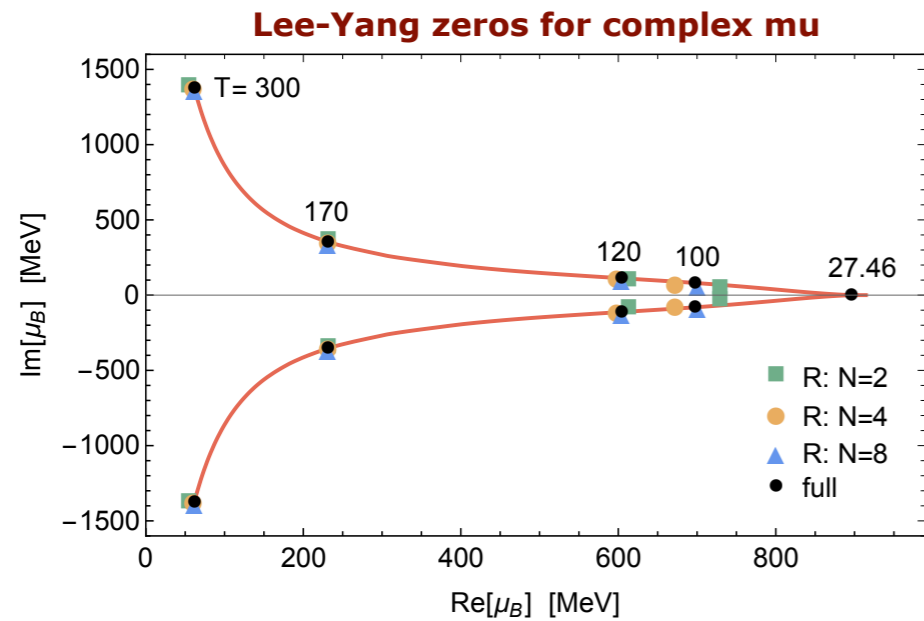
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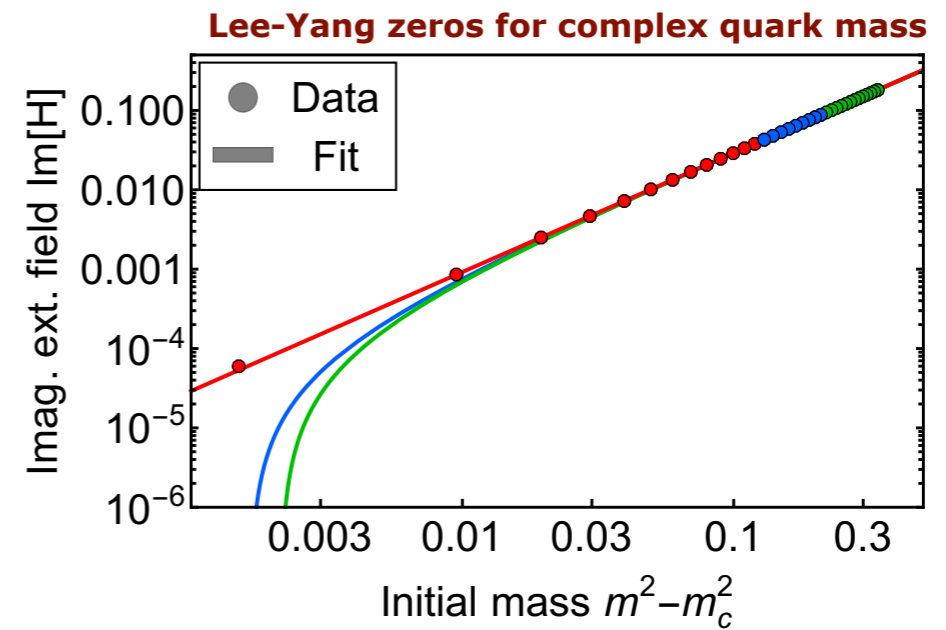
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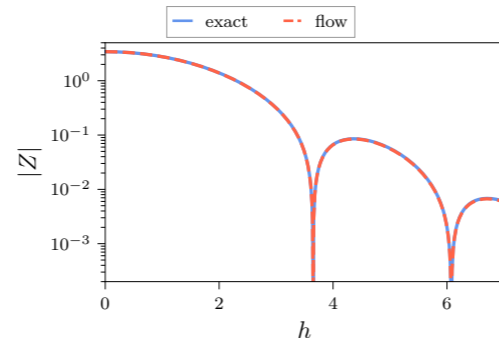
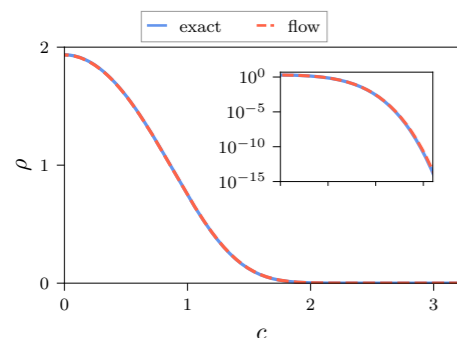


Ihssen, JMP, 2207.10057

Direct (own) computations on the lattice

Complex Langevin: Attanasio, Bauer, Kades, JMP, PoS LATTICE2021 (2022) 223

Normalising flows & DoS: JMP, Urban, 2203.01243



Mukherjee, Rennecke, Skokov, PRD 105 (2022) 014026

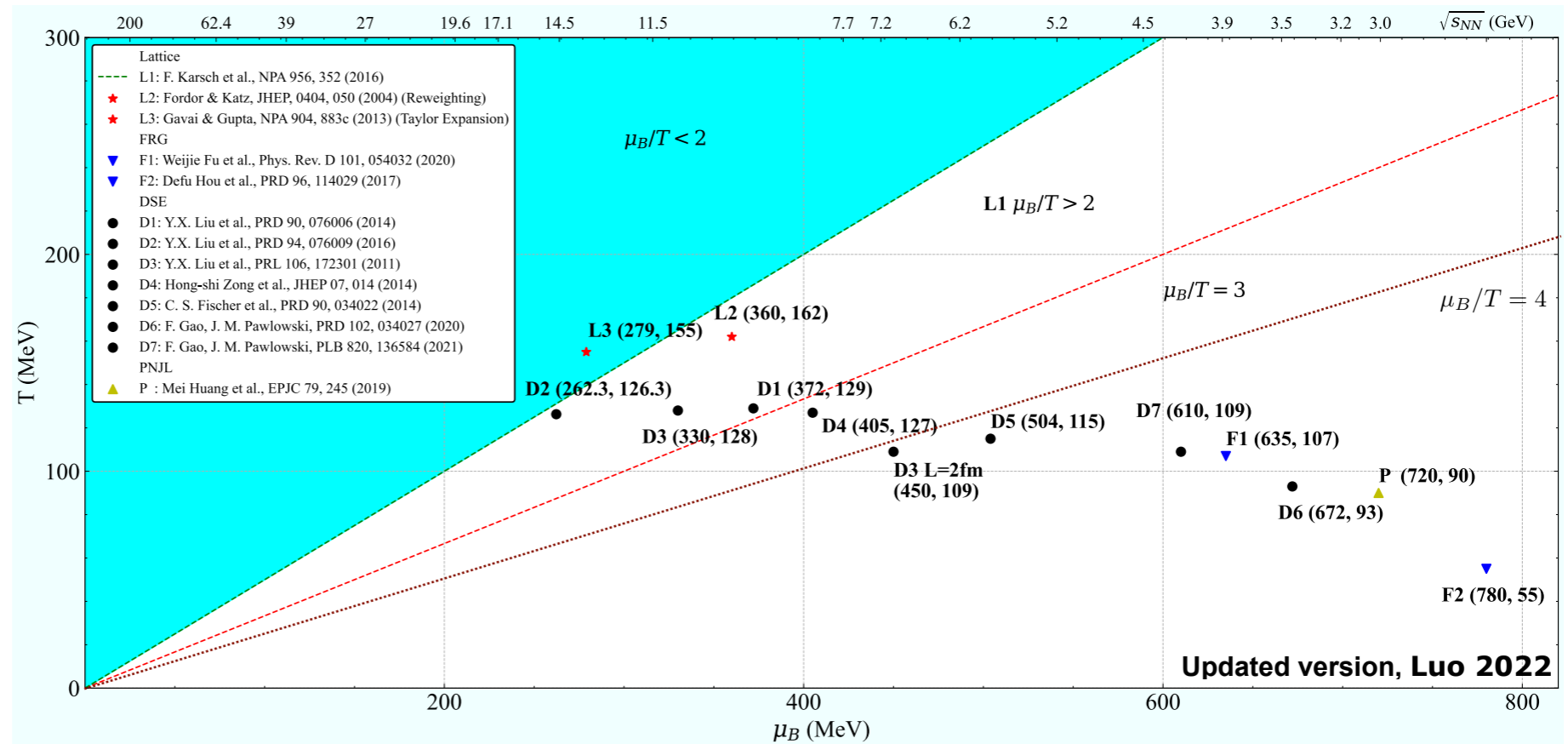
Rennecke, Skokov, AP 444 (2022) 169010

Ihssen, JMP, 2207.10057



Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

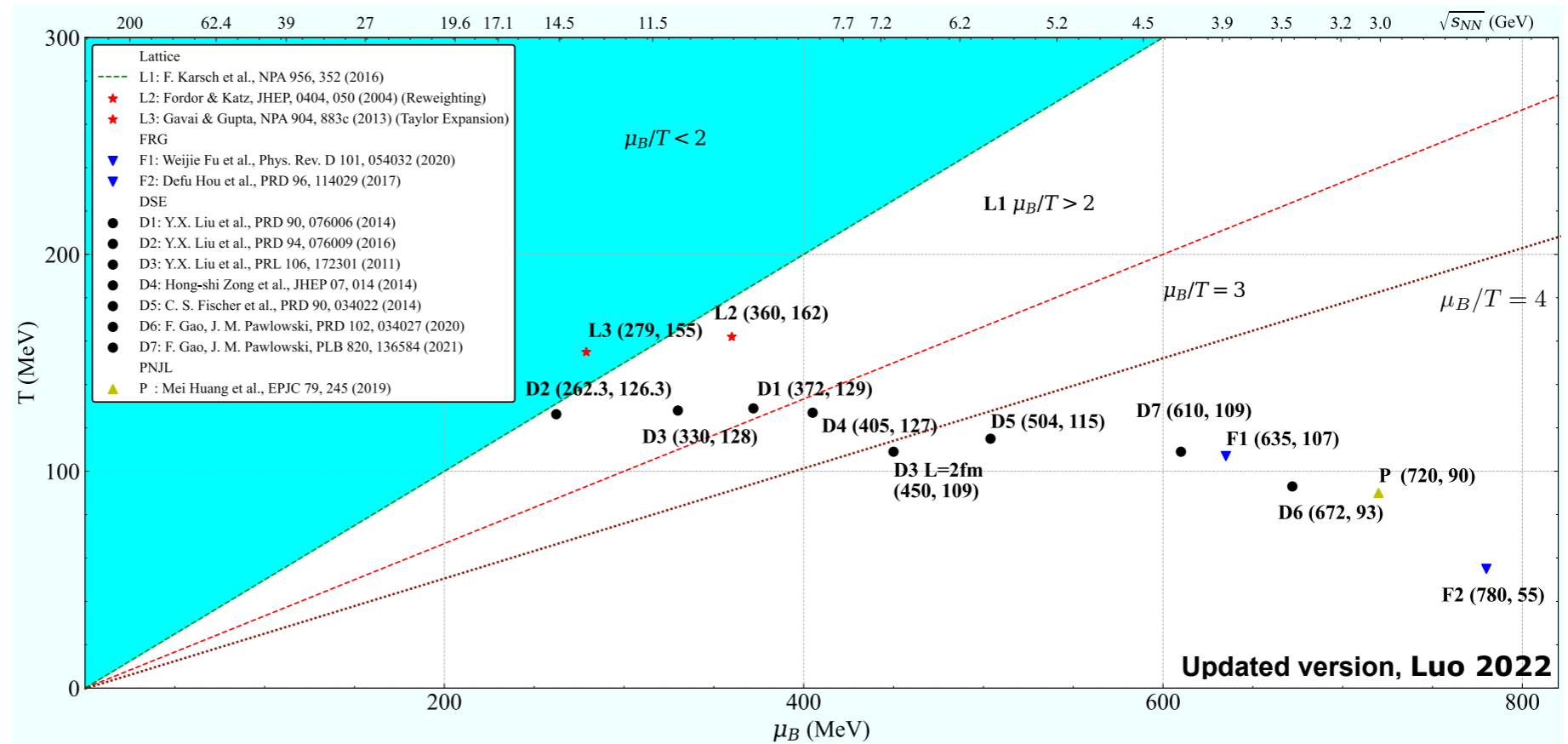
Lack of predictive power for CEP-predictions is no quality measure!

CEP is standing for 'regime with new physics'



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Common folklore since ~2004



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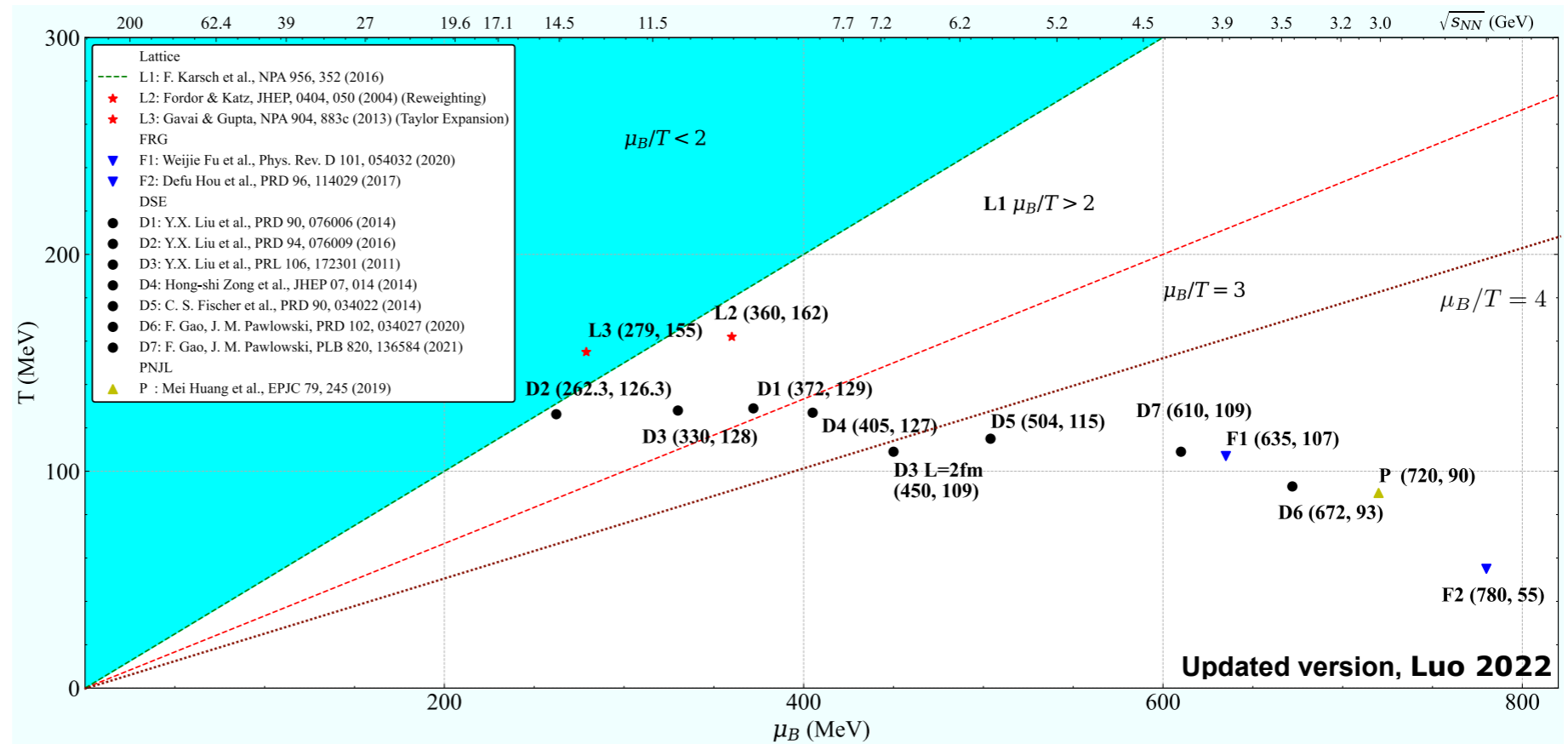
Lack of predictive power for CEP-predictions is no quality measure!

CEP is standing for 'regime with new physics'



Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

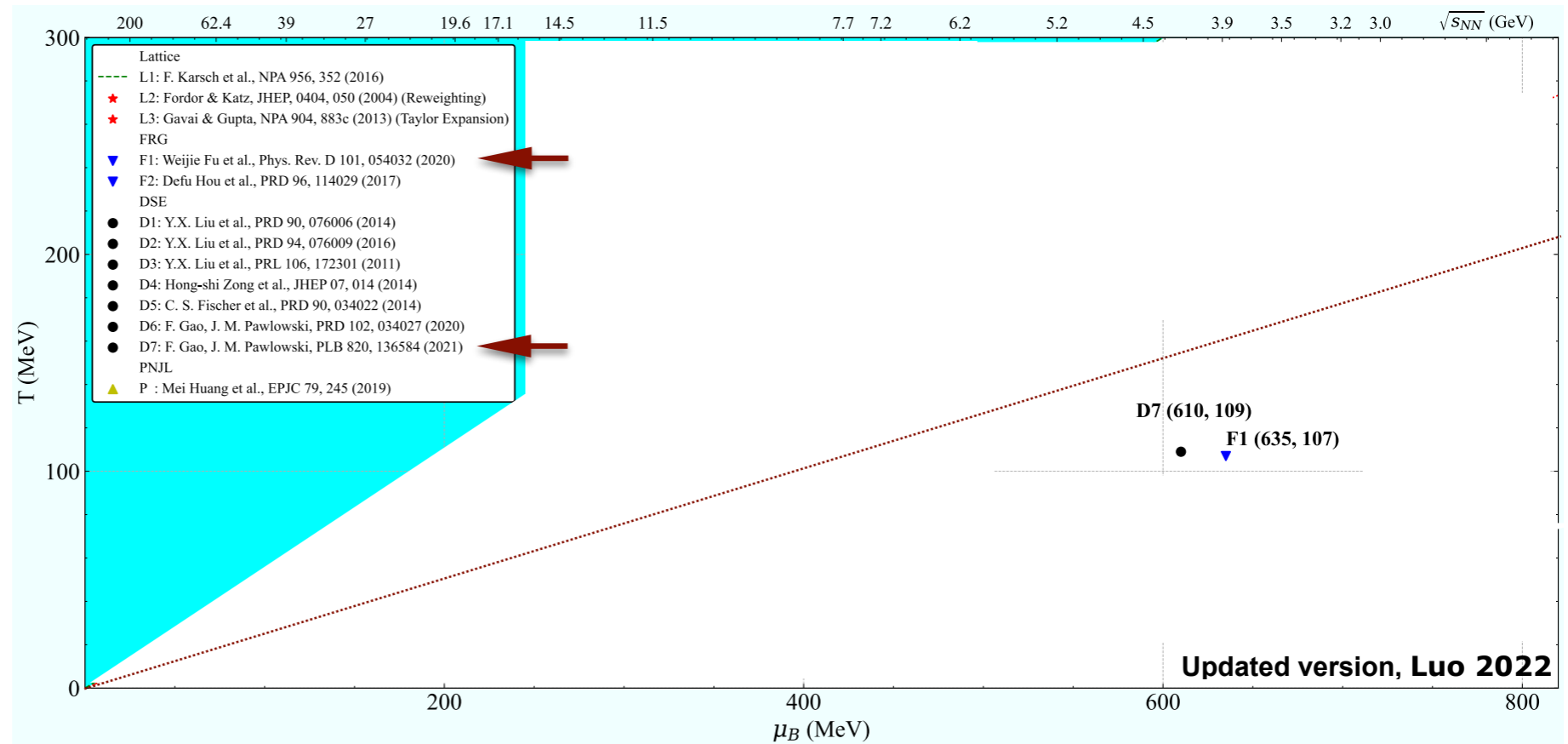
Remove CEP-predictions

- (i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)
- (ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at $\mu_B = 0$
- (iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)



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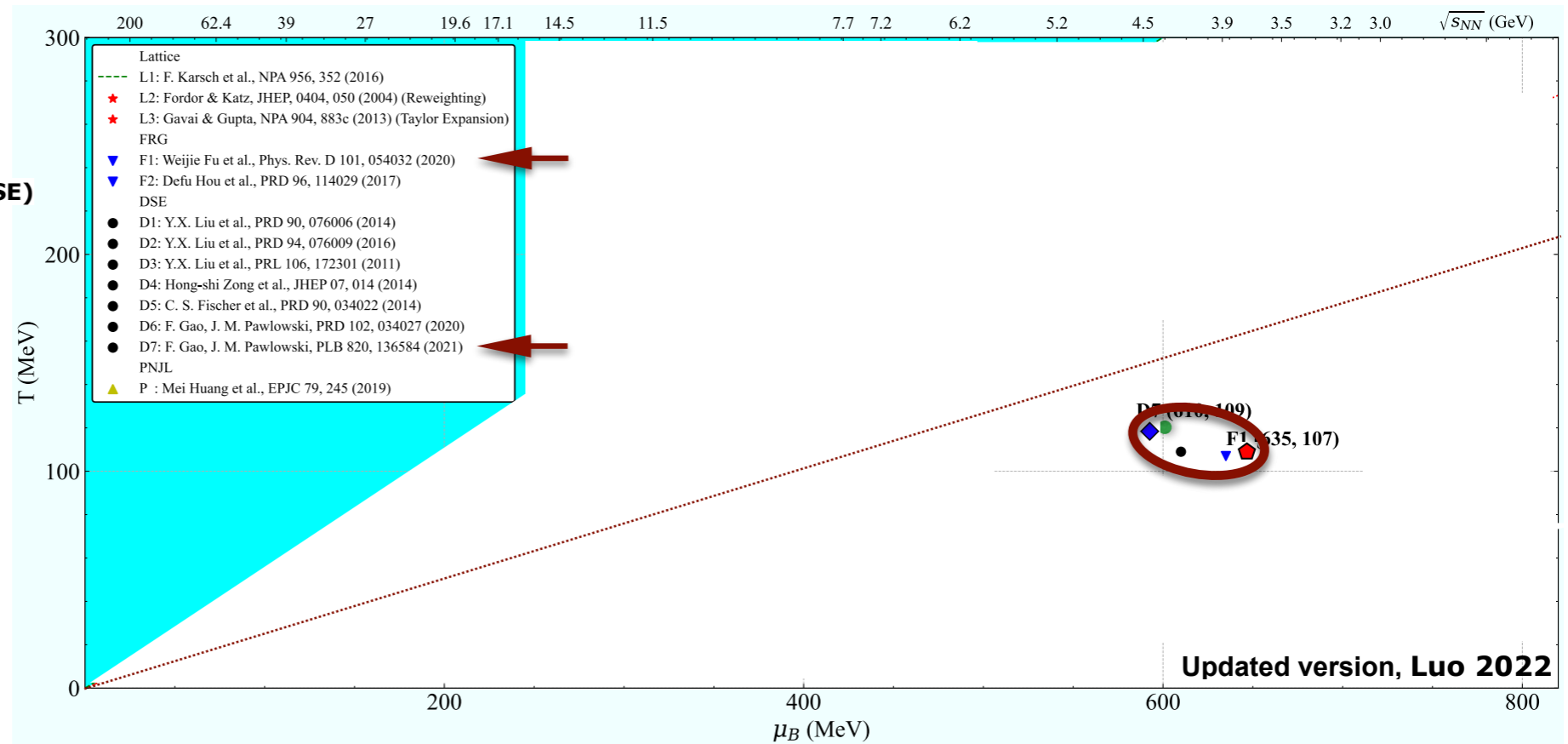
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- ◆ Gao, JMP, Schneider, in prep (DSE)
- ◆ Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)
- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)



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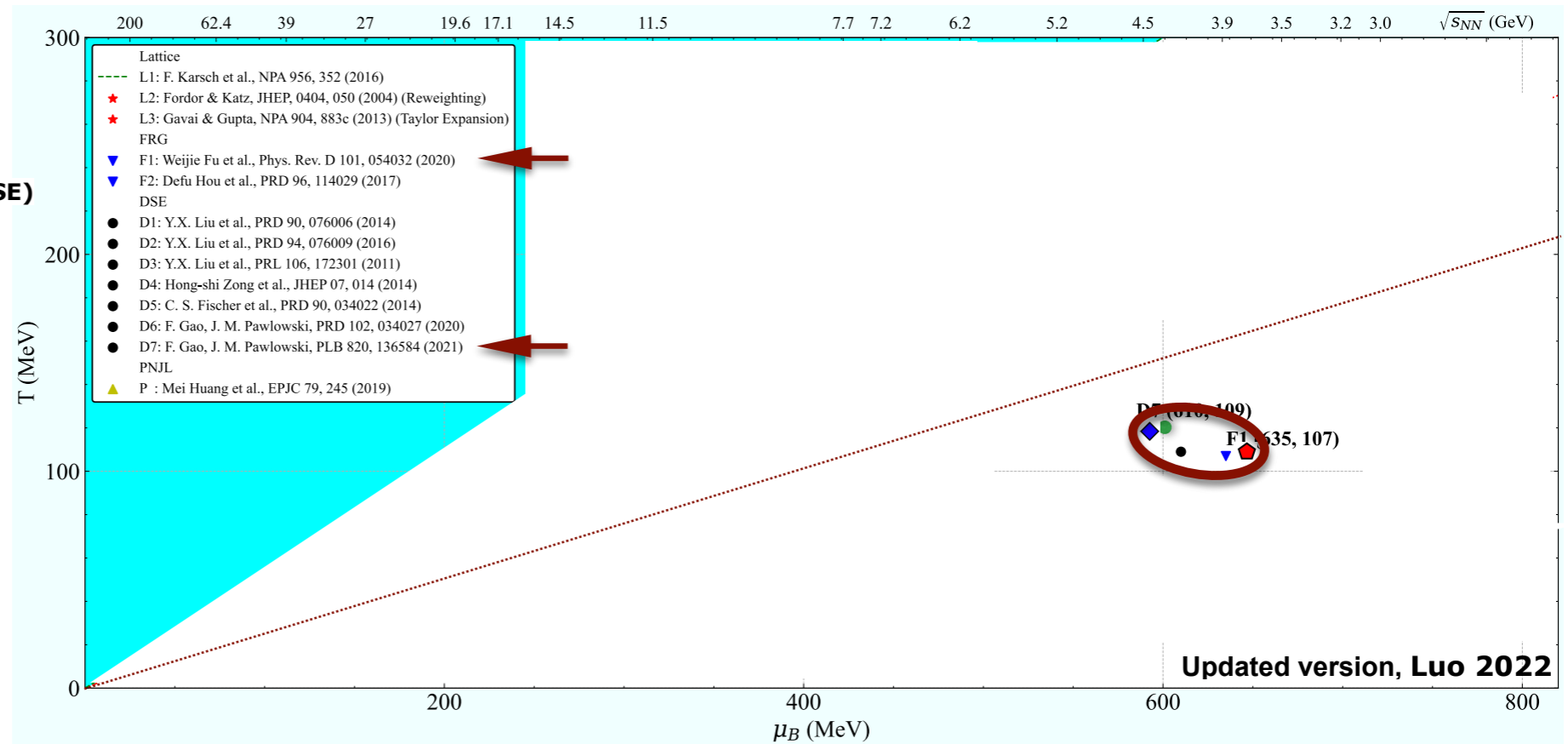
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Still uncertainties for the estimation of CP location.

Remove CEP-predictions

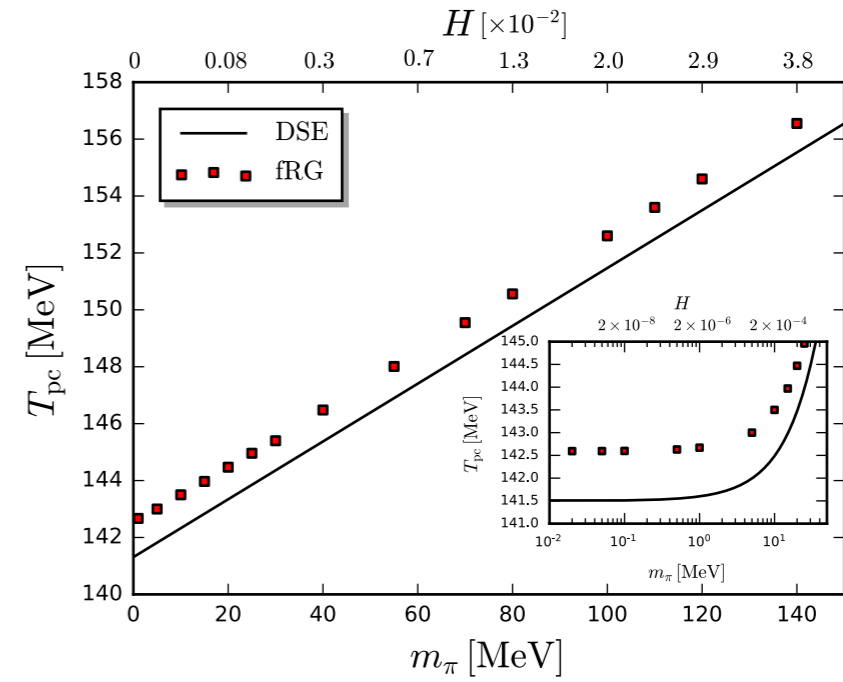
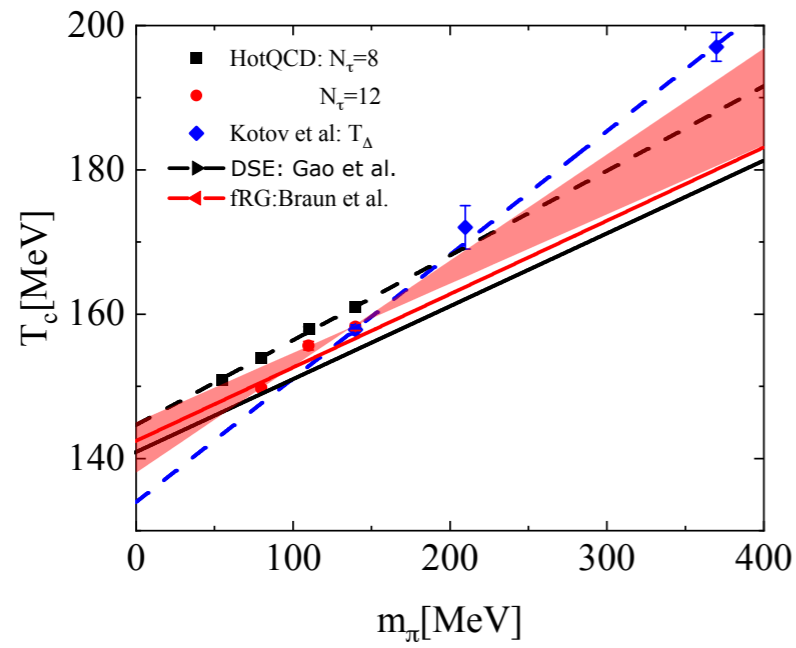
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Outline

- QCD phase structure: Where do we stand?
- QCD phase structure: observables, estimates & systematics
- Chiral dynamics & phenomenological applications
- Summary & outlook

To be (critical) or not (to be)

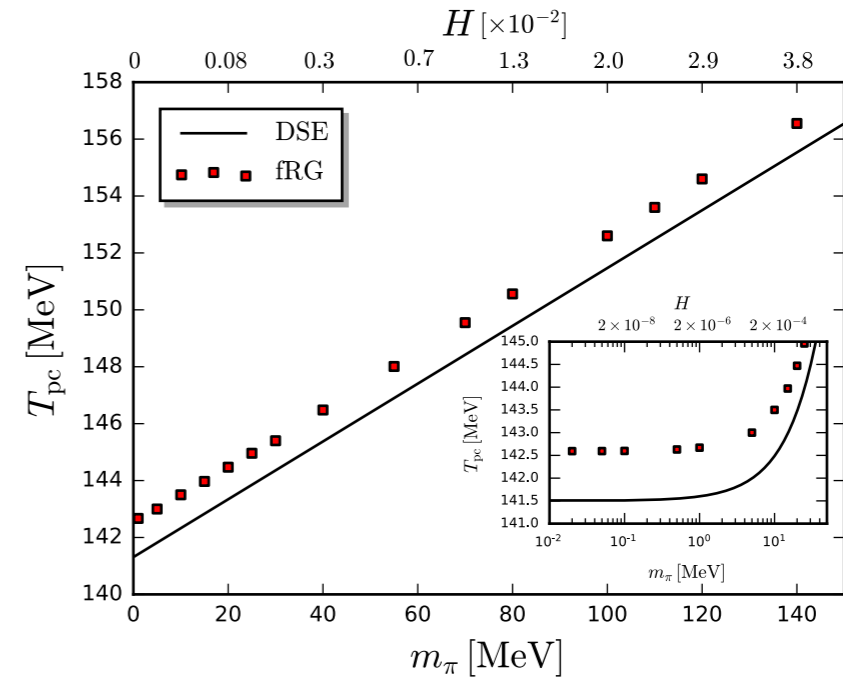
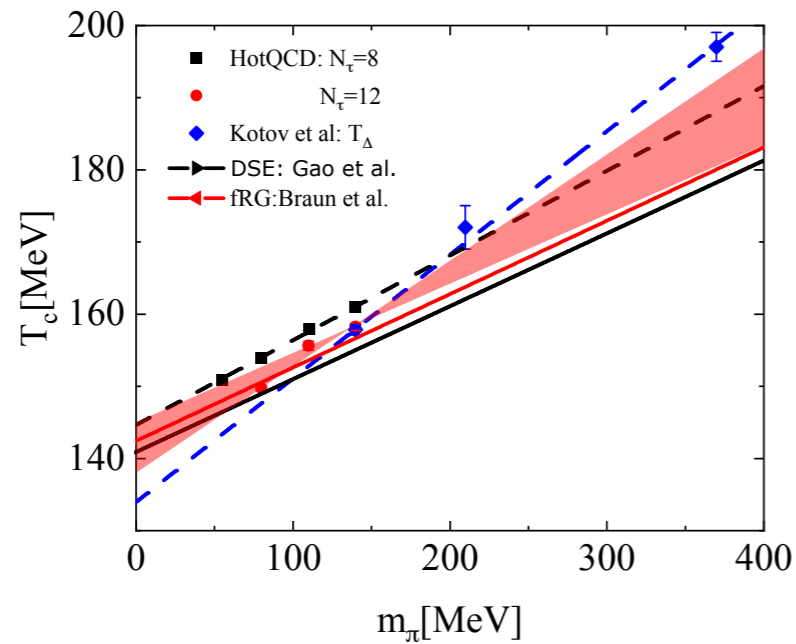
Chiral transition temperature



fQCD collaboration, in preparation

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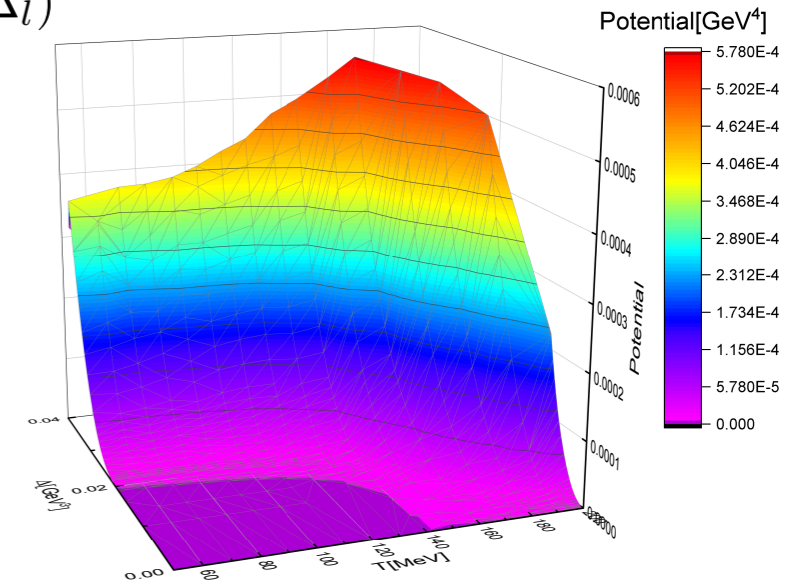
$$V_\chi^{(fRG)} \approx V_\chi^{(DSE)}$$

Order parameter potential & scaling

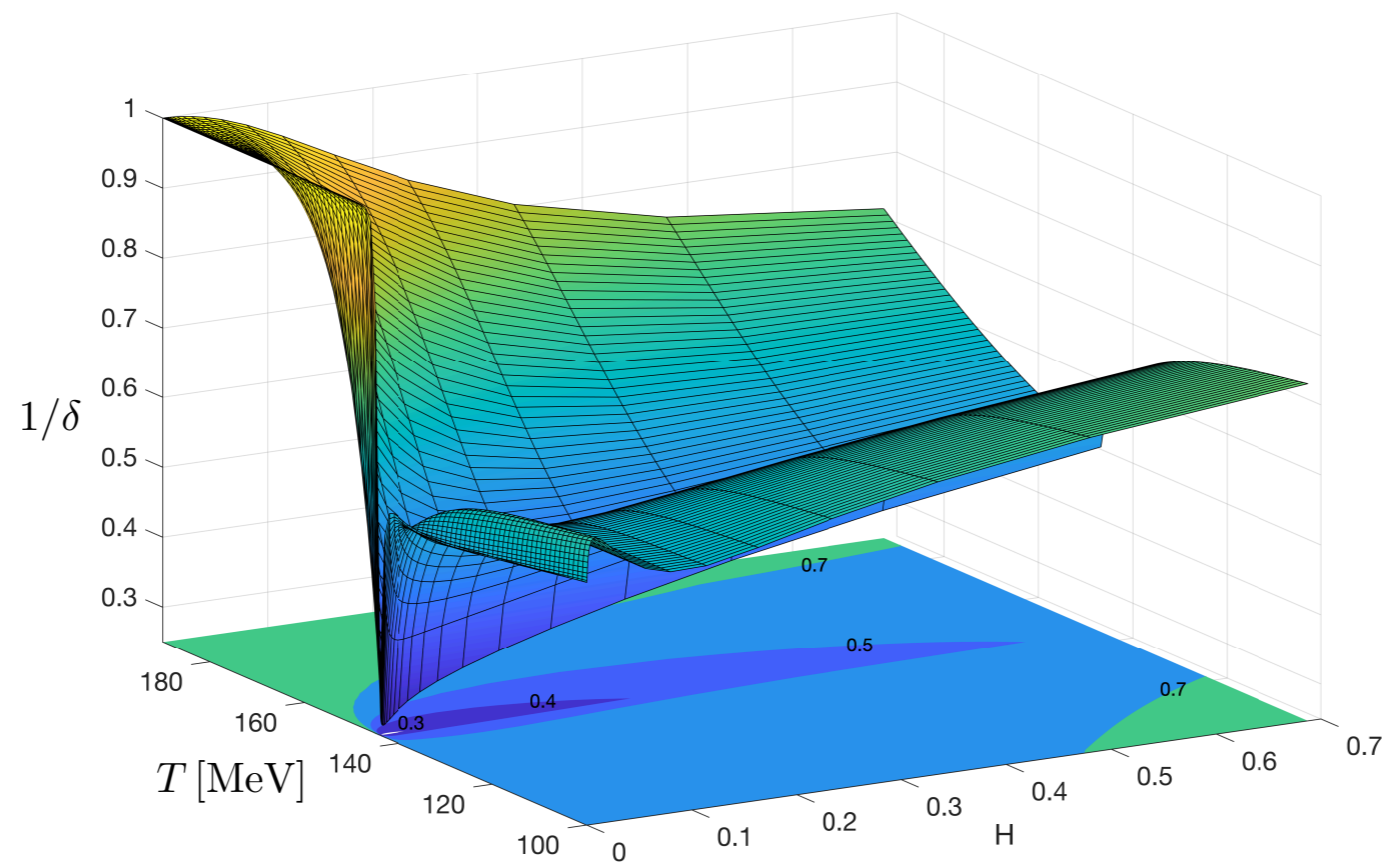
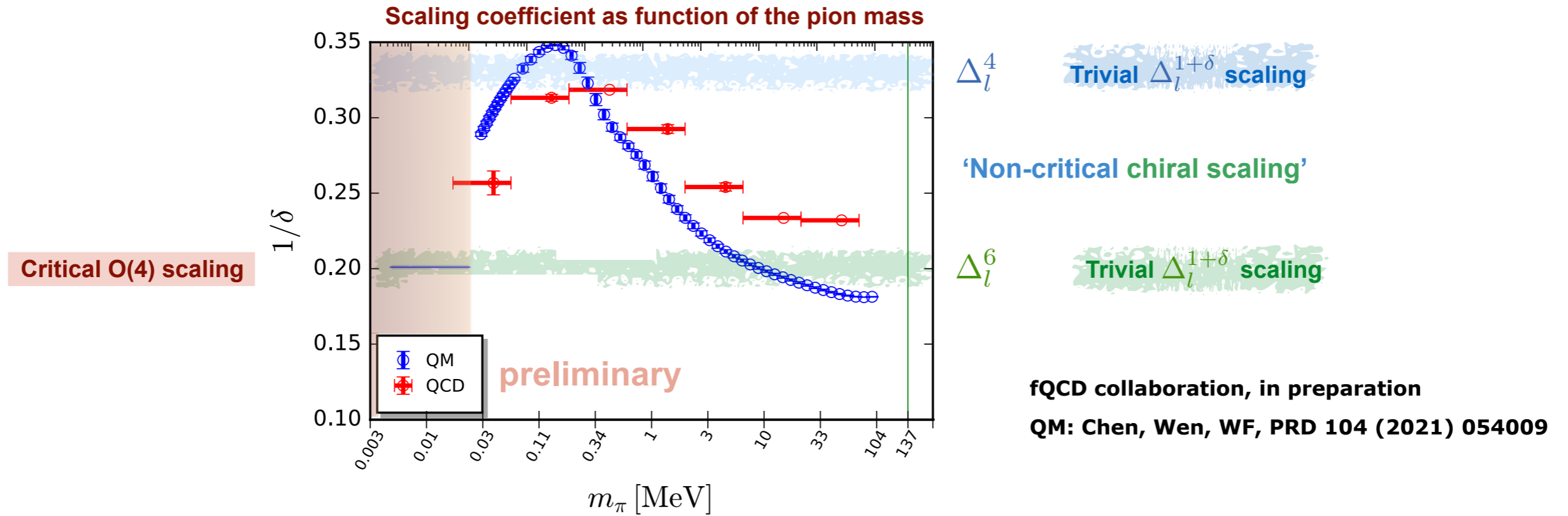
$$V_\chi \approx \Delta_l^n \iff \Delta_l(H) \propto H^{\frac{1}{n-1}}$$

$$H = \frac{m_l}{m_s}$$

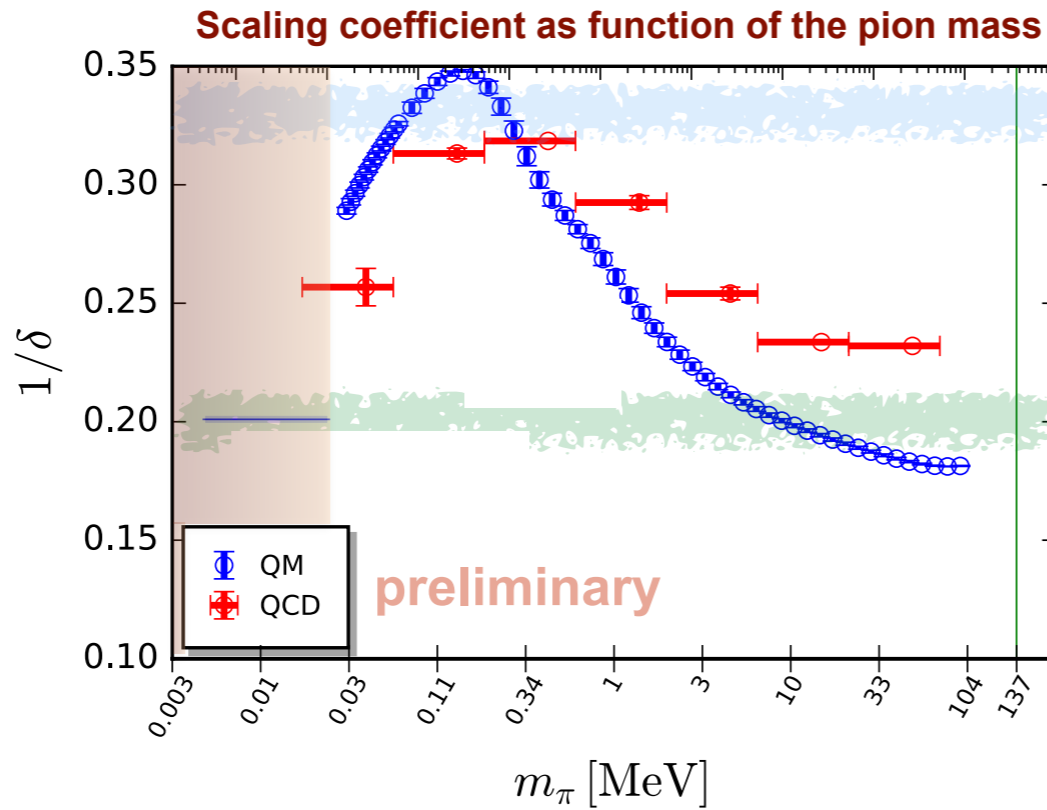
$$V_\chi(\Delta_l)$$



Chiral dynamics & quasi-massless modes



Chiral dynamics & quasi-massless modes



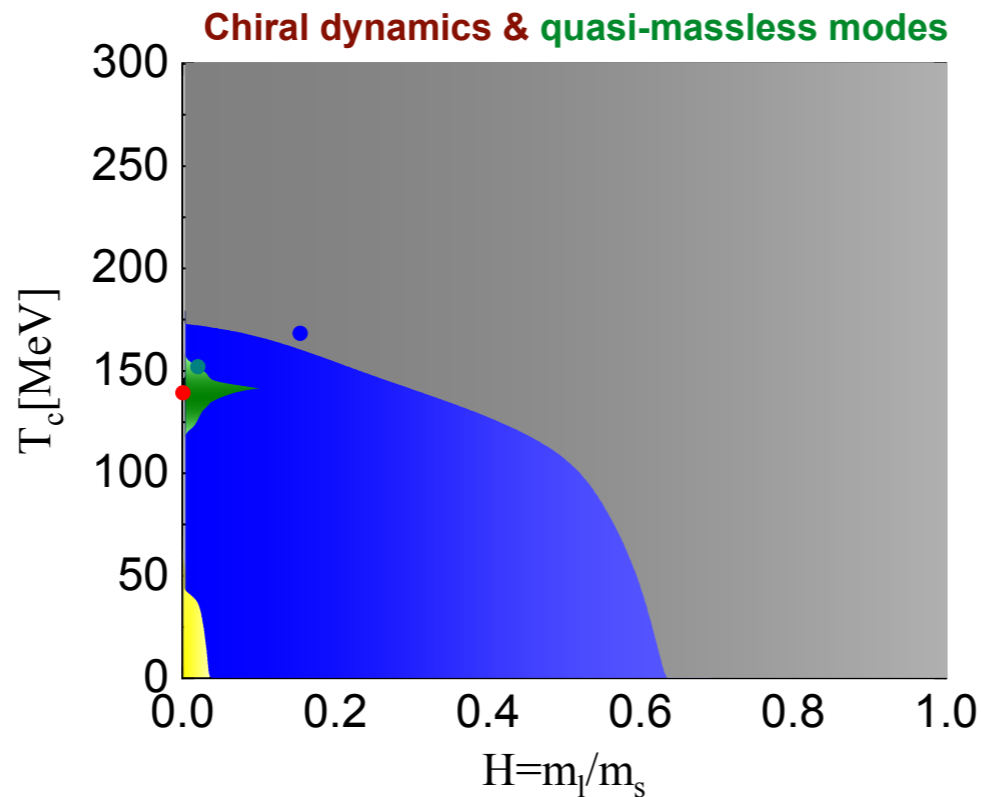
Δ_l^4 Trivial $\Delta_l^{1+\delta}$ scaling

'Non-critical chiral scaling'

Δ_l^6 Trivial $\Delta_l^{1+\delta}$ scaling

fQCD collaboration, in preparation

Critical O(4) scaling



'Non-critical chiral scaling'

Far away from the critical regime for $m_\pi \gtrsim 1 - 10$ MeV

$$\Delta_l(T, H) \approx \Delta_{l,\chi}(0) \left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H \right)$$



$$V_\chi(\Delta_l) \propto$$

$$\Delta_l^6$$

$$\Delta_l^4$$

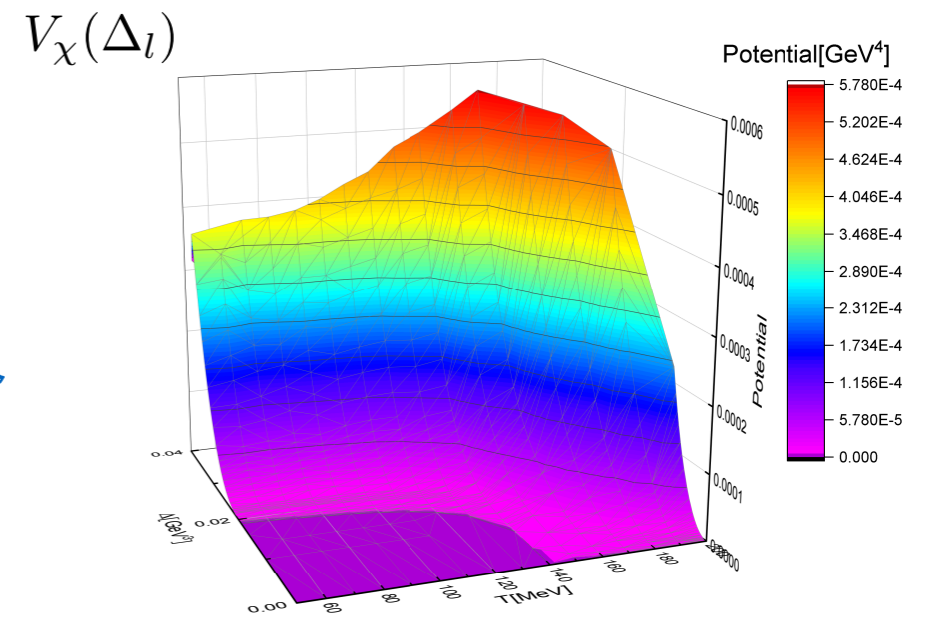
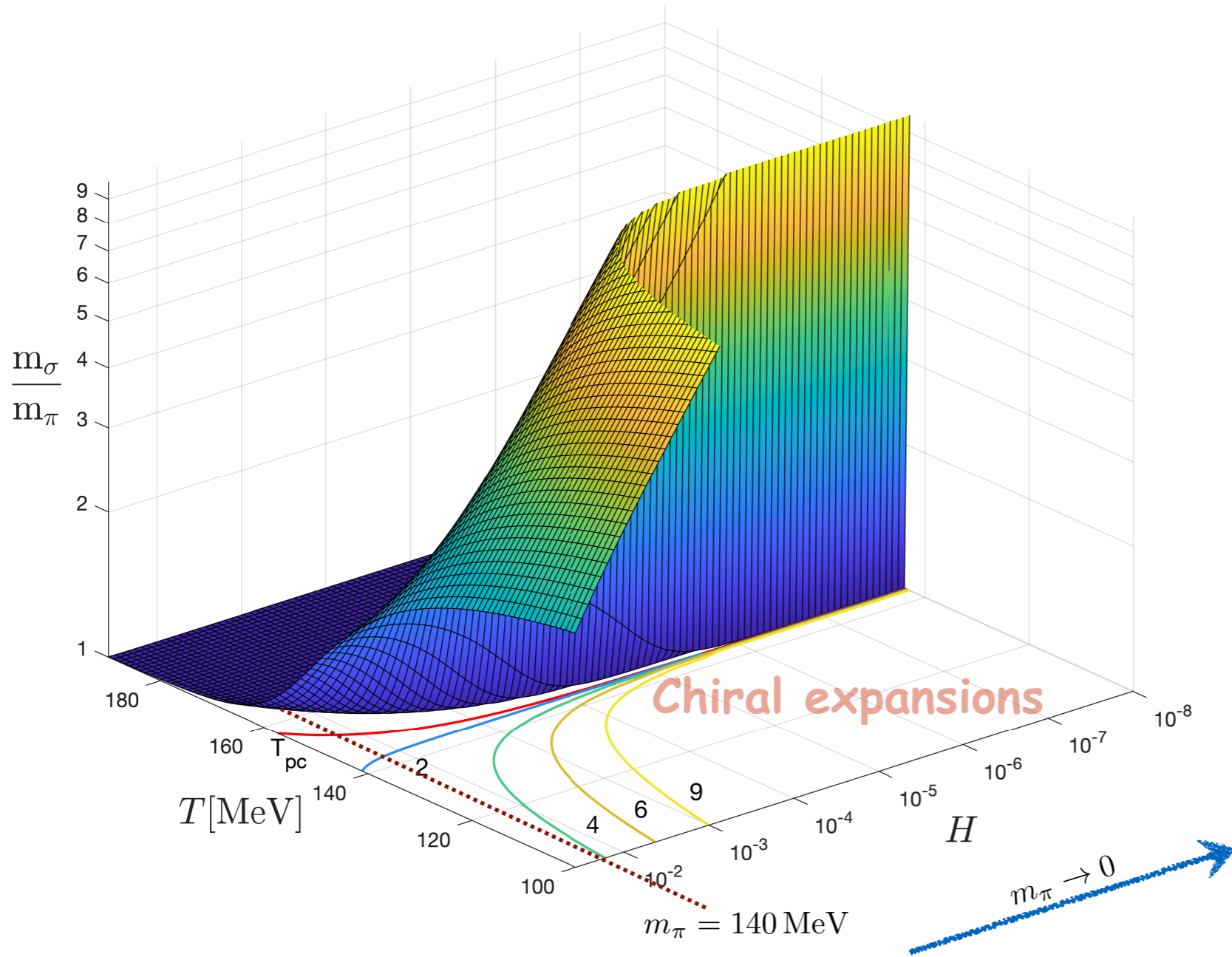
$$\Delta_l^2$$

Critical scaling



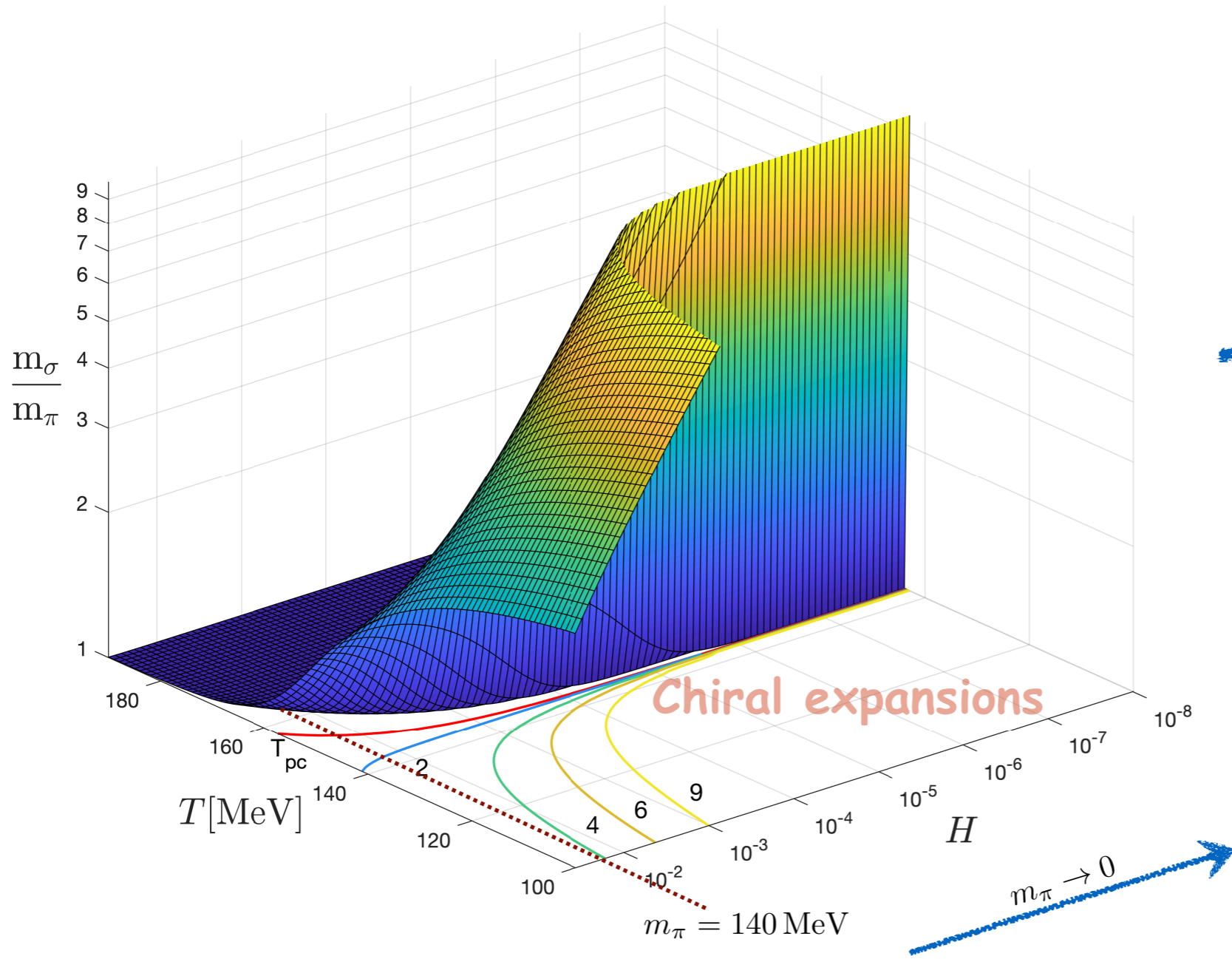
Chiral dynamics & quasi-massless modes

Measure: correlation length



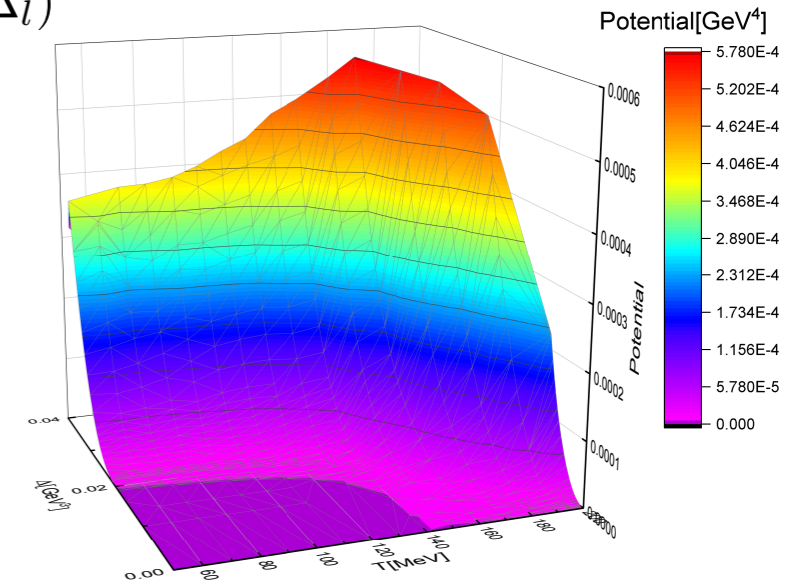
Chiral dynamics & quasi-massless modes

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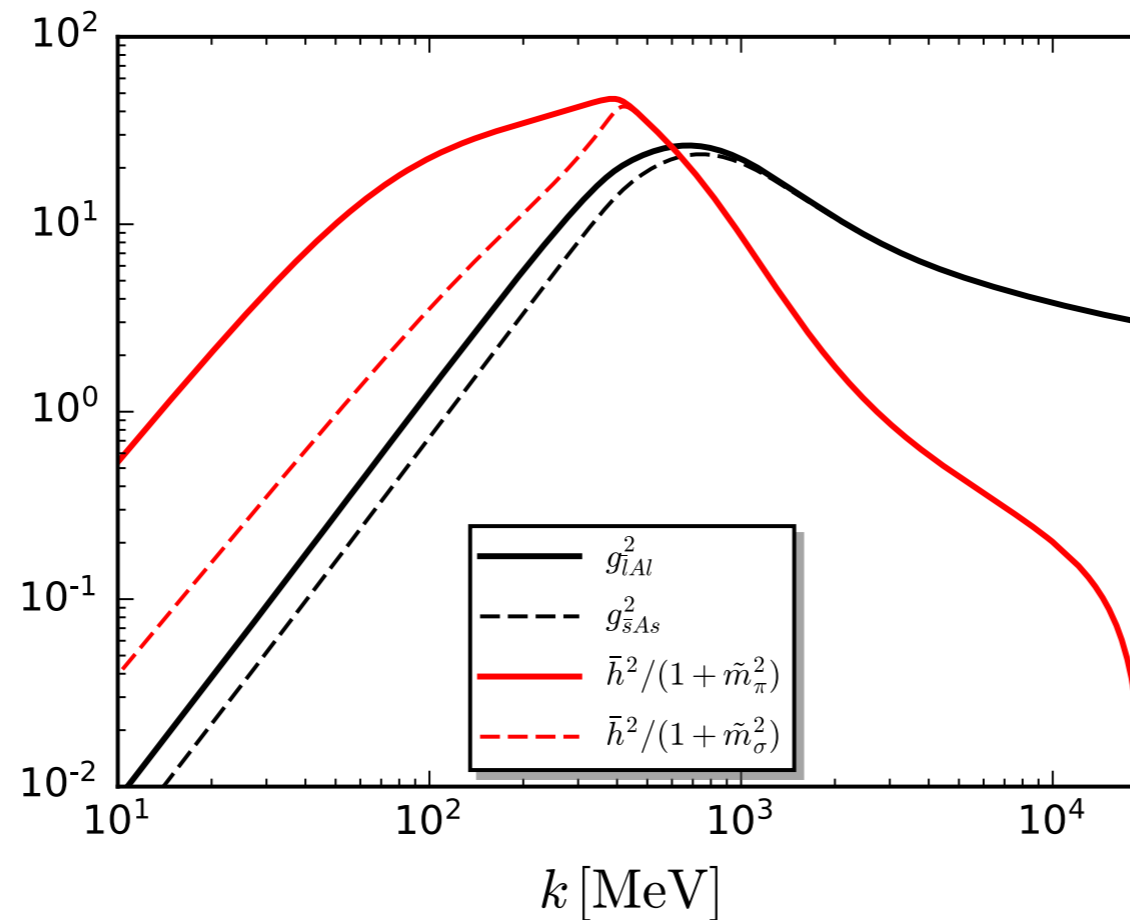
Use for chiral dynamics in heavy ion collisions

$V_\chi(\Delta_l)$



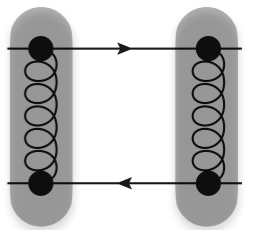
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange loop} - \text{dashed loop} - \text{solid loop} \right) + \frac{1}{2} \left(\text{blue loop} \right)$$

Sequential decoupling of gluon, quark, sigma, pion fluctuations



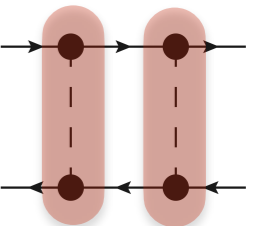
Fu, JMP, Rennecke, PRD 101, (2020) 054032

$$\frac{g_{IA}^2}{g_{sAs}^2}$$



$$\frac{\bar{h}^2}{1+m_\pi^2}$$

$$\frac{\bar{h}^2}{1+m_\sigma^2}$$



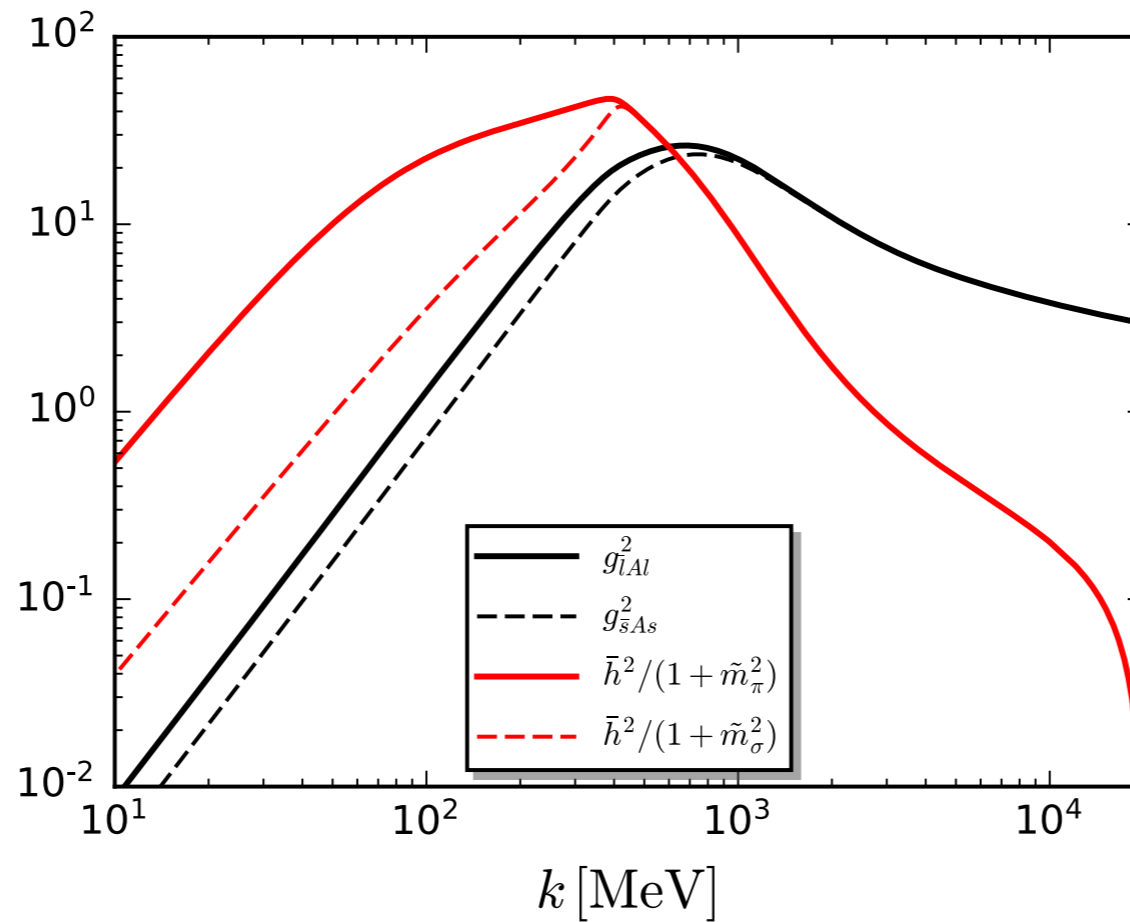
Based on:

Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

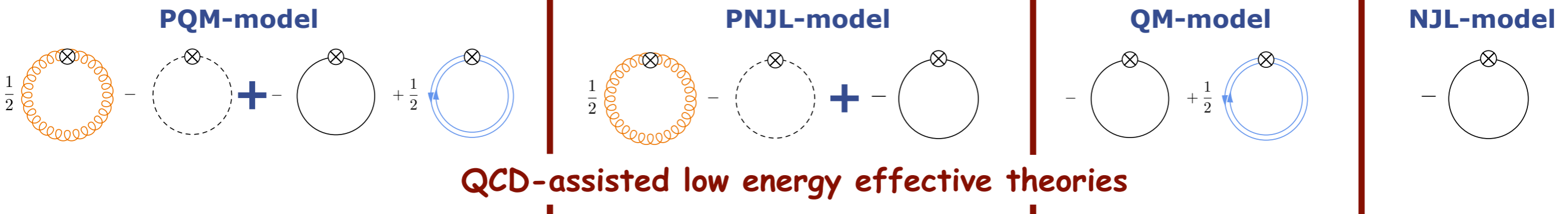
Rennecke, PRD 92 (2015) 076012

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Fu, JMP, Rennecke, PRD 101, (2020) 054032

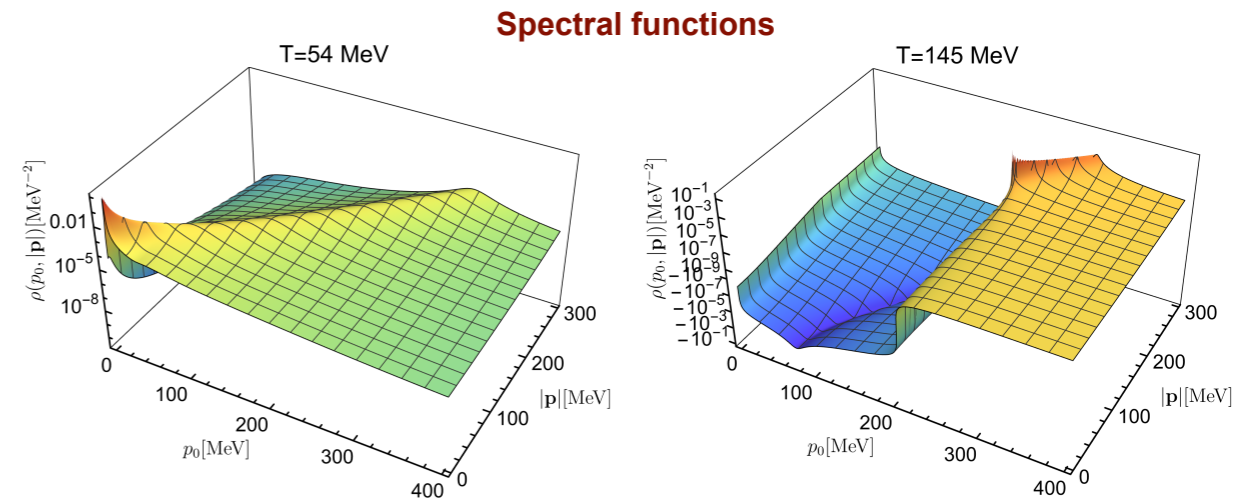
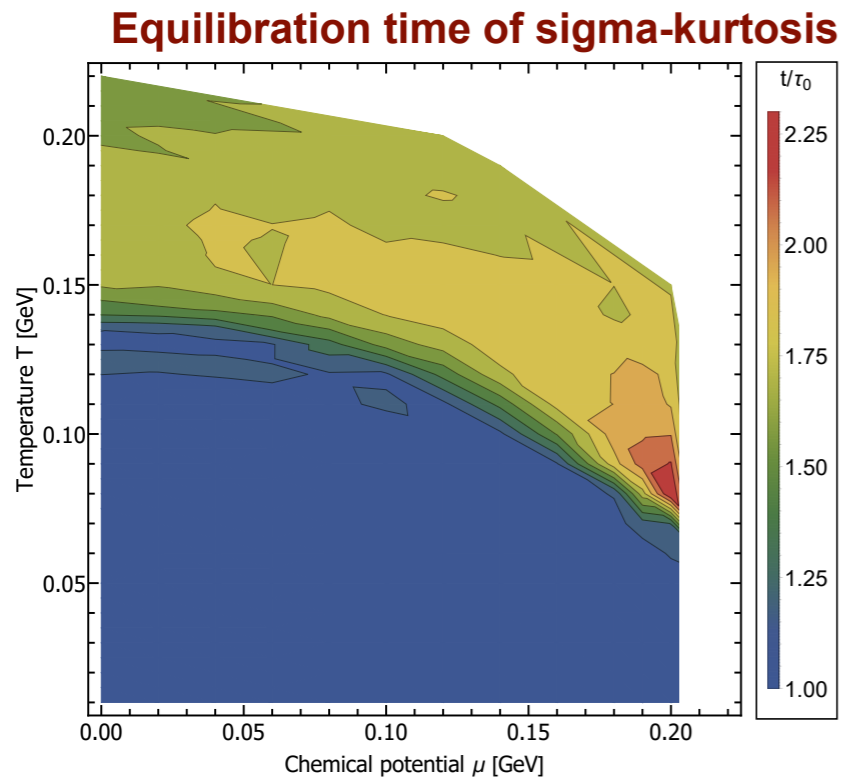


The need for dynamics

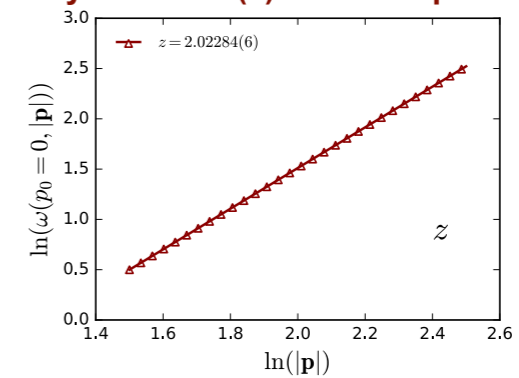
Showcases in linear sigma models

Transport with fRG spectral functions & potential

Dynamical universality



Dynamical O(4) critical exponent



$$\omega \propto (\vec{p}^2)^{\frac{z}{2}}$$

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871

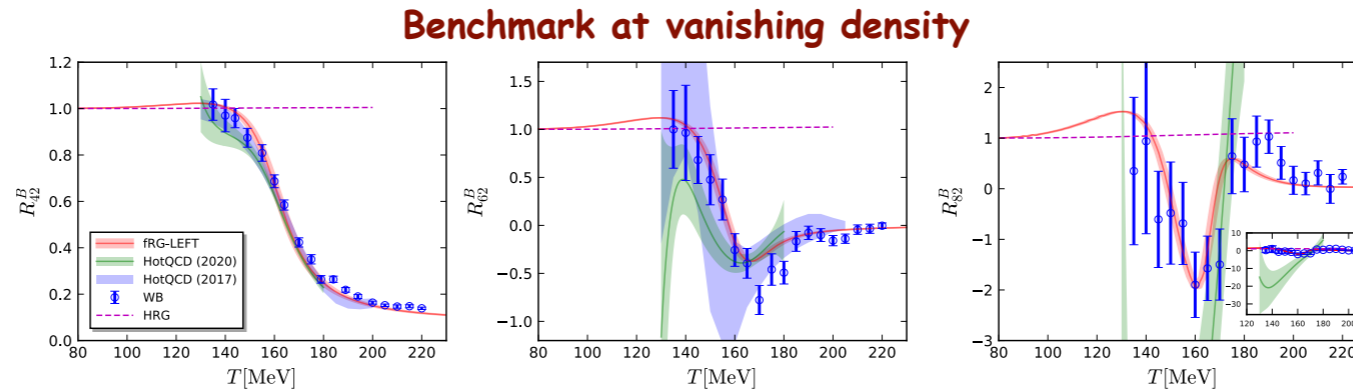
Tan, Chen, Fu, SciPost Phys. 12 (2022) 026

Model A ... H with the fRG, see talk of Johannes Roth (work in preparation)

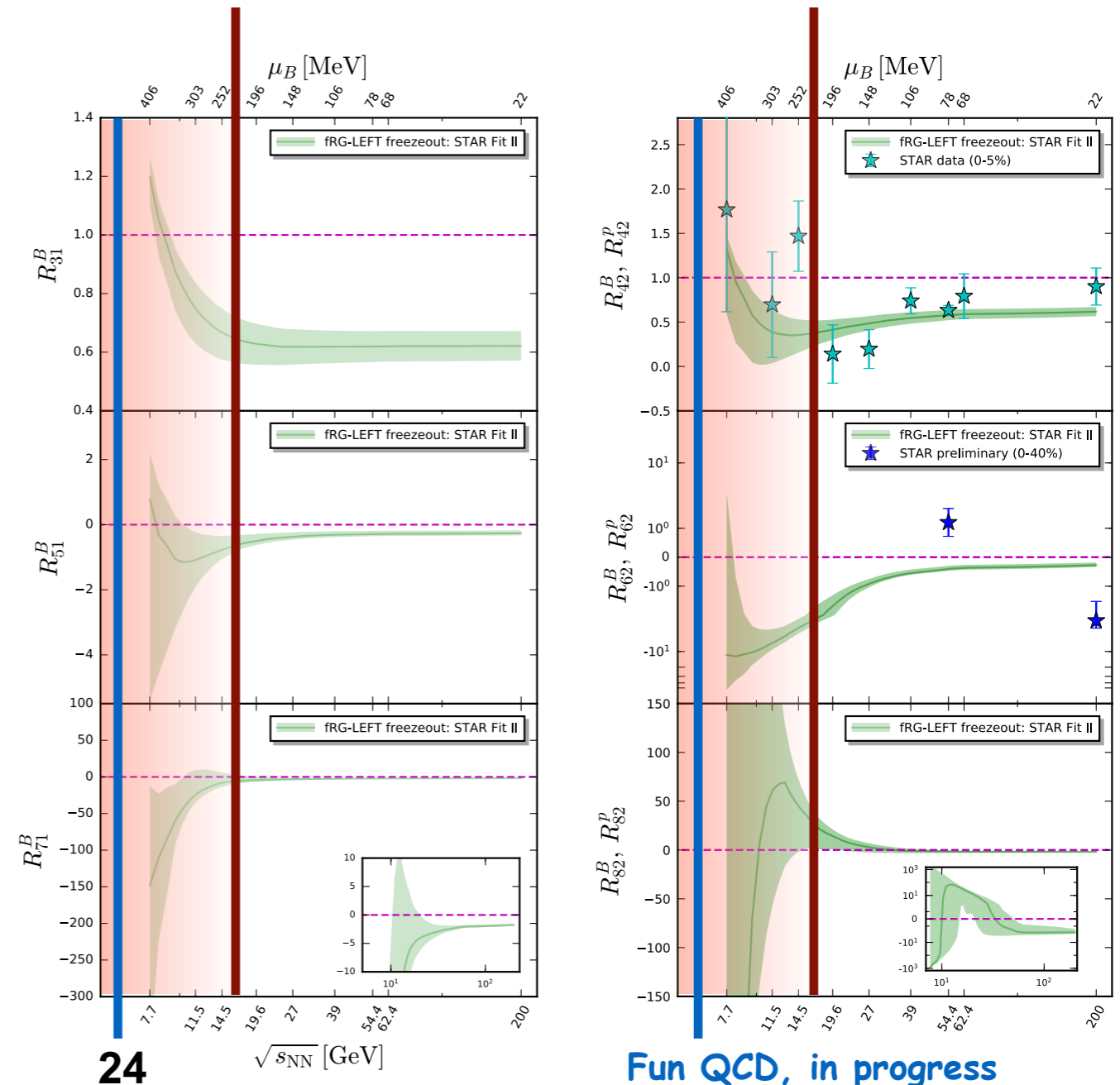
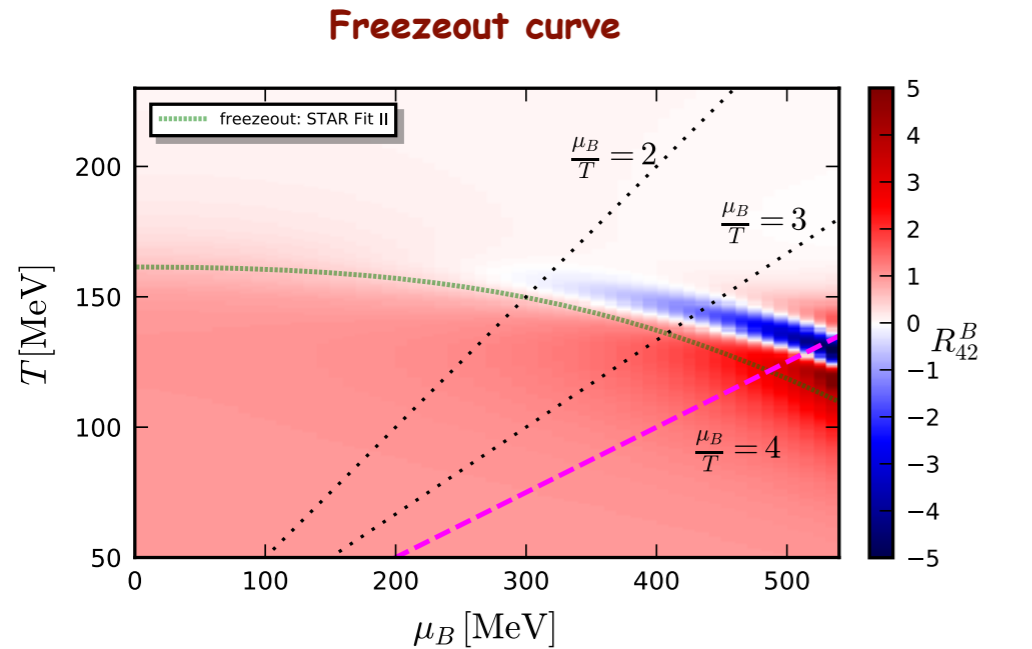
QM: Roth, Schweitzer, Rieke, von Smekal, PRD 105 (2022) 116017

Fluctuations of conserved charges

QCD-assisted LEFT



QCD-assisted LEFT



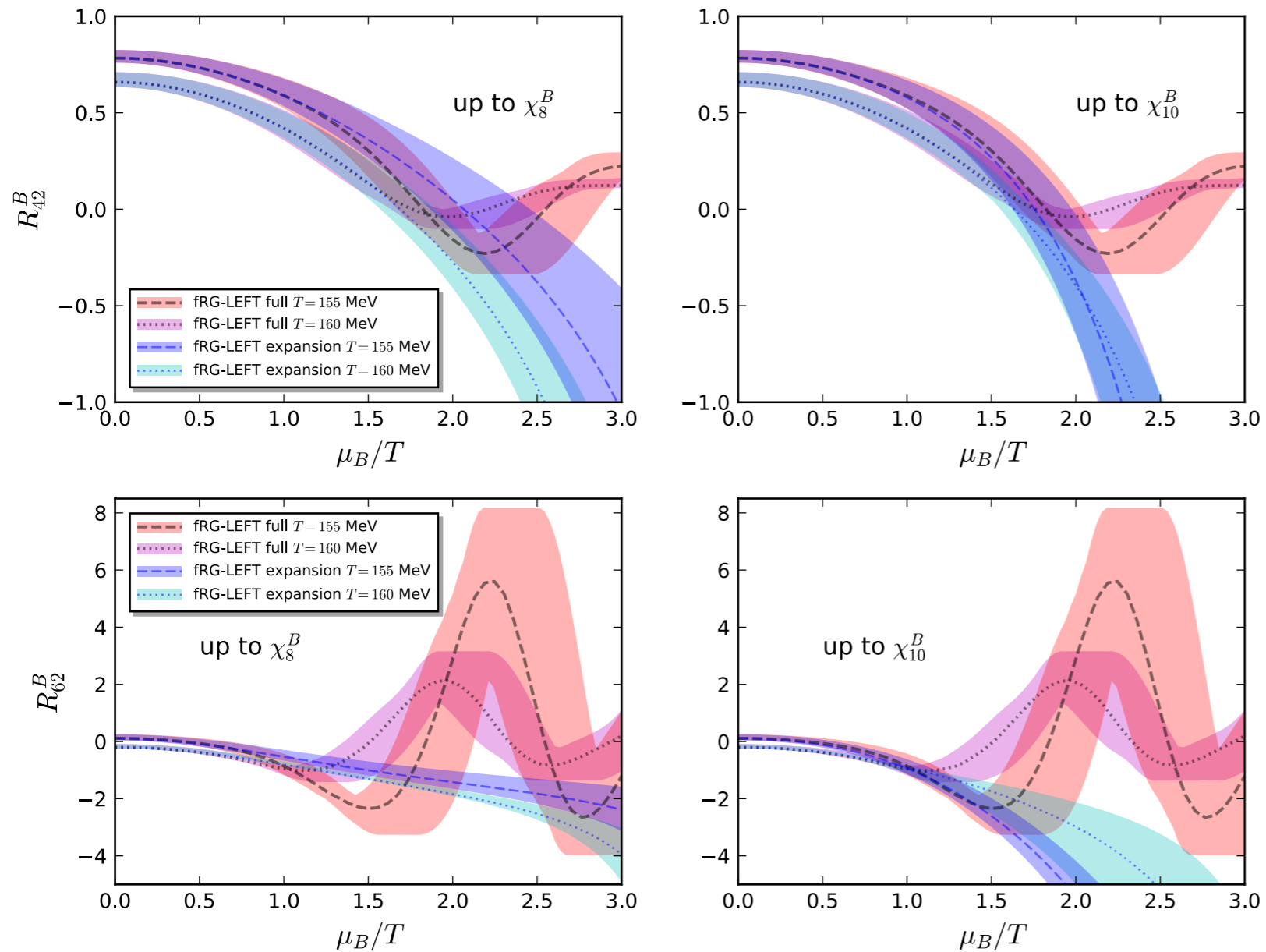
Great opportunity for a combined analysis
of high density QCD
(Exp. data + lattice QCD + functional QCD)

Fun QCD, in progress

Fluctuations of conserved charges

Fluctuation of conserved charges

QCD-assisted LEFT: Taylor expansion vs full results



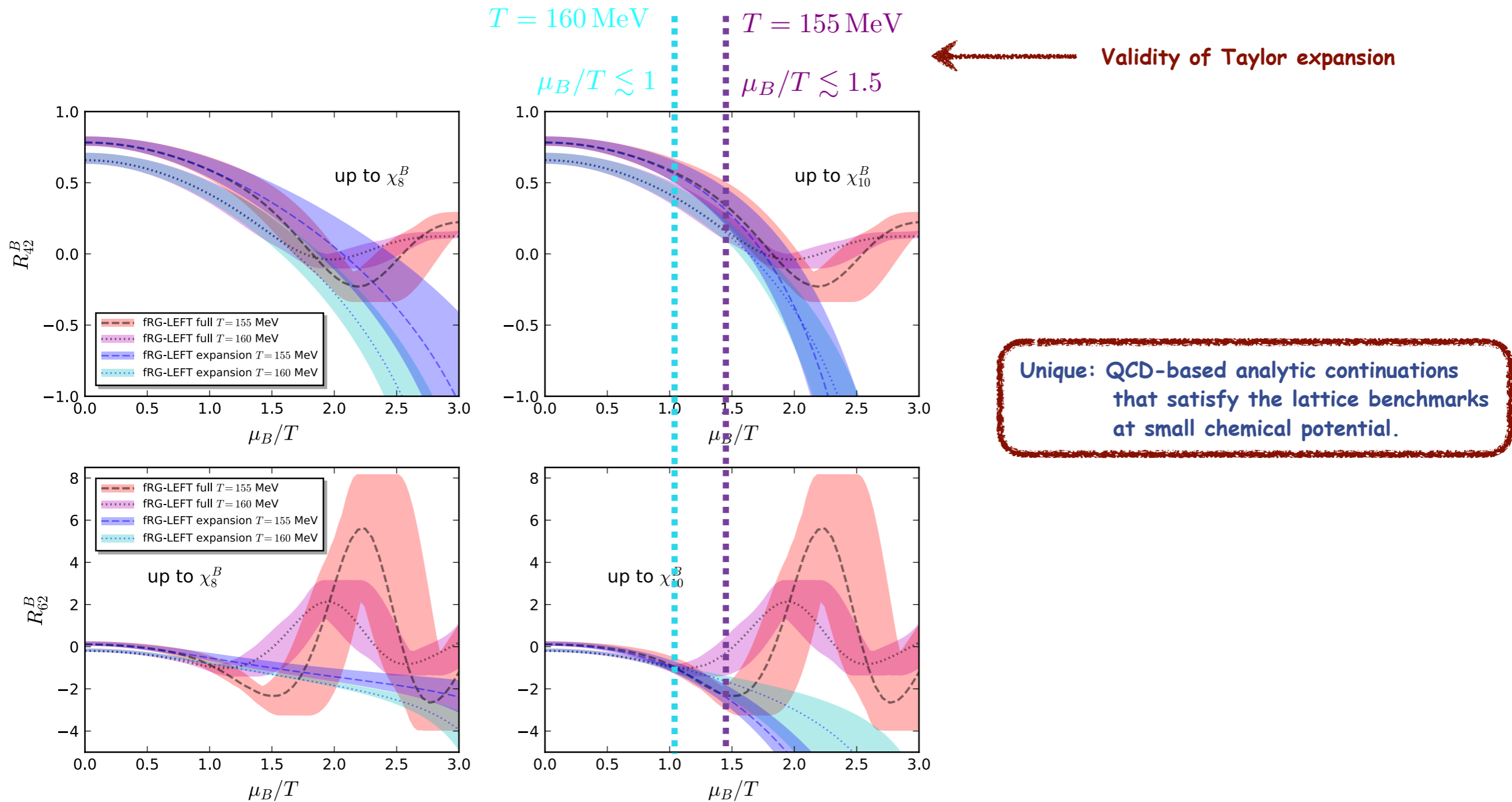
Unique: QCD-based analytic continuations that satisfy the lattice benchmarks at small chemical potential.

Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fluctuations of conserved charges

Fluctuation of conserved charges

QCD-assisted LEFT: Taylor expansion vs full results



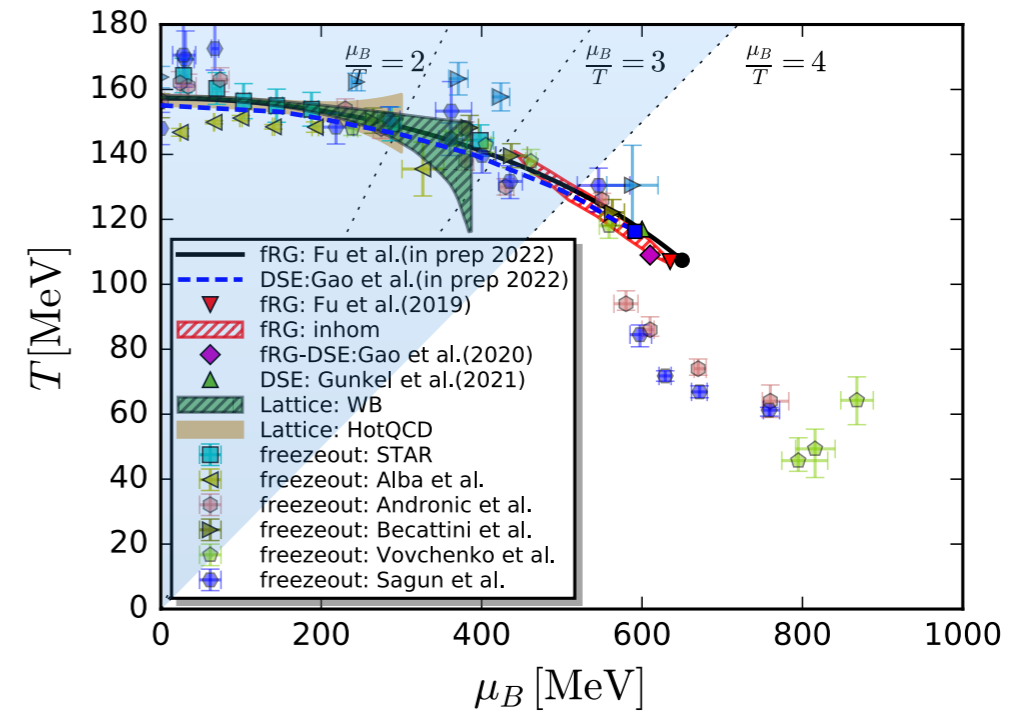
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Outline

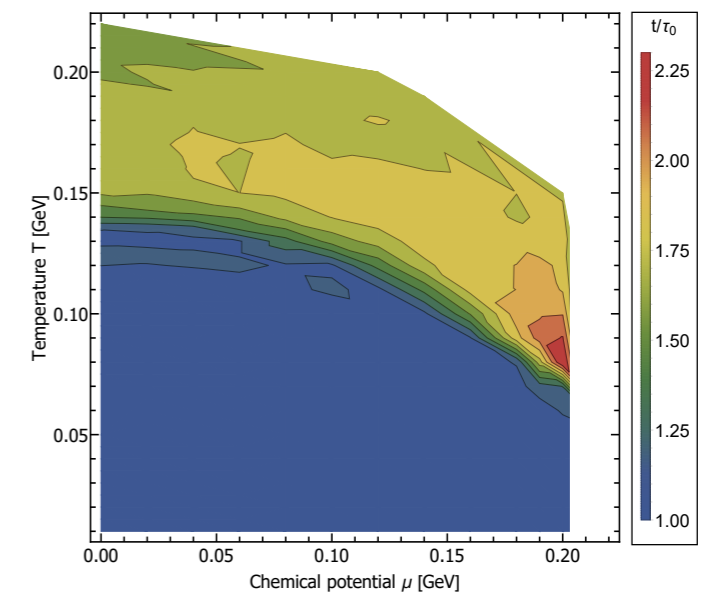
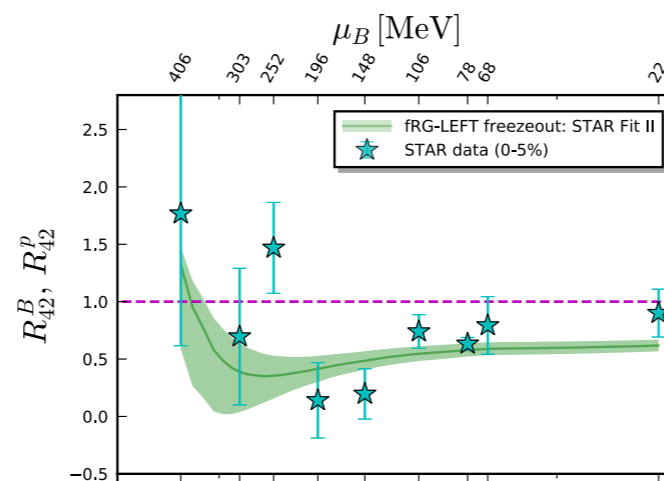
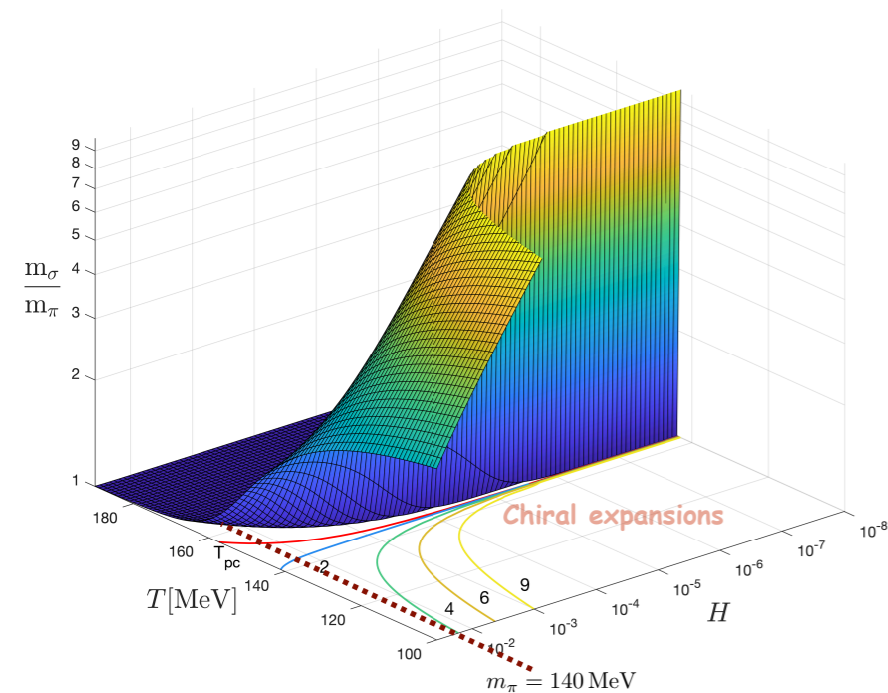
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Summary & Outlook

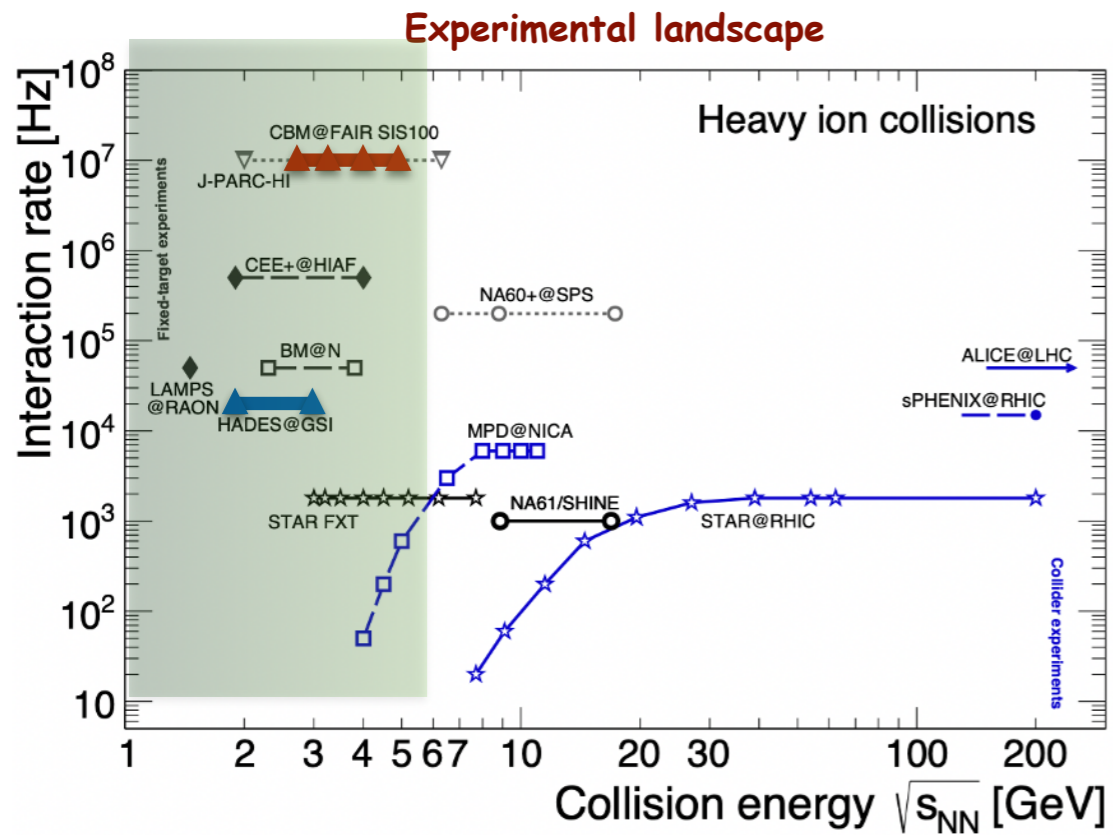
QCD phase structure and CEP estimate



Chiral dynamics & phenomenological applications



The future is bright!



Galatyuk, A982 (2019) update 2021
CBM, EPJA 53 3 (2017) 60

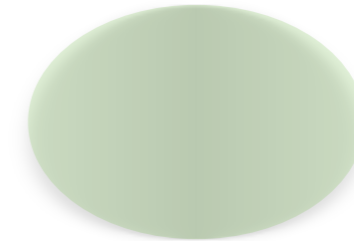
Freezeout curve

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

CBM/STAR

HADES

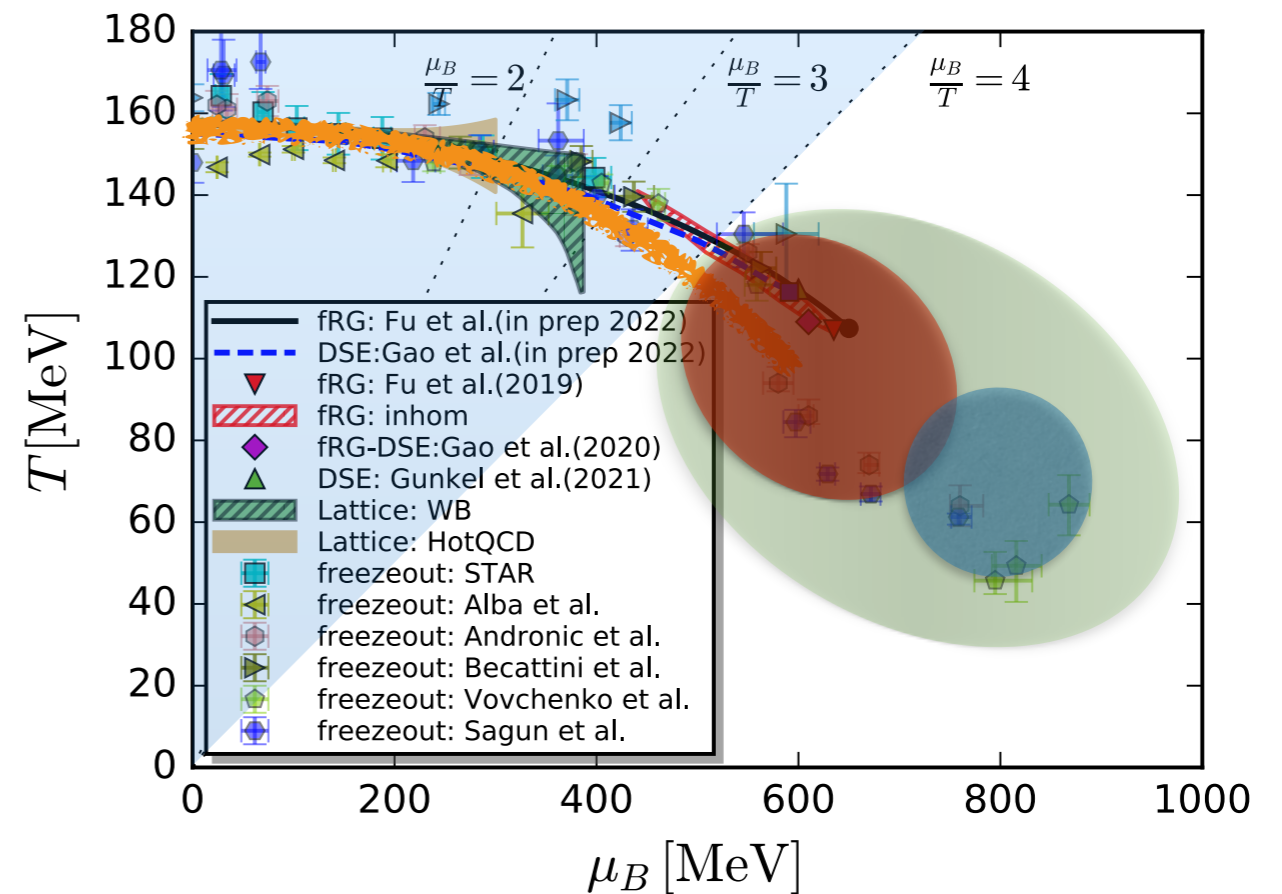
CEP or other NEW physics/phases



Fu, JMP, Rennecke, Wen, Yin, in prep.

Gao, JMP, Schneider, in prep.

Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)



Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

