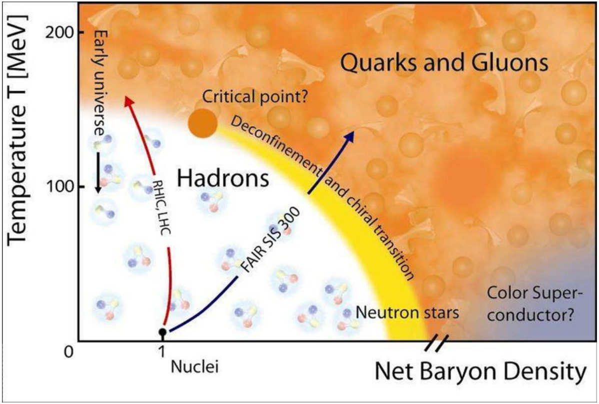
Renormalization Group Consistent Treatment of NJL Color-Superconductivity

HFHF 2023

Hosein Gholami in collaboration with M. Hofmann and M. Buballa

Sketch of QCD Phase Diagram

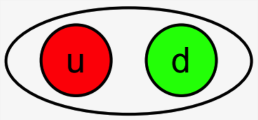


- Phase structure at large (but not asymptotically large) density and moderate T is relevant for neutron stars and neutron star mergers
- Use effective models in this regime.
  This talk: Treatment of cut-off artefacts.

Color Superconductivity

- Cooper theorem: Fermi surface unstable against finite attractive interaction of particles
- Strong interactions: Attractive diquark interaction in scalar color, flavor antitriplet channel
- Pairing of particular color-flavor combinations

2SC

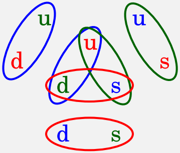


Intermediate $\mu \lesssim M_s$

$SU(2)_f \times SU(2)_c$

1 finite gap parameter Δ_2

Color-flavor-locking (CFL)



Large $\mu \gg M_s$

$SU(3)_{c+f}$

3 finite gap parameters

$\Delta_2, \Delta_5, \Delta_7$

Model

Nambu Jona-Lasinio (NJL)-type model [Klähn et al. (2007, 2013), Alaverdyan (2022)]

$$\mathcal{L} =$$

$\bar{\psi}(i\partial\!\!\!/ - m)\psi$	kinetic term
$+\, G \sum \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right]$	scalar NJL interaction
$-\, K \left[\mathrm{det_f}(\bar{\psi}(\mathbb{1}+\gamma_5)\psi) + \mathrm{det_f}(\bar{\psi}(\mathbb{1}-\gamma_5)\psi)\right]$	't Hooft (KMT) interaction
$+\, G\,\eta_D \sum (\bar{\psi}i\gamma_5\tau_A\lambda_{A'}\psi^c)(\bar{\psi}^c i\gamma_5\tau_A\lambda_{A'}\psi)$	diquark interaction

with charge conjugated spinor $\psi^c = C\bar{\psi}^T$

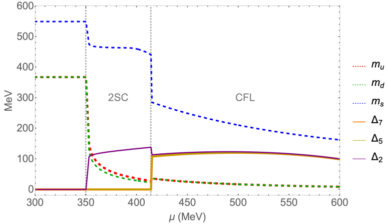
- Regularization: sharp 3-momentum cutoff $\Lambda^{'}$
- $\Lambda^{'}, G, K, m$ fitted to vacuum meson spectrum
- choose $\eta_D \sim 1$ (Fierz value is $\eta_D = \frac{3}{4}$)

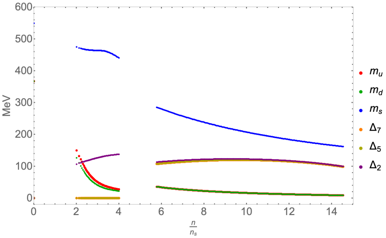
Condensates in Mean Field

Mean field approximation: Linearise theory around condensates

$$
\begin{aligned}
\phi_f &= \langle \bar{\psi}_f \psi_f \rangle && f = u, d, s \\
\Delta_A &= -2G\eta_D \langle \bar{\psi}^c \gamma_5 \tau_A \lambda_A \psi \rangle && A = 2, 5, 7
\end{aligned}
$$

- $\eta_D = 1$ at $T = 0$





- Cutoff artefact: Δ values seem to descend at high chemical potentials/number densities

Electric and Color Neutrality

- Chemical potential matrix in color-flavor space:

$$\mu_{f,c} = \mu + Q_f \mu_Q + \lambda_{3,c}\mu_3 + \lambda_{8,c}\mu_8$$

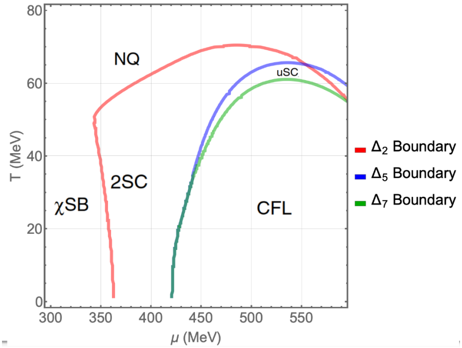
$$\text{e.g. } \mu_{u,r} = \mu + \frac{2}{3}\mu_Q + \mu_3 + \frac{1}{\sqrt{3}}\mu_8$$

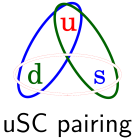
- Neutron star: Enforce charge and color-neutrality locally, i.e. for every phase:

$$\frac{\partial\Omega}{\partial\mu_Q} = \frac{\partial\Omega}{\partial\mu_3} = \frac{\partial\Omega}{\partial\mu_8} = 0$$

- Leptonic contribution: e^- and μ^- in $\beta-$equilibrium $\mu_e = \mu_\mu = -\mu_Q$
- Optimization problem with nonlinear constraints

Phase Diagram of Neutral Quark Matter

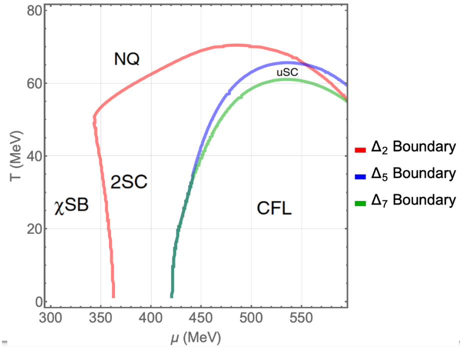


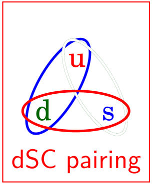


- Cutoff artefacts
  - Gaps and phase boundary to normal phase bend downwards for $\mu \sim \Lambda'$
  - Appearance of uSC phase [Fukushima 2005]
- Previous explanations for uSC: T=0 arguments $\rightarrow$ Not relevant for $T \neq 0$
- Puzzle: Absence of expected dSC phase in CFL melting pattern [Iida et al 2004]

Phase Diagram of Neutral Quark Matter

CRC-TR 211





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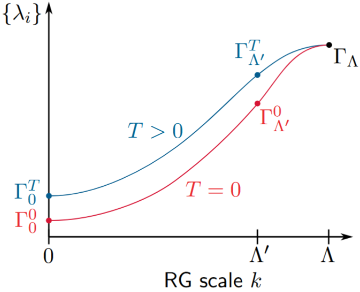
Removal of Cut-off Artefacts: RG Consistency

RG-consistency: Full quantum effective action is cutoff independent $\Lambda \frac{\mathrm{d}\Gamma}{d\Lambda} = 0$

- Idea: Flow up to higher scale Λ that is much larger than external parameters and gaps $(\Lambda \gg \mu, T, \Delta, M)$ [Braun, Leonhardt, Pawlowski, 2019]
- Trivial flow solutions for NJL type mean-field models through change of integration bounds



$$\Omega_{\text{RG}} = \Omega_{\Lambda'} + \Omega^{\Lambda}_{\text{med}} - \Omega^{\Lambda'}_{\text{med}} - (\Omega^{\Lambda}_{\text{vac}} - \Omega^{\Lambda'}_{\text{vac}})$$

RG Consistency and Divergent Medium

CRC–TR 211

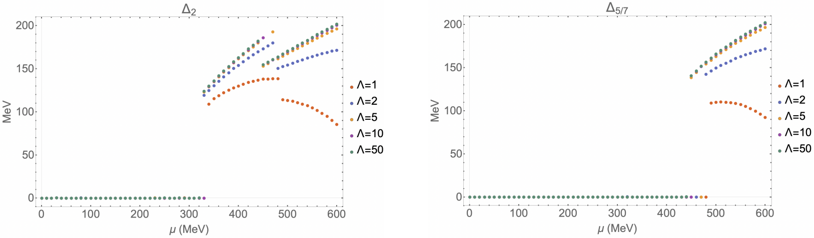
- While vacuum is unchanged, any medium divergency needs additional renormalization
- Color-superconductivity in Quark-Meson model has a zero temperature medium divergency of form $\mu^2 \Delta^2 \log \Lambda$
  Solution: "diquark chemical potential" renormalization [Braun, Leonhardt, Pawlowski, 2019]
- 3-flavour NJL color-superconductivity suffers from a divergence of quite the same form:

$$\mu^2 (\Delta_2^2 + \Delta_5^2 + \Delta_7^2) \log \Lambda$$

Our proposed solution:

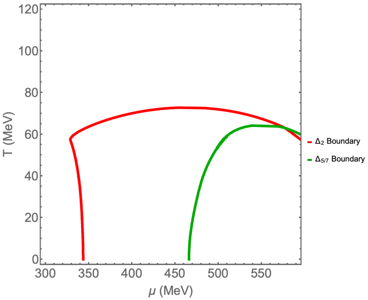
$$\Omega_{\text{RG}} = \Omega_{\Lambda'} + \Omega_{\text{med}}^{\Lambda} - \Omega_{\text{med}}^{\Lambda'} - (\Omega_{\text{vac}}^{\Lambda} - \Omega_{\text{vac}}^{\Lambda'})$$
$$- \frac{1}{2} \mu^2 \left. \frac{\partial^2 \Omega}{\partial \mu^2} \right|_{\mu, T=0}$$

Condensates in RG-Consistent Model

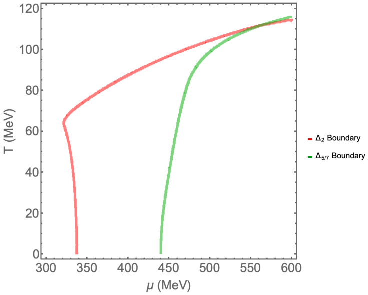


- At $\Lambda \approx 10\Lambda^{'}$, results become independent of Λ
- No bending downward $\rightarrow$ cut-off artefacts removed
- Gap values become enlarged for the same diquark coupling
- Phase boundaries to CSC phases move to lower μ

Phase Diagram in RG-consistent Model



Non-neutral, without RG $\Lambda = \Lambda'$



Non-neutral, with RG $\Lambda = 10\Lambda'$

- Phase boundary rising in μ, T-plane $\rightarrow$ cut-off artefacts are removed
- Critical temperature increases by factor 1.5-2 (for the same diquark coupling)
- Imposing neutrality conditions are not that easy due to non analytic nature of our problem

RG Consistency for Neutral CSC

- For a generalized chemical potential, the behaviour of medium divergences are not that easy to predict
- Analytically derived form of divergence

$$\left(\Delta_2^2\bar{\mu}_{ud}^2 + \Delta_5^2\bar{\mu}_{us}^2 + \Delta_7^2\bar{\mu}_{ds}^2\right)\log\Lambda$$

  with $\bar{\mu}_{ij}$ being the average chemical potential of quark flavour species i and j
- Subtraction of this term would lead to a RG-consistent model that can be used to impose neutrality conditions on

$$\Omega_{\mathsf{RG}} = \Omega_{\Lambda'} + \Omega_{\mathsf{med}}^{\Lambda} - \Omega_{\mathsf{med}}^{\Lambda'} - (\Omega_{\mathsf{vac}}^{\Lambda} - \Omega_{\mathsf{vac}}^{\Lambda'})$$
$$-\frac{2}{\pi^2}\left(\Delta_2^2\bar{\mu}_{ud}^2 + \Delta_5^2\bar{\mu}_{us}^2 + \Delta_7^2\bar{\mu}_{ds}^2\right)\log\Lambda$$

- This scheme is not exactly in accordance to a proper renormalization of a QFT $\rightarrow$ Note that we are not renormalizing a QFT!

RG Consistency for Neutral CSC

- Behaviour of divergence suggests that

$$\mathcal{L}^R \supset \frac{1}{2} Y_1(\Delta) \bar{\mu}_{ud}^2$$

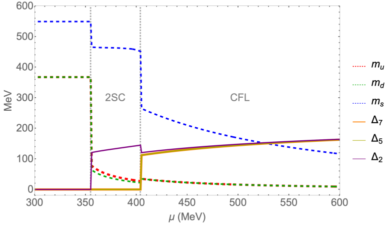
With renormalization condition $Y_1^0 = 0$.

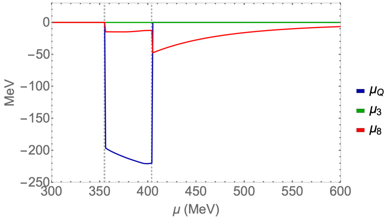
  - Renormalization $\to Y_1(\Delta) = \frac{d^2\Omega}{d\bar{\mu}_{ud}^2}\Big|_{\hat{\mu}=0} \Rightarrow \frac{1}{2}\bar{\mu}_{ud}^2 \frac{d^2\Omega}{d\bar{\mu}_{ud}^2}\Big|_{\hat{\mu}=0}$ to be subtracted from Ω
  - Y_1 shouldn't give any contributions if there isn't any ud pairing

Same scenario for other types of pairing and combinations.

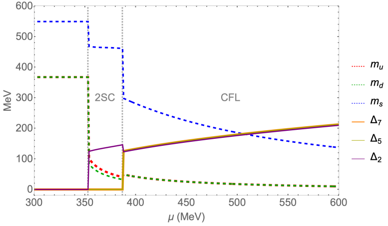
- Finding these set of Ys to satisfy mentioned conditions renormalize the medium divergence

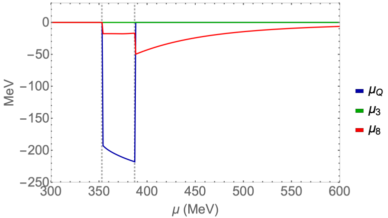
Results: Diquark Condensates





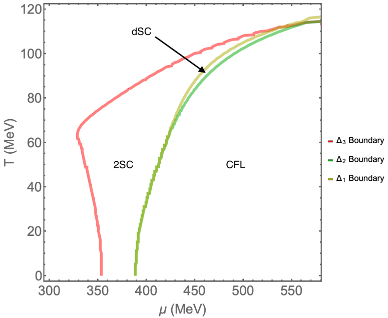
RG-Consistency($\Lambda = 10\Lambda'$)
+subtraction of divergence





RG-Consistency($\Lambda = 10\Lambda'$)
+medium renormalization

Results: Phase Diagram



- Cut-off artefacts seem to be removed
  - No bending downwards
  - No uSC phase
- Expected dSC phase appears in CFL melting: Puzzle solved...?

Summary and Outlook

Summary

- NJL color-superconductivity suffers from cut-off artefacts
- RG-consistent formulation systematically removes the cutoff artefacts and changes the phase diagram in terms of critical temperatures, diquark condensate values and phase transition points
- RG-consistent formulation for neutral CSC matter is in agreement with expected dSC phase in CFL melting pattern

Outlook

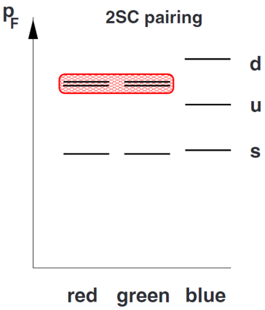
- Systematic explanation of appearance of dSC and disappearance of uSC by RG-consistent treatment
- Main interest: Study imprints of color superconductivity in neutron star mergers $\rightarrow$ Marco's talk

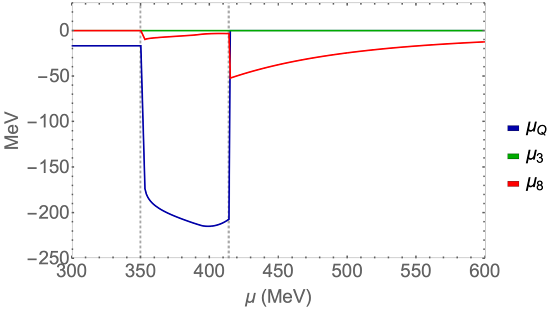
Thank you for listening!

Appendix

Charge neutrality and the 2SC phase

Neutral system: Mismatch of Fermi momenta for up and down quarks





- Electric charge neutrality suppresses pairing in the 2SC phase

RG Consistency for Neutral CSC

- Behaviour of divergence suggests that

$$\mathcal{L}^R \supset \frac{1}{2} Y_1(\Delta)\bar{\mu}_{ud}^2$$

With renormalization condition $Y_1^0 = 0$.

  - Renormalization $\rightarrow Y_1(\Delta) = \frac{d^2\Omega}{d\bar{\mu}_{ud}^2}\bigg|_{\hat{\mu}=0} \Rightarrow \frac{1}{2}\bar{\mu}_{ud}^2 \frac{d^2\Omega}{d\bar{\mu}_{ud}^2}\bigg|_{\hat{\mu}=0}$ to be subtracted from Ω
  - Y_1 shouldn't give any contributions if there isn't any ud pairing

Same scenario for other types of pairing and combinations.

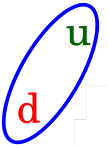
- We need to analyze the inverse propagator for possible pairings

$$S^{-1}(p) = \begin{pmatrix} \not{p} - M + \mu_{f,c}\gamma^0 & \sum_{A=2,5,7} \Delta_A \gamma_5 \tau_A \lambda_A \\ -\sum_{A=2,5,7} \Delta_A^* \gamma_5 \tau_A \lambda_A & \not{p} - M - \mu_{f,c}\gamma^0 \end{pmatrix}$$

- Inverse propagator is a 72x72 matrix that can be seperated into
  - Six 4x4 block $M1$ to $M6$
  - One 12x12 block $M7$

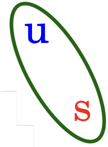
2flavour CSC blocks

$M1$ and $M2$



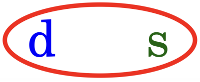
2SC$_{ud}$

$M3$ and $M4$



2SC$_{us}$

$M5$ and $M6$



2SC$_{ds}$

- Each corresponding block in Ω needs a subtraction of form $\frac{1}{2}\bar{\mu}_{ij}^2 \frac{d^2\Omega^{(*)}}{d\bar{\mu}_{ij}^2}\Big|_{\hat{\mu}=0}$
- In total for $M1$ to $M6$:

$$-\frac{1}{2}\mu_{dg}^2 \frac{\partial^2\Omega^{(1,2)}}{\partial\mu_{dg}^2}\Big|_{\hat{\mu},T=0} - \frac{1}{2}\mu_{ur}^2 \frac{\partial^2\Omega^{(3,4)}}{\partial\mu_{ur}^2}\Big|_{\hat{\mu},T=0} - \frac{1}{2}\mu_{ur}^2 \frac{\partial^2\Omega^{(5,6)}}{\partial\mu_{ur}^2}\Big|_{\hat{\mu},T=0}$$

3flavour CSC block

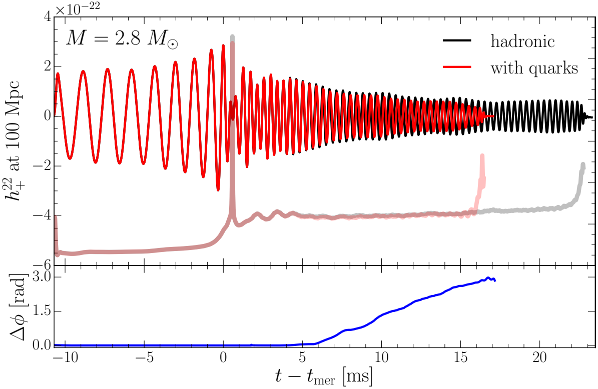
- $M7$ has a general structure form of

$$M7 \sim \begin{pmatrix} u & \Delta_2 & \Delta_5 \\ \Delta_2 & d & \Delta_7 \\ \Delta_5 & \Delta_7 & s \end{pmatrix}$$

- We need to make sure that we subtract correct terms corresponding to existing pairs.
  Example: Consider $\Delta_2 = 0 \Rightarrow$ Only us and ds pairs cause a divergence
- A weight function is needed to guarantee this
- $M7$ renormalization counter term:

$$-\frac{1}{2}\left(\frac{\mu_{ud}^2\Delta_2^2 + \mu_{us}^2\Delta_5^2 + \mu_{ds}^2\Delta_7^2}{\Delta_2^2 + \Delta_5^2 + \Delta_7^2}\right)\frac{\partial^2\Omega^{(7)}}{\partial\mu^2}\Big|_{\hat{\mu}, T=0}$$

Possible imprints of a Phase transition to quark matter in Gravitational Wave Signals from Neutron Star Mergers



[Rezzolla et al., 2019]

- Phase transition might be detected in data of postmerger signal

RG consistency with gaps

Quasiparticle spectra with gaps lead to divergence in the medium contribution, e.g.

$$\int_0^{\Lambda} \frac{\mathrm{d}^3 p}{2\pi^2}(\omega_+ + \omega_-) \sim \mu^2 \Delta^2 \log(\Lambda)$$

where $\omega_\pm = \sqrt{(\sqrt{p^2 + M^2} \pm \mu)^2 + \Delta^2}$.

- Remove divergence through counterterm $\frac{1}{2}\mu^2 \, \frac{\partial^2 \Omega}{\partial \mu^2}|_{\mu, T=0}$

$$\Omega_{\mathsf{RG}} = \Omega_{\Lambda'} + \Omega_{\mathsf{med}}^{\Lambda} - \Omega_{\mathsf{med}}^{\Lambda'} - (\Omega_{\mathsf{vac}}^{\Lambda} - \Omega_{\mathsf{vac}}^{\Lambda'}) - \frac{1}{2}\mu^2 \, \frac{\partial^2 \Omega}{\partial \mu^2}\bigg|_{\mu, T=0}$$

(Color) Superconductivity

- Cooper theorem: Fermi surface unstable against finite attractive interaction of particles
- Cooper pairing and Gapped modes in excitation spectrum below critical Temperature $T_c \simeq 0.57\Delta(T=0)$
- Strong interactions: Attractive Diquark interaction in color-, flavor antitriplet channel
- Pairing of particular color-flavor combinations

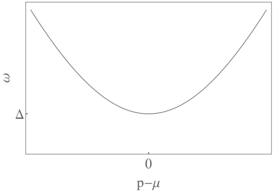


Figure: Most simple case: $\omega = \sqrt{(E-\mu)^2 + \Delta^2}$