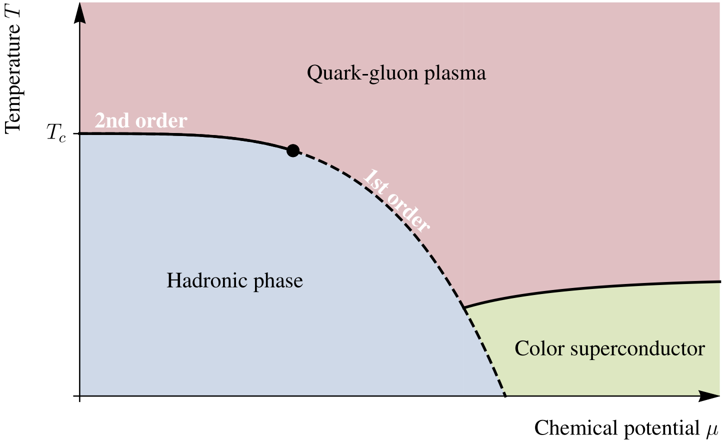
Symmetry constraints for Callan-Symanzik flows in chiral models

Sebastian Töpfel

Institute of Nuclear Physics, Theory Center
Technische Universität Darmstadt

HFHF Theory Retreat 2022
September 13, 2022, Castiglione della Pescaia, Italy

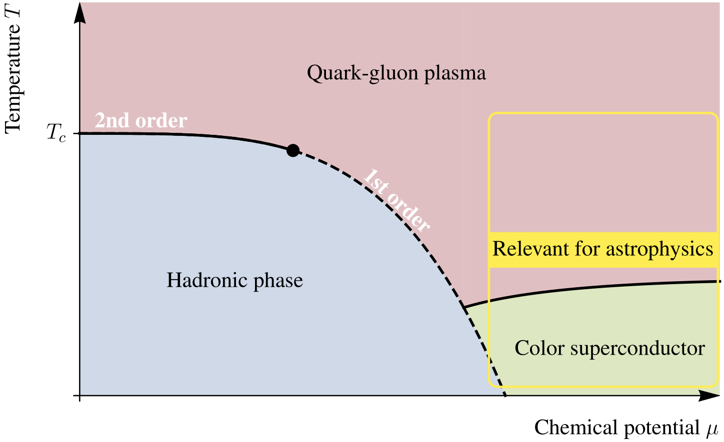
Motivation



Points of interest:

- Transition boundaries and critical point
- Equation of state (EOS)
- Spectral properties of mesons

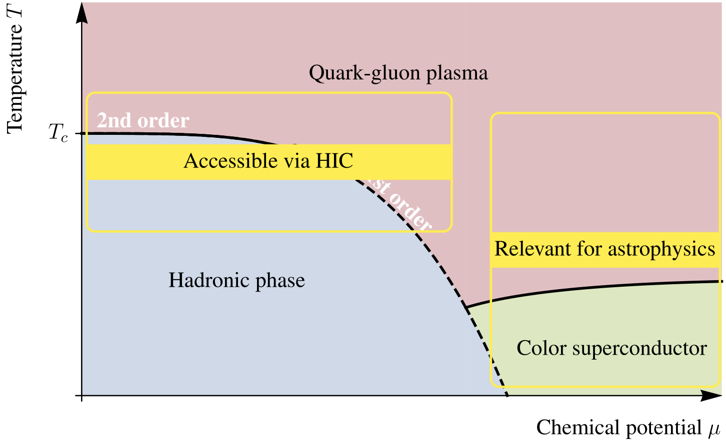
Motivation



Points of interest:

- Transition boundaries and critical point
- Equation of state (EOS)
- Spectral properties of mesons

Motivation



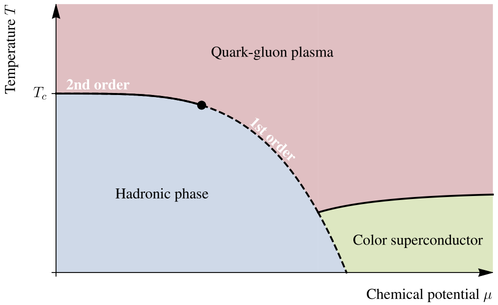
Points of interest:

- Transition boundaries and critical point
- Equation of state (EOS)
- Spectral properties of mesons

What this talk is (not)

- ✗ No EOS
- ✗ No spectral functions
- ✗ No qualitatively new phase diagram

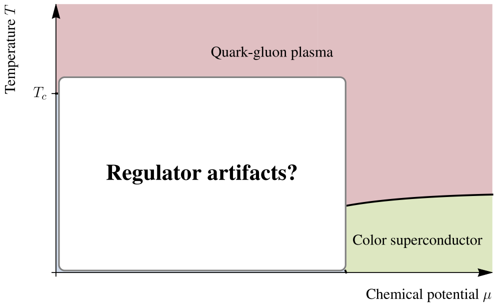
- ✓ Study regulator artifacts in phase diagram
- ✓ Systematic and consistent treatment of theoretical uncertainties



What this talk is (not)

- ✗ No EOS
- ✗ No spectral functions
- ✗ No qualitatively new phase diagram

- ✓ Study regulator artifacts in phase diagram
- ✓ Systematic and consistent treatment of theoretical uncertainties



Overview

- Basic principles of the functional renormalization group (FRG)
- The Callan-Symanzik (CS) regulator
- Implementation of the CS regulator into chiral model
- Numerical results for the phase diagram

Basics of FRG

- Suppress momentum modes below energy scale k

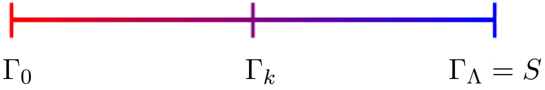
$$\mathcal{Z}_k[J] = \int_\Lambda \mathcal{D}\varphi \; \mathrm{e}^{-S[\varphi]-\Delta S_k[\varphi]+J^\intercal \cdot \varphi}$$

$$\Delta S_k[\varphi] = \frac{1}{2} \int_p \varphi(-p)^\intercal \; R_k(p) \; \varphi(p)$$

- Effective average action

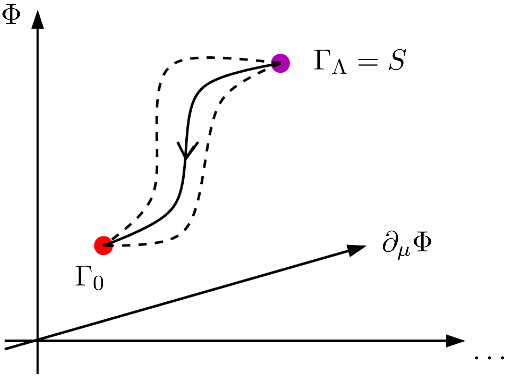
$$\Gamma_k[\Phi] = \sup_J \left\{ J^\intercal \cdot \Phi - \ln \mathcal{Z}_k[J] \right\} - \Delta S_k[\Phi]$$

$$\Phi = \langle \varphi \rangle_J$$



Basics of FRG

- Successively integrate out fluctuations from UV to IR



[H. Gies, (2012)]

- Flow equation for Γ_k [C. Wetterich, (1993)]

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2} \operatorname{Tr}\left\{ \partial_k R_k \cdot \left(\Gamma_k^{(2)}[\Phi] + R_k \right)^{-1} \right\}, \quad \Gamma_k^{(2)}[\Phi] = \left(\frac{\delta^2}{\delta \Phi^2} \Gamma_k \right) [\Phi]$$

Basics of FRG

FRG-based restrictions on the regulator function R_k:

- IR-regularization: $\lim_{p \to 0} R_k(p) > 0$
- Physical limit: $\lim_{k \to 0} R_k(p) \to 0$
- UV condition: $\lim_{k \to \Lambda} R_k(p) \to \infty$

Model-based restrictions on the regulator function R_k:

- Lorentz symmetry
- Silver-Blaze symmetry
- Chiral symmetry
- ...

Most of the times: regulator cannot preserve everything

Basics of FRG - Fermionic regulators

- BCS-type regulator for dense systems

$$R_k(\vec{p}) = -\mathrm{i} \sum_{\sigma=\pm 1} (\mu + \sigma|\vec{p}|) \; r\left(\frac{(\mu+\sigma|\vec{p}|)^2}{k^2}\right) \; P_\sigma \gamma^0$$

[J. Braun, T. Dörnfeld, B. Schallmo, ST, (2020)]

- Polynomial regulator

$$R_k(p) = -(\not{p} + \mathrm{i}\mu\gamma^0) \; \left(\sum_{n=1}^{N} \frac{1}{n!} \; \left[\frac{(p_0+\mathrm{i}\mu)^2 + \vec{p}^2}{k^2}\right]^n \right)^{-1}$$

- Mass-like Callan-Symanzik (CS) regulator

$$R_k(p) = \mathrm{i}k$$

[J. Fehre et al., (2021); J. V. Roth, (2021)]
[K. Otto, C. Busch, B.-J. Schaefer, (2022)]
[J. Braun, Y.-R. Chen, A. Geißel, ..., ST, et al. (2022)]

Also interesting...

Impact of different regulators (classes) on the phase diagram?

→ ongoing collaboration

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

TECHNISCHE UNIVERSITÄT DARMSTADT

Dirk H. Rischke

Lutz Kiefer
Fabrizio Murgana

Jens Braun

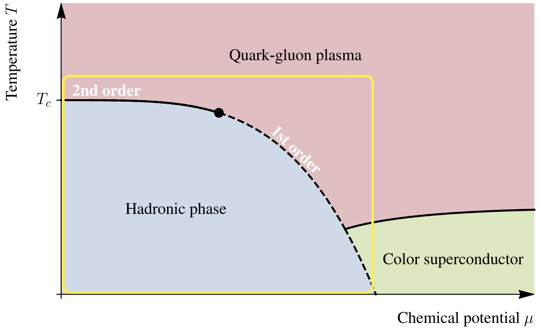
Jonas Stoll
ST
Niklas Zorbach

Stay tuned!

Callan-Symanzik regulator

Use mass-like CS regulator to study phase diagram

$$R_k(p) = \mathrm{i}k$$



- ✗ Silver-Blaze symmetry
- ✗ Chiral symmetry

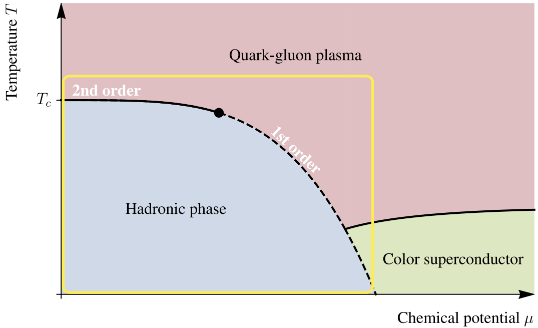
- ✓ Lorentz symmetry
- ✓ Allows for spectral representations

Callan-Symanzik regulator

Use mass-like CS regulator to study phase diagram

$$R_k(p) = \mathrm{i}k$$

Can chiral symmetry be restored?



- ✗ Silver-Blaze symmetry
- ✗ Chiral symmetry

- ✓ Lorentz symmetry
- ✓ Allows for spectral representations

Model & approach

- Chiral quark-meson model defined by action

$$S = \int \mathrm{d}^4x \, \left\{ \bar{\psi} \Big[\mathrm{i}\partial\!\!\!/ + \mathrm{i}\bar{h}\left(\sigma + \mathrm{i}\gamma^5\tau^j\pi_j\right) \Big] \psi + \phi^\intercal \, \frac{1}{2} \left[-\partial^2 + \bar{m}^2 \right] \phi \right\}, \quad \phi = \begin{pmatrix} \sigma \\ \vec{\pi} \end{pmatrix}$$

- Flow equation for Γ_k

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2} \operatorname{Tr} \left\{ \partial_k R_k \cdot \left(\Gamma_k^{(2)}[\Phi] + R_k \right)^{-1} \right\}, \quad \Phi = \begin{pmatrix} \psi \\ \bar{\psi}^\intercal \\ \phi \end{pmatrix}$$

- Truncations:

Model & approach

- Chiral quark-meson model defined by action

$$S = \int \mathrm{d}^4x \left\{ \bar{\psi} \Big[\mathrm{i}\partial\!\!\!/ + \mathrm{i}\bar{h} \left(\sigma + \mathrm{i}\gamma^5 \tau^j \pi_j \right) \Big] \psi + \phi^\intercal \, \frac{1}{2} \left[-\partial^2 + \bar{m}^2 \right] \phi \right\}, \quad \phi = \begin{pmatrix} \sigma \\ \vec{\pi} \end{pmatrix}$$

- Flow equation for Γ_k

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2} \operatorname{Tr} \left\{ \partial_k R_k \cdot \left(S^{(2)}[\Phi] + R_k \right)^{-1} \right\}, \quad \Phi = \begin{pmatrix} \psi \\ \bar{\psi}^\intercal \\ \phi \end{pmatrix}$$

- Truncations:

  1. 1-loop approximation

Model & approach

- Chiral quark-meson model defined by action

$$S = \int \mathrm{d}^4x \, \left\{ \bar{\psi} \left[\mathrm{i}\partial\!\!\!/ + \mathrm{i}\bar{h}\left(\sigma + \mathrm{i}\gamma^5 \tau^j \pi_j\right) \right] \psi + \frac{\bar{m}^2}{2} (\sigma^2 + \vec{\pi}^2) \right\}$$

- Flow equation for Γ_k

$$\partial_k \Gamma_k[\sigma, \vec{\pi}] = -\operatorname{Tr}\left\{ \partial_k R_k \cdot \left(S^{(2)}_{\bar{\psi}\psi}[\sigma, \vec{\pi}] + R_k \right)^{-1} \right\}$$

- Truncations:

  1. 1-loop approximation
  2. Drop meson loops

Model & approach

- Chiral quark-meson model defined by action

$$S = \int \mathrm{d}^4x \, \left\{ \bar{\psi} \Big[\mathrm{i}\partial\!\!\!/ + \mathrm{i}\bar{h} \left(\sigma + \mathrm{i}\gamma^5 \tau^j \pi_j \right) \Big] \psi + \frac{\overline{m}^2}{2} (\sigma^2 + \vec{\pi}^2) \right\}$$

- Flow equation for Γ_k

$$\partial_k \Gamma_k[\sigma, \vec{\pi}] = -\,\mathrm{Tr} \left\{ \partial_k R_k \cdot \left(S^{(2)}_{\bar{\psi}\psi}[\sigma, \vec{\pi}] + R_k \right)^{-1} \right\}$$

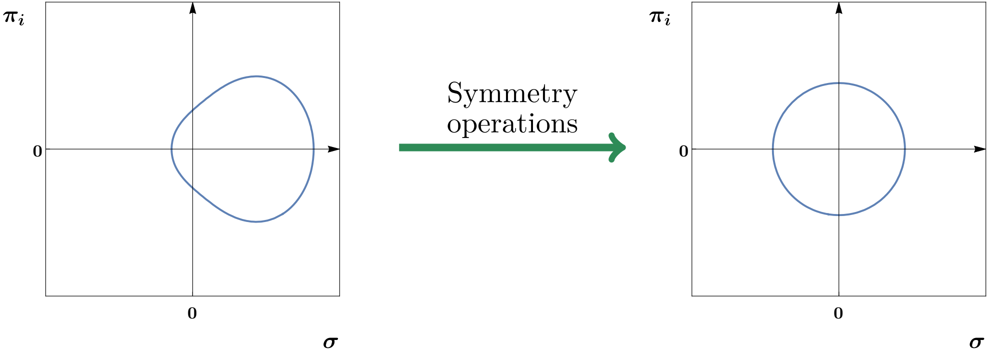
- Truncations:

  1. 1-loop approximation
  2. Drop meson loops

- CS Regulator treats σ-direction differently: $\partial_k \Gamma_k[\sigma, \vec{\pi}] = \partial_k \Gamma[\sigma + \frac{k}{\bar{h}}, \vec{\pi}]$

Restoring chiral symmetry

In bosonic field space, chiral symmetry manifests itself as an $O(4)$ symmetry.

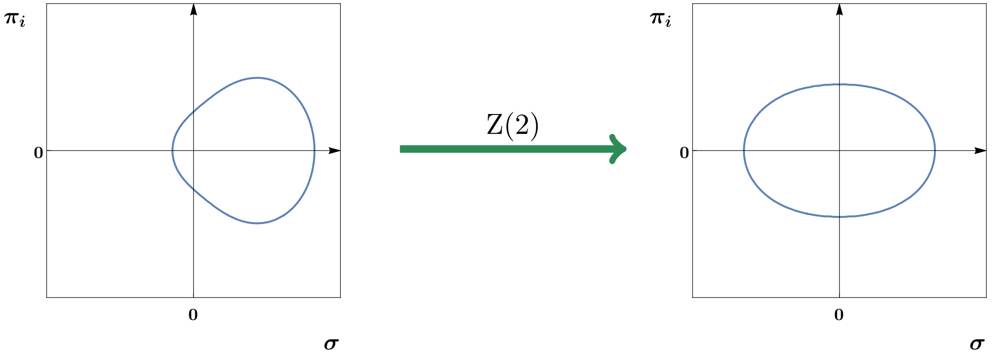


$\Gamma_k[\sigma, \vec{\pi}]$

$\hat{\mathcal{S}}\left(\Gamma_k\right)[\sigma, \vec{\pi}] = f_k(\sigma^2 + \vec{\pi}^2)$

Restoring chiral symmetry

In bosonic field space, chiral symmetry manifests itself as an $O(4)$ symmetry.

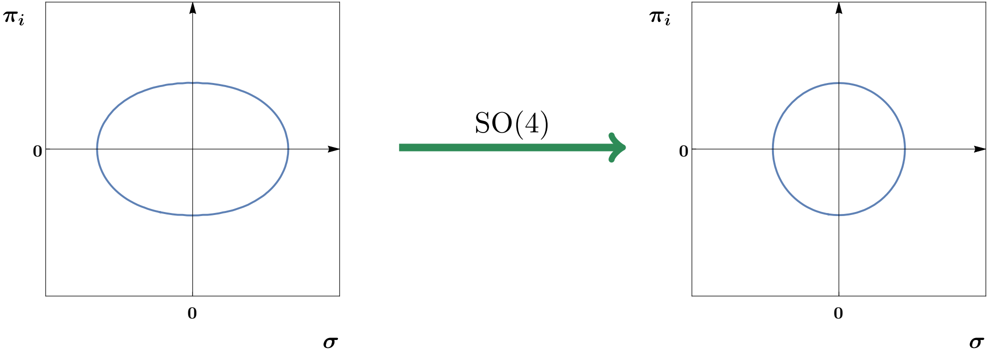


$\Gamma_k[\sigma, \vec{\pi}]$

$$\Gamma_k^{(\mathrm{refl})}[\sigma, \vec{\pi}] = \frac{\Gamma_k[\sigma, \vec{\pi}] + \Gamma_k[-\sigma, \vec{\pi}]}{2}$$

Restoring chiral symmetry

In bosonic field space, chiral symmetry manifests itself as an $O(4)$ symmetry.



$$\Gamma_k^{(\text{refl})}[\sigma, \vec{\pi}]$$

$$\Gamma_k^{(\text{ch})} = \Gamma_k^{(\text{refl})} + \text{boundary term}$$

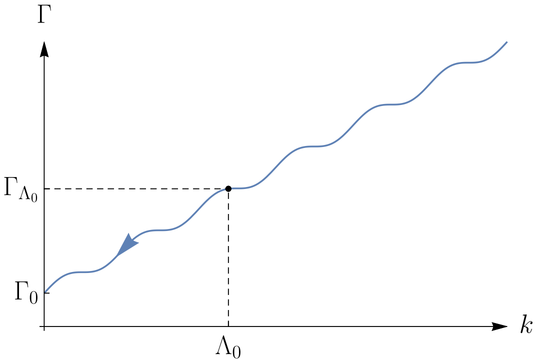
$$\text{WTI:} \quad \frac{1}{\sigma}\frac{\partial}{\partial\sigma}\Gamma_k^{(\text{ch})} \overset{!}{=} 2\frac{\partial}{\partial\vec{\pi}^2}\Gamma_k^{(\text{ch})}$$

RG consistency [J. Braun, M. Leonhardt, J. M. Pawlowski, (2018)]

- Functional approaches feature momentum cutoff

- Consistent UV completion to Γ_Λ ensures cutoff independence of Γ_0

$$\forall \Lambda \neq \Lambda_0 : \quad \frac{\mathrm{d}}{\mathrm{d}\Lambda}\Gamma_0 = 0$$



- At finite m_{ext} cutoff artifacts to be removed by UV completion up to scale Λ such that

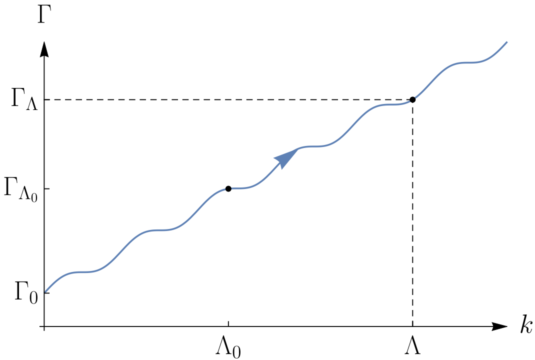
$$\forall m_{\text{ext}} \in \{T, \mu\} : \quad m_{\text{ext}} \ll \Lambda$$

RG consistency [J. Braun, M. Leonhardt, J. M. Pawlowski, (2018)]

- Functional approaches feature momentum cutoff

- Consistent UV completion to Γ_Λ ensures cutoff independence of Γ_0

$$\forall \Lambda \neq \Lambda_0 : \quad \frac{\mathrm{d}}{\mathrm{d}\Lambda}\Gamma_0 = 0$$



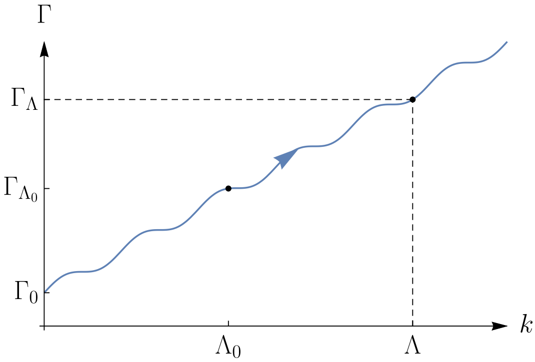
- At finite m_{ext} cutoff artifacts to be removed by UV completion up to scale Λ such that

$$\forall m_{\text{ext}} \in \{T, \mu\} : \quad m_{\text{ext}} \ll \Lambda$$

RG consistency [J. Braun, M. Leonhardt, J. M. Pawlowski, (2018)]

- Functional approaches feature momentum cutoff
- Consistent UV completion to Γ_Λ ensures

cutoff independence



- At finite m_{ext} cutoff artifacts to be removed by UV completion up to scale Λ such that

$$\forall m_{\text{ext}} \in \{T, \mu\} : \quad m_{\text{ext}} \ll \Lambda$$

Recap

We have:

- chiral model S
- flow equation $\partial_k \Gamma_k$
- Callan-Semanzik regulator R_k

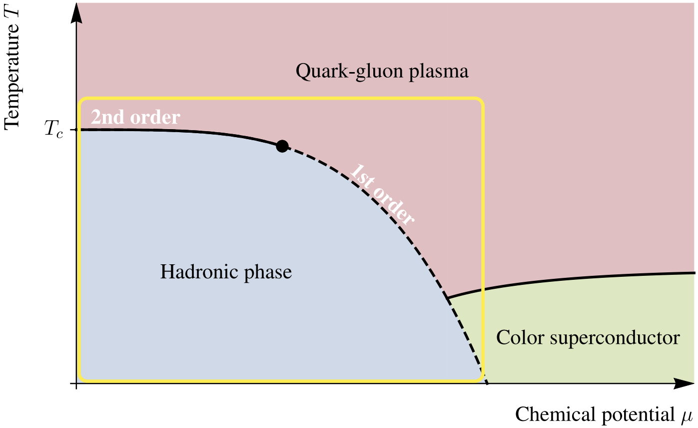
To obtain reliable physics at our level of truncation, we have to

1. restore $Z(2)$
2. restore $SO(4)$
3. implement RG consistency (cutoff independence)

Results

Order parameter: $m_\sigma^2 = “\frac{\partial^2}{\partial\sigma^2}\Gamma_0“$

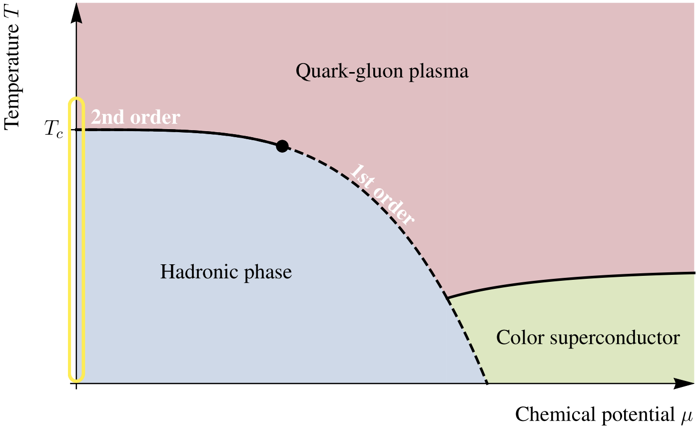
Critical temperature: $m_\sigma^2\big|_{T_c} = 0$ (chiral limit)



Results

Order parameter: $m_\sigma^2 = “\frac{\partial^2}{\partial\sigma^2}\Gamma_0“$

Critical temperature: $m_\sigma^2\big|_{T_c} = 0$ (chiral limit)

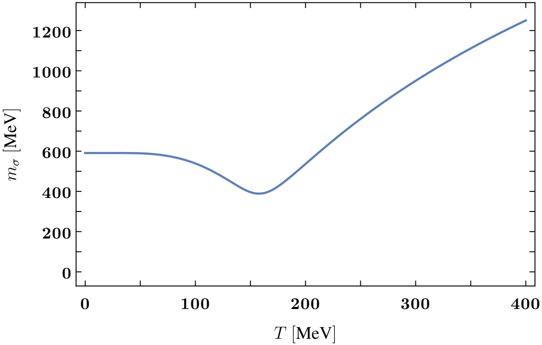


Results

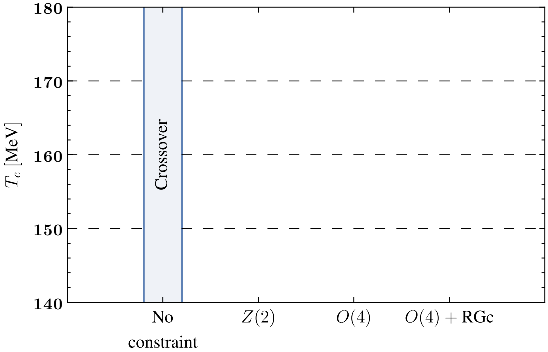
No constraint

Order parameter: $m_\sigma^2 = \frac{\partial^2}{\partial\sigma^2}\Gamma_0$

Critical temperature: $m_\sigma^2\big|_{T_c} = 0$ (chiral limit)



(a) Sigma mass m_σ at $\mu = 0$



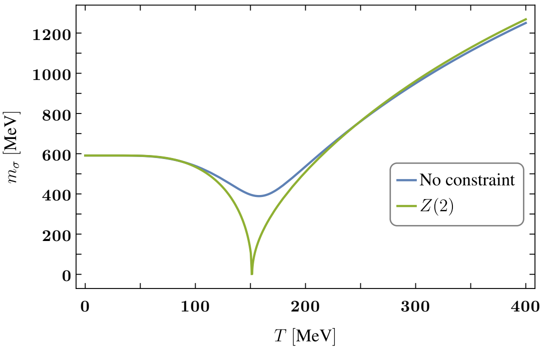
(b) Critical temperature T_c

Results

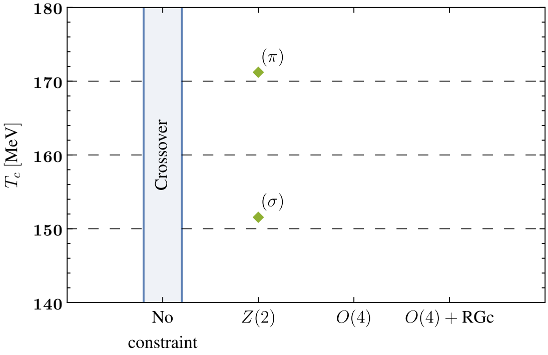
$Z(2)$ restored

Order parameter: $m_\sigma^2 = \frac{\partial^2}{\partial\sigma^2}\Gamma_0^{(\text{refl})}$

Critical temperature: $m_\sigma^2\big|_{T_c} = 0$ (chiral limit)



(a) Sigma mass m_σ at $\mu = 0$



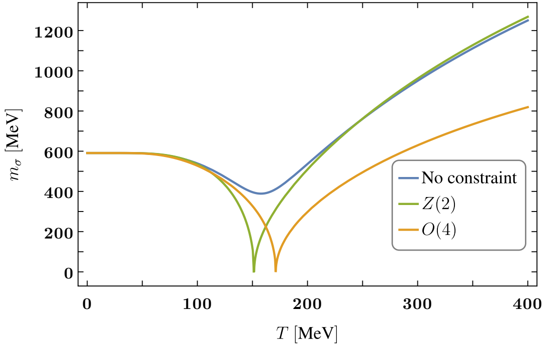
(b) Critical temperature T_c

Results

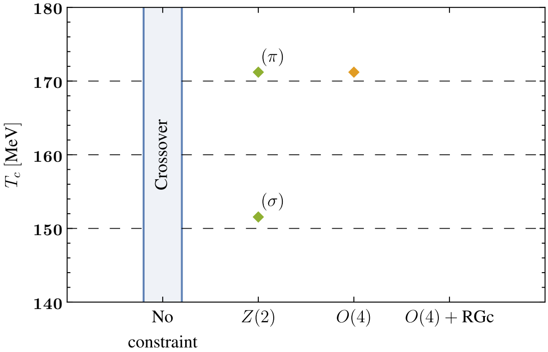
$O(4)$ restored

Order parameter: $m_\sigma^2 = \frac{\partial}{\partial\sigma}\left[2\sigma\frac{\partial}{\partial\vec{\pi}^2}\Gamma_0^{(\text{refl})}\right]$

Critical temperature: $m_\sigma^2\big|_{T_c} = 0$ (chiral limit)



(a) Sigma mass m_σ at $\mu = 0$



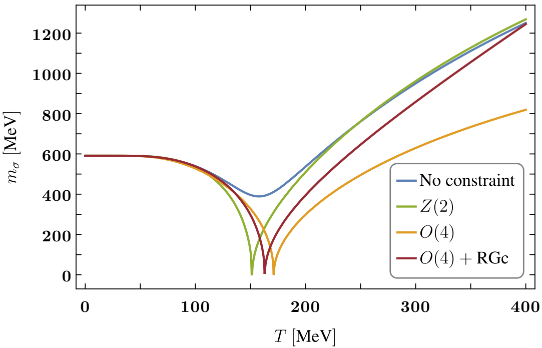
(b) Critical temperature T_c

Results

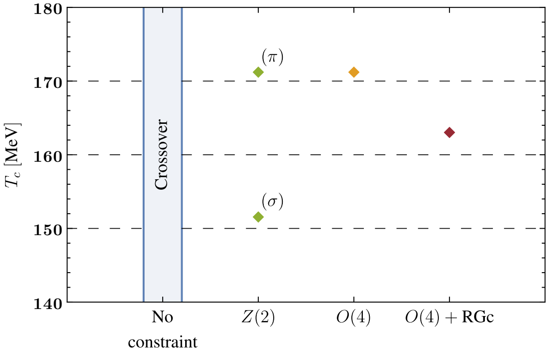
$O(4)$ + RG consistency

Order parameter: $m_\sigma^2 = \frac{\partial}{\partial\sigma}\left[2\sigma\frac{\partial}{\partial\vec{\pi}^2}\Gamma_0^{(\mathrm{refl+RGc})}\right]$

Critical temperature: $m_\sigma^2\big|_{T_c} = 0$ (chiral limit)



(a) Sigma mass m_σ at $\mu = 0$

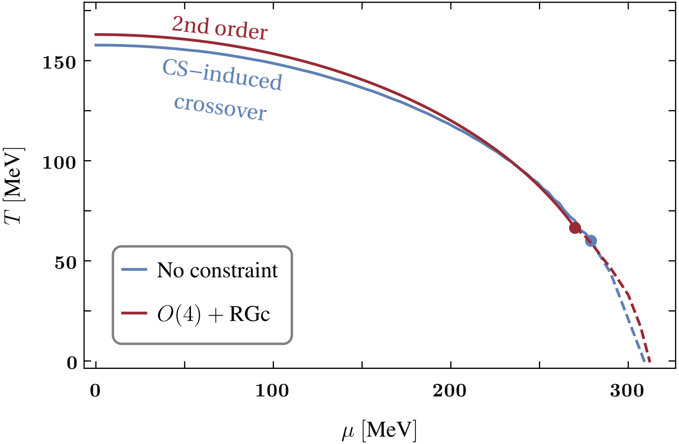


(b) Critical temperature T_c

Results

- Not vs. fully constrained: looks similar but different physics
- Physical explicit symmetry breaking expected to be "overshadowed" by regulator if no constraints are implemented

Take order parameter m_σ^2 for calculation of phase diagram



Summary and outlook

We have shown...

- how to restore chiral symmetry for CS flows
- some implications of ignoring regulator-induced symmetry breaking

We plan to...

- extend our truncations
  $\rightarrow$ include, e.g., explicit symmetry breaking and/or bosonic fluctuations
- calculate EOS
- calculate bands of theoretical uncertainties

Thank you for your attention!

HFHF
Helmholtz Forschungsakademie Hessen für FAIR

TECHNISCHE
UNIVERSITÄT
DARMSTADT