UNIVERSITÉ DE NANTES

Heavy quarks: a comparison of different theoretical approaches

Joerg Aichelin

- Why heavy hadrons are interesting ?
- Results from the different approaches
- How can we compare the approaches ?
- tool box approach -> transport coefficients?
- Influence of elementary HQ-parton interaction, of QGP expansion and of initial condition.
- What can we conclude presently?

Phys. Rev. C99,014902 (19): Yingru Xu, Steffen A. Bass, Pierre Moreau, Taesoo Song, Marlene Nahrgang, Elena Bratkovskaya, Pol Gossiaux, Jorg Aichelin, Shanshan Cao, Vincenzo Greco, Gabriele Coci, Klaus Werner

Phys. Rev. C99,054907 (19): Shanshan Cao, Gabriele Coci , Santosh Kumar Das, Weiyao Ke, Shuai Y.F. Liu, Salvatore Plumari, Taesoo Song, Yingru Xu, Jörg Aichelin, Steffen Bass, Elena Bratkovskaya, Xing Dong, Pol Bernard Gossiaux, Vincenzo Greco, Min He, Marlene Nahrgang, Ralf Rapp, Francesco Scardina, Xin-Nian Wang

Why should we study heavy hadrons ?

Light hadrons tell us T and μ at hadronization, but not properties of expanding QGP

At first glance HQs are an ideal probe for a tomography of the QGP

initially created in a hard process → accessible to pQCD calculations

high p_T HQs traverse the QGP without coming to an equilibrium with the QGP
→ preserve memory on the trajectory in the QGP
→ sensitive to the properties of the QGP during the expansion (and not only to its final state)

HQs keep their identity while traversing the QGP (in contradistinction to light quark jets)

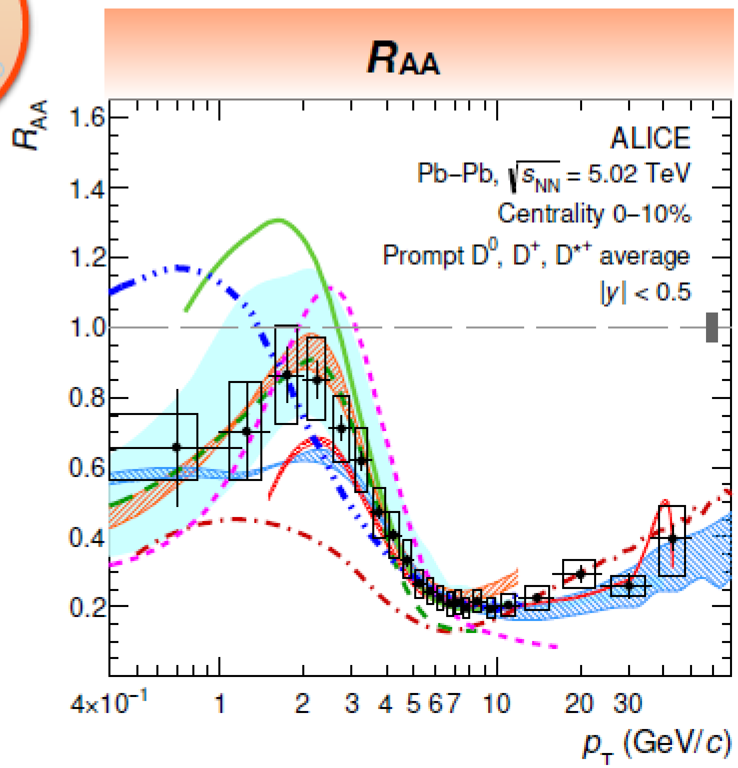
HQs interact strongly with the QGP (in contradistinction to photons)

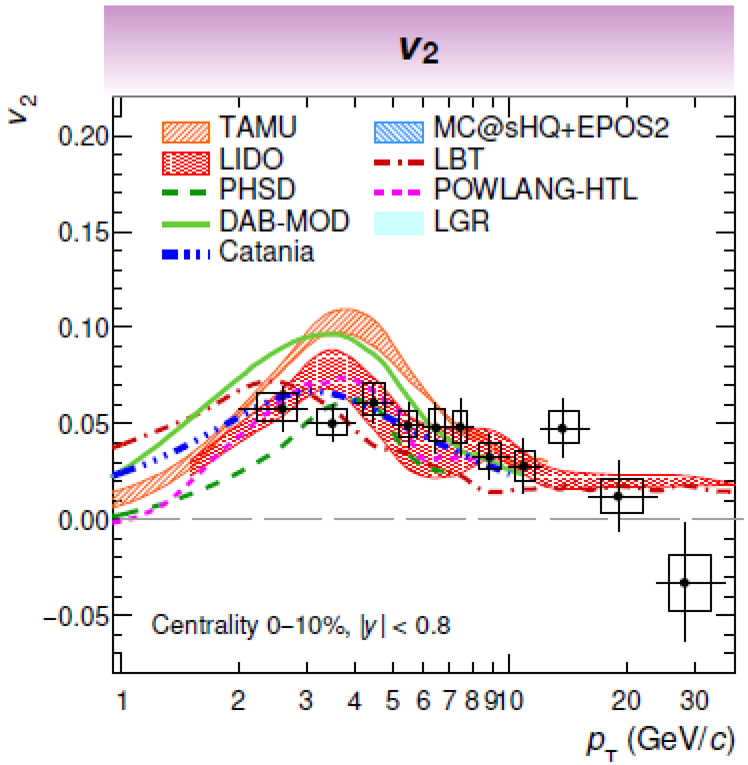
HQs are heavy and theory does not predict large changes of their mass in a QGP

But –as usual – the devil is in the details

Starting point: all models agree (more or less) with available data

Most of the models reproduce quite reasonable the experimental results !!





More difficult is to answer the question:

What does this agreement tell us?

What can we take home?

The participants and their approaches

Participants:

Catania (Santosh Das)
Duke (Yingrou Xu)
Frankfurt (PHSD) (Taesoo Song)
CCNU-LBNL (Shanshan Cao)
Nantes (PB. Gossiaux, M. Nahrgang)
TAMU (Min He)

Some key features of the participating programs:

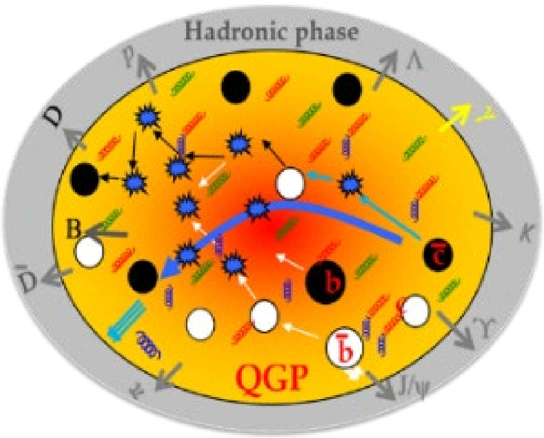
	Catania	Duke	Frankfurt(PHSD)	LBL	Nantes	TAMU
Initial HQ (p)	FONLL	FONLL	PQCD	pQCD	FONLL	
Initial HQ (x)	binary coll.	binaryy coll.	binary coll.	binary coll.		binary coll.
Initial QGP	Glauber	Trento	Lund		EPOS	
QGP	Boltzm.	Vishnu	Boltzm.	Vishnu	EPOS	2d ideal hydro
partons	mass	m=0	m(T)	m=0	m=0	m=0
formation time QGP	0.3 fm/c	0.6 fm/c	0.6 fm/c (early coll.)	0.6 fm/c	0.3 fm/c	0.4 fm/c
formation time HQ	1/(2m)	E/mT^2	0	E/mT^2	0.	0.4 fm/c
interactions in between	HQ-glasma	no	HQ-preformed plasma	no		no

GOAL: To study how the different model ingredients influence the final result by replacing the specific ingredient of a model by a common standard

- for the expansion of the QGP
- for the elementary interaction between QGP partons and HQs
- for the initial condition

The complicated trajectory of heavy quarks

The details one has to know to explore the information carried by HQs



- (p,x) distribution of the hard collisions which produce HQ (FONLL, Glauber)
- Initial (p,x) distribution of the QGP (EPOS, Trento, PHSD, Glasma)
- Formation time of heavy quarks and the QGP (when does the interactions start?)
- Expansion of the QGP ((viscous) hydrodynamics, PHSD)
- Elementary interaction between HQ and the QGP
- Hadronization of HQs to heavy mesons
- Hadronic scattering of heavy mesons

In addition there is the question which time evolution equations are appropriate to describe the heavy quarks which travers the QGP

- ❑ Fokker-Planck equation
- ❑ Boltzmann equation

How very different models can be compared?

How to compare the different approaches?

A Boltzmann equation can be (under certain conditions) converted into a Fokker-Planck equation which can be solved by a stochastic differential equation, the Langevin eq.

→ Langevin eq. for the heavy quarks is the lowest common denominator of all approaches

$$
\begin{aligned}
dx_i &= \frac{p_i}{E} dt \\
dp_i &= -\eta_D(\vec{p}, T)\ p_i\ dt + \xi_i dt
\end{aligned}
$$

ξ_i = Gaussian random variable

The whole dynamics is there casted into 3 momentum and temperature dependent functions which describe the interaction between HQ and the QGP

$$< \xi_i(t)\xi_j(t') >= (\kappa_T(\vec{p}, T)\ p_{ij}^T + \kappa_{||}(\vec{p}, T)\ p_{ij}^{||})\delta(t - t')$$

$$< \xi_i >= 0$$

$$p_{ij}^T = \delta_{ij} - \frac{p_i p_j}{p^2}\ ;\ p_{ij}^{||} = \frac{p_i p_j}{p^2}$$

η_D = drag coefficient
κ = diffusion coefficients (transversal and longitudinal)

For every transport approach these coefficients, which contain the elementary interaction between heavy quarks and partons, have been calculated and made available for the comparison.

Transport coefficients

Transport coefficients are calculated assuming that the particle interacts with particles in the thermal equilibrium

We can calculate transport coefficients from the Boltzmann eq. by

$$\langle\langle O^* \rangle\rangle \equiv \frac{1}{2E_p} \sum_{i=q,\bar{q},g} \int \frac{d^3k}{(2\pi)^3 2E} f_i(k) \int \frac{d^3k'}{(2\pi)^3 2E'}$$

$$\times \int \frac{d^3p'}{(2\pi)^3 2E'_p} \; O^* \; (2\pi)^4 \delta^{(4)}(p+k-p'-k') \frac{|M_{ic}|^2}{\gamma_c}, (13)$$

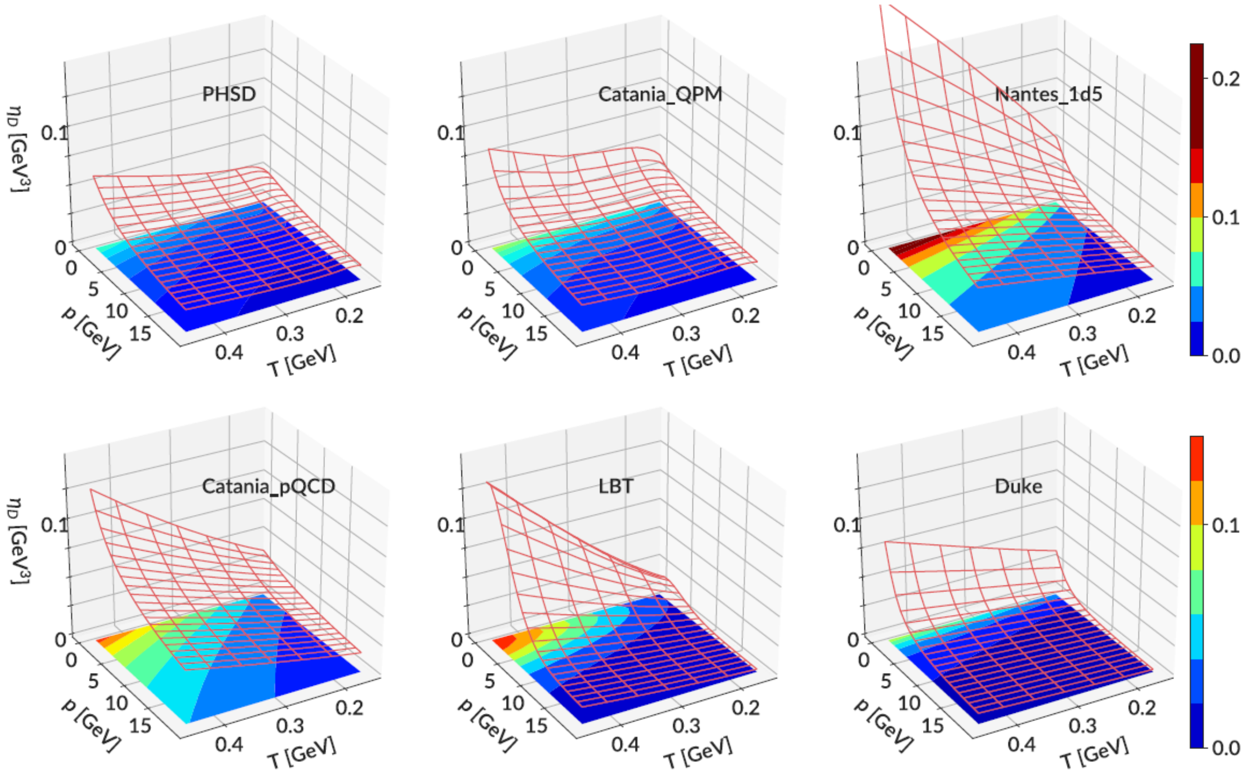
Here we need

$$\frac{d}{dt}\langle p \rangle \equiv -\eta_D \langle p \rangle,$$
$$\frac{1}{2}\frac{d}{dt}\langle (\Delta p_T)^2 \rangle \equiv \kappa_T,$$
$$\frac{d}{dt}\langle (\Delta p_z)^2 \rangle \equiv \kappa_L,$$

the transport coefficients for the Langevin equation

The drag coefficients from the different approaches

The drag coefficient η_D of the different models (standard version to describe the data)



All drag coefficients η_D decrease with p and increase withT but absolute values differ by large factors

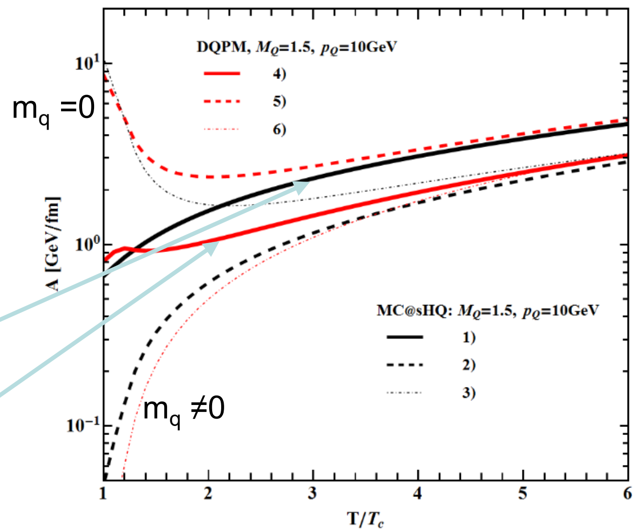
Why « pQCD » does not determine all ?

How can this happen if the if the cross sections q(g)Q -> q(g)Q are calculated in leading order pQCD?

Take a simple t-channel elastic scattering
For the calculation one has to fix:

- α_S , α_S (T), α_S (Q^2)
- masses of the incoming/outgoing QGP partons
- mass of the exchanged gluons (m_D)

		coupling	mass in gluon propagator	mass in external legs
Nantes →	1)	$\alpha(Q^2)$	$\kappa = 0.2, m_D$	$m_{q,g} = 0$
	2)	$\alpha(Q^2)$	$\kappa = 0.2, m_D$	$m_{q,g} = m_{q,g}^{DQPM}$
	3)	$\alpha(T)$	$\kappa = 0.2, m_D$	$m_{q,g} = 0$
Frankfurt/PHSD →	4)	$\alpha(T)$	m_g^{DQPM}	$m_{q,g} = m_{q,g}^{DQPM}$
	5)	$\alpha(T)$	m_g^{DQPM}	$m_{q,g} = 0$
	6)	$\alpha(Q^2)$	m_g^{DQPM}	$m_{q,g} = m_{q,g}^{DQPM}$



H. Berrehrah et al. 1604.02343,
T. Song et al. PRC 92 (2015), PRC 93 (2016)

Different choices chance the drag A for p_{HQ} = 10 GeV/c by
a factor of 100 close to T_C
a factor of 2 for 4 T_C

Why not simply ask lattice QCD ?

Can lattice QCD calculations help us to fix the transport coefficients?

Lattice:
Spatial diffusion coefficient at p=0 is defined via the spectral function $\sigma(\omega,\vec{p})$ as

$$D_s(\vec{p}=0) = lim(\omega \to 0)\frac{\sigma(\omega,\vec{p}=0)}{\omega\chi_q\pi}$$

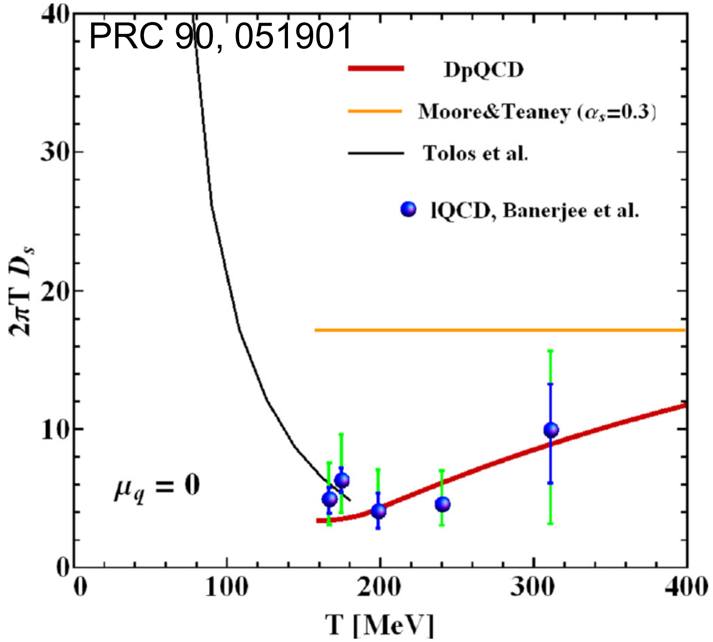
coeff
where the spectral function in obtained via the current-current correlator by

$$G(\tau,T) = \int_0^\infty \frac{d\omega}{2\pi}\,\sigma(\omega,T)\,K(\tau,\omega,T)$$

Problems/approximations:

- Euclidian time calculation
  Agreement quite reasonable
- Quenched
- No continuum extrapolation

lQCD does not cover the dynamical range needed in heavy in collisions



Dynamical models:

$$D_s = lim(\vec{p} \to 0)\frac{T}{M\eta_D}$$

η_D = A/p ; A(p,T) = drag

(PRC 71, 064904
PRC 90, 064906)

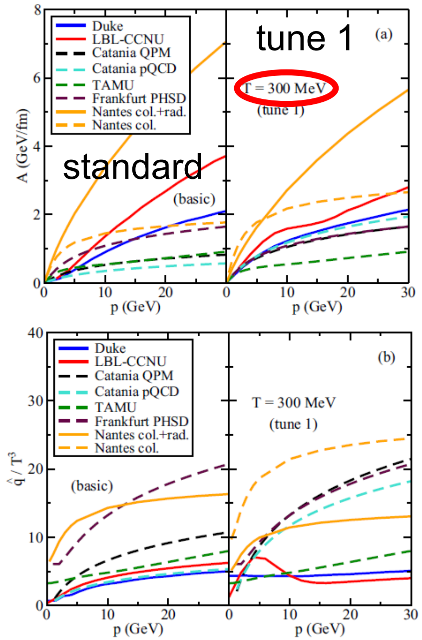
Steps of the comparison of A and qhat I

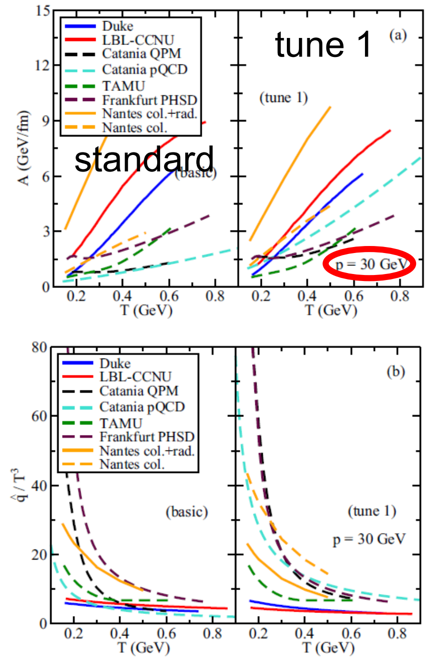
First step for the comparison:

tune the models for best agreement for R_{AA} in PbPb (2.76 ATeV) 2 GeV/c < p_T < 15 GeV/c (tune 1)

$$A = dp_L/dt, \quad \hat{q} = dp_T^2/dt$$ (for elastic collisions only)

standard version





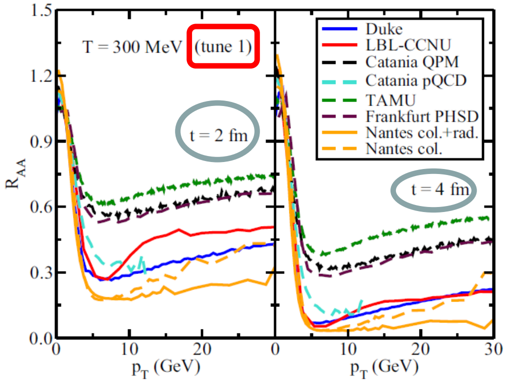
Solid lines elast coll + rad
Dashed lines elast coll

tune 1 does not really narrow the differences

Steps of the comparison A and qhat II

Second step: R_{AA} of charm quarks in a brick

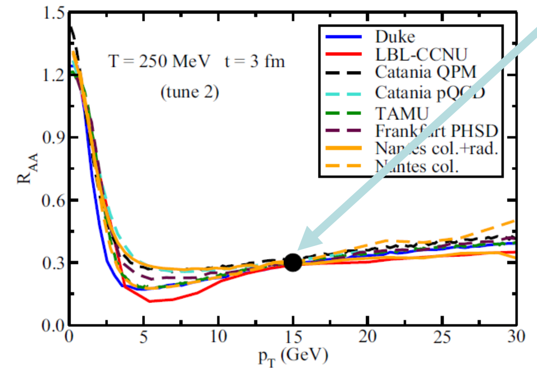
R_{AA} in static brick after 2 and 4 fm/c

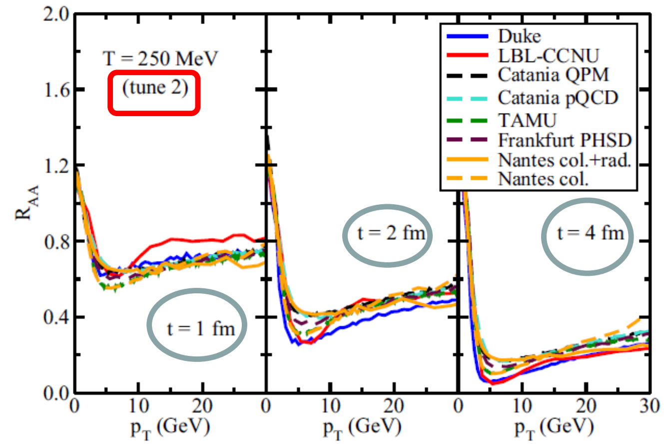


Models do what expected
Large t → small R_{AA} (final/initial)

But differences of more than a factor of two remain

Tune 2: K factors that all models agree for:
T=250 MeV
p=15 GeV/c
at t= 3 fm/c

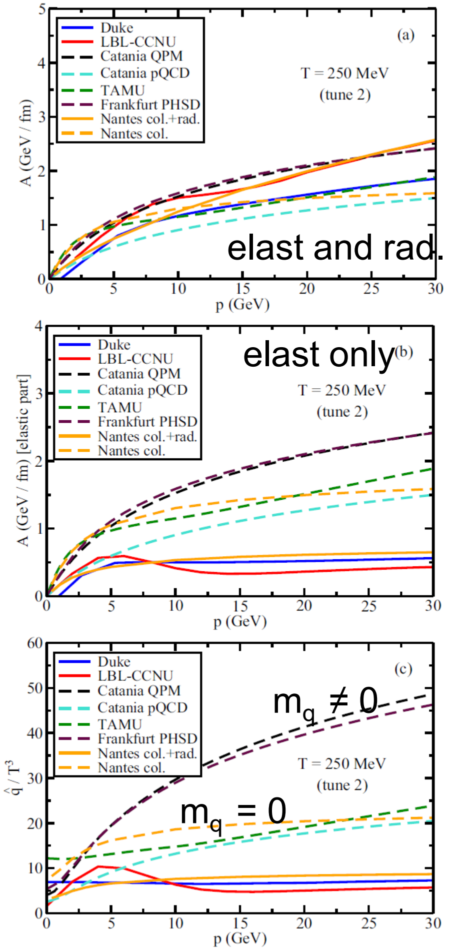




Narrows down the differences in R_{AA} between the models also at other times.

Steps of the comparison A and qhat III

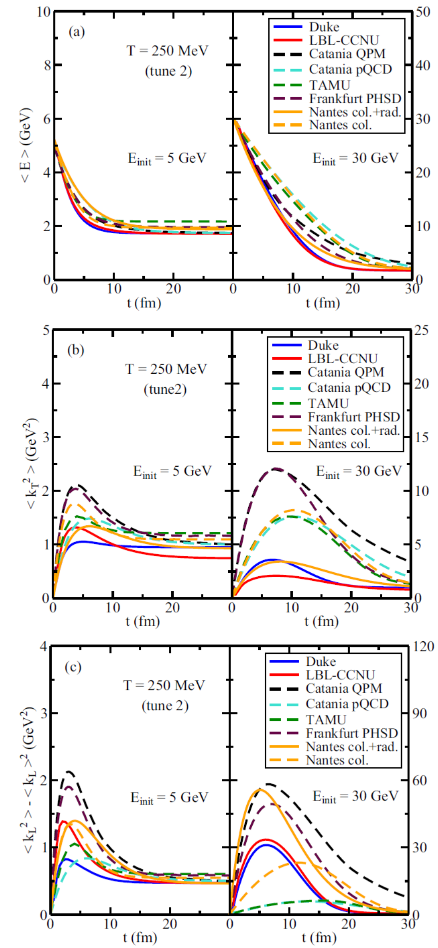
But: does not reduces substantially the difference of drag and diffusion coefficients



drag

drag

qhat



$\langle E(t) \rangle$

$\langle k_T^2 \rangle$

$\langle k_L^2 \rangle - \langle k_T^2 \rangle$

Conclusions of the brick wall comparison of elementary HQ-parton interact.:

Although all models are internally consistent (checked but not reported here)

different description of the interaction of the HQ with the QGP partons
yield different results for the transport coefficient:

- they vary by up to a factor 2
- this variation is temperature and momentum dependent
- and leads to different energy loss and p_T broadening even in a brick

the difference between different models cannot be removed by a const K-factor
to agree at one common benchmark.

Some of the origins (but not all) of the difference of drag and diffusion coefficients could be identified:

- finite parton masses (to reproduce the lattice Eq. of State)
- radiation in addition to elastic collisions

We have to better understand the interaction between HQ and the QGP. What may help:

- lattice calculations of transport coefficients
- new and better experimental data (correlations)
- modelling of (high multiplicity) small systems (pp)

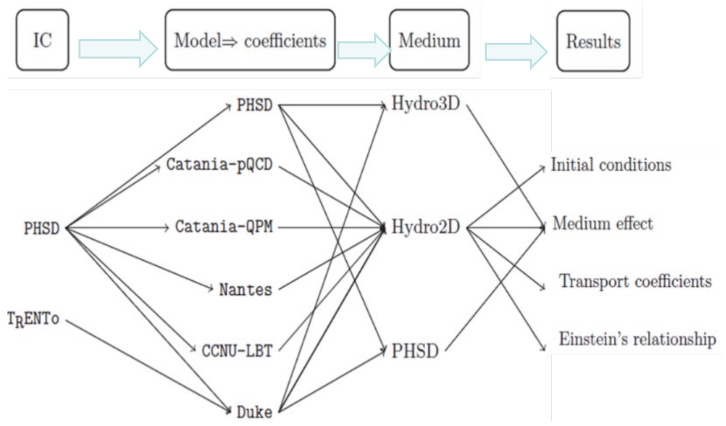
What's about other differences ?

Since all models describe the data but the transport coefficients are quite different: there must be other ingredients in the transport model which compensate for the different transport coefficients.

Possible candidates:

- Initial condition
- time evolution of the QGP
- hadronization

For this a second round of comparisons have been performed



Using a Langevin equation we can combine different

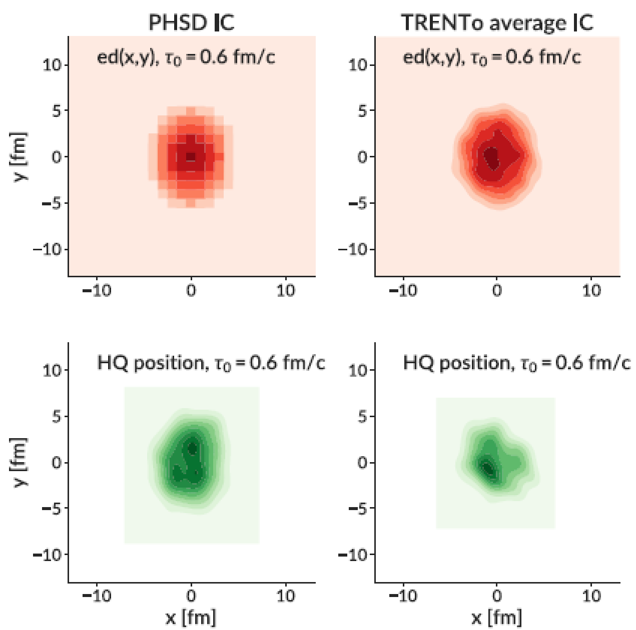
- Initial conditions
- QGP evolutions
- HQ-QGP interactions

and explore the consequences on observables

Initial conditions

Influence of the initial condition: here PHSD versus averaged Trento initial condition

RHIC $\sqrt{s}$ =200 AGeV , b= 6fm

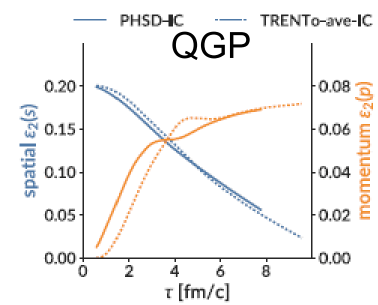


HQ: FONLL

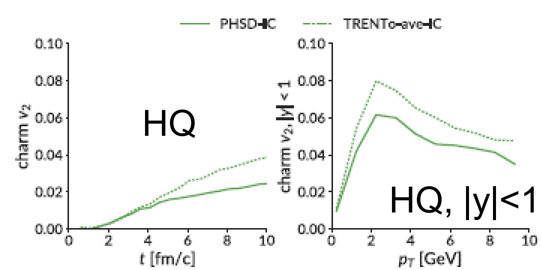
QGP formation time = 0.6 fm/c

QGP evolution: VISHNU

HQ-QGP Duke transport coeff



Vishnu



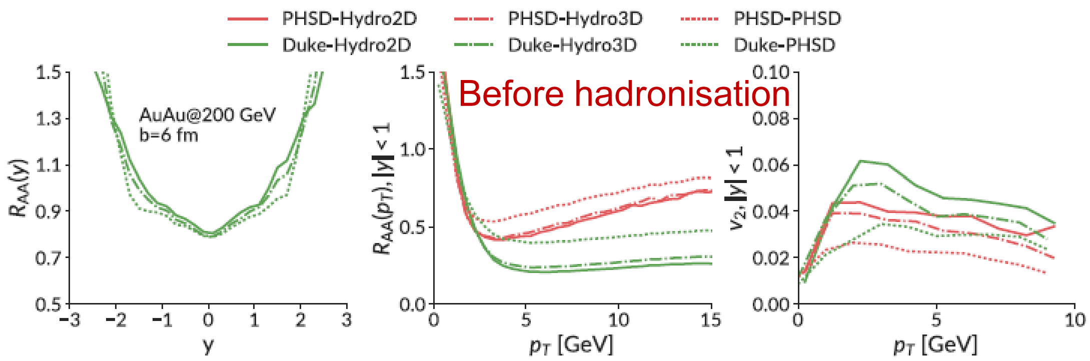
final v_2 of QGP similar but 15-20% difference for HQ v_2 due to the different time evolution

Same transport coeff. but different hydro evolutions

Same transport coeff. , different QGP time evolution

(Duke,PHSD, PHSD initial condition)

All identical besides
- transport coefficients
- Time evolution of the QGP



Duke transport coeff

Frankfurt (PHSD) transport coeff

- Rapidity distribution little affected
- 2d hydro and 3d hydro give similar results for R_{AA} but 15% difference for v_2 at |y|<1
- v_2 (Hydro) and v_2 (PHSD) differ by 20%

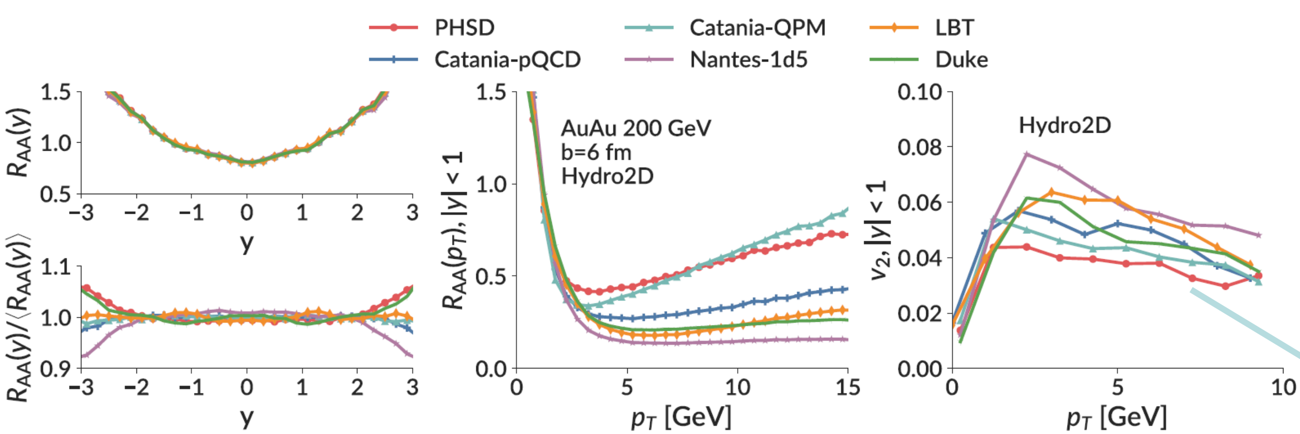
Difference due to different QGP description not as large as due to diff. transport coefficients

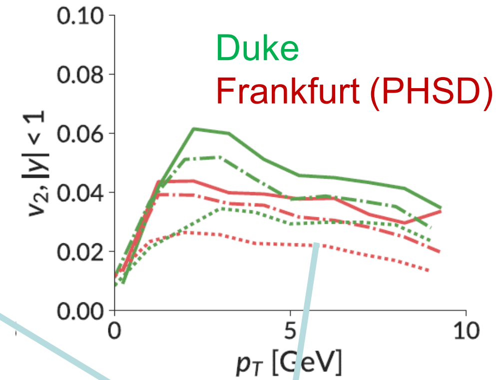
Same hydro evolution but different transport coeff.

Same QGP evolution, different transport coeff.

(Vishnu, same initial condition: PHSD)

For comparison:
different QGP evolution
same transport coefficients





R_{AA} (y) remarkably insensitive to different transport coeff.
R_{AA} (p_T) shows for large p_T large differences (already expected from brick wall study)
v_2 30% difference between for the transport coefficients of different codes

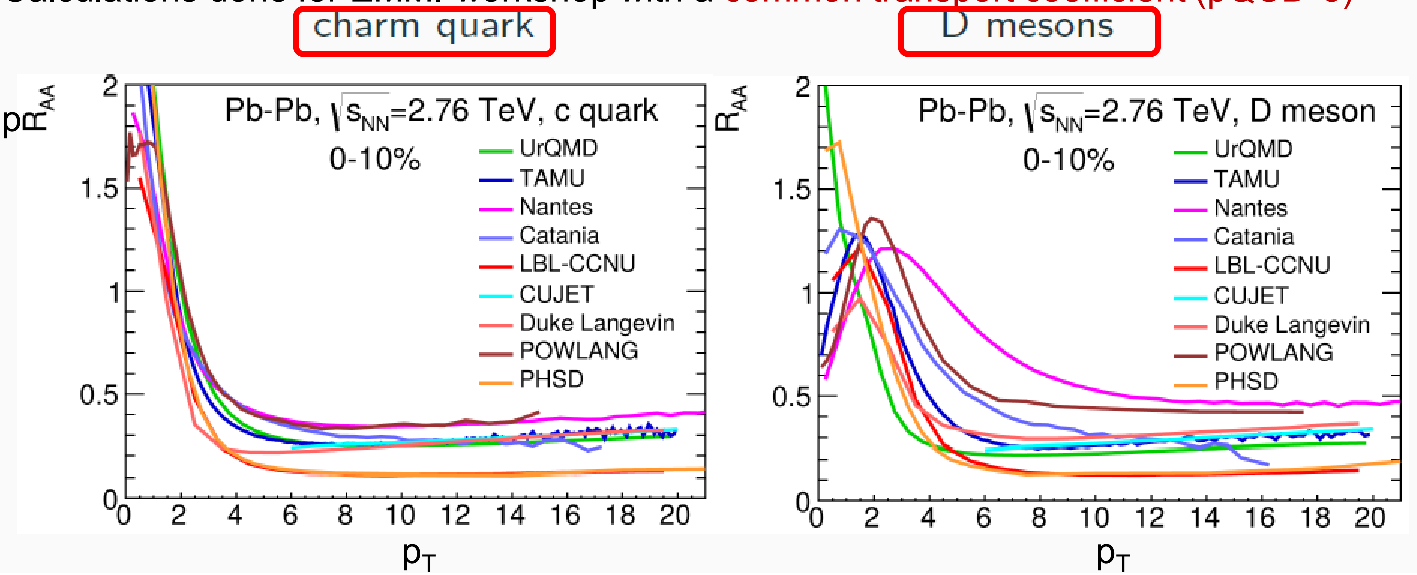
Both modifications of the same order

Hadronization

Study of the influence of the hadronization on final observables has just started:

Different hadronization mechanisms yield different R_{AA}

Calculations done for EMMI-workshop with a common transport coefficient (pQCD*5)



EMMI
NPA 979, 21

- Common fall off of $R_{AA}(p_T)$ of HQs transformed into a variety of different curves.
- Most of the approaches create a maximum of $R_{AA}(p_T)$ by hadronization (exception PHSD and UrQMD)

So a lot remains to be done

Langevin or Boltzmann? I

Which is right transport equation to describe HQ in a QGP?

In a dilute system (collision time << time between collisions) the time evolution of HQs can be described by a Boltzmann equation (BE)

$$\frac{d}{dt} f_{HQ}(\vec{p},\vec{x},t) = I_{coll} \qquad I_{coll} = \int d^3k \Big[\underbrace{w(\vec{p}+\vec{k},\vec{k})\ f_{HQ}(\vec{p}+\vec{k})}_{gain} - \underbrace{w(\vec{p},\vec{k})\ f_{HQ}(\vec{p})}_{loss} \Big]$$

Dilute -> $|M|^2$ and cross section σ can be defined. σ known -> equation can be solved

For small angle scattering

$$w(\vec{p}+\vec{k},\vec{k})\ f_{HQ}((\vec{p}+\vec{k}) \approx \left(1 + k_i \frac{\partial}{\partial p_i} + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}\right) w(\vec{p},\vec{k}) f_{HQ}((\vec{p})$$

Inserted into the Boltzmann eq. -> Fokker-Planck eq.

$$\frac{\partial}{\partial t} f_{HQ}(\vec{p},t) = \frac{\partial}{\partial p^i}\left(A^i(p,T)\ f_{HQ}(\vec{p},t)\right) + \frac{\partial}{\partial p^i}\left[B^{ij}(p,T)\ f_{HQ}(\vec{p},t)\right]\Big)$$

with

$$A_i(\vec{p}) = \int d^3k w(\vec{p},\vec{k}) k_i \ = \ A(\vec{p}) p_i \quad ; \quad B_{ij} = \int d^3k w(\vec{p},\vec{k}) k_i k_j$$

Fokker-Planck eq (FPE):

approximation of to Boltzmann (if σ known A and B can be calculated)

more general than Boltzmann equation (does not require diluteness assumption)

Langevin or Boltzmann? II

FPE would be the appropriate choice if lattice calculations give us $A^i(p,T)$ and $B^{ij}(p,T)$
Till then we can

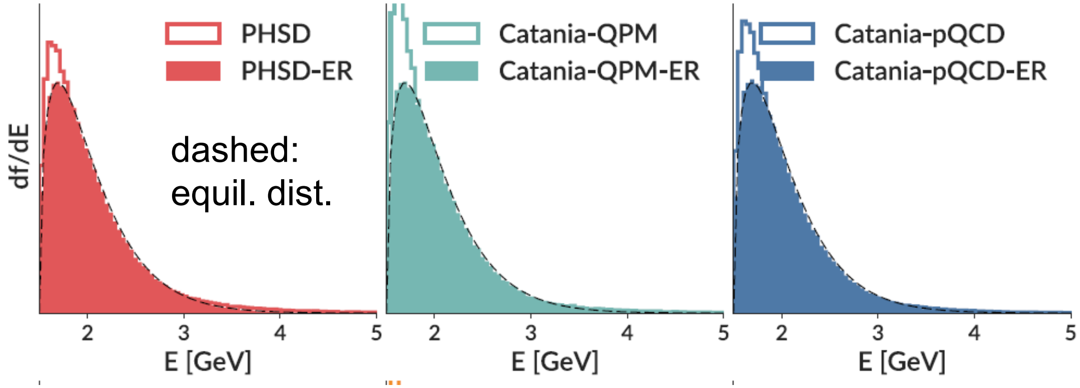
fit A and B -> Bayesian analysis
calculate A and B from the collision term of the BE

Problem : BE: For t -> ∞ $f_{HQ}(\vec{p},t)$ becomes the equilibrium distribution
FPE: For t -> ∞ $f_{HQ}(\vec{p},t)$ becomes only equilibrium distribution if the Einstein relation (here for Langevin)

$$\eta_D = \frac{\kappa_L}{2ET} - \frac{\kappa_L - \kappa_T}{p^2} - \frac{\partial \kappa_L}{\partial p^2}$$

is fulfilled (here for Jüttner distr.)

→ only two transport coefficients are independent
→ in most of the approaches the transp. coeff calculated by the BE do not fulfill the Einstein relation



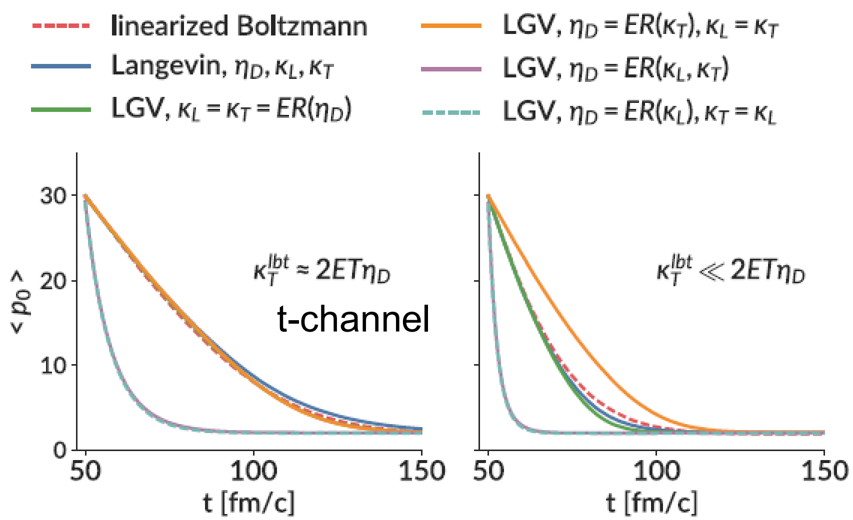
dashed: equil. dist.

Static medium

Langevin or Boltzmann? III

Not only for t -> ∞ important:

Short term behavior of the solution depends on choice of which coeff. Is considered as fct. of the others

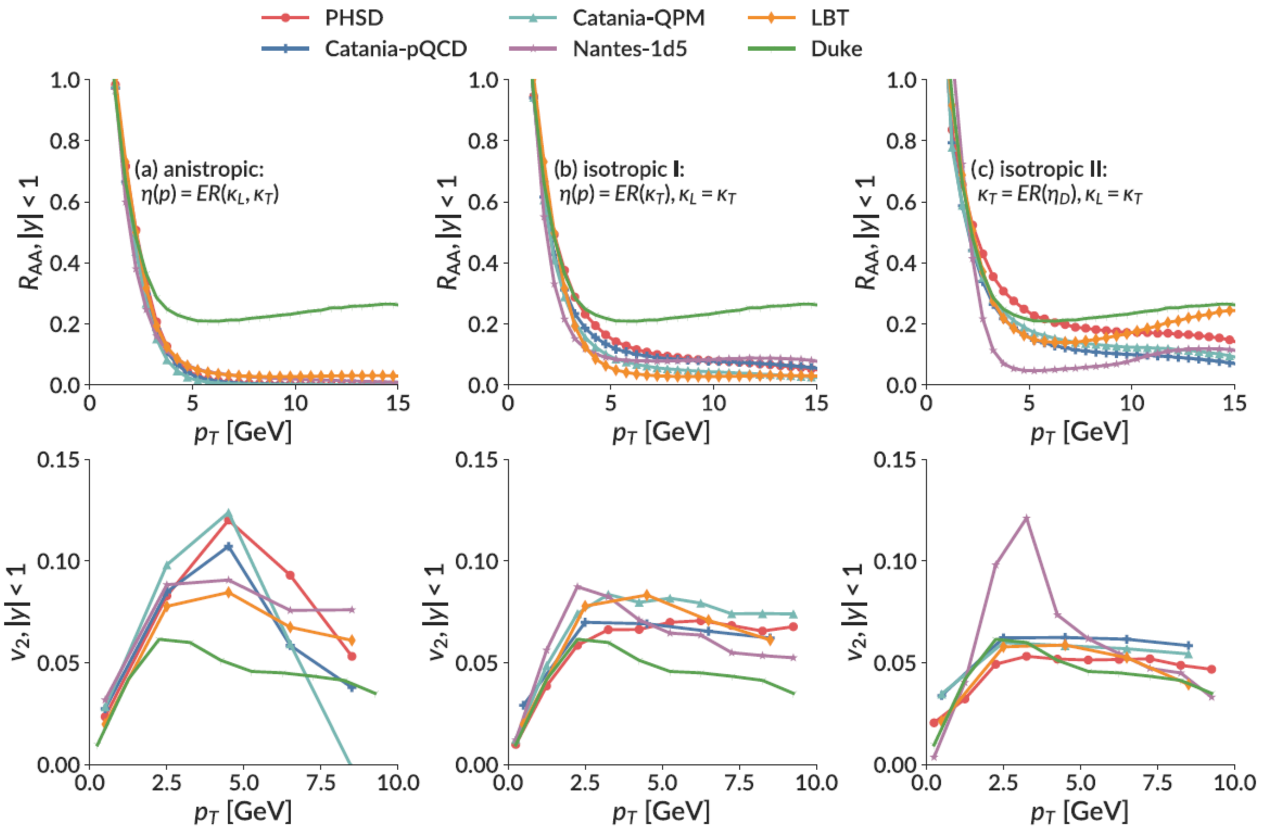


Brick wall calculation:
P_z (0) = 30 GeV/c
T = 300 MeV

Energy loss (for a brick) depends quite substantially on this choice

Langevin or Boltzmann? IV

Also in an expanding plasma the HQ observables depend on this choice:



and the calculations show differences up to 50%.

This may explain why in the past seemingly identical calculations gave different results.

Conclusions

Analyzing models for the evolution of the heavy quark distribution which agree quite well with experiments we see

HQ retain information from the initial condition up to the last stage of the HI collision -> very useful probe

the functional form of $R_{AA}(p_T)$ and $v_2(p_T)$ are reasonably reproduced

present data do not allow to disentangle the different processes which are encoded in the HQ distr.
different assumptions on QGP expansion, initial condition, HQ-QGP interactions
vary the results by up to 50% but compensate in the different programs

Our studies allowed to see the influence of different assumptions about the sub-processes
all influence the final distribution on the level of 20-50%

Three major factors for differences could be identified
- mass of the QGP partons
- the inclusion of radiative energy loss.
- others are still hidden in the transport coefficients.

In addition, if the QGP is not equilibrated, transport coefficients are modified.

Without new data (correlations) or detailed lattice calculations at finite p progress is very difficult

Perspectives

The FPE or the Langevin eq. are very useful tools to compare different models

However, because the transport coeff., calculated with the BE, do not fulfil the Einstein relation we should concentrate in future on BE approaches if we want
    to compare our results with experiments
    to relate our transport coefficient to (p)QCD processes

Before new data become available we should:

    check (more) in detail the prediction of the QGP expansion scenarios with experimental results
        in the light hadron sector to optimize
    check more in detail the hadronic rescattering (which is not negligible)
    check more in detail the hadronization process (another source of uncertainty)
                done in three week in a workshop in Paris

Transport coefficient are calculated assuming that the expanding QGP in the thermal equilibrium
If this is not the case?

We can then also calculate transport coefficients from the BE with the same formula

$$\langle\langle O^*\rangle\rangle \equiv \frac{1}{2E_p} \sum_{i=q,\bar{q},g} \int \frac{d^3k}{(2\pi)^3 2E} f_i(k) \int \frac{d^3k'}{(2\pi)^3 2E'} \times \int \frac{d^3p'}{(2\pi)^3 2E'_p} \; O^* \; (2\pi)^4 \delta^{(4)}(p+k-p'-k') \frac{|M_{ic}|^2}{\gamma_c}, (13)$$

by replacing the equilibrium f_i (k) by a one for the non-equilirium situation and obtain by

$$\frac{d}{dt}\langle p\rangle \equiv -\eta_D \langle p\rangle,$$
$$\frac{1}{2}\frac{d}{dt}\langle(\Delta p_T)^2\rangle \equiv \kappa_T,$$
$$\frac{d}{dt}\langle(\Delta p_z)^2\rangle \equiv \kappa_L,$$

the transport coefficients for the Langevin equation

Scenario I:

Non-equilibrium kinetic energy (keeping the energy density constant by changing the number density)

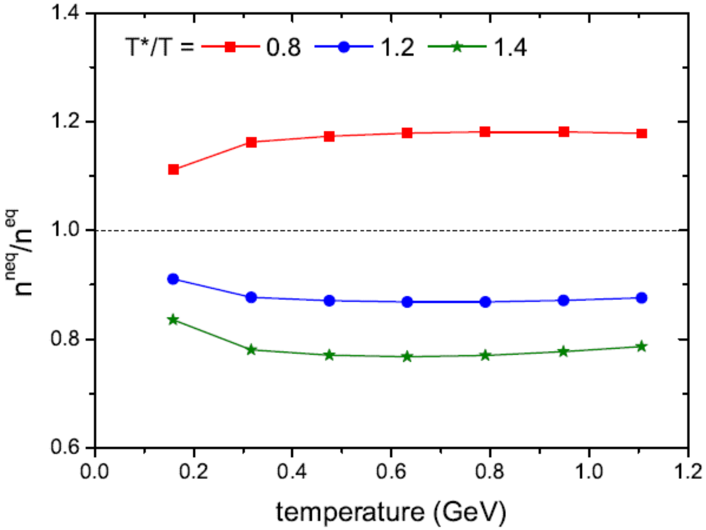
- Quite realistic scenario: spectra of measured hadrons is not thermal!!

Method: introduction of an artificial temperature T* to calculate the kinetic energy:

$$f_i(p) = \frac{D_i}{\exp(E/T^*) \pm 1} \times \frac{\epsilon(T)}{\epsilon(T^*)},$$

$$\epsilon(T) = \sum_{i=q,\bar{q},g} D_i \int \frac{d^3p}{(2\pi)^3} \frac{E}{\exp(E/T) \pm 1}$$

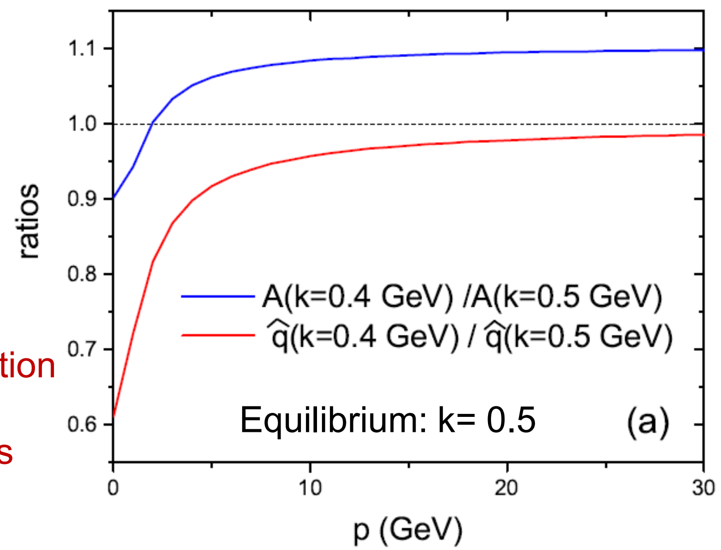
$$A(k,T) = \eta_D(k,T) \qquad \hat{q}(k,T) = \frac{E}{k_L}\kappa_T(k,T)$$



Transport coeff change by 20%

Change is strongly Momentum dependent

Nonequilibrium distribution of QGP modifies transport coefficients



Scenario I:

Non-equilibrium kinetic energy (keeping the energy density constant by changing the number density)

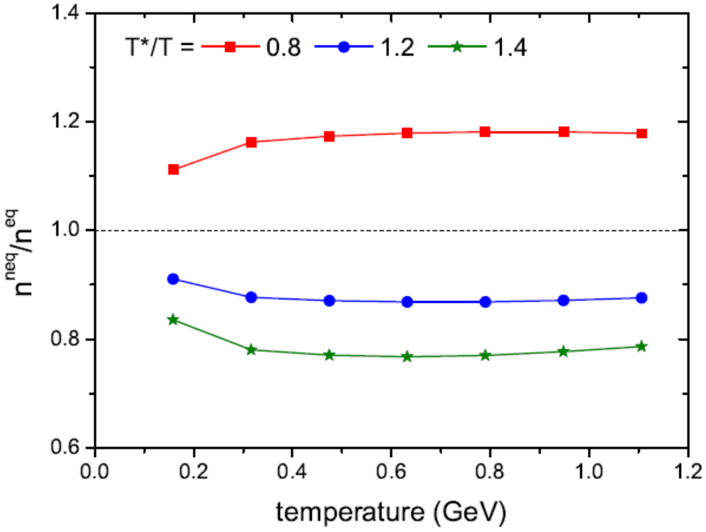
- Quite realistic scenario: spectra of measured hadrons is not thermal!!

Method: introduction of an artificial temperature T* to calculate the kinetic energy:

$$f_i(p) = \frac{D_i}{\exp(E/T^*) \pm 1} \times \frac{\epsilon(T)}{\epsilon(T^*)},$$

$$\epsilon(T) = \sum_{i=q,\bar{q},g} D_i \int \frac{d^3p}{(2\pi)^3} \frac{E}{\exp(E/T) \pm 1}$$

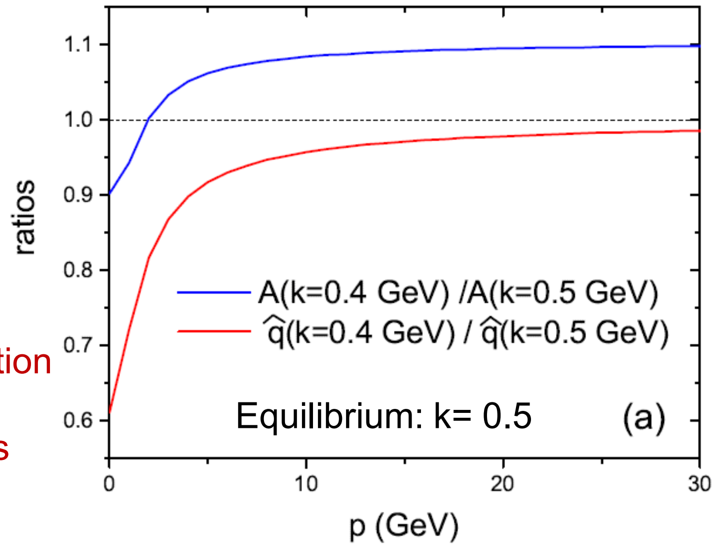
$$A(k,T) = \eta_D(k,T) \qquad \hat{q}(k,T) = \frac{E}{k_L} \kappa_T(k,T)$$



Transport coeff change by 20%

Change is strongly Momentum dependent

Nonequilibrium distribution of QGP modifies transport coefficients



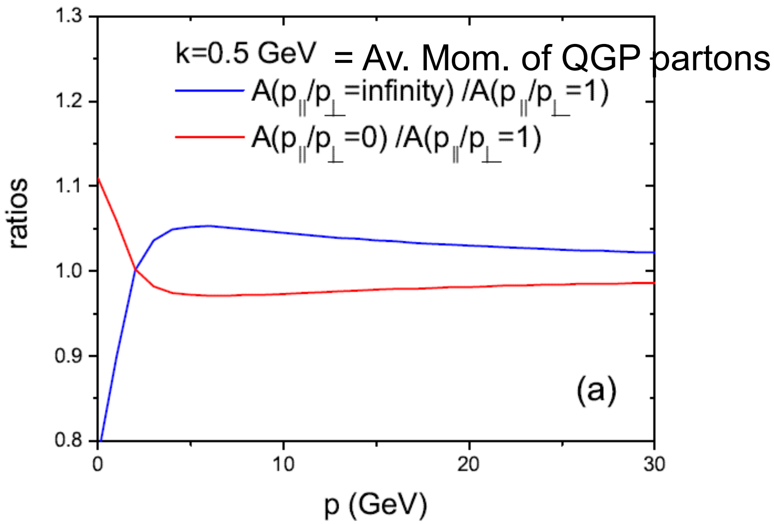
Scenario II

Anisotropic momentum distribution:

Expressed by a different pressure P in longitudinal and transverse direction:

$$P_{\|} = T_{zz} = \int \frac{d^3k}{(2\pi)^3} \frac{k_z^2}{E} f(k)$$

$$P_{\perp} = \frac{T_{xx} + T_{yy}}{2} = \int \frac{d^3k}{(2\pi)^3} \frac{k_x^2 + k_y^2}{2E} f(k)$$



Also visible modifications but effect is smaller

If the expanding plasma is not in termal equilibrium (and hadron spectra show that it is no
we expects that the
measured transport coeff. deviate from those calculated theoretically for an equilibrium Q